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Transforming Mathematics Education

ALGEBRA 1

A Learning Cycle Approach

MODULE 2

Linear & Exponential Functions

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The Mathematics Vision Project

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2.1 Connecting the Dots: Piggies and Pools

A Develop Understanding Task

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1. My little sister, Savannah, is three years old. She has a piggy bank that she wants to fill. She started with five pennies and each day when I come home from school, she is excited when I give her three pennies that are left over from my lunch money. Use a table, a graph, and an equation to create a mathematical model for the number of pennies in the piggy bank on day *n*.

2. Our family has a small pool for relaxing in the summer that holds 1500 gallons of water. I decided to fill the pool for the summer. When I had 5 gallons of water in the pool, I decided that I didn't want to stand outside and watch the pool fill, so I had to figure out how long it would take so that I could leave, but come back to turn off the water at the right time. I checked the flow on the hose and found that it was filling the pool at a rate of 2 gallons every minute. Use a table, a graph, and an equation to create a mathematical model for the number of gallons of water in the pool at *t* minutes.

3. I'm more sophisticated than my little sister so I save my money in a bank account that pays me 3% interest on the money in the account at the end of each month. (If I take my money out before the end of the month, I don't earn any interest for the month.) I started the account with $50 that I got for my birthday. Use a table, a graph, and an equation to create a mathematical model of the amount of money I will have in the account after *m* months.

4. At the end of the summer, I decide to drain the 1500 gallon swimming pool. I noticed that it drains faster when there is more water in the pool. That was interesting to me, so I decided to measure the rate at which it drains. I found that 3% was draining out of the pool every minute. Use a table, a graph, and an equation to create a mathematical model of the gallons of water in the pool at *t* minutes.

5. Compare problems 1 and 3. What similarities do you see? What differences do you notice?

6. Compare problems 1 and 2. What similarities do you see? What differences do you notice?

7. Compare problems 3 and 4. What similarities do you see? What differences do you notice?

2.1 Connecting the Dots: Piggies and Pools – Teacher Notes

A Develop Understanding Task

Special Note to Teachers: Problem number three uses the ideas of compound interest, but in an informal way. Students are expected to draw upon their past work with geometric sequences to create representations that they are familiar with. The formula for compound interest will be developed later in the course.

Purpose: This task builds upon students' experiences with arithmetic and geometric sequences to extend to the broader class of linear and exponential functions with continuous domains. The term "domain" should be introduced and used throughout the whole group discussion. Students are given contextual situations that can be modeled with either discrete and continuous linear functions, or discrete and continuous exponential functions. They are also asked to compare these types of functions using various representations.

New Vocabulary:
Domain
Discrete function
Continuous function

Core Standards Focus:

F.IF.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

F.BF.1: Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps from a calculation from a context.

F.LE.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

F.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Standards for Mathematical Practice

SMP 1 - Make sense of problems and persevere in solving them.
SMP 7 - Look for and make use of structure.

The Teaching Cycle

Launch (Whole Class):

Begin the lesson by helping students to read the four problems and understand the contexts. Since students are already comfortable with arithmetic and geometric sequences and their representations, these questions should be quite familiar, with no need for the teacher to offer a suggested path for solving them. Remind students that their mathematical models should include tables, graphs, and equations.

Explore (Small Group):

As students begin working, circulate among the groups to see that students understand the problems. Problems 1 and 2 are fairly straightforward, but there are possible interpretations that could lead to productive discussions in questions 3 and 4. In problem 3, they will need to consider that the account pays 3% on the principal and whatever interest is in the account from previous months. In problem 4, they will have to consider how to deal with the 3%. Watch for students that try subtracting 3% of the original amount each time (using linear thinking), rather than subtracting 3% of the existing amount of water in the pool at the given time.

Select student work that makes use of tables, graphs and explicit equations. Listen for students that are noticing that the graphs of #1 and #3 should be unconnected points, while #2 and #4 will be connected.

Discuss (Whole Class):

Start the discussion by asking students to present a table, graph, and equation for problem #1. Be sure that the graph is unconnected points. Ask students what they know about the relationship described in #1. They should know it to be an arithmetic sequence.

Next, present a table, graph, and equation for problem #2. This graph should be a solid line. (If no student has a solid line graph, use a graph that is otherwise correct and ask students to consider if it would be possible to have points in between the ones that have been marked, based upon the current context. Once they have discussed that the water is filling continuously, fill in the rest of the line.)

Now ask students to compare the two functions. Create a chart of similarities and differences. Students should notice that they both have a constant rate of change, both are increasing or have a positive slope. They may not have noticed differences, so this is the time to highlight the difference between a continuous context (water filling) and a discrete context where pennies are added a few at a time, with no change in between. Start with how this difference shows in the graph, then proceed to the table. Often, students choose only whole number or integer inputs for their tables. If this is the case, ask them if using some fractions or decimal numbers for inputs would make sense in each of the contexts. Introduce the idea that the inputs for a function are the domain. The input on problem #1 is the number of days. Since money is only put in once a day, then it doesn't make sense to have inputs like ½ or 3.5. That makes the domain the set of whole numbers. (Assuming she started with 5 pennies on day 0). Discuss the domain of the function generated by problem #2. Students should recognize that the domain is time, and that the water level in the pool is increasing continuously as time passes. They should also recognize that the time measurement can't be negative, so the domain in this case is real numbers greater than or equal to 0, assuming that 0 is the time that they started filling the pool.

Proceed with the discussion of #3 and #4 in a similar fashion. Again emphasize that the domain of #3 is whole numbers. Tell students that sequences have whole number domains. Functions that are not discrete are not sequences, therefore, we do not use the terms arithmetic or geometric sequences even though they may exhibit similar growth patterns. More work will be done in the next two tasks to define linear and exponential functions by their patterns of growth, so the emphasis in this task needs to be on the difference between the terms discrete and continuous.

Aligned Ready, Set, Go: ***Linear and Exponential Functions 2.1***

READY, SET, GO! Name Period Date

READY

Topic: Recognizing arithmetic and geometric sequences

Predict the next 2 terms in the sequence. State whether the sequence is arithmetic, geometric, or neither. Justify your answer.

1. 4 , -20 , 100 , -500 , . . .

2. 3 , 5 , 8 , 12 , . . .

3. 64 , 48 , 36 , 27 , . . .

4. 1.5 , 0.75 , 0 , -0.75 , . . .

5. 40 , 10 , $\frac{5}{2}$, $\frac{5}{8}$, ...

6. 1, 11, 111, 1111, . . .

7. -3.6, -5.4, -8.1, -12.15, . . .

8. -64, -47, -30, -13, . . .

9. Create a predictable sequence of at least 4 numbers that is NOT arithmetic or geometric.

SET

Topic: Discrete and continuous relationships

Identify whether the following statements represent a *discrete* or a *continuous* relationship.

10. The hair on your head grows ½ inch per month.

11. For every ton of paper that is recycled, 17 trees are saved.

12. Approximately 3.24 billion gallons of water flow over Niagara Falls daily.

13. The average person laughs 15 times per day.

14. The city of Buenos Aires adds 6,000 tons of trash to its landfills every day.

15. During the Great Depression, stock market prices fell 75%.

GO

Topic: Solving one-step equations

Either find or use the unit rate for each of the questions below.

16. Apples are on sale at the market 4 pounds for \$2.00. What is the price (in cents) for one pound?

17. Three apples weigh about a pound. About how much would one apple cost?
(Round to the nearest cent.)

18. One dozen eggs cost \$1.98. How much does 1 egg cost? (Round to the nearest cent.)

19. One dozen eggs cost \$1.98. If the charge at the register for only eggs, without tax, was \$11.88, how many dozen were purchased?

20. Best Buy Shoes had a back to school special. The total bill for four pairs of shoes came to \$69.24 (before tax.) What was the average price for each pair of shoes?

21. If you only purchased 1 pair of shoes at Best Buy Shoes instead of the four described in problem 20, how much would you have paid, based on the average price?

Solve for x. Show your work.

22. $6x = 72$

23. $4x = 200$

24. $3x = 50$

25. $12x = 25.80$

26. $\frac{1}{2}x = 17.31$

27. $4x = 69.24$

28. $12x = 198$

29. $1.98x = 11.88$

30. $\frac{1}{4}x = 2$

31. Some of the problems 22 – 30 could represent the work you did to answer questions 16 – 21. Write the number of the equation next to the story it represents.

2.2 Shh! Please Be Discreet (Discrete)!

A Solidify Understanding Task

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1. The Library of Congress in Washington D.C. is considered the largest library in the world. They often receive boxes of books to be added to their collection. Since books can be quite heavy, they aren't shipped in big boxes. If, on average, each box contains about 8 books, how many books are received by the library in 6 boxes, 10 boxes, or *n* boxes?
   a. Use a table, a graph, and an equation to model this situation.

   b. Identify the domain of the function.

2. Many of the books at the Library of Congress are electronic. If about 13 e-books can be downloaded onto the computer each hour, how many e-books can be added to the library in 3 hours, 5 hours, or *n* hours (assuming that the computer memory is not limited)?
   a. Use a table, a graph, and an equation to model this situation.

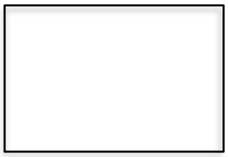
   b. Identify the domain of the function.

3. The librarians work to keep the library orderly and put books back into their proper places after they have been used. If a librarian can sort and shelve 3 books in a minute, how many books does that librarian take care of in 3 hours, 5 hours, or n hours? Use a table, a graph, and an equation to model this situation.

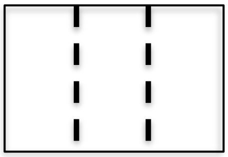
4. Would it make sense in any of these situations for there to be a time when 32.5 books had been shipped, downloaded into the computer or placed on the shelf?

5. Which of these situations (in problems 1-3) represent a discrete function and which represent a continuous function? Justify your answer.

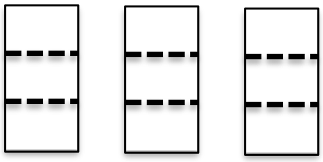
6. A giant piece of paper is cut into three equal pieces and then each of those is cut into three equal pieces and so forth. How many papers will there be after a round of 10 cuts? 20 cuts? n cuts?



Zero Cuts



One Cut



Two Cuts

a. Use a table, a graph, and an equation to model this situation.

b. Identify the domain of the function.

c. Would it make sense to look for the number of pieces of paper at 5.2 cuts? Why?

d. Would it make sense to look for the number of cuts it takes to make 53.6 papers? Why?

