June 24, 2014

mathematics vision project

Transforming Mathematics Education

Flexible & Engaging
Seamless Common Core Companion

Write your response to each question on a post-it note (you will need four post-it notes- one for each question).

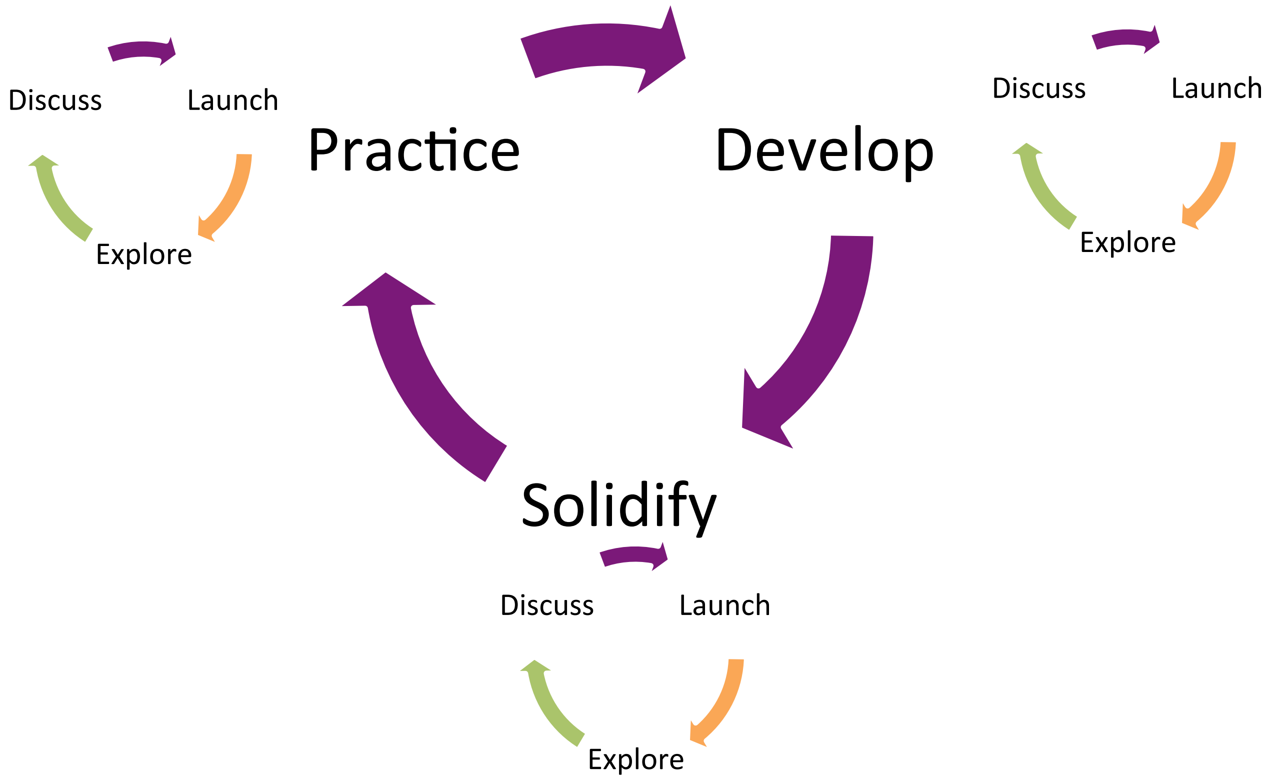
1. What can you do to create a classroom environment that promotes discourse?

2. How does understanding the learning cycle help support instructional decisions?

3. How does having a clear focus on a mathematical purpose support instructional decisions?

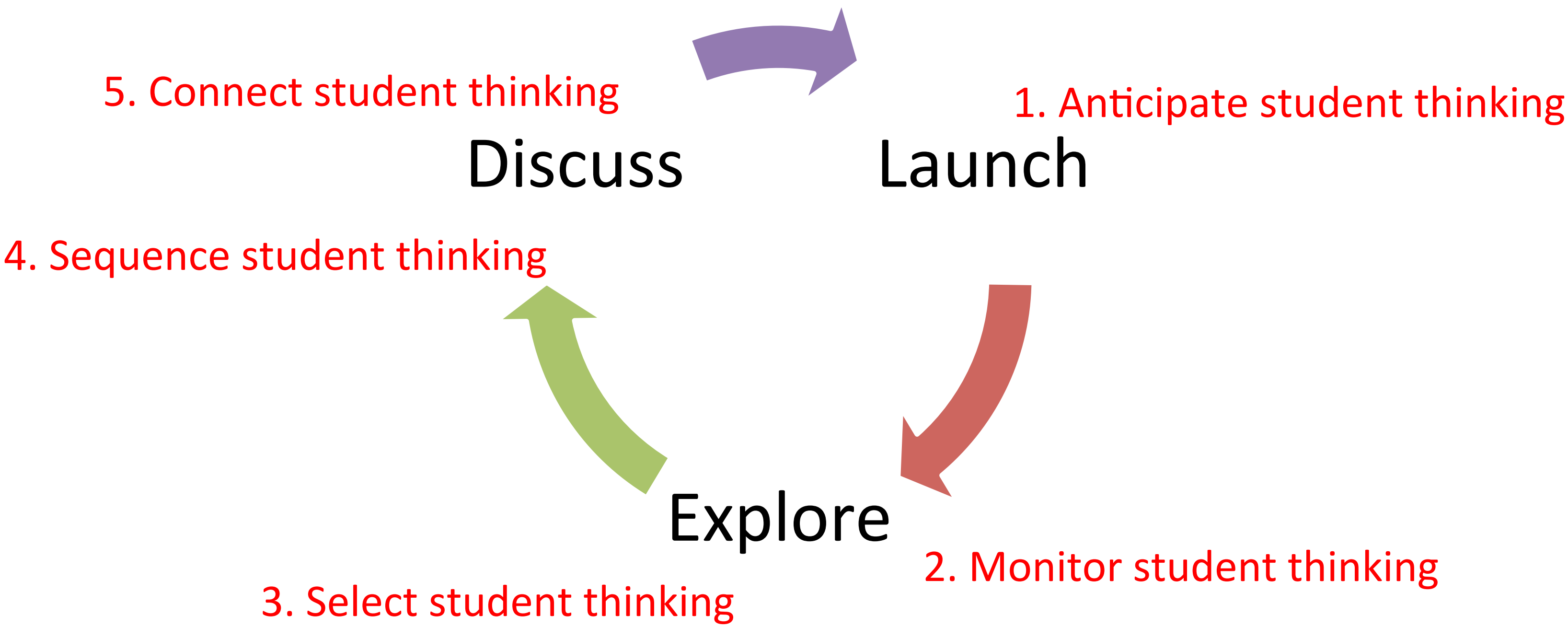
4. Share a new mathematical connection or insight you gained yesterday.

