The Secondary Mathematics III course is written to align with the third of three courses in the integrated pathway of the Common Core State Standards, as described in Appendix A. Like all courses in the integrated pathway, it contains standards from each of the conceptual categories in the standards, including:

- Number and quantity;
- Algebra;
- Functions;
- Geometry; and
- Statistics and probability.

The major purpose of Secondary Math III is for students to pull together and apply the accumulation of learning that they have from their previous courses. Students add to their catalog of functions types to include polynomial, rational, logarithmic, and trigonometric. They expand their understanding of right triangle trigonometry to include circular trigonometry and general triangles. They apply methods from probability and statistics to draw inferences and conclusions from data. And, finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. The Mathematical Practice Standards apply throughout each course and, together with the content standards, create mathematical learning experiences based upon reasoning and sensemaking, building perseverance and problem-solving skills, and rich in mathematical discourse.

The standards indicated in the CCSS with a (+) sign are addressed with additional tasks in Secondary Math III Honors. The Honors version of the course includes all the same tasks as Secondary Math III, with the additional tasks embedded into the modules where they fit conceptually.

Standards specified in the Widely Accepted Prerequisites (WAP's) included in the High School Publishers Criteria for the Common Core State Standards for Mathematics constitute the bulk of the curriculum in Secondary Math III. The F-IF standards for interpreting functions are extensively addressed in Modules 1-4 with inverse functions, logarithms, polynomial and rational functions. The functions modules also contain a number of opportunities for students to understand structure of expressions and use various algebraic forms to model a situation or to highlight a given feature, the work defined in the A-SSE standards.

In the narrative that follows, the specific approach and details of the mathematics in the curriculum is described by conceptual category in roughly the same order as the categories are addressed in the curriculum. The additional work of the Honors course is clearly identified.

Conceptual Category: Functions

The MVP curriculum takes a coherent approach to functions across grade levels. The following big ideas of functions are introduced in Secondary Math I, reinforced in Secondary Math II and fully realized with several different function types in Secondary Math III:

- Functions are categorized by their rates of change.
- The key features of functions are tools for analysis:
    - Domain and range
    - Intervals of increase and decrease

    - Maxima and minima
    - x and y intercepts
    - Continuity
- Functions can be transformed in the same, predictable way.
- Different algebraic forms of functions have purpose in different situations.
- Functions can be combined together (using basic operations or composed) to make new functions, usually retaining some of the features of both functions.

In the previous two courses, students did extensive work with linear, exponential, and quadratic functions. They also learned about piecewise and absolute value functions and were introduced to inverse functions. Secondary Math III, Module 1, Functions and Their Inverses, reviews the features of linear, exponential, and quadratic functions and general inverse relationships. The idea that the inputs and outputs are reversed in inverse functions is reinforced in the module using tables, graphs, equations, and story context. Students consider situations when the inverse is not a function and learn about invertibility. Students write equations of inverse functions, recognizing that inverse functions have inverse operations in the reverse order. As students use a story context to reason about the inverse of an exponential function, the concept of a logarithm is introduced.

Module 2, Logarithmic Functions, picks up where Module 1 leaves off. Students begin to understand logarithms by drawing upon their experiences with inverses and exponential functions to evaluate, approximate, and order logarithmic expressions such $\log_2 8$ and $\log_2 20$. Through this experience, students recognize some basic properties of logarithms like $\log_b b = 1, \log_b 1 = 0$, and $\log_b b^n = n$. They use known log values to graph logarithmic parent functions such as $y = \log_2 x$ and then use the parent functions, recognizing the vertical asymptote and the anchor point $(1,0)$ to graph transformations such as $y = 1 + \log_2(x - 3)$. The addition, subtraction, and multiplication properties for logarithms are derived from recognizing transformations of graphs of equivalent functions. Students use the log properties to write equivalent expressions and evaluate unknown log quantities using known log values. Students also solve simple exponential and log equations algebraically and using tables and graphs.