7. Medicine taken by a patient breaks down in the patient's blood stream and dissipates out of the patient's system. Suppose a dose of 60 milligrams of anti-parasite medicine is given to a dog and the medicine breaks down such that 20% of the medicine becomes ineffective every hour. How much of the 60 milligram dose is still active in the dog's bloodstream after 3 hours, after 4.25 hours, after *n* hours?

a. Use a table, a graph, and an equation to model this situation.

b. Identify the domain of the function.

c. Would it make sense to look for an amount of active medicine at 3.8 hours? Why?

d. Would it make sense to look for when there is 35 milligrams of medicine? Why?

8. Which of the functions modeled in #6 and #7 are discrete and which are continuous? Why?

9. What needs to be considered when looking at a situation or context and deciding if it fits best with a discrete or continuous model?

10. Describe the differences in each representation (table, graph, and equation) for discrete and continuous functions.

11. Which of the functions modeled in this task are linear? Which are exponential? Why?

2.2 Shh! Please Be Discreet (Discrete)! – Teacher Notes

A Solidify Understanding Task

Purpose:

The purpose of this task is for students to explicitly consider when a discrete or continuous model is appropriate for a given context. The situations in this task are designed to contrast discrete and continuous models for both linear and exponential functions. The task also provides opportunity for students to model with mathematics by connecting the type of change, either linear or exponential with the nature of that change, either discrete or continuous.

Core Standards Focus:

F-IF.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

F-BF.1a: Determine an explicit expression, a recursive process, or steps from a calculation from a context.

F.LE.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

F.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Standards for Mathematical Practice

SMP 4 - Model with mathematics.
SMP 6 - Attend to precision.

The Teaching Cycle

Launch (Whole Class):

Begin class by reviewing previous work on the idea that the domain of a function is the set of input values or independent variable. Introduce the essential question and launch the task by ensuring that students understand the context and know what they are expected to do.

Explore (Small Group):

As students are working through the task, they may need support in reading the problems and understanding the contexts, which is critical in considering whether the models should be continuous or discrete. Listen for students articulating the idea that the change is happening gradually over the interval (continuous) or all at once (discrete). Also listen for connections that can be shared during the whole group discussion of the domain and the type of model. For instance, students may argue in #3 that since the input variable is time, the model should be continuous. Others may argue that the books aren't actually placed on the shelf continuously, so the model should be discrete. These are the type of arguments that should be considered by the class to help shape the discussion of discrete and continuous.

Discuss (Whole Class):

Begin the discussion with comparing the graphs and equations for problems #1 and #2. Ask students if the points on the graph of #1 should be connected. Ask for justifications such as the idea that the number of books increases by a given amount for each box, not building up gradually. Follow by having a student share the graph and equation for #2 and ask if the points on the graph should be connected. In this case, the graph should be continuous because parts of the books are being downloaded as the time continues (the progress bar for a download is familiar to most students). Ask students how they can generally decide if the points should be connected or not.

Emphasize the terms discrete and continuous for these two types of models. Consider the domains for the two functions #1 and #2. How are the domains of these two functions different? Why?

Move the discussion to models of #3. Have a student share their model (either continuous or discrete) and ask the class for arguments as to whether or not the model should be discrete or continuous. Be sure that students discuss the idea that the domain in this case is continuous, but the outputs are not because the librarians can't shelve half of a book. This is the type of situation that is often modeled as a continuous function for the sake of simplicity. Emphasize that whenever we model a real situation, we make assumptions that should be made thoughtfully and explicitly.

The equations that are written for discrete and continuous equations are also worthy of discussion. Students should consider whether or not it makes sense to write a recursive equation for a continuous function. How would such a formula work on a continuous domain? Recursive formulas are for sequences, which are defined on whole number domains. It is difficult to determine whether an explicit equation is continuous or discrete without a particular context. It is common in the mathematical community to assume that x is a continuous variable in the absence of other information. The variable used for whole number or natural number domains is often n. This may be a useful convention for the class.

Wrap up the discussion by comparing the discrete function in #1 with the discrete function in #6. How are they alike? How are they different? Repeat by comparing the continuous functions in #2 and #7. How are they alike? How are they different?

Aligned Ready, Set, Go: ***Linear and Exponential Functions 2.2***

READY, SET, GO! Name Period Date

READY

Topic: Comparing rates of change in linear situations.

State which situation has the greatest rate of change

1. The amount of stretch in a short bungee cord stretches 6 inches when stretched by a 3 pound weight. A slinky stretches 3 feet when stretched by a 1 pound weight.

2. A sunflower that grows 2 inches every day or an amaryllis that grows 18 inches in one week.

3. Pumping 25 gallons of gas into a truck in 3 minutes or filling a bathtub with 40 gallons of water in 5 minutes.

4. Riding a bike 10 miles in 1 hour or jogging 3 miles in 24 minutes.

SET

Topic: Discrete and continuous relationships

Identify whether the following items best fit with a *__discrete__* or a *__continuous__* model. Then determine whether it is a *__linear__ (arithmetic)* or *__exponential__ (geometric)* relationship that is being described.

5. The freeway construction crew pours 300 ft of concrete in a day.

6. For every hour that passes, the amount of area infected by the bacteria doubles.

7. To meet the demands placed on them the brick layers have started laying 5% more bricks each day.

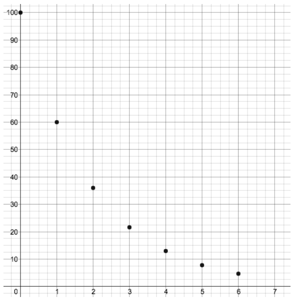
8. The average person takes 10,000 steps in a day.

9. The city of Buenos Aires has been adding 8% to its population every year.

10. At the headwaters of the Mississippi River the water flows at a surface rate of 1.2 miles per hour.

11. a. $f(n) = f(n-1) + 3; f(1) = 5$

b.



c. $g(x) = 2^x(7)$

GO

Topic: Solving one-step equations

Solve the following equations. Remember that what you do to one side of the equation must also be done to the other side. (Show your work, even if you can do these in your head.)

Example: Solve for x . $1x + 7 = 23$ *Add* -7 *to both sides of the equation.*

$$\begin{array}{l} 1x + 7 = 23 \\ \quad\; -7 = -7 \\ \hline 1x + 0 = 16 \\ \textit{Therefore } 1x = 16 \end{array}$$

Example: Solve for x. $9x = 63$ *Multiply both sides of the equation by* $\frac{1}{9}$.

$$\begin{gathered} 9x = 63 \\ \left(\frac{1}{9}\right)9x = \left(\frac{1}{9}\right)63 \\ \left(\frac{9}{9}\right)x = \frac{63}{9} \\ 1x = 7 \end{gathered}$$

Note that multiplying by $\frac{1}{9}$ *gives the same result as dividing everything by 9.*

12. $1x + 16 = 36$

13. $1x - 13 = 10$

14. $1x - 8 = -3$

15. $8x = 56$

16. $-11x = 88$

17. $425x = 850$

18. $\frac{1}{6}x = 10$

19. $-\frac{4}{7}x = -1$

20. $\frac{3}{4}x = -9$

2.3 Linear, Exponential or Neither?

A Practice Understanding Task

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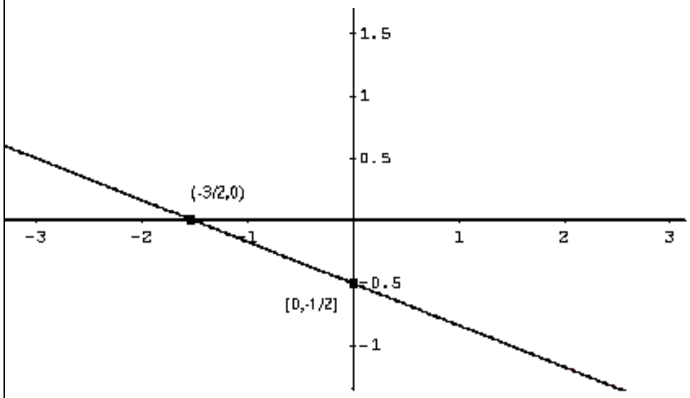
For each representation of a function, decide if the function is linear, exponential, or neither. **Give at least 2 reasons for your answer.**

1.



Linear Exponential Neither

Why?

2.

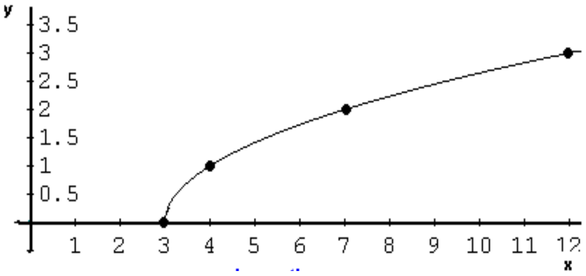
Tennis Tournament

Rounds	1	2	3	4	5
Number of Players left	64	32	16	8	4

There are 4 players remaining after 5 rounds.

Linear Exponential Neither

Why?

3. $y = 4x$	Linear Exponential Neither Why?
4. This function is decreasing at a constant rate. .	Linear Exponential Neither Why?
5. 	Linear Exponential Neither Why?

6.
A person's height as a function of a person's age (from age 0 to 100).

Linear Exponential Neither

Why?

7.

$$-3x = 4y + 7$$

Linear Exponential Neither

Why?

8.

x	***y***
-2	23
0	5
2	-13
4	-31
6	-49

Linear Exponential Neither

Why?

9.

Height in Inches	Shoe Size
62	6
74	13
70	9
67	11
53	4
58	7

Linear Exponential Neither

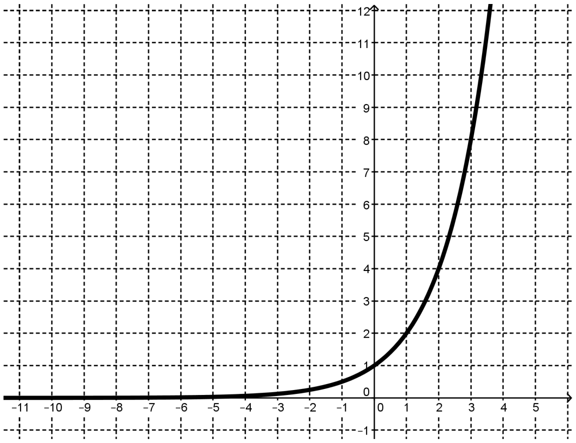
Why?

10.
The number of cell phone users in Centerville as a function of years, if the number of users is increasing by 75% each year.

Linear Exponential Neither

Why?

11.



Linear Exponential Neither

Why?

