mathematics
vision project

Transforming Mathematics Education

ALGEBRA 1

A Learning Cycle Approach

MODULE 2

Linear & Exponential Functions

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

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LINEAR AND EXPONENTIAL FUNCTIONS

2.1 Connecting the Dots: Piggies and Pools

A Develop Understanding Task

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1. My little sister, Savannah, is three years old. She has a piggy bank that she wants to fill. She started with five pennies and each day when I come home from school, she is excited when I give her three pennies that are left over from my lunch money. Use a table, a graph, and an equation to create a mathematical model for the number of pennies in the piggy bank on day n.

2. Our family has a small pool for relaxing in the summer that holds 1500 gallons of water. I decided to fill the pool for the summer. When I had 5 gallons of water in the pool, I decided that I didn't want to stand outside and watch the pool fill, so I had to figure out how long it would take so that I could leave, but come back to turn off the water at the right time. I checked the flow on the hose and found that it was filling the pool at a rate of 2 gallons every minute. Use a table, a graph, and an equation to create a mathematical model for the number of gallons of water in the pool at t minutes.

3. I'm more sophisticated than my little sister so I save my money in a bank account that pays me 3% interest on the money in the account at the end of each month. (If I take my money out before the end of the month, I don't earn any interest for the month.) I started the account with $50 that I got for my birthday. Use a table, a graph, and an equation to create a mathematical model of the amount of money I will have in the account after m months.

4. At the end of the summer, I decide to drain the 1500 gallon swimming pool. I noticed that it drains faster when there is more water in the pool. That was interesting to me, so I decided to measure the rate at which it drains. I found that 3% was draining out of the pool every minute. Use a table, a graph, and an equation to create a mathematical model of the gallons of water in the pool at t minutes.

5. Compare problems 1 and 3. What similarities do you see? What differences do you notice?

6. Compare problems 1 and 2. What similarities do you see? What differences do you notice?

7. Compare problems 3 and 4. What similarities do you see? What differences do you notice?

READY, SET, GO! Name Period Date

READY

Topic: Recognizing arithmetic and geometric sequences

Predict the next 2 terms in the sequence. State whether the sequence is arithmetic, geometric, or neither. Justify your answer.

1. 4 , -20 , 100 , -500 , . . .

2. 3 , 5 , 8 , 12 , . . .

3. 64 , 48 , 36 , 27 , . . .

4. 1.5 , 0.75 , 0 , -0.75 , . . .

5. 40 , 10 , $\frac{5}{2}$, $\frac{5}{8}$, ...

6. 1, 11, 111, 1111, . . .

7. -3.6, -5.4, -8.1, -12.15, . . .

8. -64, -47, -30, -13, . . .

9. Create a predictable sequence of at least 4 numbers that is NOT arithmetic or geometric.

SET

Topic: Discrete and continuous relationships

Identify whether the following statements represent a *discrete* or a *continuous* relationship.

10. The hair on your head grows ½ inch per month.

11. For every ton of paper that is recycled, 17 trees are saved.

12. Approximately 3.24 billion gallons of water flow over Niagara Falls daily.

13. The average person laughs 15 times per day.

14. The city of Buenos Aires adds 6,000 tons of trash to its landfills every day.

15. During the Great Depression, stock market prices fell 75%.

GO

Topic: Solving one-step equations

Either find or use the unit rate for each of the questions below.

16. Apples are on sale at the market 4 pounds for \$2.00. What is the price (in cents) for one pound?

17. Three apples weigh about a pound. About how much would one apple cost?
(Round to the nearest cent.)

18. One dozen eggs cost \$1.98. How much does 1 egg cost? (Round to the nearest cent.)

19. One dozen eggs cost \$1.98. If the charge at the register for only eggs, without tax, was \$11.88, how many dozen were purchased?

20. Best Buy Shoes had a back to school special. The total bill for four pairs of shoes came to \$69.24 (before tax.) What was the average price for each pair of shoes?

21. If you only purchased 1 pair of shoes at Best Buy Shoes instead of the four described in problem 20, how much would you have paid, based on the average price?

Solve for x. Show your work.

22. $6x = 72$

23. $4x = 200$

24. $3x = 50$

25. $12x = 25.80$

26. $\frac{1}{2}x = 17.31$

27. $4x = 69.24$

28. $12x = 198$

29. $1.98x = 11.88$

30. $\frac{1}{4}x = 2$

31. Some of the problems 22 – 30 could represent the work you did to answer questions 16 – 21. Write the number of the equation next to the story it represents.

2.2 Shh! Please Be Discreet (Discrete)!

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A Solidify Understanding Task

1. The Library of Congress in Washington D.C. is considered the largest library in the world. They often receive boxes of books to be added to their collection. Since books can be quite heavy, they aren't shipped in big boxes. If, on average, each box contains about 8 books, how many books are received by the library in 6 boxes, 10 boxes, or *n* boxes?
   a. Use a table, a graph, and an equation to model this situation.

   b. Identify the domain of the function.

2. Many of the books at the Library of Congress are electronic. If about 13 e-books can be downloaded onto the computer each hour, how many e-books can be added to the library in 3 hours, 5 hours, or *n* hours (assuming that the computer memory is not limited)?
   a. Use a table, a graph, and an equation to model this situation.

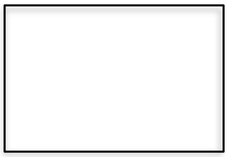
   b. Identify the domain of the function.

3. The librarians work to keep the library orderly and put books back into their proper places after they have been used. If a librarian can sort and shelve 3 books in a minute, how many books does that librarian take care of in 3 hours, 5 hours, or *n* hours? Use a table, a graph, and an equation to model this situation.

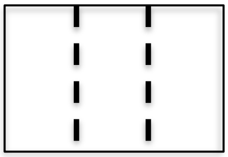
4. Would it make sense in any of these situations for there to be a time when 32.5 books had been shipped, downloaded into the computer or placed on the shelf?

5. Which of these situations (in problems 1-3) represent a discrete function and which represent a continuous function? Justify your answer.

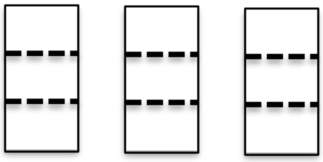
6. A giant piece of paper is cut into three equal pieces and then each of those is cut into three equal pieces and so forth. How many papers will there be after a round of 10 cuts? 20 cuts? *n* cuts?



Zero Cuts



One Cut



Two Cuts

a. Use a table, a graph, and an equation to model this situation.

b. Identify the domain of the function.

c. Would it make sense to look for the number of pieces of paper at 5.2 cuts? Why?

d. Would it make sense to look for the number of cuts it takes to make 53.6 papers? Why?

7. Medicine taken by a patient breaks down in the patient's blood stream and dissipates out of the patient's system. Suppose a dose of 60 milligrams of anti-parasite medicine is given to a dog and the medicine breaks down such that 20% of the medicine becomes ineffective every hour. How much of the 60 milligram dose is still active in the dog's bloodstream after 3 hours, after 4.25 hours, after *n* hours?

a. Use a table, a graph, and an equation to model this situation.

b. Identify the domain of the function.

c. Would it make sense to look for an amount of active medicine at 3.8 hours? Why?

d. Would it make sense to look for when there is 35 milligrams of medicine? Why?

8. Which of the functions modeled in #6 and #7 are discrete and which are continuous? Why?

9. What needs to be considered when looking at a situation or context and deciding if it fits best with a discrete or continuous model?

10. Describe the differences in each representation (table, graph, and equation) for discrete and continuous functions.

11. Which of the functions modeled in this task are linear? Which are exponential? Why?

READY, SET, GO! Name Period Date

READY

Topic: Comparing rates of change in linear situations.

State which situation has the greatest rate of change

1. The amount of stretch in a short bungee cord stretches 6 inches when stretched by a 3 pound weight. A slinky stretches 3 feet when stretched by a 1 pound weight.

2. A sunflower that grows 2 inches every day or an amaryllis that grows 18 inches in one week.

3. Pumping 25 gallons of gas into a truck in 3 minutes or filling a bathtub with 40 gallons of water in 5 minutes.

4. Riding a bike 10 miles in 1 hour or jogging 3 miles in 24 minutes.

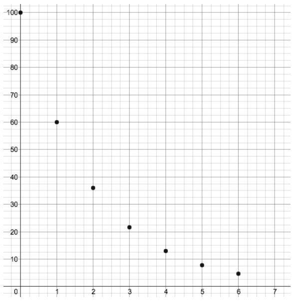
SET

Topic: Discrete and continuous relationships

Identify whether the following items best fit with a *__discrete__* or a *__continuous__* model. Then determine whether it is a *__linear__ (arithmetic)* or *__exponential__ (geometric)* relationship that is being described.

5. The freeway construction crew pours 300 ft of concrete in a day.
6. For every hour that passes, the amount of area infected by the bacteria doubles.
7. To meet the demands placed on them the brick layers have started laying 5% more bricks each day.
8. The average person takes 10,000 steps in a day.
9. The city of Buenos Aires has been adding 8% to its population every year.
10. At the headwaters of the Mississippi River the water flows at a surface rate of 1.2 miles per hour.
11. a. $f(n) = f(n-1) + 3; f(1) = 5$

b.



c. $g(x) = 2^x(7)$

GO

Topic: Solving one-step equations

Solve the following equations. Remember that what you do to one side of the equation must also be done to the other side. (Show your work, even if you can do these in your head.)

Example: Solve for x . $1x + 7 = 23$ $Add\ -7\ to\ both\ sides\ of\ the\ equation.$

$$\begin{array}{r} 1x + 7 = 23 \\ -7 = -7 \\ \hline 1x + 0 = 16 \\ Therefore\ \ 1x = 16 \end{array}$$

Example: Solve for x. $9x = 63$ $Multiply\ both\ sides\ of\ the\ equation\ by\ \frac{1}{9}.$

$$\begin{array}{c} 9x = 63 \\ \left(\frac{1}{9}\right)9x = \left(\frac{1}{9}\right)63 \\ \left(\frac{9}{9}\right)x = \frac{63}{9} \\ 1x = 7 \end{array}$$

Note that multiplying by $\frac{1}{9}$ *gives the same result as dividing everything by 9.*

12. $1x + 16 = 36$

13. $1x - 13 = 10$

14. $1x - 8 = -3$

15. $8x = 56$

16. $-11x = 88$

17. $425x = 850$

18. $\frac{1}{6}x = 10$

19. $-\frac{4}{7}x = -1$

20. $\frac{3}{4}x = -9$

2.3 Linear, Exponential or Neither?

A Practice Understanding Task

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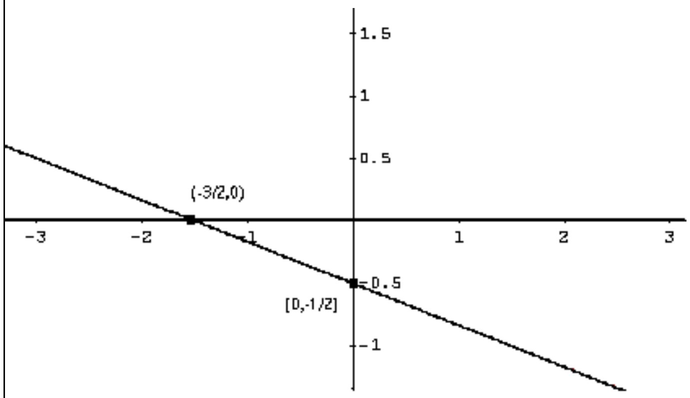
For each representation of a function, decide if the function is linear, exponential, or neither.
Give at least 2 reasons for your answer.