Comprehensive Mathematics Instruction



The *Teaching Cycle*

Connected to the 5 practices of Orchestrating Discussions

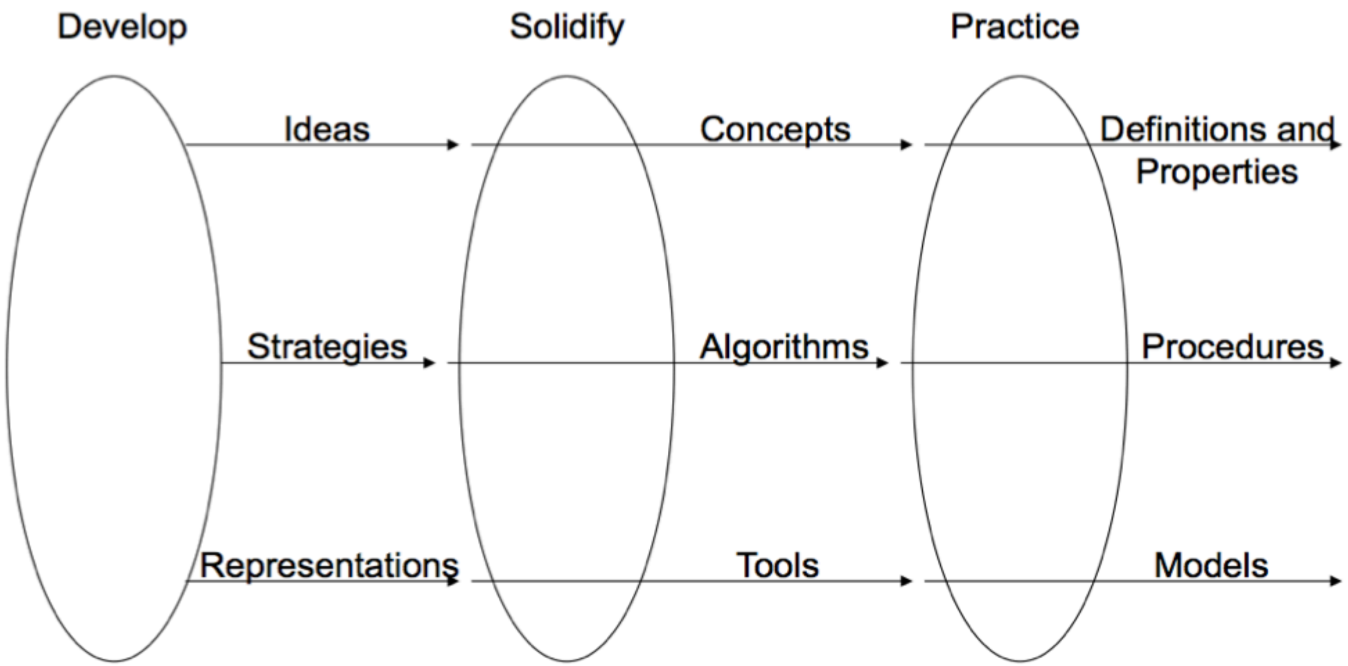


What's Your Next Step?

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Unpacking the Mathematics of the Learning Cycle

- What is the conceptual, procedural, and representational understanding that is emerging from the work in the learning cycle?
- How does the mathematical understanding change from the beginning to the end of the learning cycle?



Differentiated to meet the needs of all students

- Low-threshold, high ceiling tasks allow all students full participation in the standards
- Story contexts and visual representations support conceptual and procedural understanding
- Tasks are designed to surface students' intuitive understandings and then move them towards extensions, formalization and fluency

SM2: Reason Abstractly and Quantitatively

- Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.
- Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Module 1 Purpose and Progression

Module 1 – Getting Ready

Classroom Task: Checkerboard Borders - A Develop Understanding Task
Defining quantities and interpreting expressions (**N.Q.2, A.SSE.1**)
Ready, Set, Go Homework: Getting Ready 1.1

Classroom Task: Building More Checkerboard Borders – A Develop Understanding Task
Defining quantities and interpreting expressions (**N.Q.2, A.SSE.1**)
Ready, Set, Go Homework: Getting Ready 1.2

Classroom Task: Serving Up Symbols – A Develop Understanding Task
Interpreting expressions and using units to understand problems (**A.SSE.1, N.Q.1**)
Ready, Set, Go Homework: Getting Ready 1.3

Classroom Task: Examining Units – A Solidify Understanding Task
Using units as a way to understand problems (**N.Q.1**)
Ready, Set, Go Homework: Getting Ready 1.4

Classroom Task: Cafeteria Actions and Reactions – A Develop Understanding Task
Explaining each step in the process of solving an equation (**A.REI.1**)
Ready, Set, Go Homework: Getting Ready 1.5

Classroom Task: Elvira's Equations – A Solidify Understanding Task
Rearranging formulas to solve for a variable (**A.REI.3, A.CED.4**)

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Rearranging formulas to solve for a variable (**A.REI.3**, **A.CED.4**)
Ready, Set, Go Homework: Getting Ready 1.6

Classroom Task: Solving Equations, Literally – A Practice Understanding Task
Solving literal equations (**A.REI.1, A.REI.3, A.CED.4**)
Ready, Set, Go Homework: Getting Ready 1.7

Classroom Task: Cafeteria Conundrums – A Develop Understanding Task
Writing inequalities to fit a context (**A.REI.1, A.REI.3**)
Ready, Set, Go Homework: Getting Ready 1.8

Classroom Task: Greater Than? – A Solidify Understanding Task
Reasoning about inequalities and the properties of inequalities (**A.REI.1, A.REI.3**)
Ready, Set, Go Homework: Getting Ready 1.9

Classroom Task: Taking Sides – A Practice Understanding Task
Solving linear inequalities and representing the solution (**A.REI.1, A.REI.3**)
Ready, Set, Go Homework: Getting Ready 1.10

A.SSE.1 1. Interpret expressions that represent a quantity in terms of its context.

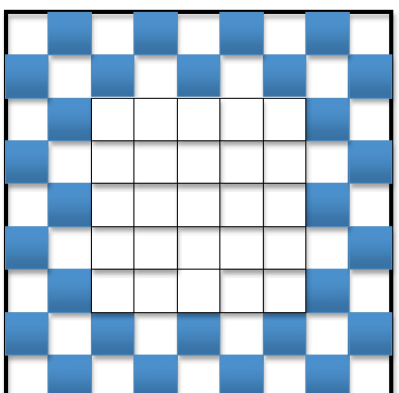
1.1 Checkerboard Borders

A Develop Understanding Task

In preparation for back to school, the school administration has planned to replace the tile in the cafeteria. They would like to have a checkerboard pattern of tiles two rows wide as a surround for the tables and serving carts.

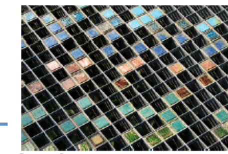
Below is an example of the boarder that the administration is thinking of using to surround a square 5 x 5 set of tiles.

A. Find the number of colored tiles in the checkerboard border. Track your thinking and find a way of calculating the number of colored tiles in the border that is quick and efficient. Be prepared to share your strategy and justify your work.



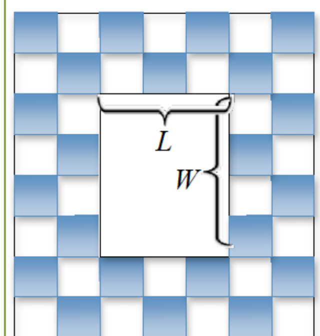
1.2 Building More Checkerboard Borders

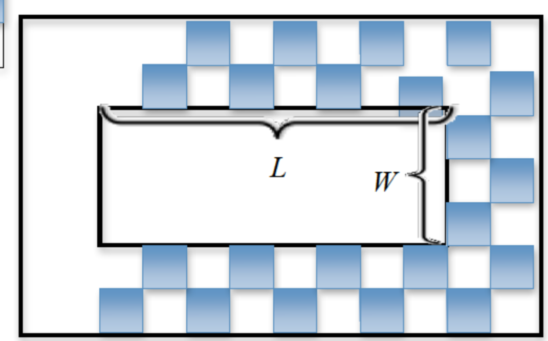
A Develop Understanding Task



As the tile workers started to look more deeply into their work they found it necessary to develop a way to quickly calculate the number of colored border tiles for not just square arrangements but also for checkerboard borders to surround any $L \times W$ rectangular tile center.

