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Practices (SMP & ETP) Progressions

Travis Lemon
lemonmath@gmail.com
@TravisLemon

Our Session Today

- Practices, Practices, Practices, ….

- 8 Effective Teaching Practices
- 8 Standards for Mathematical Practice

- Consider a framework for incorporating it all

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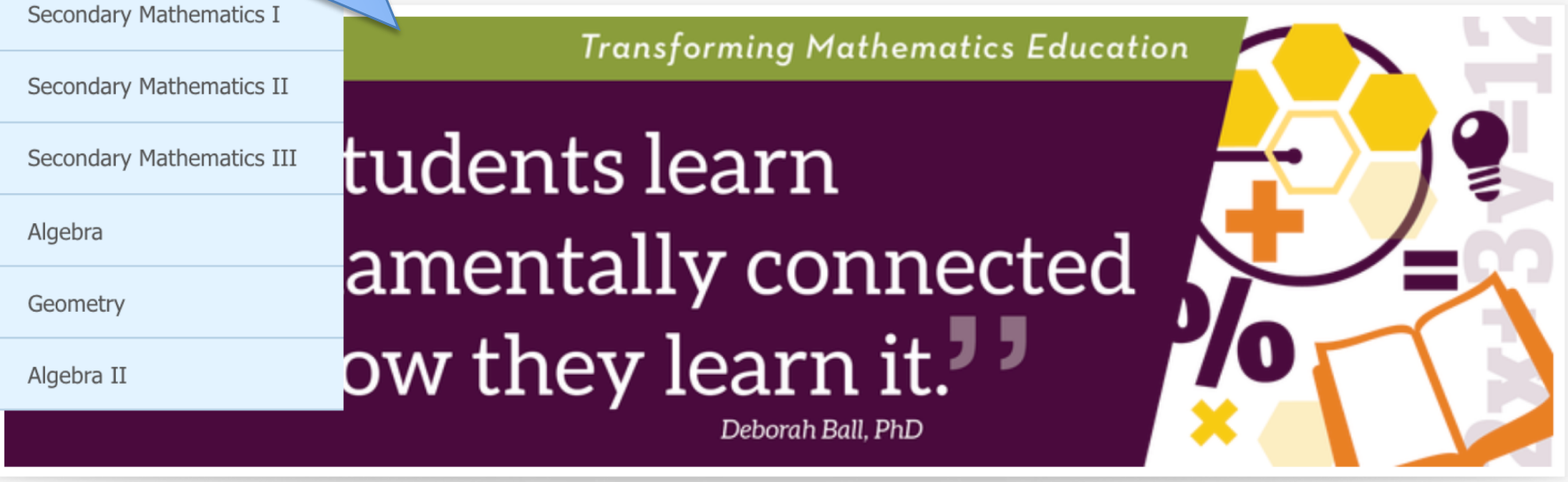
- Free, OER
- Tasks-based materials for Integrated courses and AGA
- Connected to all the practices
- Mathematicsvisionproject.org

Search

CURRICULUM DEVELOPMENT RESOURCES GET MVP UPDATES ABOUT STORE CART (0)

- Secondary Mathematics I
- Secondary Mathematics II
- Secondary Mathematics III
- Algebra
- Geometry
- Algebra II



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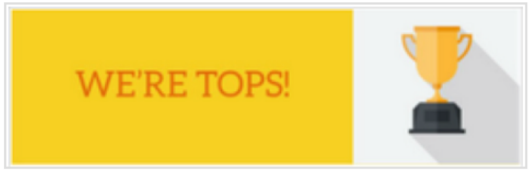
MVP team members are actively engaged in providing training and professional development to support implementation.



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MVP materials top the charts when it comes to **Alignment, Rigor & Balance** as well as **Deeper Learning**--all essential attributes of a focused, coherent and rigorous curriculum.

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Our Session Today

- Practices, Practices, Practices, ….

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- 8 Standards for Mathematical Practice

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NCTM's Mathematics Teaching Practices

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
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Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

8 and 8

Who are they for?
Which came first?
How do they relate?
What to do about it?

Standards for Mathematical Practice

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Given a specific lens of practices
Identify from the other set of practices 3 that are most critical.
Rank them in order.

8 x 8: Teacher by Student Practices

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Lens of Student Practices

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Lens of Teacher Practices

8 x 8: Teacher by Student Practices

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What patterns have emerged? What do you notice? What do you wonder? How do you feel?

8 x 8: Teacher by Student Practices

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How can we accomplish all of the practices?

- Progression
- Coherence
- Frameworks

Calls for Coherence and Progressions

***NCTM Curriculum Principle**:*

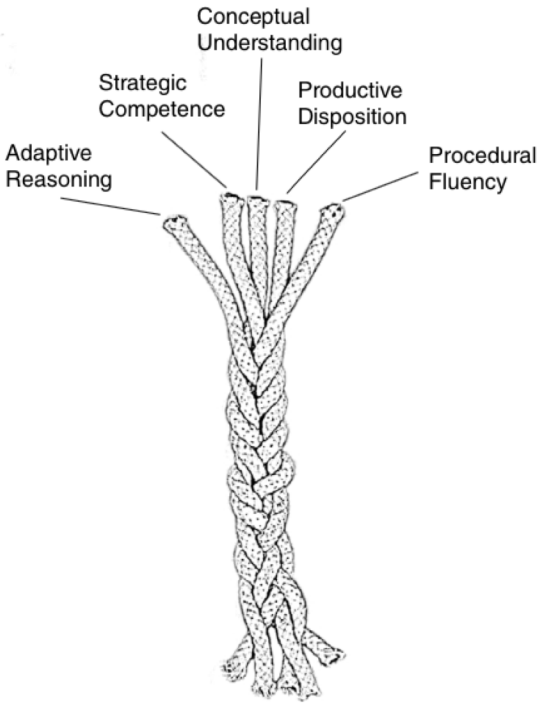
A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.

(PSSM, 2000)

Calls for Coherence and Progressions

- *conceptual understanding*—comprehension of mathematical concepts, operations, and relations
- *procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence*—ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification
- *productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

The most important observation we make about these five strands is that they are interwoven and interdependent. This observation has implications for how students acquire mathematical proficiency, how teachers develop that proficiency in their students, and how teachers are educated to achieve that goal.



Intertwined Strands of Proficiency

(Adding it Up, 2001)

Calls for Coherence and Progressions

"The Common Core State Standards in mathematics were built on progressions…informed both by research on children's cognitive development and by the logical structure of mathematics."