12. The time it takes you to get to work as a function the speed at which you drive.	Linear Exponential Neither Why?
13. $y = 7x^2$	Linear Exponential Neither Why?
14. Each point on the graph is exactly 1/3 of the previous point.	Linear Exponential Neither Why?

15. $f(1) = 7, f(2) = 7, f(n) = f(n-1) + f(n-2)$	Linear Exponential Neither Why?
16. $f(1) = 1, f(n) = \frac{2}{3} f(n-1)$	Linear Exponential Neither Why?

2.3 Linear, Exponential or Neither? – Teacher Notes

A Practice Understanding Task

Purpose:

The purpose of this task is to develop fluency in determining if a function is linear or exponential using various representations. The task also provides opportunities for discussion of features of the functions based upon the representation given.

Core Standards Focus:

F-LE1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

F-LE2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of the relation, or two input-output pairs (include reading these from a table).

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Standards for Mathematical Practice

SMP 6 – Attend to precision.

SMP 7 – Look for and make use of structure.

The Teaching Cycle:

Launch (Whole Class):

Refer the class to the linear and exponential charts made in the previous task. In this task they will be looking at a number of functions, some linear, some exponential, some neither. They need to identify what kind of function is shown in each problem and provide two reasons for their answers. One reason may be fairly easy, based upon the chart, the second one will require them to stretch a little.

Explore (Small Group):

During the small group work, listen for problems that are generating controversy. If students feel that a particular problem is too vague, ask them what information would be necessary for them to decide and why that information is important.

Discuss (Whole Class):

Start the discussion by going through each problem and asking a group to say how they categorized it and why. After each problem, ask if there was any disagreement or if another group could add another reason to support the category. If there is disagreement, ask students to present their arguments more formally and add at least one representation to support their claim. The emphasis of the discussion should be to recognize the constant rate of change to define a linear function and the equal factors over equal intervals that define an exponential function. Be sure that the class discussion includes a table, a graph, an equation, and a description or story context for both linear and exponential functions.

Aligned Ready, Set, Go: ***Linear and Exponential Functions 2.3***

READY, SET, GO! Name Period Date

READY

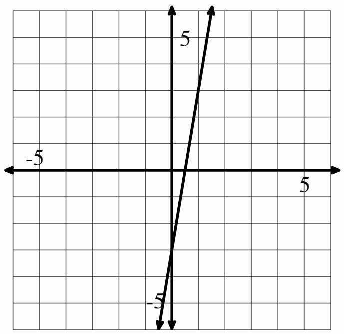
Topic: Comparing rates of change in both linear and exponential situations.

Identify whether situation "a" or situation "b" has a greater rate of change.

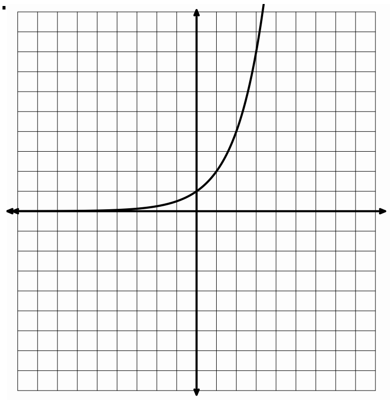
1. a.

x	y
-10	-48
-9	-43
-8	-38
-7	-33

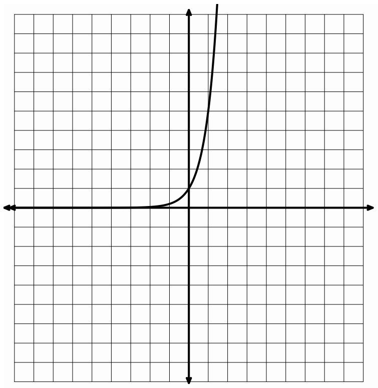
b.



2. a.



b.



3. a. Lee has $25 withheld each week from his salary to pay for his subway pass.

b. Jose owes his brother $50. He has promised to pay half of what he owes each week until the debt is paid.

4. a.

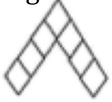
x	6	10	14	18
y	13	15	17	19

b. The number of rhombi in each shape.

Figure 1

Figure 2

Figure 3



5. a. $y = 2(5)^x$

b. In the children's book, *The Magic Pot,* every time you put one object into the pot, two of the same object come out. Imagine that you have 5 magic pots.

SET

Topic: Recognizing linear and exponential functions.

Based on each of the given representations of a function determine if it is linear, exponential or neither.

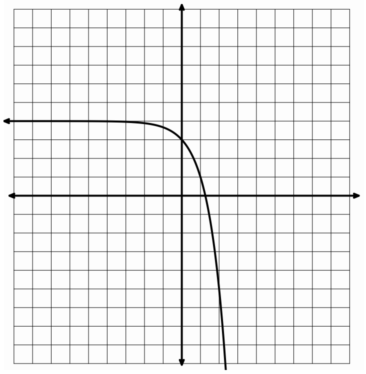
6. The population of a town is decreasing at a rate of 1.5% per year.

7. Joan earns a salary of $30,000 per year plus a 4.25% commission on sales.

8. 3x +4y = -3

9. The number of gifts received each day of "The 12 Days of Christmas" as a function of the day. ("On the 4th day of Christmas my true love gave to me, 4 calling birds, 3 French hens, 2 turtledoves, and a partridge in a pear tree.")

10.



11.

Side of a square	**Area of a square**
1 inch	1 in^2
2 inches	4 in^2
3 inches	9 in^2
4 inches	16 in^2

GO

Topic: Geometric means

For each geometric sequence below, find the missing terms in the sequence.

12.

x	1	2	3	4	5
y	2				162

13.

x	1	2	3	4	5
y	1/9			-3	

14.

x	1	2	3	4	5
y	10				0.625

15.

x	1	2	3	4	5
y	g				gz^4

16.

x	1	2	3	4	5
y	-3				-243

Find the rate of change (slope) in each of the exercises below.

17.

x	g(x)
-5	11
-3	4
-2	0.5
0	-6

18.

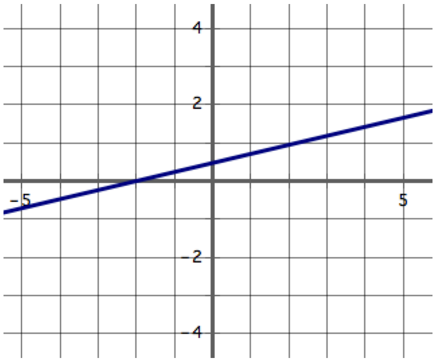
t	h(t)
3	13
8	23
18	43
23	53

19.

n	f(n)
-7	20
-5	24
-1	32
2	38

20. (2, 5) (8, 29)

21.



22. (-3, 7) (8, 29)

2.4 The In-Betweeners

A Develop Understanding Task

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Now that you've seen that there are contexts for continuous exponential functions, it's a good idea to start thinking about the numbers that fill in between the values like 2^2 and 2^3 in an exponential function. These numbers are actually pretty interesting, so we're going to do some exploring in this task to see what we can find out about these "in-betweeners".

Let's begin in a familiar place:

1. Complete the following table.

x	0	1	2	3	4
$f(x) = 4 \cdot 2^x$	4				

2. Plot these points on the graph at the end of this task, and sketch the graph of $f(x)$.

Let's say we want to create a table with more entries, maybe with a point halfway between each of the points in the table above. There are a couple of ways that we might think about it. We'll begin by letting our friend Travis explain his method.

Travis makes the following claim:

"If the function doubles each time x goes up by 1, then half that growth occurs between 0 and ½ and the other half occurs between ½ and 1. So for example, we can find the output at x = ½ by finding the average of the outputs at x = 0 and x = 1."

3. Fill in the parts of the table below that you've already computed, and then decide how you might use Travis' strategy to fill in the missing data. Also plot Travis' data on the graph at the end of the task.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
$f(x)$	4								

4. Comment on Travis' idea. How does it compare to the table generated in problem 1? For what kind of function would this reasoning work?

Miriam suggests they should fill in the data in the table in the following way:

"I noticed that the function increases by the same factor each time x goes up 1, and I think this is like what we did in *Geometric Meanies*. It seems like this property should hold over each half-interval as well."

5. Fill in the parts of the table below that you've already computed in problem 1, and then decide how you might use Miriam's new strategy to fill in the missing data. As in the table in problem 1, each entry should be multiplied by some constant factor to get the next entry, and that factor should produce the same results as those already in recorded in the table. Use this constant factor to complete the table. Also plot Miriam's data on the graph at the end of this task.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
$f(x)$	4								

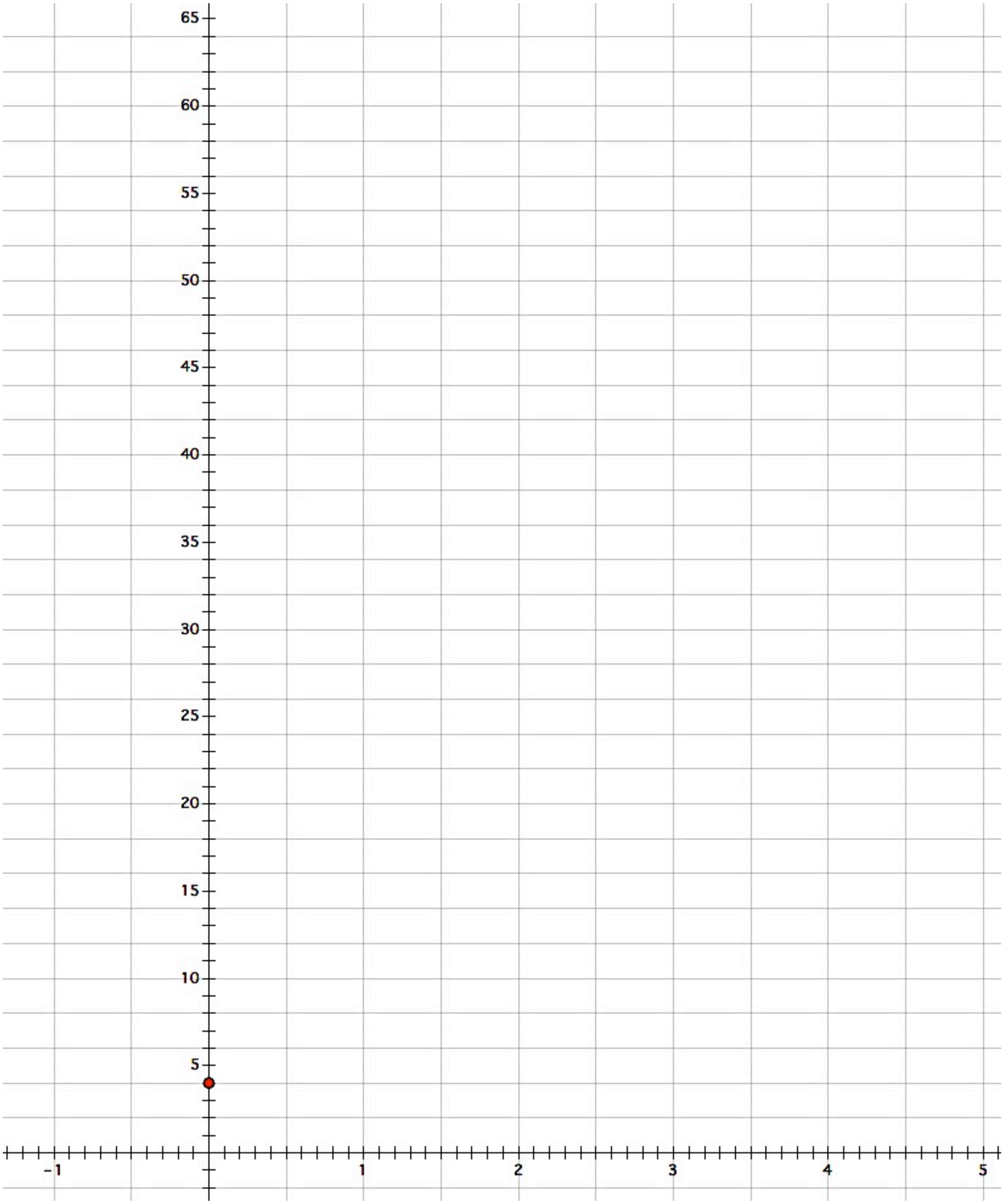
6. What if Miriam wanted to find values for the function every third of the interval instead of every half? What constant factor would she use for every third of an interval to be consistent with the function doubling as x increases by 1. Use this multiplier to complete the following table.