Find an expression to calculate the number of colored tiles in the two row checkerboard border for any rectangle. Be prepared to share your strategy and justify your work. Create models to assist you in your work.





Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

1.3 Serving Up Symbols

A Develop Understanding Task

As you look around your school cafeteria, you may see many things that could be counted or measured. To increase the efficiency of the cafeteria, the cafeteria manager, Elvira, decided to take a close look at the management of the cafeteria and think about all the components that affect the way the cafeteria runs. To make it easy, she assigned symbols for each count or measurement that she wanted to consider, and made the following table:

Symbol	Meaning (description of what the symbol means in context)	Units (what is counted or measured)
S	Number of students that buy lunch in the cafeteria each day	*students* or $\frac{students}{day}$
S_M	Number of students who have passed through a line in M minutes	
C	Number of classes per lunch period	
P	Number of lunch periods per day	
B	Number of boys that buy lunch each day	*boys* or *students* or $\frac{boys}{day}$
G	Number of girls that buy lunch each day	
F	Number of food servers in the cafeteria	
T	Total number of food items in one lunch (Each entrée, side dish, or beverage counts as 1 item.)	
M	Number of minutes passed since the beginning of the lunch period	
N_e	Number of entrees in each lunch	
N_s	Number of side dishes in each lunch	
N_b	Number of beverages in each lunch	
C_e	Cost of each entrée	
C_s	Cost of each side dish	
C_b	Cost of each beverage	
L	Number of lines in the cafeteria	
W	The number of food servers per line	
i	Average number of food items that a server can serve each minute (Each entrée, side dish, or beverage counts as 1 item.)	
H	Number of hours each food server works each day	
P_L	Price per lunch	

1.4 Examining Units

A Solidify Understanding Task

(Note: This task refers to the same set of variables as used in *Serving Up Symbols*)

Units in Addition and Subtraction

1. Why can you add $N_e + N_s + N_b$ and you can add $B + G$, but you can't add $M + W$?

2. We measure real-world quantities in units like feet, gallons, students and miles/hour (miles per hour).

   a. What units might you use to measure N_e, N_s and N_b?
   What about the sum $N_e + N_s + N_b$?

   b. What units might you use to measure B? G?
   What about the sum $B + G$?

   c. What units might you use to measure M? W?
   What about the sum $M + W$?

3. State a rule about how you might use units to help you think about what types of quantities can be added. How would you use or modify your rule to fit subtraction?

Units in Multiplication, scenario 1

1. Why can you multiply $N_e \times C_e$ and you can multiply $L \times W$, but you can't multiply $G \times C$?

2. Units in multiplication often involve rates like miles/gallon (miles per gallon), feet/second (feet per second), or students/table (students per table).

   a. What units might you use to measure N_e? C_e?
   What about the product $N_e \times C_e$?

   b. What units might you use to measure L? W?
   What about the product $L \times W$?

   c. What units might you use to measure G? C?
   What about the product $G \times C$?

3. State a rule about how you might use units to help you think about what types of quantities can be multiplied.

Understand solving equations as a process of reasoning and explain the reasoning

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

1.5 Cafeteria Actions and Reactions

A Develop Understanding Task

Elvira, the cafeteria manager, has just received a shipment of new trays with the school logo prominently displayed in the middle of the tray. After unloading 4 cartons of trays in the pizza line, she realizes that students are arriving for lunch and she will have to wait until lunch is over before unloading the remaining cartons. The new trays are very popular and in just a couple of minutes 24 students have passed through the pizza line and are showing off the school logo on the trays. At this time, Elvira decides to divide the remaining trays in the pizza line into 3 equal groups so she can also place some in the salad line and the sandwich line, hoping to attract students to the other lines. After doing so, she realizes that each of the three serving lines has only 12 of the new trays.

"That's not many trays for each line. I wonder how many trays there were in each of the cartons I unloaded?"

1. Can you help the cafeteria manager answer her question using the data in the story about each of the actions she took? Explain how you arrive at your solution.

Elvira is interested in collecting data about how many students use each of the tables during each lunch period. She has recorded some data on Post-It Notes to analyze later. Here are the notes she has recorded:

- Some students are sitting at the front table. (I got distracted by an incident in the back of the lunchroom, and forgot to record how many students.)
- Each of the students at the front table has been joined by a friend, doubling the number of students at the table.
- Four more students have just taken seats with the students at the front table.
- The students at the front table separated into three equal-sized groups and then two groups left, leaving only one-third of the students at the table.
- As the lunch period ends, there are still 12 students seated at the front table.

Elvira is wondering how many students were sitting at the front table when she wrote her first note. Unfortunately, she is not sure what order the middle three Post-It Notes were recorded in since they got stuck together in random order. She is wondering if it matters.

1.6 Elvira's Equations

A Solidify Understanding Task

(Note: This task refers to the same set of variables as used in *Serving Up Symbols*)

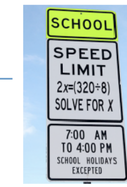
Elvira, the cafeteria manager, has written the following equation to describe a cafeteria relationship that seems meaningful to her. She has introduced a new variable A to describe this relationship.

$$A = \frac{S}{CP}$$

1. What does A represent in terms of the school and the cafeteria?
2. Using what you know about manipulating equations, solve this equation for S. Your solution will be of the form S = *an expression written in terms of the variables A, C and P.*
3. Does your expression for S make sense in terms of the meanings of the other variables? Explain why or why not.
4. Now solve the above equation for C and explain why the solution makes sense in terms of the variables.

1.7 Solving Equations, Literally

A Practice Understanding Task



Solve each of the following equations for x:

1. $\frac{3x+2}{5}=7$
2. $\frac{3x+2y}{5}=7$
3. $\frac{4x}{3}-5=11$
4. $\frac{4x}{3}-5y=11$
5. $\frac{2}{5}(x+3)=6$
6. $\frac{2}{5}(x+y)=6$
7. $2(3x+4)=4x+12$
8. $2(3x+4y)=4x+12y$

Write a verbal description for each step of the equation solving process used to solve the following equations for x. Your description should include statements about how you know what to do next. For example, you might write, "*First I* ____________, *because* ______________ . . ."

9. $\frac{ax+b}{c}-d=e$
10. $r\cdot\sqrt{\frac{mx}{n}}+s=t$

Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

1.8 Cafeteria Conundrums

A Solidify Understanding Task

Between serving and preparing delicious school lunches, our cafeteria manager, Elvira, is busy analyzing the business of running the cafeteria. We previously saw the symbols for some of the things that she measured. Now she plans to use those symbols. Help Elvira to consider the pressing questions of the lunch room.

Symbol	Meaning
S	Number of students that buy lunch in the cafeteria each day
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C_e	Cost of each entrée
C_s	Cost of each side dish
C_b	Cost of each beverage
L	Number of lines in the cafeteria
W	The number of food workers (servers) per line
i	Average number of food items that a worker can serve each minute (Each entrée, side dish, or beverage counts as 1 item.)
H	Number of hours each food worker works each day
P_L	Price per lunch

1.9 Greater Than?

A Solidify Understanding Task

For each situation you are given a mathematical statement and two expressions beneath it.

1. Decide which of the two expressions is greater, if the expressions are equal, or if the relationship cannot be determined from the statement.
2. Write an equation or inequality that shows your answer.
3. Explain why your answer is correct.

Watch out—this gets tricky!