Progression Document Introduction
http://ime.math.arizona.edu/progressions/

Calls for Coherence and Progressions

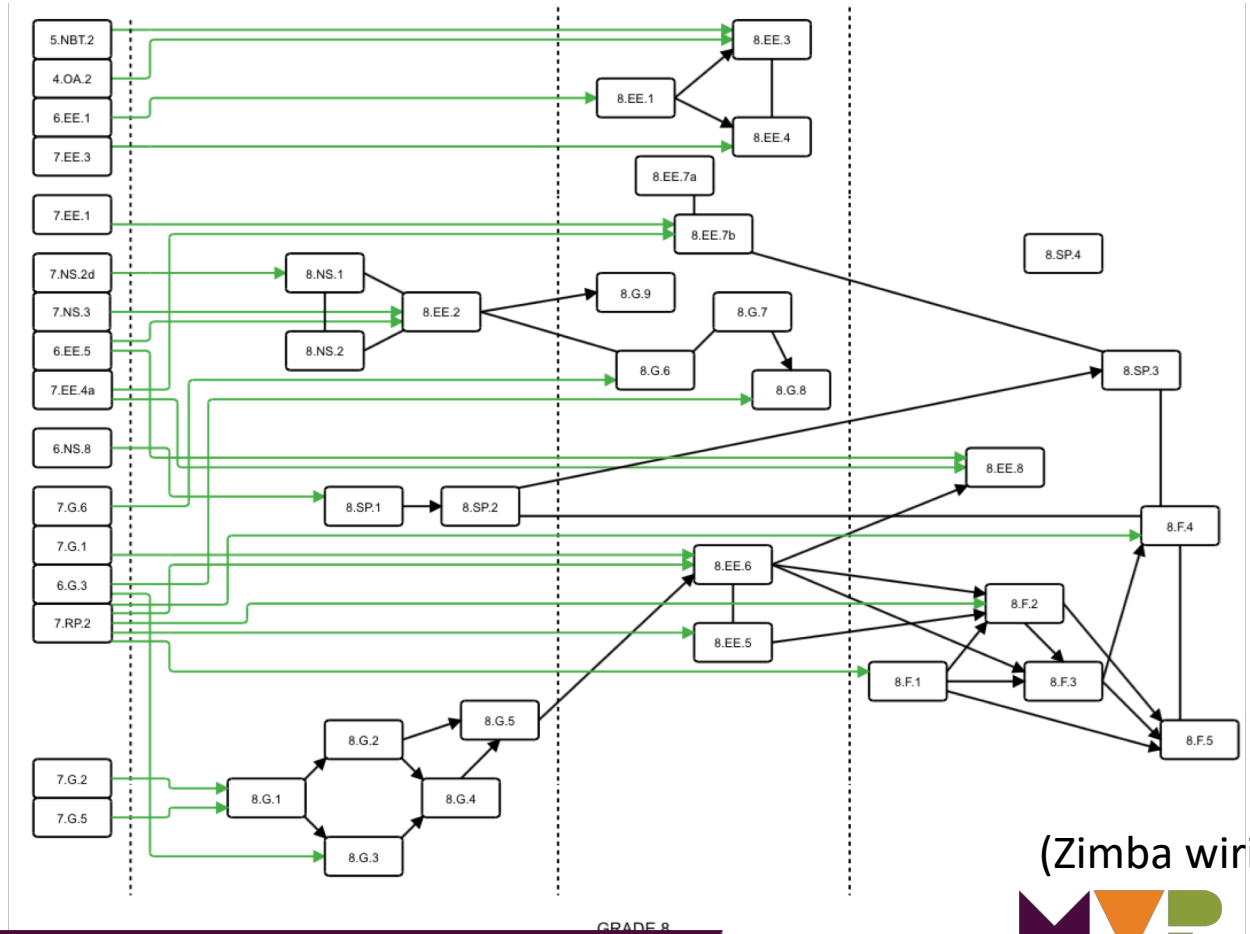
"Coherence is about making math make sense. Mathematics is not a list of disconnected tricks or mnemonics. It is an elegant subject…

(CCSSM Publisher's criteria, pg. 3)

"Fragmenting the Standards into individual standards, or individual bits of standards, erases all these relationships and produces a sum of parts that is decidedly less than the whole. "

Phil Daro, Jason Zimba, Bill McCallum

Calls for Coherence and Progressions



(Zimba wiring diagram)

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Effective Teaching Practices

- **Establish mathematics goals to focus learning.**
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- **Facilitate meaningful mathematical discourse.**
- **Pose purposeful questions.**
- **Building Procedural Fluency from Conceptual understanding**
- **Support productive struggle in learning mathematics.**
- **Elicit and use evidence of student thinking.**

(NCTM, 2014)

Effective Teaching Practices

- **Establish mathematics goals to focus learning.**
- **Implement tasks that promote reasoning and problem solving. " Student learning is greatest in classrooms where tasks consistently encourage high-level student thinking and reasoning and least in classrooms where tasks are routinely procedural in nature."**
- **Use and connect mathematical representations.**
- **Facilitate meaningful mathematical discourse.**
- **Pose purposeful questions.**
- **Build procedural fluency from conceptual understanding.**
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(NCTM, 2014)

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- **Elicit and use evidence of student thinking.**

(NCTM, 2014)

Enhancing Classroom Practice

“ Tasks … gain more traction when used within sequences of tasks that develop students’ understanding of larger mathematical ideas or processes.”

Boston, Madler & Cutone, “Implementing Tasks That Promote Reasoning and Problem Solving.” Enhancing Classroom Practice with Research Behind Principles to Actions. Reston: NCTM, 2017. 13-26)

Progressions and Coherence facilitate all of the practices!

Assist us in being more deliberate about our work.

There is a need for a framework.

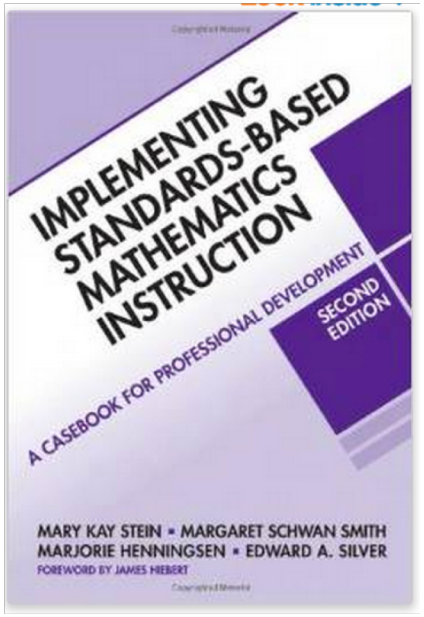
- Chazan and Ball (1999), argue that **educators are often left "with no framework for the kinds of specific, constructive pedagogical moves that teachers might make**."
- Stein et al. (2008) refer to a ***first generation*** of instructional reform from which "many teachers got the impression that in order for discussions to be focused on student thinking, they must avoid providing any substantive guidance at all," and they refer to a ***second generation*** of instructional reform "that re-asserts the critical role of the teacher in guiding mathematical discussions."

