mathematics vision project

Transforming Mathematics Education

SECONDARY MATH ONE

An Integrated Approach

MODULE 5

Systems of Equations & Inequalities

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

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MODULE 5 - TABLE OF CONTENTS

SYSTEMS OF EQUATIONS AND INEQUALITIES

5.7 Get to the Point – A Solidify Understanding Task

Solving systems of linear equations in two variables (A.REI.6)

READY, SET, GO Homework: Systems of Equations and Inequalities 5.7

5.8 Shopping for Cats and Dogs – A Develop Understanding Task

An introduction to solving systems of linear equations by elimination (A.REI.5, A.REI.6)

READY, SET, GO Homework: Systems of Equations and Inequalities 5.8

5.9 Can You Get to the Point, Too? – A Solidify Understanding Task

Solving systems of linear equations by elimination (A.REI.5, A.REI.6)

READY, SET, GO Homework: Systems of Equations and Inequalities 5.9

5.10 Taken Out of Context – A Practice Understanding Task

Working with systems of linear equations, including inconsistent and dependent systems (A.REI.6)

READY, SET, GO Homework: Systems of Equations and Inequalities 5.10

5. 1 Pet Sitters

A Develop Understanding Task

The Martinez twins, Carlos and Clarita, are trying to find a way to make money during summer vacation. When they overhear their aunt complaining about how difficult it is to find someone to care for her pets while she will be away on a trip, Carlos and Clarita know they have found the perfect solution. Not only do they have a large, unused storage shed on their property where they can house animals, they also have a spacious fenced backyard where the pets can play.

Carlos and Clarita are making a list of some of the issues they need to consider as part of their business plan to care for cats and dogs while their owners are on vacation.

- *Space*: Cat pens will require 6 ft^2 of space, while dog runs require 24 ft^2. Carlos and Clarita have up to 360 ft^2 available in the storage shed for pens and runs, while still leaving enough room to move around the cages.

- *Start-up Costs*: Carlos and Clarita plan to invest much of the $1280 they earned from their last business venture to purchase cat pens and dog runs. It will cost $32 for each cat pen and $80 for each dog run.

Of course, Carlos and Clarita want to make as much money as possible from their business, so they are trying to determine how many of each type of pet they should plan to accommodate. They plan to charge $8 per day for boarding each cat and $20 per day for each dog.

After surveying the community regarding the pet boarding needs, Carlos and Clarita are confident that they can keep all of their boarding spaces filled for the summer.

So the question is, how many of each type of pet should they prepare for? Their dad has suggested the same number of each, perhaps 12 cats and 12 dogs. Carlos thinks they should plan for more dogs, since they can charge more. Clarita thinks they should plan for more cats since they take less space and time, and therefore they can board more.

What do you think? What recommendations would you give to Carlos and Clarita, and what argument would you use to convince them that your recommendation is reasonable?

READY, SET, GO! **Name** **Period** **Date**

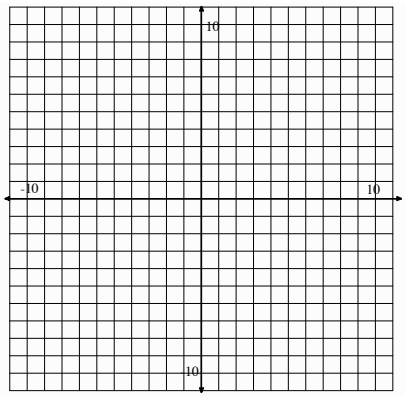
READY

Topic: Solving system of equations by graphing.

Substitute the given points into both equations to determine which ordered pair satisfies the system of linear equations. Graph both equations and label the point of intersection to verify the solution.

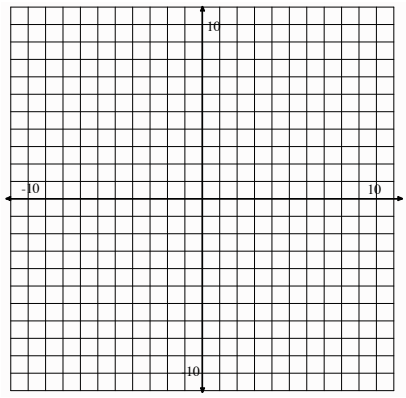
1. $y = 3x - 2 \; and \; y = \; x$

a. $(0, -2)$
b. $(2, 2)$
c. $(1, 1)$



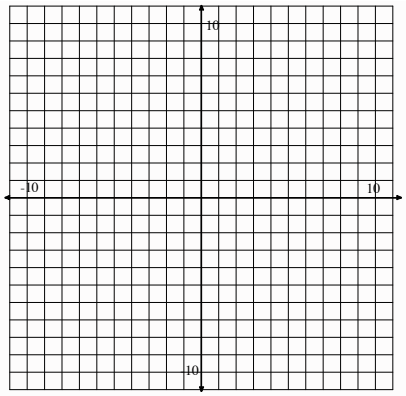
2. $y = 2x + 3 \; and \; y = \; x + 5$

a. (2, 7)
b. (-7, 11)
c. (0, 5)

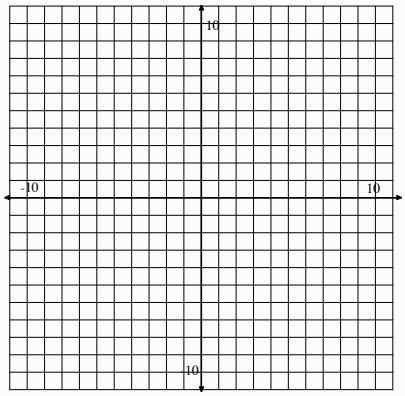


Solve the following systems by graphing. Check the solution by evaluating both equations at the point of intersection.

3. $y = x + 3 \; and \; y = -2x + 3$



4. $y = 3x - 8 \; and \; y = -x$



SET

Topic: Determining possible solutions to inequalities

5. A theater wants to take in <u>at least</u> $2000 for the matinee. Children's tickets cost $5 each and adult tickets cost $10 each. The theater can seat up to 350 people. Find five combinations of children and adult tickets that will make the $2000 goal.

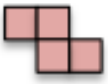
6. The Utah Jazz scored 102 points in a recent game. The team scored some 3-point shots, 2-point shots, and many free throws worth 1-point each. Find five combinations of baskets that would add up to 102 points.

7. Use as many of the following shapes in any combination as you need to try to fill in as much of the 12 by 12 grid as you can. You may rotate or reflect a shape if it helps. Write your answer showing how many of each shape you used using the letters that identify shape.

Example: 3a, 5b, 10c, 2d, 6e

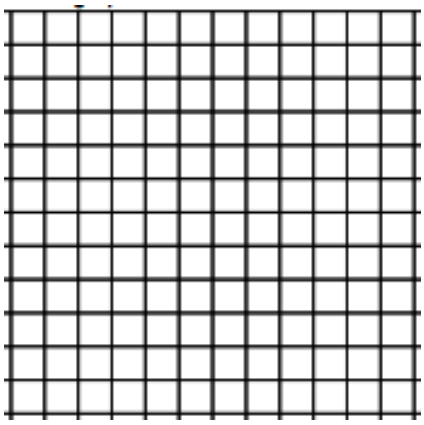
a. 

b. 

c. 

d. 

e. 

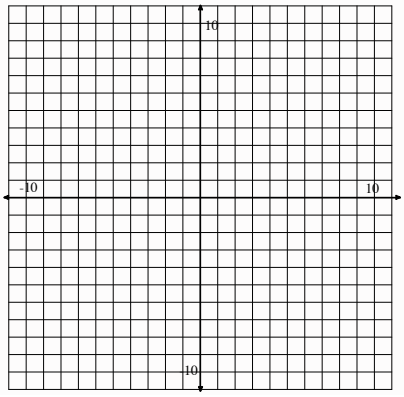


GO

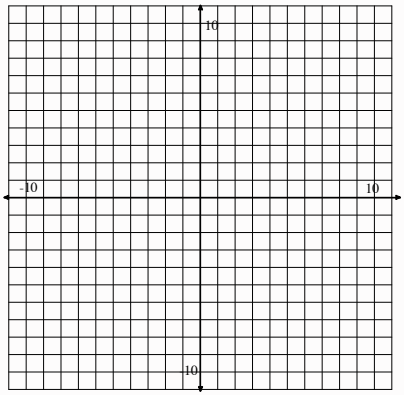
Topic: Graphing linear equations and determining if a given value is a solution, arithmetic sequences

Graph each equation below; then determine if the point (3,5) is a solution to the equation. Find two points other than (3,5) that are solutions to the equation. Show these points on the graph.

