

**Secondary One Mathematics:
An Integrated Approach
Module 2
Arithmetic and Geometric
Sequences**

By

The Mathematics Vision Project:

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Module 2 – Arithmetic and Geometric Sequences

Classroom Task: Growing Dots- A Develop Understanding Task

Representing arithmetic sequences with equations, tables, graphs, and story context (F.BF.1, F.LE.1, F.LE.2, F.LE.5)

Ready, Set, Go Homework: Sequences 1

Classroom Task: Growing, Growing Dots – A Develop Understanding Task

Representing geometric sequences with equations, tables, graphs, and story context (F.BF.1, F.LE.1, F.LE.2, F.LE.5)

Ready, Set, Go Homework: Sequences 2

Classroom Task: Scott’s Workout – A Solidify Understanding Task

Arithmetic sequences: Constant difference between consecutive terms (F.BF.1, F.LE.1, F.LE.2, F.LE.5)

Ready, Set, Go Homework: Sequences 3

Classroom Task: Don’t Break the Chain – A Solidify Understanding Task

Geometric Sequences: Constant ratio between consecutive terms (F.BF.1, F.LE.1, F.LE.2, F.LE.5)

Ready, Set, Go Homework: Sequences 4

Classroom Task: Something to Chew On – A Solidify Understanding Task

Arithmetic Sequences: Increasing and decreasing at a constant rate (F.BF.1, F.LE.1, F.LE.2, F.LE.5)

Ready, Set, Go Homework: Sequences 5

Classroom Task: Chew On This – A Solidify Understanding Task

Comparing rates of growth in arithmetic and geometric sequences (F.BF.1, F.LE.1, F.LE.2, F.LE.5)

Ready, Set, Go Homework: Sequences 6

Classroom Task: What Comes Next? What Comes Later? – A Solidify Understanding Task

Recursive and explicit equations for arithmetic and geometric sequences (F.BF.1a, F.LE.1, F.LE.2)

Ready, Set, Go Homework: Sequences 7

Classroom Task: What Does It Mean? – A Solidify Understanding Task

Using rate of change to find missing terms in an arithmetic sequence (F.LE.2, A.REI.3)

Ready, Set, Go Homework: Sequences 8

Classroom Task: Geometric Meanies – A Solidify and Practice Understanding Task

Using a constant ratio to find missing terms in a geometric sequence (F.LE.2, A.REI.3, see Math 1 note)

Ready, Set, Go Homework: Sequences 9

Classroom Task: I Know . . . What Do You Know? – A Practice Understanding Task

Developing fluency with geometric and arithmetic sequences (F.LE.2)

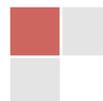
Ready, Set, Go Homework: Sequences 10

Homework Help for Students and Parents

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Core standards addressed in this unit:

F-BF: Build a function that models a relationship between two quantities.

1: Write a function that describes a relationship between two quantities.*

- a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-LE: Linear, Quadratic, and Exponential Models* (Secondary I focus is linear and exponential only)
Construct and compare linear, quadratic and exponential models and solve problems.

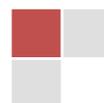
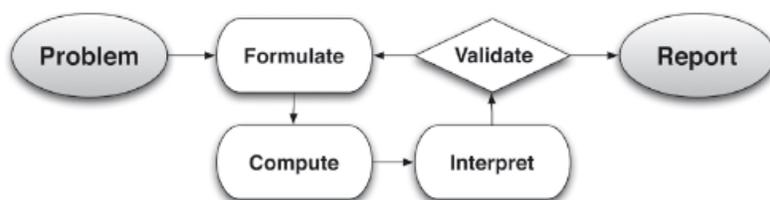
1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
 - a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
 - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
 - c. Recognize situations in which one quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expression for functions in terms of the situation they model.

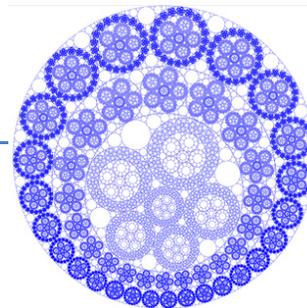
5. Interpret the parameters in a linear or exponential function in terms of a context.

Tasks in this unit also follow the structure suggested in the Modeling standard:

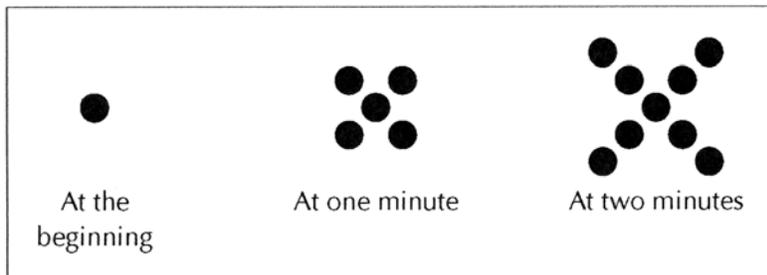


Growing Dots

A Develop Understanding Task



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1. Describe the pattern that you see in the sequence of figures above.
2. Assuming the sequence continues in the same way, how many dots are there at 3 minutes?
3. How many dots are there at 100 minutes?
4. How many dots are there at t minutes?

Solve the problems by your preferred method. Your solution should indicate how many dots will be in the pattern at 3 minutes, 100 minutes, and t minutes. Be sure to show how your solution relates to the picture and how you arrived at your solution.



Growing Dots – Teacher Notes

A Develop Understanding Task

Purpose: The purpose of this task is to develop representations for arithmetic sequences that students can draw upon throughout the module. The visual representation in the task should evoke lists of numbers, tables, graphs, and equations. Various student methods for counting and considering the growth of the dots will be represented by equivalent expressions that can be directly connected to the visual representation.

Core Standards:

F-BF: Build a function that models a relationship between two quantities.

1: Write a function that describes a relationship between two quantities.*

- a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-LE: Linear, Quadratic, and Exponential Models* (Secondary I focus in linear and exponential only)
Construct and compare linear, quadratic and exponential models and solve problems.

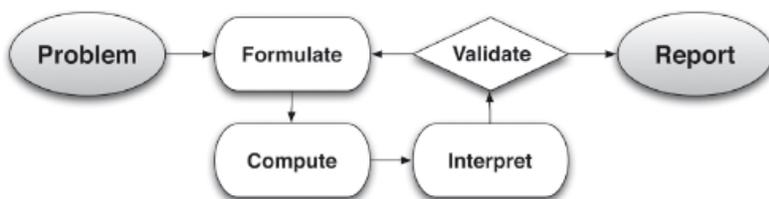
1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
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 - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expression for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

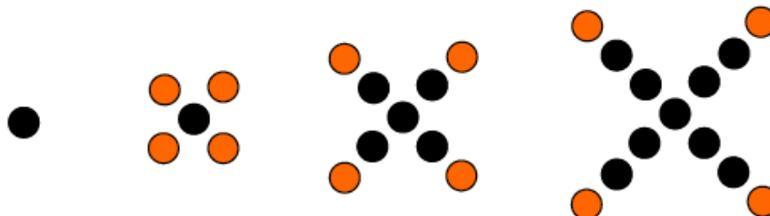
This task also follows the structure suggested in the Modeling standard:



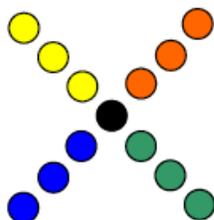
Launch (Whole Class): Start the discussion with the pattern on growing dots drawn on the board or projected for the entire class. Ask students to describe the pattern that they see in the dots (Question #1). Students may describe four dots being added each time in various ways, depending on how they see the growth occurring. This will be explored later in the discussion as students write equations, so there should not be any emphasis placed upon a particular way of seeing the growth. Ask students individually to consider and draw the figure that they would see at 3 minutes

(Question #2). Then, ask one student to draw it on the board to give other students a chance to check that they are seeing the pattern.

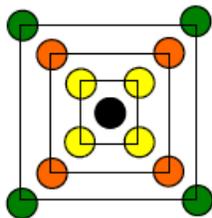
Explore (Small Group or Pairs): Ask students to complete the task. Monitor students as they work, observing their strategies for counting the dots and thinking about the growth of the figures. Some students may think about the figures recursively, describing the growth by saying that the next figure is obtained by placing four dots onto the previous figure as shown:



Some may think of the figure as four arms of length t , with a dot in the middle.



Others may use a “squares” strategy, noticing that a new square is added each minute, as shown:



As students work to find the number of dots at 100 minutes, they may look for patterns in the numbers, writing simply 1, 5, 9, . . . If students are unable to see a pattern, you may encourage them to make a table or graph to connect the number of dots with the time:

Time (Minutes)	Number of Dots
0	1
1	5
2	9
3	13
t	

Watch for students that have used a graph to show the number of dots at a given time and to help write an equation. Encourage students to connect their counting strategy to the equation that they write.

For the discussion, select a student for each of the three counting strategies shown, a table, a graph, a recursive equation, and at least one form of an explicit equation.

Discuss (Whole Group): Begin the discussion by asking students how many dots that there will be at 100 minutes. There may be some disagreement, typically between 100 and 101. Ask a student that said 101 to explain how they got their answer. If there is general agreement, move on to the discussion of the number of dots at time t .

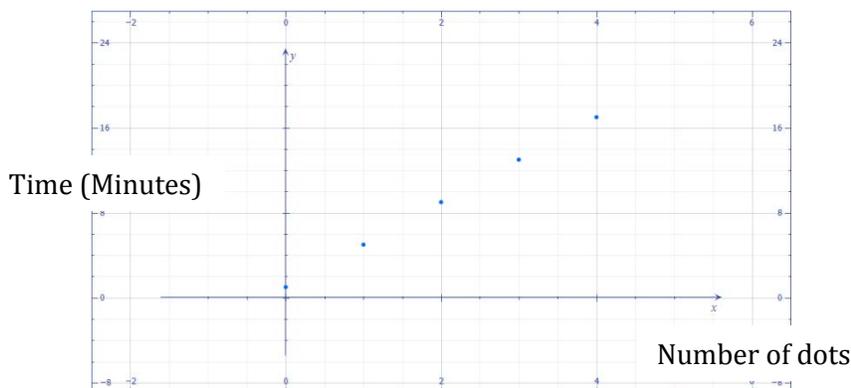
Start by asking a group to chart and explain their table. Ask students what patterns they see in the table. When they describe that the number of dots is growing by 4 each time, add a difference column to the table, as shown.

Time (Minutes)	Number of Dots	Difference
0	1	
1	5	> 4
2	9	> 4
3	13	> 4
...	...	
t		

Ask students where they see the difference of 4 occurring in the figures. Note that the difference between terms is constant each time.



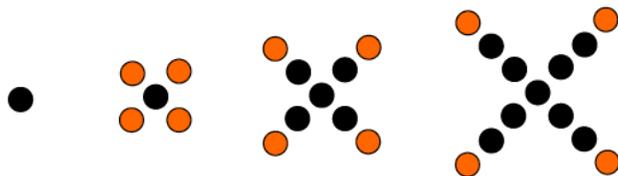
Continue the discussion by asking a group to show their graph. Be sure that it is properly labeled, as shown.