1.



Linear Exponential Neither

Why?

2.

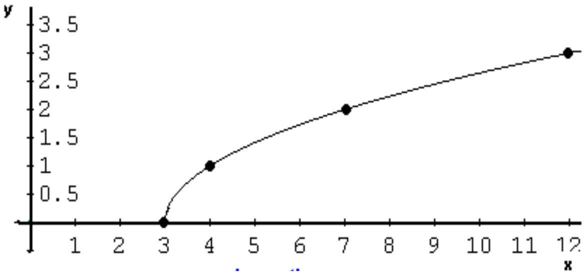
Tennis Tournament

Rounds	1	2	3	4	5
Number of Players left	64	32	16	8	4

There are 4 players remaining after 5 rounds.

Linear Exponential Neither

Why?

3. $y = 4x$	Linear Exponential Neither Why?
4. This function is decreasing at a constant rate. .	Linear Exponential Neither Why?
5. 	Linear Exponential Neither Why?

6.
A person's height as a function of a person's age (from age 0 to 100).

Linear Exponential Neither

Why?

7.

$$-3x = 4y + 7$$

Linear Exponential Neither

Why?

8.

x	*y*
-2	23
0	5
2	-13
4	-31
6	-49

Linear Exponential Neither

Why?

9.

Height in Inches	Shoe Size
62	6
74	13
70	9
67	11
53	4
58	7

Linear Exponential Neither

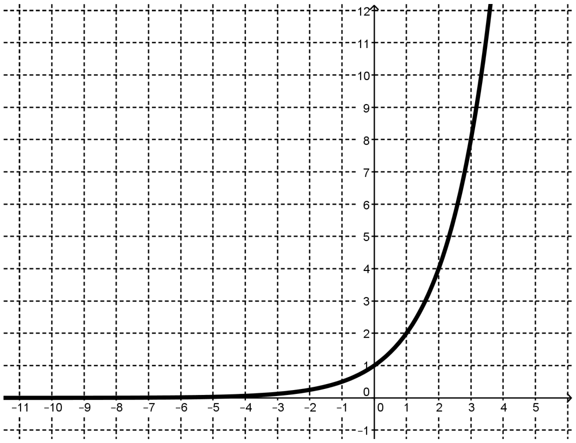
Why?

10.
The number of cell phone users in Centerville as a function of years, if the number of users is increasing by 75% each year.

Linear Exponential Neither

Why?

11.



Linear Exponential Neither

Why?

12. The time it takes you to get to work as a function the speed at which you drive.	Linear Exponential Neither Why?
13. $y = 7x^2$	Linear Exponential Neither Why?
14. Each point on the graph is exactly 1/3 of the previous point.	Linear Exponential Neither Why?

15. $f(1) = 7, f(2) = 7, f(n) = f(n-1) + f(n-2)$	Linear Exponential Neither Why?
16. $f(1) = 1, f(n) = \frac{2}{3} f(n-1)$	Linear Exponential Neither Why?

READY, SET, GO!

Name Period Date

READY

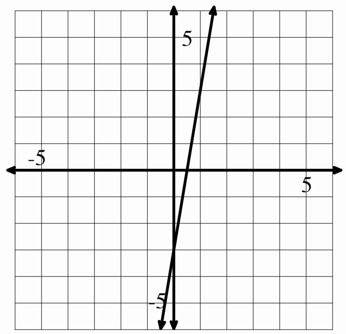
Topic: Comparing rates of change in both linear and exponential situations.

Identify whether situation "a" or situation "b" has a greater rate of change.

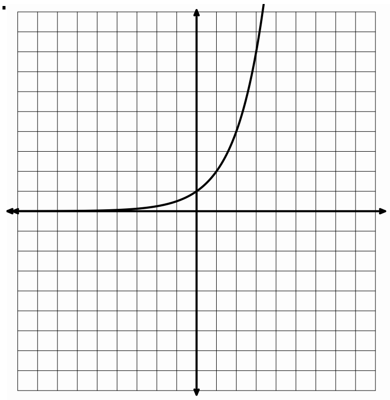
1. a.

x	y
-10	-48
-9	-43
-8	-38
-7	-33

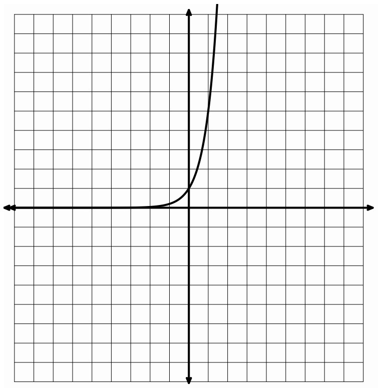
b.



2. a.



b.



3. a. Lee has $25 withheld each week from his salary to pay for his subway pass.

b. Jose owes his brother $50. He has promised to pay half of what he owes each week until the debt is paid.

4. a.

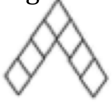
x	6	10	14	18
y	13	15	17	19

b. The number of rhombi in each shape.

Figure 1

Figure 2

Figure 3



5. a. $y = 2(5)^x$

b. In the children's book, *The Magic Pot,* every time you put one object into the pot, two of the same object come out. Imagine that you have 5 magic pots.

SET

Topic: Recognizing linear and exponential functions.

Based on each of the given representations of a function determine if it is linear, exponential or neither.

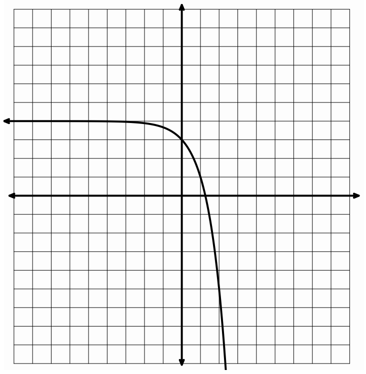
6. The population of a town is decreasing at a rate of 1.5% per year.

7. Joan earns a salary of $30,000 per year plus a 4.25% commission on sales.

8. 3x +4y = -3

9. The number of gifts received each day of "The 12 Days of Christmas" as a function of the day. ("On the 4th day of Christmas my true love gave to me, 4 calling birds, 3 French hens, 2 turtledoves, and a partridge in a pear tree.")

10.



11.

Side of a square	**Area of a square**
1 inch	1 in^2
2 inches	4 in^2
3 inches	9 in^2
4 inches	16 in^2

GO

Topic: Geometric means

For each geometric sequence below, find the missing terms in the sequence.

12.

x	1	2	3	4	5
y	2				162

13.

x	1	2	3	4	5
y	1/9			-3	

14.

x	1	2	3	4	5
y	10				0.625

15.

x	1	2	3	4	5
y	g				gz^4

16.

x	1	2	3	4	5
y	-3				-243

Find the rate of change (slope) in each of the exercises below.

17.

x	g(x)
-5	11
-3	4
-2	0.5
0	-6

18.

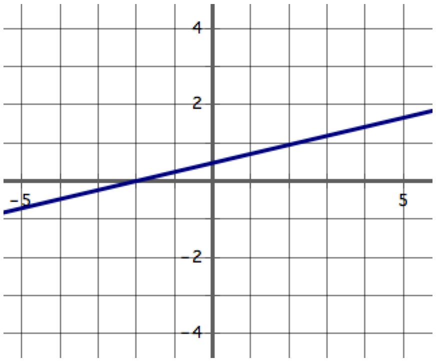
t	h(t)
3	13
8	23
18	43
23	53

19.

n	f(n)
-7	20
-5	24
-1	32
2	38

20. (2, 5) (8, 29)

21.



22. (-3, 7) (8, 29)

2.4 The In-Betweeners

A Develop Understanding Task

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Now that you've seen that there are contexts for continuous exponential functions, it's a good idea to start thinking about the numbers that fill in between the values like 2^2 and 2^3 in an exponential function. These numbers are actually pretty interesting, so we're going to do some exploring in this task to see what we can find out about these "in-betweeners".

Let's begin in a familiar place:

1. Complete the following table.

x	0	1	2	3	4
$f(x) = 4 \cdot 2^x$	4				

2. Plot these points on the graph at the end of this task, and sketch the graph of $f(x)$.

Let's say we want to create a table with more entries, maybe with a point halfway between each of the points in the table above. There are a couple of ways that we might think about it. We'll begin by letting our friend Travis explain his method.

Travis makes the following claim:

"If the function doubles each time x goes up by 1, then half that growth occurs between 0 and ½ and the other half occurs between ½ and 1. So for example, we can find the output at x = ½ by finding the average of the outputs at x = 0 and x = 1."

3. Fill in the parts of the table below that you've already computed, and then decide how you might use Travis' strategy to fill in the missing data. Also plot Travis' data on the graph at the end of the task.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
$f(x)$	4								

4. Comment on Travis' idea. How does it compare to the table generated in problem 1? For what kind of function would this reasoning work?

Miriam suggests they should fill in the data in the table in the following way:

"I noticed that the function increases by the same factor each time x goes up 1, and I think this is like what we did in *Geometric Meanies*. It seems like this property should hold over each half-interval as well."