There are some frameworks

- Levels of Cognitive demand

There are some frameworks

Lower-level demands	Higher-level demands
Memorization • Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas, or definitions to memory • Cannot be solved by using procedures, because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure • Are not ambiguous. Such tasks involve exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlies the facts, rules, formulas, or definitions being learned or reproduced	Procedures with connections • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas • Suggest, explicitly or implicitly, pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts • Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.
Procedures without connections • Are algorithmic. Use of the procedure is either specifically called for or is evident from prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done or how to do it. • Have no connection to the concepts or meaning that underlies the procedure being used • Are focused on producing correct answers instead of on developing mathematical understanding • Require no explanations or explanations that focus solely on describing the procedure that was used	Doing mathematics • Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example • Require students to explore and understand the nature of mathematical concepts, processes, or relationships • Demand self-monitoring or self-regulation of one's own cognitive processes • Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task • Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions • Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required

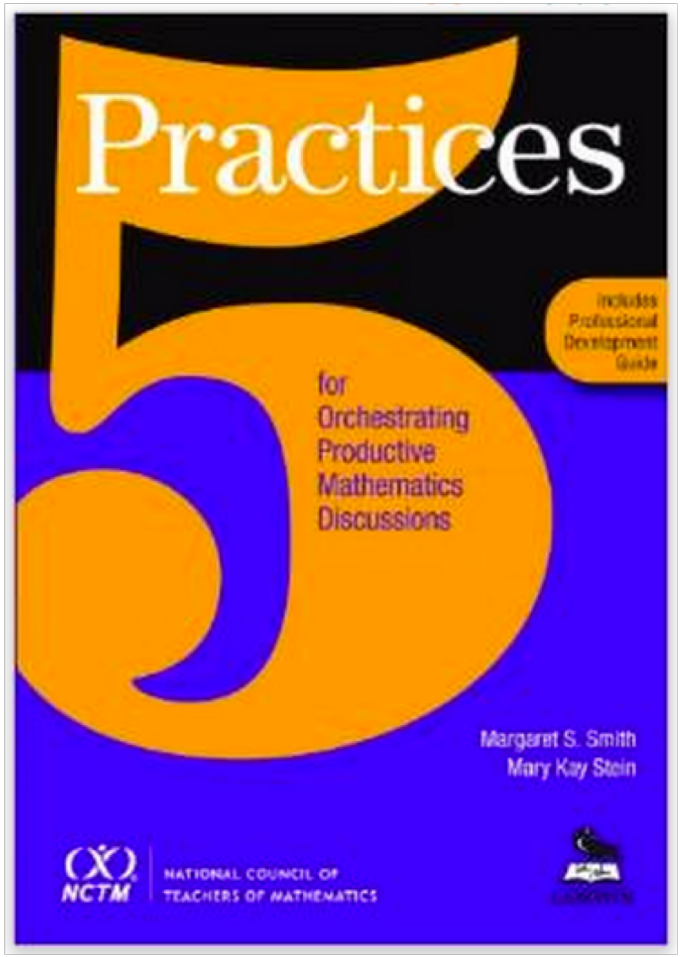


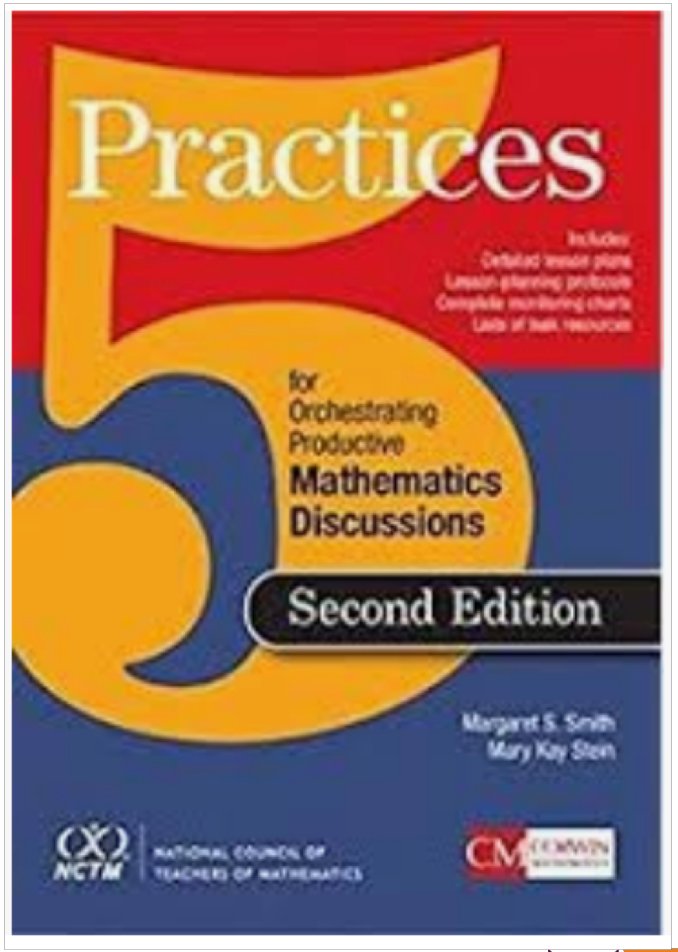
Purple Book
(NCTM, 2009)

There are some frameworks

- Levels of Cognitive demand
- 5 practices for orchestrating discussions

There are some frameworks





(NCTM, 2011, 2018)

There are some frameworks

- Levels of Cognitive demand
- 5 practices for orchestrating discussions
- 3-act tasks

There are some frameworks

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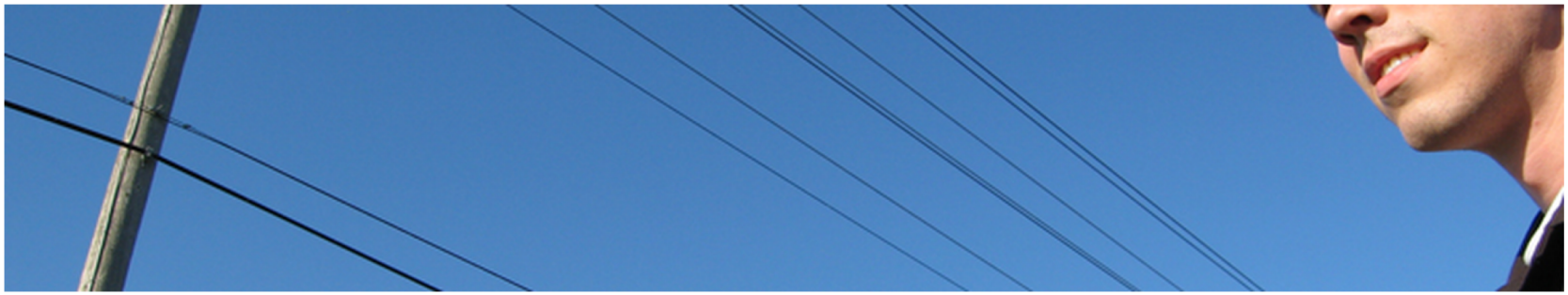

The Three Acts Of A Mathematical Story

May 11th, 2011 by Dan Meyer

2013 May 14. Here's a brief series on how to teach with three-act math tasks. It includes video.

2013 Apr 12. I've been working this blog post into curriculum ideas for a couple years now. They're all available here.

Storytelling gives us a framework for certain mathematical tasks that is both prescriptive enough to be *useful* and flexible enough to be *usable*. Many stories divide into three acts, each of which maps neatly onto these mathematical tasks.