Example:
Statement: $x = 8$
Which is greater? $x + 5$ or $3x + 2$
Answer: $3x + 2 > x + 5$ because if $x = 8$, $3x + 2 = 26$, $x + 5 = 13$ and $26 > 13$.

Try it yourself:

1. Statement: $y < x$
   Which is greater? $x - y$ or $y - x$
2. Statement: $2x - 3 > 7$
   Which is greater? 5 or x
3. Statement: $10 - 2x < 6$
   Which is greater? x or 2
4. Statement: $4x = 0$
   Which is greater? 1 or x
5. Statement: $a > 0,\ b < 0$
   Which is greater? ab or $\frac{a}{b}$

1.10 Taking Sides

A Practice Task

Joaquin and Serena work together productively in their math class. They both contribute their thinking and when they disagree, they both give their reasons and decide together who is right. In their math class right now, they are working on inequalities. Recently they had a discussion that went something like this:

> Joaquin: The problem says that "6 less than a number is greater than 4." I think that we should just follow the words and write $6 - x > 4$.
>
> Serena: I don't think that works because if x is 20 and you do 6 less than that you get $20 - 6 = 14$. I think we should write $x - 6 > 4$.
>
> Joaquin: Oh, you're right. Then it makes sense that the solution will be $x > 10$, which means we can choose any number greater than 10.

The situations below are a few more of the disagreements and questions that Joaquin and Serena have. Your job is to decide how to answer their questions, decide who is right, and give a mathematical explanation of your reasoning.

1. Joaquin and Serena are assigned to graph the inequality $x \geq -7$.
   Joaquin thinks the graph should have an open dot -7.
   Serena thinks the graph should have a closed dot at -7.
   Explain who is correct and why.
2. Joaquin and Serena are looking at the problem $3x + 1 > 0$.
   Serena says that the inequality is always true because multiplying a number by three and then adding one to it makes the number greater than zero.
   Is she right? Explain why or why not.
3. The word problem that Joaquin and Serena are working on says, "4 greater than x".
   Joaquin says that they should write: $4 > x$.
   Serena says they should write: $x + 4$.
   Explain who is correct and why.

2.1 Pet Sitters

A Develop Understanding Task

The Martinez twins, Carlos and Clarita, are trying to find a way to make money during summer vacation. When they overhear their aunt complaining about how difficult it is to find someone to care for her pets while she will be away on a trip, Carlos and Clarita know they have found the perfect solution. Not only do they have a large, unused storage shed on their property where they can house animals, they also have a spacious fenced backyard where the pets can play.

Carlos and Clarita are making a list of some of the issues they need to consider as part of their business plan to care for cats and dogs while their owners are on vacation.

- *Space*: Cat pens will require 6 ft^2 of space, while dog runs require 24 ft^2. Carlos and Clarita have up to 360 ft^2 available in the storage shed for pens and runs, while still leaving enough room to move around the cages.

- *Start-up Costs*: Carlos and Clarita plan to invest much of the $1280 they earned from their last business venture to purchase cat pens and dog runs. It will cost $32 for each cat pen and $80 for each dog run.

Of course, Carlos and Clarita want to make as much money as possible from their business, so they are trying to determine how many of each type of pet they should plan to accommodate. They plan to charge $8 per day for boarding each cat and $20 per day for each dog.

After surveying the community regarding the pet boarding needs, Carlos and Clarita are confident that they can keep all of their boarding spaces filled for the summer.

So the question is, how many of each type of pet should they prepare for? Their dad has suggested the same number of each, perhaps 12 cats and 12 dogs. Carlos thinks they should plan for more dogs, since they can charge more. Clarita thinks they should plan for more cats since they take less space and time, and therefore they can board more.

What do you think? What recommendations would you give to Carlos and Clarita, and what argument would you use to convince them that your recommendation is reasonable?

Story Contexts that Support Conceptual Understanding

Pet Sitters

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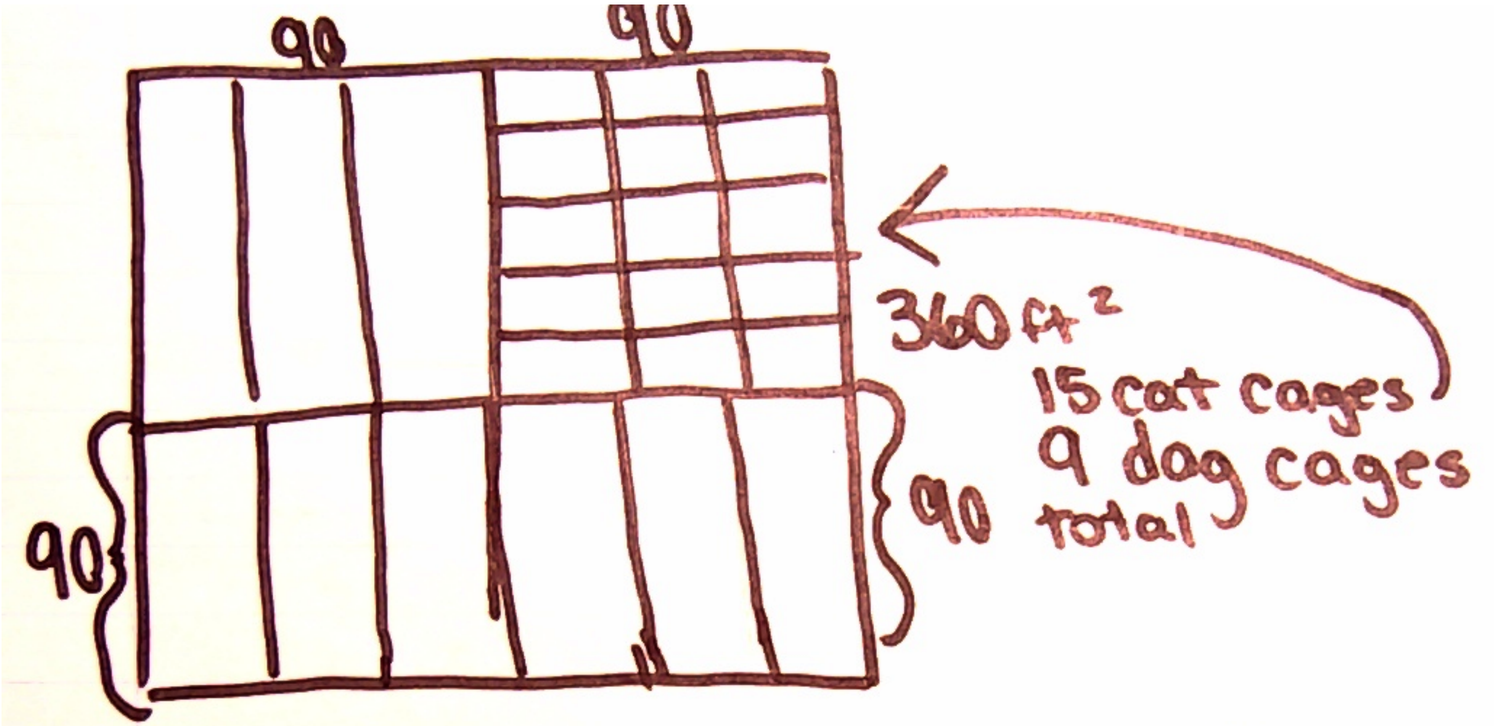
Pet Sitters: Student Work