5. Fill in the parts of the table below that you've already computed in problem 1, and then decide how you might use Miriam's new strategy to fill in the missing data. As in the table in problem 1, each entry should be multiplied by some constant factor to get the next entry, and that factor should produce the same results as those already in recorded in the table. Use this constant factor to complete the table. Also plot Miriam's data on the graph at the end of this task.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
$f(x)$	4								

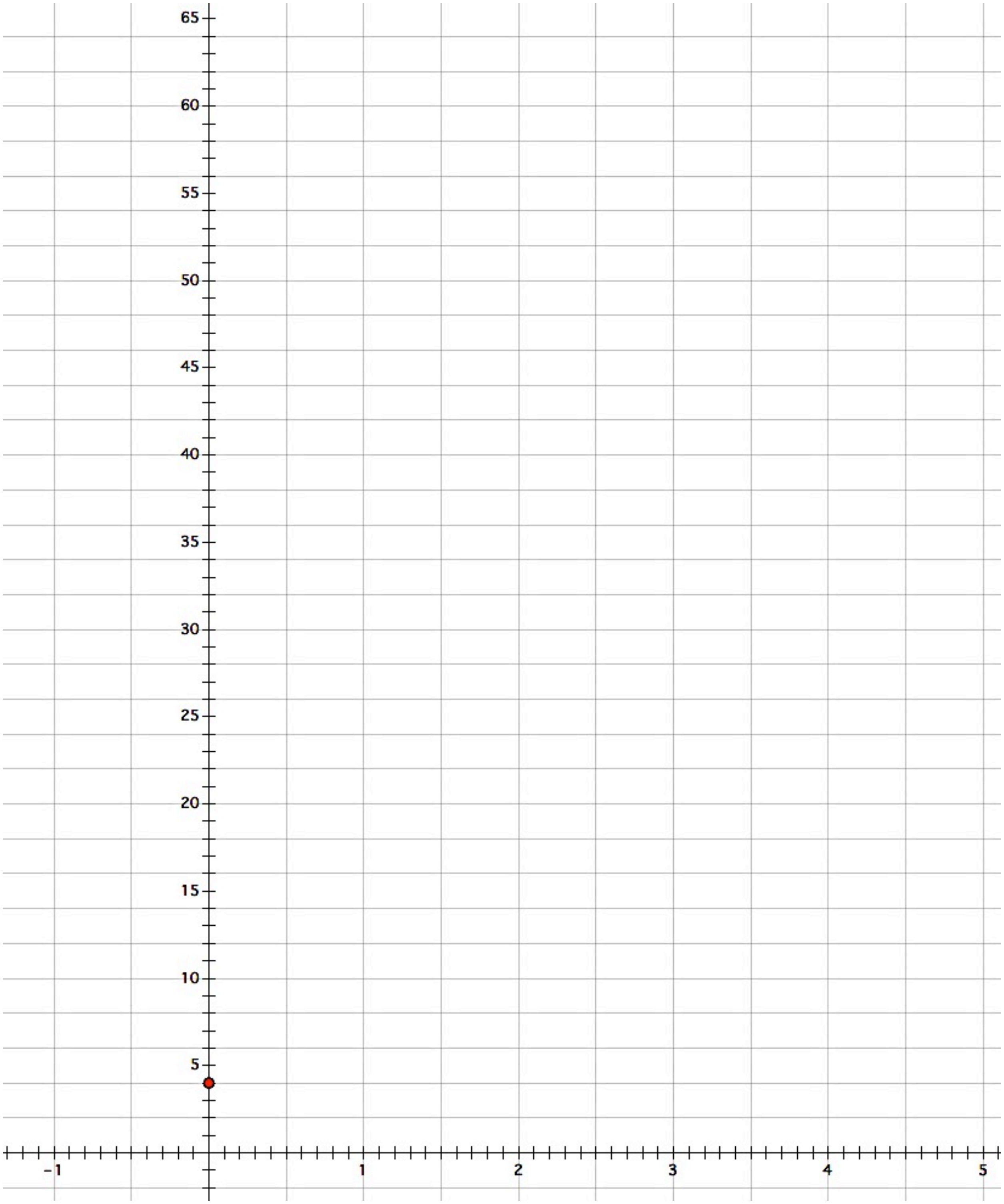
6. What if Miriam wanted to find values for the function every third of the interval instead of every half? What constant factor would she use for every third of an interval to be consistent with the function doubling as x increases by 1. Use this multiplier to complete the following table.

x	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2	$\frac{7}{3}$	$\frac{8}{3}$	3
$f(x)$	4									

7. What number did you use as a constant factor to complete the table in problem 4?

8. What number did you use as a constant factor to complete the table in problem 5?

9. Give a detailed description of how you would estimate the output value $f(x)$, for $x = 5/3$.


65
60
55
50
45
40
35
30
25
20
15
10
5
-1
1
2
3
4
5

READY, SET, GO! Name Period Date

READY

Topic: Comparing additive and multiplicative patterns.

The sequences below exemplify either an additive (arithmetic) or a multiplicative (geometric) pattern. Identify the type of sequence, fill in the missing values on the table and write an equation.

1.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	2	4	8	16	32	a.	b.	c.

d. Type of Sequence: e. Equation:

2.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	66	50	34	18	a.	b.	c.	d.

e. Type of Sequence: f. Equation:

3.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	-3	9	-27	81	a.	b.	c.	d.

e. Type of Sequence: f. Equation:

4.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	160	80	40	20	a.	b.	c.	d.

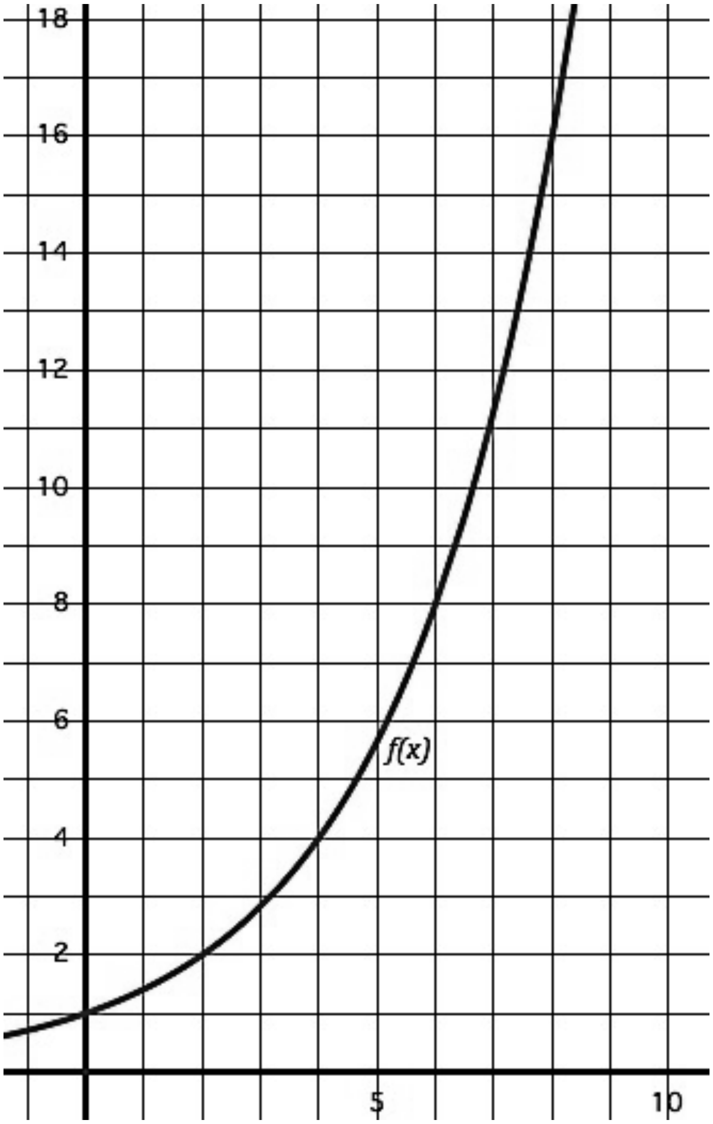
e. Type of Sequence: f. Equation:

5.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	-9	-2	5	12	a.	b.	c.	d.

e. Type of Sequence: f. Equation:

Use the graph of the function to find the desired values of the function. Also create an explicit equation for the function.



6. Find the value of *f(2)*

7. Find where *f(x) = 4*

8. Find the value of *f(6)*

9. Find where *f(x) = 16*

10. What do you notice about the way that inputs and outputs for this function relate? (Create an in-out table if you need to.)

11. What is the explicit equation for this function?

SET

Topic: Evaluate expressions with rational exponents.

Fill in the missing values of the table based on the growth that is described.

12. The growth in the table is triple at each whole year.

Years	0	$^{1}/_{2}$	1	$^{3}/_{2}$	2	$^{5}/_{2}$	3	$^{7}/_{2}$	4
bacteria	2		6						

13. The growth in the table is triple at each whole year.

Years	0	$^{1}/_{3}$	$^{2}/_{3}$	1	$^{4}/_{3}$	$^{5}/_{3}$	2	$^{7}/_{3}$	$^{8}/_{3}$
bacteria	2			6					

14. The values in the table grow by a factor of four at each whole year.

Years	0	$^{1}/_{2}$	1	$^{3}/_{2}$	2	$^{5}/_{2}$	3	$^{7}/_{2}$	4
bacteria									

GO

Topic: Simplifying Exponents

Simplify the following expressions using exponent rules and relationships. Write your answers in exponential form. (For example: $2^2 \cdot 2^5 = 2^7$)

15. $3^2 \cdot 3^5$

16. $\frac{5^3}{5^2}$

17. 2^{-5}

18. 17^0

19. $\frac{7^5}{7^2} \cdot \frac{7^3}{7^4}$

20. $\frac{3^{-2} \cdot 3^5}{3^7}$

2.5 Half Interested

A Solidify Understanding Task

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Carlos and Clarita, the Martinez twins, have run a summer business every year for the past five years. Their first business, a neighborhood lemonade stand, earned a small profit that their father insisted they deposit in a savings account at the local bank. When the Martinez family moved a few months later, the twins decided to leave the money in the bank where it has been earning 5% interest annually. Carlos was reminded of the money when he found the annual bank statement they had received in the mail.

"Remember how Dad said we could withdraw this money from the bank when we are twenty years old," Carlos said to Clarita. "We have $382.88 in the account now. I wonder how much that will be five years from now?"

1. Given the facts listed above, how can the twins figure out how much the account will be worth five years from now when they are twenty years old? Describe your strategy and calculate the account balance.

2. Carlos calculates the value of the account one year at a time. He has just finished calculating the value of the account for the first four years. Describe how he can find the next year's balance, and record that value in the table.

year	amount
0	382.88
1	402.02
2	422.12
3	443.23
4	465.39
5	

3. Clarita thinks Carlos is silly calculating the value of the account one year at a time, and says that he could have written a formula for the n^{th} year and then evaluated his formula when $n = 5$. Write Clarita's formula for the n^{th} year and use it to find the account balance at the end of year 5.

4. Carlos was surprised that Clarita's formula gave the same account balance as his year-by-year strategy. Explain, in a way that would convince Carlos, why this is so.

"I can't remember how much money we earned that summer," said Carlos. "I wonder if we can figure out how much we deposited in the account five years ago, knowing the account balance now?"