Ask students how they see the constant difference of 4 on the graph. They should recognize that the y-value increases by 4 each time, making a line with a slope of 4.

Now, move the discussion to consider the number of dots at time t , as represented by an equation. Start with a group that considered the growth as a recursive pattern, recognizing that the next term is 4 plus the previous term. They may represent the idea as: $X + 4$, with X representing the previous term. This may cause some controversy with students that wrote a different formula. Ask the group to explain their work using the figures. It may be useful to rewrite their formula with words, like:

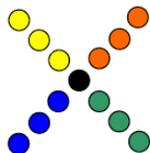
The number of dots in the current figure = the number of dots in the previous figure + 4



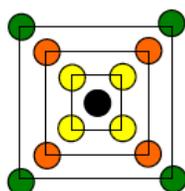
This may be written in function notation as: $f(t) = f(t - 1) + 4$. (You may choose to introduce this idea later in the discussion.)

Next ask a group that has used the “four arms strategy” to write and explain their equation. Their equation should be: $f(t) = 4t + 1$. Ask students to connect their equation to the figure. They should articulate that there is 1 dot in the middle and 4 arms, each with t dots. The 4 in the equation shows 4 groups of size t .





Next, ask a group that used the “squares” strategy to describe their equation. They may have written the same equation as the “four arms” group, but ask them to relate each of the numbers in the equation to the figures anyway. In this way of thinking about the figures, there are t groups of 4 dots, plus 1 dot in the middle. Although it is not typically written this way, this counting method would generate the equation $f(t) = t \cdot 4 + 1$.



Now ask students to connect the equations with the table and graphs. Ask them to show what the 4 and the 1 represent in the graph. Ask how they see $4t + 1$ in the table. It may be useful to show this pattern to help see the pattern between the time and the number of dots:

Time (Minutes)	Number of Dots	Difference
0	1	1
1	5	1+4
2	9	1+4+4
3	13	1+4+4+4
...	...	
t		$1+4t$

> 4
> 4
> 4

You may also point out that when the table is used to write a recursive equation like $f(t) = f(t - 1) + 4$, you may simply look down the table from one output to the next. When writing an explicit formula like $f(t) = 4t + 1$, it is necessary to look across the rows of the table to connect the input with the output.

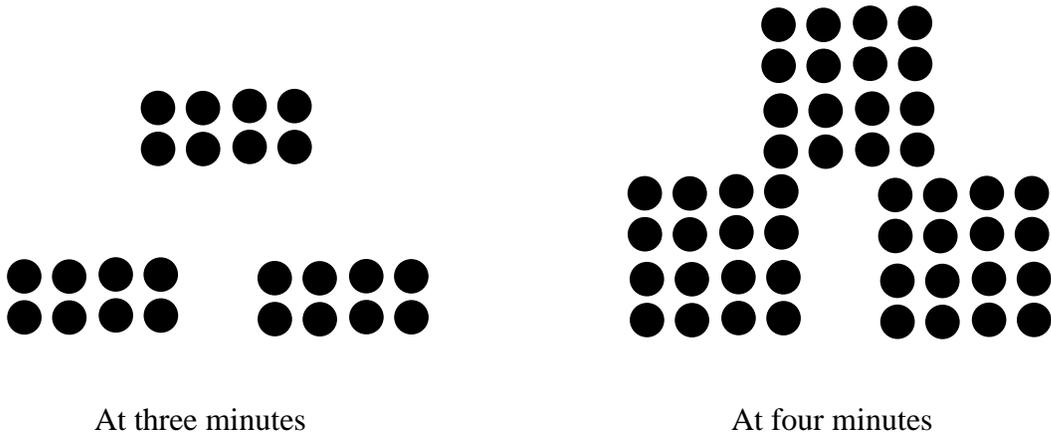
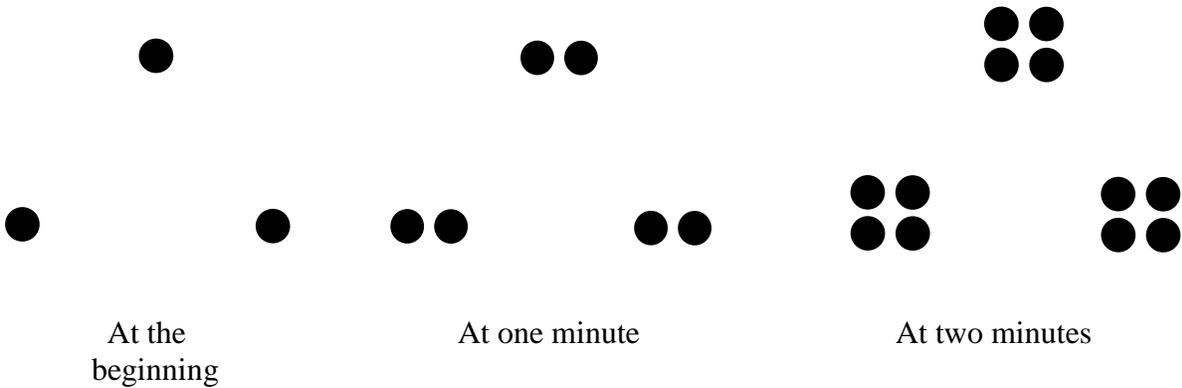
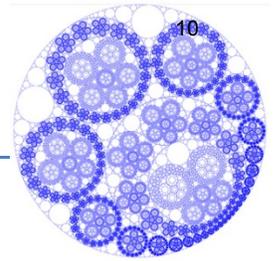
Finalize the discussion by explaining that this set of figures, equations, table, and graph represent an arithmetic sequence. An arithmetic sequence can be identified by the constant difference between consecutive terms. Tell students that they will be working with other sequences of numbers that may not fit this pattern, but tables, graphs and equations will be useful tools to represent and discuss the sequences.

Aligned Ready, Set, Go Homework: Sequences 1

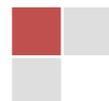


Growing, Growing Dots

A Develop Understanding Task



1. Describe and annotate the pattern of change you see in the above sequence of figures.
2. Assuming the sequence continues in the same way, how many dots are there at 5 minutes?
3. Write a recursive formula to describe how many dots there will be after t minutes?
4. Write an explicit formula to describe how many dots there will be after t minutes?



Growing, Growing Dots – Teacher Notes

Purpose: The purpose of this task is to develop representations for geometric sequences that students can draw upon throughout the module. The visual representation in the task should evoke lists of numbers, tables, graphs, and equations. Various student methods for counting and considering the growth of the dots will be represented by equivalent expressions that can be directly connected to the visual representation.

Core Standards:

F-BF: Build a function that models a relationship between two quantities.

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F-LE: Linear, Quadratic, and Exponential Models* (Secondary Mathematics I focus in linear and exponential only)

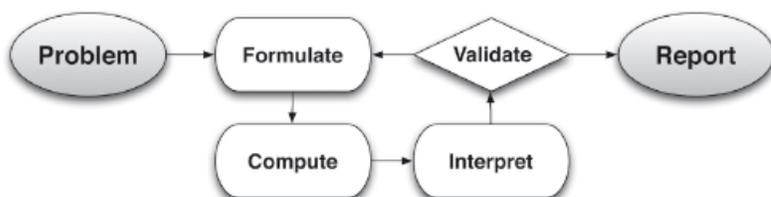
Construct and compare linear, quadratic and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
 - a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
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2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expression for functions in terms of the situation they model.

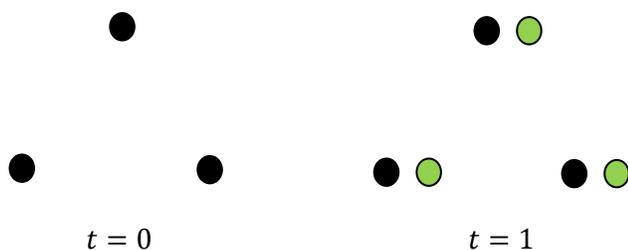
5. Interpret the parameters in a linear or exponential function in terms of a context.

This task also follows the structure suggested in the Modeling standard:

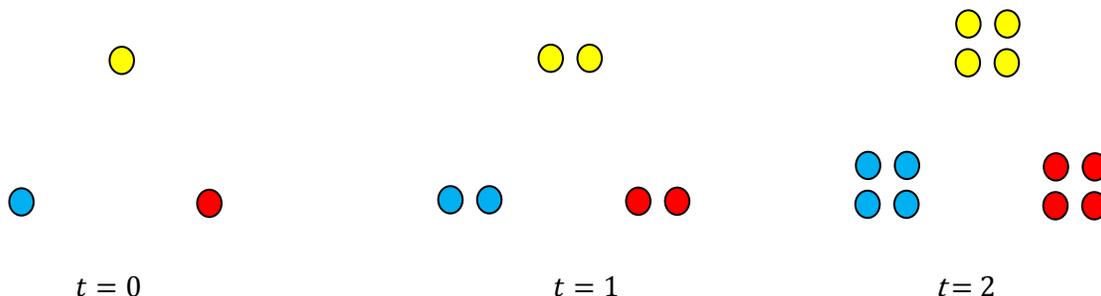


Launch (Whole Class): Start the discussion with the pattern on growing dots drawn on the board or projected for the entire class. Ask students to describe the pattern that they see in the dots (Question #1). Students may describe an increasing number of triangles being added each time or seeing three groups that each have an increasing number of dots each time, depending on how they see the growth occurring. This will be explored later in the discussion as students write equations, so there should not be any emphasis placed upon a particular way of seeing the growth. Ask students individually to consider and draw the figure that they would see at 5 minutes (Question #2). Then, ask one student to draw it on the board to give other students a chance to check that they are seeing the pattern correctly. Remind students of the work they did yesterday to write explicit and recursive formulas. These are new terms that should be reinforced at the beginning to clarify the instructions for questions 3 and 4.

Explore (Small Group or Pairs): Ask students to complete the task. Monitor students as they work, observing their strategies for counting the dots and thinking about the growth of the figures. Some students may think about the figures recursively, describing the growth by saying that the next figure is obtained doubling the previous figure as shown:



Some may think of the figure as three groups that are each doubling.



As students work to find the formulas, they may look for patterns in the numbers, writing simply 3, 6, 12, 24, 48. If students are unable to see a pattern, you may encourage them to make a table or graph to connect the number of dots with the time:

Time (Minutes)	Number of Dots
0	3
1	6
2	12
3	24
4	48

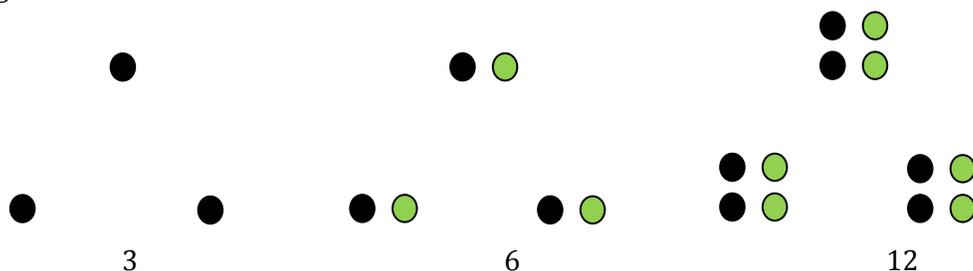


Watch for students that have used a graph to show the number of dots at a given time and to help write an equation. Encourage students to connect their counting strategy to the equation that they write.

