

SECONDARY MATH I COURSE OVERVIEW

The Secondary Mathematics I course is written to align with the first of three courses in the integrated pathway of the Common Core State Standards, as described in Appendix A. Like all courses in the integrated pathway, it contains standards from each of the conceptual categories in the standards, including:

- Number and quantity;
- Algebra;
- Functions;
- Geometry; and
- Statistics and probability.

The major purpose of Secondary Math I is to formalize and extend the mathematics that students learned in the middle grades, working with linear and exponential functions, using transformations to understand symmetries and congruence, solving systems of equations and inequalities, and analyzing data. The Mathematical Practice Standards apply throughout each course and, together with the content standards, create mathematical learning experiences based upon reasoning and sensemaking, building perseverance and problem-solving skills, and rich in mathematical discourse.

The standards indicated in the CCSS with a (+) sign are addressed with additional tasks in Secondary Math I Honors. The Honors version of the course includes all the same tasks as Secondary Math I, with the additional tasks embedded into the modules where they fit conceptually.

Standards specified in the Widely Accepted Prerequisites (WAP's) included in the High School Publishers Criteria for the Common Core State Standards for Mathematics constitute the bulk of the curriculum in Secondary Math I. The F-IF standards for interpreting functions are extensively addressed in Modules 1-3. Module 6 is foundational for all the Geometry standards, G-CO.1 G-CO.9 G-CO.10 G-SRT.B G-SRT.C, although all but G-CO-1 are topics of Secondary Math II. G-CO.1 describes understanding precise definitions of angle, circle, perpendicular line, parallel line, and line segment, which are used extensively throughout the two geometry units in Secondary Math I. Students develop a rich understanding of these terms as they use them to reason about transformations, construction, and features of triangles and quadrilaterals. All of the domains in the Algebra Conceptual Category are included in the WAP's. These domains constitute all of the work in Modules 4 and 5.

In the narrative that follows, the specific approach and details of the mathematics in the curriculum is described by conceptual category in roughly the same order as the categories are addressed in the curriculum. The additional work of the Honors course is clearly identified.

Conceptual Category: Functions

In seventh grade, students did extensive work in proportional relationships including representing them in tables, graphs, and equations, along with identifying the unit rate and determining if two quantities are proportional. In eighth grade, this work was extended as students learned to construct a function to model a linear relationship and identifying the rate of change. They graphed linear relationships, learning that the slope of the line is the same as the rate of change. They connected tables, graphs, and equations in the form $y = mx + b$.

SECONDARY MATH I
COURSE OVERVIEW

Module 1, Sequences, picks up where students left off in eighth grade, using diagrams and story contexts to introduce arithmetic sequences, identified by a constant difference, or rate of change, between terms. Arithmetic sequences are immediately contrasted with geometric sequences which have a constant ratio between terms. Module 1 is written as two intertwined learning cycles that begin by alternating from arithmetic sequence to geometric sequences, so students can compare and contrast features as they represent both types of sequences with tables, graphs, story contexts, diagrams, and equations. Students learn that both types of sequences can be increasing or decreasing, the graph of an arithmetic sequence is a line and the graph of a geometric sequence is a curve. They learn both types of sequences can be thought of recursively using the relationship from one output to the next, or explicitly using the relationship between an input and its output. The different ways of thinking of the relationships leads to different forms for the equations. Toward the end of the first module, the two types of sequences are often mixed together so that students learn to distinguish between them and represent them appropriately. They use their understanding of the different types of change in the two sequences to find missing terms.

Module 2, Linear and Exponential Functions, begins with a learning cycle that introduces contexts with continuous domains and defining linear functions as having a constant rate of change and exponential functions as having a constant ratio over equal intervals. Discrete and continuous contexts are discussed and compared so that students eventually see that arithmetic and geometric sequences are discrete linear and exponential functions. As the module continues with the second learning cycle, students compare how the different rates of growth in linear and exponential functions result in increasing exponential functions far exceeding increasing linear functions. Students are also introduced to point-slope form and learn to use different equation forms to work fluently across representations including table, graph, equation, and story context. In the Honors task in Module 2 students calculate and interpret the average rate of change of functions, using secant lines to visualize the slopes.

Module 3, Features of Functions, is the culminating functions module in Secondary Math I. In this module, students broaden their thinking about functions to relationships that are not either linear or exponential. They formalize the definition of function as a relationship where each input has a unique output. Students work with all the representations for many different functions, learning to identify features such as:

- x and y intercepts;
- Domain and range;
- Continuity;
- Intervals of increase and decrease; and
- Maxima and minima.

Modules 1, 2, and 3 form the foundation for understanding functions in all three courses in the integrated pathway. Some general concepts about functions that are established in Secondary Math 1 and used throughout the curriculum are:

1. Functions are categorized by their rates of change.
2. The key features of functions (as listed above) are tools for analysis.
3. Different forms of functions have purpose in different situations.