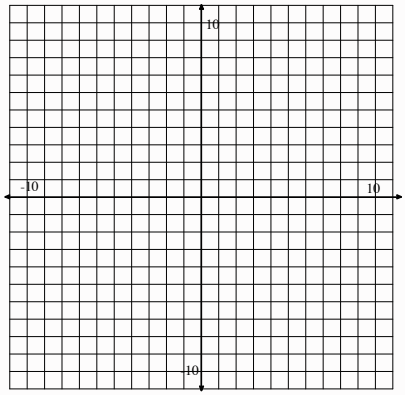
8. $y = 2x - 1$



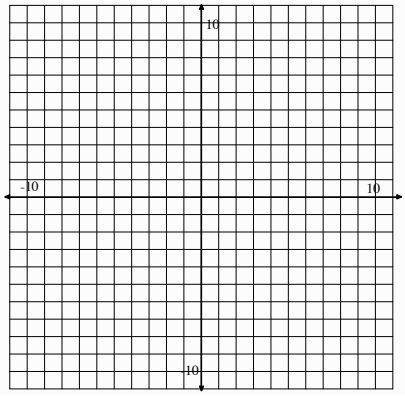
9. $y = \frac{1}{3}x + 2$



10. $y = -3x + 5$



11. $y = \frac{-3}{5}x + 4$



The tables below represent <u>different</u> arithmetic sequences. Fill in the missing numbers. Then write the explicit equation for each.

12.

term (x)	1	2	3	4
value (y)	17			-7

Equation:

13.

term(x)	1	2	3	4	5	6	7	8	9	10	11	12	13
term(y)	17												-7

Equation:

14.

term (x)	1	2	3	4	5	6	7
value (y)	17						-7

Equation:

15. Each of the sequences above begins and ends with the same number. Would the graph of each sequence represent the same line? Justify your thinking.

16. If you graphed each of these sequences and made them continuous by connecting each point, where would they intersect?

5. 2 Too Big, or Not Too Big, That Is the Question

A Solidify Understanding Task

As Carlos is considering the amount of money available for purchasing cat pens and dog runs (see below) he realizes that his father's suggestion of boarding "the same number of each, perhaps 12 cats and 12 dogs" is too big. Why?

- *Start-up Costs*: Carlos and Clarita plan to invest much of the $1280 they earned from their last business venture to purchase cat pens and dog runs. It will cost $32 for each cat pen and $80 for each dog run.

1. Find at least 5 more combinations of cats and dogs that would be "too big" based on this *Start-up Cost* **constraint.** Plot each of these combinations as points on a coordinate grid using the same color for each point.

2. Find at least 5 combinations of cats and dogs that would be "not too big" based on this *Start-up Cost* constraint. Plot each of these combinations as points on a coordinate grid using a different color for the points than you used in #1.

3. Find at least 5 combinations of cats and dogs that would be "just right" based on this *Start-up Cost* constraint. That is, find combinations of cat pens and dog runs that would cost exactly $1280. Plot each of these combinations as points on a coordinate grid using a third color.

4. What do you notice about these three different collections of points?

5. Write an equation for the line that passes through the points representing combinations of cat pens and dog runs that cost exactly $1280. What does the slope of this line represent?

Carlos and Clarita don't have to spend all of their money on cat pens and dog runs, unless it will help them maximize their profit.

6. Shade all of the points on your coordinate grid that **satisfy** the *Start-up Costs* constraint.

7. Write a mathematical rule to represent the points shaded in #6. That is, write an inequality whose **solution set** is the collection of points that satisfy the *Start-up Costs* constraint.

In addition to *start-up costs,* Carlos needs to consider how much space he has available, based on the following:

- *Space*: Cat pens will require 6 ft^2 of space, while dog runs require 24 ft^2. Carlos and Clarita have up to 360 ft^2 available in the storage shed for pens and runs, while still leaving enough room to move around the cages.

8. Write an inequality to represent the solution set for the *space* constraint. Shade the solution set for this inequality on a different coordinate grid.

What do you think? What recommendation would you give to Carlos and Clarita regarding how many cats and dogs to plan on boarding, and what argument would you use to convince them that your recommendation is reasonable?

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Determining if given values are solutions to a two variable equation.

Identify which of the given points are solutions to the following linear equations.

1. $3x + 2y = 12$
   a. $(2, 4)$
   b. $(3, 2)$
   c. $(4, 0)$
   d. $(0, 6)$

2. $5x - y = 10$
   a. $(2, 0)$
   b. $(3, 0)$
   c. $(0, -10)$
   d. $(1, 1)$

3. $-x + 6y = 10$
   a. $(-4, 1)$
   b. $(-22, -2)$
   c. $(2, 2)$
   d. $(10, 0)$

Find the value that will make each ordered pair be a solution to the given equation.

4. $x + y = 6$
   a. $(2, \_\_\_\_)$
   b. $(0, \_\_\_\_)$
   c. $(\_\_\_\_, 0)$

5. $2x + 4y = 8$
   a. $(2, \_\_\_\_)$
   b. $(0, \_\_\_\_)$
   c. $(\_\_\_\_, 0)$

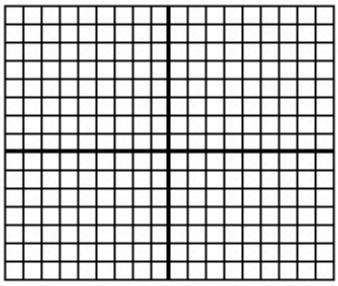
6. $3x - y = 8$
   a. $(2, \_\_\_\_)$
   b. $(0, \_\_\_\_)$
   c. $(\_\_\_\_, 0)$

SET

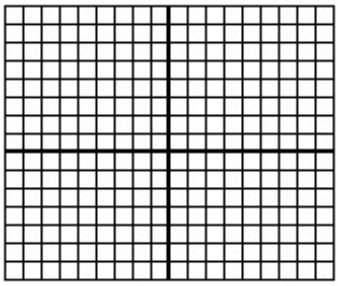
Topic: Graphing linear inequalities

Graph the following inequalities on the coordinate plane. Name one point that is a solution to the inequality and one point that is not a solution. Show algebraically and graphically that your points are correct.

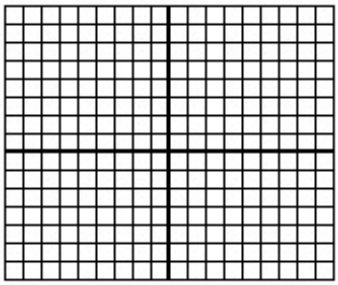
7. $y \leq 3x + 4$



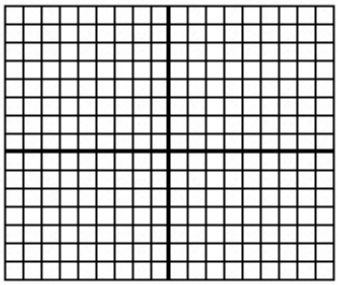
8. $y < 7x - 2$



9. $y > \frac{-3}{5}x + 2$



10. $y \geq -6$



GO

Topic: Solving inequalities

Follow the directions for each problem below. (Show your work!)

11. $10 - 3x < 28$

    a) Solve for x. Then graph the solution on the number line.

    b) Select an x-value from your graph of the solution of the inequality. Replace x in the original inequality $10 - 3x < 28$ with your chosen value. Does the inequality hold true?

    c) Select an x-value that is outside of the solution set on your graph. Replace x in the original inequality $10 - 3x < 28$ with your chosen value. Does the inequality still hold true?

12. $4x - 2y \geq 6$

    a) Solve for y.

    b) Rewrite your inequality as an equation. In other words, your solution will say y = , instead of y ≥ or y ≤. When you use the equal sign, the expression represents the equation of a line.

    c) Graph the line that goes with your equation.

    d) Name the y–intercept.

    e) Identify the slope.

    f) Select a point that is above the line. (,)

    g) Replace the x-value and y-value of your chosen point in the inequality $4x - 2y \geq 6$.

    h) Is the inequality still true?

    i) Select a point that is below the line. (,)

    j) Replace the x-value and y-value of your chosen point in the inequality $4x - 2y \geq 6$.

    k) Is the inequality still true?

    l) Explain which side of the line should be shaded.

    m) Decide whether the line should be solid or dotted. Justify your decision.

5. 3 Some of One, None of the Other

A Solidify Understanding Task

Carlos and Clarita are comparing strategies for writing equations of the boundary lines for the "Pet Sitter" constraints. They are discussing their work on the *space* constraint.

- *Space*: Cat pens will require 6 ft^2 of space, while dog runs require 24 ft^2. Carlos and Clarita have up to 360 ft^2 available in the storage shed for pens and runs, while still leaving enough room to move around the cages.

Carlos' Method: "I made a table. If I don't have any cats, then I have room for 15 dogs. If I use some of the space for 4 cats, then I can have 14 dogs. With 8 cats, I have room for 13 dogs. For each additional dog run that I don't buy, I can buy 4 more cat pens. From my table I know the y-intercept of my line is 15 and the slope is $-\frac{1}{4}$, so my equation is $y = -\frac{1}{4}x + 15$."

Clarita's Method: "I let x represent the number of cats, and y the number of dogs. Since cat pens require 6 ft^2, $6x$ represents the space used by cats. Since dog runs require 24 ft^2, $24y$ represents the amount of space used by dogs. So my equation is $6x + 24y = 360$."