For the discussion, select a student for each of the counting strategies shown, a table, a graph, a recursive equation, and at least one form of an explicit equation. Have two large charts showing the dot figures prepared in advance for students to use in explaining their counting strategies.

Discuss (Whole Group):

Begin the discussion with the group that saw the pattern as doubling the previous figure each time. Ask them to explain how they thought about the pattern and how they annotated the figures.



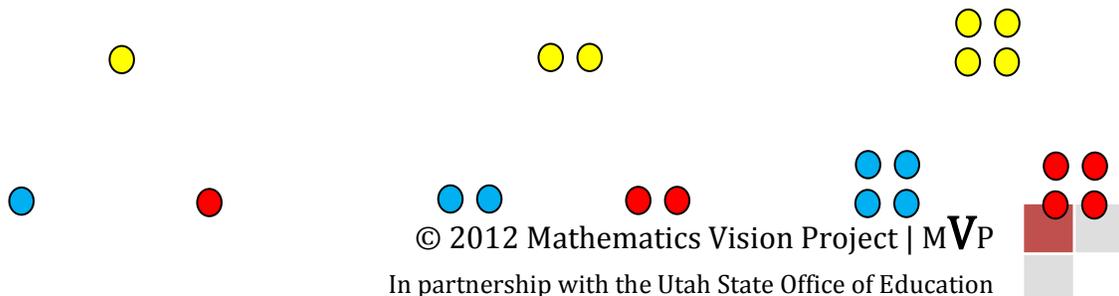
Often, students who are using this strategy will think of the number of dots, without thinking of the relationship between the number of dots and the time. If they don't mention the time at this point, be careful to point out the relationship with time when the next group presents a strategy that connects the time and the number of dots.

Ask students to describe the pattern they see and record their words:

Next figure = $2 \times$ Previous figure

Ask students to represent this idea algebraically. They may respond with expressions like: $2X$ where X is the number of dots in the previous figure. This is a chance to remind students that the instructions ask for a recursive formula that describes the number of dots at t minutes, which implies that the equation should be a function of t . If no student has written the equation in function form, help the class to understand how the equation $f(0) = 3, f(t) = 2f(t - 1)$ expresses the idea that a way to find a term at time t is to double the previous term.

Next, ask the group that saw this pattern of growth to explain the way they saw the pattern of growth.



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$t = 0$

$t = 1$

$t = 2$

Ask for a table to that shows the relationship between time and the number of dots. Ask students what patterns they see in the table. Ask students to add a difference column to the table, like they did in Growing Dots. Students may be surprised to see the difference between terms repeating the pattern in the number of dots. Ask students if they see a common difference between terms. Explain that since there is no common difference, it is not an arithmetic sequence.

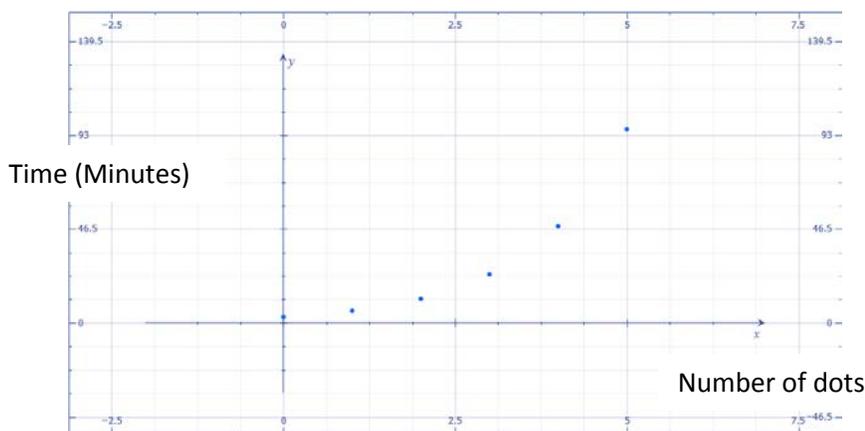
Time (Minutes)	Number of Dots	Difference
0	3	> 3
1	6	> 6
2	12	> 12
3	24	> 24
4	48	

At this point, it can be pointed out that since you get the next term by doubling the previous term, there is a common ratio between terms. Demonstrate that:

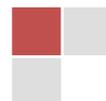
$$\frac{6}{3} = \frac{12}{6} = \frac{24}{12} = 2$$

The common ratio between terms is the identifying feature of a geometric sequence, another special type of number sequence.

Continue the discussion by asking a group to show their graph. Ask the class what they predict the graph to look like. Why would we not expect the graph to be a line? Be sure the graph it is properly labeled, as shown.



Now, move the discussion to consider the number of dots at time t , as represented by an explicit equation. Ask a group to show their explicit formula for the number of dots at time t , which is: $f(t) = 3 \cdot 2^t$.



Now ask students to connect the equations with the table and graphs. Ask them to show what the 2 and the 3 represent in the graph. Ask how they see $3 \cdot 2^t$ in the table. It may be useful to show this pattern to help see the pattern between the time and the number of dots:

Time (Minutes)	Number of Dots		Difference
0	3	3	> 3
1	6	$3 \cdot 2$	> 6
2	12	$3 \cdot 2 \cdot 2$	> 12
3	24	$3 \cdot 2 \cdot 2 \cdot 2$	> 24
4	48	$3 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	
...	...		
t	$3 \cdot 2^t$		

You may also remind students that when the table is used to write a recursive equation like $f(0) = 3, f(t) = 2f(t - 1)$ you may simply look down the table from one output to the next. When writing an explicit formula like $f(t) = 3 \cdot 2^t$, it is necessary to look across the rows of the table to connect the input with the output.

Finalize the discussion by explaining that this set of figures, equations, table, and graph represent a geometric sequence. A geometric sequence can be identified by the constant ratio between consecutive terms. Tell students that they will continue to work with sequences of numbers using tables, graphs and equations to identify and represent geometric and arithmetic sequences.

Aligned Ready, Set, Go Homework: Sequences 2



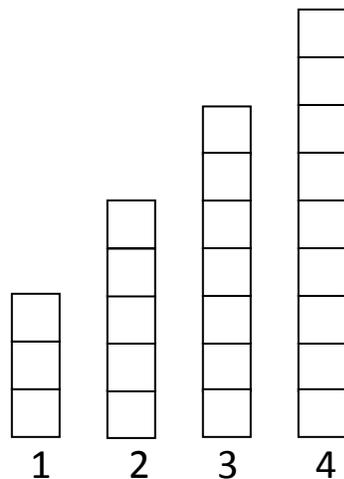
Scott's Workout

A Solidify Understanding Task



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Scott has decided to add push-ups to his daily exercise routine. He is keeping track of the number of push-ups he completes each day in the bar graph below, with day one showing he completed three push-ups. After four days, Scott is certain he can continue this pattern of increasing the number of push-ups he completes each day.



1. How many push-ups will Scott do on day 10?
2. How many push-ups will Scott do on day n ?
3. Model the number of push-ups Scott will complete on any given day. Include both explicit and recursive equations.
4. Aly is also including push-ups in her workout and says she does more push-ups than Scott because she does fifteen push-ups every day. Is she correct? Explain.



Scott's Workout – Teacher Notes

Purpose: This task is to SOLIDIFY understanding that arithmetic sequences have a constant difference between consecutive terms. The task is designed to generate tables, graphs, and both recursive and explicit formulas. The focus of the task should be to identify how the constant difference shows up in each of the representations and defines the functions as an arithmetic sequence/linear function.

Standards Focus:

F-BF: Build a function that models a relationship between to quantities.

1: Write a function that describes a relationship between two quantities.*

- a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

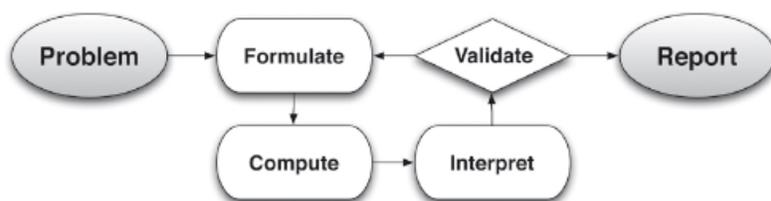
F-LE: Linear, Quadratic, and Exponential Models* (Secondary I focus in linear and exponential only)
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 - a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
 - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expression for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

This task also follows the structure suggested in the Modeling standard:



Launch (Whole Class): Remind students of the work they have done previously with Growing Dots and Growing, Growing Dots. Read Scott's Workout with the students and ask one student to model the number of push-ups that Scott will do on the fifth day. Ask students what they are observing about the pattern. Allow just a few responses so that you know that students understand the task, but avoid giving away the work of the task. Have students work on the task in small groups (2-4 students per group).

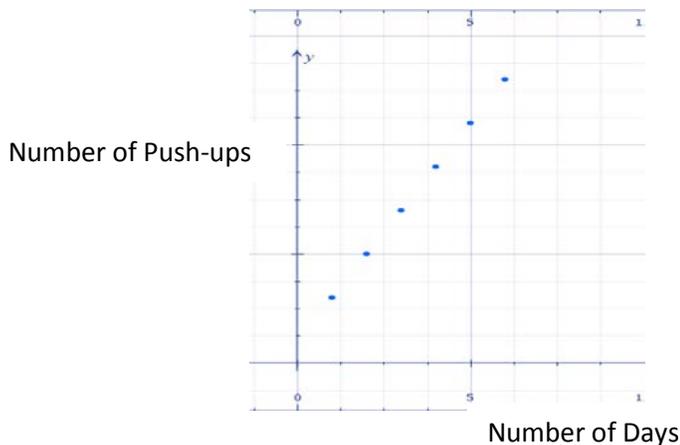
Explore (Small Group): Monitor student thinking as they work by moving from one group to another. Encourage students to use tables, graphs, and recursive and explicit equations as they work on the task. Listen to students and identify different groups to present and explain their work on one representation each. If students are having difficulty writing the equation, ask them to be sure that they have the other representations first.

Discuss (Whole Class): When the various groups are prepared to present, start the discussion with a table. Be sure that the columns of the table are labeled. After students have presented their table, ask students to identify the difference between consecutive terms and mark the table so that it looks like this:

x Days	$f(x)$ Push-ups	Difference between terms
1	3	
2	5	> 2
3	7	> 2
4	9	> 2
5	11	
...	...	
n	$3 + 2(n - 1)$	

Ask if the sequence is arithmetic or geometric based upon the table. Students should be able to identify that it is arithmetic because there is a constant difference between consecutive terms.

Next, ask the students to present the graph. The graph should be labeled and look like this:



Ask students where they see the difference between terms from the table on the graph. Identify that for each day, the number of push-ups increases by 2, so for each increase of 1 in the x value, the y value increases by 2. The students should recognize this as a slope of 2. Ask why the points in the graph are not connected. Students should be able to answer that the push-ups are assumed to be done all at once in the day. A continuous graph would suggest that the push-ups were happening for the entire time shown on the graph.