x	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2	$\frac{7}{3}$	$\frac{8}{3}$	3
$f(x)$	4									

7. What number did you use as a constant factor to complete the table in problem 4?

8. What number did you use as a constant factor to complete the table in problem 5?

9. Give a detailed description of how you would estimate the output value $f(x)$, for $x = {}^{5}\!/_{3}$.


65
60
55
50
45
40
35
30
25
20
15
10
5
-1
1
2
3
4
5

2.4 The In-Betweeners – Teacher Notes

A Develop Understanding Task

Purpose: This task surfaces the idea that data exists on the intervals between the whole number increments of a continuously increasing exponential function. Students will consider potential strategies for calculating this data at equal fractional increments so that the multiplicative pattern inherent in exponential functions is maintained. Students who are familiar with the work of such tasks as *Geometric Meanies* from the MVP Secondary Math I curriculum may choose to use a radical, such as $\sqrt{2}$ as the factor to multiply each entry in the table to get the next entry when the data is spaced in $\frac{1}{2}$ units increments, and $\sqrt[3]{2}$ when the data is spaced in $\frac{1}{3}$ unit increments. The task provides students with an opportunity to connect these multipliers with the exponents that represent the increments of x in the exponential function, pointing toward the definition for rational exponents, $b^{\frac{1}{n}} = \sqrt[n]{b}$.

Core Standards Focus:

N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

Related Standards: A.SSE.1

Standards for Mathematical Practice:

SMP 2 – Reason abstractly and quantitatively

SMP 7 – Look for and make use of structure

SMP 8 – Look for express regularity in repeated reasoning

The Teaching Cycle:

Launch (Whole Class):

As part of the launch, ask students to complete the first table and plot their data on the graph at the end of the task. This work should be easy for students to do, since they are familiar with using exponential equations like $P = 4(2)^t$ to represent exponential growth over whole number increments of time. Present the new situation—Travis and Miriam want data points at smaller increments of the input than at whole number increments—and set students to work to consider Travis' and Miriam's proposed methods for producing this data.

Explore (Small Group):

Listen for students who can argue that Travis' strategy implies that the points at the intervals in-between the whole number data points are being approximated by straight line segments, rather than by the smooth curve suggested by the data already recorded. Miriam's approach acknowledges that the curve is produced by multiplying each entry in the table by a constant factor (2, in the case of the whole number increments) to get the next data entry—a multiplicative recursive approach. Consequently, students need to find a factor that can be used over and over again to produce the values at half or third interval increments. Students may guess and check such a factor (we know it has to be larger than 1 since the function is growing, but less than 2 since the function can't double in less than a complete interval of 1 unit). Allow students to use a guess and check strategy, if that approach surfaces. Other students may recall similar work with geometric means from the task *Geometric Meanies* in the MVP Secondary Math I curriculum. This might lead them to propose $\sqrt{2}$ and $\sqrt[3]{2}$ as the common ratios for the half-hour and third-hour increments, respectively. Since rational exponents have not been introduced to students it is not anticipated that this idea will come up in the exploration. If it does, select students to present their reasoning during the whole class discussion.

Discuss (Whole Class):

Focus the discussion on the table of data for the half interval increments. Ask students what they think about Travis' strategy. Press students to acknowledge that since the function is obviously not linear, we would not suspect it to be linear on smaller subintervals of a unit. Ask students to predict when most of the growth might occur during the first interval—during the first half of the interval or during the second half of the interval—and justify their reasons for thinking so. Listen for arguments that suggest that the growth is constantly increasing, suggesting that as time passes the growth should be greater when examining equal intervals of x.

Move on to Miriam's approach and have students present the factor they used as the constant multiplier. If available in the student work, have a student present first who used a guess and test strategy, perhaps closing in on a decimal number such as 1.412 as a factor that produces half-interval values that are consistent with the full interval values. Then select a student to present who used $\sqrt{2}$ as the factor. Press for a justification such as, "We need a number that, when multiplied by itself, gives us 2, which fits the definition of a square root." Next, turn students' attention to the graph and their equation, and asked if anybody tried to plot a point on the graph where $x = \frac{1}{2}$ by substituting ½ into the exponential function for x? Point out that the calculator yields a result that is consistent with the value obtained when multiplying by the square root of 2, that is, $4 \cdot \sqrt{2} = 4(2)^{1/2}$ based on evidence obtained from a calculator, and from our reasoning about this situation.

Depending on your class you might want to end the discussion at this point, or continue to pursue this new type of exponent. The next task, *Half Interested*, continues to pursue this idea of using a fraction as an exponent. You might choose to continue this discussion at this point in time if there are students who can argue that $2^{1/2}$ can reasonably be defined $\sqrt{2}$, since by properties of exponents $2^{1/2} \cdot 2^{1/2} = 2^1 = 2$ in the same way that $\sqrt{2} \cdot \sqrt{2} = 2$. You can return to this idea as part of discussion of the next task if students have not yet surfaced this idea in their thinking.

Aligned Ready, Set, Go: Linear and Exponential Functions 2.4

READY, SET, GO! Name Period Date

READY

Topic: Comparing additive and multiplicative patterns.

The sequences below exemplify either an additive (arithmetic) or a multiplicative (geometric) pattern. Identify the type of sequence, fill in the missing values on the table and write an equation.

1.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	2	4	8	16	32	a.	b.	c.

d. Type of Sequence: e. Equation:

2.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	66	50	34	18	a.	b.	c.	d.

e. Type of Sequence: f. Equation:

3.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	-3	9	-27	81	a.	b.	c.	d.

e. Type of Sequence: f. Equation:

4.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	160	80	40	20	a.	b.	c.	d.

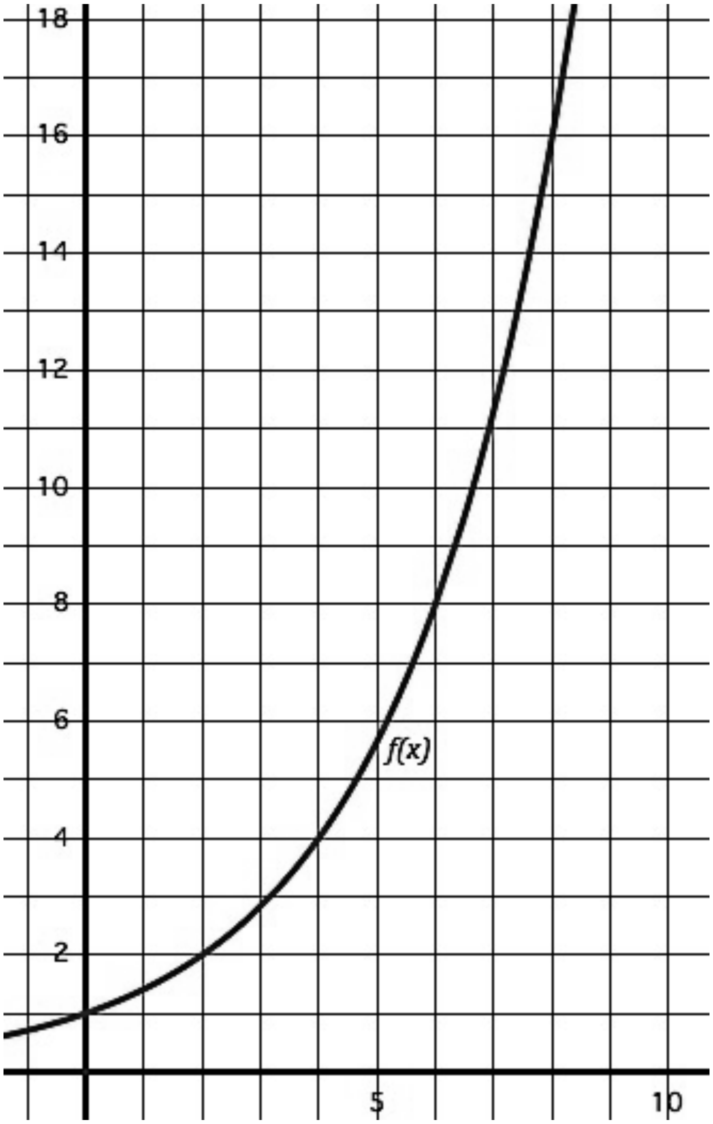
e. Type of Sequence: f. Equation:

5.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	-9	-2	5	12	a.	b.	c.	d.

e. Type of Sequence: f. Equation:

Use the graph of the function to find the desired values of the function. Also create an explicit equation for the function.



6. Find the value of *f(2)*

7. Find where *f(x) = 4*

8. Find the value of *f(6)*

9. Find where *f(x) = 16*

10. What do you notice about the way that inputs and outputs for this function relate? (Create an in-out table if you need to.)

11. What is the explicit equation for this function?

SET

Topic: Evaluate expressions with rational exponents.

Fill in the missing values of the table based on the growth that is described.