My name is **Dan Meyer** and I like to teach.

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There are some frameworks

- Levels of Cognitive demand
- 5 practices for orchestrating discussions
- 3-act tasks
- And more…

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- **These are great!**

There are some frameworks

- Levels of Cognitive demand
- 5 practices for orchestrating discussions
- 3-act tasks
- And more….

- **These are great!**
- **They have all assisted with reform.**
- **However, applied at the task level.** **(A single task)**

Calls for Coherence and Progressions

- The CCSSM publisher's criteria, Principles to Actions, Catalyzing Change and others, wouldn't urge us to do more if we had arrived.
- We need a sustained, persistent press for student thinking, development of conceptual understanding and procedural fluency, productive struggle that occurs on a daily basis.
- The effort to implement a ***task*** needs to lead to the implementation of a ***progression of tasks*** and a ***curriculum*** that is coherent, rigorous and focused.

Calls for Coherence and Progressions

NCTM Curriculum Principle:

A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.

(PSSM, 2000)

8 and 8

8 for teachers related to teaching
8 for students related to doing mathematics

Standards for Mathematical Practice

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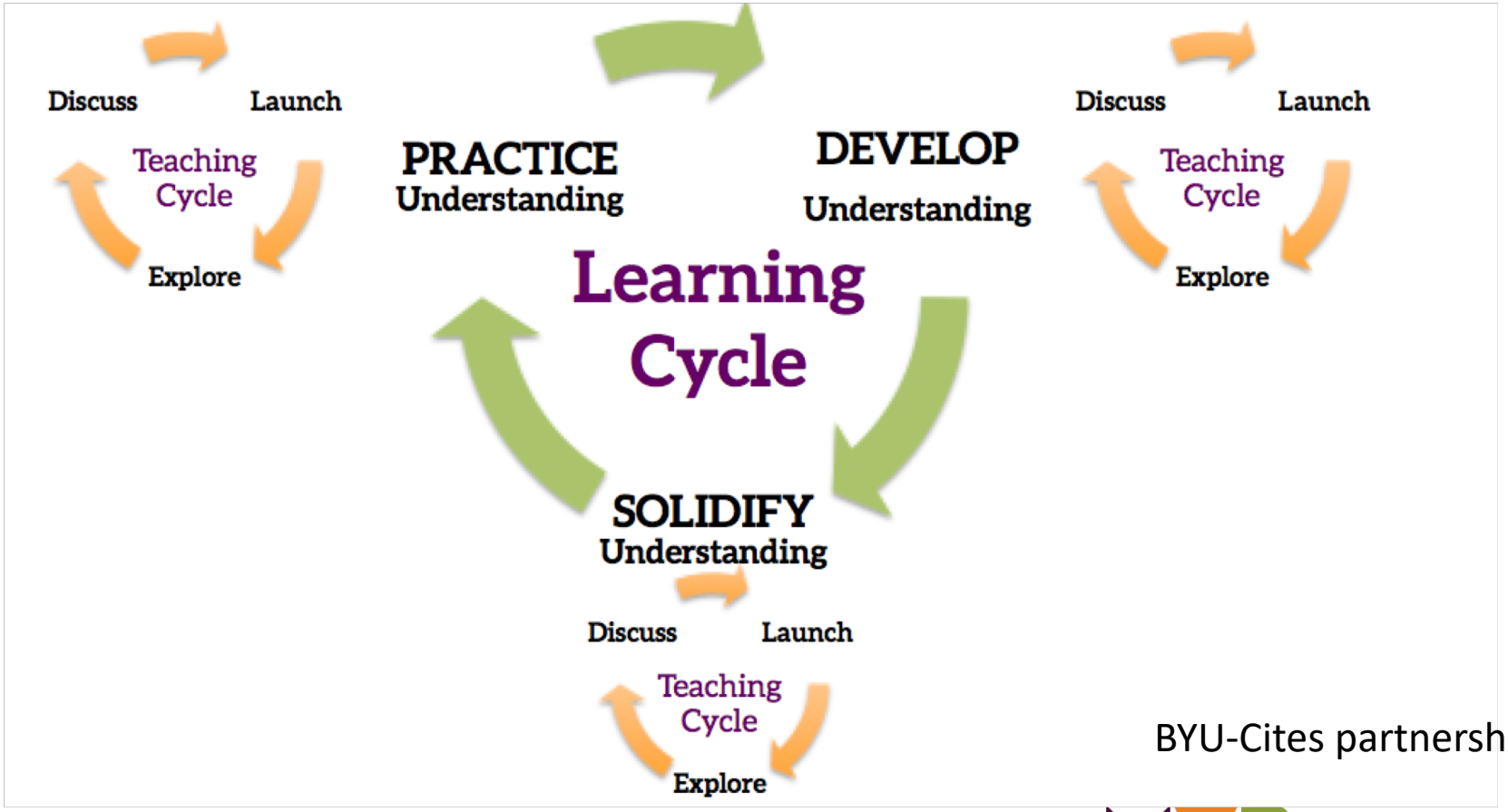
Seems sensible to have a framework that connects the 8 with the 8.

And at the same time promotes progression!

A FRAMEWORK for Coherence and Progression:

The Comprehensive Mathematics Instruction Framework

The framework on which MVP curriculum is built



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A FRAMEWORK for Progressions

- When it comes to all of the practices we have to consider, what advantage is there to a framework containing cycles?