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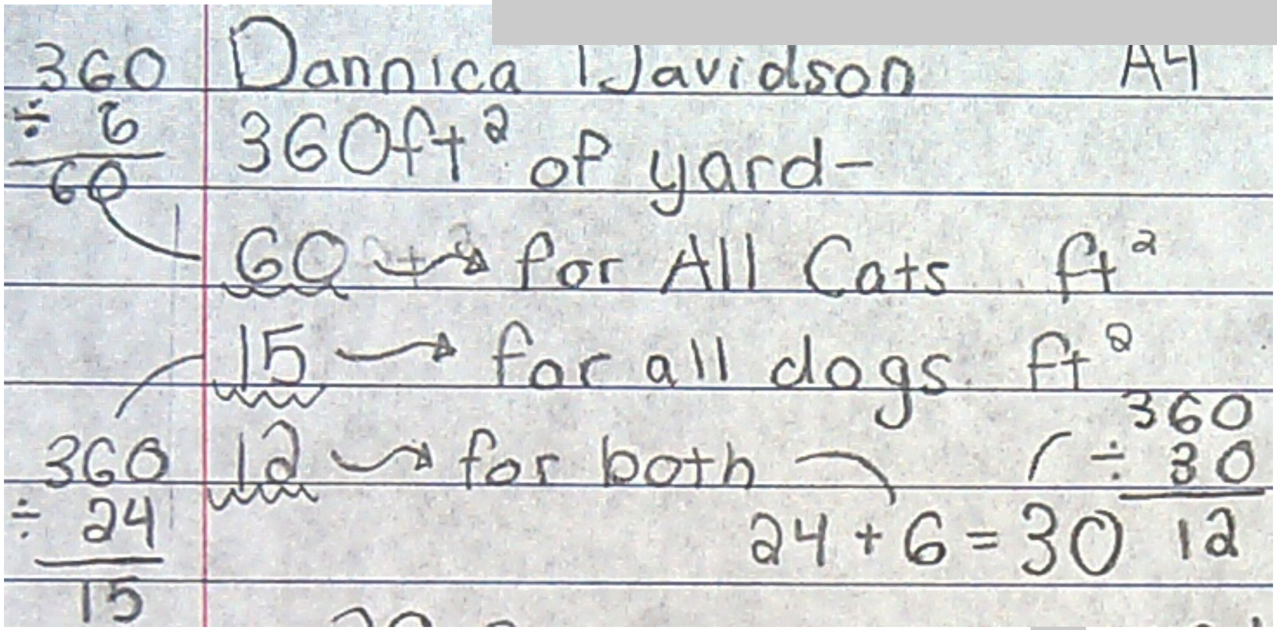
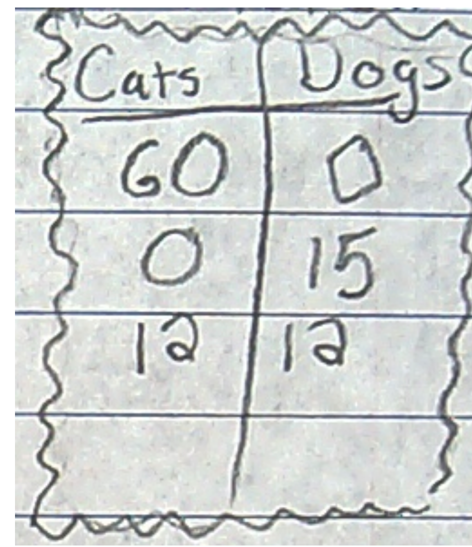




Pet Sitters: Student Work

Space

Cats	Dogs
60	0
0	15
30	7
20	10
12	12

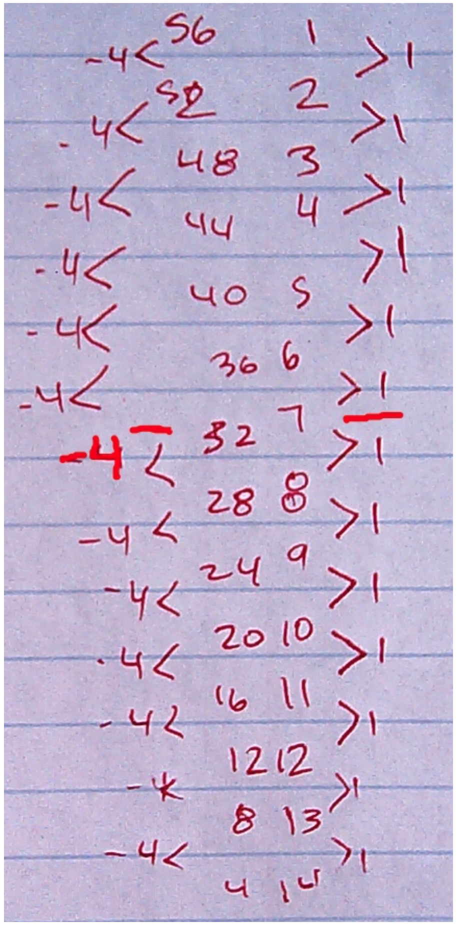
Earn

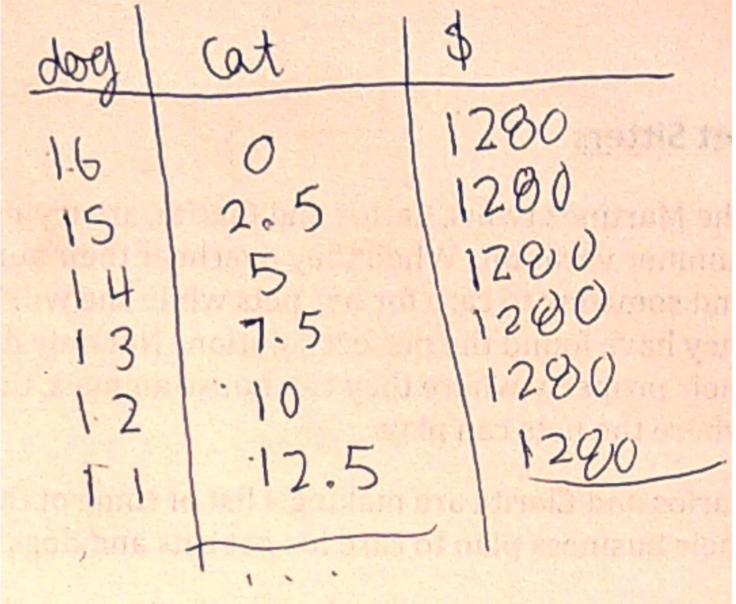
Cats	Dog
3,360	0
0	2,100
1,680	980
1,120	1,400
672	1,680

Cost

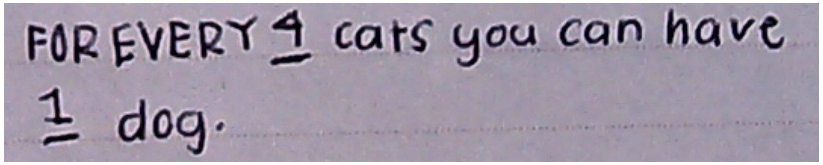
CAT'S	Dogs
1,920	0 = 1920
0	1,200 = 1,200
960	560 = 1,520
640	800 = 1,440
384	960 = 1,344