Next, ask students for their recursive equations. Students may have written any of these equations:

$$\text{Number of push-ups today} = \text{Number of push-ups yesterday} + 2$$

Or:

$$\text{Next term} = \text{Previous term} + 2$$

Or:

$$y = X + 2 \text{ (where } X \text{ is a way to denote the previous term).}$$

Ask how they see this equation in their table and their graph. On the table, they should point out that as you move from one row to the next, you add 2 to the previous term. They should be able to demonstrate a similar idea on the graph as you move from one y -value to the next.

Ask if anyone has written their recursive equation in function form. If no one has written the equation in function form, explain that the more formal method of writing the equation is: $f(1) = 3, f(n) = f(n - 1) + 2$. This form still denotes the idea that the current term is 2 more than the previous term. Ask students how they can identify that this is an arithmetic sequence using the recursive equation. The answer should be that the constant difference of 2 between terms shows up in the equation as adding 2 to get the next term. Also note that to use a recursive formula you have to know the previous term. That means that when you have a recursive formula for an arithmetic sequence, you need to provide the first term as part of the formula.

Conclude the discussion with the explicit equation, $f(n) = 3 + 2(n - 1)$. Although this equation could be simplified, it is useful to consider it in this form. Ask students how they used the table to write this equation. How does this formula show the constant difference between terms? Also ask, "If you are looking for the 10th term, what number will you multiply by 2?" Help students to connect that the $(n - 1)$ in the formula tells them that the number they multiply by 2 is one less than the term they are looking for. Can they explain that using the table or graph?

Conclude the lesson by asking students to compare recursive and explicit formulas. What information do you need to use either type of formula? What are the advantages of each? What ideas about arithmetic sequences are highlighted in each?

Aligned Ready, Set, Go Homework: Sequences 3

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Don't Break the Chain



Maybe you've received an email like this before:

Hi! My name is Bill Weights, founder of Super Scooper Ice Cream. I am offering you a gift certificate for our signature "Super Bowl" (a \$4.95 value) if you forward this letter to 10 people.

When you have finished sending this letter to 10 people, a screen will come up. It will be your Super Bowl gift certificate. Print that screen out and bring it to your local Super Scooper Ice Cream store. The server will bring you the most wonderful ice cream creation in the world—a Super Bowl with three yummy ice cream flavors and three toppings!

This is a sales promotion to get our name out to young people around the country. We believe this project can be a success, but only with your help. Thank you for your support.

Sincerely,

Bill Weights
Founder of Super Scooper Ice Cream

These chain emails rely on each person that receives the email to forward it on. Have you ever wondered how many people might receive the email if the chain remains unbroken? To figure this out, assume that it takes a day for the email to be opened, forwarded, and then received by the next person. On day 1, Bill Weights starts by sending the email out to his 8 closest friends. They each forward it to 10 people so that on day 2, it is received by 80 people. The chain continues unbroken.

1. How many people will receive the email on day 7?
2. How many people will receive the email on day n ? Explain your answer with as many representations as possible.
3. If Bill gives away a Super Bowl that costs \$4.95 to every person that receives the email during the first week, how much will he have spent?



Don't Break the Chain – Teacher Notes

Purpose: This task is to SOLIDIFY understanding that geometric sequences have a constant ratio between consecutive terms. The task is designed to generate tables, graphs, and both recursive and explicit formulas. The focus of the task should be to identify how the constant ratio shows up in each of the representations.

Core Standards:

F-BF: Build a function that models a relationship between two quantities.

- 1: Write a function that describes a relationship between two quantities.*
 - a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

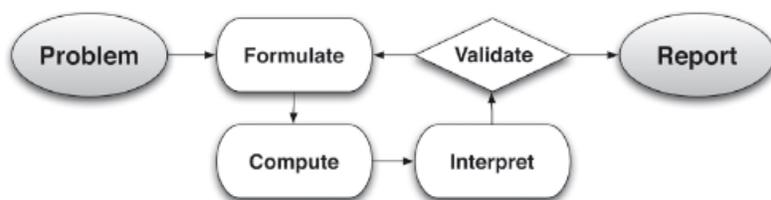
F-LE: Linear, Quadratic, and Exponential Models* (Secondary I focus in linear and exponential only)
Construct and compare linear, quadratic and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
 - a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
 - c. Recognize situations in which one quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expression for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

This task also follows the structure suggested in the Modeling standard:



Launch (Whole Class): Remind students of the work they have done previously with Growing Dots and Growing, Growing Dots. Without handing the task to students, read “Don’t Break the Chain” past the end of the letter to the point where it says that Bill starts by sending the letter to 8 people.

Ask student how many people will receive the email the next day if the chain is unbroken. Ask students what they are observing about the pattern. You may wish to draw a diagram that show the first 8 and then each of them sending the letter to 10 more so that students see that this is not a situation where they are simply adding 10 each time. Be sure to label the diagram to show that there were 8 people on day 1 and 80 on day 2 to avoid confusion about the time. Have students work on the task in small groups (2-4 students per group). Access to graphing calculators will facilitate graphing the function.

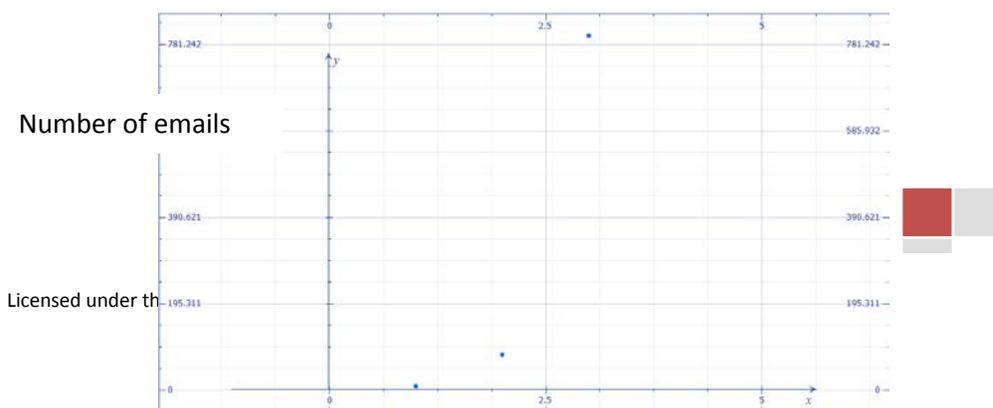
Explore (Small Group): Monitor student thinking as they work by moving from one group to another. Encourage students to use tables, graphs, and recursive and explicit equations as they work on the task. Listen to students and identify different groups to present and explain their work on one representation each. If students are having difficulty writing the equations, ask them to be sure that they have the other representations first. The focus of the task is questions 1 and 2. The third question requires students to sum the first 7 terms of the sequence. While it gives an interesting result, it is an extension for those who finish quickly. When the class is finished with the first two questions, call the class back for a whole group discussion.

Discuss (Whole Class): Start the discussion with a table. Be sure that the columns of the table are labeled. After students have presented their table, ask students to identify the difference between consecutive terms and mark the table so that it looks like this:

x Days	$f(x)$ # of emails	Difference
1	8	> 72
2	80	> 720
3	800	> 7,200
4	8000	
5	80,000	
...	...	

Ask if the sequence is arithmetic based upon the table. Students should be able to identify that it is not arithmetic because there is no constant difference between consecutive terms. Ask, "What other patterns do you see in the table?" Students should notice that the next term is obtained by multiplying by 10. Ask students to test if there is a constant ratio between consecutive terms. They should recognize the ratio of 10. Confirm that the constant ratio between consecutive terms means that this is a geometric sequence.

Next, ask the students to present the graph. The graph should be labeled and look like this:



Ask students what was difficult about creating the graph. They will probably point out that the y -values were increasing so quickly that it was difficult to scale the graph. Ask if the points on the graph make a line. Help them to see that the points make a curve; the graph is not linear. There is a constant difference between terms. Identify that for each day, the number of emails multiplies by 10, so for each increase of 1 in the x value, the y value is 10 times bigger. Ask students which graph this most resembles: Growing Dots or Growing, Growing Dots. Help them to identify the characteristic shape of an increasing exponential function.

Next, ask students for their recursive equations. Students may have written any of these equations:

$$\text{Number of emails today} = 10 * \text{Number of emails yesterday}$$

Or: $\text{Next term} = 10 * \text{Previous term}$

Or: $y = 10X$ (where X is a way to denote the previous term).

Ask how they see this equation in their table and their graph. On the table, they should point out that as you move from one row to the next, you multiply the previous term by 10. They should be able to demonstrate a similar idea on the graph as you move from one y -value to the next.

Ask if anyone has written their recursive equation in function form. If no one has written the equation in function form, explain that the more formal method of writing the equation is: $f(1) = 8, f(n) = 10f(n - 1)$. Ask students how this formula is different than the recursive formula for Scott's workout, an arithmetic sequence. Ask students how they can identify that this is a geometric sequence using the recursive equation. The answer should be that the constant ratio of 10 between terms shows up in the equation as multiplying by 10 to get the next term. Also note that to use a recursive formula you have to know the previous term. That means that when you a recursive formula for an arithmetic sequence, you need to provide the first term as part of the formula.

Conclude the discussion with the explicit formula, $f(n) = 8(10^{n-1})$. Many students may find it difficult to write the pattern that they see in this form. First, it may be hard to see that they need to use an exponent, and then it may be hard to see how the exponent relates to the term number. It may be helpful to refer back to the table, adding the third column shown here.

x Days	$f(x)$ Number of emails	$f(x)$ Number of emails	$f(x)$ Number of emails
1	8	8	$8(10^0)$
2	80	$8 \cdot 10$	$8(10^1)$
3	800	$8 \cdot 10 \cdot 10$	$8(10^2)$
4	8,000	$8 \cdot 10 \cdot 10 \cdot 10$	$8(10^3)$



5	80,000	$8 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	$8(10^4)$
...	
n			$8(10^{n-1})$

Ask students to notice how many times 10 is used as a power and compare it to the number of days. Remind them that to write an explicit formula they need to read across the table and relate the x value to $f(x)$. You may wish to ask students to help you complete a fourth column that shows the powers of 10. Help students to connect that the $(n - 1)$ in the formula tells them the number of times they multiply by 10 is one less than the term they are looking for. Ask students to relate the formula to graph.

Conclude the lesson by asking students to compare the recursive and explicit formulas for arithmetic and geometric sequences. How are the recursive formulas for geometric and arithmetic sequences alike? How are they different? How are the explicit formulas for geometric and arithmetic sequence alike? How are they different?

Aligned Ready, Set, Go Homework: Sequences 4



Something to Chew On

A Solidify Understanding Task



The Food-Mart grocery store has a candy machine like the one pictured here. Each time a child inserts a quarter, 7 candies come out of the machine. The machine holds 15 pounds of candy. Each pound of candy contains about 180 individual candies.

1. Represent the number of candies in the machine for any given number of customers. About how many customers will there be before the machine is empty?
2. Represent the amount of money in the machine for any given number of customers.
3. To avoid theft, the store owners don't want to let too much money collect in the machine, so they take all the money out when they think the machine has about \$25 in it. The tricky part is that the store owners can't tell how much money is actually in the machine without opening it up, so they choose when to remove the money by judging how many candies are left in the machine. About how full should the machine look when they take the money out? How do you know?