In Math II, students will learn two more general concepts about functions that will help them to develop a “catalog” of function types that they can work with fluently and flexibly, recognizing consistencies and differences among functions.

Conceptual Category: Number and Quantity

In eighth grade, students learned about the properties of exponents and were introduced to integer exponents. Although there are no new standards relating to exponents, the work with geometric sequences and exponential functions in Modules 1 and 2 provides opportunity to reinforce students understanding of integer exponents and increase their skill in using them. Continuous exponential functions provide a context for beginning to think about the outputs that lie between integer exponents, which will be further explored when students are introduced to rational exponents in Secondary Math II.

Students in eighth grade learn about square roots in the context of finding sides of a right triangle using the Pythagorean Theorem. They learn that numbers that are not rational, such as $\sqrt{2}$, are irrational. In Secondary Math I, Module 8, students will again use the Pythagorean Theorem to derive the distance formula and square roots to describe distances.

The three standards in the Numbers and Quantity conceptual category in Secondary Math I focus on using and interpreting units, defining quantities for modeling, and using appropriate levels of accuracy, based on measurement limitations. These three standards are touched upon throughout Modules 1 and 2 as students model various contexts with linear and exponential functions. Working with units and defining quantities are directly addressed in Module 4, Equations and Inequalities, where students use units in combinations to define new variables for use in modeling with equations and inequalities, and interpret expressions that are the result of combining units.

The last learning cycle in Module 4, Equations and Inequalities, of the Honors course contains additional tasks that involve students in organizing information in matrices. The operations of addition, subtraction, and multiplication with matrices are imbedded in story contexts that help students to understand the appropriate dimensions for each operation and why the operations on matrices work as they do.

In the Honors course, Module 8 contains five additional tasks about vectors and matrices. Quantities that can be represented using vectors are introduced. Multiplication and addition properties of matrices are explored. Students learn to find determinants and use matrix multiplication to rotate vectors and images.

Conceptual Category: Algebra

The grade 8 standards provide extensive background for students in solving single variable equations, including those that require multiple steps and using the Distributive Property. Module 4, Equations and Inequalities, builds on students’ experience solving equations that have numeric solutions to solve literal equations, with one variable in terms of another. The approach to algebra throughout the curriculum is to motivate algebraic work through context. In the first learning cycle of Module 4, Equations and Inequalities, story context is used to support students in reasoning about what algebraic steps would be appropriate and why the steps make sense. The story contexts that have been provided in Module 4 help students to meet the standards which require

them to solve literal equations and to justify each step in solving an equation or inequality. In the second learning cycle of module 4, story contexts are used to reason about the rules for solving inequalities, writing inequalities, and to provide a means for discussion about common misconceptions in writing and using single variable inequalities.

Module 5, Systems of Equations and Inequalities, has two learning cycles, built around a common story context that is used throughout the module. The first learning cycle begins by making the representations, tables, graphs, equations, and diagrams, needed for the rest of the module available. The learning cycle proceeds by carefully developing the concepts and associated procedures for finding solutions to linear inequalities. The meaning of a constraint, the idea that the solutions to a linear inequality form a half plane, and interpreting and using standard form of the equation of a line to graph boundaries are addressed in this learning cycle. Extending these ideas to consider two linear inequalities as a system and the idea that the solutions to a system of inequalities must meet all constraints is explored in the second learning cycle. Students find solutions to systems of inequalities and write a system of inequalities given a solution.

The third learning cycle of Module 5 addresses solving systems of linear equations. The conceptual development for the meaning of a solution to a system of equations is provided in eighth grade, along with some experience in solving a simple linear system algebraically with substitution and by finding the intersection of two lines. The third learning cycle in Module 5 builds on this experience to develop the procedure for solving a system of equations by elimination. The procedure is developed using a story context related to the rest of the module so that students think about matching one of the unknown quantities in the two equations and then finding the difference between what is left to get a solution for one of the variables. The process is carefully built conceptually and then reinforced to be a procedure that students can perform fluently. The final task of the module explores systems of equations that are inconsistent or dependent, giving each of the terms meaning in the story context. In the Honors course, there are two additional tasks in Module 5 that introduce solving systems of equations using row reduction of matrices.

Conceptual Category: Geometry

The standards for geometry in the integrated pathway are carefully designed to allow students to experiment and construct general ideas about shapes and how they transform in eighth grade, moving towards formalizing definitions of rigid transformations and congruence in Secondary Math I through reasoning with diagrams, and then proving theorems and formalizing definitions of dilation and similarity in Secondary Math II. True to the vision of the standards, the MVP curriculum takes a transformational approach to the standards, developing transformations and construction as tools for reasoning and proof that are used in addition to the traditional axiomatic tools of geometry. The curriculum provides students many opportunities to use their intuitive understanding about geometry and experiment with compass, protractor, patty paper, rulers, graph paper, dynamic geometry software and other physical tools to make and justify conjectures.

