The Secondary Mathematics II course is written to align with the second of three courses in the integrated pathway of the Common Core State Standards, as described in Appendix A. Like all courses in the integrated pathway, it contains standards from each of the conceptual categories in the standards, including:

- Number and quantity;
- Algebra;
- Functions;
- Geometry; and
- Statistics and probability.

The focus of Secondary Math II is on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships from Secondary Mathematics I. The need for extending the set of rational numbers arises and real and complex numbers are introduced so that all quadratic equations can be solved. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. Circles, with their quadratic algebraic representations, round out the course. The Mathematical Practice Standards apply throughout each course and, together with the content standards, create mathematical learning experiences based upon reasoning and sensemaking, building perseverance and problem solving skills, and rich in mathematical discourse.

The standards indicated in the CCSS with a (+) sign are addressed with additional tasks in Secondary Math II Honors. The Honors version of the course includes all the same tasks as Secondary Math II, with the additional tasks embedded into the modules where they fit conceptually.

Standards specified in the Widely Accepted Prerequisites (WAP's) included in the High School Publishers Criteria for the Common Core State Standards for Mathematics constitute the bulk of the curriculum in Secondary Math II. The F-IF standards for interpreting quadratic functions are extensively addressed in Modules 1-2. Module 5 contains the emphasized Geometry standards, G-CO.1 G-CO.9 G-CO.10, which address proving statements about lines, angles, and triangles. Module 6 contains G-SRT.B G-SRT.C addressing the topics of similarity and right triangle trigonometry. Students develop a rich understanding of these terms as they use them to reason about transformations, construction, and features of triangles and quadrilaterals. All of the domains in the Algebra Conceptual Category are included in the WAP's. These domains constitute all of the work in Modules 3 and much of Module 2.

In the narrative that follows, the specific approach and details of the mathematics in the curriculum is described by conceptual category in roughly the same order as the categories are addressed in the curriculum. The additional work of the Honors course is clearly identified.

Conceptual Category: Functions

In Math I, students did extensive work with linear and exponential functions. They learned to work flexibly and fluently among representations for these two function types including tables, graphs, equations (both recursive and explicit), diagrams, and story contexts. Students used rates of change to distinguish between linear and exponential functions and to compare their behavior for large values of x. They learned that an arithmetic sequence is a discrete linear function and that a geometric sequence is a discrete exponential function.

The definition of function as a relationship in which each input has a unique output was formalized in Secondary Math I. Besides their work with linear and exponential functions, students learned that functions can be models for many situations and that in each situation it is useful to identify features such as:

- x and y intercepts;
- Domain and range;
- Continuity;
- Intervals of increase and decrease; and
- Maxima and minima.

They were also introduced to the idea that functions can be transformed, although they used only vertical transformations.

Module 1, Quadratic Functions, picks up where students left off with linear and exponential functions, using the same types of diagrams and representations to introduce quadratic functions. The entire module focuses on the features of quadratic functions, comparing quadratics to other functions, and representing quadratic functions. The first tasks in Module 1 use diagrams to help students write equations, both explicit and recursive, for quadratic functions. They use tables to identify the rate of change, noticing that the second difference is constant, making the first difference linear in a quadratic function. This idea is extended to the entire class of polynomials in Secondary Math III. In the first two tasks, students are also introduced to a related idea: that a quadratic function can be the sum of an arithmetic sequence or linear function. As the module advances, students use quadratic functions to model continuous contexts with a quadratic curve that has a maximum. There are lessons in the module that compare linear and quadratic functions and exponential and quadratic functions. The final task of the module asks students to distinguish between linear, exponential, and quadratic, given a single representation for the function, to create other representations for the function, and to identify the rate of change exhibited by the function.

Module 2, Structures of Expressions, is both a functions module and an algebra module. It is designed to extend students' knowledge of quadratic functions and to reinforce two big ideas of functions:

- Functions can be transformed in the same, predictable way.
- Different algebraic forms have purpose in different situations.

The module begins with students using technology to explore transformations of the graph of $f(x) = x^2$. They learn about horizontal and vertical translations, reflections over the x-axis, and vertical stretching or shrinking. These transformations are combined and students learn to identify the vertex, line of symmetry, reflection, and vertical stretch factor from equations in the form: $f(x) = a(x - h)^2 + k$. The curriculum encourages students to use a quick-graph method to be able to fluently identify the features and produce an accurate graph of any equation in vertex form.