5. Carlos continued to use his strategy to extend his table year-by-year back five years. Explain what you think Carlos is doing to find his table values one year at a time, and continue filling in the table until you get to -5, which Carlos uses to represent "five years ago".

6. Clarita evaluated her formula for n = -5. Again Carlos is surprised that they get the same results. Explain why Clarita's method works.

year	amount
-5	
-4	
-3	
-2	
-1	364.65
0	382.88
1	402.02
2	422.12
3	443.23
4	465.39
5	

Clarita doesn't think leaving the money in the bank for another five years is such a great idea, and suggests that they invest the money in their next summer business, *Curbside Rivalry* (which, for now, they are keeping top secret from everyone, including their friends). "We'll have some start up costs, and this will pay for them without having to withdraw money from our other accounts."

Carlos remarked, “But we’ll be withdrawing our money halfway through the year. Do you think we’ll lose out on this year’s interest?”

“No, they’ll pay us a half-year portion of our interest,” replied Clarita.

“But how much will that be?” asked Carlos.

7. Calculate the account balance and how much interest you think Carlos and Clarita should be paid if they withdraw their money ½ year from now. Remember that they currently have $382.88 in the account, and that they earn 5% annually. Describe your strategy.

Carlos used the following strategy: He calculated how much interest they should be paid for a full year, found half of that, and added that amount to the current account balance.

Clarita used this strategy: She substituted ½ for *n* in the formula $A = 382.88(1.05)^n$ and recorded this as the account balance.

8. This time Carlos and Clarita didn’t get the same result. Whose method do you agree with and why?

Clarita is trying to convince Carlos that her method is correct. “Exponential rules are multiplicative, not additive. Look back at your table. We will earn $82.51 in interest during the next four years. If your method works we should be able to take half of that amount, add it to the amount we have now, and get the account balance we should have in two years, but it isn’t the same.”

9. Carry out the computations that Clarita suggested and compare the result for year 2 using this strategy as opposed to the strategy Carlos originally used to fill out the table.

10. The points from Carlos' table (see question 2) have been plotted on the graph at the end of this task, along with Clarita's function. Plot the value you calculated in question 9 on this same graph. What does the graph reveal about the differences in Carlos' two strategies?

11. Now plot Clarita's and Carlos' values for ½ year (see question 8) on this same graph.

"Your data point seems to fit the shape of the graph better than mine," Carlos conceded, "but I don't understand how we can use ½ as an exponent. How does that find the correct factor we need to multiply by? In your formula, writing $(1.05)^5$ means multiply by 1.05 five times, and writing $(1.05)^{-5}$ means divide by 1.05 five times, but what does $(1.05)^{1/2}$ mean?"

Clarita wasn't quite sure how to answer Carlos' question, but she had some questions of her own. She decided to jot them down, including Carlos' question:

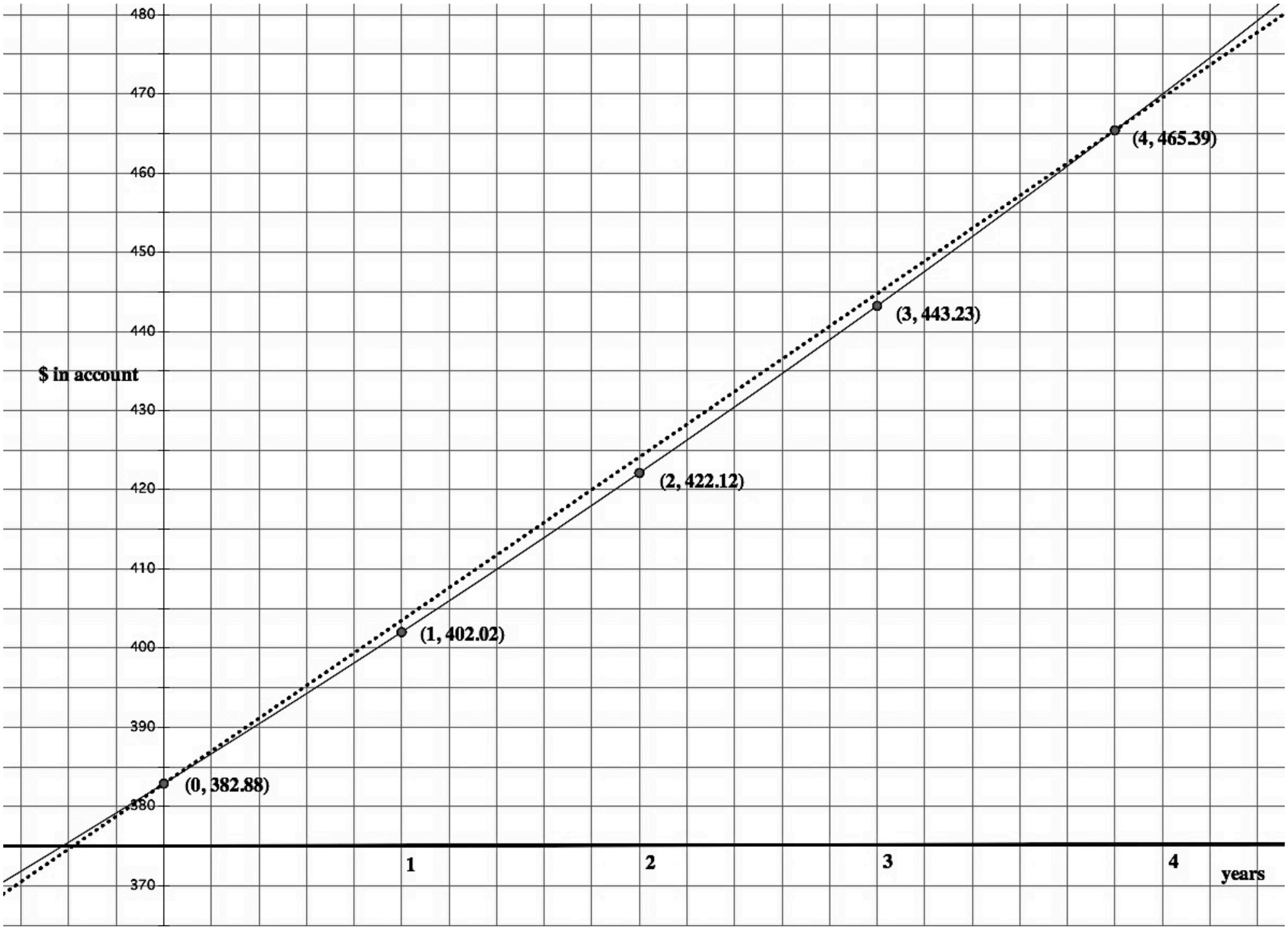
- What numerical amount do we multiply by when we use $(1.05)^{1/2}$ as a factor?
- What happens if we multiply by $(1.05)^{1/2}$ and then multiply the result by $(1.05)^{1/2}$ again? Shouldn't that be a full year's worth of interest? Is it?
- If multiplying by $(1.05)^{1/2} \cdot (1.05)^{1/2}$ is the same as multiplying by 1.05, what does that suggest about the value of $(1.05)^{1/2}$?

12. Answer each of Clarita's questions listed above as best as you can.

As Carlos is reflecting on this work, Clarita notices the date on the bank statement that started this whole conversation. "This bank statement is three months old!" she exclaims. "That means the bank will owe us ¾ of a year's interest."

"So how much interest will the bank owe us then?" asked Carlos.

13. Find as many ways as you can to answer Carlos' question: How much will their account be worth in ¾ of a year (nine months) if it earns 5% annually and is currently worth $382.88?



READY, SET, GO! Name Period Date

READY

Topic: Simplifying Radicals

A very common radical expression is a square root. One way to think of a square root is the number that will multiply by itself to create a desired value. For example: $\sqrt{2}$ *is the number that will multiply by itself to equal 2. And in like manner* $\sqrt{16}$ *is the number that will multiply by itself to equal 16, in this case the value is 4 because 4 x 4 = 16. (When the square root of a square number is taken you get a nice whole number value. Otherwise an irrational number is produced.)*

This same pattern holds true for other radicals such as cube roots and fourth roots and so forth. For example: $\sqrt[3]{8}$ *is the number that will multiply by itself three times to equal 8. In this case it is equal to the value of 2 because* $2^3 = 2 \times 2 \times 2 = 8$.

With this in mind radicals can be simplified. See the examples below.

Example 1: Simplify $\sqrt{20}$	*Example 2*: Simplify $\sqrt[5]{96}$
$\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{2 \cdot 2 \cdot 5} = 2\sqrt{5}$	$\sqrt[5]{96} = \sqrt[5]{2^5 \cdot 3} = 2\sqrt[5]{3}$

Simplify each of the Radicals

1. $\sqrt{40}$
2. $\sqrt{50}$
3. $\sqrt[3]{16}$
4. $\sqrt{72}$
5. $\sqrt[4]{81}$
6. $\sqrt{32}$
7. $\sqrt[5]{160}$
8. $\sqrt{45}$
9. $\sqrt[3]{54}$

SET

Topic: Finding arithmetic and geometric means and making meaning of rational exponents.

You may have found arithmetic and geometric means in your prior work. Finding arithmetic and geometric means requires finding values of a sequence between given values from non-consecutive terms. In each of the sequences below determine the means and show how you found them.

Find the *arithmetic* means for the following. Show your work.

10.

x	1	2	3
y	5		11

11.

x	1	2	3	4	5
y	18	a.	b.	c.	-10

12.

x	1	2	3	4	5	6	7
y	12	a.	b.	c.	d.	e.	-6

Find the *geometric* means for the following. Show your work.

13.

x	1	2	3
y	3		12

14.

x	1	2	3	4
y	7	a.	b.	875

15.

x	1	2	3	4	5	6
y	4	a.	b.	c.	d.	972

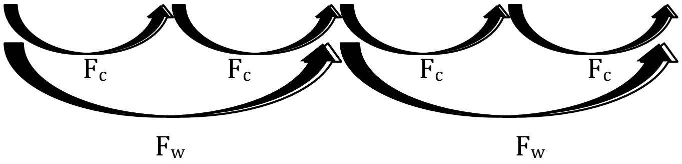
Fill in the tables of values and find the factor used to move between whole number values, F_w, as well as the factor, F_c, used to move between each column of the table.