The Honors tasks in Module 2 introduce exponential functions with base e and natural logarithm functions. Students model continuous growth situations and solve equations using natural logs. More complicated exponential and log equations are introduced with support for solving them, along with analysis of common misconceptions.

Module 3, Polynomial Functions, begins with a task that links linear, quadratic, and cubic functions together by highlighting the rates of change of each function type and using a story context to show that a linear function is the sum of a constant, a quadratic function is the accumulation or sum of a linear function, and a cubic function is the sum of a quadratic function. Students generalize the pattern they see that linear functions have a constant first difference, quadratic functions have a constant second difference, and cubic functions have a constant third difference, to predict that the pattern continues for quartic polynomials and the rest of the polynomial family. As the module proceeds, students graph $y = x^3$, identify its features, and transform the graph. They use the Fundamental Theorem of Algebra and their previous experience with quadratics to identify the number of possible roots, both real and complex, for a given polynomial. Students employ polynomial operations such as division to find roots and write equations using given roots.

Students compare end behavior of polynomials and learn to predict the end behavior. In the context of predicting end behavior for even and odd-powered polynomials, they are introduced to the concept of even and odd functions. The module ends with students synthesizing their understanding of end behavior and roots to write equations and graph polynomials given various information.

In Module 3, students saw that dividing two polynomials sometimes resulted in a remainder, which could be written as a fraction. In Module 4, Rational Expressions and Functions, students work with the fractions that are ratios of polynomials, rational expressions and functions. The module begins with a story context to provide conceptual understanding of the simple rational function, $y = \frac{1}{x}$, for both large values of x and for x approaching zero. The context provides meaning for both the horizontal and vertical asymptotes and supports students in thinking about dividing by fractions. The module continues with students transforming the graph of $y = \frac{1}{x}$ and then introduces more complicated rational functions. The rational functions are categorized by comparing the degree of the numerator and the denominator so that students learn to predict horizontal, slant, and vertical asymptotes, and find x-intercepts. Students develop a strategy for determining and keeping track of the behavior of the function near the vertical asymptotes so that they can easily graph any rational function. Technology is used throughout the module to support students in making conjectures and reasoning about rational functions.

The next functions module is Module 6, Modeling Periodic Behavior. In this module students use a Ferris wheel as a context for constructing conceptual understanding of circular trigonometry. They begin by calculating heights on the Ferris wheel, progress to calculating the heights at a given time on the Ferris wheel, and then, graphing the heights to show a sine function. This progression of concepts takes students from static right triangle trigonometry that they learned in Secondary Math II to defining a more dynamic sine function in terms of angles of rotation. At the end of the learning cycle, students also consider the shadows cast at a giving time, which produces a cosine function. The second learning cycle of the module re-establishes and extends students' understanding of radians, which were introduced in Secondary Math II. They find arc lengths on concentric circles and compare the ratio of arc length to radius on the circles, noticing that they are same for a given angle of rotation. This learning cycle culminates in defining sine and cosine on the unit circle, in terms of angles given in radians.

When graphing the heights on the Ferris wheel in Module 6, students considered the effect of changing the height of the center of the wheel, resulting in a vertical shift of the graph. In Module 7, Trigonometric Functions, Equations, and Identities, students work with more trigonometric graphs, beginning with the familiar Ferris wheel context. In this case, the Ferris wheel is used to introduce a horizontal shift of the graph. The first learning cycle continues with more graphical modeling using sine and cosine, with students learning to work fluently and flexibly with all the transformations of the graphs of the function. An additional task in the Honors course extends the tangent function for angles of rotations, and introduces secant, cosecant, and cotangent functions.

In Secondary Math II, students discovered the Pythagorean Identity for sine and cosine. In the second learning cycle of Module 7, students use diagrams to find more identities including the cofunction identities and the identities that are related to odd and even functions. After making

arguments to establish the basic identities, students use the identities to rewrite trigonometric expressions. In the Honors course, students also develop and use the addition and double angle identities to solve equations. Module 7 continues in the Honors course to introduce inverse trig functions with restricted domains.