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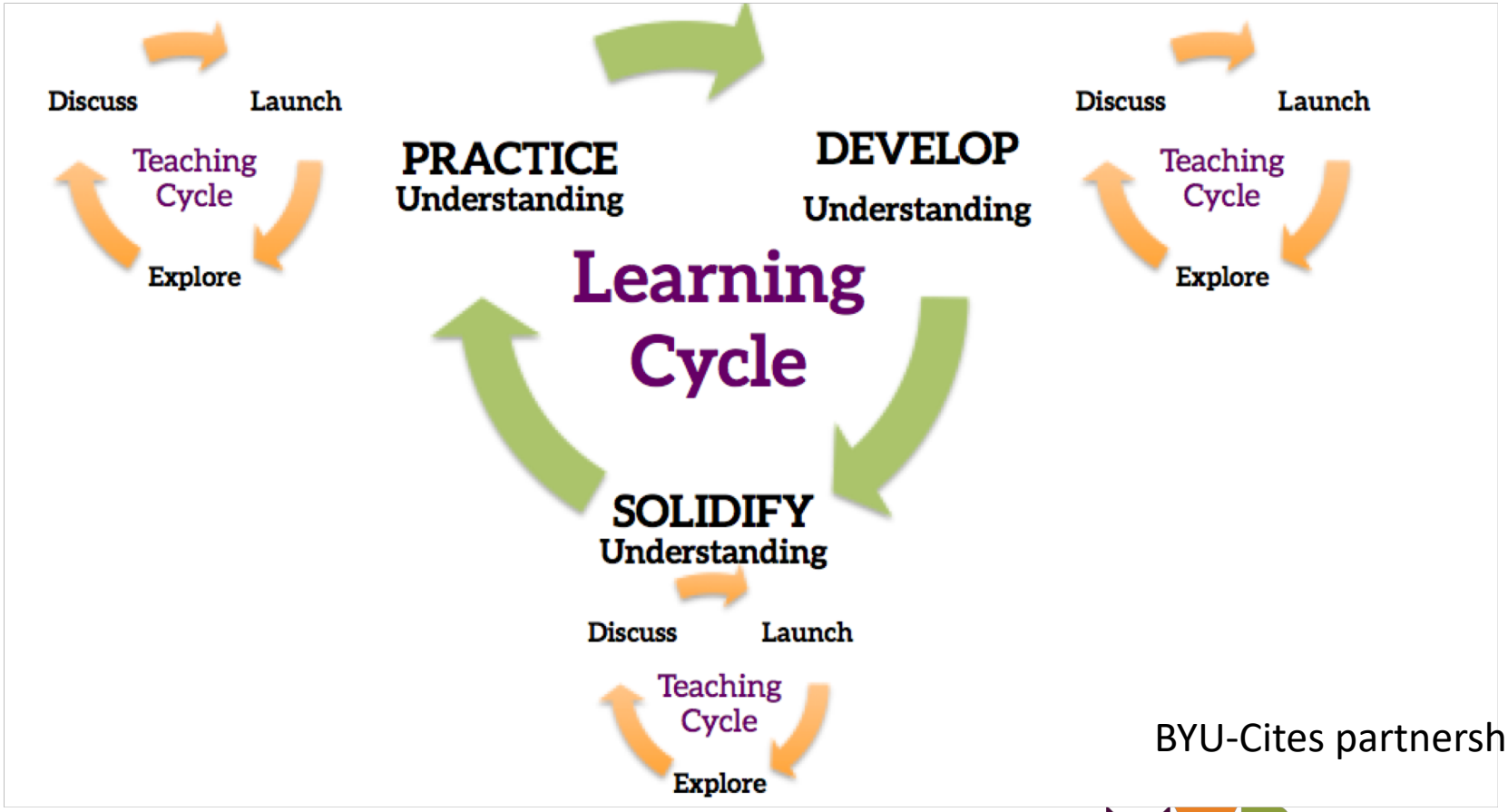

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A FRAMEWORK for a Lesson or TASK:

Moving from a conceptual foundation to procedural fluency

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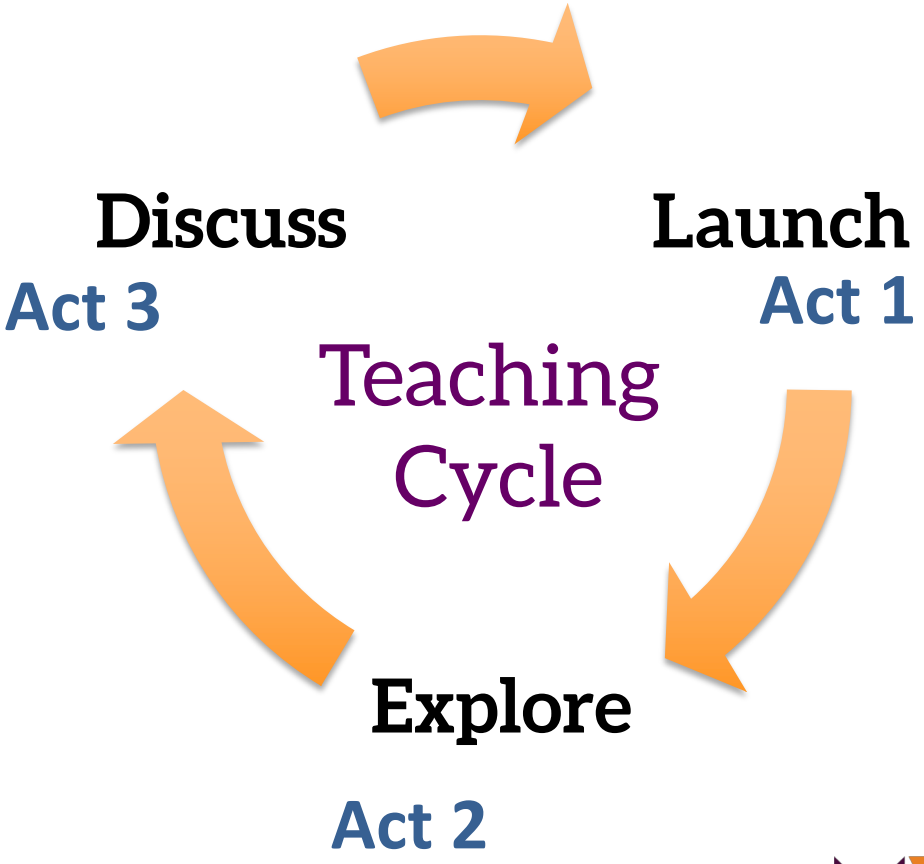