16.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
y	4		16		

d. $F_w =$

e. $F_c =$

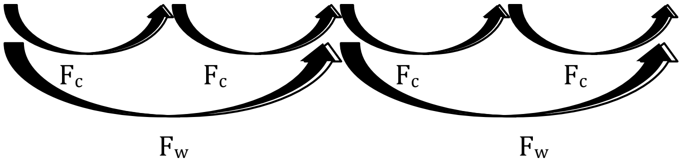


17.

x	0	½	1	3/2	2
y	4		8		

d. F_w =

e. F_c =

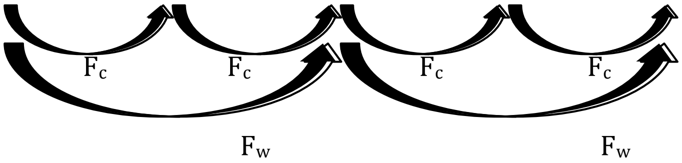


18.

x	0	½	1	3/2	2
y	5		15		

d. F_w =

e. F_c =



GO

Topic: Evaluating Functions

Either find or use the unit rate for each of the questions below.

19. $f(x) = 2x - 7$

a. Find $f(-3)$

b. Find $f(x) = 21$

c. Find $f\left(\frac{1}{2}\right)$

20. $g(x) = 3^x(2)$

a. Find $g(-4)$

b. Find $g(x) = 162$

c. Find $g\left(\frac{1}{2}\right)$

21. $I(t) = 210(1.08^t)$

a. Find $I(12)$

b. Find $I(t) = 420$

c. Find $I\left(\frac{1}{2}\right)$

22. $h(x) = x^2 + x - 6$

a. Find $h(-5)$

b. Find $h(x) = 0$

c. Find $h\left(\frac{1}{2}\right)$

23. $k(x) = -5x + 9$

a. Find $k(-7)$

b. Find $k(x) = 0$

c. Find $k\left(\frac{1}{2}\right)$

24. $m(x) = (5^x)2$

a. Find $m(-2)$

b. Find $m(x) = 1$

c. Find $m\left(\frac{1}{2}\right)$

2.6 More Interesting

A Solidify Understanding Task

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Carlos now knows he can calculate the amount of interest earned on an account in smaller increments than one full year. He would like to determine how much money is in an account each month that earns 5% annually with an initial deposit of $300.

He starts by considering the amount in the account each month during the first year. He knows that by the end of the year the account balance should be $315, since it increases 5% during the year.

1. Complete the table showing what amount is in the account each month during the first twelve months.

deposit											1 year
$300											$315

2. What number did you multiply the account by each month to get the next month's balance?

Carlos knows the exponential equation that gives the account balance for this account on an annual basis is $A = 300(1.05)^t$. Based on his work finding the account balance each month, Carlos writes the following equation for the same account: $A = 300(1.05^{1/12})^{12t}$.

3. Verify that both equations give the same results. Using the properties of exponents, explain why these two equations are equivalent.

4. What is the meaning of the 12*t* in this equation?

Carlos shows his equation to Clarita. She suggests his equation could also be approximated by $A = 300(1.004)^{12t}$, since $(1.05)^{\frac{1}{12}} \approx 1.004$. Carlos replies, "I know the 1.05 in the equation $A = 300(1.05)^t$ means I am earning 5% interest annually, but what does the 1.004 mean in your equation?"

5. Answer Carlos' question. What does the 1.004 mean in $A = 300(1.004)^{12t}$?

The properties of exponents can be used to explain why $[(1.05)^{\frac{1}{12}}]^{12t} = 1.05^t$. Here are some more examples of using the properties of exponents with rational exponents. For each of the following, simplify the expression using the properties of exponents, and explain what the expression means in terms of the context.

6. $(1.05)^{\frac{1}{12}} \cdot (1.05)^{\frac{1}{12}} \cdot (1.05)^{\frac{1}{12}}$

7. $[(1.05)^{\frac{1}{12}}]^6$

8. $(1.05)^{-\frac{1}{12}}$

9. $(1.05)^2 \cdot (1.05)^{\frac{1}{4}}$

10. $\dfrac{(1.05)^2}{(1.05)^{\frac{1}{2}}}$

11. Use $\left[(1.05)^{\frac{1}{12}}\right]^{12} = 1.05$ to explain why $(1.05)^{\frac{1}{12}} = \sqrt[12]{1.05}$

READY, SET, GO!

Name Period Date

READY

Topic: Meaning of Exponents

In the table below there is a column for the exponential form, the meaning of that form, which is a list of factors and the standard form of the number. Fill in the form that is missing.

Exponential form	List of factors	Standard Form
5^3	$5 \cdot 5 \cdot 5$	125
1a.	$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$	b.
2. 2^{10}	a.	b.
3a.	b.	81
4. 11^5	a.	b.
5a.	$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$	b.
6a.	b.	625

Provide at least three other equivalent forms of the exponential expression. Use rules of exponents such as $3^5 \cdot 3^6 = 3^{11}$ and $(5^2)^3 = 5^6$ as well as division properties and others.

	1st Equivalent Form	2nd Equivalent Form	3rd Equivalent Form
7. $2^{10} =$			
8. $3^7 =$			
9. $13^{-8} =$			
10. $7^{\frac{1}{3}} =$			
11. $5^1 =$			

SET

Topic: Finding equivalent expressions and functions

Determine whether all three expressions in each problem below are equivalent. Justify why or why they are not equivalent.

12.	$5(3^{x-1})$	$15(3^{x-2})$	$\frac{5}{3}(3^{x})$
13.	$64\ (2^{-x})$	$\frac{64}{2^{x}}$	$64\left(\frac{1}{2}\right)^{x}$
14.	3(x-1)+4	3x - 1	3(x-2) +7
15.	$50(2^{x+2})$	$25(2^{2x+1})$	$50(4^{x})$
16.	$30(1.05^{x})$	$30\left(1.05^{\frac{1}{7}}\right)^{7x}$	$30\left(1.05^{\frac{x}{2}}\right)^{2}$
17.	$20\ (1.1^{x})$	$20\ (1.1^{-1})^{-1x}$	$20\left(1.1^{\frac{1}{5}}\right)^{5x}$

GO

Topic: Using rules of exponents

Simplify each expression. Your answer should still be in exponential form.

18. $7^3 \cdot 7^5 \cdot 7^2$

19. $(3^4)^5$

20. $(5^3)^4 \cdot 5^7$

21. $x^3 \cdot x^5$

22. x^{-b}

23. $x^a \cdot x^b$

24. $(x^a)^b$

25. $\frac{y^a}{y^b}$

26. $\frac{(y^a)^c}{y^b}$

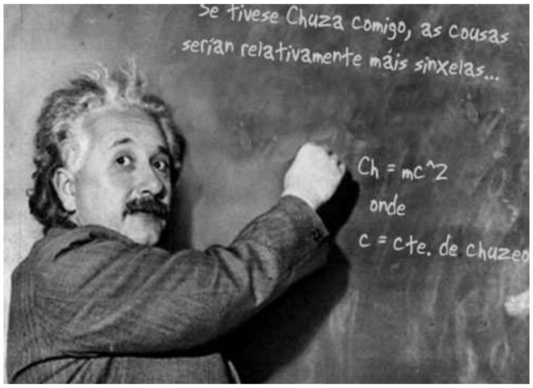
27. $\frac{(3^4)^6}{3^7}$

28. $\frac{r^5 s^3}{r s^2}$

29. $\frac{x^5 y^{12} z^0}{x^8 y^9}$

2.7 Radical Ideas

A Practice Understanding Task



Now that Tia and Tehani know that $a^{m/n} = \left(\sqrt[n]{a}\right)^m$ they are wondering which form, radical form or exponential form, is best to use when working with numerical and algebraic expressions.

Tia says she prefers radicals since she understands the following properties for radicals (and there are not too many properties to remember):

If *n* is a positive integer greater than 1 and both *a* and *b* are positive real numbers then,

1. $\sqrt[n]{a^n} = a$
2. $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
3. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Tehania says she prefers exponents since she understands the following properties for exponents (and there are more properties to work with):

1. $a^m \cdot a^n = a^{m+n}$
2. $\left(a^m\right)^n = a^{mn}$
3. $(ab)^n = a^n \cdot b^n$
4. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
5. $\frac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$
6. $a^{-n} = \frac{1}{a^n}$

DO THIS: Illustrate with examples and explain, using the properties of radicals and exponents, why $a^{1/n} = \sqrt[n]{a}$ and $a^{m/n} = \left(\sqrt[n]{a}\right)^m$ are true identities.

Using their preferred notation, Tia might simplify $\sqrt[3]{x^8}$ as follows:

$$\sqrt[3]{x^8} = \sqrt[3]{x^3 \cdot x^3 \cdot x^2} = \sqrt[3]{x^3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x^2} = x \cdot x \cdot \sqrt[3]{x^2} = x^2 \cdot \sqrt[3]{x^2}$$

(Tehani points out that Tia also used some exponent rules in her work.)

On the other hand, Tehani might simplify $\sqrt[3]{x^8}$ as follows:

$$\sqrt[3]{x^8} = x^{8/3} = x^{2+2/3} = x^2 \cdot x^{2/3} \text{ or } x^2 \cdot \sqrt[3]{x^2}$$

For each of the following problems, simplify the expression in the ways you think Tia and Tehani might do it.

Original expression	**What Tia and Tehani might do to simplify the expression:**
$\sqrt{27}$	Tia's method
	Tehani's method
$\sqrt[3]{32}$	Tia's method
	Tehani's method

$\sqrt{20x^7}$	Tia's method
	Tehani's method
$\sqrt[3]{\frac{16xy^5}{x^7y^2}}$	Tia's method
	Tehani's method

READY, SET, GO! Name Period Date

READY

Topic: Evaluating functions and creating tables of values.

Fill in the table of values for each function, look for patterns and connections in your work.

Remember that output values for a function come from evaluating the function using the given input value and following the order of operations.

1. $f(x) = 3x + 5$

x	$f(x)$
-1	
0	
1	
2	
3	

2. $g(x) = 3(x - 1) + 5$

x	$g(x)$
-1	
0	
1	
2	
3	

3. $h(x) = 3(x - 2) + 5$

x	$h(x)$
-1	
0	
1	
2	
3	

4. $k(x) = 3(x + 1) + 5$

x	$k(x)$
-1	
0	
1	
2	
3	

5. $p(x) = 2^x(3)$

x	$p(x)$
-1	
0	
1	
2	
3	

6. $q(x) = 2^{(x-1)}(3)$

x	$q(x)$
-1	
0	
1	
2	
3	

7. $r(x) = 2^{(x-2)}(3)$

x	$r(x)$
-1	
0	
1	
2	
3	

8. $s(x) = 2^{(x+1)}(3)$

x	$s(x)$
-1	
0	
1	
2	
3	

9. $h(x) = -5x + 2$

x	$t(x)$
-1	
0	
1	
2	
3	

10. $h(x) = -5(x - 1) + 2$

x	$u(x)$
-1	
0	
1	
2	
3	

11. $h(x) = -5(x - 2) + 2$

x	$v(x)$
-1	
0	
1	
2	
3	

12. $h(x) = -5(x + 1) + 2$

x	$w(x)$

SET

Topic: Radical notation and rational exponents

Each of the expressions below can be written using either radical notation, $\sqrt[n]{a^m}$ or rational exponents $a^{\frac{m}{n}}$. Rewrite each of the given expressions in the form that is missing. Express in most simplified form.

