STRUCTURE OF THE CURRICULUM

Each curriculum in the Mathematics Vision Project materials is composed of two main components, the **classroom experience**, which is designed around the implementation of a specific type of task and the aligned **"Ready, Set, Go!" homework assignment**. Each task is accompanied by a set of teacher notes. The teacher notes identify the purpose of the lesson and describe the steps the teacher can take during the classroom experience to ensure that students engage in a rich learning event. Tasks are to be done in class and should not be assigned as homework. There is an aligned "Ready, Set, Go!" homework assignment for each task. It is the independent practice. Homework serves the student as a type of formative assessment. It is while doing the homework that the student can discern for himself if the mathematics done in class can be performed independently.

The MVP **classroom experience** begins by confronting students with an engaging task and then invites them to grapple with solving it. As students' ideas emerge, take form, and are shared, the teacher orchestrates the student discussions and explorations towards a focused mathematical goal. As conjectures are made and explored, they evolve into mathematical concepts that the community of learners begins to embrace as effective strategies for analyzing and solving problems. These strategies are eventually solidified into a body of practices and mathematical habits that belong to the students, because they were developed by the students, as an outcome of their own creative and logical thinking. This is how students learn mathematics. They learn by doing mathematics. They learn by needing mathematics. They learn by verbalizing the way they see the mathematical ideas connect and by listening to how their peers perceived the problem. Students then own the mathematics because it is a collective body of knowledge that they have developed over time through guided exploration.

This process describes the **Learning Cycle**, an instructional framework that allows students to build mathematical knowledge over time. This framework is flexible. Every progression does not follow the pattern of develop, solidify, practice. For instance, the first module on quadratics begins with a Develop Understanding Task. Many aspects of the definition of a quadratic surface in that task. Five solidify tasks follow the first task. Each of the Solidify tasks extends one of the key concepts that surfaced in the

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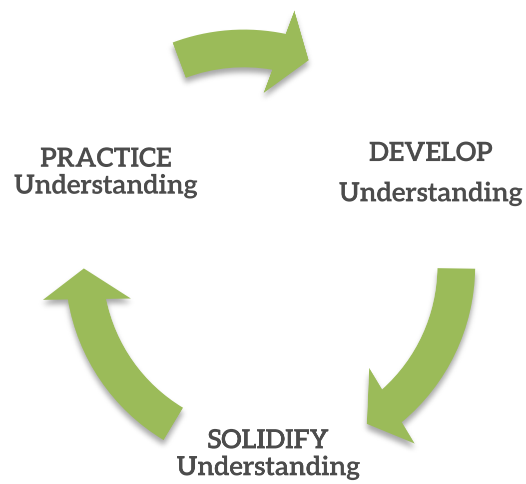
beginning Develop Understanding Task. The module ends with a Practice Understanding Task that pulls all of the key concepts together into a complete definition of quadratic.

The Learning Cycle

The diagram at the right illustrates the Comprehensive Mathematics Instructional Framework (CMI) around which the MVP curriculum has been developed. Every task in the curriculum is identified as one of the following:

- Develop Understanding Task
- Solidify Understanding Task
- Practice Understanding Task

A learning cycle begins with a single term, *develop*, which refers to bringing student thinking to the surface by activating prior knowledge, intuition, and insights to make sense of a problem.



The Learning Cycle

Develop Understanding Tasks are intended to generate ideas, strategies, and representations related to a new mathematical topic. Develop tasks contain multiple entry points for students, so that all students are able to use their intuition and logic to make sense of the problem and devise a strategy for organizing the information. In the second phase of the learning cycle, students will engage in Solidify Understanding tasks that will allow them to examine and extend the mathematical thinking that rose to the surface in the Develop Understanding task. The learning cycle will conclude with a Practice Understanding Task. It focuses students' attention on becoming fluent with the mathematics of the unit and refining the mathematics into formal definitions, properties, procedures, and models that are consistent with practices that exist outside the classroom.

In the *CMI Framework* the progression of the mathematics through the *learning cycle* is mapped out along a continuum of conceptual, procedural and representational understandings using the *Continuum of Mathematical Understanding*.

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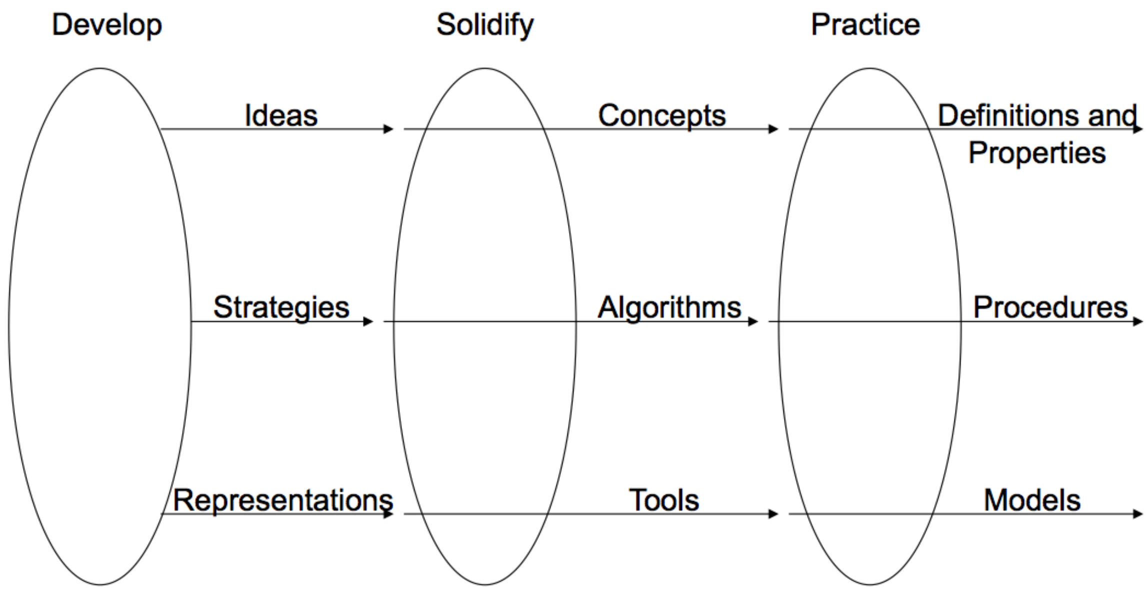
Continuum of Mathematical Understanding



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Mathematical understanding encompasses at least three connected but distinct domains as represented by the horizontal lines of the continuum: conceptualizing mathematics, doing mathematics, and representing mathematics. Mathematical understanding progresses continually along this continuum, but it is useful to note three sets of distinct landmarks of progression along the continuum that are associated with each of the three phases of the *Learning Cycle*. Emerging mental images are fragile as they are surfaced during students' initial experiences with tasks designed to elicit those images (*Develop Understanding*). In the continuum we refer to these fragile images as ideas, strategies, and representations. These ideas, strategies and representations need to be examined for accuracy and completeness, as well as extended and connected through multiple exposures and experiences until they become more tangible, solid and useful (*Solidify Understanding*). In the *CMI Framework*, ideas that have been examined for the understanding they reveal are called concepts; strategies that can be articulated and replicated are called algorithms; and useful representations are called tools. Once understanding has been developed and solidified, it needs further

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refinement to become fluent and applicable to new situations and contexts (*Practice Understanding*). In the *CMI Framework* refined concepts become the definitions or properties of formal mathematics; algorithms that can be carried out flexibly and fluently are called procedures; and representations that embody essential mathematical understandings (either conceptual or procedural) are called models, such as "an area model for multiplication" or "the number line as a model of the set of real numbers." These definitions and properties, procedures, and models must be consistent with the broader mathematical "community of practice" that exists outside of the classroom.

