Secondary One Mathematics: An Integrated Approach
Get Ready Module

By

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www.mathematicsvisionproject.org

In partnership with the
Utah State Office of Education

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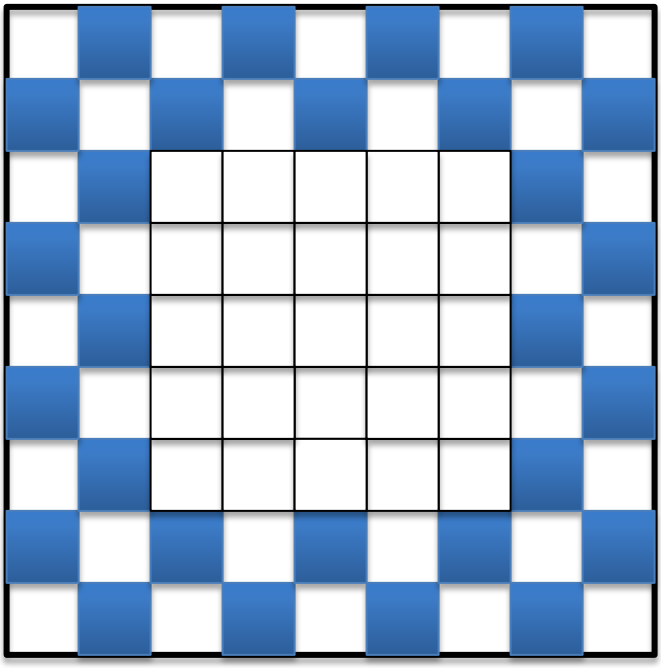
Checkerboard Borders

A Develop Understanding Task

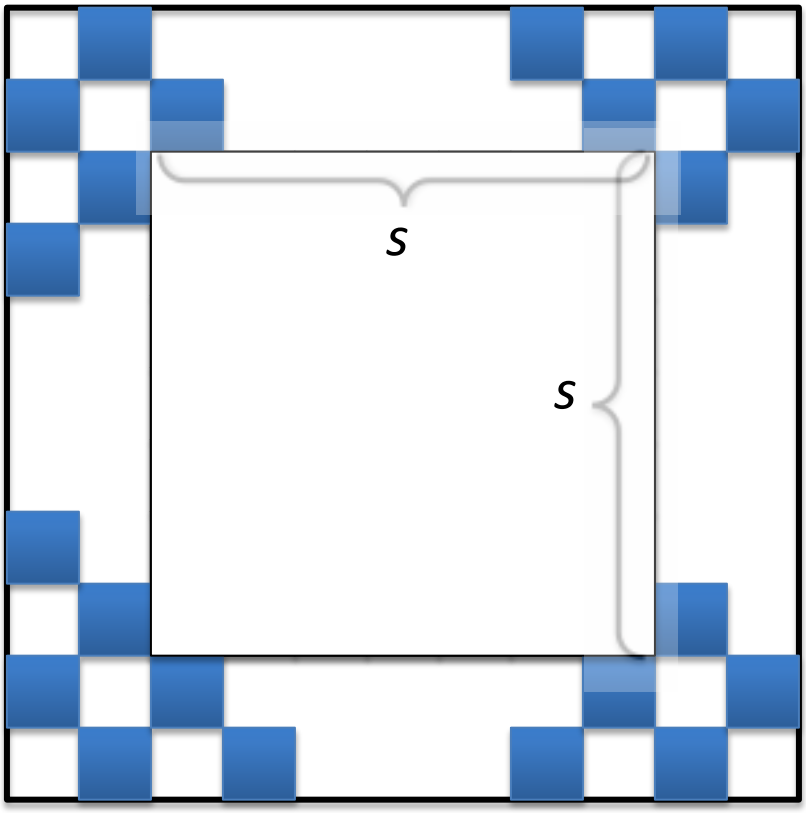
In preparation for back to school, the school administration has planned to replace the tile in the cafeteria. They would like to have a checkerboard pattern of tiles two rows wide as a surround for the tables and serving carts.

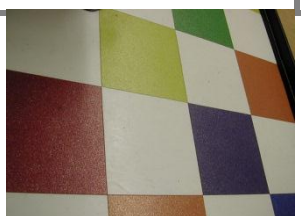
Below is an example of the border that the administration is thinking of using to surround a square 5 x 5 set of tiles.

A. Find the number of colored tiles in the checkerboard border. Track your thinking and find a way of calculating the number of color tiles in the border that is quick and efficient. Be prepared to share your strategy and justify your work.



B. The contractor that was hired to lay the tile in the cafeteria is trying to generalize a way to calculate the number of colored tiles needed for a checker board border surrounding a square of tiles with dimensions S x S. Find an expression for the number of colored border tiles needed for any S x S square center.





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Ready, Set, Go!

Ready

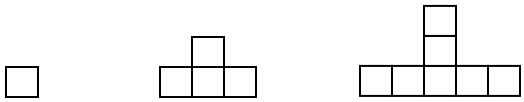
Topic: Solve one variable equations

Determine the value of x that makes each equation true.

1. $6x = 18$
2. $3x - 10 = 2$
3. $8x - 10 = x + 11$
4. $5x - 7 = 7x - 17$
5. $3x + 9 = 44 - 2x$
6. $3x + 6 = x + 2$

Set

Topic: Create and solve equations in one variable.



Step 1 Step 2 Step 3

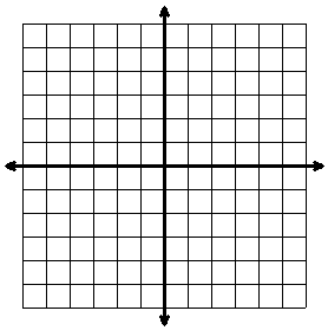
7. Each square represents one tile, how many total tiles are in Step 5? Step 6?
8. How can you determine the number of tiles in Step 25?
9. Write a rule to predict the total number of tiles for any step. Show how your rule relates to the pattern.
10. Try to think of a different rule that you can use to predict the total number of tiles for any step. Show how your rule relates to the pattern.
11. Andrew also solved this problem and came up with following equation: s = 1 + 3(n-1). How does each piece of his expression show up in the pattern?
12. Tami came up with the equation s = 3n – 2. How does each piece of her expression show up in the pattern?

Go

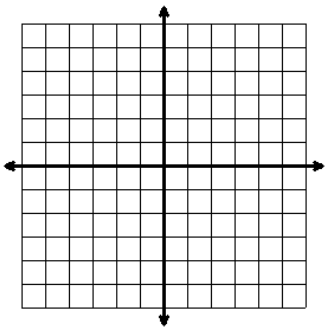
Topic: Graph linear equations

For the following problems, two points and a slope are given. Use the graph to plot these points, draw the line, and clearly label the slope.

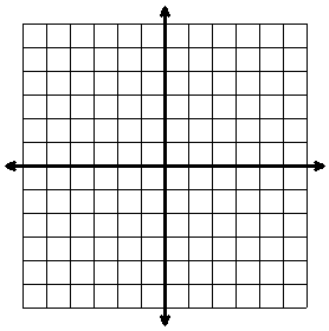
13. (2, -1) and (4, 2); m = $\frac{3}{2}$



14. (-2, 1) and (2, 5); m = 1

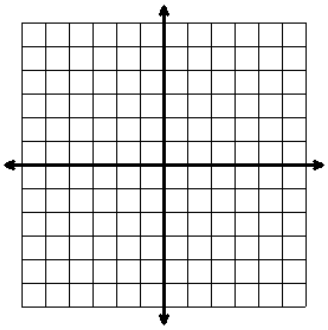


15. (0, 0) and (3, 6); m = 2

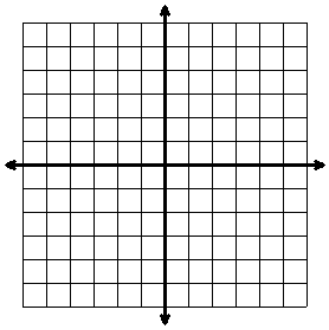


For the following problems, two points are given. Use the graph to plot these points and determine the slope.

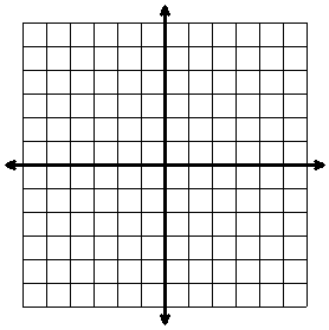
16. (-3, 0) and (0, 5); m =



17. (-2, -1) and (-4, 4); m =



18. (0, 3) and (1, 6); m =



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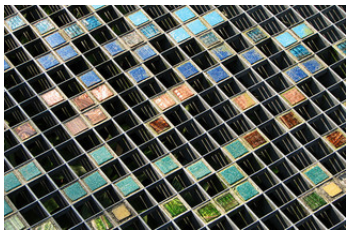
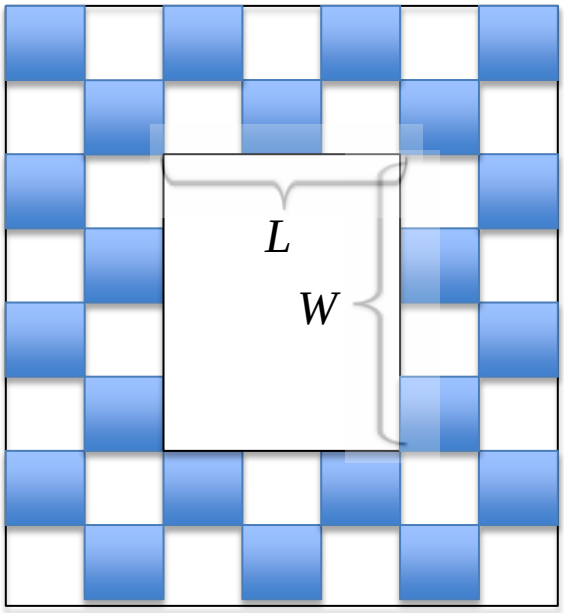


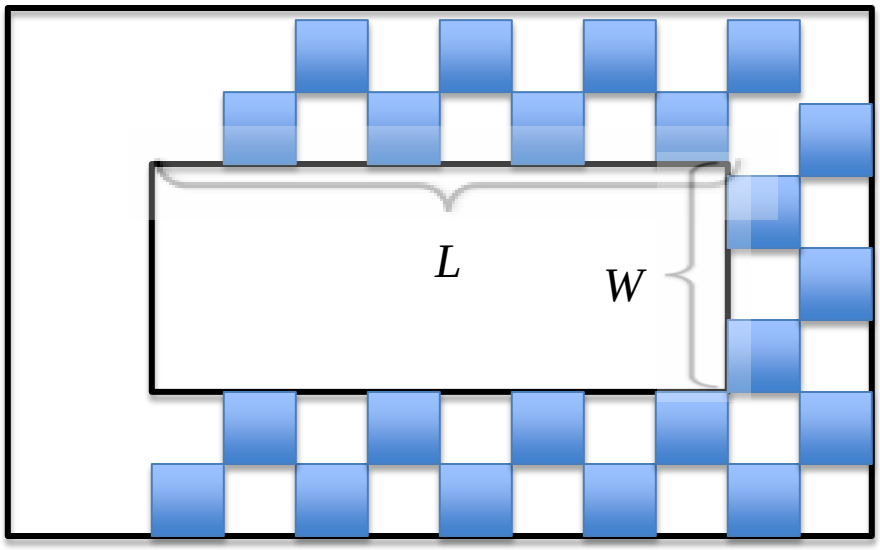
Building More Checkerboard Borders

A Develop Understanding Task

As the tile workers started to look more deeply into their work they found it necessary to develop a way to quickly calculate the number of colored border tiles for not just square arrangements but also for checkerboard borders to surround any *L* X *W* rectangular tile center.

Find an expression to calculate the number of colored tiles in the two row checkerboard border for any rectangle. Be prepared to share your strategy and justify your work. Create models to assist you in your work.





Ready, Set, Go!

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Ready

Topic: Solve one variable equations

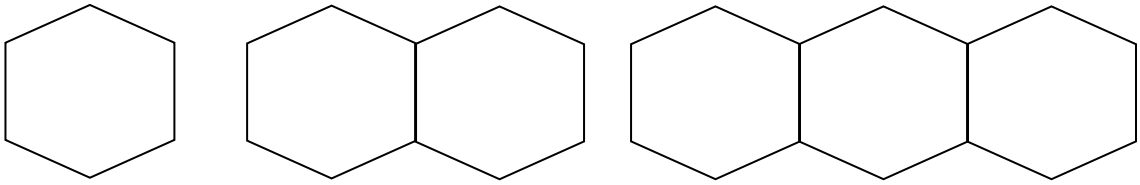
Solve the following equations for the unknown variable.

1. $4(x+3)=1$
2. $q-13=-13$
3. $21s=3$
4. $\frac{7f}{11}=\frac{7}{11}$
5. $5q-7=\frac{2}{3}$
6. $8x-(3x+2)=1$

Set

Topic: Create and solve equations in one variable.

For the growing pattern below, each line segment is one unit in length.