1. Since both equations represent the same information, they must be equivalent to each other.

   a. Show the steps you could use to turn Clarita's equation into Carlos' equation. Explain why you can do each step.

   b. Show the steps you could use to turn Carlos' equation into Clarita's. Explain why you can do each step.

2. Use both Carlos' and Clarita's methods to write the equation of the boundary line for the *start-up costs* constraint.

   - *Start-up Costs*: Carlos and Clarita plan to invest much of the $1280 they earned from their last business venture to purchase cat pens and dog runs. It will cost $32 for each cat pen and $80 for each dog run.

3. Show the steps you could use to turn Clarita's *start-up costs* equation into Carlos' equation. Explain why you can do each step.

4. Show the steps you could use to turn Carlos' *start-up costs* equation into Clarita's. Explain why you can do each step.

In addition to writing an equation of the boundary lines, Carlos and Clarita need to graph their lines on a coordinate grid.

Carlos' equations are written in **slope-intercept form**. Clarita's equations are written in **standard form**. Both forms are ways of writing **linear equations**.

Both Carlos and Clarita know they only need to plot two points in order to graph a line.

5. Carlos' strategy: How might Carlos use his slope-intercept form, $y = -\frac{1}{4}x + 15$, to plot two points on his line?

6. Clarita's strategy: How might Clarita use her standard form, $6x + 24y = 360$, to plot two points on her line? (Clarita is really clever, so she looks for the two easiest points she can find.)

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Determining points that are solutions to a system of equations.

Three points are given. Each point is a solution to at least one of the equations. Just one point satisfies <u>both</u> equations. (This is the solution to the system!) Find and justify which point is a solution to both equations. Also justify which points are not solutions.

1. $\begin{cases} y = 2x - 3 \\ y = -x + 3 \end{cases}$
   a. (-2 , 5)
   b. (2 , 1)
   c. (4 , 5)

2. $\begin{cases} y = 3x + 3 \\ y = -x + 3 \end{cases}$
   a. (-1 , 0)
   b. (6 , -3)
   c. (0 , 3)

3. $\begin{cases} y = 2 \\ y = -4x - 6 \end{cases}$
   a. (7 , 2)
   b. (2 , -14)
   c. (-2 , 2)

4. $\begin{cases} y = 2x + 4 \\ x + y = -5 \end{cases}$
   a. (1 , 6)
   b. (-3 , -2)
   c. (-3 , 2)

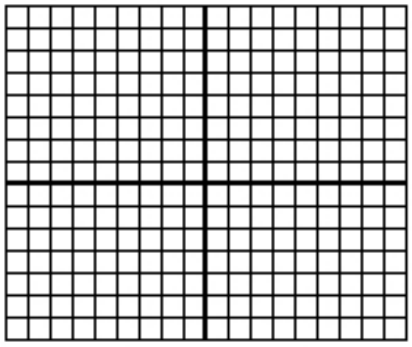
SET

Topic: Graphing linear equations written in standard form

<u>Graph</u> the following equations by finding the <u>x-intercept and the y-intercept</u>.

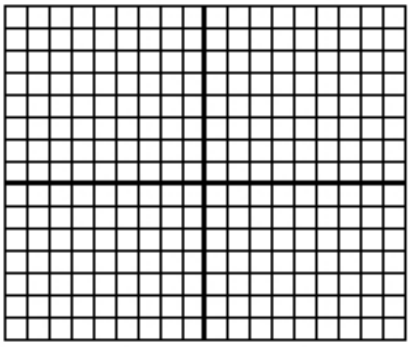
5. $5x - 2y = 10$

x-intercept: y-intercept:



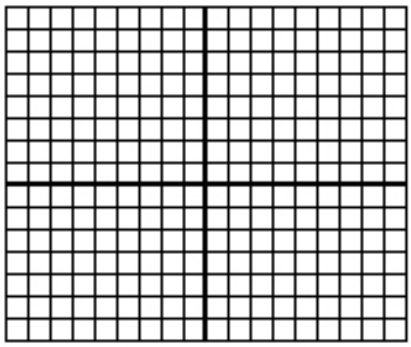
6. $3x - 6y = 24$

x-intercept: y-intercept:



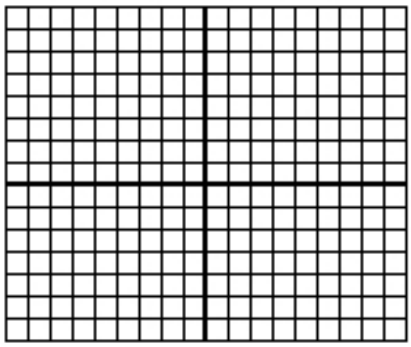
7. $6x + 2y = 18$

x-intercept: y-intercept:



8. $-2x + 7y = -14$

x-intercept: y-intercept:



GO

Topic: Adding and multiplying fractions

Add. Reduce your answers but leave as improper fractions where applicable.

9. $\frac{3}{4}+\frac{1}{8}$

10. $\frac{3}{5}+\frac{7}{10}$

11. $\frac{2}{3}+\frac{1}{4}$

12. $\frac{4}{7}+\frac{8}{21}$

Multiply. Reduce your answers but leave as improper fractions where applicable.

13. $\frac{3}{4}\times\frac{2}{9}$

14. $\frac{4}{7}\times\frac{7}{10}$

15. $\frac{5}{4}\times\frac{2}{9}$

16. $\frac{3}{7}\times\frac{8}{21}$

5. 4 Pampering and Feeding Time

A Practice Understanding Task

Carlos and Clarita have been worried about space and start-up costs for their pet sitters business, but they realize they also have a limit on the amount of time they have for taking care of the animals they board. To keep things fair, they have agreed on the following time constraints.

- *Feeding Time*: Carlos and Clarita estimate that cats will require 6 minutes twice a day—morning and evening—to feed and clean their litter boxes, for a total of 12 minutes per day for each cat. Dogs will require 10 minutes twice a day to feed and walk, for a total of 20 minutes per day for each dog. Carlos can spend up to 8 hours each day for the morning and evening feedings, but needs the middle of the day off for baseball practice and games.

- *Pampering Time*: The twins plan to spend 16 minutes each day brushing and petting each cat, and 20 minutes each day bathing or playing with each dog. Clarita needs time off in the morning for swim team and evening for her art class, but she can spend up to 8 hours during the middle of the day to pamper and play with the pets.

Write inequalities for each of these additional time constraints. Shade the solution set for each constraint on separate coordinate grids.

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Writing linear equations in standard form and slope-intercept form.

Rewrite the given equation so that they are in slope-intercept form. $(\boldsymbol{y = mx + b})$

1. $7x - 14y = -56$
2. $-8x - 2y = 6$
3. $15x + 9y = 45$

Rewrite the given equations so that they are in standard form.
(Ax + By = C, where A, B, and C are whole numbers and A is positive.)

4. $y = 7x - 3$
5. $y = 2x + 9$
6. $y = -4x - 11$
7. $y = \frac{1}{2}x + 8$
8. $y = \frac{3}{5}x - 2$
9. $y = -\frac{1}{6}x + \frac{2}{3}$

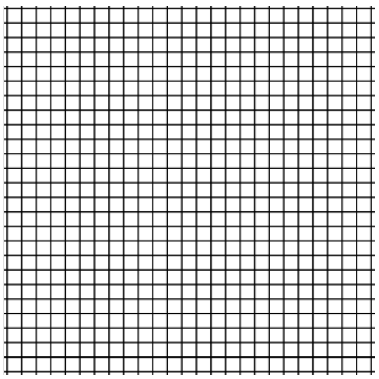
SET

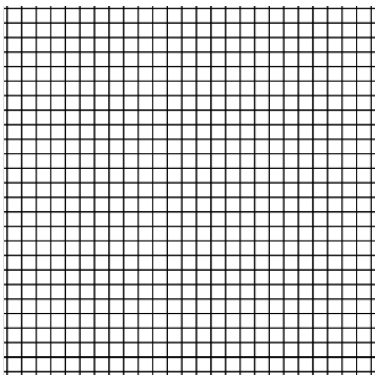
Topic: Writing inequalities from a real world problem. Graphing inequalities.

10. On a final for a creative writing course, Ben was required to write a combination of at least 10 poems or paragraphs. Ben knew that each poem would take him 30 minutes to write while a paragraph would only take 10 minutes. Ben was given two hours to complete the exam.

    a. Write an inequality to model each constraint. (Hint: One constraint is time and the other is the number of needed items. Let x be the number of poems written and y be the number of paragraphs written.)

    b. Graph each inequality on a separate coordinate grid and shade the solution set for each.





GO

Topic: Substituting a value to check if it's a solution

Determine whether $h = 3$ is a solution to each problem.