The *CMI Framework* supports teachers in enacting the NCTM effective teaching practice: *Build Procedural Fluency from Conceptual Understanding.* However, the framework implies that the end-goal of mathematical instruction is not just procedural fluency; it also includes a deeper conceptual understanding of the properties and definitions on which procedures are based, and an ability to draw upon mathematical models more flexibly and fluently when representing one's mathematical understanding. The *Learning Cycle* component of the framework supports teachers in making curricular decisions that move students from individually-constructed ideas, strategies and representations towards a community of shared definitions, properties, procedures and models. The *Continuum of Mathematical Understanding* component of the framework emphasizes that there are multiple domains of mathematical understanding that need to be developed, solidified and practiced: the conceptual domain, which provides students with *ways of thinking about mathematics*; the procedural domain, which provides students with *ways of doing mathematics*; and the representational domain, which provides students with *ways of making one's thinking visible*. Together, both components of the *CMI Framework* promote student thinking to the forefront of mathematics instruction and highlight the decision-making role of the teacher in effectively selecting and sequencing tasks that build mathematical understanding and fluency over time.

Each module in the **MVP** educational program has been carefully designed and sequenced with rich mathematical tasks that have been formulated to generate the mathematical concepts within the core curriculum. Careful attention has been placed upon the way mathematical knowledge emerges, is

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extended, and then becomes efficient, flexible, and accurate. Some tasks are developmental tasks while others are for solidifying or practicing the concepts. The sequencing of the tasks encourages students to notice relationships and make connections between the concepts. In this way, students perceive mathematics as a coherent whole.

While the classroom experience is predominantly geared towards improving students' reasoning and sense-making skills, MVP regards mathematical understanding and procedural skill as being equally important. Hence, the "**Ready, Set, Go**!" **homework assignments** are focused on students practicing procedural skills and organizing principles to add structure to the ideas developed during the classroom experience**.** As in any discipline, practice is the refining element that brings fluency and agility to the skills of the participant. The **Ready** and the **Go** sections of the homework assignments have been designed to spiral a review of content, while the **Set** section focuses on consolidating the mathematics addressed in class that day. Each time a student engages in the homework assignment, it is expected that he or she will have the opportunity to reflect on the new learning from class and will practice the retrieval of ideas from the body of learning that has been growing over the school year, and even prior to the current school year. Recent research on learning has identified reflection and retrieval practice as being two key ingredients for durable learning. True learning should be long lasting and should grow out of previous understandings, extending over years of study. Hence, the "**Go!"** sections of the **"Ready, Set, Go!" homework assignments** will contain topics from previous lessons and prior years of mathematics instruction. Together the **classroom experience** and the **"Ready, Set, Go!" homework assignments** offer a powerful blend of new learning and maintained proficiency.

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The Teaching Cycle

The Learning Cycle depicts how students become proficient in the mathematics overtime. Each task represents at least one day of instruction. Therefore, a Learning Cycle may extend over several days or weeks of classroom instruction, however, each day the teacher is expected to frame the lesson around **The Teaching Cycle.** This cycle also has three components: **Launch, Explore**, and **Discuss.**

The Teaching Cycle may seem to be simple, but it involves careful preparation and then deliberate implementation by the instructor.

Launch: How will you . . .

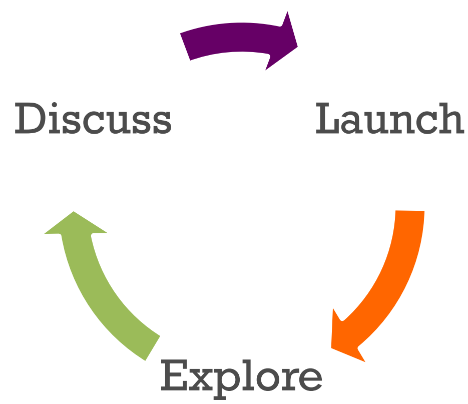
- hook and motivate students?
- provide schema for the task?
- describe the expectations for the finished task?

Explore: What will you . . .

- look for and listen for as you observe?
- accept as evidence of understanding?
- ask to stimulate, redirect, focus, and extend mathematical thinking?

Discuss: How will you . . .

- select which students will present their solutions and strategies?
- determine what ideas to pursue?
- decide whether to contribute to the discourse or allow students to continue to struggle to make sense of a concept?



The Teaching Cycle

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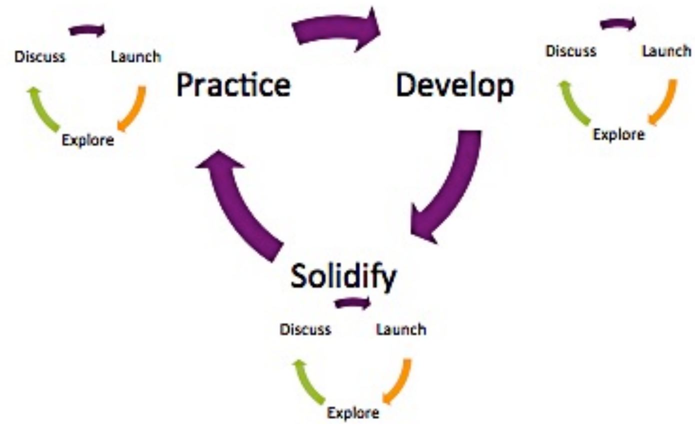
The diagram to the right depicts how the two instructional frameworks, the **Teaching Cycle** and the **Learning Cycle**, fit together. The **Teaching Cycle** occurs each day in the classroom, while the **Learning Cycle** extends over days and possibly weeks as the unit develops.



The MVP curriculum and the CMI instructional framework together reflect current research on teaching and learning. Research in both cognitive science and mathematics education supports changes in the roles of the learner and the teacher. During instruction, students need to be developing specific reasoning habits that will serve them in other disciplines, real life, and their future careers. It is the teacher's role to provide opportunities for students to develop these skills. The CMI model provides a framework for both the teacher and the student to improve teaching and learning in the classroom.