Module 6, Transformations and Symmetry, builds on students' experiences with rigid motion in earlier grades to formalize the definitions of translation, rotation, and reflection. In the first learning cycle which focuses on the definitions of the rigid transformations, students discover features such as:

SECONDARY MATH I
COURSE OVERVIEW

- In a translation, the corresponding points from the pre-image to the image form segments that are congruent and parallel.
- In a rotation, corresponding points from the pre-image to the image lie along concentric arcs.
- In a reflection, the line of reflection is the perpendicular bisector of corresponding points from the pre-image to the image.

Students use these features to perform translations and to determine what translations have been performed, given an image and corresponding pre-image. Students' observations about the rigid transformations give purpose and meaning to vocabulary words such as parallel, perpendicular, bisect, concentric, etc.

In the second learning cycle of Module 6, students use their understanding of rigid transformations to find the rotations and reflections that carry a figure onto itself. They explore the symmetries and diagonals of quadrilaterals and regular polygons, making and justifying conjectures about the relationship between the number of lines of symmetry and the number of sides of a regular polygon. At the end of the learning cycle, students make conjectures and classify quadrilaterals based upon symmetries and see that the classifications turn out to be the same as when the quadrilaterals are classified by angles and sides. Students justify their conjecture using their knowledge of the transformations.

Module 7, Congruence, Construction, and Proof, begins by developing constructions as another tool to be used to reason about figures and to justify properties of shapes. Individual constructions are not taught for the sake of memorizing a series of steps, but rather to reason using known properties of shapes such as circles. Many of the constructions, such as the angle bisector, the perpendicular bisector, and the midpoint, flow from constructing a rhombus in the first task. In the second task, students reason to perform the construction of a parallelogram, an inscribed hexagon, and a square. At the end of the module students use triangle congruence and rigid transformation to examine and justify why given compass and straight-edge constructions result in the desired figures.

Students enter Secondary Math I with experience from grade 8 in using rigid transformations to experimentally determine if two figures are congruent. The work with congruence in Module 7 begins with students experimenting to find a general sequence of rigid transformations that will map a figure onto another if they are congruent. Students find that they can generally translate to get a pair of matching vertices, rotate to make a pair of corresponding sides coincide, then reflect to make the rest of the figure coincide. As the learning cycle continues, students use this sequence of transformations (or an equivalent sequence that they have found) to show the triangle congruence properties of ASA, AAS, SSS, and SAS. They also learn that two consecutive sides and an angle (SSA) are not enough to guarantee congruence between two triangles. After determining the congruence criteria of triangles, students use the criteria along with the rigid transformations to justify properties of quadrilaterals, such as the diagonals of a rectangle are congruent.

In Module 8, Connecting Algebra and Geometry, students use the Pythagorean Theorem to find the distance between two points and to derive the distance formula. The idea that parallel lines have the same slope and the slopes of perpendicular lines are negative reciprocals is introduced in Module 6, and then proven in Module 8. Students use the distance formula, their knowledge of slopes, and the features of quadrilaterals, to prove that a given figure is a particular type of quadrilateral, like a parallelogram. Module 8 concludes with a learning cycle that introduces the

idea that functions, like geometric figures, can be transformed. In Secondary Math I, students learn about vertical translations of functions; other transformations such as reflection, dilation, and horizontal translations are added in Secondary Math II. The idea that all functions can be transformed in the same, predictable way is one of the big ideas about functions that is introduced in Secondary Math I and used consistently throughout the curriculum.

Conceptual Category: Statistics and Probability

The first learning cycle in Module 9, Modeling Data, addresses representing data in dot plots, histograms, and box plots, and analyzing the data with appropriate summary statistics for center, shape, and spread and identifying the existence of extreme data points. They compare data sets to draw conclusions and justify arguments based upon story context. This work extends the experience that students had in grades 6-8 where they informally described both center and spread.

The module progresses to using two-way frequency tables for bivariate data, analyzing joint and marginal relative frequencies to draw conclusions about the data. Students work with scatter plots and technology to construct meaning for the correlation coefficient, recognizing that as the correlation coefficient becomes closer to 1 or -1, the relationship is more linear. Students learn about the line of best fit and interpret the meaning of the slope and y-intercept of the line of best fit in context. As part of this work, they encounter situations that show that correlation is not the same as causation. The second learning cycle ends with students learning about residuals and how residual plots help to determine if a linear model is the most appropriate for the data. The tasks in Module 9 are designed to promote argumentation based on reasoning and statistical principles, involving students in interesting contexts using real data.