11. $3(h-4) = -3$

12. $3h = 2(h+2) - 1$

13. $2h - 3 = h + 6$

14. $3h > -3$

15. $\frac{3}{5} \leq h \times \frac{1}{5}$

16. $\frac{3}{5} > h \times \frac{1}{6}$

Determine the value of x that makes each equation true.

17. $4x - 2 = 8$

18. $3(x+5) = 20$

19. $2x + 3 = 2x - 5$

20. $4(6x-1) = 3(8x+5) - 19$

5. 5 All For One, One For All

A Solidify Understanding Task

Carlos and Clarita have found a way to represent combinations of cats and dogs that satisfy each of their individual "Pet Sitter" constraints, but they realize that they need to find combinations that satisfy all of the constraints simultaneously. Why?

1. Begin by listing the **system of inequalities** you have written to represent the *start-up costs* and *space* "Pet Sitter" constraints.

2. Find at least 5 combinations of cats and dogs that would satisfy both of the constraints represented by this system of inequalities. How do you know these combinations work?

3. Find at least 5 combinations of cats and dogs that would satisfy one of the constraints, but not the other. For each combination, explain how you know it works for one of the inequalities, but not for the other?

4. Shade a region on the coordinate grid that would represent the **solution set to the system of inequalities.** Explain how you found the region to shade.

5. Rewrite your systems of inequalities to include the additional constraints for *feeding time* and *pampering time*.

6. Find at least 5 combinations of cats and dogs that would satisfy all of the constraints represented by this new system of inequalities. How do you know these combinations work?

7. Find at least 5 combinations of cats and dogs that would satisfy some of the constraints, but not all of them. For each combination, explain how you know it works for some inequalities, but not for others?

8. Shade a region of the coordinate grid that would represent the solution set to the system of inequalities consisting of all 4 "Pet Sitter" constraints. Explain how you found the region to shade.

9. Shade a region in quadrant 1 of the coordinate grid that would represent all possible combinations of cats and dogs that satisfy the 4 "Pet Sitter" constraints. This set of points is referred to as the **feasible region** since Carlos and Clarita can feasibly board any of the combinations of cats and dogs represented by the points in this region without exceeding any of their constraints on time, money or space.

10. How is the feasible region shaded in #9 different from the solution set to the system of inequalities shaded in #8?

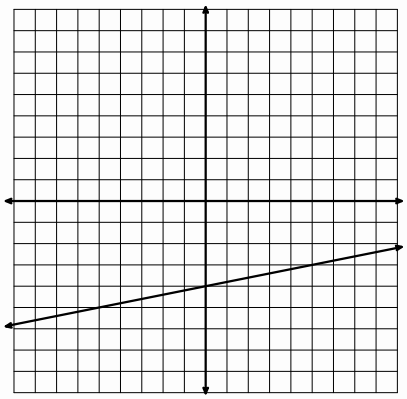
READY, SET, GO! **Name** **Period** **Date**

READY

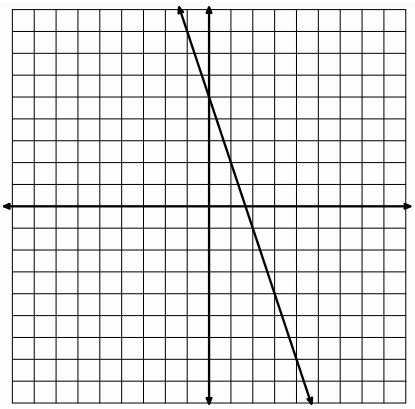
Topic: Graphing two variable inequalities.

For each inequality and graph, pick a point and use it to determine which half-plane should be shaded; then shade the correct half-plane.

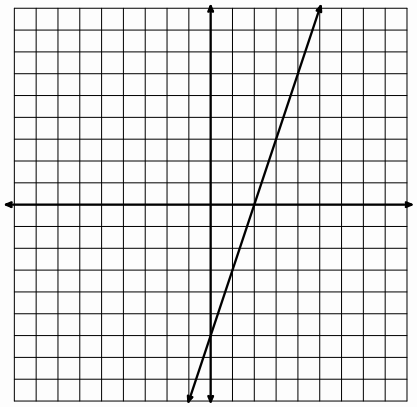
1. $y \leq \frac{1}{5}x - 4$



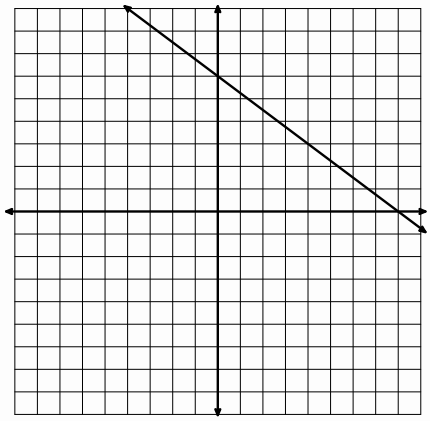
2. $y \geq -3x + 5$



3. $5x - 2y \leq 10$



4. $3x + 4y \geq 24$

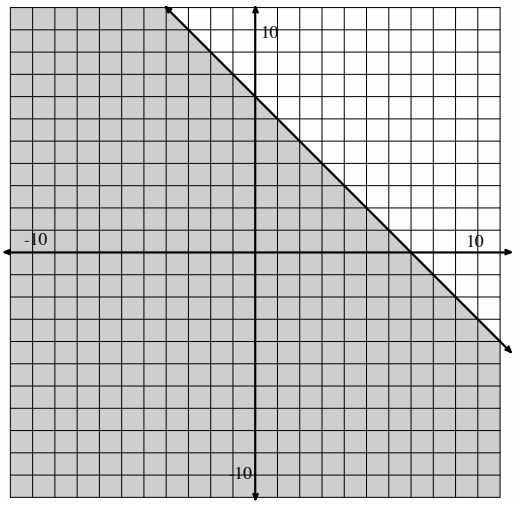


SET

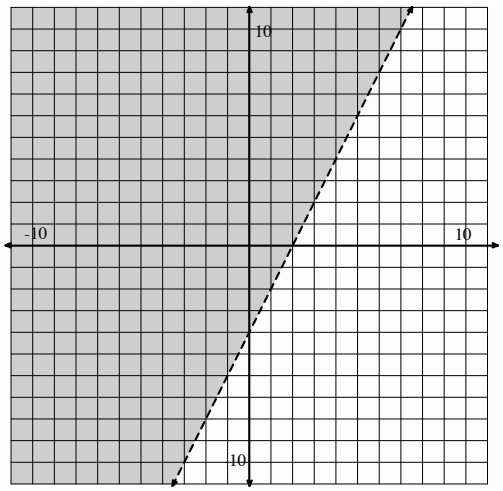
Topic: Writing two variable inequalities

Use the graph to write the inequality that represents the shaded region.

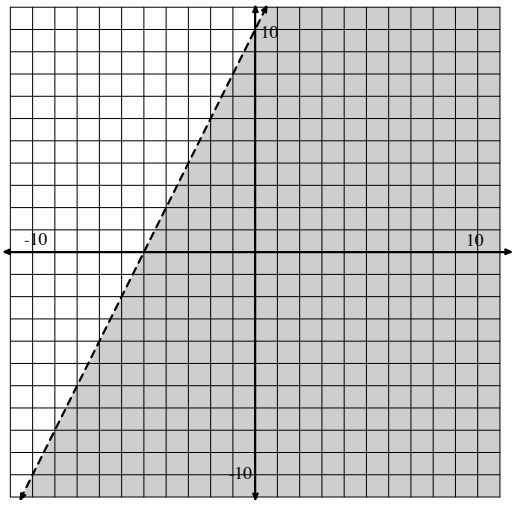
5.



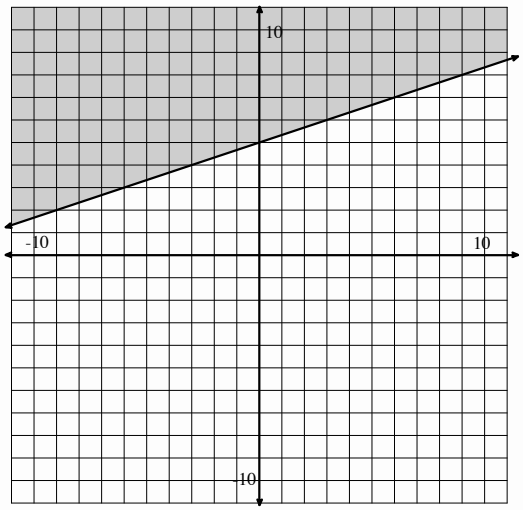
6.



7.



8.



GO

Topic: Proportional relationships

For each proportional relationship below, one representation is provided. Create the remaining representations and explain any connections you notice between representations.