The Comprehensive Mathematics Instruction Model		
	Teacher's role	**Student's role**
Develop Understanding	Focus learning on the goal of the task; provide experiences using rich tasks; support productive struggle; elicit and use evidence of student thinking to orchestrate discussions using the 5 practices*	Make sense of the context, organize information, notice patterns, make conjectures, invent strategies, create arguments, engage in mathematical discourse
Solidify Understanding	Focus learning on the goal of the task; provide experiences using rich tasks; support productive struggle; elicit and use evidence of student thinking to orchestrate discussions using the 5 practices*	See structure; see regularities; attend to precision; create and critique arguments; adopt strategies, use multiple representations, engage in mathematical discourse
Practice Understanding	Provide a vehicle for practice; provide feedback; clarify misconceptions; confirm mathematical and symbolic language; elicit and use evidence of student thinking to orchestrate discussions using the 5 practices*	Reason quantitatively; work towards efficiency, flexibility, accuracy; apply (model with mathematics)

**Five Practices for Orchestrating Productive Mathematical Discussion* – 2nd Edition, Margaret S. Smith and Mary K. Stein, NCTM, 2018

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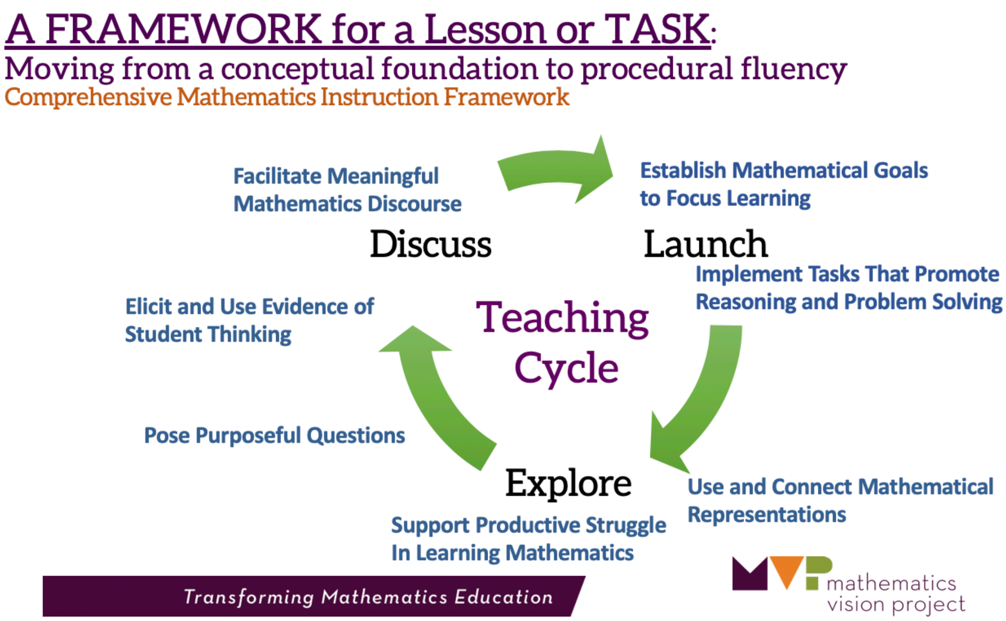
The eight effective teaching practices, as articulated in the NCTM publication *Principles To Actions, Ensuring Mathematical Success for All* (2014), describe a framework for improving instructional practice. The following figure shows how these eight practices can be incorporated into the Teaching Cycle. Note that seven of the practices fit naturally around the Teaching Cycle and can be implemented during each day of instruction, while building procedural fluency from conceptual understanding is a curriculum practice that describes the process of creating deep learning over time.



The *Launch, Explore, Discuss* sequence of the Teaching Cycle is the visible form of the daily, classroom experience. Yet, to make deep learning occur in the classroom, the teacher must carefully prepare for each aspect of the lesson. During the *Launch* the teacher must set the stage by informing students of the situation and the expectations of the task. During the *Explore* phase, as students are reasoning through the task, the teacher is busy moving from student to student, clarifying student questions and encouraging student work. As the teacher monitors student effort, he is also selecting and sequencing which work will move student

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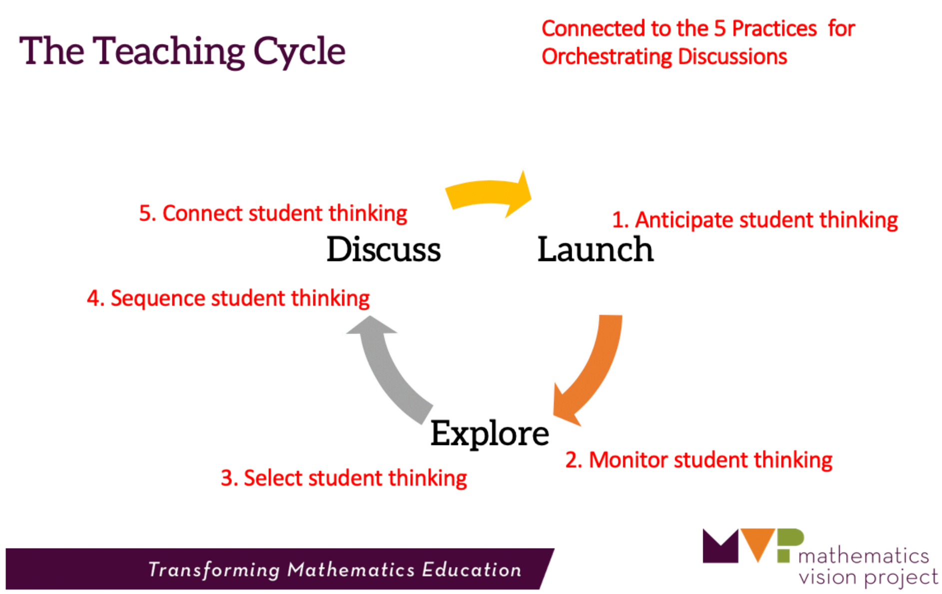
thinking towards the purpose of the lesson. During the *Discuss* phase, selected students share their mathematical thinking and strategies, while members of the class listen, question, and record strategies and key concepts. Throughout the lesson, it is the obligation of the teacher to connect the mathematics so that students leave class with the big ideas of the intended mathematical lesson. The following figure depicts how the framework of the five practices for orchestrating discourse fit within the Teaching Cycle. (Adapted from *Five Practices for Orchestrating Productive Mathematical Discussion* – Second Edition, Margaret S. Smith and Mary K. Stein, NCTM, 2018)



**Five Practices for Orchestrating Productive Mathematical Discussion* – 2^{nd} Edition, Margaret S. Smith and Mary K. Stein, NCTM, 2018

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FEATURES

Each module begins with an **annotated table of contents** which identifies the key concepts that will be the focus of the module and the core standards that will be addressed. A set of teacher notes accompanies each task. The teacher notes outline each step of the lesson while following the framework of the **Teaching Cycle**. All of the teacher notes follow the same basic outline as described below:

The Enhanced Teacher Notes include:

Purpose: Paying attention to the purpose of the task will help the teacher stay true to the progression of the module and refrain from trying to accomplish too much within the task.