Something to Chew On – Teacher Notes

Purpose: This task introduces a decreasing arithmetic sequence to further solidify the idea that arithmetic sequences have a constant difference between consecutive terms. Again, connections should be made among all representations: table, graph, recursive and explicit formulas. The emphasis should be on comparing increasing and decreasing arithmetic sequences through the various representations.

Core Standards:

F-BF: Build a function that models a relationship between two quantities.

1: Write a function that describes a relationship between two quantities.*

- a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-LE: Linear, Quadratic, and Exponential Models* (Secondary I focus in linear and exponential only)
Construct and compare linear, quadratic and exponential models and solve problems.

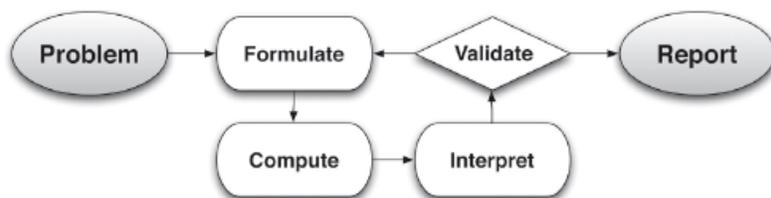
1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
 - a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
 - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expression for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

This task also follows the structure suggested in the Modeling standard:



Launch (Whole class): Before handing out the task, ask students to define an arithmetic sequence. Later we will say that it is a linear function with the domain of positive integers. Right now, expect students to identify the constant rate of change or constant difference between consecutive terms. Ask students to give a few examples of arithmetic sequences. Since the only sequences they have seen up to this point have been increasing, expect them to add a number to get to the next term. Then, wonder out loud whether or not it would be an arithmetic sequence if a number is subtracted to get the next term. Don't answer the question or solicit responses.

Read the opening paragraph of the task and be sure that all students understand how the candy machines work; when a quarter is inserted, 7 candies come out. Read the first prompt in the task and discuss what it means to "Represent the number of candies in the machine on a given day." Explain that their representations should include tables, graphs, and equations. As soon as students understand the task, set them to work.

Explore (Small groups): To begin the task, students will need to decide how to set up their tables or graphs. Tables should be set with customers as the independent variable and the number of candies as the dependent variable. To represent candies vs customers, then students must calculate how many candies in the machine when it is full. From there they will be able to build their tables and graphs by subtracting 7 for each customer. It would be appropriate to have graphing calculators available for this task.

The second prompt (#2) is similar to the increasing geometric sequences that students have previously modeled in Scott's Workout and Growing Dots. Again, encourage as many representations as possible. When you find that most students are finished with number 1 and 2. Number 3 is an extension provided for differentiation, but not the focus of the task for most students.

Monitor the group work with particular focus on the work in #1. Have one group prepared to present the table and one group present the graph. Another group can present both forms of the equations from #1. You may choose to have the presenters begin to draw their tables and graphs while the other groups finish their work. Also select just one group to do all of problem #2 on the board for comparison.

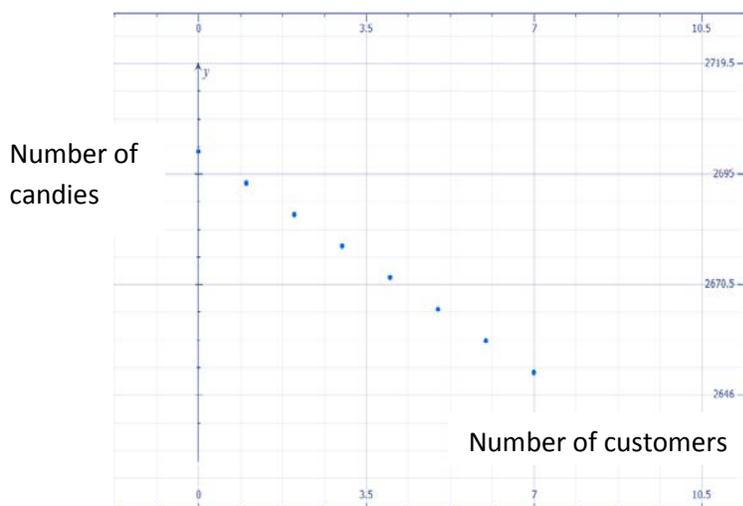
Discuss (Whole Group): Start the discussion by repeating the question that was stated in the launch: Can you form an arithmetic sequence by subtracting a number from each term to get the next term? Ask the group to present the table that they made for #1, which should look something like this, although students may not have included the first difference at the side. If not, add it in as part of the discussion.



# of Customers	# of Candies	Difference
0	2700	> -7
1	2693	> -7
2	2686	> -7
3	2679	> -7
4	2672	> -7
5	2665	> -7
6	2658	> -7
7	2651	> -7
8	2644	
9	2637	
10	2632	
...	...	
n	$2700 - 7n$	

Ask students what they notice about the table. Ask, “Does the table show a constant difference between terms? What is the constant difference? Does this table represent an arithmetic sequence?”

Next, discuss the graph, which should be properly labeled and look something like this:



The constant difference between terms has been demonstrated in the table. Now ask students how this graph of an arithmetic sequence is like the other arithmetic sequences that we have studied in Scott’s Workout and Growing Dots. They should identify that the points form a line and the graph is not continuous. How is this graph different? It is decreasing at a constant rate, rather than increasing at a constant rate.

Ask students to compare the recursive formulas for both #1 and #2. Encourage them to use function notation only, so that their formulas look like:



$$1. f(0) = 2700, f(n) = f(n - 1) - 7 \quad 2. f(1) = .25, f(n) = f(n - 1) + .25$$

Ask students, “Based on the recursive formula, is #2 an arithmetic sequence? Why or why not?”
Expect students to answer that +.25 shows that each term is increasing by a constant amount.

Now ask students to compare the explicit formulas for both #1 and #2. In function notation, they should be like:

$$1. f(n) = 2700 - 7n \quad 2. f(n) = .25n$$

Ask students how they can identify an arithmetic sequence from an explicit equation.

Wrap up the discussion by completing the following chart. Be sure to include ideas about both increasing and decreasing sequences in all representations.

How to Identify an Arithmetic Sequence	
In a table:	
In a graph:	
In a recursive formula:	
In an explicit formula:	

Aligned Ready, Set, Go Homework: Sequences 5



Chew on This

A Solidify Understanding Task



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Mr. and Mrs. Gloop want their son, Augustus, to do his homework every day. Augustus loves to eat candy, so his parents have decided to motivate him to do his homework by giving him candies for each day that the homework is complete. Mr. Gloop says that on the first day that Augustus turns in his homework, he will give him 10 candies. On the second day he promises to give 20 candies, on the third day he will give 30 candies, and so on.

1. Write both a recursive and an explicit formula that shows the number of candies that Augustus earns on any given day with his father's plan.
2. Use a formula to find how many candies Augustus will have on day 30 in this plan.

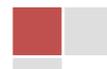
Augustus looks in the mirror and decides that he is gaining weight. He is afraid that all that candy will just make it worse, so he tells his parents that it would be ok if they just give him 1 candy on the first day, 2 on the second day, continuing to double the amount each day as he completes his homework. Mr. and Mrs. Gloop like Augustus' plan and agree to it.

3. Model the amount of candy that Augustus would get each day he reaches his goals with the new plan.
4. Use your model to predict the number of candies that Augustus would earn on the 30th day with this plan.
5. Write both a recursive and an explicit formula that shows the number of candies that Augustus earns on any given day with this plan.



Augustus is generally selfish and somewhat unpopular at school. He decides that he could improve his image by sharing his candy with everyone at school. When he has a pile of 100,000 candies, he generously plans to give away 60% of the candies that are in the pile each day. Although Augustus may be earning more candies for doing his homework, he is only giving away candies from the pile that started with 100,000. (He's not that generous.)

6. Model the amount of candy that would be left in the pile each day.
7. How many pieces of candy will be left on day 8?
8. When would the candy be gone?



Chew On This – Teacher Notes

Purpose: The purpose of this task is to solidify and extend the idea that geometric sequences have a constant ratio between consecutive terms to include sequences that are decreasing ($0 < r < 1$). The common ratio in one geometric sequence is a whole number and in the other sequence it is a percent. This task contains an opportunity to compare the growth of arithmetic and geometric sequences. This task also provides practice in writing and using formulas for arithmetic sequences.

Core Standards:

F-BF: Build a function that models a relationship between to quantities.

1: Write a function that describes a relationship between two quantities.*

- a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

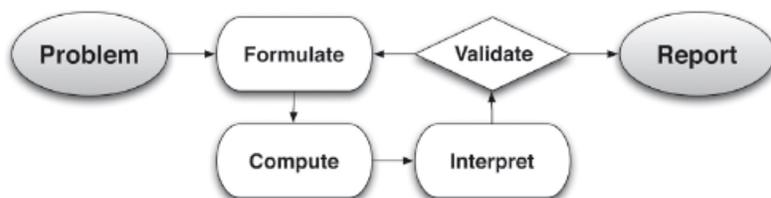
F-LE: Linear, Quadratic, and Exponential Models* (Secondary I focus in linear and exponential only)
Construct and compare linear, quadratic and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
 - a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
 - c. Recognize situations in which one quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expression for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

This task also follows the structure suggested in the Modeling standard:



Launch (Whole Group): Begin by reading the first part of the task, leading to questions 1 and 2. Be sure that students understand the problem situation and then ask what type of sequence would model this situation and why? Students should be able to identify that this is an arithmetic sequence because the number of candies increases by 10 each day that Augustus meets his goals. Ask students to work on #1 individually and then call on students to share their responses. The formulas should be:

$$f(n) = 10n \quad \text{and} \quad f(1) = 10, f(n) = f(n - 1) + 10$$

Read #2 together and ask students which formula will be easier to use to figure out how many candies there will be on day 30. Confirm that the explicit formula will be better for this purpose since we don't know the 29th term so that it can be used in the recursive formula.

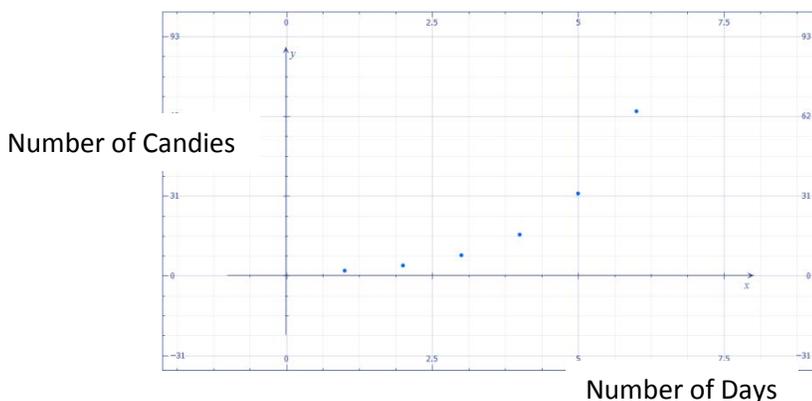
Read the information provided to answer questions 3, 4, and 5. After checking to see that students understand the problem situation, ask them to answer #3 and #4 individually. This will be a good assessment opportunity to see what students understand about their previous work, since this is very similar to the geometric sequences that they have already modeled. After they have had some time, they should be able to produce properly labeled tables, graphs and equations like:

# of Days	# of Candies
1	2
2	4
3	8
4	16
5	32
...	...
n	2^n

Common ratio, $r = 2$

Explicit formula: $f(n) = 2^n$

Recursive formula: $f(1) = 2, f(n) = 2f(n - 1)$



When students have shared their work, ask them to predict which plan will give Augustus more candies on day 30 and why? Then have student use a formula to find the number of candies on day 30 and compare it to what they found in #2. They will probably be surprised to find that in the second plan, Augustus will have 1,073,741,824. Ask students, “Why the huge difference in results? What is it about a geometric sequence like this that makes it grow so quickly?”