Pet Sitters: Student Work





dog	cat	$
16	0	1280
15	2.5	1280
14	5	1280
13	7.5	1280
12	10	1280
11	12.5	1280



Pet Sitters: Student Work

What do you think? What recommen
what argument would you use to con

dogs cats

space $y = 4x + 60$ cats

dogs

cost $y = 40 - 2.5x$

Student #1

Student #2

amnt. of cats

ace equation: $y = 360 - ((6 \cdot c) + (24 \cdot d))$

cost equation: $y = (c \cdot 32) + (d \cdot 80)$

Dogs equashion: $y = -80x + 1280$

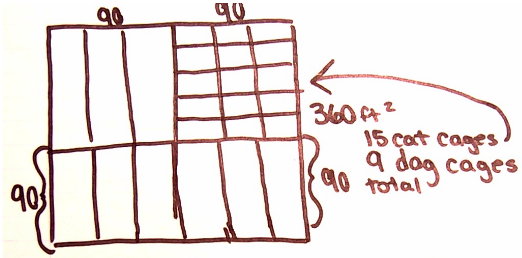
Cats equashion: $y = -32x + 1280$

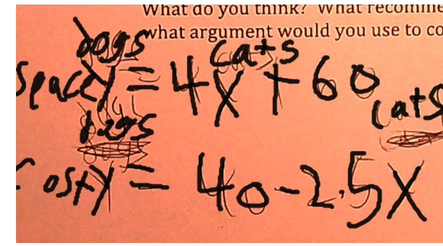
Student #3

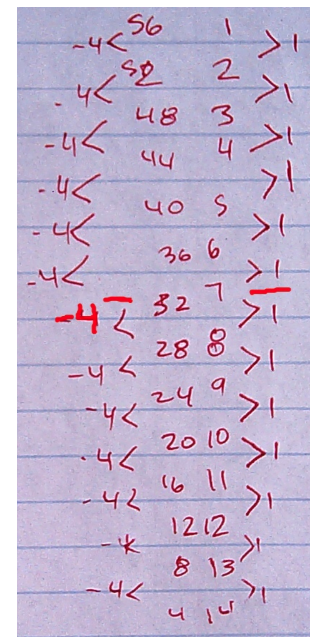
Space : Dog equashion: $y = -24x + 360$

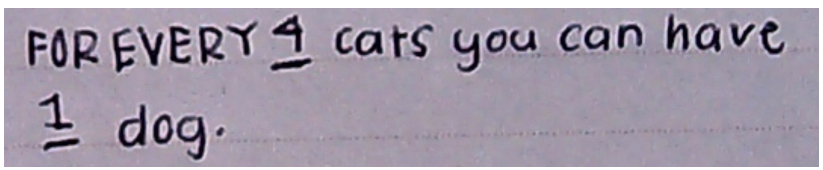
cats equashion: $y = -6x + 360$

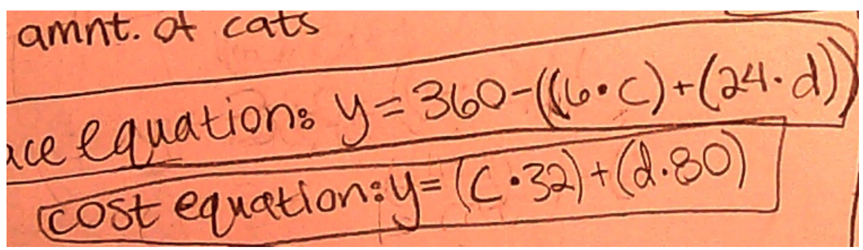
Student Work Demonstrates Multiple Access Points



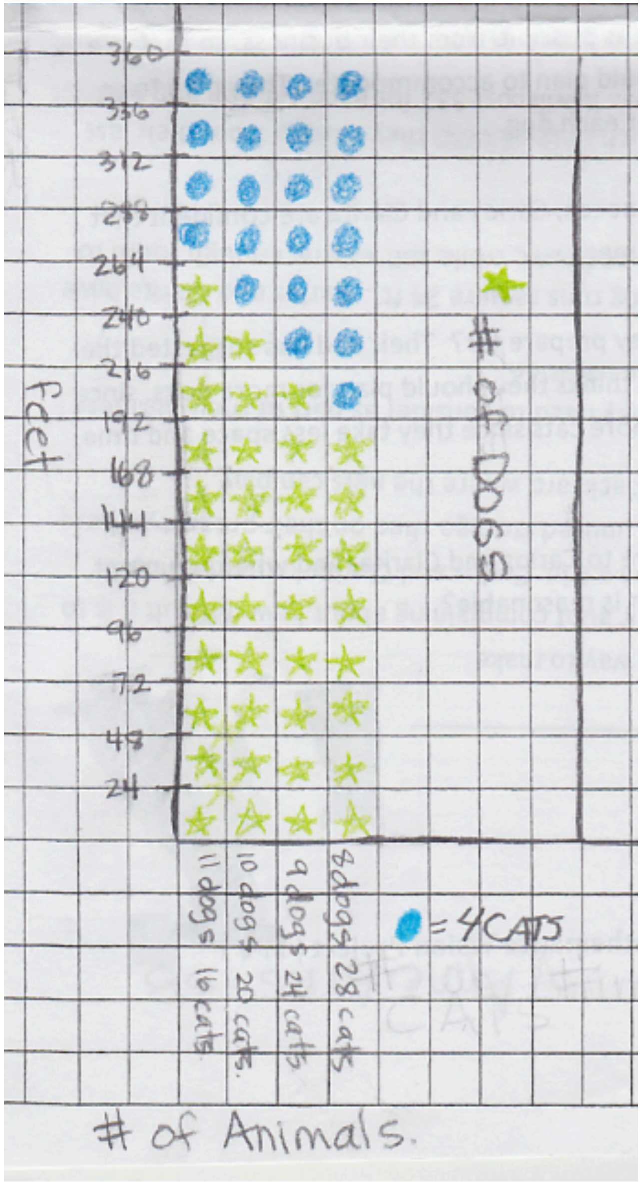








How is this student thinking about the data?



Teacher's Role in Supporting Student Understanding

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Teacher's Role Supported with Teacher Notes

Teacher notes identify the focus for each task and provides full description of the lesson including anticipating student work, and ideas for facilitating discussions.

2.1 Pet Sitters – Teacher Notes

A Develop Understanding Task

Purpose: As students work with the context of making recommendations for how many dogs and cats Carlos and Clarita should plan to accommodate, they will surface many ideas, strategies and representations related to solving systems of equations and inequalities. For example, they will explore the notion of *constraints* since in this task the number of each type of pet that can be accommodated is limited by space and money, but many different combinations of dogs and cats are possible. They may consider the notion of a *system of equations* since each constraint (space, start-up costs) allows for a different set of possibilities—a particular combination of dogs and cats may satisfy one constraint but not another—so both constraints must be considered simultaneously. Finally, they may surface the notion of *a system of inequalities* since Carlos and Clarita don't have to use up all of the available space or money, implying that each constraint may be represented by an inequality.

Core Standards Focus:

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

Related Standards: N.Q.2, A.REI.12

Launch (Whole Class):

After reading and discussing the "Pet Sitters" scenario, challenge students to come up with a combination of dogs and cats that would yield the highest daily income. Give students a few minutes to work independently to find the income for a particular combination of dogs and cats. After a few minutes have students compare their daily income with others and then work with a partner to improve their initial guesses

Explore (Small Group):

For students who don't know where to begin, have them determine the daily income if Carlos and Clarita follow their dad's advice of boarding "the same number of each, perhaps 12 cats and 12

Students who focus on just one constraint and find combinations of cats and dogs that satisfy that constraint have naturally simplified the larger problem into a more manageable task. They will gain valuable insight into the mathematical work of this module, and need not be pressed at this time to consider both constraints, or the bigger issue of maximizing the profit. Press for the work you feel your students can handle, and use this task to assess what ideas, strategies and representations will be available for future work in this module. For example, a student who visualizes the space constraint by drawing possible layouts of how the shed might be occupied with cat pens and dog runs is noticing that there are many such combinations that use up the total available space.