Core Standards Focus: The MVP authors have taken a "multi-tasking approach" to the standards. While one task may focus on more than one standard, several tasks may hi-light a single standard. In this way a set of interrelated ideas or a sequence of strategies and skills can be fused into a meaningful whole. This "multi-tasking approach" to the standards also gives students multiple opportunities to master the standards.

Related Standards: The focus of a lesson may be on a specific standard, yet doing the mathematics may require students to draw on related standards.

Standards for Mathematical Practice: It is possible and even likely that students will implement all of the Standards for Mathematical Practice within a given lesson, however, different types of tasks naturally elicit certain practices. Those that seem to be the most likely to be drawn upon in the lesson have been identified in the teacher notes.

Essential question for students: Since all of the tasks are inquiry based, the essential question has been formulated to direct students' attention towards the purpose of the lesson without explicitly revealing the key ideas and strategies they should be producing.

The Teaching Cycle:

Launch (whole class): Suggestions for introducing the lesson to the students. Sometimes this is relating a story, while other times it's working the first problem together. The prompts for the tasks often involve a lot of reading. It is the teacher's obligation to make sure that students understand what they are expected to do or produce during the Explore stage of the learning.

Explore (small groups): While students are exploring, the teacher will be monitoring the individual students and groups, looking for student strategies that will promote the discussion about the

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mathematics of the task. This is also a time during which the teacher can assess what previously learned skills the students are bringing to the task. The teacher notes will make suggestions of what the teacher should be looking for during the Explore session.

Discuss: Here the teacher will find suggestions for orchestrating the discussion in order to achieve the purpose of the lesson. This is the time when key connections need to be made.

Exit ticket for students: An exit slip can aid the teacher in checking for understanding. The items in the exit ticket could also be used as a warm-up in the subsequent lesson.

Instructional Supports

ELL and equity suggestions: Equitable mathematics teaching maintains high standards of learning for all students. Instruction should affirm students' mathematical identities by honoring the multiple resources of mathematical learning present in the classroom. By following the plan of instruction included in the teacher notes, students' different mathematical strengths are used as a resource for learning. Additional strategies for providing equal opportunities for learning are offered where appropriate.

Interventions: These suggestions may lower the threshold for the task to accommodate students who don't know how to begin thinking about the task.

Challenge activity: The challenge activity is to provide a "high ceiling" for students who have finished early or need to be encouraged to think more deeply about the mathematics. Sometimes the last question in the task provides that extension, and it is not essential that it be completed by all students.

Additional Resources for Teachers: This could be a variety of things depending on the lesson. For instance, an app using GeoGebra has been developed for the rubber-band activity in the first task of Module 2 in the geometry course.

Sentence frame cards are available as an aid for students. The cards are intended to assist students in becoming self-directed thinkers by guiding their thinking and prompting the language needed for discourse about their mathematical work. The cards are structured around the Eight Student Practices for Mathematical Thinking. The cards are intended to support all learners, but they are particularly useful in supporting learners who struggle with language.

Answer Key for each task:

The suggested mathematical approach for some of these tasks may require teachers

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to look at the mathematics from a different perspective than they have ever done before. The best way to prepare to teach a task is to work the problem from the standpoint of the student. The answer key is provided as reassurance for the teacher.

Answer Key for each Ready, Set, Go! Homework assignment

Additional Resources for Teachers and Students

The website www.rsgsupport.org contains a support video to match each Ready, Set, Go Homework assignment.

The Helps, Hints, and Explanations book provides an explanation for each type of homework problem and usually a worked example or two with annotation. Each "Ready, Set, Go!" homework assignment has an accompanying explanation in the Helps, Hints, and Explanations book.

Assessments and Tools for the PLC

The assessment resources provide a more complete assessment package including: quick quizzes, self-assessments, performance tasks, and a bank of items that cover the topics of the module. When these tools are combined with the exit tickets and the formative assessment available from listening to students as they work on the tasks, teachers can really know what their students understand and can do.

The assessment book includes per module:

- Quick quizzes (mid-unit checks for understanding)
- Student self-assessments (identifies what the student should know and be able to do as he progresses through the module)
- A Module test
- A Performance task with teacher notes and a scoring rubric

The **quick quizzes** are short, multiple-choice and short answer assessments that give a snapshot of what students have learned in the module. They are designed to be given after a learning cycle is completed (in most cases), so the number of quick quizzes in a module varies. The quick

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quizzes should be just that: quick. They can be given at the beginning or end of a period, still allowing time for other work to occur.

The **self-assessments** are a tool designed to help students know what they should be learning and to reflect on their progress. Like the quizzes, the self-assessments usually occur at the end of each learning cycle. They identify the mathematics that students should have learned and ask students to provide evidence (from their homework, their work on tasks, or problems from other assessments) that shows how well they have learned it, and to write what they will do to increase their understanding. The idea is to help students develop a growth mindset and gain ownership of their learning.

The **Module Tests**, provide a bank of items that can be used to design a summative assessment that reflects the work of the class. Some teachers like to create tests that have both the performance task and some more traditional items to ensure that all the standards of the module are assessed. There are about as many ways to assess as there are teachers, and most of the methods have merit. The key is to use all assessments as checkpoints to make instructional adjustments that will increase student learning.

The final tool is the **performance task**. There is one performance task provided for each module. This task incorporates the most important ideas of the module and asks students to use them flexibly. These tasks also provide an opportunity for students to communicate mathematically, using proper vocabulary and notation. An answer key and grading rubric are provided for each task, along with instructions for launching the task so that the task is accessible for each and every student. Some teachers like to give students the opportunity to work these tasks in pairs, mirroring the classroom experience. Others prefer to ask students to work individually to ensure that the results give a clear picture of what each student can do on their own.

The PLC tools include **The Essentials Tracker** and **The Power of the Module**.

The Essentials Tracker is a grid connecting the standards and the tasks. When a standard is addressed in a task, it is indicated with one of three letters, *D* for developing, *S* for solidifying, or *P* for practicing. This helps teachers to see that the standards are addressed in more than one task. It also helps teachers to set an appropriate level of

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expectation for students relative to the standard. *D*, for developing, indicates students' first exposure to the ideas and/or procedures of the standard, so teachers can expect new ideas to surface, although students may lack the notation or vocabulary that will be developed later. At the *S* level, for solidifying, students will be sorting through ideas that have previously been surfaced, with support for examining and extending their understanding and clarifying their procedures. If the standard is addressed at the *P* level, for practice, then students should be working on becoming efficient, accurate, and flexible as they demonstrate mastery of the standard.