Students soon experience equations that are not in vertex form, which may require them to change forms. Completing the square is introduced as a method for this purpose, using area models so that students have a visual model to rely on for the procedure. Students are also introduced to factoring using area models and come to understand that factored form can also be useful for graphing quadratics using the x-intercepts and the symmetry of the parabola to find the vertex. The only Honors task in the module addresses factoring trinomials in which the lead term has a coefficient that is not equal to 1. The module ends with students learning to be efficient in identifying the form that will be easiest to use in a given situation and moving flexibly between forms of quadratic equations. The idea that different quadratic forms are useful in graphing is extended in Module 3, Solving Quadratic and Other Equations, when students use different forms of quadratic equations to find the roots or solve quadratic equations of a single variable.

After all the work with quadratics in Modules 1-3, Module 4, More Functions, More Features, shifts focus to consider piecewise functions, absolute value functions, and inverse functions. Understanding of piecewise functions is built from students' understanding of graphs and the stories that they can tell. Students learn to write functions for contexts in which rates change, making a piecewise function. Point-slope form of the equation of a line is frequently used and connected to students' previous experience with transformation of functions. Students write and linear absolute value functions as piecewise functions and learn to graph linear absolute value functions with transformations. They also graph non-linear absolute value functions to more deeply understand the meaning of absolute and the effect it has when composed with other functions.

The last learning cycle in Module 4 introduces students to inverse functions. It begins with a context in which two people each keep track of their own bike ride in different units, one in minutes per mile, one in miles per minute. As students model the two different methods for thinking about the bike rides, they notice that the inputs and outputs are reversed, making the graphs reflections over the $y = x$ line. These initial ideas about inverses are generalized in subsequent tasks in the module so that students learn to graph and write the inverse for simple functions. In Secondary Math III, students delve more deeply into inverses, writing inverses for more complicated functions, understanding invertibility, and extending the general ideas of inverses to find the inverse of an exponential function, which a logarithmic function.

Conceptual Category: Number and Quantity

In eighth grade, students learned about the properties of exponents and were introduced to integer exponents. Students used integer exponents in Secondary Math I as they created tables and wrote equations for geometric sequences and exponential functions in Modules 1 and 2. In Secondary Math II, Module 3, Solving Quadratic and Other Equations, exponential functions provide a context for thinking about the outputs that lie between integer exponents, and find that they can be named with rational exponents. In the first learning cycle of Module 3, rational exponents are connected to roots, which students learned about in eighth grade and used with the distance formula in Secondary Math I. As they work with rational exponents and roots, students find that the rules are directly analogous. In the final task of the learning cycle, students use both rational exponents and roots to work with the same expression, with the goal being to find the form that is most efficient or useful for a given expression and operation.

As students are solving quadratic equations Module 3, they encounter equations that give solutions like: $x = 3 \pm \sqrt{-4}$. Students know from previous experience that square roots are undefined for negative values, so these solutions present a problem. This problem is resolved by introducing imaginary numbers and using them to write solutions to quadratic equations.

Conceptual Category: Algebra

The overall approach in the MVP curriculum to algebra is to give algebraic work meaning and purpose by embedding it in story context, modeling, and functions. This approach is evident in Module 2, Structures of Expressions, which is described in the Functions section, and is also illustrated by Module 3, Solving Quadratic and Other Equations. Module 3 uses students' experience with graphing quadratics to derive a method for finding roots and x-intercepts that becomes the quadratic formula. Students learn to use the quadratic formula, along with other methods such as factoring or taking the square root of both sides to solve equations and inequalities accurately and efficiently.

In the Honors tasks in Module 3 students solve quadratic inequalities. The tasks also extend the algebraic work with imaginary numbers to the complex plane, representing both the complex numbers and their operations. If students have been in Secondary Math I Honors, they have learned to use matrices. In Module 3 of Secondary Math II Honors, they use inverse matrices to solve systems of equations.

Conceptual Category: Geometry

The standards for geometry in the integrated pathway are carefully designed to allow students to experiment and construct general ideas about shapes and how they transform in eighth grade, moving towards formalizing definitions of rigid transformations and congruence in Secondary Math I through reasoning with diagrams, and then proving theorems and formalizing definitions of dilation and similarity in Secondary Math II. True to the vision of the standards, the MVP curriculum takes a transformational approach to the standards, developing transformations and construction as tools for reasoning and proof that are used in addition to the traditional axiomatic tools of geometry. The curriculum provides students many opportunities to use their intuitive understanding about geometry and experiment with compass, protractor, patty paper, rulers, graph paper, dynamic geometry software and other physical tools to make and justify conjectures.

In Secondary Math II, Geometry begins with Module 5, Geometric Figures. Formal proof is introduced in this module, beginning with students understanding the ways of knowing continuum:

1. Based on authority
2. Based on experience with a few examples
3. Based on reasoning from a diagram
4. Based on statements accepted as true by the community of practice, including postulates, definitions and theorems.