Step 1 Step 2 Step 3

7. How much total perimeter in Step 5? Step 6? (Remember to focus on the perimeter.)
8. How can you determine the amount of perimeter in Step 25?
9. Write a rule to predict the total amount of perimeter for any step. Show how your rule relates to the pattern.
10. Marsha also solved this problem and came up with following equation: p = 1 + 5n -(n-1).

    How does each piece of her expression show up in the pattern?

11. Tyler came up with the equation p = 6n – 2(n-1).

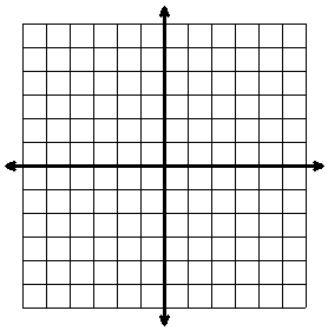
    How does each piece of his expression show up in the pattern?

Go

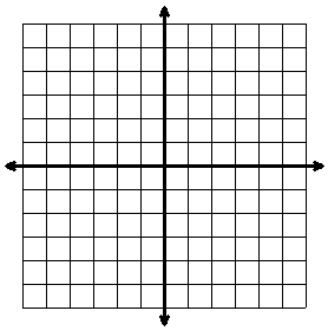
Topic: Graphing linear equations.

For problems 12 and 13, the y-intercept and the slope of a line are given. Graph the line on the coordinate axes, clearly labeling the slope and y-intercept.

12. (0, 2); m = $\frac{3}{4}$



13. (0, -3); m = 4

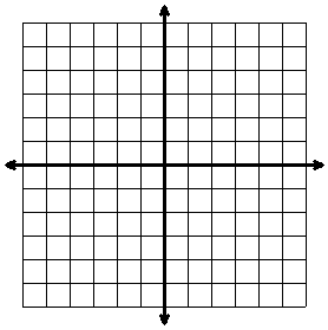


The equations below are represented in the above graphs. Explain how the slope and y-intercept show up in both the graphical and algebraic representations.

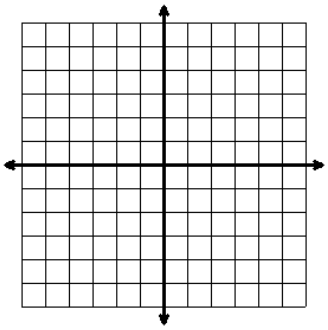
$$y = \frac{3}{4}x + 2 \qquad\qquad y = 4x - 3$$

For problems 3-5, graph the following equations on the provided coordinate axes.

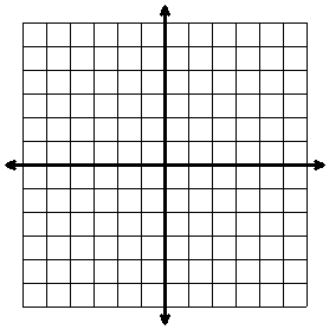
14. y = 2x-1



15. y = $\frac{1}{3}$ x +2



16. y = -3x + 5



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3. http://www.youtube.com/watch?v=WXzpisUh0AU

Serving Up Symbols

A Develop Understanding Task

As you look around your school cafeteria, you may see many things that could be counted or measured. To increase the efficiency of the cafeteria, the cafeteria manager, Elvira, decided to take a close look at the management of the cafeteria and think about all the components that affect the way the cafeteria runs. To make it easy, she assigned symbols for each count or measurement that she wanted to consider, and made the following table:

Symbol	Meaning
S	Number of students that buy lunch in the cafeteria each day
S_C	Average number of students per class that eat lunch each day
S_M	Number of students who have passed through a line in M minutes
C	Number of classes per lunch period
P	Number of lunch periods per day
B	Number of boys that buy lunch each day
G	Number of girls that buy lunch each day
F	Number of food servers in the cafeteria
T	Total number of food items in one lunch (Each entrée, side dish, or beverage counts as 1 item.)
M	Number of minutes passed since the beginning of the lunch period
N_e	Number of entrees in each lunch
N_s	Number of side dishes in each lunch
N_b	Number of beverages in each lunch
C_e	Cost of each entrée
C_s	Cost of each side dish
C_b	Cost of each beverage
L	Number of lines in the cafeteria
W	The number of food workers (servers) per line
i	Average number of food items that a worker can serve each minute (Each entrée, side dish, or beverage counts as 1 item.)
H	Number of hours each food worker works each day
P_L	Price per lunch

Using the given symbols, it is possible to write many different algebraic expressions.

1. Using these symbols, what would the expression $\frac{G+B}{C\times P}$ mean?

2. Using these symbols, what would the expression $S + F + L$ mean?

Elvira hopes to use the symbols in the chart to come up with some meaningful expressions that will allow her to analyze her cafeteria. Your job is to help her by writing as many expressions as you can and describe what they mean. Put each of your expressions in the following chart, adding lines if you need to:

Expression	Description

Write an expression for the average number of lunches served in a line each day.

Write an expression for the total price of the items served in a line.

Ready, Set, Go!

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Ready

Topic: Substitution

Evaluate the following expressions for $a = -3, b = 2, c = 5, and\ d = -4$.

1. $2a + 3b$

2. $4c + d$

3. $5ac - 2b$

4. $\frac{2a}{c-d}$

5. $\frac{3b}{d}$

6. $\frac{a-4b}{3c+2d}$

The weekly cost (c) of manufacturing remote controls (r) is given by the formula $c = 2000 + 3r$, where the cost is given in dollars.

7. What is the cost of producing 1000 remote controls?

8. What is the cost of producing 2000 remote controls?

9. What is the cost of producing 2500 remote controls?

Set

Topic: Solving one variable equations

Solve each equation, justifying each step you use.

10.

$3x = 15$	Justification

11.

$x - 10 = 2$	Justification

12.

$-16 = x + 11$	Justification

13.

$6 - x = 10$	Justification

14.

$6x + 3 = 15$	Justification

15.

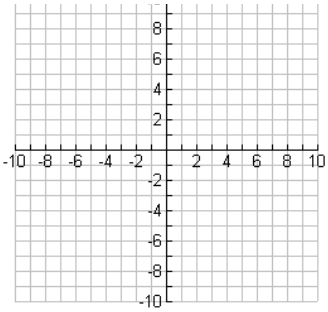
$3x - 10 = 2$	Justification

Go

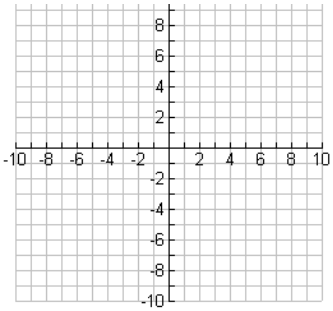
Topic: Graph linear equations

Graph the following equations on the provided coordinate axes.

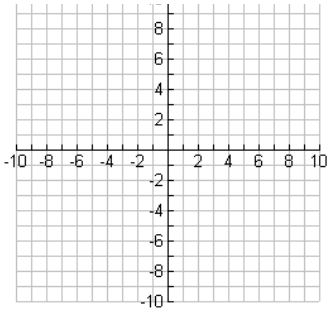
16. $y = \frac{-3}{5}x + 7$



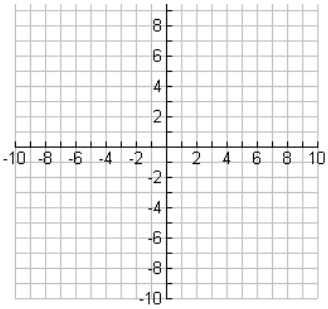
17. $y = -2x + 1$



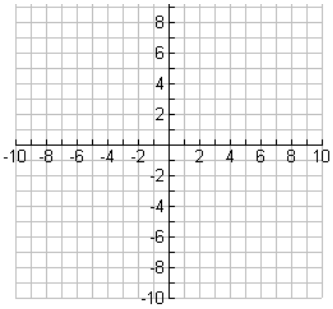
18. $y = \frac{5}{8}x + 1$



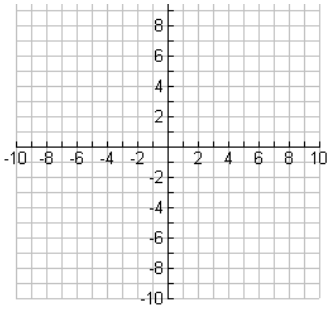
19. $y = \frac{6}{7}x$



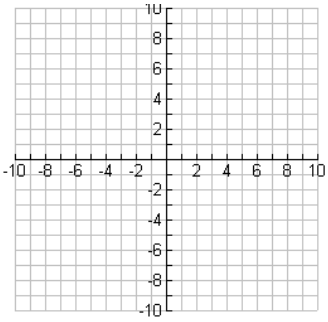
20. $y = x - 3$



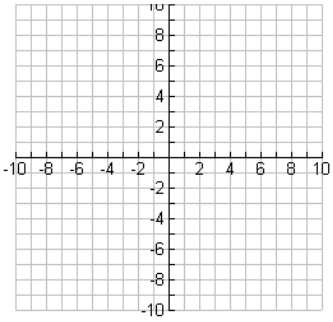
21. $y = 4x$



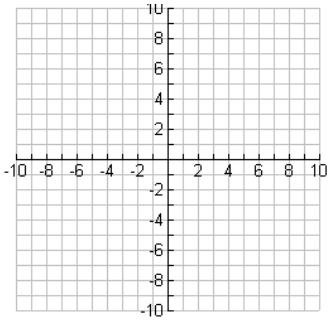
22. $y = -x - 6$



23. $y = 3x + 2$



24. $y = x$



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Examining Units

A Solidify Understanding Task

(Note: This task refers to the same set of variables as used in *Serving Up Symbols*)

Units in Addition and Subtraction

1. Why can you add $N_e + N_s + N_b$ and you can add $B + G$, but you can't add $M + W$?

2. We measure real-world quantities in units like feet, gallons, students and miles/hour (miles per hour).

   a. What units might you use to measure N_e, N_s and N_b?
   What about the sum $N_e + N_s + N_b$?

   b. What units might you use to measure B? G?
   What about the sum $B + G$?

   c. What units might you use to measure M? W?
   What about the sum $M + W$?

3. State a rule about how you might use units to help you think about what types of quantities can be added. How would you use or modify your rule to fit subtraction?

Units in Multiplication, scenario 1

1. Why can you multiply $N_e \times C_e$ and you can multiply $L \times W$, but you can't multiply $G \times C$?

2. Units in multiplication often involve rates like miles/gallon (miles per gallon), feet/second (feet per second), or students/table (students per table).

   a. What units might you use to measure N_e? C_e?
   What about the product $N_e \times C_e$?

b. What units might you use to measure L? W?
What about the product $L \times W$?

c. What units might you use to measure G? C?
What about the product $G \times C$?

3. State a rule about how you might use units to help you think about what types of quantities can be multiplied.

Units in Multiplication, scenario 2

1. Let l represent the length of the cafeteria in feet and w represent its width in feet. What does
$l + w + l + w$ represent? What about $l \times w$?

2. Why can we add $l + w$ <u>and</u> multiply $l \times w$? What is it about these variables that allow them to be added or multiplied?

3. How might you modify you rule for using units to guide your thinking when multiplying?

Units in Division, scenario 1

1. What are the units for the **dividend** (what you are dividing up), the **divisor** (what you are dividing by) and the **quotient** (the result of the division) in the following expressions:

a. $\frac{S}{P}$

b. $\frac{F}{L}$

c. $\frac{S}{F}$

d. $\frac{S_M}{M}$

2. State a rule about the units in division problems like those represented above.

Units in Division, scenario 2

1. What are the units for the dividend (what you are dividing up), the divisor (what you are dividing by) and the quotient (the result of the division) in the following expressions:

   a. $\frac{F}{W}$

   b. $\frac{P_L}{T}$

2. State a rule about the units in division problems like those represented above.

Getting Ready | 4

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Ready

Topic: Solve and justify one variable equations

Solve each equation, justifying each step you use.

1.

$8x - 10 = x + 11$	Justification

2.

$5p - 2 = 32$	Justification

3.

$10(y + 5) = 10$	Justification

4.

$3x + 9 = 44 - 2x$	Justification

Set

Topic: Understanding variables

5. Use the task Serving Up Symbols to complete the table below.

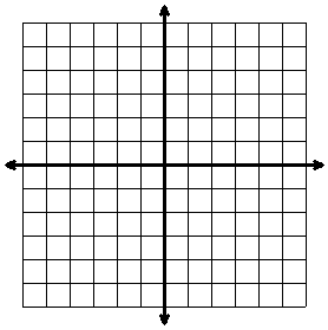
Expression	**Description**
$\frac{C}{L}$	
$\frac{C}{W}$	
$\frac{F}{L}$	
	Total beverages served in the cafeteria per day
	Average number of food items per week
	The average number of food items served per minute

Go

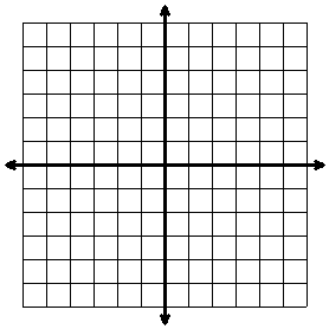
Topic: Graph linear equations

Graph each equation.