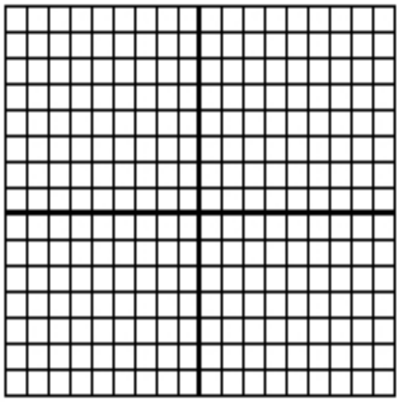
	Radical Form	Exponential Form
13.	$\sqrt[3]{5^2}$	
14.		$16^{\frac{3}{4}}$
15.	$\sqrt[3]{5^7 \cdot 3^5}$	
16.		$9^{\frac{2}{3}} \cdot 9^{\frac{4}{3}}$
17.	$\sqrt[5]{x^{13}y^{21}}$	
18.	$\sqrt[3]{27a^5b^2}$	
19.	$\sqrt[5]{\frac{32x^{13}}{243y^{15}}}$	
20.		$9^{\frac{3}{2}}s^{\frac{6}{3}}t^{\frac{1}{2}}$

GO

Topic: x-intercepts and y-intercepts for linear and exponential functions

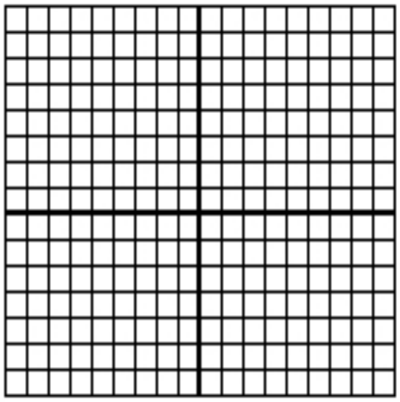
Given the function, find the x-intercept (s) and y-intercept if they exist and then use them to graph a sketch of the function.

23. $f(x) = 5(x - 4) - 10$



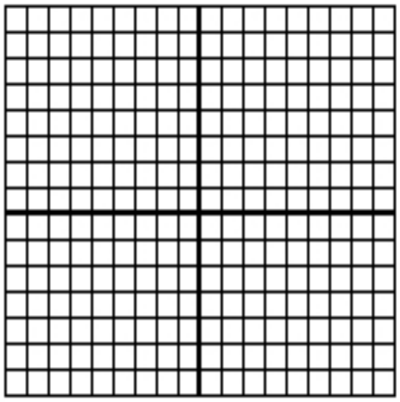
a. x-intercept(s): b. y-intercept:

24. $g(x) = 5(2^{x-1})$



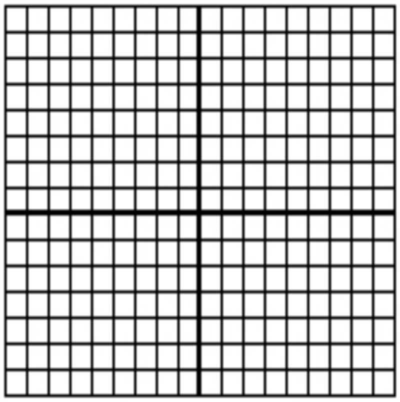
a. x-intercept(s): b. y-intercept:

25. $h(x) = -2(x + 3)$



a. x-intercept(s): b. y-intercept:

26. $k(x) = 2x - 8$



a. x-intercept(s): b. y-intercept:

2.8 Getting Down to Business

A Solidify Understanding Task

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Calcu-rama had a net income of 5 million dollars in 2010, while a small competing company, Computafest, had a net income of 2 million dollars. The management of Calcu-rama develops a business plan for future growth that projects an increase in net income of 0.5 million per year, while the management of Computafest develops a plan aimed at increasing its net income by 15% each year.

a. Create standard mathematical models (table, graph and equations) for the projected net income over time for both companies. (Attend to precision and be sure that each model is accurate and labeled properly so that it represents the situation.)

b. Compare the two companies. How are the representations for the net income of the two companies similar? How do they differ? What relationships are highlighted in each representation?

c. If both companies were able to meet their net income growth goals, which company would you choose to invest in? Why?

d. When, if ever, would your projections suggest that the two companies have the same net income? How did you find this? Will they ever have the same net income again?

e. Since we are creating the models for these companies we can choose to have a discrete model or a continuous model. What are the advantages or disadvantages for each type of model?

2.8

READY, SET, GO! Name Period Date

READY

Topic: Comparing arithmetic and geometric sequences.

The first and fifth terms of a sequence are given. Fill in the missing numbers if it is an arithmetic sequence. Then fill in the numbers if it is a geometric sequence.

Example:

+80 +80 +80 +80

Arithmetic	4	84	164	244	324
Geometric	4	12	36	108	324

× 3 × 3 × 3 × 3

1.

Arithmetic	3				48
Geometric	3				48

2.

Arithmetic	-6250				-10
Geometric	-6250				-10

3.

Arithmetic	-12				-0.75
Geometric	-12				-0.75

SET

Topic: Distinguishing specifics between sequences and linear or exponential functions.

Answer the questions below with respect to the relationship between sequences and the larger families of functions.

4. If a relationship is modeled with a continuous function which of the domain choices is a possibility?
A. $\{x \mid x \in R, x \geq 0\}$ B. $\{x \mid x \in W\}$ C. $\{x \mid x \in Z, x \geq 0\}$ D. $\{x \mid x \in N\}$

5. Which one of the options below is the mathematical way to represent the Natural Numbers?
A. $\{x \mid x \in R, x \geq 0\}$ B. $\{x \mid x \in Q, x \geq 0\}$ C. $\{x \mid x \in Z, x \geq 0\}$ D. $\{x \mid x \in N\}$

6. Only one of the choices below would likely be used for a continuous exponential model, which one?
   A. $f(x) = f(x-1)\cdot 4, f(1) = 3$
   B. $g(x) = 4^x(5)$
   C. $h(t) = 3t - 5$
   D. $k(n) = k(n-1) - 5, k(1) = 32$

7. Only one of the choices below would likely be used for a continuous linear model, which one is it?
   A. $f(x) = f(x-1)\cdot 4, f(1) = 3$
   B. $g(x) = 4^x(5)$
   C. $h(t) = 3t - 5$
   D. $k(n) = k(n-1) - 5, k(1) = 32$

8. Which domain choice would be most appropriate for an arithmetic or geometric sequence?
   A. $\{x \mid x \in R, x \geq 0\}$
   B. $\{x \mid x \in Q, x \geq 0\}$
   C. $\{x \mid x \in Z, x \geq 0\}$
   D. $\{x \mid x \in N\}$

9. What attributes will arithmetic or geometric sequences always have?
   (There could be more than one correct choice. Circle all that apply.)
   A. Continuous
   B. Discrete
   C. Domain: $\{x \mid x \in N\}$
   D. Domain: $\{x \mid x \in R\}$
   E. Negative x-values
   F. Something constant
   G. Recursive Rule

10. What type of sequence fits with linear mathematical models?

    What is the difference between this sequence type and the overarching umbrella of linear relationships? (Use words like discrete, continuous, domain and so forth in your response.)

11. What type of sequence fits with exponential mathematical models?

    What is the difference between this sequence type and the overarching umbrella of exponential relationships? (Use words like discrete, continuous, domain and so forth in your response.)

GO

Topic: Writing explicit equations for linear and exponential models.

Write the explicit equations for the tables and graphs below. This is something you really need to know. Persevere and do all you can to figure them out. Remember the tools we have used. (#21 is bonus give it a try.)

12.

x	$f(x)$
2	-4
3	-11
4	-18
5	-25

13.

x	$f(x)$
-1	2/5
0	2
1	10
2	50

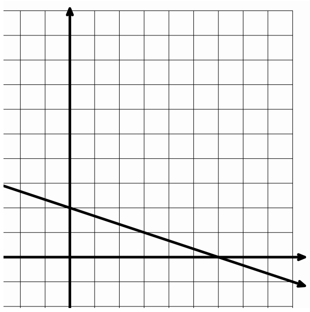
14.

x	$f(x)$
2	-24
3	-48
4	-96
5	-192

15.

x	$f(x)$
-4	81
-3	27
-2	9
-1	3

16



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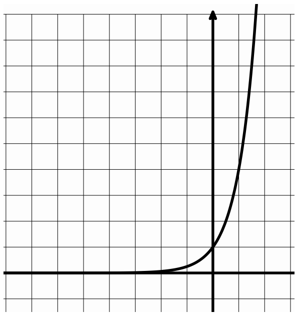

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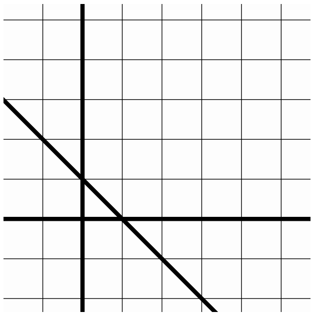

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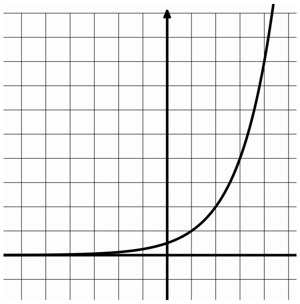

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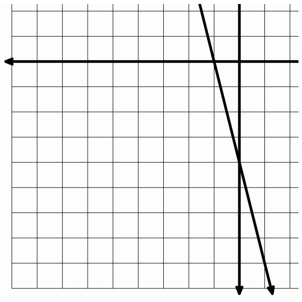
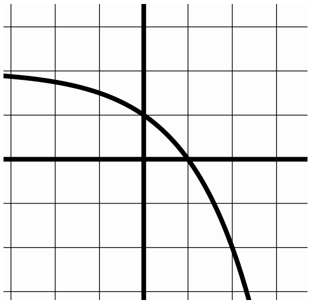


21.