The Power of the Module shows the focus or target for each task in the module and the topics of each section of the homework: *Ready* (to prepare for upcoming tasks), *Set* (to solidify the work done in the task), and *Go* (to reinforce previously-learned skills). This tool can help teachers see the opportunities for recall and rehearsal built into the program, along with the progression of mathematical ideas in the tasks. It also provides a "quick glance" or overview of the module, which will help teachers anticipate upcoming mathematical content. By working the tasks, then creating and discussing the **Power of a Module** outlines as a team, teachers will come to trust the materials and understand the progressions of mathematics that students will have the opportunity learn.

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TECHNOLOGY

Technology is an important tool to be used as part of the MVP curriculum. In their description of the CCSS Standard for Mathematical Practice 5, "Use appropriate tools strategically," the core authors specifically name graphing calculators, computer algebra systems, statistical packages, and dynamic geometry software. They suggest that these tools could be used by students to explore and deepen their understanding of concepts, analyze graphs of functions, visualize mathematical models, and test various assumptions and compare predictions with data. Tasks in MVP provide opportunities for using technological tools in each of the ways described. The use of calculators may also help students to quickly make calculations so that their attention remains focused on the analytical work of the task. The curriculum is designed so that students may use technology that is widely available including graphing calculators or free computer apps such as Desmos or Geogebra. Making technology an integral tool for mathematical thinking enriches the work and provides students with opportunities to engage with SMP 5.

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The Geometry course is written to align with the second of three courses in the traditional pathway of the Common Core State Standards, as described in Appendix A. Each of the three courses, Algebra I, Geometry, and Algebra II contain standards from statistics and probability. The two Algebra courses contain the bulk of the work in number and quantity, algebra, and functions. The Geometry course covers geometry standards from a transformational perspective and includes right triangle trigonometry, and conics.

The fundamental purpose of the course in Geometry is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Geometry is addressed from a transformational perspective, the definitions and features of translations, rotations, reflections, and dilations becoming important tools for reasoning and proof. Students use constructions as an additional tool for understanding. Congruence and similarity are defined with transformations and students explore symmetry as a way of classifying quadrilaterals. The study of similarity leads to an understanding of right triangle trigonometry and extended to finding angles and sides for general triangles. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The Mathematical Practice Standards apply throughout each course and, together with the content standards, create mathematical learning experiences based upon reasoning and sensemaking, building perseverance and problem solving skills, and rich in mathematical discourse.

The standards indicated in the CCSS with a (+) sign are addressed with additional tasks in Geometry Honors. The Honors version of the course includes all the same tasks as Geometry, with the additional tasks embedded into the modules where they fit conceptually.

Geometry standards specified in the Widely Accepted Prerequisites (WAP's) included in the High School Publishers Criteria for the Common Core State Standards for Mathematics are fully represented in this course. Modules 1 and 2 contain the emphasized Geometry standards, G-CO.1 G-CO.9 G-CO.10, which address proving statements about lines, angles, and triangles. Module 4 contains G-SRT.B G-SRT.C, addressing the topics of similarity and right triangle trigonometry. Students develop a rich understanding of geometric terms as they use them to reason about transformations, construction, and features of triangles and quadrilaterals.

In the narrative that follows, the specific approach and details of the mathematics in the curriculum is described by conceptual category in roughly the same order as the categories are addressed in the curriculum. The additional work of the Honors course is clearly identified.

Conceptual Category: Geometry

The standards for geometry are carefully designed to allow students to experiment and construct general ideas about shapes and how they transform in eighth grade. In Geometry, the standards move towards formalizing definitions of rigid transformations and congruence through reasoning with diagrams, and then proving theorems and formalizing definitions of dilation and similarity. True to the vision of the standards, the MVP curriculum takes a transformational approach to the standards, developing transformations and construction as tools for reasoning and proof that are used in addition to the traditional axiomatic tools of geometry. The curriculum provides students many opportunities to use their intuitive understanding about geometry and to experiment with

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compass, protractor, patty paper, rulers, graph paper, dynamic geometry software and other physical tools to make and justify conjectures.

Module 1, Transformations and Symmetry, builds on students' experiences with rigid motion in earlier grades to formalize the definitions of translation, rotation, and reflection. In the first learning cycle which focuses on the definitions of the rigid transformations, students discover features such as:

- In a translation, the corresponding points from the pre-image to the image form segments that are congruent and parallel.
- In a rotation, corresponding points from the pre-image to the image lie along concentric arcs.
- In a reflection, the line of reflection is the perpendicular bisector of corresponding points from the pre-image to the image.

Students use these features to perform translations and to determine what translations have been performed, given an image and corresponding pre-image. Students' observations about the rigid transformations give purpose and meaning to vocabulary words such as parallel, perpendicular, bisect, concentric, etc.

In the second learning cycle of Module 1, students use their understanding of rigid transformations to find the rotations and reflections that carry a figure onto itself. They explore the symmetries and diagonals of quadrilaterals and regular polygons, making and justifying conjectures about the relationship between the number of lines of symmetry and the number of sides of a regular polygon. At the end of the learning cycle, students make conjectures and classify quadrilaterals based upon symmetries and see that the classifications turn out to be the same as when the quadrilaterals are classified by angles and sides. Students justify their conjectures using their knowledge of the transformations.

Module 2, Congruence, Construction, and Proof, begins by developing constructions as another tool to be used to reason about figures and to justify properties of shapes. Individual constructions are not taught for the sake of memorizing a series of steps, but rather to reason using known properties of shapes such as circles. Many of the constructions, such as the angle bisector, the perpendicular bisector, and the midpoint, flow from constructing a rhombus in the first task. In the second task, students reason to perform the construction of a parallelogram, an inscribed hexagon, and a square. At the end of the module students use triangle congruence and rigid transformation to examine and justify why given compass and straight-edge constructions result in the desired figures.

Students enter Geometry with experience from grade 8 in using rigid transformations to experimentally determine if two figures are congruent. The work with congruence in Module 2 begins with students experimenting to find a general sequence of rigid transformations that will map a figure onto another if they are congruent. Students find that they can generally translate to get a pair of matching vertices, rotate to make a pair of corresponding sides coincide, then reflect to make the rest of the figure coincide. As the learning cycle continues, students use this sequence of transformations (or an equivalent sequence that they have found) to show the triangle congruence properties of ASA, AAS, SSS, and SAS. They also learn that two consecutive sides and an angle (SSA) are not enough to guarantee congruence between two triangles. After determining the congruence

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criteria of triangles, students use the criteria along with the rigid transformations to justify properties of quadrilaterals, such as the diagonals of a rectangle are congruent.

Formal proof is introduced in Module 3, Geometric Figures, beginning with students understanding the ways of knowing continuum:

1. Based on authority
2. Based on experience with a few examples
3. Based on reasoning from a diagram
4. Based on statements accepted as true by the community of practice, including postulates, definitions and theorems.