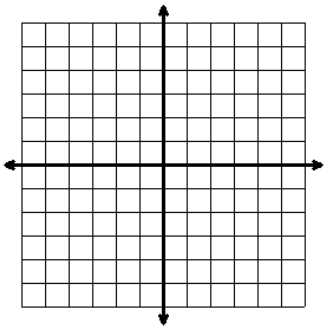
6. $y=3x+1$



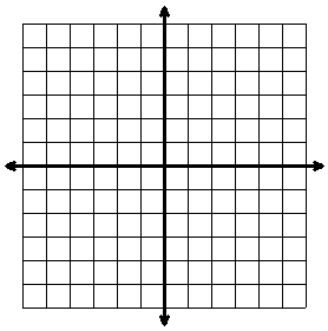
7. $y=-2x+3$



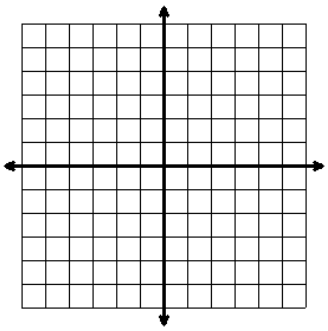
8. $y=\frac{1}{2}x-5$



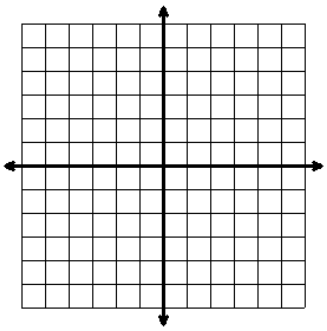
9. $y=-\frac{2}{3}x+2$



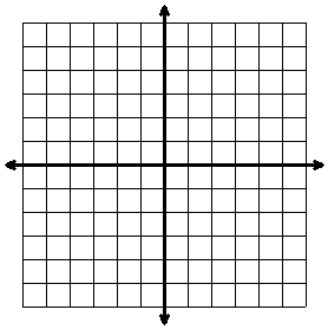
10. $y=2x-1$



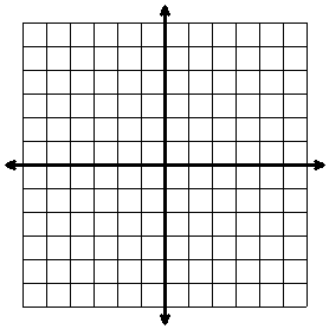
11. $y=-\frac{1}{2}+4$



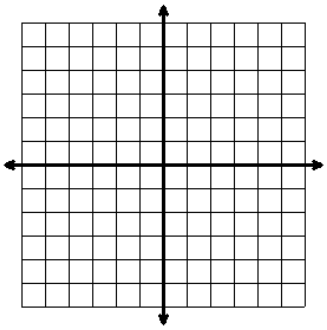
12. $y=4x+2$



13. $y = 2x$



14. $y=-3x+5$



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http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphing-a-line-in-slope-intercept-form

http://www.youtube.com/watch?v=WXzpisUh0AU

Cafeteria Actions and Reactions

A Develop Understanding Task

Elvira, the cafeteria manager has just received a shipment of new trays with the school logo prominently displayed in the middle of the tray. After unloading 4 cartons of trays in the pizza line, she realizes that students are arriving for lunch and she will have to wait until lunch is over before unloading the remaining cartons. The new trays are very popular and in just a couple of minutes 24 students have passed through the pizza line and are showing off the school logo on the trays. At this time, Elvira decides to divide the remaining trays in the pizza line into 3 equal groups so she can also place some in the salad line and the sandwich line, hoping to attract students to the other lines. After doing so, she realizes that each of the three serving lines has only 12 of the new trays.

"That's not many trays for each line. I wonder how many trays there were in each of the cartons I unloaded?"

1. Can you help the cafeteria manager answer her question using the data in the story about each of the actions she took? Explain how you arrive at your solution.

Elvira is interested in collecting data about how many students use each of the tables during each lunch period. She has recorded some data on Post-It Notes to analyze later. Here are the notes she has recorded:

- Some students are sitting at the front table. (I got distracted by an incident in the back of the lunchroom, and forgot to record how many students.)
- Each of the students at the front table has been joined by a friend, doubling the number of students at the table.
- Four more students have just taken seats with the students at the front table.
- The students at the front table separated into three equal-sized groups and then two groups left, leaving only one-third of the students at the table.
- As the lunch period ends, there are still 12 students seated at the front table.

Elvira is wondering how many students were sitting at the front table when she wrote her first note. Unfortunately, she is not sure what order the middle three Post-It Notes were recorded in since they got stuck together in random order. She is wondering if it matters.

2. Does it matter which order the notes were recorded in? Determine how many students were originally sitting at the front table based on the sequence of notes that appears above. Then rearrange the middle three notes in a different order and determine what the new order implies about the number of students seated at the front table at the beginning.

3. Here are three different equations that could be written based on a particular sequence of notes. Examine each equation, and then list the order of the five notes that is represented by each equation. Find the solution for each equation.

   - $\frac{2(x+4)}{3}=12$

   - $2\left(\frac{x}{3}+4\right)=12$

   - $\frac{2x+4}{3}=12$

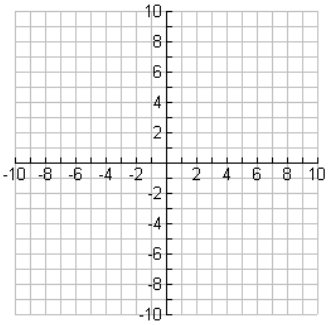
Ready, Set, Go!

Ready

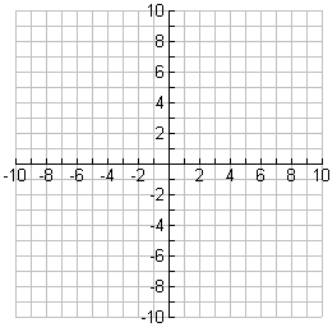
Topic: Solutions to an equation

Graph the following equations using the coordinate graph, and then determine if the given point is a solution to the equation.

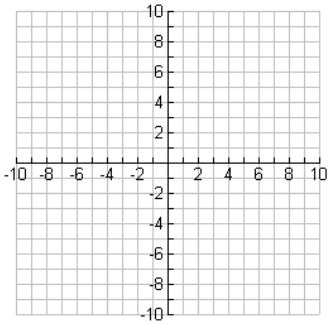
1. $y = 5x - 2$ pt: (1, 3)



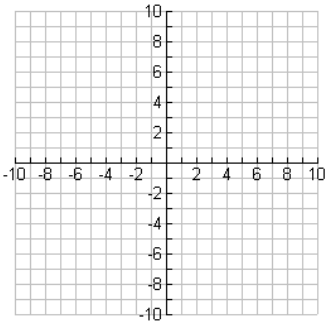
2. $y = \frac{-1}{2}x + 8$ pt: (0, 4)



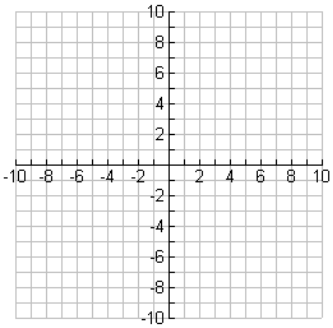
3. $y = x + 4$ pt: (-2, 2)



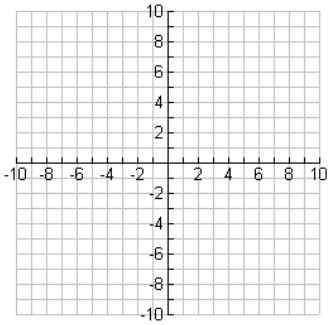
4. $y = x + 2$ pt: (-3, 0)



5. $y = \frac{5}{2}x - 7$ pt: (2, -2)



6. $y = \frac{-4}{3}x$ pt: (4, -16)



Set

Topic: Solutions to an equation

7. The solution to an equation is n = -5. The equation has parentheses on at least one side of the equation and has variables on both sides of the equation. What might the equation be?

8. Create a two-step equation that is true by expanding the given solution using properties of equality. Draw a model to represent your expanded equation.

   a. $x = 3$ b. $m = -2$ c. $a = 0$

9. Without solving, determine if the two expressions are equivalent. Explain your reasoning.
   a. $14 - (3a + 2) = 14 - 3a - 2$

10. Without solving, determine if the two expressions are equivalent. Explain your reasoning.

    a. $4a - 10 = 2(2a - 5)$

11. Without solving, determine if these two equations have the same solution. $3(x - 5) = 35 \; and \; 3x - 5 = 35$. Why or why not?

12. Which of the following expressions are equivalent? Justify.

    $\frac{4t-10}{2}$ $\frac{4t}{2} - 10$ $2t - 10$ $4t - 5$

13. The solution to an equation is n = -5. The equation has parentheses on at least one side of the equation and has variables on both sides of the equation. What might the equation be?

14. Create a two-step equation that is true by expanding the given solution using properties of equality. Draw a model to represent your expanded equation.

    a. $x = 3$ $m = -2$ $a = 0$

Go

Topic: Solutions to an equation

Check whether the given number is a solution to the corresponding equation.

15. $a = -3; 4a + 3 = -9$

16. $x = \frac{4}{3}; \frac{3}{4}x + \frac{1}{2} = \frac{3}{2}$

17. $y = 2; 2.5y - 10.0 = -0.5$

18. $z = -5; 2(5 - 2z) = 20 - 2(z - 1)$

Need Help? Check out these related videos:

http://www.khanacademy.org/math/algebra/solving-linear-equations/v/solving-equations-1

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphing-a-line-in-slope-intercept-form

http://www.youtube.com/watch?v=WXzpisUh0AU

http://patrickjmt.com/an-intro-to-solving-linear-equations-what-does-it-mean-to-be-a-solution/

Elvira's Equations

A Solidify Understanding Task

(Note: This task refers to the same set of variables as used in *Serving Up Symbols*)

Elvira, the cafeteria manager, has written the following equation to describe a cafeteria relationship that seems meaningful to her. She has introduced a new variable *A* to describe this relationship.

$$A = \frac{S}{CP}$$

1. What does *A* represent in terms of the school and the cafeteria?

2. Using what you know about manipulating equations, solve this equation for *S.* Your solution will be of the form *S* = *an expression written in terms of the variables A, C* and *P.*

3. Does your expression for *S* make sense in terms of the meanings of the other variables? Explain why or why not.

4. Now solve the above equation for *C* and explain why the solution makes sense in terms of the variables.

Here is another one of Elvira's equations.

$$T_S = \frac{S(N_e + N_s + N_b)}{i}$$

5. What does T_S represent in terms of the school and the cafeteria?

6. Using what you know about manipulating equations, solve this equation for *S.*

7. Does your expression for *S* make sense in terms of the meanings of the other variables? Explain why or why not.

8. Now solve the above equation for N_e and explain why the solution makes sense in terms of the variables.

Ready, Set, Go!

Ready

Topic: Solve literal equations

Solve a linear equation involving variable coefficients using the properties of equality. Isolate each equation to highlight the variable x. Justify each step.

1. $ax = d$
2. $b + cx = d$
3. $ab + cx = d$

Set

Topic: Solve literal equations or writing linear equations in slope-intercept form

Rearrange the following equations to solve for y (slope-intercept form).

4. $4x + y = 3$
5. $2y = 6x + 9$
6. $5x - 2y = 10$

7. $3x + 6y = 25$
8. $x - 8y = 12$
9. $3x - 7y = 20$

Go

Topic: Create and solve equations for real world problems

Create an equation that describes each of the situations below, then solve.

10. The cost of a birthday party at Classic Boon is $200 plus $4 per person. The cost for Fletcher's party came to $324. How many people came to his party?

11. A cell phone company charges $55 per month for unlimited minutes plus $0.25 per text sent. If the charges to Dayne's cell phone for last month came to $100, how many texts did Aly send?

12. Aly has baked an apple pie and wants to sell it in her bakery. She is going to cut it into 12 slices and sell them individually. She wants to sell it for three times the cost of making it. The ingredients cost $8.50, and she allowed $1.25 to cover the cost of electricity to bake it. Find the values for each of the following questions:

a) What is the amount Aly will charge for each slice of pie?
b) What is the total amount she will gross if she sells the entire pie?
c) What will be the profit if she sells the entire pie?

Need Help? Check out these related videos:

1. Solve and justify two step equations using properties of equality
   http://www.youtube.com/watch?v=WXzpisUh0AU
2. Rearrange equations to highlight a variable: solve literal equations
   http://www.khanacademy.org/math/algebra/solving-linear-equations/v/example-of-solving-for-a-variable

Solving Equations, Literally

A Practice Understanding Task

Solve each of the following equations for *x*:

1. $\frac{3x+2}{5}=7$

2. $\frac{3x+2y}{5}=7$

3. $\frac{4x}{3}-5=11$

4. $\frac{4x}{3}-5y=11$

5. $\frac{2}{5}(x+3)=6$

6. $\frac{2}{5}(x+y)=6$

7. $2(3x+4)=4x+12$

8. $2(3x+4y)=4x+12y$

Write a verbal description for each step of the equation solving process used to solve the following equations for *x*. Your description should include statements about how you know what to do next. For example, you might write, "*First I* _____, *because* _____. . ."