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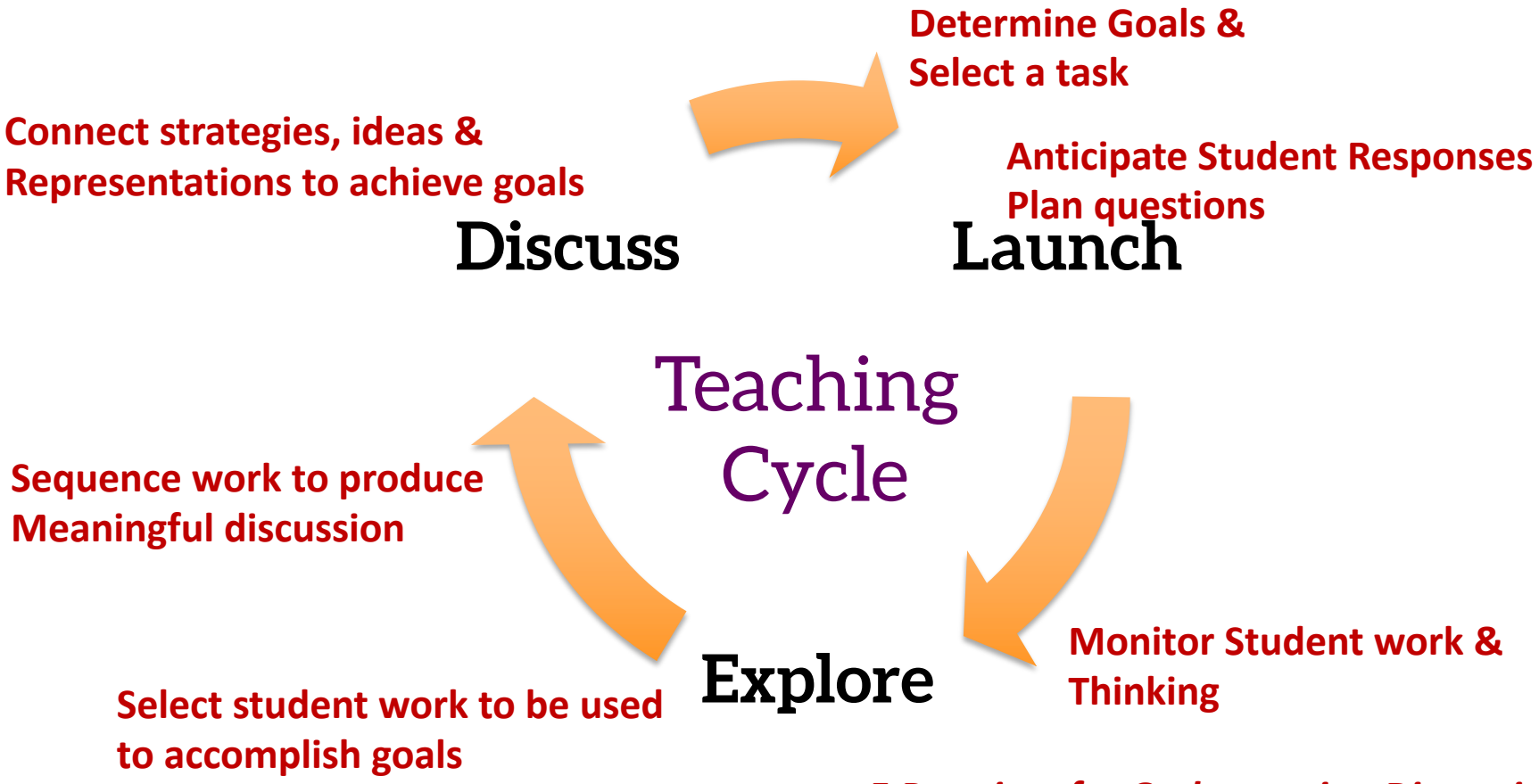

5 Practices for Orchestrating Discussions

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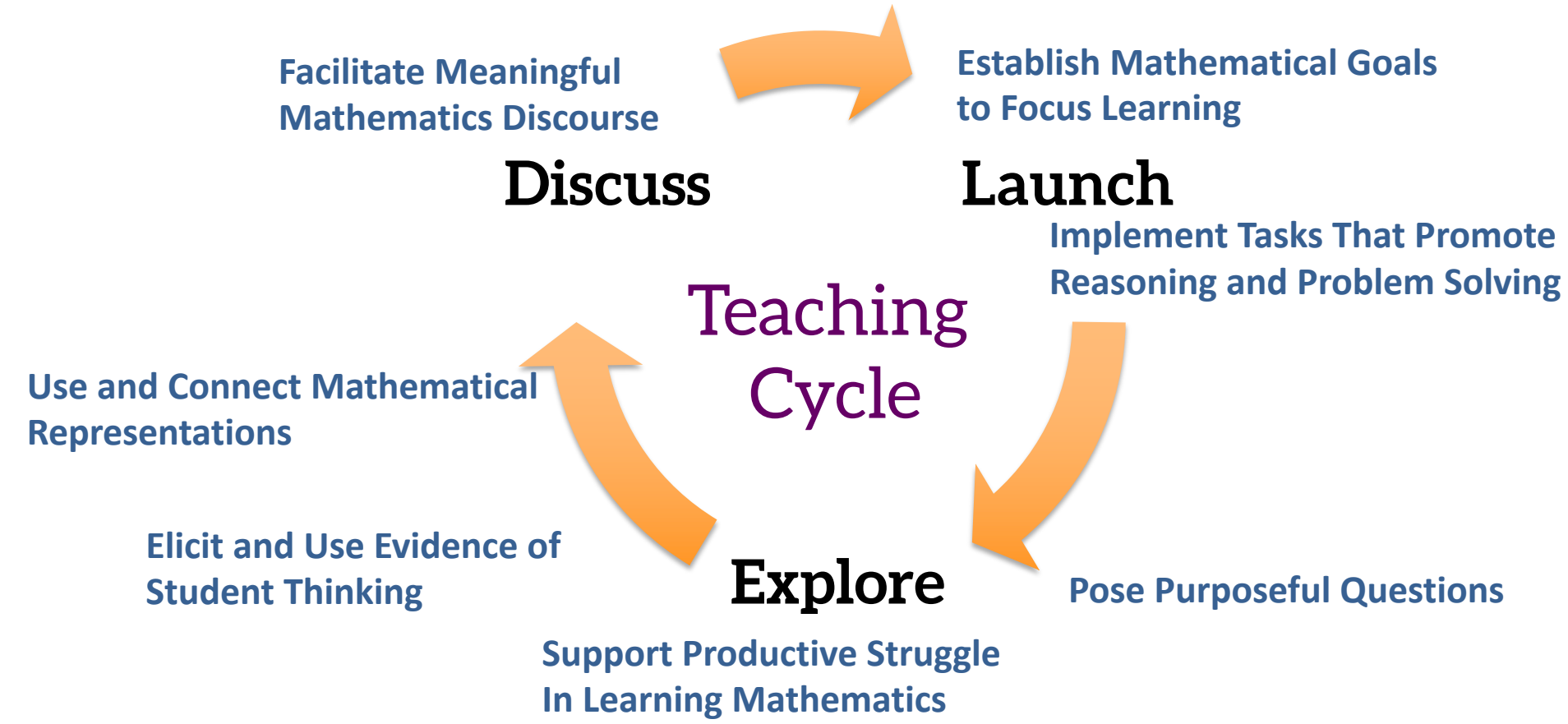