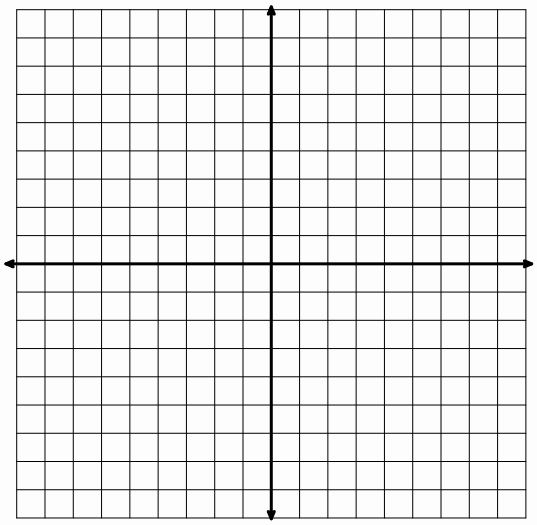
9. **Equation**:

Table

Days	Cost
1	8
2	16
3	24
4	32

Create a context

Graph



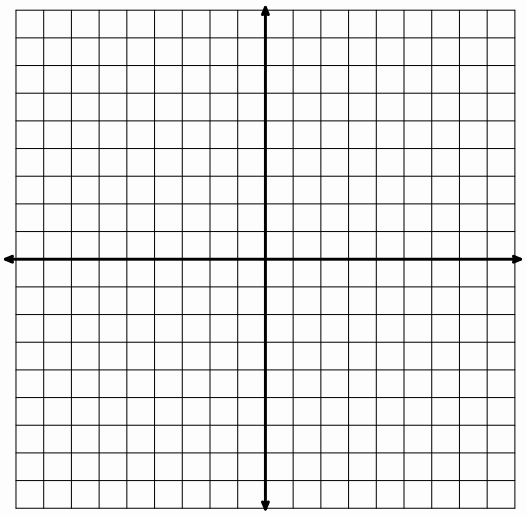
10. **Equation**:

Table

Create a context

Claire earns $9 per week allowance.

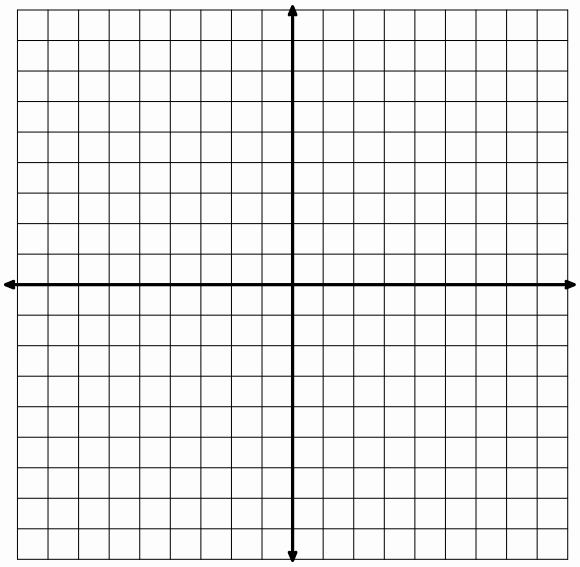
Graph



11. **Equation**: $y = 3x$

Table

Graph



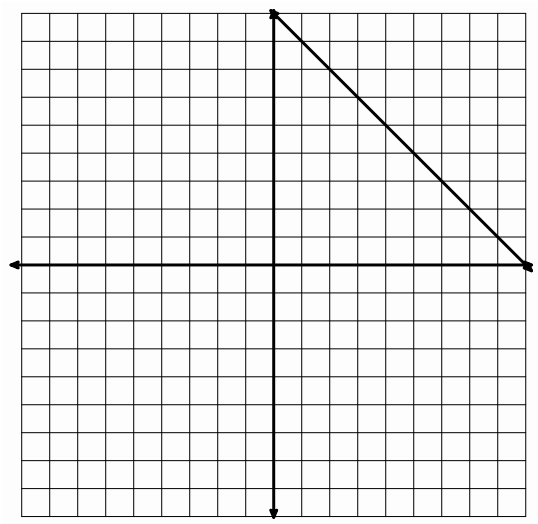
Create a Context

12. **Equation**:

Table

Seconds	Sand

Graph



Create a Context

5. 6 More or Less

A Practice Understanding Task

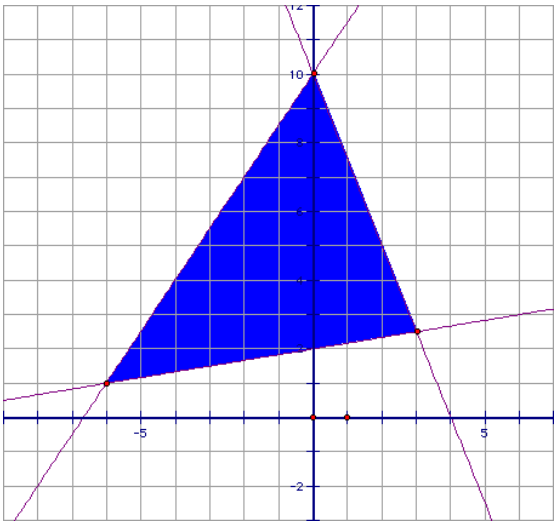
Solve the following systems of inequalities:

1. $\begin{cases} -5x + 3y \le 45 \\ 2x + 3y > 24 \end{cases}$

2. $\begin{cases} -10x + 6y \le 90 \\ 6x + 9y > 36 \end{cases}$

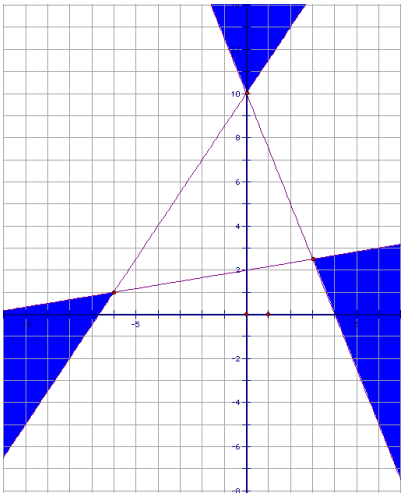
3. Is the point (-3, 10) a solution to the system in problem #1? Why or why not?

4. Write the system of inequalities whose solution set is shown below:



5. Amanda is examining Frank's work on #4, when she exclaims, "You have written all of your inequalities backwards. The solution set to your system would look like this."

What do you think about Amanda's statement?



Carlos and Clarita have found a cat food that seems to appeal to even the most finicky of cats, *Figaro Flakes.* They want to mix it with a less expensive cat food, *Tabitha Tidbits,* to make an affordable, but tasty cat food.

> *Tabitha Tidbits* contains 4 grams of protein and 6 grams of fat per scoop. *Figaro Flakes* contains 12 grams of protein and 4 grams of fat per scoop. Carlos wants to make a meal for cats that contains at least 8 grams of protein and no more than 6 grams of fat per scoop.

6. Write and solve a system of inequalities that Carlos can use to determine possible combinations of *Tabitha Tidbits* and *Figaro Flakes* that will satisfy both of these constraints.

7. Based on your work, suggest at least 3 different "recipes" using each type of cat food that meets Carlos' nutritional goals. For example, would 1 scoop of *Tabitha Tidbits* and ½ scoop of *Figaro Flakes* be an acceptable meal?

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Using substitution to find a missing value.

Substitute the given value of x into the equation to find the value of y.

1. $5x - 9y = 73; x = 2$
2. $-4x + 9y = 16; x = 5$
3. $3x - 8y = 1; x = -5$
4. $-14x + 5y = 51; x = 1$
5. $9x - 7y = 21; x = 0$
6. $12x - 15y = -42;\ x = \frac{1}{4}$

Use the given value to find the value of the other variable that is not provided.

7. $5a + 2b = -37$

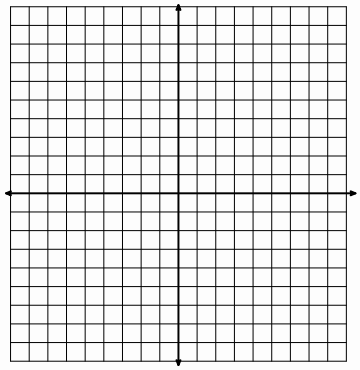
   $b = -1$
8. $13f - 7g = 10$

   $f = -3$
9. $2m + 3z = -22$

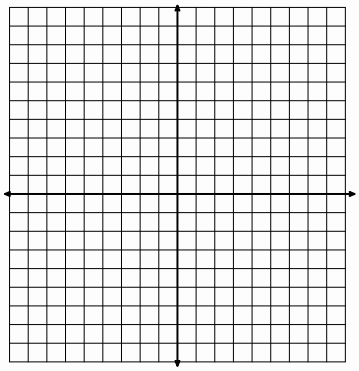
   $z = -9$

SET

Topic: Examining the impact of the direction of the inequality symbol



10. Graph $y > \frac{3}{4}x - 2$ and $y < \frac{3}{4}x + 3$ on the grid at the right.
11. What is the relationship between the two lines in your graph?
12. Name 3 points that satisfy both inequalities.
13. Now, graph $y < \frac{3}{4}x - 2$ and $y > \frac{3}{4}x + 3$ on the next grid at the right.