Explore (Small Group): Before allowing students to work in their groups on the last part of the problem, check to be sure they understand the problem situation. The pile of candy starts with 100,000. When 60% of the candy is removed on day 1 there are 40,000 candies. Students need to be clear that they are keeping track of the number of candies that remain in the pile, not the number of candies that are removed. There are several different strategies for figuring this out, but that should be part of the conversation in the small groups. As you monitor student work at the

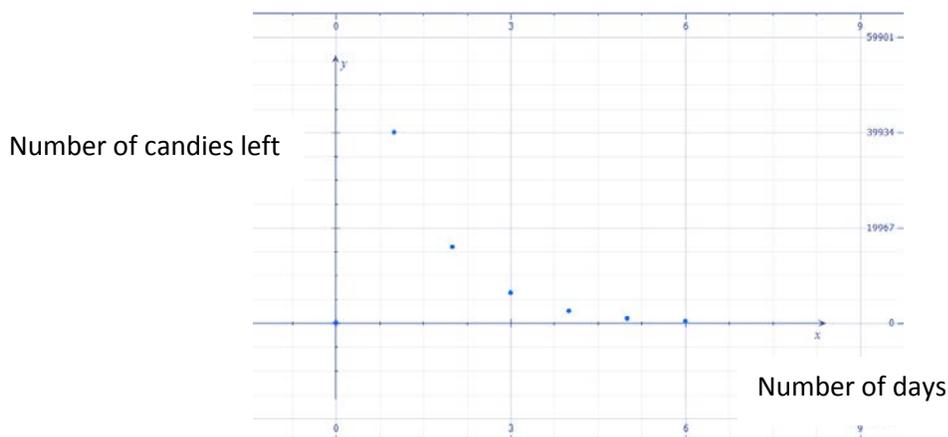
beginning of the task, make sure that they are finding some way to calculate the number of candies left in the pile. Let students work in their groups to decide how to organize and analyze this information. Students have experience in creating tables and graphs in these types of situations, although you should check to be sure that they have scaled their graphs appropriately. Watch for groups that recognize that are able to write formulas and recognize that this is a geometric sequence.

Discuss (Whole Class): Start the discussion with students presenting a table. Ask students if they check for a common difference or common ratio between terms. Note that the common ratio to the side of the table. Ask students, what kind of sequence this must be, given a common ratio between terms. Ask, “How is this geometric sequence different than others we have seen?”

Number of Days	Number of Candies Left
1	40000
2	16000
3	6400
4	2560
5	1024
6	410
7	164
8	66

Common ratio, $r = 0.4$

Continue the discussion with the graph of the sequence.



How is the graph different than other geometric sequences we have seen? What is it about the sequence that causes the behavior of the graph?

Next, ask for the recursive formula. If students have identified the common ratio, they should be able to write: $f(1) = 40,000$, $f(n) = .4f(n - 1)$. Students that calculated each term of the sequence by subtracting 60% of the previous term may have struggled with this formula. Emphasize the usefulness of the table in finding the common ratio for writing formulas.

The explicit formula, $f(n) = 100,000(0.4^n)$ will probably be difficult for students to produce. Help them to recall that in past geometric sequences that the number in the formula that is raised to a power is the common ratio. Then they need to think about the exponent—is it n or $(n-1)$? Discuss ways that they can test the exponent. Finally, they need to consider the multiplier, in this case 100,000. Where does it come from in the problem, both in this case and in previous geometric sequence problems like Growing, Growing Dots? Help students to generalize to make their work in writing explicit formulas easier.

Wrap up the discussion by completing the following chart:

Wrap up the discussion by completing the following chart. Be sure to include ideas about both increasing and decreasing sequences in all representations.

How to Identify a Geometric Sequence	
In a table:	
In a graph:	
In a recursive formula:	
In an explicit formula:	

Place the chart next to the arithmetic sequence chart that was completed at the end of the discussion in “Something To Chew On”. Keep the two charts visible for reference throughout the unit.

Aligned Ready, Set, Go Homework: Sequences 6



What Comes Next? What Comes Later?

A Practice Understanding Task



For each of the following tables,

- describe how to find the next term in the sequence,
- write a recursive rule for the function,
- describe how the features identified in the recursive rule can be used to write an explicit rule for the function, and
- write an explicit rule for the function.
- identify if the function is arithmetic, geometric or neither

Example:

x	y
0	5
1	8
2	11
3	14
4	?
...	...
n	?

- To find the next term: add 3 to the previous term
- Recursive rule: $f(0) = 5, f(n) = f(n - 1) + 3$
- To find the n^{th} term: start with 5 and add 3 n times
- Explicit rule: $f(n) = 5 + 3n$
- Arithmetic, geometric, or neither? Arithmetic

Function A

x	y
1	5
2	10
3	20
4	40
5	?
...	...
n	?

1. To find the next term: _____
2. Recursive rule: _____
3. To find the n^{th} term: _____
4. Explicit rule: _____
5. Arithmetic, geometric, or neither? _____

Function B

x	y
0	3
1	4
2	7
3	12
4	19
5	?
...	...
n	?

6. To find the next term: _____
7. Recursive rule: _____
8. To find the n^{th} term: _____
9. Explicit rule: _____
10. Arithmetic, geometric, or neither? _____



Function C

x	y
1	3
2	5
3	9
4	17
5	33
6	?
...	...
n	?

11. To find the next term: _____
12. Recursive rule: _____
13. To find the n^{th} term: _____
14. Explicit rule: _____
15. Arithmetic, geometric, or neither? _____

Function D

x	y
1	-8
2	-17
3	-26
4	-35
5	-44
6	-53
...	...
n	

16. To find the next term: _____
17. Recursive rule: _____
18. To find the n^{th} term: _____
19. Explicit rule: _____
20. Arithmetic, geometric, or neither? _____

Function E

x	y
1	2
2	-6
3	18
4	-54
5	162
6	-486
...	...
n	

21. To find the next term: _____
22. Recursive rule: _____
23. To find the n^{th} term: _____
24. Explicit rule: _____
25. Arithmetic, geometric, or neither? _____

Function F

x	y
0	1
1	$1\frac{3}{5}$
2	$2\frac{1}{5}$
3	$2\frac{4}{5}$
4	$3\frac{2}{5}$
5	4
...	...
n	

26. To find the next term: _____
27. Recursive rule: _____
28. To find the n^{th} term: _____
29. Explicit rule: _____
30. Arithmetic, geometric, or neither? _____



x	y
1	10
2	2
3	$\frac{2}{5}$
4	$\frac{2}{25}$
5	$\frac{2}{125}$
6	$\frac{2}{625}$
...	...
n	

Function G

31. To find the next term: _____

32. Recursive rule: _____

33. To find the n^{th} term: _____

34. Explicit rule: _____

35. Arithmetic, geometric, or neither? _____

x	y
1	-1
2	0.2
3	-0.04
4	0.008
5	-0.0016
6	0.00032
...	...
n	

Function H

36. To find the next term: _____

37. Recursive rule: _____

38. To find the n^{th} term: _____

39. Explicit rule: _____

40. Arithmetic, geometric, or neither? _____



What Comes Next, What Comes Later? – Teacher Notes

Purpose: The purpose of this task is to practice writing recursive and explicit formulas for arithmetic and geometric sequences from a table. This task also provides practice in using tables to identify when a sequence is arithmetic, geometric, or neither. The task extends students' experiences with sequences to include geometric sequences with alternating signs, and more work with fractions and decimal numbers in the sequences.

Core Standards:

F-BF: Build a function that models a relationship between two quantities.

1: Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-LE: Linear, Quadratic, and Exponential Models* (Secondary I focus in linear and exponential only)
Construct and compare linear, quadratic and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Launch (Whole Class): Remind students of the work done in previous tasks in this module. Ask, “What strategies are you working on for writing formulas for arithmetic and geometric sequences?” Students may offer some ideas that will be helpful in this task. Hand out the task and work through the example with the class. It will be important to clarify the two prompts that ask students to “describe” what they are seeing in the table so that they understand that they are to write a verbal description before they write the formula.

Explore (Small Group): Monitor student work on the task. Functions A and B should be familiar and accessible for most students. Function C is more challenging; although students may recognize an exponential growth pattern, it is not a geometric sequence. They may find difficulty writing formulas. If you find that a group is entirely stuck on the problem, you might ask them to move on to some of the others and go back to finish Function C. Watch their work on Function E to be sure that they consider the alternating sign in their formula. If they haven't included it in the formula, ask them how their formulas will produce the negative sign on some of the terms. Allow them to work to find a way to include the negative sign in their formula. Watch for misconceptions that might make for a productive discussion and listen for students generalizing strategies for writing equations. Identify groups to present their work for Functions D and E that have shown the first



difference on their tables and can articulate how they have used the common difference or common ratio to write the equations.

Discuss (Whole Class): Start the discussion with the student presentation of their work on Function D. Their work will probably look something like:

x	y	Difference
1	-8	> -9
2	-17	> -9
3	-26	> -9
4	-35	
5	-44	
6	-53	
...	...	
n		

Function D

To find the next term: Add -9 to the previous term

Recursive rule: $f(1) = -8, f(n) = f(n - 1) - 9$

To find the n^{th} term: Start with -8 and add -9 $n-1$ times

Explicit rule: $f(n) = -8 - 9(n - 1)$

Arithmetic, geometric, or neither? Arithmetic

Ask students how they were able to use the common difference of -9 in the formulas. Ask how the first term, -8, shows up in each formula.

Next, ask a group to show their work with Function E. It should look something like this:

x	y	Difference
1	2	> -8
2	-6	> 24
3	18	> -72
4	-54	
5	162	
6	-486	
...	...	
n		

Common ratio -3

Function E

To find the next term: Multiply the previous term by -3

Recursive rule: $f(1) = 2, f(n) = -3f(n - 1)$

To find the n^{th} term: start with 2 and multiply by -3 $n-1$ times

Explicit rule: $f(n) = 2(-3)^{n-1}$

Arithmetic, geometric, or neither? Geometric

Ask students how they were able to use the common ratio of -3 in the formulas. Then ask how the first term, 2, appears in the formulas.