Students experience each of the ways of knowing in the module, learning to evaluate the strength of a mathematical argument. The fourth way of knowing, which is mathematical proof, has traditionally been taught using the two-column proof format. As suggested by the CCSS, the MVP curriculum also introduces other forms of proof including paragraph proofs and flow proofs. With the addition of transformation and construction as tools for creating geometric arguments, and the availability of more open forms of proof, geometric proofs become accessible to all students. Once students have been introduced to different ways of reasoning and making arguments in Module 3, they use their understanding of congruence and the congruent triangle criteria from Module 2 to prove statements about other figures including equilateral triangles and various quadrilaterals. Many of the ideas about congruence, symmetry, and properties of quadrilaterals that were surfaced in Module 2 are proved in Module 3.

Module 4, Similarity and Right Triangle Trigonometry, introduces the last of the transformations, dilation. A big idea of Module 4 is that two figures are similar if a sequence of rigid transformation and dilations exists that maps one figure onto the other. Students begin the module by learning about the features of a dilation, including the effects of changing the scale factor and/or point of dilation. They use the definition of a dilation to establish the AA similarity criterion and understand that corresponding sides of similar figures will be proportional. In the second learning cycle of the module, students prove theorems about the angles that occur when two parallel lines are cut by a transversal. They also develop a method for finding the midpoint or dividing a segment into other proportional pieces. In the final task of the learning cycle, students use similarity to prove the Pythagorean Theorem and find geometric means in right triangles.

The third learning cycle of Module 4 explores right triangle trigonometry. The definitions of sine, cosine, and tangent are introduced, and students use relationships between sine and cosine to construct the Pythagorean Identity for sine and cosine. They solve right triangles in both abstract and real-world situations.

Module 5, Circles: A Geometric Perspective, is composed of four learning cycles. In the first learning cycle, students use rotations and perpendicular bisectors to find the center of a circle. The task also introduces the terms associated with circles, including arcs, chords, secants, tangent, radius, diameter, etc. Students use these terms throughout the module as they explore features and develop conjectures about circles. The learning cycle proceeds with students showing that all circles are similar and making conjectures about central angles, inscribed angles, and circumscribed angles.

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The second learning cycle in Module 5 builds on the circle relationships that students have learned so far in the module to develop a formula for the perimeter and area of a regular polygon. Using intuitive ideas of limits, students extend these formulas to understand the formulas for the circumference and area of a circle.

The third learning cycle addresses relationships among central angles, radii, arcs, and sectors. Students calculate arc length and the area of a sector. Students learn that radians are another way to describe angles and to make conversions between degrees to radians. Radians are introduced in Geometry as part of understanding proportional relationships in circles. Radians are not used in circular trigonometry until Algebra II.

The final learning cycle in Module 5 is an intuitive approach to volume of prisms, pyramids, and cylinders. Students informally consider dissection as a method for deriving volume formulas for solid figures and to understand Cavalieri's Principle for calculating the volume of oblique geometric solids.

In Module 6, Connecting Algebra and Geometry, students use the Pythagorean Theorem to find the distance between two points and to derive the distance formula. The idea that parallel lines have the same slope and the slopes of perpendicular lines are negative reciprocals is introduced in Module 1 and then proven in Module 6. Students use the distance formula, their knowledge of slopes, and the features of quadrilaterals, to prove that a given figure is a particular type of quadrilateral, like a parallelogram.
The second learning cycle of Module 6 takes an algebraic approach to solving problems with circles, parabolas, and in the Honors course, ellipses and hyperbolas. The module includes several hands-on explorations to develop the equations for circles, parabolas, and ellipses. Students build a circle from right triangles and use the Pythagorean Theorem to derive the equation of a circle. They use the equation of a circle to determine if a given point is on a circle, to graphs circles, and to write equations given specific information about a circle.

The module continues with students examining parabolas as a set of points defined by a focus and directrix. Students construct parabolas using this definition and discover relationships that help them to write equations. Students consider both parabolas with a horizontal directrix and those with a vertical directrix. They compare parabolas considered from a geometric perspective to their previous experience in Algebra I with parabolas from a functions perspective. The module also includes Honors tasks that involve students in deriving and using the equation of an ellipse and the equation of a hyperbola.

The final learning cycle of Module 6 in the Geometry Honors course In the Honors course contains additional tasks about vectors and matrices. Quantities that can be represented using vectors are introduced. Students explore both the multiplication and addition properties of matrices. Students also learn to find determinants and use matrix multiplication to rotate vectors and images.

The first learning cycle in Module 7, Geometric Modeling, begins with students visualizing two-dimensional cross sections of three-dimensional objects and solids of rotation. They learn to

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approximate the volume of an irregular solid by decomposing it into cylinders, frustrums, and cones with volumes that can be easily calculated.

In the second learning cycle of Module 7, students extend their understanding of right triangle trigonometry from Module 4 to general triangles. They begin with a study of special right triangles and proceed to finding the sides and angles of some triangles by decomposing them into right triangles so that the Pythagorean Theorem or right triangle trigonometry can be used. This strategy supports students in deriving the Law of Cosines and the Law of Sines, which they then apply to finding sides and angles for triangles. In the final task of the module, students explore the ambiguous case of Law of Sines and develop formulas for the area of triangles using the Law of Sines and the Law of Cosines.

Conceptual Category: Statistics and Probability

Students do a great deal of work in probability in grade 7, which advances in the Geometry course. From seventh grade, they have experience developing probability models and testing the models with experiments. They learned that probabilities are numbers between 0 and 1, with events becoming more likely as the probability approaches 1. They have represented sample spaces for simple situations and used simulations to determine frequencies for compound events.

Module 8, Probability, extends students' work in representing and analyzing data to understand concepts in probability. In the module, students use representations such as tree diagrams, Venn diagrams, and two-way frequency tables to draw conclusions about the likelihood of an event. Students learn about conditional probability, writing statements, using both words and probability notation, about real situations. They use probability statements to complete Venn diagrams and use them to draw conclusions. They understand terms such as mutually exclusive, union, and intersection, in contexts that give them meaning and help to visualize the terms with diagrams. Students learn about independence and how to determine if events are independent using a Venn diagram, a tree diagram, or a two-way frequency table.

