The Algebra I course is written to align with the first of three courses in the traditional pathway of the Common Core State Standards, as described in Appendix A. Each of the three courses, Algebra I, Geometry, and Algebra II contain standards from statistics and probability. The two Algebra courses contain the bulk of the work in number and quantity, algebra, and functions. The Geometry course covers geometry standards from a transformational perspective and includes right triangle trigonometry and conics.

The major purpose of Algebra I is to formalize and extend the mathematics that students learned in the middle grades, working with linear and exponential functions, solving systems of equations and inequalities, and analyzing data. The Mathematical Practice Standards apply throughout each course and, together with the content standards, create mathematical learning experiences based upon reasoning and sensemaking, building perseverance and problem-solving skills, and rich in mathematical discourse.

The standards indicated in the CCSS with a (+) sign are addressed with additional tasks in Algebra I Honors. The Honors version of the course includes all the same tasks as Algebra I, with the additional tasks embedded into the modules where they fit conceptually.

Standards specified in the Widely Accepted Prerequisites (WAP's) included in the High School Publishers Criteria for the Common Core State Standards for Mathematics constitute the bulk of the curriculum in Algebra I. The F-IF standards for interpreting functions are extensively addressed in Modules 1-3 and 6-8. All of the domains in the Algebra Conceptual Category are included in the WAP's. These domains constitute all the work in Modules 4 and 5.

In the narrative that follows, the specific approach and details of the mathematics in the curriculum is described by conceptual category in roughly the same order as the categories are addressed in the curriculum. The additional work of the Honors course is clearly identified.

Conceptual Category: Functions

In seventh grade, students did extensive work in proportional relationships including representing them in tables, graphs, and equations, along with identifying the unit rate and determining if two quantities are proportional. In eighth grade, this work was extended as students learned to construct a function to model a linear relationship and identifying the rate of change. They graphed linear relationships, learning that the slope of the line is the same as the rate of change. They connected tables, graphs, and equations in the form $y = mx + b$.

Module 1, Sequences, picks up where students left off in eighth grade, using diagrams and story contexts to introduce arithmetic sequences, identified by a constant difference, or rate of change, between terms. Arithmetic sequences are immediately contrasted with geometric sequences which have a constant ratio between terms. Module 1 is written as two intertwined learning cycles that begin by alternating from arithmetic sequence to geometric sequences, so students can compare and contrast features as they represent both types of sequences with tables, graphs, story contexts, diagrams, and equations. Students learn that both types of sequences can be increasing or decreasing, the graph of an arithmetic sequence is a line and the graph of a geometric sequence is a curve. They learn both types of sequences can be thought of recursively using the relationship from one output to the next, or explicitly using the relationship between an input and its output. The different ways of thinking of the relationships leads to different forms for the equations. Toward the end of the first module, the two types of sequences are often mixed together so that students

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learn to distinguish between them and represent both types appropriately. They use their understanding of the different rates of change in the two sequences to find missing terms.

Module 2, Linear and Exponential Functions, begins with a learning cycle that introduces contexts with continuous domains and defining linear functions as having a constant rate of change and exponential functions as having a constant ratio over equal intervals. Discrete and continuous contexts are discussed and compared so that students eventually see that arithmetic and geometric sequences are discrete linear and exponential functions. As the module continues with the second learning cycle, students compare how the different rates of growth in linear and exponential functions result in increasing exponential functions far exceeding increasing linear functions for large values of x. Students are also introduced to point-slope form and learn to use different equation forms to work fluently across representations including table, graph, equation, and story context. In the Honors task in Module 2 students calculate and interpret the average rate of change of functions, using secant lines to visualize the slopes.

Module 3, Features of Functions, generalizes many of the ideas about functions that students have worked with in the specific cases of linear and exponential functions. In this module, students broaden their thinking about functions to relationships that are not either linear or exponential. They formalize the definition of function as a relationship where each input has a unique output. Students work with all the representations for many different functions, learning to identify features such as:

- x and y intercepts;
- Domain and range;
- Continuity;
- Intervals of increase and decrease; and
- Maxima and minima.

Modules 1, 2, and 3 form the foundation for understanding functions throughout their high school experience. Some general concepts about functions that are established in Algebra I and used throughout the curriculum are:

1. Functions are categorized by their rates of change.
2. The key features of functions (as listed above) are tools for analysis.
3. Different forms of functions have purpose in different situations.
4. Functions can be transformed in a consistent, predictable way.
5. Functions can be combined together, either with arithmetic operations or by composition.