2.9 Making My *Point*

A Solidify Understanding Task

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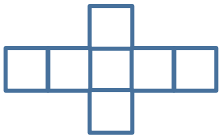
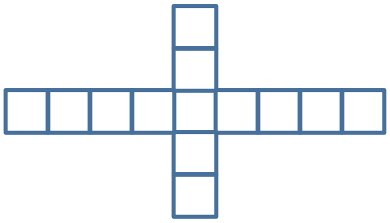
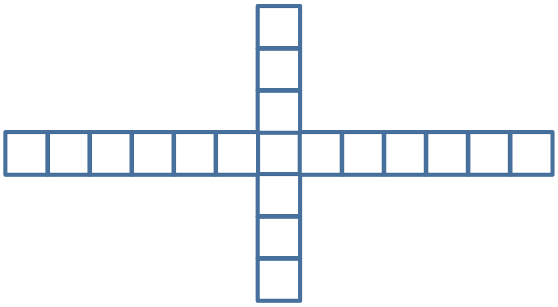
Zac and Sione were working on predicting the number of quilt blocks in this pattern:







When they compared their results, they had an interesting discussion:

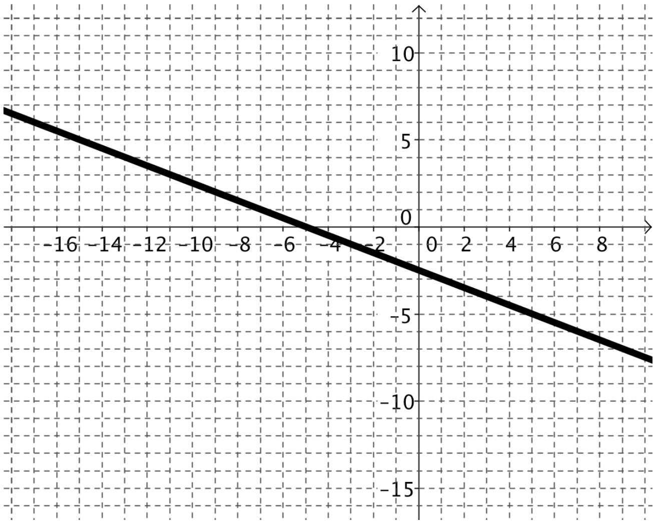
Zac: I got $y = 6n + 1$ because I noticed that 6 blocks were added each time so the pattern must have started with 1 block at n = 0.

Sione: I got $y = 6(n - 1) + 7$ because I noticed that at n = 1 there were 7 blocks and at n = 2 there were 13, so I used my table to see that I could get the number of blocks by taking one less than the n, multiplying by 6 (because there are 6 new blocks in each figure) and then adding 7 because that's how many blocks in the first figure. Here's my table:

1	2	3	n
7	13	19	$6(n - 1) + 7$

1. What do you think about the strategies that Zac and Sione used? Are either of them correct? Why or why not? Use as many representations as you can to support your answer.

The next problem Zac and Sione worked on was to write the equation of the line shown on the graph below.



When they were finished, here is the conversation they had about how they got their equations:

Sione: It was hard for me to tell where the graph crossed the y axis, so I found two points that I could read easily, (-9, 2) and (-15, 5). I figured out that the slope was - $\frac{1}{2}$ and made a table and checked it against the graph. Here's my table:

x	-15	-13	-11	-9	n
$f(x)$	5	4	3	2	$-\frac{1}{2}(n+9)+2$

I was surprised to notice that the pattern was to start with the n, add 9, multiply by the slope and then add 2.

I got the equation: $f(x) = -\frac{1}{2}(x+9)+2$.

Zac: Hey—I think I did something similar, but I used the points, (7,-6) and (9,-7).

I ended up with the equation: $f(x) = -\frac{1}{2}(x-9)-7$. One of us must be wrong because yours says that you add 9 to the n and mine says that you subtract 9. How can we both be right?

2. What do you say? Can they both be right? Show some mathematical work to support your thinking.

Zac: My equation made me wonder if there was something special about the point (9, -7) since it seemed to appear in my equation $f(x) = -\frac{1}{2}(x - 9) - 7$ when I looked at the number pattern. Now I'm noticing something interesting—the same thing seems to happen with your equation, $f(x) = -\frac{1}{2}(x + 9) + 2$ and the point (-9, 2)

3. Describe the pattern that Zac is noticing.

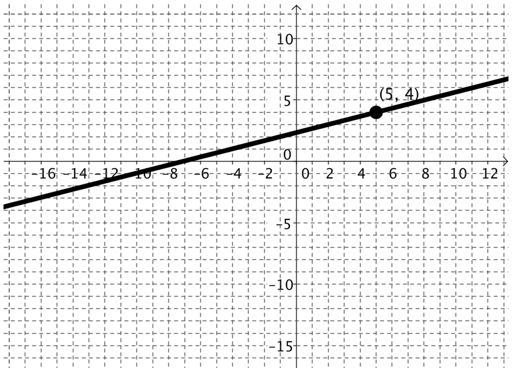
4. Find another point on the line given above and write the equation that would come from Zac's pattern.

5. What would the pattern look like with the point (a, b) if you knew that the slope of the line was m?

6. Zac challenges you to use the pattern he noticed to write the equation of line that has a slope of 3 and contains the point (2,-1). What's your answer?

   Show a way to check to see if your equation is correct.

7. Sione challenges you to use the pattern to write the equation of the line graphed below, using the point (5, 4).



Show a way to check to see if your equation is correct.

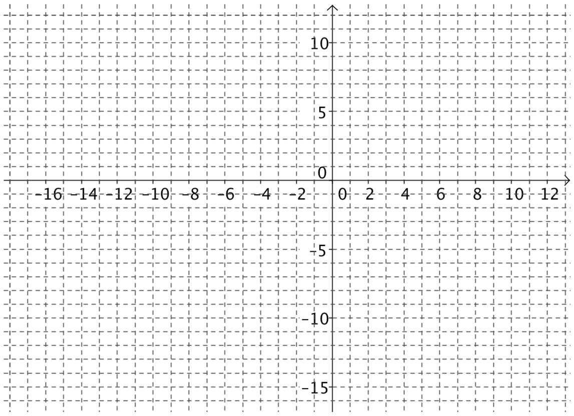
8. **Zac:** "I'll bet you can't use the pattern to write the equation of the line through the points (1,-3) and (3,-5). Try it!"

Show a way to check to see if your equation is correct.

9. **Sione:** I wonder if we could use this pattern to graph lines, thinking of the starting point and using the slope. Try it with the equation: $f(x) = -2(x+1) - 3$.

Starting point: Slope:

Graph:



10. Zac wonders, “What is it about lines that makes this work?” How would you answer Zac?

11. Could you use this pattern to write the equation of any linear function? Why or why not?

READY, SET, GO! Name Period Date

READY

Topic: Writing equations of lines.

Write the equation of a line in slope-intercept form: y = mx + b, using the given information.

1. m = -7, b = 4
2. m = 3/8, b = -3
3. m = 16, b = -1/5

Write the equation of the line in point-slope form: $y = m(x - x_1) + y_1$, using the given information.

4. m = 9, (0. -7)
5. m = 2/3, (-6, 1)
6. m = -5, (4, 11)

7. (2,-5) (-3, 10)
8. (0, -9) (3, 0)
9. (-4, 8) (3, 1)

SET

Topic: Graphing linear and exponential functions

Make a graph of the function based on the following information. Add your axes. Choose an appropriate scale and label your graph. Then write the equation of the function.

10. The beginning value is 5 and its value is 3 units smaller at each stage. Equation:	11. The beginning value is 16 and its value is ¼ smaller at each stage. Equation:
12. The beginning value is 1 and its value is 10 times as big at each stage. Equation:	13. The beginning value is -8 and its value is 2 units larger at each stage. Equation:

GO

Topic: Equivalent equations

Prove that the two equations are equivalent by simplifying the equation on the right side of the equal sign. The justification in the example is to help you understand the steps for simplifying. You do NOT need to justify your steps.

Example:	Justification
$2x - 4 = 8 + x - 5x + 6(x - 2)$	Add $x - 5x$ and distribute the 6 over $(x - 2)$
$= 8 - 4x + 6x - 12$	Combine like terms.
$= -4 + 2x$	
$2x - 4 = 2x - 4$	Commutative property of addition

14. $x - 5 = 5x - 7 + 2(3x + 1) - 10x$

15. $6 - 13x = 24 - 10(2x + 8) + 62 + 7x$

16. $14x + 2 = 2x - 3(-4x - 5) - 13$

17. $x + 3 = 6(x + 3) - 5(x + 3)$

18. $4 = 7(2x + 1) - 5x - 3(3x + 1)$

19. $x = 12 + 8x - 3(x + 4) - 4x$

20. Write an expression that equals $(x - 13)$. It must have at least two sets of parentheses and one minus sign. Verify that it is equal to $(x - 13)$.

2.10 Form Follows Function

A Practice Understanding Task

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In our work so far, we have worked with linear and exponential equations in many forms. Some of the forms of equations and their names are:

Linear Functions	
Equation	**Name**
$y = \frac{1}{2}x + 1$	Slope Intercept Form $y = mx + b$, where m is the slope and b is the y-intercept
$y = \frac{1}{2}(x - 4) + 3$	Point Slope Form $y = m(x - x_1) + y_1$, where m is the slope and (x_1, y_1) the coordinates of a point on the line
$f(0) = 1, f(n) = f(n-1) + \frac{1}{2}$	Recursion Formula $f(n) = f(n-1) + \text{D}$, Given an initial value *f(a)* D = constant difference in consecutive terms (used only for discrete functions)

Exponential Functions	
Equation	**Name**
$y = 10(3)^x$	Explicit Form $y = a(b)^x$
$f(0) = 10, f(n+1) = 3f(n)$	Recursion Formula $f(n+1) = Rf(n)$ Given an initial value *f(a)* *R* = constant ratio between consecutive terms (used only for discrete functions)

Knowing a number of different forms for writing and graphing equations is like having a mathematical toolbox. You can select the tool you need for the job, or in this case, the form of the equation that makes the job easier. Any master builder will tell you that the more tools you have the better. In this task, we'll work with our mathematical tools to be sure that we know how to use them all efficiently. As you model the situations in the following problems, think about the important information in the problem and the conclusions that can be drawn from it. Is the function linear or exponential? Does the problem give you the slope, a point, a ratio, a y-intercept? Is the function discrete or continuous? This information helps you to identify the best tools and get to work!

1. In his job selling vacuums, Joe makes \$500 each month plus \$20 for each vacuum he sells. Write the equation that describes Joe's monthly income I as a function of the n, the number of vacuums sold.

   Name the form of the equation you wrote and why you chose to use that form.

   This function is: linear exponential neither (choose one)

   This function is: continuous discrete neither (choose one)

2. Write the equation of the line with a slope of -1 through the point (-2, 5)

   Name the form of the equation you wrote and why you chose to use that form.