12. The growth in the table is triple at each whole year.

Years	0	$^1/_2$	1	$^3/_2$	2	$^5/_2$	3	$^7/_2$	4
bacteria	2		6						

13. The growth in the table is triple at each whole year.

Years	0	$^1/_3$	$^2/_3$	1	$^4/_3$	$^5/_3$	2	$^7/_3$	$^8/_3$
bacteria	2			6					

14. The values in the table grow by a factor of four at each whole year.

Years	0	$^1/_2$	1	$^3/_2$	2	$^5/_2$	3	$^7/_2$	4
bacteria									

GO

Topic: Simplifying Exponents

Simplify the following expressions using exponent rules and relationships. Write your answers in exponential form. (For example: $2^2 \cdot 2^5 = 2^7$)

15. $3^2 \cdot 3^5$

16. $\frac{5^3}{5^2}$

17. 2^{-5}

18. 17^0

19. $\frac{7^5}{7^2} \cdot \frac{7^3}{7^4}$

20. $\frac{3^{-2} \cdot 3^5}{3^7}$

2.5 Half Interested

A Solidify Understanding Task

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Carlos and Clarita, the Martinez twins, have run a summer business every year for the past five years. Their first business, a neighborhood lemonade stand, earned a small profit that their father insisted they deposit in a savings account at the local bank. When the Martinez family moved a few months later, the twins decided to leave the money in the bank where it has been earning 5% interest annually. Carlos was reminded of the money when he found the annual bank statement they had received in the mail.

"Remember how Dad said we could withdraw this money from the bank when we are twenty years old," Carlos said to Clarita. "We have $382.88 in the account now. I wonder how much that will be five years from now?"

1. Given the facts listed above, how can the twins figure out how much the account will be worth five years from now when they are twenty years old? Describe your strategy and calculate the account balance.

2. Carlos calculates the value of the account one year at a time. He has just finished calculating the value of the account for the first four years. Describe how he can find the next year's balance, and record that value in the table.

year	amount
0	382.88
1	402.02
2	422.12
3	443.23
4	465.39
5	

3. Clarita thinks Carlos is silly calculating the value of the account one year at a time, and says that he could have written a formula for the n^{th} year and then evaluated his formula when $n = 5$. Write Clarita's formula for the n^{th} year and use it to find the account balance at the end of year 5.

4. Carlos was surprised that Clarita's formula gave the same account balance as his year-by-year strategy. Explain, in a way that would convince Carlos, why this is so.

"I can't remember how much money we earned that summer," said Carlos. "I wonder if we can figure out how much we deposited in the account five years ago, knowing the account balance now?"

5. Carlos continued to use his strategy to extend his table year-by-year back five years. Explain what you think Carlos is doing to find his table values one year at a time, and continue filling in the table until you get to -5, which Carlos uses to represent "five years ago".

6. Clarita evaluated her formula for *n* = -5. Again Carlos is surprised that they get the same results. Explain why Clarita's method works.

year	amount
-5	
-4	
-3	
-2	
-1	364.65
0	382.88
1	402.02
2	422.12
3	443.23
4	465.39
5	

Clarita doesn't think leaving the money in the bank for another five years is such a great idea, and suggests that they invest the money in their next summer business, *Curbside Rivalry* (which, for now, they are keeping top secret from everyone, including their friends). "We'll have some start up costs, and this will pay for them without having to withdraw money from our other accounts."

Carlos remarked, "But we'll be withdrawing our money halfway through the year. Do you think we'll lose out on this year's interest?"

"No, they'll pay us a half-year portion of our interest," replied Clarita.

"But how much will that be?" asked Carlos.

7. Calculate the account balance and how much interest you think Carlos and Clarita should be paid if they withdraw their money ½ year from now. Remember that they currently have \$382.88 in the account, and that they earn 5% annually. Describe your strategy.

Carlos used the following strategy: He calculated how much interest they should be paid for a full year, found half of that, and added that amount to the current account balance.

Clarita used this strategy: She substituted ½ for *n* in the formula $A = 382.88(1.05)^n$ and recorded this as the account balance.

8. This time Carlos and Clarita didn't get the same result. Whose method do you agree with and why?

Clarita is trying to convince Carlos that her method is correct. "Exponential rules are multiplicative, not additive. Look back at your table. We will earn \$82.51 in interest during the next four years. If your method works we should be able to take half of that amount, add it to the amount we have now, and get the account balance we should have in two years, but it isn't the same."

9. Carry out the computations that Clarita suggested and compare the result for year 2 using this strategy as opposed to the strategy Carlos originally used to fill out the table.

10. The points from Carlos' table (see question 2) have been plotted on the graph at the end of this task, along with Clarita's function. Plot the value you calculated in question 9 on this same graph. What does the graph reveal about the differences in Carlos' two strategies?

11. Now plot Clarita's and Carlos' values for ½ year (see question 8) on this same graph.

"Your data point seems to fit the shape of the graph better than mine," Carlos conceded, "but I don't understand how we can use ½ as an exponent. How does that find the correct factor we need to multiply by? In your formula, writing $(1.05)^5$ means multiply by 1.05 five times, and writing $(1.05)^{-5}$ means divide by 1.05 five times, but what does $(1.05)^{1/2}$ mean?"

Clarita wasn't quite sure how to answer Carlos' question, but she had some questions of her own. She decided to jot them down, including Carlos' question:

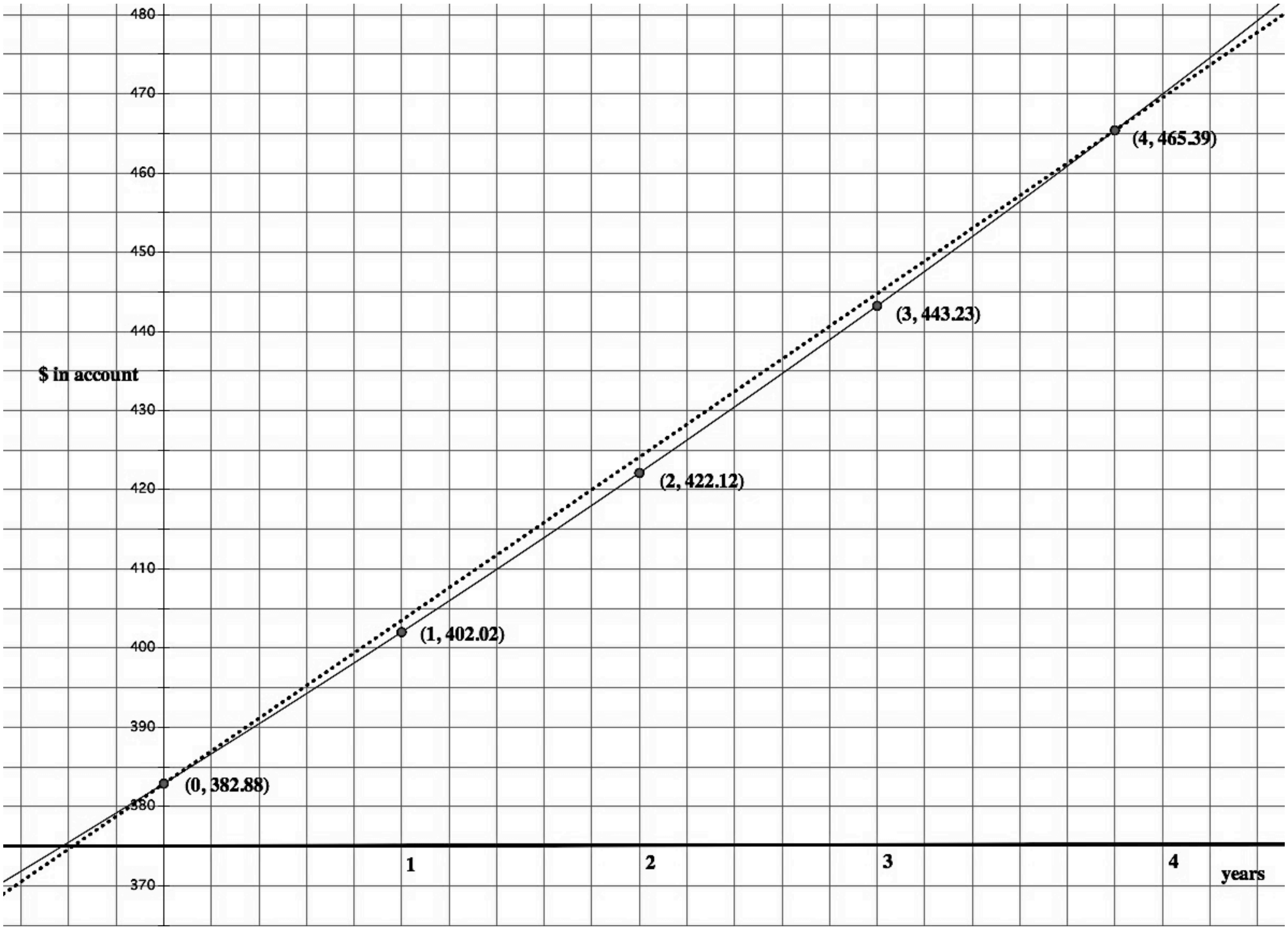
- What numerical amount do we multiply by when we use $(1.05)^{1/2}$ as a factor?
- What happens if we multiply by $(1.05)^{1/2}$ and then multiply the result by $(1.05)^{1/2}$ again? Shouldn't that be a full year's worth of interest? Is it?
- If multiplying by $(1.05)^{1/2} \cdot (1.05)^{1/2}$ is the same as multiplying by 1.05, what does that suggest about the value of $(1.05)^{1/2}$?

12. Answer each of Clarita's questions listed above as best as you can.

As Carlos is reflecting on this work, Clarita notices the date on the bank statement that started this whole conversation. "This bank statement is three months old!" she exclaims. "That means the bank will owe us ¾ of a year's interest."

"So how much interest will the bank owe us then?" asked Carlos.

13. Find as many ways as you can to answer Carlos' question: How much will their account be worth in ¾ of a year (nine months) if it earns 5% annually and is currently worth $382.88?



2.5 Half Interested – Teacher Notes

A Solidify Understanding Task

Purpose: In the context of predicting the account balance at different times for an account earning 5% interest annually, students examine the role of positive and negative integer exponents as well as the need for rational exponents. Tables, graphs and reasoning based on the definition of radicals and rules of exponents are used to attach meaning to using fractions such as ½ or ¾ as exponents.