In Algebra II, students will develop a “catalog” of function types that they can work with fluently and flexibly, recognizing consistencies and differences among functions.

Module 6, Quadratic Functions, picks up where students left off with linear and exponential functions, using the same types of diagrams and representations to introduce quadratic functions. The entire module focuses on the features of quadratic functions, comparing quadratics to other functions, and representing quadratic functions. The first tasks in Module 6 use diagrams to help students write equations, both explicit and recursive, for quadratic functions. They use tables to identify the rate of change, noticing that the second difference is constant, making the first difference linear in a quadratic function. This idea is extended to the entire class of polynomials in

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Algebra II. In the first two tasks, students are also introduced to a related idea: that a quadratic function can be the sum of an arithmetic sequence or linear function. As the module advances, students use quadratic functions to model continuous contexts with a quadratic curve that has a maximum. There are lessons in the module that compare linear and quadratic functions and exponential and quadratic functions. The final task of the module asks students to distinguish between linear, exponential, and quadratic, given a single representation for the function, to create other representations for the function, and to identify the rate of change exhibited by the function.

Module 7, Structures of Expressions, is both a functions module and an algebra module. It is designed to extend students' knowledge of quadratic functions and to reinforce two big ideas of functions:

- Functions can be transformed in a consisten, predictable way.
- Different algebraic forms have purpose in different situations.

The module begins with students using technology to explore transformations of the graph of $f(x) = x^2$. They learn about horizontal and vertical translations, reflections over the x-axis, and vertical stretching or shrinking. These transformations are combined and students learn to identify the vertex, line of symmetry, reflection, and vertical stretch factor from equations in the form: $f(x) = a(x - h)^2 + k$. The curriculum encourages students to use a quick-graph method to be able to fluently identify the features of a quadratic function and produce an accurate graph of any equation in vertex form.

Students soon experience equations that are not in vertex form, which may require them to change forms. Completing the square is introduced as a method for this purpose, using area models so that students have a visualization to rely on for the procedure. Students are also introduced to factoring using area models and come to understand that factored form can also be useful for graphing quadratics using the x-intercepts and symmetry of the parabola to find the vertex. The only Honors task in the module addresses factoring trinomials in which the lead term has a coefficient that is not equal to 1. The module ends with students learning to be efficient in identifying the form that will be easiest to use in a given situation and moving flexibly between forms of quadratic equations for both graphing and solving. The idea that different quadratic forms are useful in graphing is extended to solving equations when students use different forms of quadratic equations to find the roots or solve quadratic equations of a single variable.

After all the work with linear, exponential, and quadratic functions in previous modules, Module 8, More Functions, More Features, shifts focus to consider piecewise functions, absolute value functions, and inverse functions. Understanding of piecewise functions is built from students' understanding of graphs and the stories that they can tell. Students learn to write functions for contexts in which rates change, making a piecewise function. Point-slope form of the equation of a line is frequently used and connected to students' previous experience with transformation of functions. Students write linear absolute value functions as piecewise functions and learn to graph linear absolute value functions with transformations. They also graph non-linear absolute value functions to more deeply understand the meaning of absolute value and the effect it has when composed with other functions.

The last learning cycle in Module 8 introduces students to inverse functions. It begins with a context in which two people each keep track of their own bike rides in different units, one in

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minutes per mile, one in miles per minute. As students model the two different methods for thinking about the bike rides, they notice that the inputs and outputs are reversed, making the graphs reflections over the $y = x$ line. These initial ideas about inverses are generalized in subsequent tasks in the module so that students learn to graph and write the inverse for simple functions. In Algebra II, students delve more deeply into inverses, writing inverses for more complicated functions, understanding invertibility, and extending the general ideas of inverses to find the inverse of an exponential function, which a logarithmic function.

Conceptual Category: Number and Quantity

In eighth grade, students learned about the properties of exponents and were introduced to integer exponents. The work with geometric sequences and exponential functions in Modules 1 and 2 provides opportunity to reinforce students understanding of integer exponents and increase their skill in using them. Continuous exponential functions provide a context for beginning to think about the outputs that lie between integer exponents, which will be further explored when students are introduced to rational exponents in Algebra II.