   This function is: linear exponential neither (choose one)

   This function is: continuous discrete neither (choose one)

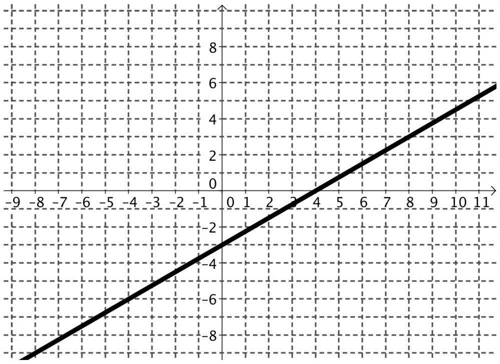
3. Write the equation of the geometric sequence with a constant ratio of 5 and a first term of -3.

Name the form of the equation you wrote and why you chose to use that form.

This function is: linear exponential neither (choose one)

This function is: continuous discrete neither (choose one)

3. Write the equation of the function graphed below:



Name the form of the equation you wrote and why you chose to use that form.

This function is: linear exponential neither (choose one)

This function is: continuous discrete neither (choose one)

4. The population of the resort town of Java Hot Springs in 2003 was estimated to be 35,000 people with an annual rate of increase of about 2.4%. Write the equation that models the number of people in Java Hot Springs, with t = the number of years from 2003?

Name the form of the equation you wrote and why you chose to use that form.

This function is: linear exponential neither (choose one)

This function is: continuous discrete neither (choose one)

5. Yessica’s science fair project involved growing some seeds to see what fertilizer made the seeds grow fastest. One idea she had was to use an energy drink to fertilize the plant. (She thought that if they make people perky, they might have the same effect on plants.) This is the data that shows the growth of the seed each week of the project.

Week	1	2	3	4	5
Height (cm)	1.7	2.9	4.1	5.3	6.5

Write the equation that models the growth of the plant over time.

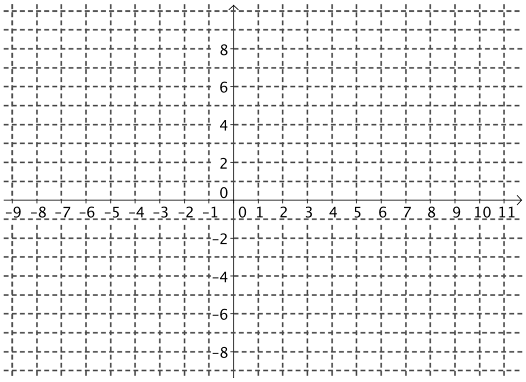
Name the form of the equation you wrote and why you chose to use that form.

This function is: linear exponential neither (choose one)

This function is: continuous discrete neither (choose one)

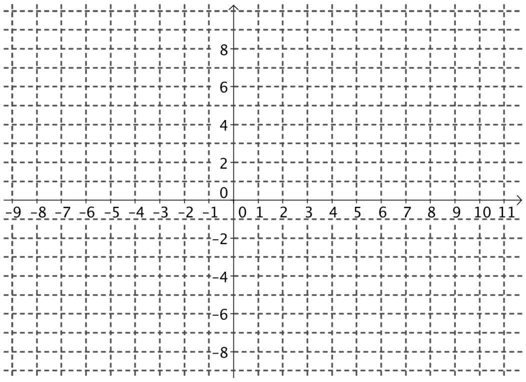
An equation gives us information that we can use to graph the function. Pick out the important information given in each of the following equations and use the information to graph the function.

6. $y = \frac{1}{2}x - 5$



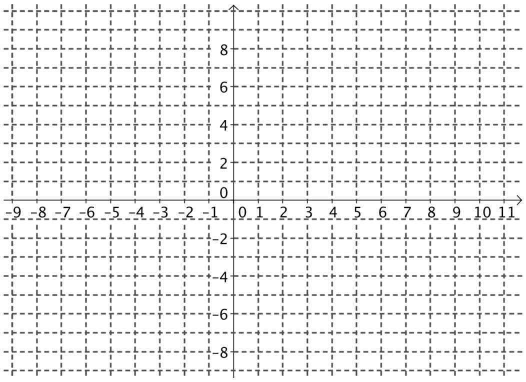
What do you know from the equation that helps you to graph the function?

7. $y = 2^n$



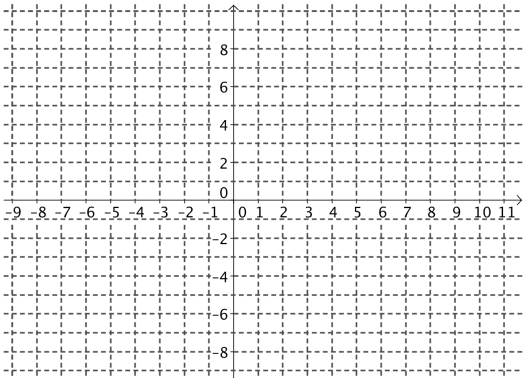
What do you know from the equation that helps you to graph the function?

8. $y = -2(x + 6) + 8$



What do you know from the equation that helps you to graph the function?

9. $f(1) = -5, f(n) = f(n - 1) + 1$



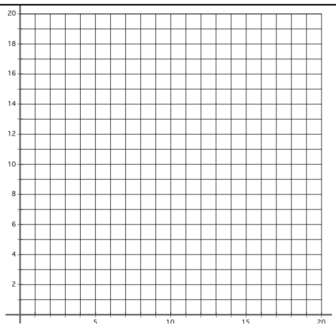
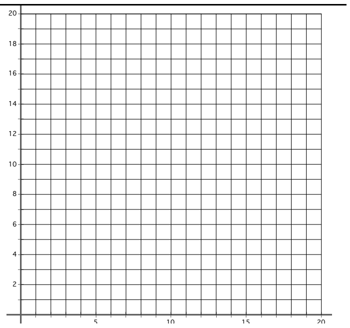
What do you know from the equation that helps you to graph the function?

READY, SET, GO! Name Period Date

READY

Topic: Comparing linear and exponential models.

Comparing different characteristics of each type of function by filling in the cells of each table as completely as possible.

	$y = 4 + 3x$	$y = 4(3^x)$
1. Type of growth		
2. What kind of sequence corresponds to each model?		
3. Make a table of values	x | y	x | y
4. Find the rate of change		
5. Graph each equation. Compare the graphs. What is the same? What is different?		
6. Find the y-intercept for each function.		

7. Find the y-intercepts for the following equations

a) $y = 3x$

b) $y = 3^x$

8. Explain how you can find the y-intercept of a linear equation and how that is different from finding the y-intercept of a geometric equation.

SET

Topic: Efficiency with different forms of linear and exponential functions.

For each exercise or problem below use the given information to determine which of the forms would be the most efficient to use for what is needed. (See task 2.6, Linear: slope-intercept, point-slope form, recursive, Exponential: explicit and recursive forms)

9. Jasmine has been working to save money and wants to have an equation to model the amount of money in her bank account. She has been depositing $175 a month consistently, she doesn't remember how much money she deposited initially, however on her last statement she saw that her account has been open for 10 months and currently has $2475 in it. Create an equation for Jasmine.

Which equation form do you chose? **Write the equation.**

10.

The table below shows the number of rectangles created every time there is a fold made through the center of a paper. Use this table for each question.

Folds	Rectangles
1	2
2	4
3	8
4	16

A. Find the number or rectangles created with 5 folds.

Which equation form do you chose? **Write the equation.**

B. Find the number of rectangles created with 14 folds.

Which equation form do you chose? **Write the equation.**

11. Using a new app that I just downloaded I want to cut back on my calorie intake so that I can lose weight. I currently weigh 90 kilograms, my plan is to lose 1.2 kilograms a week until I reach my goal. How can I make an equation to model my weight loss for the next several weeks.

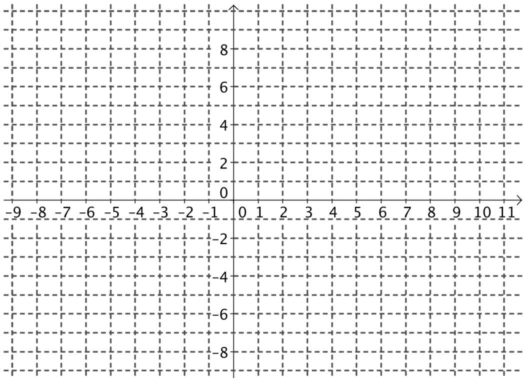
Which equation form do you chose? **Write the equation.**

12. Since Scott started doing his work out plan Janet has been inspired to set her self a goal to do more exercise and walk a little more each day. She has decided to walk 10 meters more every day. On the day 20 she walked 800 meters. How many meters will she walk on day 21? On day 60?

Which equation form do you chose? **Write the equation.**

For each equation provided state what information you see in the equation that will help you graph it, then graph it. Also, use the equation to fill in any four coordinates on the table.

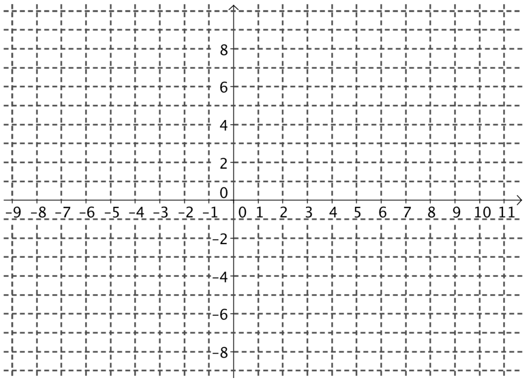
13. $y = \left(\frac{1}{2}\right)^n 8$



What do you know from the equation that helps you to graph the function?

14. $y = 5(x - 2) - 6$

What do you know from the equation that helps you to graph the function?



GO

Topic: Solving one-step equations with justification.

Recall the two properties that help us solve equations.

The Additive property of equality states:
You can add any number to both sides of an equation and the equation will still be true.

The Multiplicative **property of equality states:**
You can multiply any number to both sides of an equation and the equation will still be true.

Solve each equation. Justify your answer by identifying the property(s) you used to get it.

Example 1:

$x - 13 = 7$	Justification
$+13 \quad +13$	additive property of equality
$x + 0 = 20$	addition
$x = 20$	additive identity (You added 0 and got x.)

Example 2:

$5x = 35$	Justification
$\frac{5}{5}x = \frac{35}{5}$	multiplicative property of equality (multiplied by $\frac{1}{5}$)
$1x = 7$	multiplicative identity (A number multiplied by its reciprocal = 1)

15. $3x = 15$ Justification

16. $x - 10 = 2$ Justification

17. $-16 = x + 11$ Justification

18. $6 + x = 10$ Justification

19. $6x = 18$ Justification

20. $-3x = 2$ Justification