If time allows, you may wish to have some of the others presented. If not, move to asking students to generalize their thinking about writing formulas for arithmetic and geometric functions with the following chart:



	How do you use	
	Common ratio or difference	First term
Arithmetic Sequence – Recursive Formula		
Arithmetic Sequence – Explicit Formula		
Geometric Sequence – Recursive Formula		
Geometric Sequence – Explicit Formula		

Ask if there are any other strategies that they have found for writing formulas and record them beneath the chart. Leave the chart in view for students to use in upcoming tasks.

Aligned Ready, Set, Go Homework: Sequences 7



What Does It Mean?

A Solidify Understanding Task



Each of the tables below represents an arithmetic sequence. Find the missing terms in the sequence, showing your method.

x	1	2	3
y	5		11

x	1	2	3	4	5
y	18				-10

x	1	2	3	4	5	6	7
y	12						-6

Describe your method for finding the missing terms. Will the method always work? How do you know?



Here are a few more arithmetic sequences with missing terms. Complete each table, either using the method you developed previously or by finding a new method.

x	1	2	3	4
y	50			86

x	1	2	3	4	5	6
y	40					10

x	1	2	3	4	5	6	7	8
y	-23							5

The missing terms in an arithmetic sequence are called “arithmetic means”. For example, in the problem above, you might say, “Find the 6 arithmetic means between -23 and 5”. Describe a method that will work to find arithmetic means and explain why this method works.



What Does It Mean? - Teacher Notes

Purpose: The purpose of this task is to solidify student understanding of arithmetic sequences to find missing terms in the sequence. Students will draw upon their previous work in using tables and writing explicit formulas for arithmetic sequences. The task will also reinforce their fluency in solving equations in one variable.

Core Standards:

A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters.

Clusters with Instructional Notes: Solve equations and inequalities in one variable.

Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.

Launch (Whole Class): Refer students to the previous work with arithmetic sequences. A brief discussion of the chart that was created as part of the “What Come Next, What Comes Later” task may be useful in drawing students’ attention to the role of the first term and the common difference in writing explicit formulas.

Explore Part 1 (Small Group): Give students the task and be sure that they understand the instructions. Tell them that they are looking for a strategy for solving these problems that works all the time, so they should be paying attention to their methods. The task builds in complexity from the problems on side one to the problems on side 2. Because the problems on the first side are all asking students to find an odd number of terms, most students will probably average the first and last terms to find the middle term, and then average the next two, and so on until the table is completed. Some students may choose to write an equation and use it to find the missing terms. Others may use guess and check to find the common difference to get to each term. Watch for the various strategies and note the students that you will select for the discussion of the first part. Let students work on the task until they complete the first page, and then call them back for a short discussion.

Discuss (Whole Class): Begin the discussion by asking a student that used guess and check to explain his/her strategy. Ask the class what understanding of arithmetic sequences is necessary to use this strategy? They should answer that it relies on knowing when the common difference is known, it can be added to a term to get the next term. This is using the reasoning that we use to write recursive formulas. Next, ask the student that used an averaging method to explain his/her method. Ask the class why this method works. Now ask the class what the advantages are to each of these methods. Will they both work every time? If students haven’t noticed that all of these examples are working with an odd number of missing terms, ask them what would happen if you



needed to use one of these strategies to find an even number of missing terms. Don't discuss the answer as a class, just use it as a teaser for the next section.

Explore (Small Group): Have students work on side 2 of the task, with the same instructions about keeping track of their strategies. Monitor students as they work noting the strategies that they are trying. Some will continue to try guess and check, although it becomes more difficult. You might encourage them to look for a more reliable method, just in case the numbers are too messy to guess at. The method of taking a simple average will not work well for these problems either, so students may become stuck. You might ask them what they would need to know and how they would use it to fill in the missing terms. When they know that they need the difference between terms, then ask if they can think of a way to find it without guessing. This may help them to think of writing an equation to find the difference. Listen to students as they explain their thinking about the formula and be prepared to ask students to present their various strategies.

Discuss (Whole Group): Since most students will use the same method for all three problems, start the discussion by talking about the second problem. A couple of useful ideas that could emerge from the discussion might be:

x	1	2	3	4	5	6
y	40					10

One way to think about setting up an equation to find the difference is to say: I know that I need to start at 40 and add the difference (d), 5 times to get to 10. So, I wrote the equation:

$$40 + 5d = 10$$

Another strategy is to think more about the explicit formulas that we have written in the past as a general formula: $f(n) = f(1) + d(n - 1)$. Students probably won't use this notation, but might say something like, "We learned in the last task that when you write an explicit formula to get any term, you multiply the common difference by 1 less than the term you're looking for and add the first term." If they apply the idea like a general formula they would write:

$$10 = 40 + d(6 - 1)$$

Either way they think about it, students can get the common difference and add it to the term they know to get the next term. Ask students if the strategy of writing an equation works with the three problems on side 1 and give them a chance to convince themselves that it will.

Finally, ask students to amend, edit, or add to their strategy and explanation on the last question. Conclude the discussion by having a few students share what they have written with the class.

Aligned Ready, Set, Go Homework: Sequences 8





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Geometric Meanies

Each of the tables below represents a geometric sequence. Find the missing terms in the sequence, showing your method.

Table 1

x	1	2	3
y	3		12

Is the missing term that you identified the only answer? Why or why not?

Table 2

x	1	2	3	4
y	7			875

Are the missing terms that you identified the only answers? Why or why not?

Table 3

x	1	2	3	4	5
y	6				96



Are the missing terms that you identified the only answers? Why or why not?

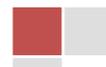
Table 4

x	1	2	3	4	5	6
y	4					972

Are the missing terms that you identified the only answers? Why or why not?

A. Describe your method for finding the geometric means.

B. How can you tell if there will be more than one solution for the geometric means?



Geometric Meanies – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to solidify student understanding of geometric sequences to find missing terms in the sequence. Students will draw upon their previous work in using tables and writing explicit formulas for geometric sequences.

Core Standards:

A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters.

Clusters with Instructional Notes: Solve equations and inequalities in one variable.

Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.

Launch (Whole Class): Explain to students that today’s puzzles involve finding missing terms between two numbers in a geometric sequence. These numbers are called “geometric means”. Ask them to recall the work that they have done previously with arithmetic means. Ask, “What information do you think will be useful for finding geometric means? Some may remember that the first term and the common difference were important for arithmetic means; similarly the first term and the common ratio may be important for geometric means.

Ask, “How do you predict the methods for finding arithmetic and geometric means to be similar?” Ideally, students would think to use the common ratio to multiply to get the next term in the same way that they added the common difference to get the next term. They may also think about writing the equation using the number of “jumps” that it takes to get from the first term to the next term that they know.

Ask, “How do you think the methods will be different?” Ideally, students will recognize that they have to multiply rather than simply add. They may also think that the equations have exponents in them because the formulas that they have written for geometric sequences have exponents in them.

Explore (Small Group): The problems in this task get larger and require students to solve equations of increasingly higher order. Some students will start with a guess and check strategy, which may work for many of these. Even if it is working, you may choose to ask them to work on a strategy that is more consistent and will work no matter what the numbers are. Students are likely to try the same strategies as they used for arithmetic sequences. These strategies will be successful if they think to multiply by the common ratio, rather than add the common difference. If they are writing equations, they may have similar thinking to this:

Table 2



x	1	2	3	4
y	7			875

“I need to start at 7 and multiply by a number, n , to get to the second term, then multiply by n again to get to the third term, and by n again to end up at 875. So, I will write the equation:

$$7 \cdot r \cdot r \cdot r = 875 \text{ or } 7r^3 = 875$$

Some students may have difficulty solving the equation that they write. It may be helpful to get them to divide by 7 and then think about the number that can be raised to the third power to get 125. Calculators will also be helpful if students understand how to take roots of numbers. All of the numbers used in this task are straightforward, making them accessible for most students to think about without a calculator. Watch for students that can explain the strategy shown above, along with other productive strategies so that you can highlight their work in the discussion.

Along with completing the table, students are asked if the solution they found is the only solution. This is a reminder for them to think of both the positive and negative solution if the exponent is even. As you are monitoring student work, watch to see if they are answering this prompt appropriately. If not, you may want to ask a question like, “I see that you have $r^2 = 4$, $r = 2$. Is there another number that you can multiply by itself to get 4?”

Discuss (Whole Class): Start the discussion with Table 1. Select a student that guessed at the common ratio and ask how he/she figured it out and then how they used the common ratio to find the missing term. Since most guessers only get the positive ratio (2), you will probably have students that want to add the negative solution (-2) also. Great! Ask how they figured the solution out and to verify for the class that a common ratio of -2 also produces a term that works in the sequence. Some students may have simply reasoned that -2 would work as a common ratio. Be sure to select a student that has written an equation like: $3r^2 = 12$. Use this as an opportunity for students to see that the algebraic solution to this equation is $r = \pm 2$.

Next, ask a student to write and explain an equation for Table 2, as shown above. As they solve the equation, $7r^3 = 875$, $r^3 = 125$, ask students if there is more than one number that can be cubed to obtain 125? They should decide that 5 is a solution, but -5 is not. Have a student demonstrate using the common ratio that was found to complete the table.

Now that students have worked a couple of problems as a class, they have had a chance to check their work, ask them to reconsider the generalizations at the end of the task. Give students a few minutes to modify their responses to questions A and B. Then, ask a few students to share their procedure for finding geometric means and knowing the number of solutions with the entire class.

Aligned Ready, Set, Go Homework: Sequences 9



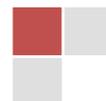
I Know . . . What Do You Know?

A Practice Task



In each of the problems below I share some of the information that I know about a sequence. Your job is to add all the things that you know about the sequence from the information that I have given. Depending on the sequence, some of things you may be able to figure out for the sequence are: a table, a graph, an explicit equation, a recursive equation, the constant ratio or constant difference between consecutive terms, any terms that are missing, the type of sequence, or a story context. Try to find as many as you can for each sequence, but you must have at least 4 things for each.

1. I know that: the recursive formula for the sequence is $f(1) = -12$, $f(n) = f(n - 1) + 4$
What do you know?
2. I know that: the first 5 terms of the sequence are 0, -6, -12, -18, -25 . . .
What do you know?
3. I know that: the explicit formula for the sequence is $f(n) = -10(3)^n$
What do you know?
4. I know that: The first 4 terms of the sequence are 2, 3, 4.5, 6.75 . . .
What do you know?
5. I know that: the sequence is arithmetic and $f(3) = 10$ and $f(7) = 26$
What do you know?



6. I know that: the sequence is a model for the perimeter of the following figures:

Figure 1

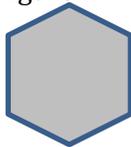


Figure 2

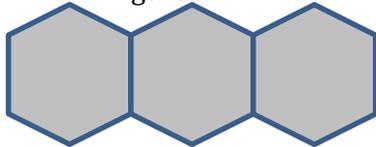
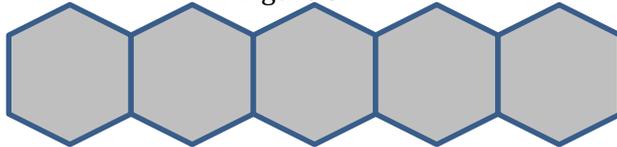


Figure 3



Length of each side = 1

What do you know?