Core Standards Focus:

N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

Related Standards: A.SSE.1

Standards for Mathematical Practice:

- **SMP 1 – Make sense of problems and persevere in solving them**
- **SMP 2 – Reason abstractly and quantitatively**
- **SMP 3 – Construct viable arguments and critique the reasoning of others**
- **SMP 4 – Model with mathematics**
- **SMP 6 – Attend to precision**
- **SMP 7 – Look for and make use of structure**

The Teaching Cycle:

Launch (Whole Class):

Read through the context at the beginning of the task, making sure students are aware that we are working with an exponential context that has both a future and a past. This will allow positive, negative and rational exponents to make sense in the context. Point out that Carlos is using a recursive approach to extend the exponential growth, while Clarita is using an explicit function. In this module students are learning that a recursive approach is appropriate for discrete data, but not for continuous data. This issue will surface for Carlos as he tries to reason through the data that exists between the whole number increments of time. Point out to students that the work in this task is similar to work in the previous task, and then set them to work on the different elements of the story.

Explore (Small Group):

Students should make good progress on these problems through question 10, since the work and reasoning is very similar to the work in the previous task, *The In-betweeners.* Give appropriate support, as needed, particularly to the idea that a negative integer exponent implies dividing by a factor of 1.05 as many times as represented by the magnitude of the exponent.

If the discussion in the previous task surfaced the idea of using a fraction as an exponent—and its meaning as a radical—then the rest of the task should confirm students thinking. If the previous discussion did not make this connection, then this task should do so now. Listen for students making sense of Clarita's questions and identify students who can relate the idea that $(1.05)^{1/2} \cdot (1.05)^{1/2} = (1.05)^1 = 1.05$ is consistent with the additive properties of exponents, and implies that $(1.05)^{1/2} = \sqrt{1.05}$ based on the definition of square root.

Discuss (Whole Class):

Start the whole class discussion by focusing on question 12, students' answers to Clarita's questions. Make sure the conversation solidifies the reasonableness of using a fraction, such as $1/n$, to represent an nth root. Discuss the third-of-an-hour table in the previous task to add additional support to this claim.

Next, discuss question 13, (how to find the value of the account after ¾ of a year has elapsed), along with question 8 from the previous task, (how would you find the value at $x = 5/3$ for the function $f(x) = 4 \cdot 2^x$). Help students recognize that, based on the properties of exponents,

$2^{1/3} \cdot 2^{1/3} \cdot 2^{1/3} \cdot 2^{1/3} \cdot 2^{1/3}$ can be written as $2^{5/3}$ and gives the same result as $\left(\sqrt[3]{2}\right)^5$, or that $\left(\sqrt[4]{1.05}\right)^3$ gives the same result as $1.05^{3/4}$. Allow students to calculate these values using technology and show that the results are consistent with the contexts provided in these two tasks.

Aligned Ready, Set, Go: Linear & Exponential Functions 2.5

2.5

READY, SET, GO! Name Period Date

READY

Topic: Simplifying Radicals

A very common radical expression is a square root. One way to think of a square root is the number that will multiply by itself to create a desired value. For example: $\sqrt{2}$ *is the number that will multiply by itself to equal 2. And in like manner* $\sqrt{16}$ *is the number that will multiply by itself to equal 16, in this case the value is 4 because 4 x 4 = 16. (When the square root of a square number is taken you get a nice whole number value. Otherwise an irrational number is produced.)*

This same pattern holds true for other radicals such as cube roots and fourth roots and so forth. For example: $\sqrt[3]{8}$ *is the number that will multiply by itself three times to equal 8. In this case it is equal to the value of 2 because* $2^3 = 2 \times 2 \times 2 = 8$.

With this in mind radicals can be simplified. See the examples below.

Example 1: Simplify $\sqrt{20}$	*Example 2*: Simplify $\sqrt[5]{96}$
$\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{2 \cdot 2 \cdot 5} = 2\sqrt{5}$	$\sqrt[5]{96} = \sqrt[5]{2^5 \cdot 3} = 2\sqrt[5]{3}$

Simplify each of the Radicals

1. $\sqrt{40}$
2. $\sqrt{50}$
3. $\sqrt[3]{16}$
4. $\sqrt{72}$
5. $\sqrt[4]{81}$
6. $\sqrt{32}$
7. $\sqrt[5]{160}$
8. $\sqrt{45}$
9. $\sqrt[3]{54}$

SET

Topic: Finding arithmetic and geometric means and making meaning of rational exponents.

You may have found arithmetic and geometric means in your prior work. Finding arithmetic and geometric means requires finding values of a sequence between given values from non-consecutive terms. In each of the sequences below determine the means and show how you found them.

Find the *arithmetic* means for the following. Show your work.

10.

x	1	2	3
y	5		11

11.

x	1	2	3	4	5
y	18	a.	b.	c.	-10

12.

x	1	2	3	4	5	6	7
y	12	a.	b.	c.	d.	e.	-6

Find the *geometric* means for the following. Show your work.

13.

x	1	2	3
y	3		12

14.

x	1	2	3	4
y	7	a.	b.	875

15.

x	1	2	3	4	5	6
y	4	a.	b.	c.	d.	972

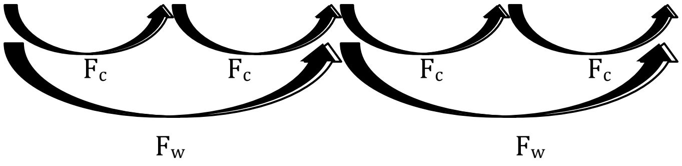
Fill in the tables of values and find the factor used to move between whole number values, F_w, as well as the factor, F_c, used to move between each column of the table.

16.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
y	4		16		

d. $F_w =$

e. $F_c =$

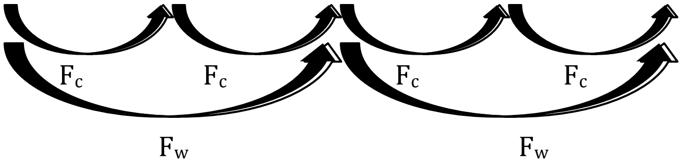


17.

x	0	½	1	3⁄2	2
y	4		8		

d. F_w =

e. F_c =

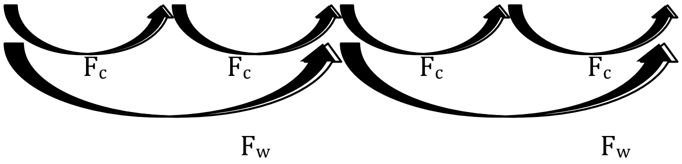


18.

x	0	½	1	3⁄2	2
y	5		15		

d. F_w =

e. F_c =



GO

Topic: Evaluating Functions

Either find or use the unit rate for each of the questions below.

19. $f(x) = 2x - 7$	20. $g(x) = 3^x(2)$	21. $I(t) = 210(1.08^t)$
a. Find $f(-3)$	a. Find $g(-4)$	a. Find $I(12)$
b. Find $f(x) = 21$	b. Find $g(x) = 162$	b. Find $I(t) = 420$
c. Find $f\left(\frac{1}{2}\right)$	c. Find $g\left(\frac{1}{2}\right)$	c. Find $I\left(\frac{1}{2}\right)$
22. $h(x) = x^2 + x - 6$	23. $k(x) = -5x + 9$	24. $m(x) = (5^x)2$
a. Find $h(-5)$	a. Find $k(-7)$	a. Find $m(-2)$
b. Find $h(x) = 0$	b. Find $k(x) = 0$	b. Find $m(x) = 1$
c. Find $h\left(\frac{1}{2}\right)$	c. Find $k\left(\frac{1}{2}\right)$	c. Find $m\left(\frac{1}{2}\right)$

2.6 More Interesting

A Solidify Understanding Task

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Carlos now knows he can calculate the amount of interest earned on an account in smaller increments than one full year. He would like to determine how much money is in an account each month that earns 5% annually with an initial deposit of $300.

He starts by considering the amount in the account each month during the first year. He knows that by the end of the year the account balance should be $315, since it increases 5% during the year.

1. Complete the table showing what amount is in the account each month during the first twelve months.

deposit											1 year
$300											$315

2. What number did you multiply the account by each month to get the next month's balance?

Carlos knows the exponential equation that gives the account balance for this account on an annual basis is $A = 300(1.05)^t$. Based on his work finding the account balance each month, Carlos writes the following equation for the same account: $A = 300(1.05^{1/12})^{12t}$.

3. Verify that both equations give the same results. Using the properties of exponents, explain why these two equations are equivalent.

4. What is the meaning of the $12t$ in this equation?

Carlos shows his equation to Clarita. She suggests his equation could also be approximated by $A = 300(1.004)^{12t}$, since $(1.05)^{1/12} \approx 1.004$. Carlos replies, "I know the 1.05 in the equation $A = 300(1.05)^t$ means I am earning 5% interest annually, but what does the 1.004 mean in your equation?"

5. Answer Carlos' question. What does the 1.004 mean in $A = 300(1.004)^{12t}$?

The properties of exponents can be used to explain why $[(1.05)^{1/12}]^{12t} = 1.05^t$. Here are some more examples of using the properties of exponents with rational exponents. For each of the following, simplify the expression using the properties of exponents, and explain what the expression means in terms of the context.

6. $(1.05)^{1/12} \cdot (1.05)^{1/12} \cdot (1.05)^{1/12}$

7. $[(1.05)^{1/12}]^6$

8. $(1.05)^{-1/12}$

9. $(1.05)^2 \cdot (1.05)^{1/4}$

10. $\dfrac{(1.05)^2}{(1.05)^{1/2}}$

11. Use $\left[(1.05)^{1/12}\right]^{12} = 1.05$ to explain why $(1.05)^{1/12} = \sqrt[12]{1.05}$

2.6 More Interesting – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to verify that the properties of exponents students know for integer exponents also work for rational exponents. In the context of writing exponential equations to represent the amount of interest earned over smaller intervals of time than annually, students will solidify their understanding of working with rational exponents in conjunction with the properties of exponents.

Core Standards Focus:

N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.

N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

Related Standards: A.SSE.1

Standards for Mathematical Practice:

SMP 2 – Reason abstractly and quantitatively

SMP 4 – Model with mathematics

SMP 7 – Look for and make use of structure

The Teaching Cycle:

Launch (Whole Class):

Since this task is a continuation of the context and the mathematical work of the previous task, *Half Interested*, students should be able to begin work immediately on the task.

Explore (Small Group):

Students should draw upon their understanding of positive whole number exponents to make sense of the work of this task. Listen for how they reason about each of the problems, and if they can relate each to the following properties of exponents. Identify any of these rules that seem problematic for students to discuss during the whole class discussion.