14. Can you name 3 points that satisfy both inequalities for this system?
15. Compare the graph for problem 10 with the graph for problem 13. How are they the same?

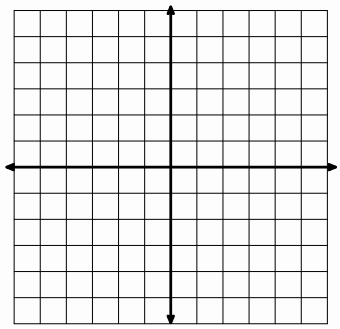
    How are they different?

GO

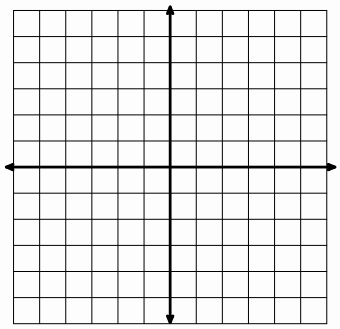
Topic: Graphing linear inequalities

Graph each inequality.

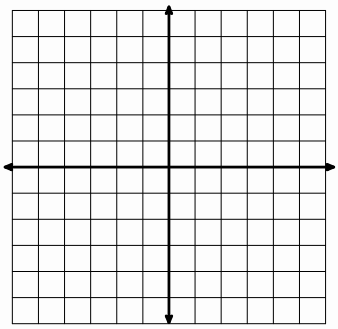
16. $y \leq 3x - 4$



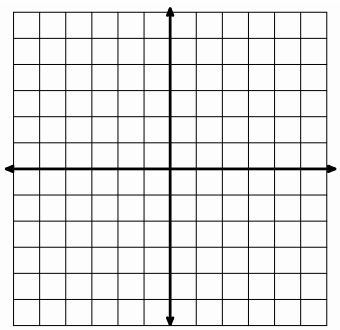
17. $y \leq -2x + 3$



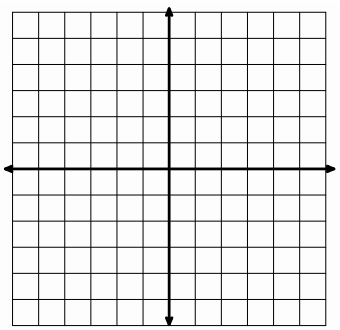
18. $y > 4x - 3$



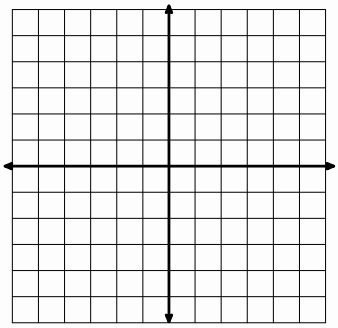
19. $3x + 4y < 12$



20. $6x + 8y \leq 24$



21. $5x + 3y \leq 15$



5. 7 Get to the Point

A Solidify Understanding Task

Carlos and Clarita need to clean the storage shed where they plan to board the pets. They have decided to hire a company to clean the windows. After collecting the following information, they have come to you for help deciding which window cleaning company they should hire.

- *Sunshine Express Window Cleaners* charges $50 for each service call, plus $10 per window.
- "*Pane"less Window Cleaners* charges $25 for each service call, plus $15 per window.

1. Which company would you recommend, and why? Prepare an argument to convince Carlos and Clarita that your recommendation is reasonable. (It is always more convincing if you can support your claim in multiple ways. How might you support your recommendation using a table? A graph? Algebra?)

Your presentation to Carlos reminds him of something he has been thinking about—how to find the coordinates of the points where the boundary lines in the "Pet Sitter" constraints intersect. He would like to do this algebraically since he thinks guessing the coordinates from a graph might be less accurate.

2. Write equations for the following two constraints.

   - *Space*
   - *Start-up Costs*

   Find where the two lines intersect algebraically. Record enough steps so that someone else can follow your strategy.

3. Now find the point of intersection for the two time constraints.

   - *Feeding Time*
   - *Pampering Time*

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Pythagorean theorem

An easy way to check if a triangle contains a 90° angle (also called a right triangle) is to use the Pythagorean theorem. You may remember the theorem as $a^2 + b^2 = c^2$, where c is the length of the longest side (the hypotenuse) and a and b are the lengths of the two shorter sides.

Identify which lengths make a right triangle. Example: Given 5, 12, 13
Replace $a, b,$ and c with the numbers $(5^2 + 12^2 = 13^2) \rightarrow (25 + 144 = 169) \rightarrow (169 = 169)$
Since 169 = 169, a triangle with side lengths of 5, 12, and 13 must be a right triangle.

Do these numbers represent the sides of a right triangle? Write YES in the boxes that apply.

1. 9, 40, 41	2. 3, 4, 5	3. 6, 7, 8	4. 20, 21, 29
5. 9, 12, 15	6. 10, 11, 15	7. 6, 8, 10	8. 8, 15, 17

SET

Topic: Solving systems of equations using substitution.

Solve each system of equations using substitution. Check your solution in both equations.

In this problem, substitute $(x + 1)$ in place of y in the second equation.

9. $\begin{cases} y = x + 1 \\ x + 2y = 8 \end{cases}$

In this problem, substitute $(3 + y)$ in place of x in the first equation.

10. $\begin{cases} y + 2x = 7 \\ x = 3 + y \end{cases}$

11. $\begin{cases} x = 9 + 2y \\ 3x + 5y = 20 \end{cases}$

12. $\begin{cases} y = 2x - 4 \\ 3y + 21x = 15 \end{cases}$

13. $\begin{cases} x = -1 - 2y \\ 3x + 5y = -1 \end{cases}$

14. $\begin{cases} y = 2x - 3 \\ x + y = -5 \end{cases}$

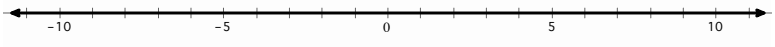
15. Tickets to a concert cost \$10 in advance and \$15 at the door. If 120 tickets were sold for a total of \$1390, how many of the tickets were purchased in advance?

GO

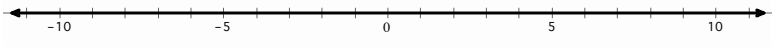
Topic: Solving one variable inequalities

Solve the following inequalities. Write the solution set in *interval notation* and graph the solution set on a number line.

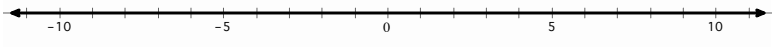
16. $4x + 10 < 2x + 14$



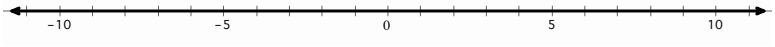
17. $2x + 6 > 55 - 5x$



18. $2(\frac{x}{4} + 3) > 6(x - 1)$



19. $9x + 4 \leq -2(x + \frac{1}{2})$



Solve each inequality. Give the solution in *set builder* notation (e.g. $\{x \in \mathbb{R} | x < 2\}$).

20. $-\frac{x}{3} > -\frac{10}{9}$

21. $5x > 8x + 27$

22. $\frac{x}{4} > \frac{5}{4}$

23. $3x - 7 \geq 3(x - 7)$

24. $2x < 7x - 36$

25. $5 - x < 9 + x$

5.8 Shopping for Cats and Dogs

A Develop Understanding Task

Clarita is upset with Carlos because he has been buying cat and dog food without recording the price of each type of food in their accounting records. Instead, Carlos has just recorded the total price of each purchase, even though the total cost includes more that one type of food. Carlos is now trying to figure out the price of each type of food by reviewing some recent purchases.

As Carlos is examining the first set of purchases he realizes that he can figure out the cost of the individual items just by reasoning about the numbers and the assumption that the price of each item remained the same for each shopping trip. Clarita is surprised that Carlos can find the individual prices without using tables, graphs or algebra.

See if you can reason about these shopping scenarios as well as Carlos by figuring out the cost of each item purchased, without using tables, graphs or equations.

1. One week Carlos bought 3 bags of *Tabitha Tidbits* and 4 bags of *Figaro Flakes* for $43.00. The next week he bought 3 bags of *Tabitha Tidbits* and 6 bags of *Figaro Flakes* for $54.00. Based on this information, figure out the price of one bag of each type of cat food. Explain your reasoning.

2. One week Carlos bought 2 bags of *Brutus Bites* and 3 bags of *Lucky Licks* for $42.50. The next week he bought 5 bags of *Brutus Bites* and 6 bags of *Lucky Licks* for $94.25. Based on this information, figure out the price of one bag of each type of dog food. Explain your reasoning.

3. Carlos purchased 6 dog leashes and 6 cat brushes for $45.00 for Clarita to use while pampering the pets. Later in the summer he purchased 3 additional dog leashes and 2 cat brushes for $19.00. Based on this information, figure out the price of each item. Explain your reasoning.