The three standards in the Numbers and Quantity conceptual category in Algebra I focus on using and interpreting units, defining quantities for modeling, and using appropriate levels of accuracy, based on measurement limitations. These three standards are touched upon throughout Modules 1 and 2 as students model various contexts with linear and exponential functions. Working with units and defining quantities are directly addressed in Module 4, Equations and Inequalities, where students use units in combinations to define new variables for use in modeling with equations and inequalities, and interpret expressions that are the result of combining units.

The last learning cycle in Module 4, Equations and Inequalities, of the Honors course contains additional tasks that involve students in organizing information in matrices. The operations of addition, subtraction, and multiplication with matrices are imbedded in story contexts that help students to understand the appropriate dimensions for each operation and why the operations on matrices work as they do.

Conceptual Category: Algebra

The grade 8 standards provide extensive background for students in solving single variable equations, including those that require multiple steps and using the Distributive Property. Module 4, Equations and Inequalities, builds on students' experience solving equations that have numeric solutions to solve literal equations, with one variable in terms of another. The approach to algebra throughout the curriculum is to motivate algebraic work through context. In the first learning cycle of Module 4, Equations and Inequalities, story context is used to support students in reasoning about what algebraic steps would be appropriate and why the steps make sense. The story contexts that have been provided in Module 4 help students to meet the standards which require them to solve literal equations and to justify each step in solving an equation or inequality. In the second learning cycle of Module 4, story contexts are used to reason about the rules for solving inequalities, writing inequalities, and to provide a means for discussion about common misconceptions in writing and using single variable inequalities.

Module 5, Systems of Equations and Inequalities, has two learning cycles, built around a common story context that is used throughout the module. The first learning cycle begins by making the representations, tables, graphs, equations, and diagrams, needed for the rest of the module

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available. The learning cycle proceeds by carefully developing the concepts and associated procedures for finding solutions to linear inequalities. The meaning of a constraint, the idea that the solutions to a linear inequality form a half plane, and interpreting and using standard form of the equation of a line to graph boundaries are addressed in this learning cycle. Extending these ideas to consider two linear inequalities as a system and the idea that the solutions to a system of inequalities must meet all constraints is explored in the second learning cycle. Students find solutions to systems of inequalities and write a system of inequalities given a solution.

The third learning cycle of Module 5 addresses solving systems of linear equations. The conceptual development for the meaning of a solution to a system of equations is provided in eighth grade, along with some experience in solving a simple linear system algebraically with substitution and by finding the intersection of two lines. The third learning cycle in Module 5 builds on this experience to develop the procedure for solving a system of equations by elimination. The procedure is developed using a story context related to the rest of the module so that students think about matching one of the unknown quantities in the two equations and then finding the difference between what is left to get a solution for one of the variables. The process is carefully built conceptually and then reinforced to be a procedure that students can perform fluently. The final task of the module explores systems of equations that are inconsistent or dependent, giving each of the terms meaning in the story context. In the Honors course, there are two additional tasks in Module 5 that introduce solving systems of equations using row reduction of matrices.

The overall approach in the MVP curriculum to algebra is to give algebraic work meaning and purpose by embedding it in story context, modeling, and functions. This approach is evident in Module 7, Structures of Expressions, which is described in the Functions section. Module 7 uses students' experience with graphing quadratics to derive a method for finding roots and x-intercepts that becomes the quadratic formula. Students learn to use the quadratic formula, along with other methods such as factoring or taking the square root of both sides to solve equations and inequalities accurately and efficiently.

Conceptual Category: Statistics and Probability

The first learning cycle in Module 9, Modeling Data, addresses representing data in dot plots, histograms, and box plots, and analyzing the data with appropriate summary statistics for center, shape, and spread, and identifying the existence of extreme data points. They compare data sets to draw conclusions and justify arguments based upon story context. This work extends the experience that students had in grades 6-8 where they informally described both center and spread.

The module progresses to using two-way frequency tables for bivariate data, analyzing joint and marginal relative frequencies to draw conclusions about the data. Students work with scatter plots and technology to construct meaning for the correlation coefficient, recognizing that as the correlation coefficient becomes closer to 1 or -1, the relationship is more linear. Students learn about the line of best fit and interpret the meaning of the slope and y-intercept of the line of best fit in context. As part of this work, they encounter situations that show that correlation is not the same as causation. The second learning cycle ends with students learning about residuals and how residual plots help to determine if a linear model is the most appropriate for the data. The tasks in Module 9 are designed to promote argumentation based on reasoning and statistical principles, involving students in interesting contexts using real data.

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