Watch for students who focus on the following ideas, since they underpin essential strategies in future tasks: (This list is sequenced from most likely to less likely to occur; don't be concerned if not all of these ideas are present in your class, since future tasks will elicit each of these ways of thinking about the system of constraints.)

- Students who calculate the intercepts of the constraints; that is, students who consider how they might use up all of the money or all of the space, by boarding just cats or just dogs.
- Students who make note of an "exchange rate" between cats and dogs in terms of either of the constraints. For example, four cats use the same space as one dog.
- Students who create charts to keep track of the combinations of dogs and cats they have tried. Such charts will probably include columns to track the number of dogs, the number of cats, and the money earned. Students will also have to keep track of whether a particular combination of dogs and cats satisfies each of the constraints.
- Students who plot combinations on a coordinate grid to keep track of the combinations they have tested.
- Students who try to write equations or inequalities to represent the constraints.

Discuss (Whole Class):

Begin with a combination of dogs and cats that worked, a second combination that worked and

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Thinking Through a Module of Instruction

- Work the task. Pay attention to the purpose and the goal. Ask yourself, what mathematics is developed, solidified or practiced in this task?
- Complete the "Module at a Glance" form.
- Make a poster labeled with the number and title of the task. Illustrate the mathematical focus of the task. Look at the task before and the task that follows in order to highlight the mathematics that this task contributes to the learning progression.

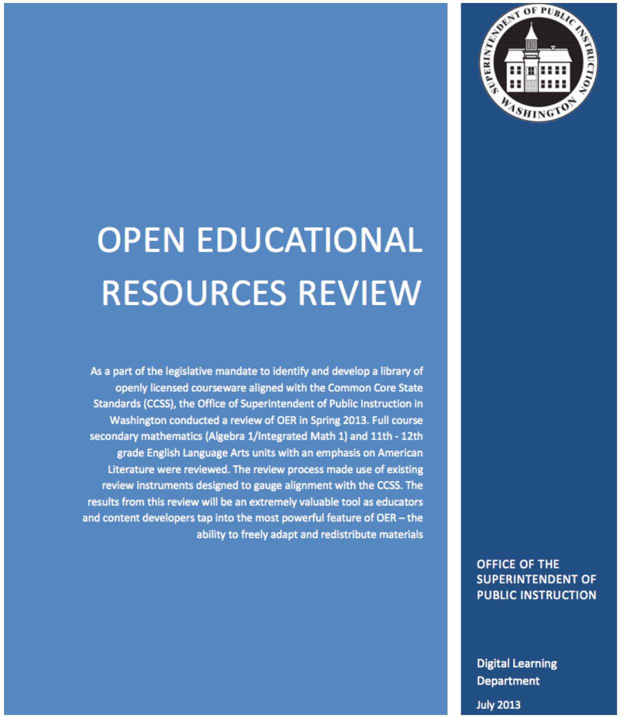
Thinking Through a Module of Instruction

Tracking Student Thinking in MVP Secondary I, Module 2H: ***Systems of Equations and Inequalities (Honors)***

Cluster titles ⇒	Create equations that describe numbers or relationships						Solve systems of equations		Represent and solve systems of equations and inequalities graphically		
Description of the nature of student thinking relative to this standard ⇒ in this task ⇓ *(see sample descriptors below*)*	Create equations in two or more variables to represent relationships between quantities (ACED.2)	Graph equations in two variables on coordinate axes with labels and scales (ACED.2)	Represent constraints by equations or inequalities (A.CED.3)	Represent constraints by systems of equations and/or inequalities (A.CED.3)	Interpret solutions to a system of equations and/or inequalities as viable or nonviable options in a modeling context (A.CED.3)	Rearrange formulas to highlight a quantity of interest (A.CED.4)	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. (A.REI.5)	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables (A.REI.6)	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) (A.REI.12)	Graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes (A.CED.4)	Solve systems of equations with matrices (UT Honors)
Pet Sitters											
Too Big or Not Too Big											
Some of One, None of the Other											
Pampering and Feeding Time											
All For One, One For All											
Get to the Point											
Shopping for Cats and Dogs											

MVP leads the way in Open Education Resources

Independent Review of MVP: Washington State, July 2013



OPEN EDUCATIONAL RESOURCES REVIEW

As a part of the legislative mandate to identify and develop a library of openly licensed courseware aligned with the Common Core State Standards (CCSS), the Office of Superintendent of Public Instruction in Washington conducted a review of OER in Spring 2013. Full course secondary mathematics (Algebra 1/Integrated Math 1) and 11th - 12th grade English Language Arts units with an emphasis on American Literature were reviewed. The review process made use of existing review instruments designed to gauge alignment with the CCSS. The results from this review will be an extremely valuable tool as educators and content developers tap into the most powerful feature of OER – the ability to freely adapt and redistribute materials

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Digital Learning Department

July 2013

Independent Review of Secondary Mathematics I

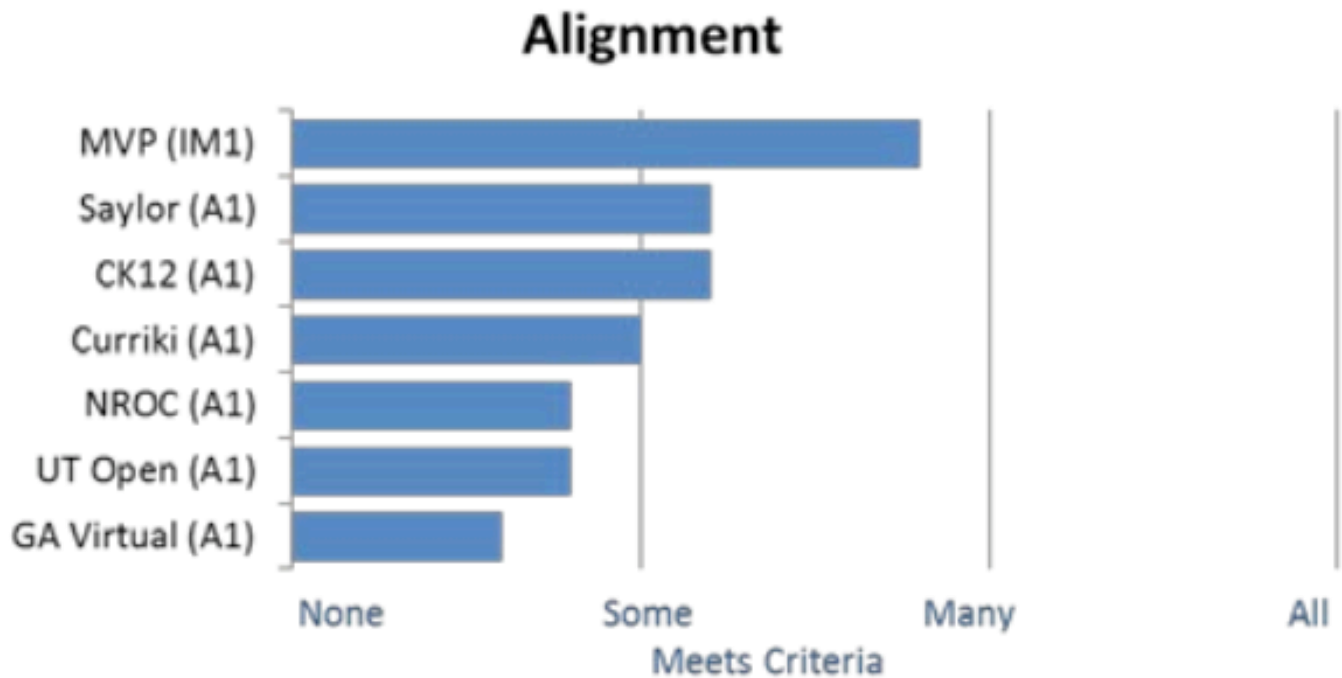

Figure 7. EQuIP. This scale looks at the alignment of a selected unit in the materials to the CCSS.