Module 8, Modeling with Functions, focuses on a big idea of functions: Functions can be combined together (using basic operations or composed) to make new functions, usually retaining some of the features of both functions. The module begins with students taking a closer look at transformations using tables. They compare function notation with geometric notation. As the learning cycle proceeds, students combine functions using basic operations, noticing and predicting the graphs. They model complex situations by combining functions with arithmetic operations. In the second learning cycle, composition of functions is introduced and students do more modeling combining a variety of function types using both composition and arithmetic operations. In the Honors course, students learn about parametric functions, both graphing and writing parametric equations.

Conceptual Category: Number and Quantity

Most of the standards in Number and Quantity are addressed in Secondary Math I and II. In Secondary Math III, extend the work done with complex numbers and quadratic functions in Secondary Math II to higher-powered polynomials. In Module 3, Polynomial Functions, students use the Fundamental Theorem of Algebra to predict the number of roots of a polynomial. They find real and complex roots. They use the relationship between roots and factors to write equations of polynomials in factored and standard form, given a known root.

In Module 7, Trigonometric Functions, Equations, and Identities, in the Honors course, students learn about polar coordinates. They write complex numbers in polar form and use them to multiply, divide and find complex roots.

Conceptual Category: Algebra

In Module 3, Polynomial Functions, students learn to identify and classify polynomials. They compare polynomials to integers and learn that their structures are analogous. Students perform the basic operations of addition, subtraction, multiplication and division with conceptual connections made to how the operations work with integers. Students extend their work with area models for two binomials from Secondary Math II to higher powered expressions. They also learn to expand binomials using patterns in Pascal's Triangle. Besides performing the basic operations with polynomials, students are introduced to the idea of closure. They construct arguments about statements regarding closure of the set of polynomials under given operations and learn that polynomials are closed under the same operations as integers.

In Module 4, students compare rational expressions to rational numbers and perform operations on rational expressions using the same properties. Students learn to simplify rational expressions and to rewrite rational expressions given in the form $a(x)/b(x)$ into the form $q(x) + r(x)/b(x)$ to facilitate graphing the function using transformations or other strategies, depending on the relationship between $a(x)$ and $b(x)$. Students model real situations using rational functions and

solve rational equations. They learn to recognize extraneous solutions and to interpret solutions based upon the context.

Conceptual Category: Geometry

The geometry of Secondary Math III is primarily focused on using the principles of geometry, including transformation, to model real situations. The first learning cycle in Module 5, Geometric Modeling, begins with students visualizing two-dimensional cross sections of three-dimensional objects and solids of rotation. They learn to approximate the volume of an irregular solid by decomposing it into cylinders, frustrums, and cones with volumes that can be easily calculated.

In the second learning cycle of Module 5, students extend their understanding of right triangle trigonometry from Secondary Math II to general triangles. They begin with a study of special right triangles and proceed to finding the sides and angles of some triangle by decomposing them into right triangles so that the Pythagorean Theorem or right triangle trigonometry can be used. This strategy supports students in deriving the Law of Cosines and the Law of Sines, which they then apply to finding sides and angles for triangles. In the final task of the module, students explore the ambiguous case of Law of Sines and develop formulas for the area of triangles using the Law of Sines and the Law of Cosines.

Conceptual Category: Statistics and Probability

In Secondary Math III, students combine all their experience with data and probability from previous courses to make inferences and draw conclusions from data. Module 9, Statistics, begins with a learning cycle where students construct the concept of normal distributions, understanding the effect of modifying either the mean or standard deviation. Students learn to compare distributions using z-scores, and to determine whether a particular point in a normal distribution is typical or unusual. The module continues by introducing methods of sampling and comparing the validity of each method for selecting a sample that is representative of the population. Students learn about different study methods and select an appropriate study type and sampling method for a given parameter of interest. In the last task, students draw conclusions about the likelihood of a given event, based on a simulation.