9. $\frac{ax+b}{c}-d=e$

10. $r\cdot\sqrt{\frac{mx}{n}+s}=t$

Ready, Set, Go!

Ready

Topic: Inequalities

1. Use the inequality 4 < 6 to complete each row in the table.

Apply each operation to the original inequality 4 < 6	**Result**	**Is the inequality still true?**
Add 4 to both sides		
Add -4 to both sides		
Subtract 10 from both sides		
Multiply both sides by 4		
Divide both sides by 2		
Multiply both sides by -3		
Divide both sides by -2		

In general, what operations, when performed on an inequality, *reverse* the inequality?

Set

Topic: Solve literal equations

Solve for the indicated variable.

2. Solve the following equation to isolate F: $C = \frac{5}{9}(F - 32)$
3. For $V = \frac{1}{3}\pi r^2 h$, rewrite the formula to isolate the variable h.
4. The area formula of a regular polygon is $A = \frac{1}{2}Pa$. The variable a represents the apothem and P represents the perimeter of the polygon. Rewrite the equation to highlight the value of the perimeter, P.
5. The equation $y = mx + b$ is the equation of a line. Isolate the variables m.
6. The equation $y = mx + b$ is the equation of a line. Isolate the variable x.

7. $Ax + By = C$ is the standard form for a line. Isolate the equation for *x.*

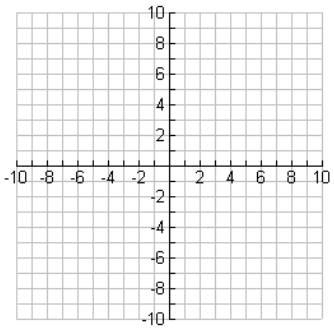
8. $Ax + By = C$ is the standard form for a line. Isolate the equation for *y.*

Go

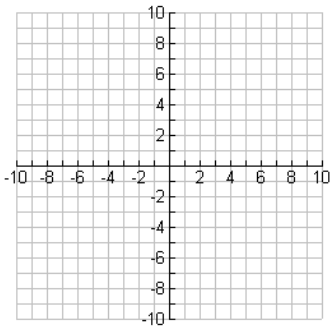
Topic: Solve systems of linear equations

Solve linear equations and pairs of simultaneous linear equations (simple, with a graph only). Justify the solution numerically.

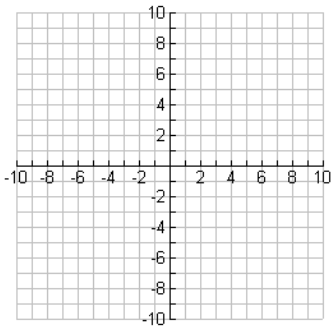
9. $y = x + 3 \; and \; y = -x + 3$



10. $y = 3x - 6 \; and \; y = -x + 6$



11. $2x = 4 \; and \; y = -3$



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http://www.khanacademy.org/math/algebra/solving-linear-equations/v/solving-for-a-variable

http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/solving-linear-systems-by-graphing

Cafeteria Conundrums

A Solidify Understanding Task

Between serving and preparing delicious school lunches, our cafeteria manager, Elvira, is busy analyzing the business of running the cafeteria. We previously saw the symbols for some of the things that she measured. Now she plans to use those symbols. Help Elvira to consider the pressing questions of the lunch room.

Symbol	Meaning
S	Number of students that buy lunch in the cafeteria each day
S_C	Average number of students per class that eat lunch each day
S_m	Number of students who have passed through a line in *m* minutes
C	Number of classes per lunch period
P	Number of lunch periods per day
B	Number of boys that buy lunch each day
G	Number of girls that buy lunch each day
F	Number of food servers in the cafeteria
T	Total number of food items in one lunch (Each entrée, side dish, or beverage counts as 1 item.)
M	Number of minutes passed since the beginning of the lunch period
N_e	Number of entrees in each lunch
N_s	Number of side dishes in each lunch
N_b	Number of beverages in each lunch
C_e	Cost of each entrée
C_s	Cost of each side dish
C_b	Cost of each beverage
L	Number of lines in the cafeteria
W	The number of food workers (servers) per line
i	Average number of food items that a worker can serve each minute (Each entrée, side dish, or beverage counts as 1 item.)
H	Number of hours each food worker works each day
P_L	Price per lunch

Write equations or inequalities to express some of the conditions that Elvira sees in the cafeteria.

1. Each lunch can have no more than 3 side dishes.

2. More boys eat school lunch than girls.

3. There can be no more than 7 food items in each lunch.

4. In each lunch, there are 3 more side dishes than entrees and twice as many beverages as entrees. Write an inequality in **one variable** that shows that the total number of food items in a lunch cannot be more than 7.

5. The cost of food in the lunch is the total of the cost of the entrée, the side dishes, and the beverages. Write an inequality that shows that the cost of the food in the lunch must be less than $1.50.

6. To meet district guidelines, the total price of a lunch must be more than $2.25, but less than $3.50.

7. Elvira knows that the number of lines that she can open in the cafeteria depends on how many food servers she has in the cafeteria each day and how many workers are needed in each line. Write an inequality that shows this relationship.

8. Food workers are paid $11.50 per hour. Elvira can't spend more than $500 per day on employees. Write an inequality that relates the number of food workers to the amount spent each day on employees.

9. Elvira knows that the money she gets from selling lunches has to be greater than her costs.
    a. Write an expression for the cost of employees each day
    b. Write an expression for the cost of food each day
    c. Write an expression that shows that the total cost of food and employees each day must be less than the amount she brings in from selling lunches.

Ready, Set, Go!

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Ready

Topic: Solving equations

Jesse was asked to solve an algebra problem. She submitted the following solution

$$4(x+3)=1$$

$$4x+3=1$$

$$4x=-2$$

$$x=-2$$

1. Is Jesse's solution correct?
2. If it is correct, justify each step of her solution.
3. If it is incorrect, correct her solution, and explain to Jesse what she did wrong.

Set

Topic: Creating and solving real world problems

4. Jade is stranded downtown with only $10 to get home. Taxis cost $0.75 per mile, but there is an additional $2.35 hire charge. Write a formula and use it to calculate how many miles she can travel with her money.

5. Jasmin's Dad is planning a surprise birthday party for her. He will hire a bouncy castle, and will provide party food for all the guests. The bouncy castle costs $150 for the afternoon, and the food will cost $3 per person. Andrew, Jasmin's Dad, has a budget of $300. Write an equation and use it to determine the maximum number of guests he can invite.

6. Jane is baking cookies for a large party. She has a recipe that will make one batch of two dozen cookies, and she decides to make five batches. To make five batches, she finds that she will need 12.5 cups of flour and 15 eggs. Write an equation to describe each of the following situations. Then solve the problem.

   a. How many cookies will she make in all?

   b. How many cups of flour go into one batch?

   c. How many eggs go into one batch?

   d. If Jane only has a dozen eggs on hand, how many more does she need to make five batches?

   e. If she doesn't go out to get more eggs, how many batches can she make? How many cookies will that be?

Go

Topic: Solve systems of equations

Solve the following systems of equations by graphing. You may use a graphing calculator.

7. Mary's car has broken down and it will cost her $1200 to get it fixed—or, for $4500, she can buy a new, more efficient car instead. Her present car uses about $2000 worth of gas per year, while gas for the new car would cost about $1500 per year. After how many years would the total cost of fixing the car equal the total cost of replacing it?

8. Juan is considering two cell phone plans. The first company charges $120 for the phone and $30 per month for the calling plan that Juan wants. The second company charges $40 for the same phone but charges $45 per month for the calling plan that Juan wants. After how many months would the total cost of the two plans be the same?

9. A tortoise and hare decide to race 30 feet. The hare, being much faster, decides to give the tortoise a 20 foot head start. The tortoise runs at 0.5 feet/sec and the hare runs at 5.5 feet per second. How long until the hare catches the tortoise?

Need Help? Check out these related videos:

http://www.youtube.com/watch?v=EWcllbr8Hqs&feature=related

http://www.youtube.com/watch?v=ldYGiXSHa_Q

Greater Than?

A Solidify Understanding Task

For each situation you are given a mathematical statement and two expressions beneath it.

1. Decide which of the two expressions is greater, if the expressions are equal, or if the relationship cannot be determined from the statement.
2. Write an equation or inequality that shows your answer.
3. Explain why your answer is correct.

Watch out—this gets tricky!

Example:
Statement: $x = 8$
Which is greater? $x + 5$ or $3x + 2$
Answer: $3x + 2 > x + 5$ because if $x = 8$, $3x + 2 = 26$, $x + 5 = 13$ and $26 > 13$.

Try it yourself:

1. Statement: $y < x$
   Which is greater? $x - y$ or $y - x$

2. Statement: $a > 0$, $b < 0$
   Which is greater? ab *or* $\frac{a}{b}$

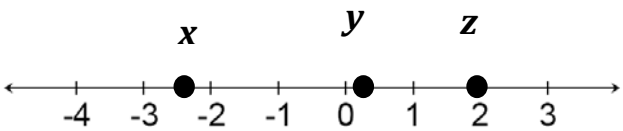
3. Statement: $2x - 3 > 7$
   Which is greater? 5 or x

4. Statement: $10 - 2x < 6$
   Which is greater? x or 2

5. Statement: $4x = 0$
   Which is greater? 1 or x

6. Statement: n is an integer
   Which is greater? n or $-n$

7. Statement:



   Which is greater? 1 or yz

8. Statement: Use the number line in #7 and $x < w < y$
   Which is greater? w or $-y$

9. Statement: $0 < x < 10$ and $0 < y < 12$
   Which is greater? x or y

10. Statement: $3^{n+2} = 27$
    Which is greater? n or 3

11. Statement: $x > 0, y > 0\ and\ \frac{x}{y} > 2$
    Which is greater? $2y$ or x

12. Statement: $5 > 4$
    Which is greater? $5x$ or $4x$

13. Statement $x > y$
    Which is greater? $x + a \; or \; y + a$

14. Statement: $5 > 4$
    Which is greater? $\frac{5}{x}$ or $\frac{4}{x}$

15. Statement: $x > y$ and $a > b$
    Which is greater? $x - a$ or $y - b$

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Ready, Set, Go!

Ready

Topic: Solve inequalities

Solve the following inequalities for x.

1. $2x - 9 < 3$

2. $4x - 3 > 13$

3. $6x - 4 < 26$

Topic: Create and solve the equations for the following problems.

4. Virginia's Painting Service charges $10 per job and $0.20 per square foot. If Virginia earns $50 for painting one job, how many square feet did she paint at the job?

5. Renting the ice-skating rink for Dayne's birthday party costs $200 plus $4 per person. If the cost was $324, how many people were at Dayne's birthday party?

Set

Topic: Solve inequalities

Solve each inequality. Write the solution as an inequality.

6. $x + 15 < 12$

7. $x - 4 \geq 13$

8. $9x > -\frac{3}{4}$

9. $x - 12 \geq 80$

10. $3x - 7 \geq 3(x - 7)$

Solve each inequality and graph the solution on the number line.

11. $x - 2 \leq 1$

12. $x - 8 > -20$

Solve each inequality. Write the solution as an inequality and graph it.

13. $3x \leq 6$

14. $\frac{x}{5} > -\frac{3}{10}$

15. $-10x > 250$

16. $\frac{x}{-7} \geq -5$

Solve each multi-step inequality.

17. $x - 5 > 2x + 3$

18. $2(x-3) \leq 3x-2$

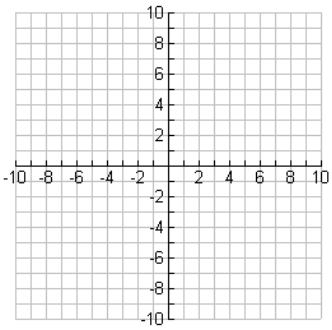
19. $\frac{3(x-4)}{12} \leq \frac{2x}{3}$

Go

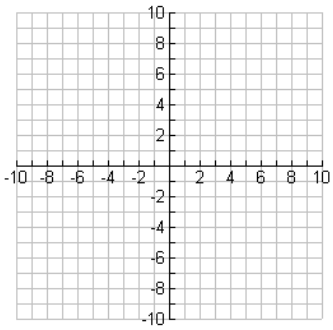
Topic: Solve systems of linear equations

Solve linear equations and pairs of simultaneous linear equations (simple, with a graph only). Justify the solution numerically.

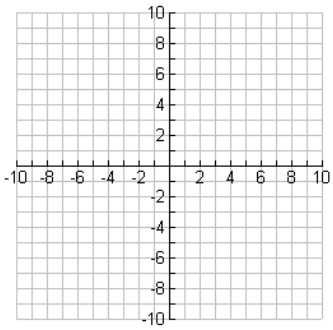
20. $y = -x + 5$
$-x + y = 1$



21. $x + 2y = 8$
$5x + 2y = 0$



22. $3x + 2y = 12$
$4x - y = 5$



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http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/solving-linear-systems-by-graphing

Taking Sides

A Practice Task

Joaquin and Serena work together productively in their math class. They both contribute their thinking and when they disagree, they both give their reasons and decide together who is right. In their math class right now, they are working on inequalities. Recently they had a discussion that went something like this:

> Joaquin: The problem says that "6 less than a number is greater than 4." I think that we should just follow the words and write $6 - x > 4$.
>
> Serena: I don't think that works because if x is 20 and you do 6 less than that you get 20 – 6 = 14. I think we should write $x - 6 > 4$.
>
> Joaquin: Oh, you're right. Then is makes sense that the solution will be $x > 10$, which means we can choose any number greater than 10.