Students experience each of the ways of knowing in the module, learning to evaluate the strength of a mathematical argument. The fourth way of knowing, which is mathematical proof, has traditionally been taught using the two-column proof format. As suggested by the CCSS, the MVP curriculum also introduces other forms of proof including paragraph proofs and flow proofs. With the addition of transformation and construction as tools for creating geometric arguments and the availability of more open forms of proof, geometric proofs become accessible to all students. Once students have been introduced to different ways of reasoning and making arguments in Module 5, they use their understanding of congruence and the congruent triangle criteria from Secondary Math I to prove statements about other figures including equilateral triangles and various quadrilaterals. Many of the ideas about congruence, symmetry, and properties of quadrilaterals that were surfaced in Secondary Math I are proved in Module 5 of Secondary Math II.

Module 6, Similarity and Right Triangle Trigonometry, introduces the last of the transformations, dilation. A big idea of Module 6 is that two figures are similar if a sequence of rigid transformation and dilations exists that maps one figure onto the other. Students begin the module by learning about the features of a dilation, including the effects of changing the scale factor and/or point of dilation. They use the definition of a dilation to establish the AA similarity criterion and understand that corresponding sides of similar figures will be proportional. In the second learning cycle of the module, students prove theorems about the angles that occur when two parallel lines are cut by a transversal. They also develop a method for finding the midpoint or dividing a segment into other proportional pieces. In the final task of the learning cycle, students use similarity to prove the Pythagorean Theorem and find geometric means in right triangles.

The third learning cycle of Module 6 explores right triangle trigonometry. The definitions of sine, cosine, and tangent are introduced, and students use relationships between sine and cosine to construct the Pythagorean Identity for sine and cosine. They solve right triangles in both abstract and real-world situations.

Module 7, Circles: A Geometric Perspective, is composed of four learning cycles. In the first learning cycle, students use rotations and perpendicular bisectors to find the center of a circle. The task also introduces the terms associated with circles, including arcs, chords, secants, tangent, radius, diameter, etc. Students use these terms throughout the module as they explore features and develop conjectures about circles. The learning cycle proceeds with students showing that all circles are similar and making conjectures about central angles, inscribed angles and circumscribed angles.

The second learning cycle in Module 7 builds on the circle relationships that students have learned so far in the module to develop a formula for the perimeter and area of a regular polygon. Using intuitive ideas of limits, students extend these formulas to understand the formulas for the circumference and area of a circle.

The third learning cycle addresses relationships among central angles, radii, arcs, and sectors. Students calculate arc length and the area of a sector. Students learn that radians are another way to describe angles and to make conversions between degrees to radians. Radians are introduced in Secondary Math II as part of understanding proportional relationships in circles. Radians are not used in circular trigonometry until Secondary Math III.

The final learning cycle in Module 7 is an intuitive approach to volume of prisms, pyramids, and cylinders. Students informally consider dissection as a method for deriving volume formulas for solid figures and to understand Cavalieri's Principle for calculating the volume of oblique geometric solids.

Module 8, Circles and Other Conics, takes an algebraic approach to solving problems with circles, parabolas, and in the Honors course, ellipses and hyperbolas. The module includes several hands-on explorations to develop the equations for circles, parabolas, and ellipses. In the first learning cycle, students build a circle from right triangles and use the Pythagorean Theorem to derive the equation of a circle. They use the equation of a circle to determine if a given point is on a circle, to graphs circles, and to write equations given specific information about a circle.

The second learning cycle focuses on parabolas as a set of points defined by a focus and directrix. Students construct parabolas using this definition and discover relationships that help them to write equations. Students consider both parabolas with a horizontal directrix and those with a vertical directrix. They compare parabolas considered from a geometric perspective to their previous experience with parabolas from a functions perspective. The module also includes Honors tasks that involve students in deriving and using the equation of an ellipse and the equation of a hyperbola.

Conceptual Category: Statistics and Probability

Students do a great deal of work in probability in grade 7, which informs the work of Secondary Math II. From seventh grade, they have experience developing probability models and testing the models with experiments. They learned that probabilities are numbers between 0 and 1, with events becoming more likely as the probability approaches 1. They represented sample spaces for simple situations and used simulations to determine frequencies for compound events.

Module 9, Probability, extends students' work in representing and analyzing data to understand concepts in probability. In the module, students use representations such as tree diagrams, Venn diagrams, and two-way frequency tables to draw conclusions about the likelihood of an event. Students learn about conditional probability, writing statements, using both words and probability notation, about real situations. They use probability statements to complete Venn diagrams and use them to draw conclusions. They understand terms such as mutually exclusive, union, and intersection, in contexts that give them meaning and help them to visualize the terms with diagrams. Students learn about independence and how to determine if events are independent using a Venn diagram, a tree diagram, or a two-way frequency table.