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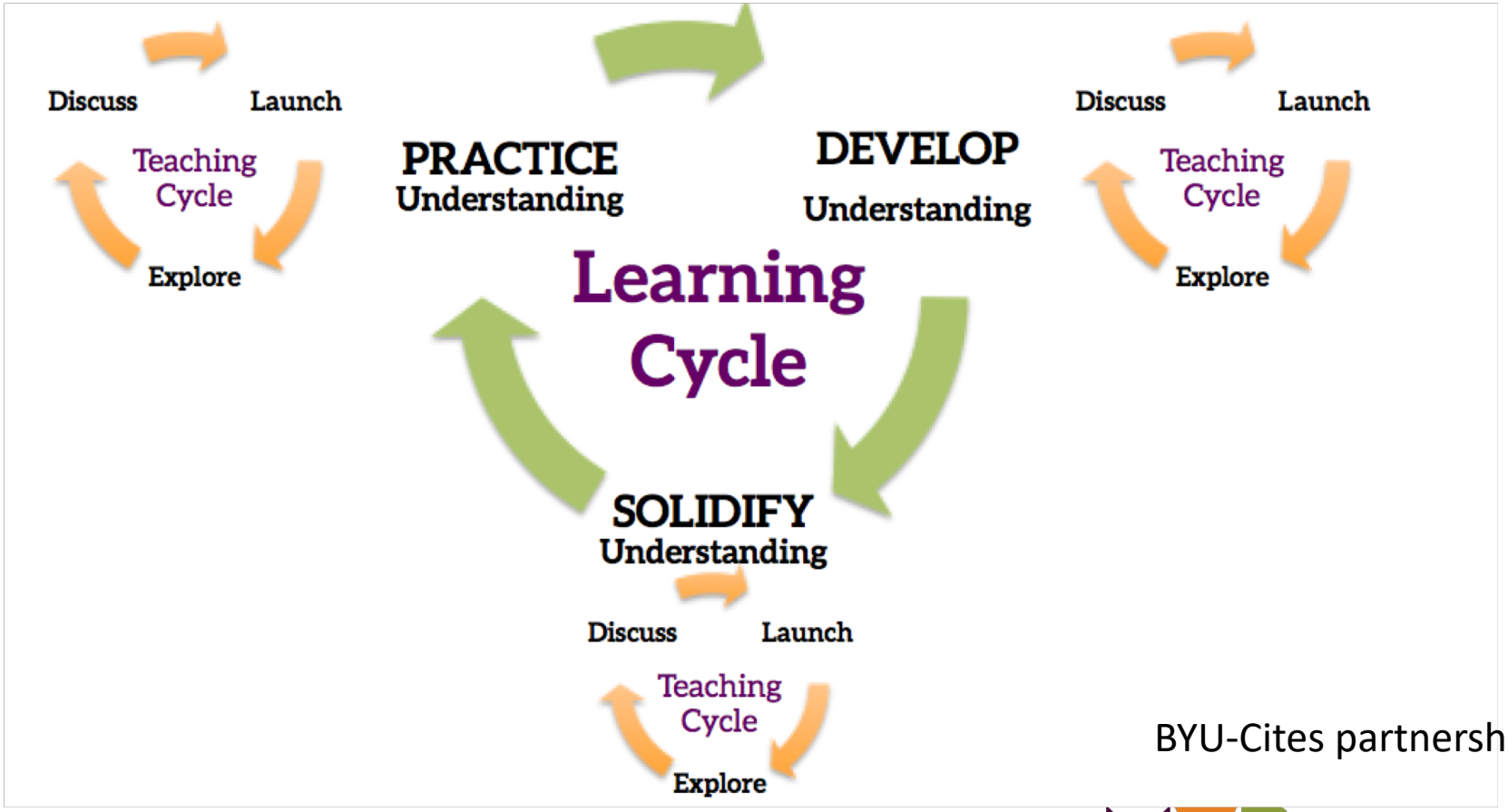

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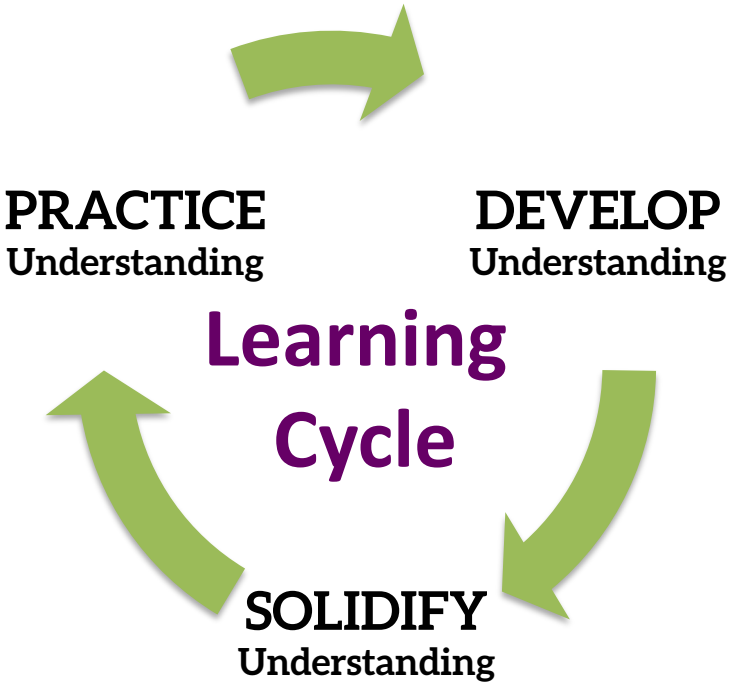
A FRAMEWORK for Task Sequencing:

Moving from a conceptual foundation to procedural fluency

Comprehensive Mathematics Instruction Framework



- ***Develop Understanding*** tasks surface student thinking
- ***Solidify Understanding*** tasks examine and extend
- ***Practice Understanding*** tasks build fluency