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Module 1 Transformations and Symmetry	**3 weeks of instruction**
1.1 Leaping Lizards! – A Develop Understanding Task Developing the definitions of the rigid-motion transformations: translations, reflections and rotations (G.CO.1, G.CO.4, G.CO.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
1.2 Is It Right? – A Solidify Understanding Task Examining the slope of perpendicular lines (G.CO.1, G.GPE.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
1.3 Leap Frog – A Solidify Understanding Task Determining which rigid-motion transformations will carry one image onto another congruent image (G.CO.4, G.CO.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
1.4 Leap Year – A Practice Understanding Task Writing and applying formal definitions of the rigid-motion transformations: translations, reflections and rotations (G.CO.1, G.CO.2, G.CO.4, G.GPE.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 1 & Self-Assessment (formative)	**20 minutes**
1.5 Symmetries of Quadrilaterals – A Develop Understanding Task Finding rotational symmetry and lines of symmetry in special types of quadrilaterals (G.CO.3, G.CO.6)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
1.6 Symmetries of Regular Polygons – A Solidify Understanding Task Examining characteristics of regular polygons that emerge from rotational symmetry and lines of symmetry (G.CO.3, G.CO.6)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
1.7 Quadrilaterals—Beyond Definition - A Practice Understanding Task Comparing rates of growth in arithmetic and geometric sequences (F.BF.1, F.LE.1, F.LE.2)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 2 & Self-Assessment (formative)	**20 minutes**
Module 1 Assessment and Performance Assessment	**1 - 80 minute period for both** **2 – 45 to 50 minute periods each**

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Module 2 Congruence, Construction and Proof	**3 weeks of instruction**
2.1 Under Construction – A Develop Understanding Task Exploring compass and straightedge constructions to construct rhombuses and squares (G.CO.12, G.CO.13)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
2.2 More Things Under Construction – A Develop Understanding Task Exploring compass and straightedge constructions to construct parallelograms, equilateral triangles and inscribed hexagons (G.CO.12, G.CO.13)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 1 & Self-Assessment (formative)	**20 minutes**
2.3 Can You Get There From Here? – A Develop Understanding Task Describing a sequence of transformations that will carry congruent images onto each other (G.CO.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
2.4 Congruent Triangles– A Solidify Understanding Task Establishing the ASA, SAS and SSS criteria for congruent triangles (G.CO.6, G.CO.7, G.CO.8)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
2.5 Congruent Triangles to the Rescue – A Practice Understanding Task Identifying congruent triangles and using them to justify claims (G.CO.7, G.CO.8)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
2.6 Justifying Constructions – A Solidify Understanding Task Examining why compass and straightedge constructions produce the desired results (G.CO.12, G.CO.13)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 2 & Self-Assessment (formative)	**20 minutes**
Module 2 Test & Performance Assessment	**1 – 45 to 50 minute period each**

Module 3 Geometric Figures	**4 weeks of instruction**
3.1 How Do You Know That? – A Develop Understanding Task An introduction to proof illustrated by the triangle interior angle sum theorem (G.CO.10)	**1 - 80 minute period** **2 – 45 to 50 minute periods**

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3.2 Do You See What I See? – A Develop Understanding Task Reasoning from a diagram to develop proof-like arguments about lines and angles, triangles and parallelograms (G.CO.9, G.CO.10, G.CO.11)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
3.3 It's All in Your Head – A Solidify Understanding Task Organizing proofs about lines, angles and triangles using flow diagrams and two-column proof formats (G.CO.9, G.CO.10)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
3.4 Parallelism Preserved and Protected – A Solidify Understanding Task Examining parallelism from a transformational perspective (G.CO.9)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
3.5 Claims and Conjectures – A Solidify Understanding Task Generating conjectures from a diagram about lines, angles and triangles (G.CO.9, G.CO.10)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
3.6 Justification and Proof – A Practice Understanding Task Writing formal proofs to prove conjectures about lines, angles and triangles (G.CO.9, G.CO.10)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 1 & Self-Assessment (formative)	**20 minutes**
3.7 Parallelogram Conjectures and Proof – A Solidify Understanding Task Proving conjectures about parallelograms (G.CO.11)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
3.8 Guess My Parallelogram – A Practice Understanding Task Identifying parallelograms from information about the diagonals (G.CO.11)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
3.9 Centers of a Triangle– A Practice Understanding Task Reading and writing proofs about the concurrency of medians, angle bisectors and perpendicular bisectors of the sides of a triangle (G.CO.10)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 2 & Self-Assessment (formative)	**20 minutes**
Module 3 Test & Performance Assessment	**1 – 45 to 50 minute period each**

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Module 4 Similarity and Right Triangle Trigonometry	**5 weeks of instruction**
4.1 Photocopy Faux Pas – A Develop Understanding Task Describing the essential features of a dilation (G.SRT.1)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
4.2 Triangle Dilations – A Solidify Understanding Task Examining proportionality relationships in triangles that are known to be similar to each other based on dilations (G.SRT.2, G.SRT.4)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
4.3 Similar Triangles and Other Figures – A Solidify Understanding Task Comparing definitions of similarity based on dilations and relationships between corresponding sides and angles (G.SRT.2, G.SRT.3)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
4.4 Cut by a Transversal – A Solidify Understanding Task Examining proportionality relationships of segments when two transversals intersect sets of parallel lines (G.SRT.4)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
4.5 Measured Reasoning – A Practice Understanding Task Applying theorems about lines, angles and proportional relationships when parallel lines are crossed by multiple transversals (G.CO.9, G.CO.10, G.SRT.4, G.SRT.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 1 & Self-Assessment (formative)	**20 minutes**
4.6 Yard Work in Segments – A Solidify Understanding Task Applying understanding of similar and congruent triangles to find the midpoint or any point on a line segment that partitions the segment into a given ratio (G.GPE.6)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
4.7 Pythagoras by Proportions – A Practice Understanding Task Using similar triangles to prove the Pythagorean theorem and theorems about geometric means in right triangles (G.SRT.4, G.SRT.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 2 & Self-Assessment (formative)	**20 minutes**
4.8 Are Relationships Predictable? – A Develop Understanding Task Developing an understanding of right triangle trigonometric relationships based on similar triangles (G.SRT.6, G.SRT.8)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
4.9 Relationships with Meaning– A Solidify Understanding Task	**1 - 80 minute period** **2 – 45 to 50 minute periods**

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Finding relationships between the sine and cosine ratios for right triangles, including the Pythagorean identity (G.SRT.6, G.SRT.7, F.TF.8)	
4.10 Finding the Value of a Relationship– A Solidify Understanding Task Solving for unknowns in right triangles using trigonometric ratios (G.SRT.7, G.SRT.8)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
4.11 Solving Right Triangles Using Trigonometric Relationships– A Practice Understanding Task Setting up and solving right triangles to model real world contexts (G.SRT.6, G.SRT.7, F.TF.8)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 3 & Self-Assessment (formative)	**20 minutes**
Module 4 Test & Performance Assessment	**1 – 45 to 50 minute period each**

Module 5 Circles: A Geometric Perspective	**5 weeks of instructions**
5.1 Centered – A Develop Understanding Task Searching for centers of rotation using perpendicular bisectors as a tool (G.C.2)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
5.2 Circle Dilations – A Solidify Understanding Task Proving circles similar (G.C.1)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
5.3 Cyclic Polygons – A Solidify Understanding Task Examining relationships between central angles, inscribed angles, circumscribed angles and their arcs (G.C.2, G.C.3, G.C.4)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
5.4 Planning the Gazebo – A Practice Understanding Task Developing formulas for perimeter and area of regular polygons (G.GMD.1)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
5.5 From Polygons to Circles – A Solidify Understanding Task Justifying formulas for circumference and area of circles using intuitive limit arguments (G.GMD.1)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
5.6 Circular Reasoning – A Practice Understanding Task Applying and practicing circle relationships (G.C.2)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 1 & Self-Assessment (formative)	**20 minutes**