The situations below are a few more of the disagreements and questions that Joaquin and Serena have. Your job is to decide is to answer their questions, decide who is right and give a mathematical explanation of your reasoning.

1. Joaquin and Serena are assigned to graph the inequality $x \geq -7$.
   Joaquin thinks the graph should have an open dot -7.
   Serena thinks the graph should have a closed dot at -7.
   Explain who is correct and why.

2. Joaquin and Serena are looking at the problem $3x + 1 > 0$
   Serena says that the inequality is always true because multiplying a number by three and then adding one to it makes the number greater than zero.
   Is she right? Explain why or why not.

3. The word problem that Joaquin and Serena are working on says, "4 greater than x".
   Joaquin says that they should write: $4 > x$.
   Serena says they should write: $4 + x$.
   Explain who is correct and why.

4. Joaquin is thinking hard about equations and inequalities and comes up with this idea:
   If $45 + 47 = t$, then $t = 45 + 47$.
   So, if $45 + 47 < x$, then $x < 45 + 47$.
   Is he right? Explain why or why not.

5. Joaquin's question in #4 made Serena think about other similarities and differences in equations and inequalities. Serena wonders about the equation $-\frac{x}{3} = 4$ and the inequality $-\frac{x}{3} > 4$. Explain to Serena ways that solving these two problems are alike and ways that they are different. How are the solutions to the problems alike and different?

6. Joaquin solved $-15q \leq 135$ by adding 15 to each side of the inequality. Serena said that he was wrong. Who do you think is right and why?

   Joaquin's solution was $q \leq 150$. He checked his work by substituting 150 for q in the original inequality. Does this prove that Joaquin is right? Explain why or why not.

   Joaquin is still skeptical and believes that he is right. Find a number that satisfies his solution but does not satisfy the original inequality.

7. Serena is working is checking her work with Joaquin and finds that they disagree on a problem. Here's what Serena wrote:

$$3x + 3 \leq -2x + 5$$
$$3x \leq -2x + 2$$
$$x \leq 2$$

   Is she right? Explain why or why not?

8. Joaquin and Serena are having trouble solving $-4(3m - 1) \geq 2(m + 3)$. Explain how they should solve the inequality, showing all the necessary steps and identifying the properties you would use.

9. Joaquin and Serena know that some equations are true for any value of the variable and some equations are never true, no matter what value is chosen for the variable. They are wondering about inequalities. What could you tell them about the following inequalities? Do they have solutions? What are they? How would you graph their solutions on a number line?

   a. $4s + 6 \geq 6 + 4s$

   b. $3r + 5 > 3r - 2$

   c. $4(n + 1) < 4n - 3$

10. The partners are given the literal inequality $ax + b > c$ to solve for x. Joaquin says that he will solve it just like an equation. Serena says that he needs to be careful because if a is a negative number, the solution will be different. What do you say? What are the solutions for the inequality?

Ready, Set, Go!

Ready

Topic: Solving equations and inequalities

1. The local amusement park sells summer memberships for $50 each. Normal admission to the park costs $25; admission for members costs $15.

   a. If Darren wants to spend no more than $100 on trips to the amusement park this summer, how many visits can he make if he buys a membership with part of that money?

   b. How many visits can he make if he does not?

   c. If he increases his budget to $160, how many visits can he make as a member?

   d. How many can he make as a non-member?

2. Jae just took a math test with 20 questions, each worth an equal number of points. The test is worth 100 points total.

   a. Write an equation relating the number of questions Jae got right to the total score he will get on the test.

   b. If a score of 70 points earns a grade of $C-$, how many questions would Jae need to get right to get a $C-$ on the test?

   c. If a score of 83 points earns a grade of B, how many questions would Jae need to get right to get a B on the test?

   d. Suppose Jae got a score of 60% and then was allowed to retake the test. On the retake, he got all the questions right that he got right the first time, and also got half the questions right that he got wrong the first time. What percent did Jae get right on the retake?

Set

Topic: Solve and justify one variable inequalities

Solve each inequality, justifying each step you use.

3.

$x - 5 < 35$	Justification

4.

$x + 68 \geq 75$	Justification

5.

$2x - 4 \leq 10$	Justification

6.

$5 - 4x \leq 17$	Justification

7.

$\frac{x}{-3} > -\frac{10}{9}$	Justification

8.

$2(x - 3) \leq 3x - 2$	Justification

Solve each inequality and graph the solution on the number line.

9. $x - 8 > -20$

10. $x + 11 > 13$

Solve each multi-step inequality.

11. $4x + 3 < -1$

12. $4 - 6x \leq 2(2x + 3)$

13. $5(4x + 3) \geq 9(x - 2) - x$

14. $\frac{2}{3}x - \frac{1}{2}(4x - 1) \geq x + 2(x - 3)$

Topic: Solve literal equations

15. Solve the following equation to isolate F: $C = \frac{5}{9}(F - 32)$

16. For $V = \frac{1}{3}\pi r^2 h$, rewrite the formula to isolate the variable h.

17. The area formula of a regular polygon is $A = \frac{1}{2}Pa$. The variable a represents the apothem and P represents the perimeter of the polygon. Rewrite the equation to highlight the value of the perimeter, P.

18. The equation $y = mx + b$ is the equation of a line. Isolate the variables m.

19. The equation $y = mx + b$ is the equation of a line. Isolate the variable x.

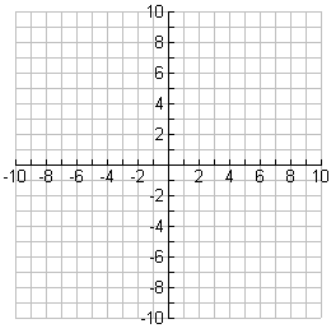
20. $Ax + By = C$ is the standard form for a line. Isolate the equation for x.

21. $Ax + By = C$ is the standard form for a line. Isolate the equation for y.

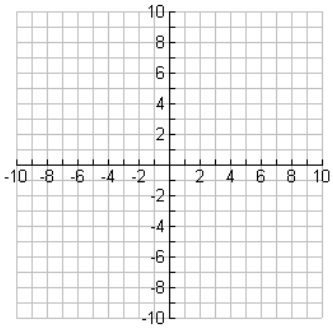
Go

Topic: Solve systems of equations

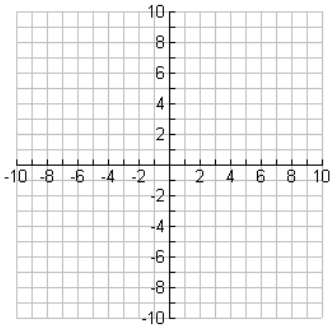
22. $y = -x + 5$
$-x + y = 1$



23. $x + 2y = 8$
$5x + 2y = 0$



24. $3x + 2y = 12$
$4x - y = 5$



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HOMEWORK HELP: Getting Ready Unit

Skills students will be working on:

1. Solve and justify one step equations using properties of equality
   http://www.khanacademy.org/math/algebra/solving-linear-equations/v/solving-equations-1
2. Graph linear equations in slope intercept form
   http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphing-a-line-in-slope-intercept-form
3. Solve and justify two step equations using properties of equality
   http://www.youtube.com/watch?v=WXzpisUh0AU
4. Rearrange equations to highlight a variable
   http://www.khanacademy.org/math/algebra/solving-linear-equations/v/example-of-solving-for-a-variable
5. Equations and Inequalities: Solve equations and inequalities and check whether a given number is a solution

1. One-Step Equations

Objectives

- Solve an equation using addition.
- Solve an equation using subtraction.
- Solve an equation using multiplication.
- Solve an equation using division.

Concept

Introduction

Nadia is buying a new mp3 player. Peter watches her pay for the player with a $100 bill. She receives $22.00 in change, and from only this information, Peter works out how much the player cost. How much was the player?

In algebra, we can solve problems like this using an **equation**. An **equation** is an algebraic expression that involves an **equals** sign. If we use the letter x to represent the cost of the mp3 player, we can write the equation $x + 22 = 100$. This tells us that the value of the player **plus** the value of the change received is **equal** to the $100 that Nadia paid.

Another way we could write the equation would be $x = 100 - 22$. This tells us that the value of the player is **equal** to the total amount of money Nadia paid . This equation is mathematically equivalent to the first one, but it $(100 - 22)$ is easier to solve.

In this chapter, we will learn how to solve for the variable in a one-variable linear equation. **Linear equations** are equations in which each term is either a constant, or a constant times a single variable (raised to the first power). The term linear comes from the word line, because the graph of a linear equation is always a line.

We'll start with simple problems like the one in the last example.

Solving Equations Using Addition and Subtraction

When we work with an algebraic equation, it's important to remember that the two sides have to stay equal for the equation to stay true. We can change the equation around however we want, but whatever we do to one side of the equation, we have to do to the other side. In the introduction above, for example, we could get from the first equation to the second equation by subtracting 22 from both sides:

$$\begin{aligned} x + 22 &= 100 \\ x + 22 - 22 &= 100 - 22 \\ x &= 100 - 22 \end{aligned}$$

Similarly, we can add numbers to each side of an equation to help solve for our unknown.

Example 1

$x - 3 = 9$ *Solve .*

Solution

To solve an equation for x, we need to $x-$ **isolate** that is, we need to get it by itself on one side of the equals sign. Right now our x has a 3 subtracted from it. To reverse this, we'll add 3—but we must add 3 to **both sides.**

$$\begin{aligned} x - 3 &= 9 \\ x - 3 + 3 &= 9 + 3 \\ x + 0 &= 9 + 3 \\ x &= 12 \end{aligned}$$

Example 2

$z - 9.7 = -1.026$ *Solve*

Solution

It doesn't matter what the variable is—the solving process is the same.

$$\begin{aligned} z - 9.7 &= -1.026 \\ z - 9.7 + 9.7 &= -1.026 + 9.7 \\ z &= 8.674 \end{aligned}$$

Make sure you understand the addition of decimals in this example!

Example 3

Solve $x + \frac{4}{7} = \frac{9}{5}$.

Solution

To isolate x, we need to subtract $\frac{4}{7}$ from both sides.

$$\begin{aligned} x + \frac{4}{7} &= \frac{9}{5} \\ x + \frac{4}{7} - \frac{4}{7} &= \frac{9}{5} - \frac{4}{7} \\ x &= \frac{9}{5} - \frac{4}{7} \end{aligned}$$

Now we have to subtract fractions, which means we need to find the LCD. Since 5 and 7 are both prime, their lowest common multiple is just their product, 35.

$$\begin{aligned} x &= \frac{9}{5} - \frac{4}{7} \\ x &= \frac{7 \cdot 9}{7 \cdot 5} - \frac{4 \cdot 5}{7 \cdot 5} \\ x &= \frac{63}{35} - \frac{20}{35} \\ x &= \frac{63 - 20}{35} \\ x &= \frac{43}{35} \end{aligned}$$

Make sure you're comfortable with decimals and fractions! To master algebra, you'll need to work with them frequently.

Solving Equations Using Multiplication and Division

Suppose you are selling pizza for $1.50 a slice and you can get eight slices out of a single pizza. How much money do you get for a single pizza? It shouldn't take you long to figure out that you get $8 \times \$1.50 = \$12.00$. You solved this problem by multiplying. Here's how to do the same thing algebraically, using x to stand for the cost in dollars of the whole pizza.

Example 4

Solve $\frac{1}{8} \cdot x = 1.5$.

Our x is being multiplied by one-eighth. To cancel that out and get x by itself, we have to multiply by the reciprocal, 8. Don't forget to multiply **both sides** of the equation.

$$8\left(\frac{1}{8} \cdot x\right) = 8(1.5)$$
$$x = 12$$

Example 5

Solve . . $\frac{9x}{5} = 5$

$$\frac{5}{9}\left(\frac{9x}{5}\right) = \frac{5}{9}(5)$$
$$x = \frac{25}{9}$$

Example 6

Solve . $0.25x = 5.25$

0.25 is the decimal equivalent of one fourth, so to cancel out the 0.25 factor we would multiply by 4.

$$4(0.25x) = 4(5.25)$$
$$x = 21$$

Solving by division is another way to isolate x. Suppose you buy five identical candy bars, and you are charged $3.25. How much did each candy bar cost? You might just divide $3.25 by 5, but let's see how this problem looks in algebra.

Example 7

Solve.

To cancel the 5, we divide both sides by 5.

$$\frac{5x}{5} = \frac{3.25}{5}$$
$$x = 0.65$$

Example 8

Solve .

Divide both sides by 7.

$$x = \frac{5}{11.7}$$
$$x = \frac{5}{77}$$

Example 9

$1.375x = 1.2$ *Solve* .

Divide by 1.375

$$x = \frac{1.2}{1.375}$$
$$x = 0.8\overline{72}$$

Notice the bar above the final two decimals; it means that those digits recur, or repeat. The full answer is 0.872727272727272....