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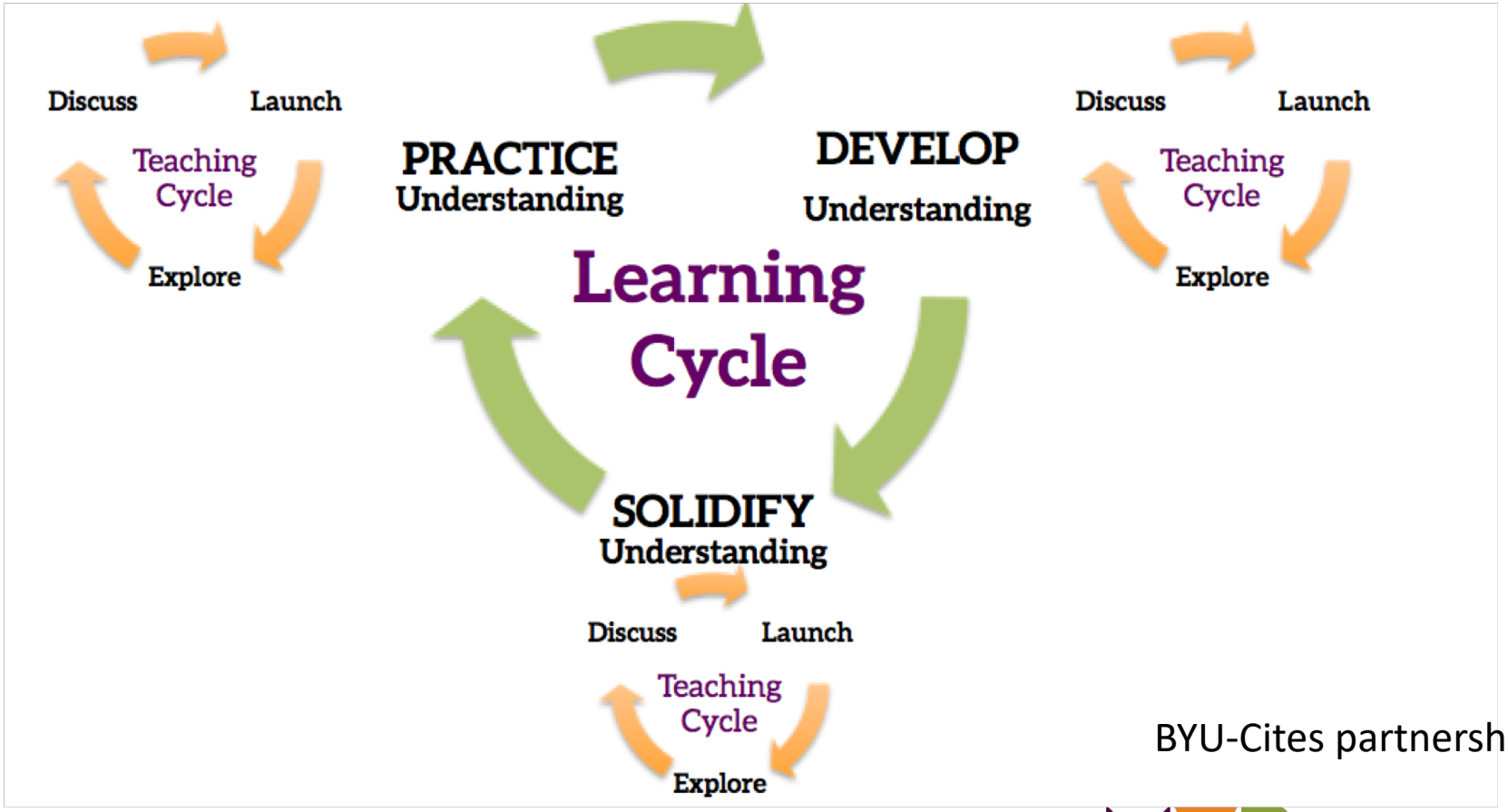

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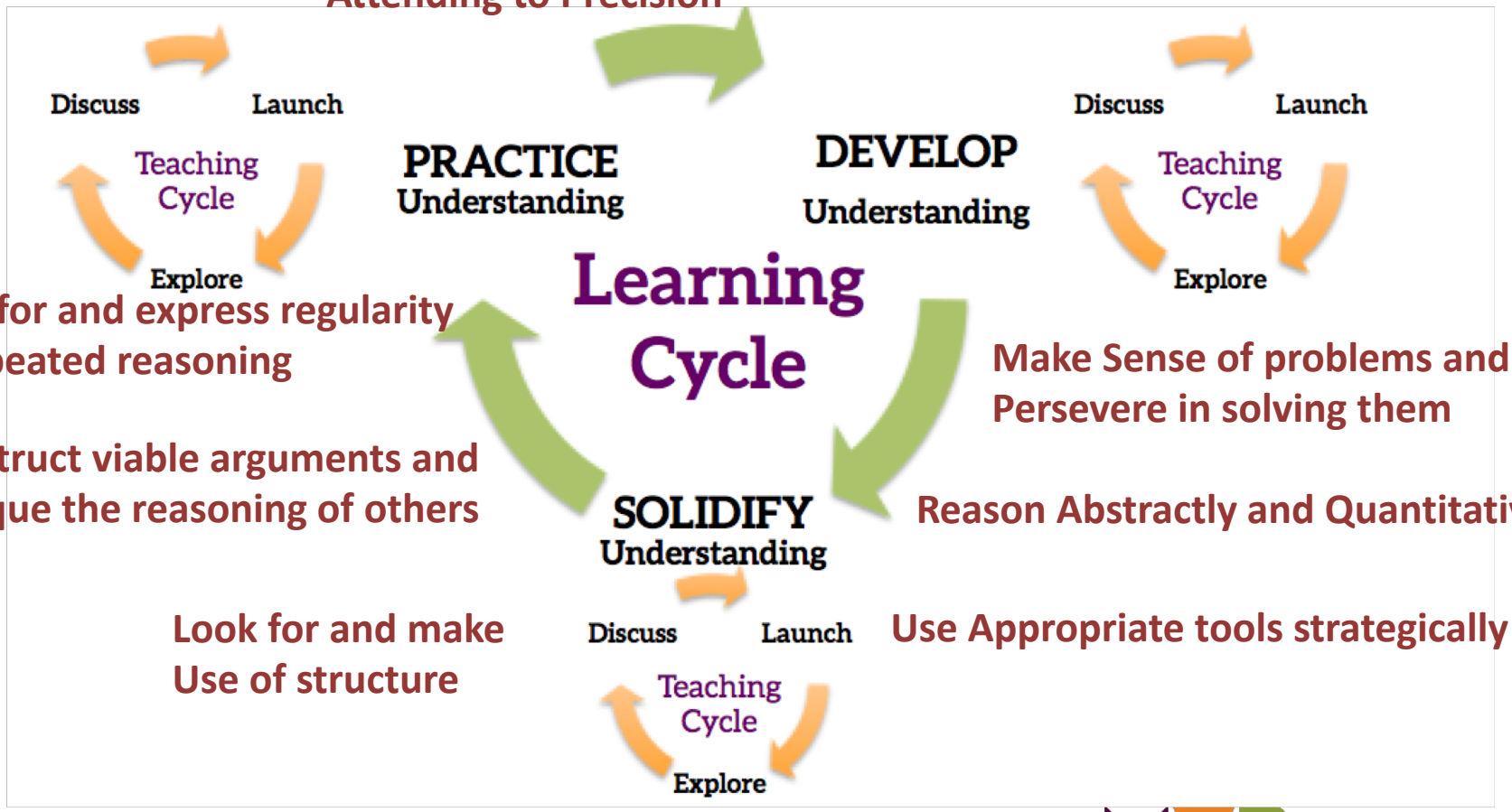

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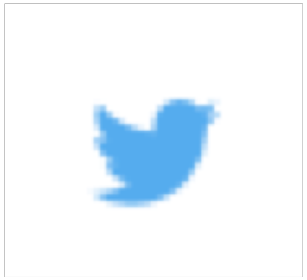

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