Independent Review of Secondary Mathematics I

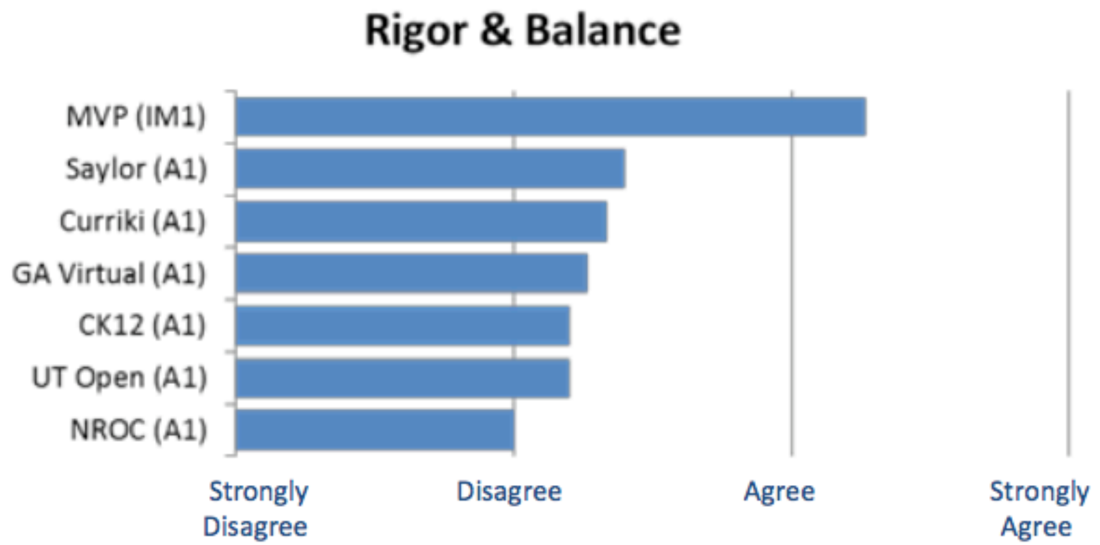

Figure 9. Publishers' Criteria. This scale measures whether the materials pursue with equal intensity conceptual understanding, procedural skill and fluency, and applications. MVP was designed from the ground up to be more aligned with shifts in thinking, including rigor and balance.

Independent Review of Secondary Mathematics I

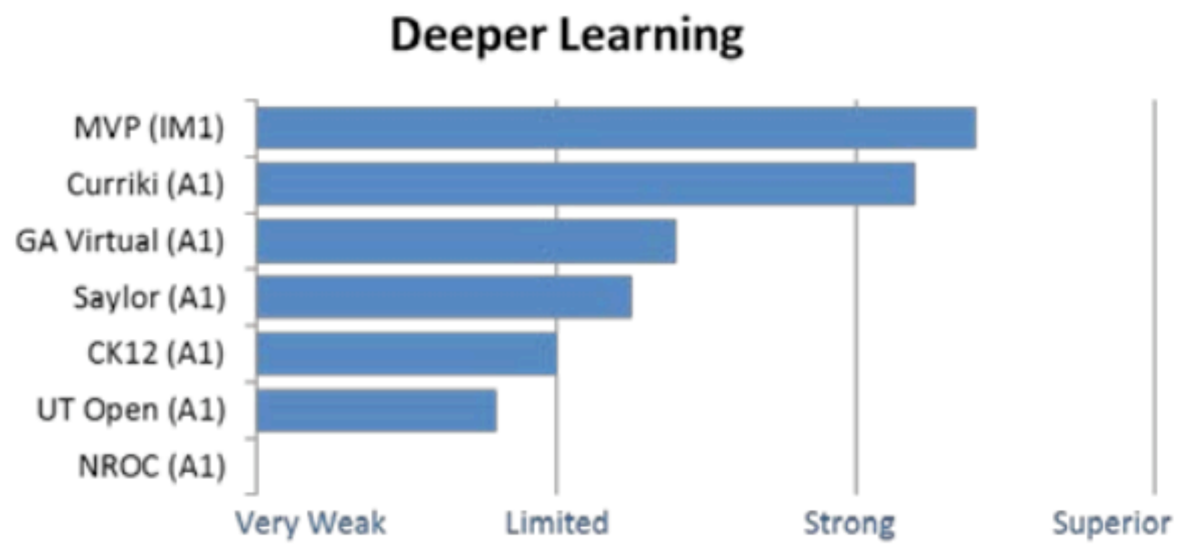

Figure 10. Achieve OER. This scale measures the unit's ability to engage learners in one or deeper learning skills, including think critically and solve complex problems, reason abstractly, construct viable arguments and apply discrete knowledge and skills to real-world situations.

Independent Review of Secondary Mathematics I

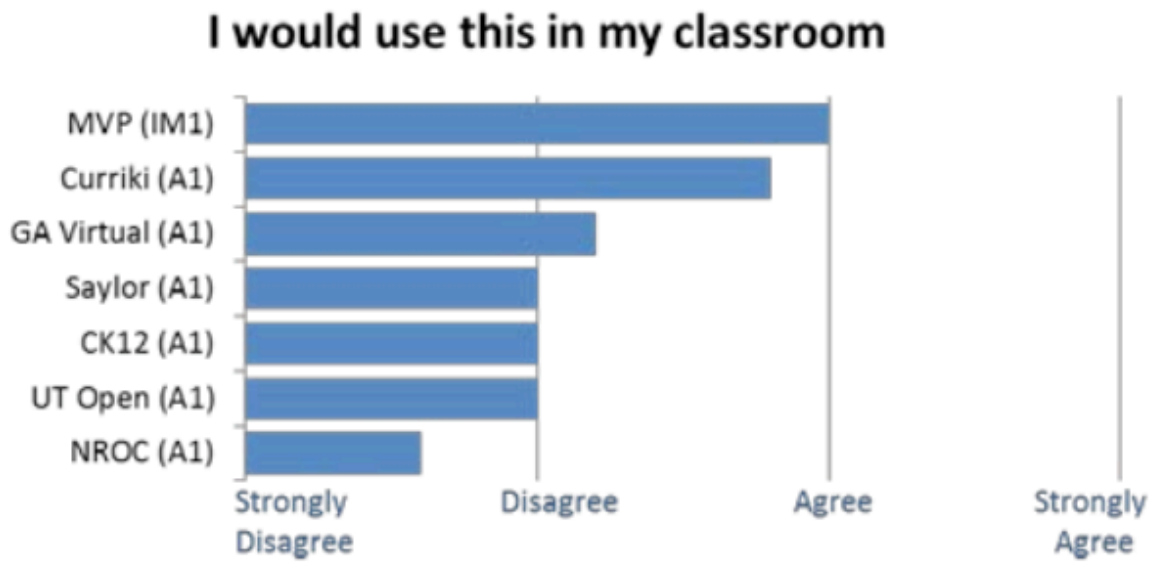

Figure 5. Reviewer Comments. Note that reviewers indicated they would use MVP and Curriki in their classrooms. It is also important to note that only the open source NROC book was reviewed. NROC has a much more robust product offered through Hippocampus.org, but it did not meet all the criteria for consideration in the review.

What's Your Next Step?

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June 24, 2014

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