4. One week Carlos bought 2 packages of dog bones and 4 packages of cat treats for $18.50. Because the finicky cats didn't like the cat treats, the next week Carlos returned 3 unopened packages of cat treats and bought 2 more packages of dog bones. After being refunded for the cat treats, Carlos only had to pay $1.00 for his purchase. Based on this information, figure out the price of each item. Explain your reasoning.

5. Carlos has noticed that because each of his purchases have been somewhat similar, it has been easy to figure out the cost of each item. However, his last set of receipts has him puzzled. One week he tried out cheaper brands of cat and dog food. On Monday he purchased 3 small bags of cat food and 5 small bags of dog food for $22.75. Because he went through the small bags quite quickly, he had to return to the store on Thursday to buy 2 more small bags of cat food and 3 more small bags of dog food, which cost him $14.25. Based on this information, figure out the price of each bag of the cheaper cat and dog food. Explain your reasoning.

Summarize the strategies you have used to reason about the price of individual items in the problems given above. What are some key ideas that seem helpful?

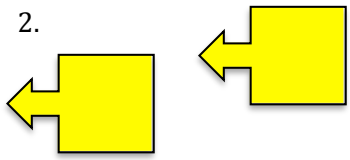
READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Transformations

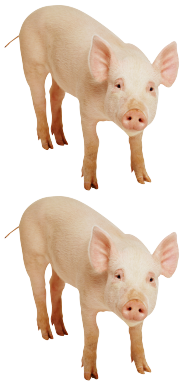
Each set of pictures below shows a transformation from a "pre-image" to an "image." The word *transformation* in mathematics refers to how a figure can be moved. You might know transformations as "slide, flip, or turn" or by their formal names. Identify the transformation between the two figures by writing *slide, flip,* or *turn* next to the pair of images.

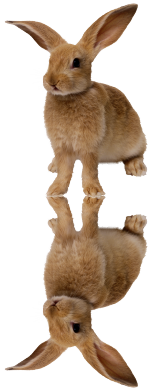
1. 

2. 

3. 

4. 

5. 

6. 

SET

Topic: Using equivalence to solve systems of equations

7. Mike and Gavin visited the amusement park with their science class on a school physics day. Their tickets were stamped each time they went on a ride. Mike and Gavin spent the entire afternoon going on their two favorite rides, the Rollercoaster and the Gravity free-fall. At the end of the day their tickets were stamped as shown.

   What was the cost to ride on the Rollercoaster?

   What was the cost to ride on the Gravity free-fall?

Physics Day Rides

Rollercoaster	√ √ √
Gravity free-fall	√ √ √
Total	$36.00

Mike's ticket

Physics Day Rides

Rollercoaster	√ √
Gravity free-fall	√ √ √
Total	$29.50

Gavin's ticket

8. Mallory and Meg also attended the physics day with their school. Their favorite rides were Splash Mountain and the Centrifugal Chamber. Here are their stamped tickets at the end of the day. Find the cost of each ride.

Mallory's ticket

Physics Day Rides

Splash Mtn. √ √ √ √

Centrifugal Ch. √ √

Total $39.50

Meg's ticket

Physics Day Rides

Splash Mtn. √ √ √

Centrifugal Ch. √ √ √ √

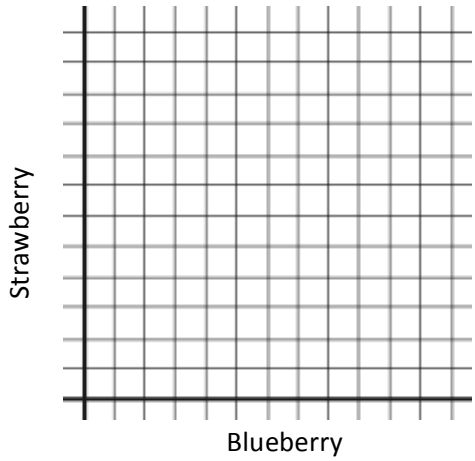
Total $42.75

GO

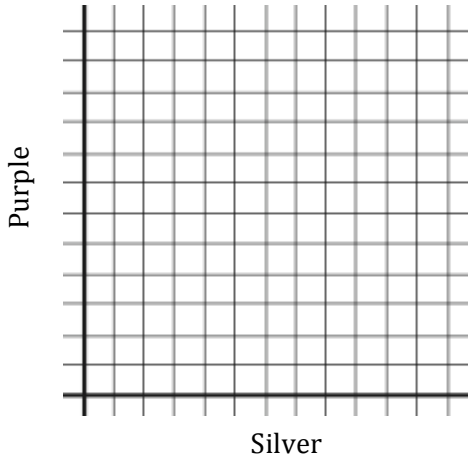
Topic: Graphing two variable, linear inequalities

<u>Write</u> an inequality to describe the given context. <u>Graph</u> each inequality on the provided grid. <u>Then make a table that shows at least 3 possible combinations that will work for each situation.</u>

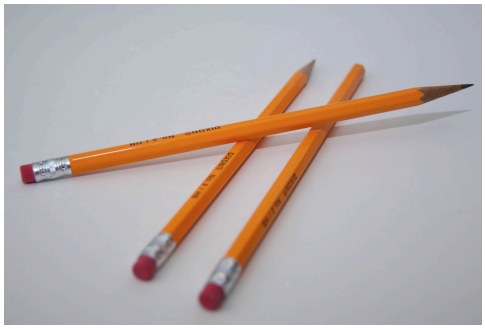
9. Dion has enough money to buy up to eight yogurts. His favorite flavors are blueberry and strawberry.



10. Shaniqua is buying a balloon bouquet. Her favorite colors are silver and purple. The silver balloons cost $1.00 and the purple balloons cost $0.80. How many of each color of balloon can she put in her bouquet if she doesn't spend more than $20 on her bouquet?



5.9 Can You Get to the Point, Too?



A Solidify Understanding Task

Part 1

In "Shopping for Cats and Dogs," Carlos found a way to find the cost of individual items when given the purchase price of two different combinations of those items. He would like to make his strategy more efficient by writing it out using symbols and algebra. Help him formalize his strategy by doing the following:

- For each scenario in "Shopping for Cats and Dogs" write a **system of equations** to represent the two purchases.
- Show how your strategies for finding the cost of individual items could be represented by manipulating the equations in the system. Write out intermediate steps symbolically, so that someone else could follow your work.
- Once you find the price of one of the items in the combination, show how you would find the price of the other item.

Part 2

Writing out each system of equations reminded Carlos of his work with solving systems of equations graphically. Show how the following scenario from "Shopping for Cats and Dogs" can be represented graphically, and how the cost of each item shows up in the graphs.

> *Carlos purchased 6 dog leashes and 6 cat brushes for $45.00 for Clarita to use while pampering the pets. Later in the summer he purchased 3 additional dog leashes and 2 cat brushes for $19.00. Based on this information, figure out the price of each item.*

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Matching definitions of geometric figures.

Match the name of the figure with its geometric definition.

a. isosceles triangle	b. equilateral triangle	c. scalene triangle	d. right triangle
e. rectangle	f. rhombus	g. square	h. trapezoid

1. _____ A quadrilateral with only one pair of parallel sides.
2. _____ All of the sides of this triangle are the same length.
3. _____ All of the sides of this quadrilateral are the same length.
4. _____ This triangle has exactly one right angle.
5. _____ This quadrilateral has four right angles.
6. _____ None of the sides of this triangle are the same length.
7. _____ This quadrilateral is both #3 and #5.
8. _____ Only two sides of this triangle are the same length.

SET

Topic: Solving systems of equations by elimination

Solve each system of equations using *elimination of a variable.* Check your solution.

9. $\begin{cases} 2x + y = 3 \\ 2x + 2y = 2 \end{cases}$

10. $\begin{cases} 2x + 5y = 3 \\ x + 5y = 6 \end{cases}$

11. $\begin{cases} 2x + 0.5y = 3 \\ x + 2y = 8.5 \end{cases}$

12. $\begin{cases} 3x + 5y = -1 \\ x + 2y = -1 \end{cases}$

13. $\begin{cases} 3x + 5y = -3 \\ x + 2y = -\frac{4}{3} \end{cases}$

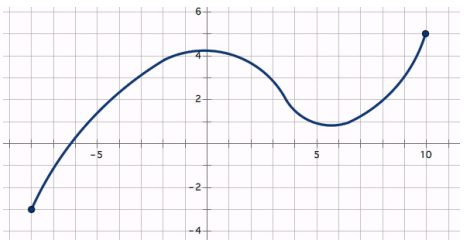
14. A 150-yard pipe is cut to provide drainage for two fields. If the length of one piece (a) is three yards less than twice the length of the second piece (b), what are the lengths of the two pieces?

GO

Topic: Identifying functions

For each graph determine if the relationship represents a function.
If it is a function, write yes. If it is not a function, explain why it is not.

15.