To see more examples of one - and two-step equation solving, watch the Khan Academy video series starting at http://www.youtube.com/watch?v=bAerID24QJ0.

Solve Real-World Problems Using Equations

Example 10

In the year 2017, Anne will be 45years old. In what year was Anne born?

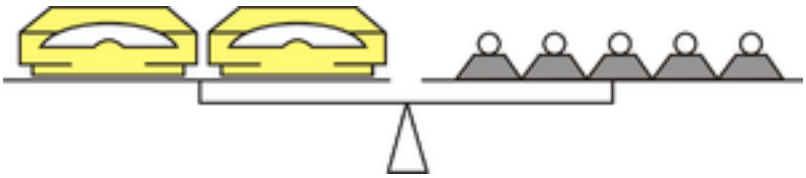
The unknown here is the year Anne was born, so that's our variable x. Here's our equation:

$$\begin{aligned} x + 45 &= 2017 \\ x + 45 - 45 &= 2017 - 45 \\ x &= 1972 \end{aligned}$$

Anne was born in 1972.

Example 11

A mail order electronics company stocks a new mini DVD player and is using a balance to determine the shipping weight. Using only one-pound weights, the shipping department found that the following arrangement balances:



How much does each DVD player weigh?

Solution

Since the system balances, the total weight on each side must be equal. To write our equation, we'll use x for the weight of one DVD player, which is unknown. There are two DVD players, weighing a total of $2x$ pounds, on the left side of the balance, and on the right side are 5 1-pound weights, weighing a total of 5 pounds. So our equation is $2x = 5$. Dividing both sides by 2 gives us $x = 2.5$.

Each DVD player weighs 2.5 pounds.

Example 12

In 2004, Takeru Kobayashi of Nagano, Japan, ate 53.5 hot dogs in 12 minutes. This was 3 more hot dogs than his own previous world record, set in 2002. Calculate:

a) *How many minutes it took him to eat one hot dog.*

b) *How many hot dogs he ate per minute.*

c) *What his old record was.*

Solution

a) We know that the total time for 53.5 hot dogs is 12 minutes. We want to know the time for one hot dog, so that's x. Our equation is $53.5x = 12$. Then we divide both sides by 53.5 to get

$x = \frac{12}{53.5}$, or $x = 0.224\ minutes$.

We can also multiply by 60 to get the time in seconds; 0.224 minutes is about 13.5 seconds. So that's how long it took Takeru to eat one hot dog.

b) Now we're looking for hot dogs per minute instead of minutes per hot dog. We'll use the variable y instead of x this time so we don't get the two confused. 12 minutes, times the number of hot dogs per minute, equals the total number of hot dogs, so $12y = 53.5$. Dividing both sides by 12 gives us $y = \frac{53.5}{12}$, or $y = 4.458$ hot dogs per minute.

c) We know that his new record is 53.5, and we know that's three more than his old record. If we call his old record z, we can write the following equation: $z + 3 = 53.5$. Subtracting 3 from both sides gives us $z = 50.5$. So Takeru's old record was 50.5 hot dogs in 12 minutes.

Lesson Summary

- An equation in which each term is either a constant or the product of a constant and a single variable is a **linear equation**.
- We can add, subtract, multiply, or divide both sides of an equation by the same value and still have an equivalent equation.
- To solve an equation, **isolate** the unknown variable on one side of the equation by applying one or more arithmetic operations to both sides.

2. Forms of Linear Equations

Objectives

- Write equations in slope-intercept form.
  .

Concept

Introduction

We saw in the last chapter that many real-world situations can be described with linear graphs and equations. In this chapter, we'll see how to find those equations in a variety of situations.

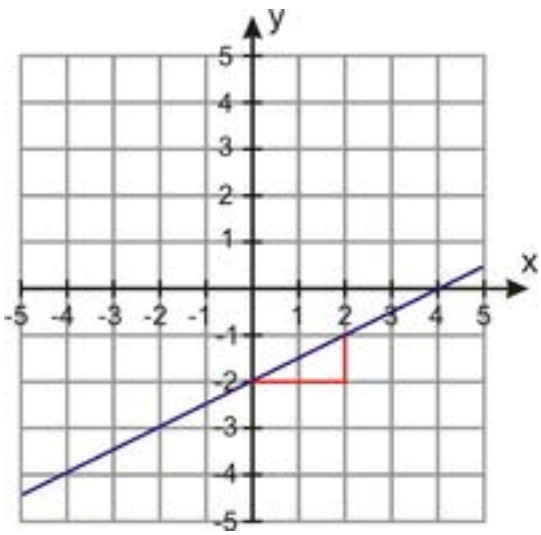
Write an Equation Given Slope and $y-$Intercept

You've already learned how to write an equation in slope–intercept form: simply start with the general equation for the slope-intercept form of a line, $y = mx + b$, and then plug the given values of m and b into the equation. For example, a line with a slope of 4 and a $y-$intercept of -3 would have the equation .

$$y = 4x - 3$$

If you are given just the graph of a line, you can read off the slope and $y-$intercept from the graph and write the equation from there. For example, on the graph below you can see that the line rises by 1 unit as it moves 2 units to the right, so its slope is $\frac{1}{2}$. Also, you can see that the $y-$intercept is -2, so the equation of the line is .

$$y = \frac{1}{2}x - 2$$



Write an Equation Given the Slope and a Point

Often, we don't know the value of the $y-$intercept, but we know the value of y for a non-zero value of x. In this case, it's often easier to write an equation of the line in **point-slope form.** An equation in point-slope form is written as $y - y_0 = m(x - x_0)$, where m is the slope and (x_0, y_0) is a point on the line.

Example 1

A line has a slope of $\frac{3}{5}$, and the point (2, 6) is on the line. Write the equation of the line in point-slope form.

Solution

Start with the formula $y - y_0 = m(x - x_0)$.

Plug in $\frac{3}{5}$ for m, 2 for x_0 and 6 for y_0.

The equation in point-slope form is $y - 6 = \frac{3}{5}(x - 2)$.

Notice that the equation in point-slope form is not solved for y. If we did solve it for y, we'd have it in $y-$intercept form. To do that, we would just need to distribute the $\frac{3}{5}$ and add 6 to both sides. That means that the equation of this line in slope-intercept form is , or simply $y = \frac{3}{5}x - \frac{6}{5} + 6$ $y = \frac{3}{5}x + \frac{24}{5}$.

Write an Equation Given Two Points

Point-slope form also comes in useful when we need to find an equation given just two points on a line.

For example, suppose we are told that the line passes through the points (-2, 3) and (5, 2). To find the equation of the line, we can start by finding the slope.

Starting with the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, we plug in the $x-$ and $y-$values of the two points to get $m = \frac{2-3}{5-(-2)} = \frac{-1}{7}$. We can plug that value of m into the point-slope formula to get $y - y_0 = -\frac{1}{7}(x - x_0)$.

Now we just need to pick one of the two points to plug into the formula. Let's use (5, 2); that gives us . $y - 2 = -\frac{1}{7}(x - 5)$

What if we'd picked the other point instead? Then we'd have ended up with the equation $y - 3 = -\frac{1}{7}(x + 2)$, which doesn't look the same. That's because there's more than one way to write an equation for a given line in point-slope form. But let's see what happens if we solve each of those equations for y.

Starting with $y - 2 = -\frac{1}{7}(x - 5)$, we distribute the $-\frac{1}{7}$ and add 2 to both sides. That gives us $y = -\frac{1}{7}x + \frac{5}{7} + 2$, or $y = -\frac{1}{7}x + \frac{19}{7}$.

On the other hand, if we start with $y-3=-\frac{1}{7}(x+2)$, we need to distribute the $-\frac{1}{7}$ and add 3 to both sides. That gives us , $y=-\frac{1}{7}x-\frac{2}{7}+3$ which also simplifies $y=-\frac{1}{7}x+\frac{19}{7}$ to .

So whichever point we choose to get an equation in point-slope form, the equation is still mathematically the same, and we can see this when we convert it to $y-$intercept form.

Example 2

A line contains the points (3, 2) and (-2, 4). Write an equation for the line in point-slope form; then write an equation in $y-$intercept form.

Solution

Find the slope of the line: $m=\frac{y_2-y_1}{x_2-x_1}=\frac{4-2}{-2-3}=-\frac{2}{5}$

Plug in the value of the slope: . $y-y_0=-\frac{2}{5}(x-x_0)$

Plug point (3, 2) into the equation: $y-2=-\frac{2}{5}(x-3)$.

The equation in point-slope form is $y-2=-\frac{2}{5}(x-3)$.

To convert to $y-$intercept form, simply solve for y:

$$y-2=-\frac{2}{5}(x-3)\rightarrow y-2=-\frac{2}{5}x-\frac{6}{5}\rightarrow y=-\frac{2}{5}x-\frac{6}{5}+2=-\frac{2}{5}x+\frac{4}{5}.$$

The equation in $y-$intercept form is $y=-\frac{2}{5}x+\frac{4}{5}$ **.**

3. Two-Step Equations

Objectives

- Solve a two-step equation using addition, subtraction, multiplication, and division.
- Solve a two-step equation by combining like terms.
- Solve real-world problems using two-step equations.

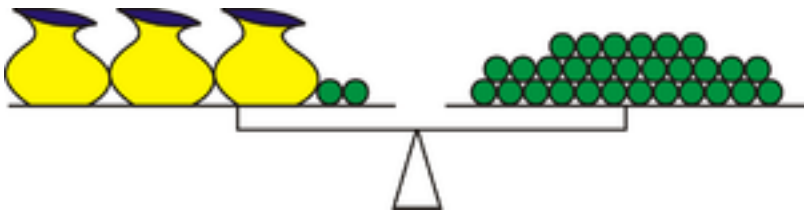
Concept

Solve a Two-Step Equation

We've seen how to solve for an unknown by isolating it on one side of an equation and then evaluating the other side. Now we'll see how to solve equations where the variable takes more than one step to isolate.

Example 1

Rebecca has three bags containing the same number of marbles, plus two marbles left over. She places them on one side of a balance. Chris, who has more marbles than Rebecca, adds marbles to the other side of the balance. He finds that with 29 marbles, the scales balance. How many marbles are in each bag? Assume the bags weigh nothing.



Solution

We know that the system balances, so the weights on each side must be equal. If we use x to represent the number of marbles in each bag, then we can see that on the left side of the scale we have three bags (each containing x marbles) plus two extra marbles, and on the right side of the scale we have 29 marbles. The balancing of the scales is similar to the balancing of the following equation.

$$3x + 2 = 29$$

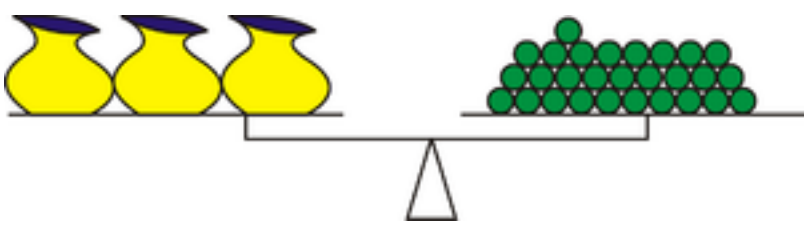
"Three bags plus two marbles **equals** 29 marbles"

To solve for x, we need to first get all the variables (terms containing an x) alone on one side of the equation. We've already got all the x's on one side; now we just need to isolate them.

$$\begin{aligned} 3x + 2 &= 29 \\ 3x + 2 - 2 &= 29 - 2 && \text{Get rid of the 2 on the left by subtracting it from both sides.} \\ 3x &= 27 \\ \frac{3x}{3} &= \frac{27}{3} && \text{Divide both sides by 3.} \\ x &= 9 \end{aligned}$$

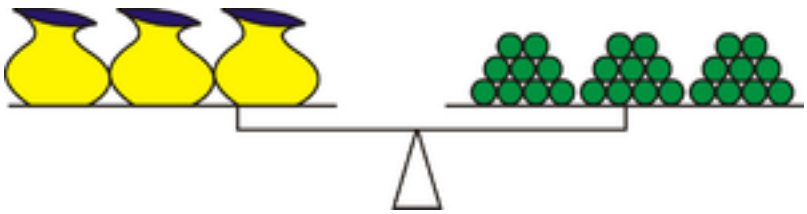
There are nine marbles in each bag.

We can do the same with the real objects as we did with the equation. Just as we subtracted 2 from both sides of the equals sign, we could remove two marbles from each side of the scale. Because we removed the same number of marbles from each side, we know the scales will still balance.



Then, because there are three bags of marbles on the left-hand side of the scale, we can divide the marbles on the right-hand side into three equal piles. You can see that there are nine marbles in each.

*Three bags of marbles **balances** three piles of nine marbles.*



So each bag of marbles balances nine marbles, meaning that each bag contains nine marbles.

Check out http://www.mste.uiuc.edu/pavel/java/balance/ for more interactive balance beam activities!

Example 2

Solve $6(x+4)=12$.

This equation has the x buried in parentheses. To dig it out, we can proceed in one of two ways: we can either distribute the six on the left, or divide both sides by six to remove it from the left. Since the right-hand side of the equation is a multiple of six, it makes sense to divide. That gives us $x+4=2$. Then we can subtract 4 from both sides to get $x=-2$.