1. $a^m \cdot a^n = a^{m+n}$
2. $\left(a^m\right)^n = a^{mn}$
3. $(ab)^n = a^n \cdot b^n$
4. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
5. $\dfrac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$
6. $a^{-n} = \dfrac{1}{a^n}$

Discuss (Whole Class):

If necessary, illustrate each of these properties of exponents with examples using positive integer exponents, such as $2^3 \cdot 2^5 = (2\cdot 2\cdot 2)(2\cdot 2\cdot 2\cdot 2\cdot 2) = 2^8$ or $\frac{2^5}{2^3} = \frac{2\cdot 2\cdot 2\cdot 2\cdot 2}{2\cdot 2\cdot 2} = 2^2$. The question now is, "Do these properties that were developed with positive integer exponents still hold when the exponents are rational numbers?" The answer to this question is "yes" if rational number exponents are defined to mean $a^{m/n} = \left(\sqrt[n]{a}\right)^m$.

Aligned Ready, Set, Go: Linear & Exponential Functions 2.6

READY, SET, GO! Name Period Date

READY

Topic: Meaning of Exponents

In the table below there is a column for the exponential form, the meaning of that form, which is a list of factors and the standard form of the number. Fill in the form that is missing.

Exponential form	List of factors	Standard Form
5^3	$5 \cdot 5 \cdot 5$	125
1a.	$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$	b.
2. 2^{10}	a.	b.
3a.	b.	81
4. 11^5	a.	b.
5a.	$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$	b.
6a.	b.	625

Provide at least three other equivalent forms of the exponential expression. Use rules of exponents such as $3^5 \cdot 3^6 = 3^{11}$ and $\left(5^2\right)^3 = 5^6$ as well as division properties and others.

	1st Equivalent Form	2nd Equivalent Form	3rd Equivalent Form
7. $2^{10} =$			
8. $3^7 =$			
9. $13^{-8} =$			
10. $7^{\frac{1}{3}} =$			
11. $5^1 =$			

SET

Topic: Finding equivalent expressions and functions

Determine whether all three expressions in each problem below are equivalent. Justify why or why they are not equivalent.

12. $5(3^{x-1})$ $15(3^{x-2})$ $\frac{5}{3}(3^{x})$

13. $64\ (2^{-x})$ $\frac{64}{2^{x}}$ $64\left(\frac{1}{2}\right)^{x}$

14. 3(x-1)+4 3x - 1 3(x-2) +7

15. $50(2^{x+2})$ $25(2^{2x+1})$ $50(4^{x})$

16. $30(1.05^{x})$ $30\left(1.05^{\frac{1}{7}}\right)^{7x}$ $30\left(1.05^{\frac{x}{2}}\right)^{2}$

17. $20\ (1.1^{x})$ $20\ (1.1^{-1})^{-1x}$ $20\left(1.1^{\frac{1}{5}}\right)^{5x}$

GO

Topic: Using rules of exponents

Simplify each expression. Your answer should still be in exponential form.

18. $7^3 \cdot 7^5 \cdot 7^2$

19. $(3^4)^5$

20. $(5^3)^4 \cdot 5^7$

21. $x^3 \cdot x^5$

22. x^{-b}

23. $x^a \cdot x^b$

24. $(x^a)^b$

25. $\frac{y^a}{y^b}$

26. $\frac{(y^a)^c}{y^b}$

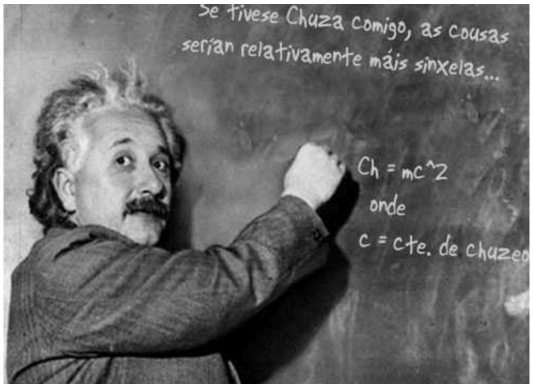
27. $\frac{(3^4)^6}{3^7}$

28. $\frac{r^5 s^3}{r s^2}$

29. $\frac{x^5 y^{12} z^0}{x^8 y^9}$

2.7 Radical Ideas

A Practice Understanding Task



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Now that Tia and Tehani know that $a^{m/n} = \left(\sqrt[n]{a}\right)^m$ they are wondering which form, radical form or exponential form, is best to use when working with numerical and algebraic expressions.

Tia says she prefers radicals since she understands the following properties for radicals (and there are not too many properties to remember):

If n is a positive integer greater than 1 and both a and b are positive real numbers then,

1. $\sqrt[n]{a^n} = a$
2. $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
3. $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Tehania says she prefers exponents since she understands the following properties for exponents (and there are more properties to work with):

1. $a^m \cdot a^n = a^{m+n}$
2. $\left(a^m\right)^n = a^{mn}$
3. $(ab)^n = a^n \cdot b^n$
4. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
5. $\dfrac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$
6. $a^{-n} = \dfrac{1}{a^n}$

DO THIS: Illustrate with examples and explain, using the properties of radicals and exponents, why $a^{1/n} = \sqrt[n]{a}$ and $a^{m/n} = \left(\sqrt[n]{a}\right)^m$ are true identities.

Using their preferred notation, Tia might simplify $\sqrt[3]{x^8}$ as follows:

$$\sqrt[3]{x^8} = \sqrt[3]{x^3 \cdot x^3 \cdot x^2} = \sqrt[3]{x^3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x^2} = x \cdot x \cdot \sqrt[3]{x^2} = x^2 \cdot \sqrt[3]{x^2}$$

(Tehani points out that Tia also used some exponent rules in her work.)

On the other hand, Tehani might simplify $\sqrt[3]{x^8}$ as follows:

$$\sqrt[3]{x^8} = x^{8/3} = x^{2+2/3} = x^2 \cdot x^{2/3} \text{ or } x^2 \cdot \sqrt[3]{x^2}$$

For each of the following problems, simplify the expression in the ways you think Tia and Tehani might do it.

Original expression	What Tia and Tehani might do to simplify the expression:
$\sqrt{27}$	Tia's method
	Tehani's method
$\sqrt[3]{32}$	Tia's method
	Tehani's method

$\sqrt{20x^7}$	Tia's method
	Tehani's method
$\sqrt[3]{\frac{16xy^5}{x^7y^2}}$	Tia's method
	Tehani's method

2.7 Radical Ideas – Teacher Notes

A Practice Understanding Task

Purpose: This task provides opportunities for students to become fluent converting between exponential and radical representations of expressions, as well as using the rules of exponents to simplify exponential and radical expressions.

Core Standards Focus:

N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.

N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Algebra 1 Note for N.RN.1 and N.RN.2

These standards (N.RN.1 and N.RN.2) should occur before discussing exponential functions with continuous domains.

Related Standards: A.SSE.1

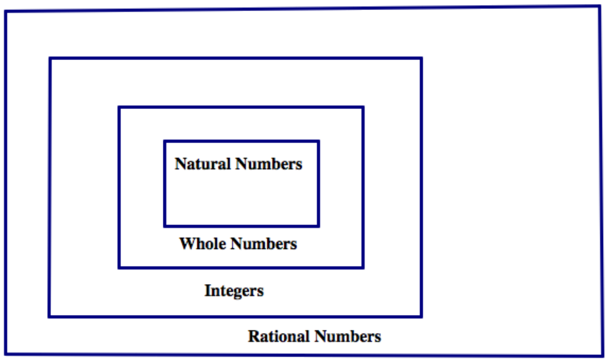
Standards for Mathematical Practice:

- **SMP 2 – Reason abstractly and quantitatively**
- **SMP 8 – Look for express regularity in repeated reasoning**

The Teaching Cycle:

Launch (Whole Class):

Draw a Venn diagram of the relationship of subsets of the rational numbers as follows:



With student input, list a few examples of numbers that fit within each set. Point out to students that their understanding of which of these numbers can meaningfully be used as exponents has expanded over their years of experience working with exponents. Initially, only natural numbers made sense as exponents, since the exponent represented how many times the base was used as a factor (e.g., $5^3 = 5 \times 5 \times 5$).

Later, we expanded the set of reasonable exponents to include 0 and the negative integers. The reasonableness of treating such numbers as exponents was reinvestigated in the task *Half Interested* where negative integer exponents were used to represent an interest-bearing account balance *n years ago* rather than *n years from now*, giving meaning to 1.05^{-3} as dividing by the factor 1.05 three times (or multiplying by the factor $\frac{1}{1.05}$ three times). In that context, 0 would be used as an exponent to represent the current amount in the account when no time has elapsed. Since we want the account balance to stay the same at time $t = 0$, the factor $(1.05)^0$ would have to equal 1. Point out that this interpretation of 0 as an exponent is also consistent with the properties of exponents observed when using only natural numbers as exponents. For example, we want the additive property of exponents, $a^m \cdot a^n = a^{m+n}$, which was evident as a true observation for natural number exponents, to still hold with the extended set of exponents. Consequently, $a^n \cdot a^{-n} = a^0$ and $a^n \cdot a^{-n} = \frac{a^n}{a^n} = 1$, which implies $a^0 = 1$ in order to maintain consistency with the properties of exponents.

Now we have extended the set of reasonable exponents to include rational numbers. As we have observed in the past few tasks, the properties of exponents remain consistent if we define $a^{1/n} = \sqrt[n]{a}$

For example, $a^{1/3} \cdot a^{1/3} \cdot a^{1/3} = a^{1/3+1/3+1/3} = a^1 = a$ based on the properties of exponents in the same way as $\sqrt[3]{a} \cdot \sqrt[3]{a} \cdot \sqrt[3]{a} = a$ based on the definition of radicals.

After this brief review of how we have extended the meaning of exponents to include integer and rational exponents, review all of the properties of radicals and exponents listed on the first page of the task, and give students a few minutes to work on the DO THIS exploration following the list of properties. Allow students to share their examples showing connections between the properties of radicals and exponents. After sharing a few examples, discuss Tia' and Tehani's preferred methods and have students begin working on the problems in the task.

Explore (Small Group):
If necessary, suggest that students might want to decompose numbers under the radical into their prime factorizations, or perhaps look for ways to decompose a number into factors that include perfect squares or perfect cubes, as needed. Listen for students who can make connections between Tia' and Tehani's strategies, such as Tia decomposes factors into powers of *n* (e.g., perfect squares or cubes), while Tehani divides exponents to find how many groups of factors of size *n* can be formed, and how many factors are left over as factors in the radicand.

As you monitor student work, watch for any algebraic procedures that need to be discussed as a whole class.