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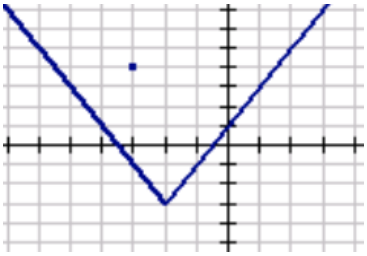

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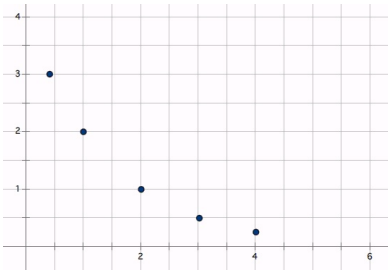

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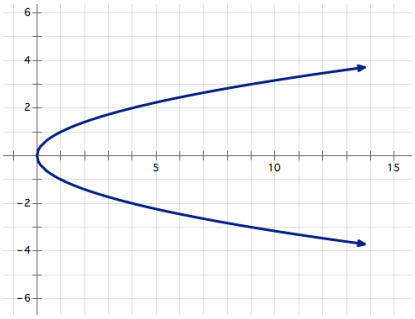

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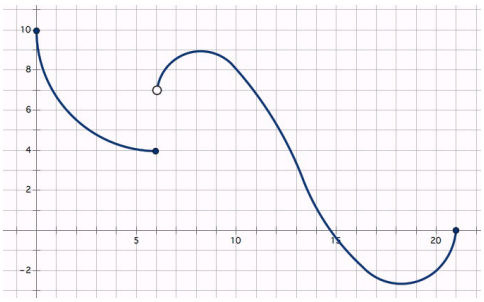

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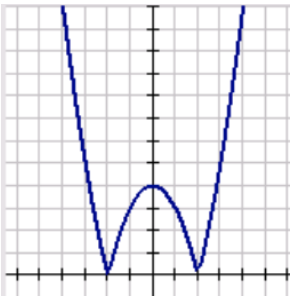


5.10 Taken Out of Context

A Practice Understanding Task

Write a shopping scenario similar to those in "Shopping for Cats and Dogs" to fit each of the following systems of equations. Then use the elimination of variables method you invented in "Can You Get to the Point, Too" to solve the system. Some of the systems may have interesting or unusual solutions. See if you can explain them in terms of the shopping scenarios you wrote.

1. $\begin{cases} 3x+4y=23 \\ 5x+3y=31 \end{cases}$

2. $\begin{cases} 2x+3y=14 \\ 4x+6y=28 \end{cases}$

3. $\begin{cases} 3x+2y=20 \\ 9x+6y=35 \end{cases}$

4. $\begin{cases} 4x+2y=8 \\ 5x+3y=9 \end{cases}$

5. Three of Carlos and Clarita's friends are purchasing school supplies at the bookstore. Stan buys a notebook, three packages of pencils and two markers for \$7.50. Jan buys two notebooks, six packages of pencils and five markers for \$15.50. Fran buys a notebook, two packages of pencils and two markers for \$6.25. How much do each of these three items cost?

   Explain in words or with symbols how you can use your intuitive reasoning about these purchases to find the price of each item.

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: System of inequalities

For each of the systems of inequalities, determine if the given coordinates are solutions to the system. (Show your work.)

1. $\begin{cases} y \le 3x - 5 \\ y \ge x + 2 \end{cases}$	2. $\begin{cases} y > -2x + 9 \\ y \ge 5x - 6 \end{cases}$	3. $\begin{cases} y < -\frac{1}{2}x + 9 \\ y > 6x - 10 \end{cases}$
a. $(6, 10)$ b. $(1, 4)$ c. $(8, 15)$	a. $(-2, -5)$ b. $(-1, 12)$ c. $(5, 0)$	a. $(-2, -5)$ b. $(7, 3)$ c. $(-8, 10)$

SET

Topic: Determining the number of solutions in a system of equations

Write each equation in slope-intercept form. Based on slope-intercept form of the equations determine whether the system of equations has zero, one, or infinitely many solutions. How do you know?

4. $3x - 4y = 13$ $y = -3x - 7$	5. $3x - 3y = 3$ $x - y = 1$	6. $0.5x - y = 30$ $0.5x - y = -30$	7. $4x - 2y = -2$ $3x + 2y = -12$
How many solutions? How do you know?	How many solutions? How do you know?	How many solutions? How do you know?	How many solutions? How do you know?

Solve each system. Write your solution as an ordered pair or indicate if it has no solutions or infinitely many solutions.

8. $\begin{cases} \boldsymbol{x + 4y = 6} \\ \boldsymbol{x + y = 3} \end{cases}$

9. $\begin{cases} \boldsymbol{2x + y = 5} \\ \boldsymbol{y = x - 4} \end{cases}$

10. $\begin{cases} \boldsymbol{y = 2x + 1} \\ \boldsymbol{2x - y + 1 = 0} \end{cases}$

11. $\begin{cases} 4y - 5x = 9 \\ x - 4y = 11 \end{cases}$

12. $\begin{cases} y = x - 1 \\ -x + y = 4 \end{cases}$

13. $\begin{cases} -2x + 5y = -1 \\ 3x + 2y = 11 \end{cases}$

14. $\begin{cases} -3x + 4y = 12 \\ 2x + y = -8 \end{cases}$

15. $\begin{cases} 9x - 3y = 3 \\ 3x + 8y = -17 \end{cases}$

16. $\begin{cases} -7x + y = -2 \\ 7x - y - 2 = 0 \end{cases}$

17. $\begin{cases} 2y = x + 2 \\ -\frac{1}{2}x + y = 1 \end{cases}$

18. $\begin{cases} 2y = 2x - 2 \\ -\frac{1}{2}x + \frac{1}{2}y = 1 \end{cases}$

19. $\begin{cases} -2y = 4x + 2 \\ 8x - 4y = -4 \end{cases}$

20. $\begin{cases} x + y = 2x + 5 \\ x + y = 6y - 9 \end{cases}$

21. $\begin{cases} 5x = -y \\ 5x + 2y = 30 \end{cases}$

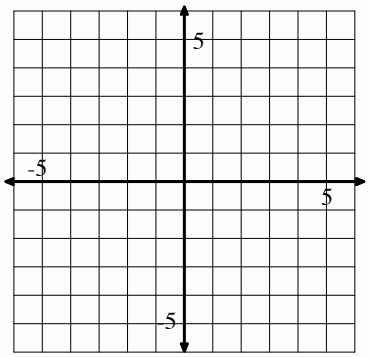
22. $\begin{cases} 3x + 8y = 9y - 6 \\ 9x - 3y = 3 \end{cases}$

GO

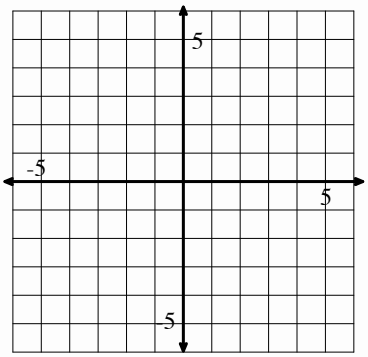
Topic: graphing two variable inequalities

Graph the following inequalities. Justify the region you shade by showing at least one point in the region as being a solution to each inequality.

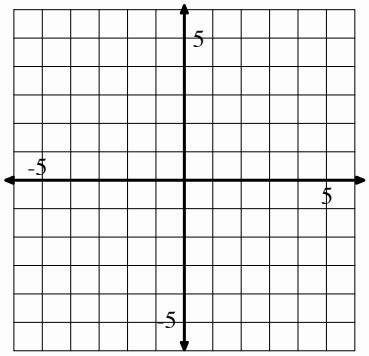
23. $3x - 4y \geq 12$



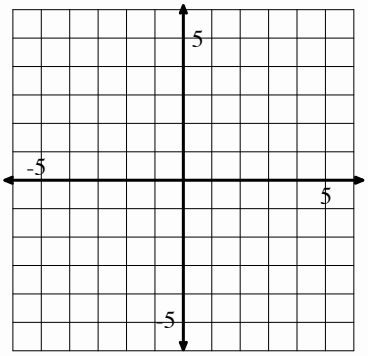
24. $x + 6y < 6$



25. $6x + 5y > 1$

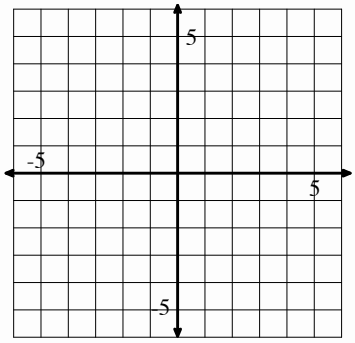


26. $x - \frac{1}{2}y \geq 3$



27. On the same set of axes graph $y < x + 2$ and $y > x + 5$.

Do the solution sets of these two inequalities share any points? Explain.



28. On the same set of axes graph $y < x + 2$ and $y < x + 5$.

Do the solution sets of these two inequalities share any points? Explain.