Example 3

$\frac{x-3}{5}=7$ *Solve* .

It's always a good idea to get rid of fractions first. Multiplying both sides by 5 gives us $x-3=35$, and then we can add 3 to both sides to get $x=38$.

Example 4

$\frac{5}{9}(x+1)=\frac{2}{7}$ *Solve* .

First, we'll cancel the fraction on the left by multiplying by the reciprocal (the multiplicative inverse).

$$\begin{aligned}\frac{9}{5}\cdot\frac{5}{9}(x+1) &= \frac{9}{5}\cdot\frac{2}{7}\\ (x+1) &= \frac{18}{35}\end{aligned}$$

Then we subtract 1 from both sides. ($\frac{35}{35}$ is equivalent to 1.)

$$\begin{aligned}x+1 &= \frac{18}{35}\\ x+1-1 &= \frac{18}{35}-\frac{35}{35}\\ x &= \frac{18-35}{35}\\ x &= \frac{-17}{35}\end{aligned}$$

These examples are called **two-step equations**, because we need to perform two separate operations on the equation to isolate the variable.

Solve a Two-Step Equation by Combining Like Terms

When we look at a linear equation we see two kinds of terms: those that contain the unknown variable, and those that don't. When we look at an equation that has an x on both sides, we know that in order to solve it, we need to get all the x—terms on one side of the equation. This is called **combining like terms**. The terms with an x in them are **like terms** because they contain the same variable (or, as you will see in later chapters, the same combination of variables).

Like Terms	**Unlike Terms**
$4x$, $10x$, $-3.5x$, and $\frac{x}{12}$	$3x$ and $3y$
$3y$, $0.000001y$, and y	$4xy$ and $4x$
xy, $6xy$, and $2.39xy$	$0.5x$ and 0.5

To add or subtract like terms, we can use the Distributive Property of Multiplication.

$$\begin{aligned} 3x + 4x &= (3+4)x = 7x \\ 0.03xy - 0.01xy &= (0.03 - 0.01)xy = 0.02xy \\ -y + 16y + 5y &= (-1+16+5)y = 10y \\ 5z + 2z - 7z &= (5+2-7)z = 0z = 0 \end{aligned}$$

To solve an equation with two or more like terms, we need to combine the terms first.

Example 5

$(x+5) - (2x-3) = 6$ *Solve* .

There are two like terms: the x and the $-2x$ (don't forget that the negative sign applies to everything in the parentheses). So we need to get those terms together. The associative and distributive properties let us rewrite the equation as $x + 5 - 2x + 3 = 6$, and then the commutative property lets us switch around the terms to get $x - 2x + 5 + 3 = 6$, or $(x - 2x) + (5 + 3) = 6$.

$(x - 2x)$ is the same as $(1-2)x$, or $-x$, so our equation becomes $-x + 8 = 6$

Subtracting 8 from both sides gives us $-x = -2$.

And finally, multiplying both sides by -1 gives us $x = 2$.

Example 6

$\frac{x}{2} - \frac{x}{3} = 6$ *Solve* .

This problem requires us to deal with fractions. We need to write all the terms on the left over a common denominator of six.

$\frac{3x}{6} - \frac{2x}{6} = 6$

Then we subtract the fractions to get . $\frac{x}{6} = 6$

Finally we multiply both sides by 6 to get $x = 36$.

Solve Real-World Problems Using Two-Step Equations

The hardest part of solving word problems is translating from words to an equation. First, you need to look to see what the equation is asking. What is the **unknown** for which you have to solve? That will be what your **variable** stands for. Then, follow what is going on with your variable all the way through the problem.

Example 7

An emergency plumber charges $65 as a call-out fee plus an additional $75 per hour. He arrives at a house at 9:30 and works to repair a water tank. If the total repair bill is $196.25, at what time was the repair completed?

In order to solve this problem, we collect the information from the text and convert it to an equation.

Unknown: time taken in hours – this will be our x

The bill is made up of two parts: a call out fee and a per-hour fee. The call out is a flat fee, and independent of x—it's the same no matter how many hours the plumber works. The per-hour part depends on the number of hours (x). So the total fee is $65 (no matter what) plus $\$75x$ (where x is the number of hours), or $65 + 75x$.

Looking at the problem again, we also can see that the total bill is $196.25. So our final equation is $196.25 = 65 + 75x$.

Solving for x:

$$\begin{aligned} 196.25 &= 65 + 75x && \text{Subtract 65 from both sides.} \\ 131.25 &= 75x && \text{Divide both sides by 75.} \\ 1.75 &= x && \text{The job took 1.75 hours.} \end{aligned}$$

Solution

The repair job was completed 1.75 hours after 9:30, so it was completed at 11:15AM.

Example 8

When Asia was young her Daddy marked her height on the door frame every month. Asia's Daddy noticed that between the ages of one and three, he could predict her height (in inches) by taking her age in months, adding 75 inches and multiplying the result by one-third. Use this information to answer the following:

a) *Write an equation linking her predicted height, h, with her age in months, m.*

b) *Determine her predicted height on her second birthday.*

c) *Determine at what age she is predicted to reach three feet tall.*

Solution

a) To convert the text to an equation, first determine the type of equation we have. We are going to have an equation that links **two variables**. Our unknown will change, depending on the information we are given. For example, we could solve for height given age, or solve for age given height. However, the text gives us a way to determine **height**. Our equation will start with "$h =$".

The text tells us that we can predict her height by taking her age in months, adding 75, and multiplying by $\frac{1}{3}$. So our equation is , or . $h = (m + 75) \cdot \frac{1}{3}$ $h = \frac{1}{3}(m + 75)$

b) To predict Asia's height on her second birthday, we substitute $m = 24$ into our equation (because 2 years is 24 months) and solve for h.

$$h = \frac{1}{3}(24 + 75)$$
$$h = \frac{1}{3}(99)$$
$$h = 33$$

Asia's height on her second birthday was predicted to be 33 inches.

c) To determine the predicted age when she reached three feet, substitute $h = 36$ into the equation and solve for m.

$$36 = \frac{1}{3}(m + 75)$$
$$108 = m + 75$$
$$33 = m$$

Asia was predicted to be 33 months old when her height was three feet.

Example 9

To convert temperatures in Fahrenheit to temperatures in Celsius, follow the following steps: Take the temperature in degrees Fahrenheit and subtract 32. Then divide the result by 1.8 and this gives the temperature in degrees Celsius.

a) *Write an equation that shows the conversion process.*

b) *Convert 50 degrees Fahrenheit to degrees Celsius.*

c) *Convert 25 degrees Celsius to degrees Fahrenheit.*

d) *Convert -40 degrees Celsius to degrees Fahrenheit.*

a) The text gives the process to convert Fahrenheit to Celsius. We can write an equation using two variables. We will use f for temperature in Fahrenheit, and c for temperature in Celsius.

First we take the temperature in Fahrenheit and subtract 32.	$f - 32$
Then divide by 1.8.	$\frac{f-32}{1.8}$
This equals the temperature in Celsius.	$c = \frac{f-32}{1.8}$

In order to convert from one temperature scale to another, simply substitute in for whichever temperature you know, and solve for the one you don't know.

b) To convert 50 degrees Fahrenheit to degrees Celsius, substitute $f = 50$ into the equation.

$$c = \frac{50-32}{1.8}$$
$$c = \frac{18}{1.8}$$
$$c = 10$$

50 degrees Fahrenheit is equal to 10 degrees Celsius.

c) To convert 25 degrees Celsius to degrees Fahrenheit, substitute $c = 25$ into the equation:

$$25 = \frac{f - 32}{1.8}$$
$$45 = f - 32$$
$$77 = f$$

25 degrees Celsius is equal to 77 degrees Fahrenheit.

d) To convert -40 degrees Celsius to degrees Fahrenheit, substitute $c = -40$ into the equation.

$$-40 = \frac{f - 32}{1.8}$$
$$-72 = f - 32$$
$$-40 = f$$

-40 degrees Celsius is equal to -40 degrees Fahrenheit. (No, that's not a mistake! This is the one temperature where they are equal.)

Lesson Summary

- Some equations require more than one operation to solve. Generally it, is good to go from the outside in. If there are parentheses around an expression with a variable in it, cancel what is outside the parentheses first.
- Terms with the same variable in them (or no variable in them) are **like terms. Combine like terms** (adding or subtracting them from each other) to simplify the expression and solve for the unknown.

5. Equations and Inequalities

Objectives

- Write equations and inequalities.
- Check solutions to equations.
- Check solutions to inequalities.
- Solve real-world problems using an equation.

Concept

Introduction

In algebra, an **equation** is a mathematical expression that contains an equal sign. It tells us that two expressions represent the same number. For example, $y = 12x$ is an equation. An **inequality** is a mathematical expression that contains inequality signs. For example, $y \leq 12x$ is an inequality. Inequalities are used to tell us that an expression is either larger or smaller than another expression. Equations and inequalities can contain both **variables** and **constants**.

Variables are usually given a letter and they are used to represent unknown values. These quantities can change because they depend on other numbers in the problem.

Constants are quantities that remain unchanged. Ordinary numbers like $2,\ -3,\ \frac{3}{4},$ and π are constants.

Equations and inequalities are used as a shorthand notation for situations that involve numerical data. They are very useful because most problems require several steps to arrive at a solution, and it becomes tedious to repeatedly write out the situation in words.

Write Equations and Inequalities

Here are some examples of equations:

$$3x - 2 = 5 \qquad x + 9 = 2x + 5 \qquad \frac{x}{3} = 15 \qquad x^2 + 1 = 10$$

To write an inequality, we use the following symbols:

> **greater than**

$\geq$ **greater than or equal to**

< **less than**

$\leq$ **less than or equal to**

$\neq$ **not equal to**

Here are some examples of inequalities:

$$3x < 5 \qquad 4 - x \leq 2x \qquad x^2 + 2x - 1 > 0 \qquad \frac{3x}{4} \geq \frac{x}{2} - 3$$

The most important skill in algebra is the ability to translate a word problem into the correct equation or inequality so you can find the solution easily. The first two steps are **defining the variables** and **translating** the word problem into a mathematical equation.

Defining the variables means that we assign letters to any unknown quantities in the problem.

Translating means that we change the word expression into a mathematical expression containing variables and mathematical operations with an equal sign or an inequality sign.

Example 1

Define the variables and translate the following expressions into equations.

a) A number plus 12 is 20.

b) 9 less than twice a number is 33.

c) $20 was one quarter of the money spent on the pizza.

Solution

a) **Define**

Let $n =$ the number we are seeking.

Translate

A number plus 12 is 20.

$$n + 12 = 20$$

b) **Define**

Let $n =$ the number we are seeking.

Translate

9 less than twice a number is 33.

This means that twice the number, minus 9, is 33.

$$2n - 9 = 33$$

c) **Define**

Let $m =$ the money spent on the pizza.

Translate

$20 was one quarter of the money spent on the pizza.

$$20 = \frac{1}{4}m$$

Often word problems need to be reworded before you can write an equation.

Example 2

Find the solution to the following problems.

a) Shyam worked for two hours and packed 24 boxes. How much time did he spend on packing one box?

b) After a 20% discount, a book costs $12. How much was the book before the discount?

Solution

a) **Define**

Let $t =$ time it takes to pack one box.

Translate

Shyam worked for two hours and packed 24 boxes. This means that two hours is 24 times the time it takes to pack one box.

$$2 = 24t$$

Solve

$$t = \frac{2}{24} = \frac{1}{12} \text{ hours}$$

$$\frac{1}{12} \times 60 \text{ minutes} = 5 \text{ minutes}$$

Answer

Shyam takes 5 minutes to pack a box.

b) **Define**

Let $p =$ the price of the book before the discount.

Translate

After a 20% discount, the book costs $12. This means that the price minus 20% of the price is $12.

$$p - 0.20p = 12$$

Solve

$$p - 0.20p = 0.8p, \text{ so } 0.8p = 12$$

$$p = \frac{12}{0.8} = 15$$

Answer

The price of the book before the discount was $15.

Check

If the original price was $15, then the book was discounted by 20% of $15, or $3. $\$15 - 3 = \$12$. **The answer checks out**.

Example 3

Define the variables and translate the following expressions into inequalities.

a) The sum of 5 and a number is less than or equal to 2.

b) The distance from San Diego to Los Angeles is less than 150 miles.

c) Diego needs to earn more than an 82 on his test to receive a B in his algebra class.

d) A child needs to be 42 inches or more to go on the roller coaster.

Solution

a) **Define**

Let $n =$ the unknown number.

Translate

$5 + n \leq 2$

b) **Define**

Let $d =$ the distance from San Diego to Los Angeles in miles.

Translate

$d < 150$

c) **Define**

Let $x =$ Diego's test grade.

Translate

$x > 82$

d) **Define**

Let $h =$ the height of child in inches.

Translate:

$h \geq 42$

Check Solutions to Equations

You will often need to check solutions to equations in order to check your work. In a math class, checking that you arrived at the correct solution is very good practice. We check the solution to an equation by replacing the variable in an equation with the value of the solution. A solution should result in a true statement when plugged into the equation.