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5.7 Pied! – A Develop Understanding Task Using proportional reasoning to calculate arc length and area of sectors (G.C.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
5.8 Madison's Round Garden – A Practice and Develop Understanding Task Using the ratio of arc length to radius to develop radians as a way of measuring angles (G.C.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
5.9 Rays and Radians – A Solidify and Practice Understanding Task Converting between degree measure and radian measure of an angle (G.C.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
5.10 Sand Castles – A Solidify Understanding Task Examining the proportionality relationships of lengths, areas and volumes when geometric figures are scaled up (G.GMD.1, G.GMD.3)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
5.11 Footprints in the Sand– A Develop Understanding Task Examining informal, dissection arguments for the volume formulas of prisms, pyramids and cylinders (G.GMD.1, G.GMD.3)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
5.12 Cavelieri to the Rescue – A Solidify Understanding Task Examining informal, dissection arguments based on Cavelieri's principle for the volume formulas of oblique prisms, pyramids and cylinders (G.GMD.2)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 2 & Self-Assessment (formative)	**20 minutes**
Module 5 Test & Performance Assessment	**1 – 45 to 50 minute period each**

Module 6 Connecting Algebra and Geometry	**6 Weeks of Instruction (honors)** **4 Weeks of Instruction (non-honors)**
6.1 Go the Distance – A Develop Understanding Task Using coordinates to find distances and determine the perimeter of geometric shapes (G.GPE.7)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
6.2 Slippery Slopes – A Solidify Understanding Task Proving slope criteria for parallel and perpendicular lines (G.GPE.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**

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6.3 Prove It! – A Practice Understanding Task Using coordinates to algebraically prove geometric theorems (G.GPE.4)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 1 & Self-Assessment (formative)	**20 minutes**
6.4 Circling Triangles (or Triangulating Circles)– A Solidify Understanding Task Deriving the equation of a circle using the Pythagorean Theorem (G.GPE.1)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
6.5 Getting Centered– A Solidify Understanding Task Complete the square to find the center and radius of a circle given by an equation (G.GPE.1)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
6.6 Circle Challenges – A Practice Understanding Task Writing the equation of a circle given various information (G.GPE.1)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 2 & Self-Assessment (formative)	**20 minutes**
6.7 Directing Our Focus – A Develop Understanding Task Derive the equation of a parabola given a focus and directrix (G.GPE.2)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
6.8 Functioning with Parabolas – A Solidify Understanding Task Connecting the equations of parabolas to prior work with quadratic functions (G.GPE.2)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
6.9 Turn It Around – A Solidify Understanding Task Writing the equation of a parabola with a vertical directrix, and constructing an argument that all parabolas are similar (G.GPE.2)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 3 & Self-Assessment (formative)	**20 minutes**
6.10H Operating on a Shoestring – A Solidify Understanding Task Exploring features of ellipses and writing the equation of an ellipse using the fact that the sum of the distances from the foci is constant. (G.GPE.3) (+)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
6.11H What Happens If …? – A Solidify Understanding Task	**1 - 80 minute period** **2 – 45 to 50 minute periods**

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Exploring features of hyperbolas writing the equation of a hyperbola using the fact that the difference of the distances from the foci is constant. (G.GPE.3) (+)	
6.12H The Arithmetic of Vectors – A Solidify Understanding Task Defining and operating with vectors as quantities with magnitude and direction (N.VM.1, N.VM.2, N.VM.3, N.VM.4, N.VM.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
6.13H Transformations and Matrices – A Solidify Understanding Task Using matrix multiplication to reflect and rotate vectors and images (N.VM.11, N.VM.12)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
6.14H Plane Geometry – A Practice Understanding Task Solving problems involving quantities that can be represented by vectors (N.VM.3, N.VM.4a, N.VM.12)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 4 & Self-Assessment (formative)	**20 minutes**
Module 6 Test & Performance Assessment	**1 – 45 to 50 minute period each**

Module 7 Modeling with Geometry	**3 weeks of instruction**
7.1 Any Way You Slice It – A Develop Understanding Task Visualizing two-dimensional cross sections of three dimensional objects (G.GMD.4)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
7.2 Any Way You Spin It – Develop Understanding Task Visualizing solids of revolution (G.GMD.4)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
7.3 Take Another Spin – A Solidify Understanding Task Approximating volumes of solids of revolution with cylinders and frustums (G.MG.1, G.GMD.4)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
7.4 You Nailed It! – A Practice Understanding Task Solving problems using geometric modeling (G.MG.1, G.MG.2, G.MG.3)	**1 - 80 minute period** **2 – 45 to 50 minute periods**

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Quick Quiz 1 & Self-Assessment (formative)	**20 minutes**
7.5 Special Rights– A Solidify Understanding Task Examining the relationship of sides in special right triangles (G.SRT.11)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
7.6 More Than Right – A Develop Understanding Task Developing strategies for solving non-right triangles (G.SRT.10, G.SRT.11)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
7.7 Justify the Laws – A Solidify Understanding Task Examining the Law of Cosines and the Law of Sines (G.SRT.10, G.SRT.11)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
7.8 Triangle Areas by Trig – A Practice Understanding Task Finding the missing sides, angles and areas of general triangles (G.SRT.9, G.SRT.10, G.SRT.11)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 2 & Self-Assessment (formative)	**20 minutes**
Module 7 Test & Performance Assessment	**1 – 45 to 50 minute period each**

Module 8 Probability	**3 weeks of instruction**
8.1 TB or Not TB – A Develop Understanding Task Estimating conditional probabilities and interpreting the meaning of a set of data (S.CP.6, S.MD.7+)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
8.2 Chocolate Versus Vanilla – A Solidify Understanding Task Examining conditional probability using multiple representations (S.CP.6)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
8.3 Fried Freddy's – A Solidify Understanding Task Using sample to estimate probabilities (S.CP.2, S.CP.6)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
8.4 Visualizing with Venn– A Practice Understanding Task Creating Venn diagram's using data while examining the addition rule for probability (S.CP.6, S.CP.7)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 1 & Self-Assessment (formative)	**20 minutes**

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8.5 Freddy Revisited – A Solidify Understanding Task Examining independence of events using two-way tables (S.CP.2, S.CP.3, S.CP.4, S.CP.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
8.6 Striving for Independence – A Practice Understanding Task Using data in various representations to determine independence (S.CP.2, S.CP.3, S.CP.4, S.CP.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Module 8 Test & Performance Assessment	**1 – 45 to 50 minute period each**

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