7. I know that: it is a sequence where $a_1 = 5$ and the constant ratio between terms is -2 .

What do you know?

8. I know that: the sequence models the value of a car that originally cost \$26,500, but loses 10% of its value each year.

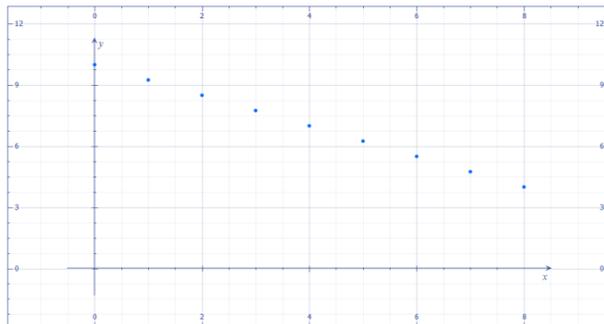
What do you know?

9. I know that: the first term of the sequence is -2 , and the fifth term is $-\frac{1}{8}$.

What do you know?

10. I know that: a graph of the sequence is:

What do you know?



I Know . . . What Do You Know? – Teacher Notes

Purpose: The purpose of this task is to practice working with geometric and arithmetic sequences. This is the final task in the module and is intended to help students develop fluency in using various representations for sequences.

Core Standards:

F-LE2.: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Launch (Whole Class): The task gives students some information about a sequence and asks them to complete the remaining information. Ask students what are some things that we can know about a sequence. This question should generate a list like:

- A table with 5 terms of the sequence
- A graph of the sequence
- An explicit formula for the sequence
- A recursive formula for the sequence
- The common ratio or common difference between terms
- Whether the sequence is geometric, arithmetic or neither

If the class doesn't identify all of these representations, add the missing ones to the list. Hand out the task and tell students that they will be trying to figure out all the things that can be known from the information given.

Explore (Small Group): Monitor students as they work. After most students have completed a substantial portion of the task, begin assigning one group per problem to make a large chart and present it to the class. Each chart should have all the information listed above about the sequence that they have been assigned. (In each case, some of the information was given.) Ask students to be prepared to discuss how they found each piece of information.

Discuss (Whole Group): Ask each group to present their work with the sequence one at a time. Encourage the groups that are not presenting to be checking their work and asking questions to the presenters. In the interest of time, you may choose just a few groups to present, possibly #6 and #8, and then have the other groups post their work. You could then organize a "gallery stroll" in which students went from one chart to the next, comparing their work to the chart and formulating questions for the groups. When they have had a chance to go to each chart, facilitate a discussion where students can ask their questions to the group that developed the chart.

Aligned Ready, Set, Go Homework: Sequences 10

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Module 2 Assessment: Geometric and Arithmetic Sequences

Multiple Choice

- Find the 125 term of the sequence 5, 11, 17, 23, ...
 - 755
 - 155
 - 749
 - 739
- The number of fish in Fish Lake is decreasing by 40% per year because of the increase of pollution in the lake. If there are about 75,000 fish in the lake now, predict the number of fish in the lake after 7 years.
 - 3500
 - 2100
 - 1200
 - 300

- Find a recursive formula for the sequence:

n	1	2	3	4
$f(n)$	12	18	27	40.5

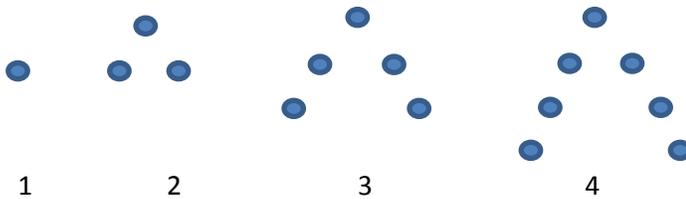
- $f(1) = 12, f(n) = f(n - 1) + 6$
 - $f(1) = 12, f(n) = 3f(n - 1)$
 - $f(1) = 12, f(n) = 1.5f(n - 1)$
 - $f(1) = 12, f(n) = f(n - 1) + 9$
- The common difference in the arithmetic sequence 2, -6, -14, -22, ... is
 - 8
 - 16
 - 3
 - 8
 - Abbie decides to start doing crunches as part of her daily workout. She decides to do 15 crunches the first day and then increase the number 4 each day after that. Write an explicit formula for the number of crunches Abbie will do on day n .
 - $f(n) = 15 + 4n$
 - $f(n) = 15 + 4(n - 1)$
 - $f(n) = 4 + 15n$
 - $f(n) = 4 + 15(n - 1)$
 - Find three geometric means between 2 and 2592.
 - 3, 120, 1500
 - 16, 92, 624
 - 12, 72, 432
 - 649.5, 1297, 1944.5

7. Which of the following is a formula for the sequence:

n	$f(n)$
1	8
2	-20
3	50
4	-125

- a. $f(n) = 8 - 2.5(n - 1)$
 b. $f(n) = 8 + (-2.5)^{n+1}$
 c. $f(n) = 8(-2.5)^{n-1}$
 d. $f(n) = (-2.5)8^{n-1}$
8. What is the 6th term of the geometric sequence in which $(1) = -4$ and $f(n) = 5f(n - 1)$?
 a. -5,120
 b. -12,500
 c. 20,480
 d. -62,500
9. What are the three arithmetic means between 18 and -10?
 a. 14, 10, 6
 b. 6, 0, -6
 c. 11, 4, -3
 d. 9, 4.5, -3

10. Given the pattern of dots below:



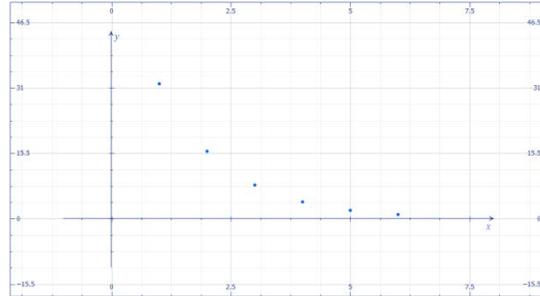
If the pattern continues, how many dots will be in the 14th figure?

- a. 25
 b. 26
 c. 27
 d. 29
11. What is a rule for the n th term of the geometric sequence with $f(3) = -12$ and common ratio $r = 3$?
- a. $f(n) = -\frac{4}{3}(3)^{n-1}$
 b. $f(n) = -4(3)^{n-1}$
 c. $f(n) = -\frac{3}{4}(3)^{n-1}$
 d. $f(n) = 4(3)^{n-1}$

12. What is the fourth term of the sequence defined by the recursive rule $f(1) = 3$ and $f(n) = f(n - 1) - 7$

- a. -11
- b. -18
- c. -1029
- d. 7203

13. Given the graph:



Which of the following statements is true?

- a. This is a geometric sequence with a common ratio of 2.
 - b. This is a geometric sequence with a common ratio $\frac{1}{2}$.
 - c. This is an arithmetic sequence with a common difference of 2.
 - d. This is an arithmetic sequence with a common difference of -2.
14. Find the missing term in the geometric sequence: $-64, 16, -4, 1, \dots$
- a. -0.25
 - b. 0.25
 - c. 4
 - d. -4

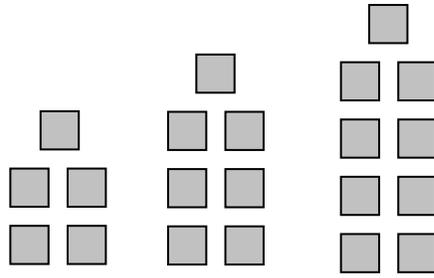
15. Which of the following is not a geometric sequence?

- a. $1, 4, 16, 64, \dots$
- b. $4, 2, 1, \frac{1}{2}, \dots$
- c. $3, 6, 12, 24, \dots$
- d. $6, 4\frac{1}{2}, 3, 1\frac{1}{2}, \dots$

Short Answer

1. An arithmetic sequence has a first term of 0 and a 9th term of 12. If 27 is an output of the sequence, which term number is it? Show and explain how you know you have the correct term number.
2. If the sequence 10, $2x$, $3x$, ... is geometric, what are the values of $2x$ and $3x$? Show all of your work or explain your thinking clearly.
3. Identify which of the following situations can be modeled with an arithmetic sequence and which can be modeled with a geometric sequence.
 - a. Raul invests \$1000 in a savings account that earns 5% interest compounded annually.
 - b. Saul puts \$1000 in cash in a safety deposit box at the bank. Each month, he adds an additional \$50 to the amount in the safety deposit box.
 - c. Yolanda interviewed for a job that promised her a starting salary of \$32,000 with a \$1250 raise at the end of each year.
 - d. A rock climber climbs 75 feet in the first half-hour of climbing. In each succeeding half-hour, the climber only achieves 90% of the height achieved in the previous half-hour.

4.



Tower 0

Tower 1

Tower 2

Tower 3

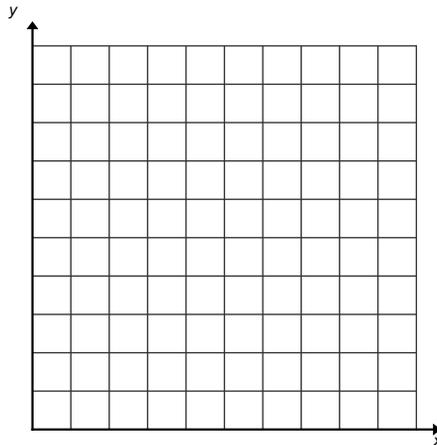
Tower 4

A. Draw **Tower 0** and **Tower 4** in the space above.

B. Create a **table, graph, and equation** for this geometric pattern in the space below.

Table:

Graph



Equation:

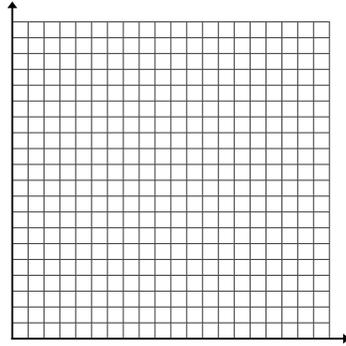
Constructed Response

1. Rocky the Squirrel saves the same number of acorns each week.
 - a. What type of sequence represents his total number of saved acorns? How can you tell?
 - b. After week 4, he had 23 acorns, and after week 8, he had 35 acorns. How many did he start with? Show your table and general rule.
 - c. When will he have more than 100 acorns?
 - d. If Boris steals 25% of the acorns each week, how many acorns will Rocky have after week 5?

2. Finish the table and find an equation for each of the following sequences. Neatly sketch their graphs showing important points.

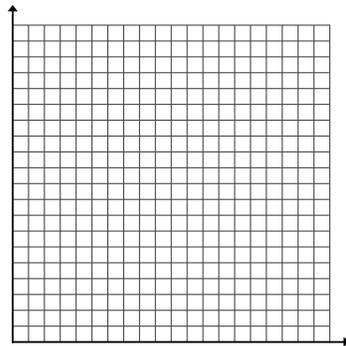
a.

n	1	2		4	5
$t(n)$	36	27	18		



b.

n	1	2	3	4	5
$t(n)$	36	18	9		



c. What can you say about the value of the 100th term of each sequence? Which one will be larger? How do you know?