Example 4

Check that the given number is a solution to the corresponding equation.

a) $y = -1;\ 3y + 5 = -2y$

b) $z = 3;\ z^2 + 2z = 8$

c) $x = -\frac{1}{2};\ 3x + 1 = x$

Solution

Replace the variable in each equation with the given value.

a)
$$\begin{aligned} 3(-1) + 5 &= -2(-1) \\ -3 + 5 &= 2 \\ 2 &= 2 \end{aligned}$$

This is a true statement. This means that $y = -1$ is a solution to $3y + 5 = -2y$.

b)
$$\begin{aligned} 3^2 + 2(3) &= 8 \\ 9 + 6 &= 8 \\ 15 &= 8 \end{aligned}$$

This is not a true statement. This means that $z = 3$ is **not a solution** to $z^2 + 2z = 8$.

c)
$$\begin{aligned} 3\left(-\frac{1}{2}\right) + 1 &= -\frac{1}{2} \\ \left(-\frac{3}{2}\right) + 1 &= -\frac{1}{2} \\ -\frac{1}{2} &= -\frac{1}{2} \end{aligned}$$

This is a true statement. This means that $x = -\frac{1}{2}$ is a solution to $3x + 1 = x$.

Check Solutions to Inequalities

To check the solution to an inequality, we replace the variable in the inequality with the value of the solution. A solution to an inequality produces a true statement when substituted into the inequality.

Example 5

Check that the given number is a solution to the corresponding inequality.

a) $a = 10;\ 20a \leq 250$

b) $b = -0.5;\ \frac{3-b}{b} > -4$

c) $x = \frac{3}{4};\ 4x + 5 \leq 8$

Solution

Replace the variable in each inequality with the given value.

a)
$$\begin{aligned} 20(10) &\leq 250 \\ 200 &\leq 250 \end{aligned}$$

This statement is true. This means that $a = 10$ is a solution to the inequality $20a \leq 250$.

Note that $a = 10$ is not the only solution to this inequality. If we divide both sides of the inequality by 20, we can write it as $a \leq 12.5$. This means that any number less than or equal to 12.5 is also a solution to the inequality.

b)
$$\begin{aligned} \frac{3-(-0.5)}{(-0.5)} &> -4 \\ \frac{3+0.5}{-0.5} &> -4 \\ -\frac{3.5}{0.5} &> -4 \\ -7 &> -4 \end{aligned}$$

This statement is false. This means that $b = -0.5$ is not a solution to the inequality $\frac{3-b}{b} > -4$.

c)
$$\begin{aligned} 4\left(\frac{3}{4}\right) + 5 &\geq 8 \\ 3 + 5 &\geq 8 \\ 8 &\geq 8 \end{aligned}$$

This statement is true. It is true because this inequality includes an equals sign; since 8 is equal to itself, it is also "greater than or equal to" itself. This means that $x = \frac{3}{4}$ is a solution to the inequality $4x + 5 \leq 8$.

Solve Real-World Problems Using an Equation

Let's use what we have learned about defining variables, writing equations and writing inequalities to solve some real-world problems.

Example 6

Tomatoes cost $0.50 each and avocados cost $2.00 each. Anne buys six more tomatoes than avocados. Her total bill is $8. How many tomatoes and how many avocados did Anne buy?

Solution

Define

Let $a =$ the number of avocados Anne buys.

Translate

Anne buys six more tomatoes than avocados. This means that $a + 6 =$ the number of tomatoes.

Tomatoes cost $0.50 each and avocados cost $2.00 each. Her total bill is $8. This means that .50 times the number of tomatoes plus 2 times the number of avocados equals 8.

$$\begin{aligned} 0.5(a+6)+2a &= 8 \\ 0.5a+0.5\cdot 6+2a &= 8 \\ 2.5a+3 &= 8 \\ 2.5a &= 5 \\ a &= 2 \end{aligned}$$

Remember that $a =$ the number of avocados, so Anne buys two avocados. The number of tomatoes is $a+6=2+6=8$.

Answer

Anne bought 2 avocados and 8 tomatoes.

Check

If Anne bought two avocados and eight tomatoes, the total cost is:

$(2\times 2)+(8\times 0.5)=4+4=8$. **The answer checks out.**

Example 7

To organize a picnic Peter needs at least two times as many hamburgers as hot dogs. He has 24 hot dogs. What is the possible number of hamburgers Peter has?

Solution

Define

$x =$ Let number of hamburgers

Translate

Peter needs at least two times as many hamburgers as hot dogs. He has 24 hot dogs.

This means that twice the number of hot dogs is less than or equal to the number of hamburgers.

$$2 \times 24 \leq x, \text{ or } 48 \leq x$$

Answer

Peter needs at least 48 hamburgers.

Check

48 hamburgers is twice the number of hot dogs. So more than 48 hamburgers is more than twice the number of hot dogs. **The answer checks out**.

Additional Resources

For more practice solving inequalities, check out http://www.aaastudy.com/equ725x7.htm.

7. Variable Expressions

Objectives

- Evaluate algebraic expressions.
- Evaluate algebraic expressions with exponents.

Concept

Introduction - The Language of Algebra

No one likes doing the same problem over and over again—that's why mathematicians invented algebra. Algebra takes the basic principles of math and makes them more general, so we can solve a problem once and then use that solution to solve a group of similar problems.

In arithmetic, you've dealt with numbers and their arithmetical operations (such as $+, -, \times, \div$). In algebra, we use symbols called **variables** (which are usually letters, such as $x, y, a, b, c, \ldots$) to represent numbers and sometimes processes.

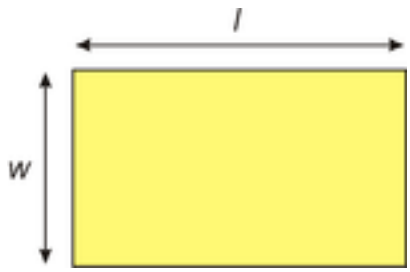
For example, we might use the letter x to represent some number we don't know yet, which we might need to figure out in the course of a problem. Or we might use two letters, like x and y, to show a relationship between two numbers without needing to know what the actual numbers are. The same letters can represent a wide range of possible numbers, and the same letter may represent completely different numbers when used in two different problems.

Using variables offers advantages over solving each problem "from scratch." With variables, we can:

- Formulate arithmetical laws such as $a + b = b + a$ for all real numbers a and b.
- Refer to "unknown" numbers. For instance: find a number x such that $3x + 1 = 10$.
- Write more compactly about functional relationships such as, "If you sell " x tickets, then your profit will be $3x - 10$ dollars, or "$f(x) = 3x - 10$," where "f" is the profit function, and x is the input (i.e. how many tickets you sell).

Example 1

Write an algebraic expression for the perimeter and area of the rectangle below.



To find the perimeter, we add the lengths of all 4 sides. We can still do this even if we don't know the side lengths in numbers, because we can use variables like l w and to represent the unknown length and width. If we start at the top left and work clockwise, and if we use the letter P to represent the perimeter, then we can say:

$$P = l + w + l + w$$

We are adding 2 l's and 2 w's, so we can say that:

$$P = 2 \cdot l + 2 \cdot w$$

It's customary in algebra to omit multiplication symbols whenever possible. For example, $11x$ means the same thing as $11 \cdot x$ or $11 \times x$. We can therefore also write:

$$P = 2l + 2w$$

Area is *length multiplied by width*. In algebraic terms we get:

$$A = l \times w \rightarrow A = l \cdot w \rightarrow A = lw$$

Note: $2l + 2w$ by itself is an example of a **variable expression**; $P = 2l + 2w$ is an example of an **equation**. The main difference between expressions and **equa**tions is the presence of an **equa**ls sign (=).

In the above example, we found the simplest possible ways to express the perimeter and area of a rectangle when we don't yet know what its length and width actually are. Now, when we encounter a rectangle whose dimensions we do know, we can simply substitute (or **plug in**) those values in the above

equations. In this chapter, we will encounter many expressions that we can evaluate by plugging in values for the variables involved.

Evaluate Algebraic Expressions

When we are given an algebraic expression, one of the most common things we might have to do with it is **evaluate** it for some given value of the variable. The following example illustrates this process.

Example 2

Let $x = 12$. *Find the value of* $2x - 7$.

To find the solution, we substitute 12 for x in the given expression. Every time we see x, we replace it with 12.

$$\begin{aligned} 2x - 7 &= 2(12) - 7 \\ &= 24 - 7 \\ &= 17 \end{aligned}$$

Note: At this stage of the problem, we place the substituted value in parentheses. We do this to make the written-out problem easier to follow, and to avoid mistakes. (If we didn't use parentheses and also forgot to add a multiplication sign, we would end up turning $2x$ into 212 instead of 2 times 12!)

Example 3

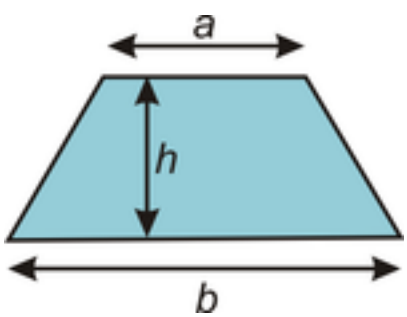
$y = -2$. *Let Find the* $\frac{7}{y} - 11y + 2$ *value of*

Solution

$$\begin{aligned}\frac{7}{(-2)} - 11(-2) + 2 &= -3\frac{1}{2} + 22 + 2 \\ &= 24 - 3\frac{1}{2} \\ &= 20\frac{1}{2}\end{aligned}$$

Many expressions have more than one variable in them. For example, the formula for the perimeter of a rectangle in the introduction has two variables: length (l) and width (w). In these cases, be careful to substitute the appropriate value in the appropriate place.

Example 4



The area of a trapezoid is given by the equation $A = \frac{h}{2}(a+b)$. *Find the area of a trapezoid with bases* $a = 10\ cm$ *and* $b = 15\ cm$ *and height* $h = 8\ cm$.

To find the solution to this problem, we simply take the values given for the $a,\ b,$ variables and h, and plug them in to the expression for A:

$$\begin{aligned}A &= \frac{h}{2}(a+b) && \text{Substitute } 10 \text{ for } a,\ 15 \text{ for } b, \text{ and } 8 \text{ for } h. \\ A &= \frac{8}{2}(10+15) && \text{Evaluate piece by piece. } 10+15=25;\ \frac{8}{2}=4. \\ A &= 4(25) = 100\end{aligned}$$

Solution: *The area of the trapezoid is 100 square centimeters.*

Evaluate Algebraic Expressions with Exponents

Many formulas and equations in mathematics contain exponents. Exponents are used as a short-hand notation for repeated multiplication. For example:

$$2 \cdot 2 = 2^2$$
$$2 \cdot 2 \cdot 2 = 2^3$$

The exponent stands for how many times the number is used as a factor (multiplied). When we deal with integers, it is usually easiest to simplify the expression. We simplify:

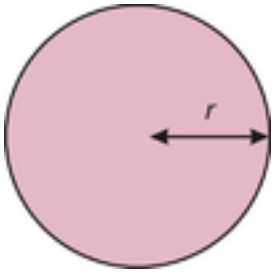
$$2^2 = 4$$
$$2^3 = 8$$

However, we need exponents when we work with variables, because it is much easier to write x^8 than $x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$.

To evaluate expressions with exponents, substitute the values you are given for each variable and simplify. It is especially important in this case to substitute using parentheses in order to make sure that the simplification is done correctly.

For a more detailed review of exponents and their properties, check out the video at http://www.mathvids.com/lesson/mathhelp/863-exponents---basics.

Example 5



The area of a circle is given by the formula $A = \pi r^2$*. Find the area of a circle with radius* $r = 17\ inches$*.*

Substitute values into the equation.

$$A = \pi r^2 \qquad \text{Substitute } 17 \text{ for } r.$$
$$A = \pi(17)^2 \qquad \pi \cdot 17 \cdot 17 \approx 907.9202\ldots \text{ Round to 2 decimal places.}$$

The area is approximately 907.92 square inches.

Example 6

Find the value of $\frac{x^2y^3}{x^3+y^2}$ *, for* $x = 2$ $y = -4$ *and .*

Substitute the values of x y and in the following.

$$\frac{x^2y^3}{x^3+y^2} = \frac{(2)^2(-4)^3}{(2)^3+(-4)^2} \qquad \text{Substitute } 2 \text{ for } x \text{ and } -4 \text{ for } y.$$
$$\frac{4(-64)}{8+16} = \frac{-256}{24} = \frac{-32}{3} \qquad \text{Evaluate expressions: } (2)^2 = (2)(2) = 4 \text{ and}$$
$$(2)^3 = (2)(2)(2) = 8.\ (-4)^2 = (-4)(-4) = 16 \text{ and}$$
$$(-4)^3 = (-4)(-4)(-4) = -64.$$

Example 7

The height (h) *of a ball in flight is given by the formula* $h = -32t^2 + 60t + 20$*, where the height is given in feet and the time* (t) *is given in seconds. Find the height of the ball at time* $t = 2\ seconds$*.*

Solution

$$\begin{aligned} h &= -32t^2 + 60t + 20 \\ &= -32(2)^2 + 60(2) + 20 \qquad \text{Substitute } 2 \text{ for } t. \\ &= -32(4) + 60(2) + 20 \\ &= 12 \end{aligned}$$

The height of the ball is 12 feet.