Discuss (Whole Class):
As needed, discuss the algebra of specific problems. Most questions can be resolved by rewriting expression involving radicals as expressions involving rational exponents and using the properties of exponents to simplify the expression. Help students see the power of exponential form. Other techniques for working with radicals might surface and can be discussed; for example, looking for ways to meaningfully decompose the expression in the radicand.

Aligned Ready, Set, Go: Linear and Exponential Functions 2.7

READY, SET, GO! Name Period Date

READY

Topic: Evaluating functions and creating tables of values.

Fill in the table of values for each function, look for patterns and connections in your work.

Remember that output values for a function come from evaluating the function using the given input value and following the order of operations.

1. $f(x) = 3x + 5$

x	$f(x)$
-1	
0	
1	
2	
3	

2. $g(x) = 3(x - 1) + 5$

x	$g(x)$
-1	
0	
1	
2	
3	

3. $h(x) = 3(x - 2) + 5$

x	$h(x)$
-1	
0	
1	
2	
3	

4. $k(x) = 3(x + 1) + 5$

x	$k(x)$
-1	
0	
1	
2	
3	

5. $p(x) = 2^x(3)$

x	$p(x)$
-1	
0	
1	
2	
3	

6. $q(x) = 2^{(x-1)}(3)$

x	$q(x)$
-1	
0	
1	
2	
3	

7. $r(x) = 2^{(x-2)}(3)$

x	$r(x)$
-1	
0	
1	
2	
3	

8. $s(x) = 2^{(x+1)}(3)$

x	$s(x)$
-1	
0	
1	
2	
3	

9. $h(x) = -5x + 2$

x	$t(x)$
-1	
0	
1	
2	
3	

10. $h(x) = -5(x - 1) + 2$

x	$u(x)$
-1	
0	
1	
2	
3	

11. $h(x) = -5(x - 2) + 2$

x	$v(x)$
-1	
0	
1	
2	
3	

12. $h(x) = -5(x + 1) + 2$

x	$w(x)$

SET

Topic: Radical notation and rational exponents

Each of the expressions below can be written using either radical notation, $\sqrt[n]{a^m}$ or rational exponents $a^{\frac{m}{n}}$. Rewrite each of the given expressions in the form that is missing. Express in most simplified form.

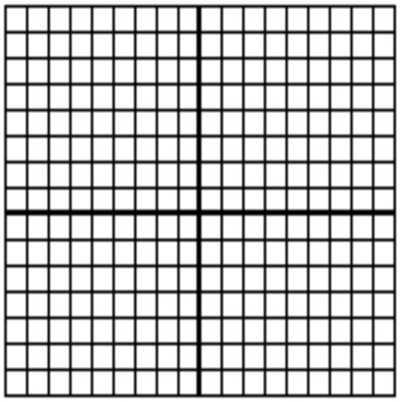
	Radical Form	Exponential Form
13.	$\sqrt[3]{5^2}$	
14.		$16^{\frac{3}{4}}$
15.	$\sqrt[3]{5^7 \cdot 3^5}$	
16.		$9^{\frac{2}{3}} \cdot 9^{\frac{4}{3}}$
17.	$\sqrt[5]{x^{13}y^{21}}$	
18.	$\sqrt[3]{27a^5b^2}$	
19.	$\sqrt[5]{\frac{32x^{13}}{243y^{15}}}$	
20.		$9^{\frac{3}{2}}s^{\frac{6}{3}}t^{\frac{1}{2}}$

GO

Topic: x-intercepts and y-intercepts for linear and exponential functions

Given the function, find the x-intercept (s) and y-intercept if they exist and then use them to graph a sketch of the function.

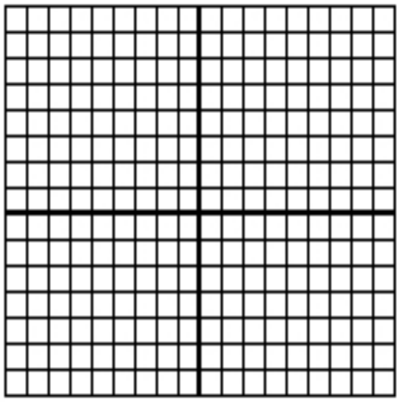
23. $f(x) = 5(x - 4) - 10$



a. x-intercept(s):

b. y-intercept:

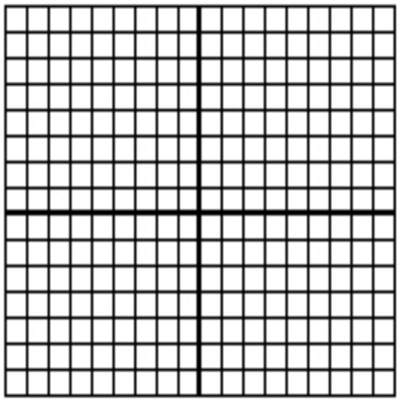
24. $g(x) = 5(2^{x-1})$



a. x-intercept(s):

b. y-intercept:

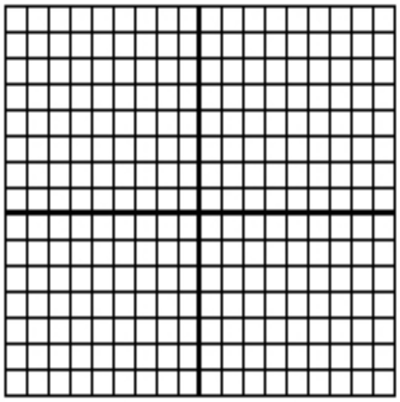
25. $h(x) = -2(x + 3)$



a. x-intercept(s):

b. y-intercept:

26. $k(x) = 2x - 8$



a. x-intercept(s):

b. y-intercept:

2.8 Getting Down to Business

A Solidify Understanding Task

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Calcu-rama had a net income of 5 million dollars in 2010, while a small competing company, Computafest, had a net income of 2 million dollars. The management of Calcu-rama develops a business plan for future growth that projects an increase in net income of 0.5 million per year, while the management of Computafest develops a plan aimed at increasing its net income by 15% each year.

a. Create standard mathematical models (table, graph and equations) for the projected net income over time for both companies. (Attend to precision and be sure that each model is accurate and labeled properly so that it represents the situation.)

b. Compare the two companies. How are the representations for the net income of the two companies similar? How do they differ? What relationships are highlighted in each representation?

c. If both companies were able to meet their net income growth goals, which company would you choose to invest in? Why?

d. When, if ever, would your projections suggest that the two companies have the same net income? How did you find this? Will they ever have the same net income again?

e. Since we are creating the models for these companies we can choose to have a discrete model or a continuous model. What are the advantages or disadvantages for each type of model?

2.8 Getting Down to Business – Teacher Notes

A Solidify Understanding Task

Note: Use of technology tools such as graphing calculators is recommended for this task.

Purpose:

The purpose of this task is to compare the rates of growth of an exponential and a linear function. The task provides an opportunity to look at the growth of an exponential and a linear function for large values of *x*, showing that increasing exponential functions become much larger as *x* increases. This task is a good opportunity to model functions using technological tools and to discuss how to set appropriate viewing windows for functions. The task also leads to a discussion of whether this particular situation should be modeled using discrete or continuous functions.

Core Standards Focus:

F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F-LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

★For F.IF.7a, 7e, and 9 focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y=3^n$ and $y=100\cdot2^n$.

Standards for Mathematical Practice

SMP 4 – Model with mathematics.

SMP 5 – Use appropriate tools strategically.

The Teaching Cycle

Launch (Whole Class):

Start with a short discussion of the context to be sure that students understand the problem situation. As part of this discussion, clarify the choice of units and scale. Students may choose to use 5 million with million being the unit or as 5,000,000 in their equations. (They will probably find it easier to use millions as a unit, but they will need to interpret the scale on their graphs and be consistent in their equations.) Be sure that students understand terms like "net income" so that they know what the problem is asking. When students understand the problem, set them to work on the task, starting with parts a, b, and c.

Explore (Small Group):

Monitor students as they work on the task. Be prepared to redirect students that may not think of one function as linear, based on the constant growth, and the other as exponential based on the 15% growth factor. Be sure that students have discussed their answers to "c" before returning to the whole group discussion. The discussion for "d" will follow later.

Discuss and Re-launch (Whole Class):

Have a group that has written the explicit and recursive equations correctly present their work. Ask the class which company has a linear model and which has an exponential model and how can they tell from both recursive and explicit forms of the function rules. Ask how the growth pattern shows up in the equations. Finally ask if the functions should be modeled as discrete or continuous. Ask why the companies might choose a discrete or continuous model. They may choose a continuous model because they feel that the net income is increasing on a steady basis across the year, so it makes sense to fill in all the points on the graph and use an explicit formula. They may choose a discrete model because there are fluctuations in income during the year, with the net income increasing. If they can't predict the fluctuations, they may choose to use a discrete function, modeling with just one point each year.

Once students have discussed the equations, ask students to focus on the explicit equations and complete part "d" of the task. Encourage the use of technology, either graphing calculators or computers with programs with graphing capabilities.

When students have completed their work, ask a group to present their tables showing the projected net income of the two businesses. Ask how they could find where the net income of the two businesses would be the same using their tables. Then have a group present their graphs and demonstrate how to find the year where Computafest exceeds the net income of Calcu-rama. (You may ask how to use the equations to find where the net incomes will be equal, but students will not be able to find an analytic solution to the equation.)

Conclude the task with a discussion of the end behavior of the two functions. How much will each company be making in 10 years, 20 years, etc.? Trace the graphs and look at the difference between the net incomes over time. Ask why an exponential function becomes so much larger than a linear function over time. A big idea here is that exponential growth depends on the amount available at any given time, so the more available, the bigger the increase. In the early years when the company is small, an increase of 15% adds a small amount. As the company grows, 15% of the income becomes larger and larger, making the company grow by more each year. In contrast, linear growth has the same increase every time no matter how much is available.

Aligned Ready, Set, Go: ***Linear and Exponential Functions 2.8***

READY, SET, GO!

Name Period Date

READY

Topic: Comparing arithmetic and geometric sequences.

The first and fifth terms of a sequence are given. Fill in the missing numbers if it is an arithmetic sequence. Then fill in the numbers if it is a geometric sequence.

+80 +80 +80 +80

Example:

Arithmetic	4	84	164	244	324
Geometric	4	12	36	108	324

x 3 x 3 x 3 x 3

1.

Arithmetic	3				48
Geometric	3				48

2.

Arithmetic	-6250				-10
Geometric	-6250				-10

3.

Arithmetic	-12				-0.75
Geometric	-12				-0.75

SET

Topic: Distinguishing specifics between sequences and linear or exponential functions.

Answer the questions below with respect to the relationship between sequences and the larger families of functions.

4. If a relationship is modeled with a <u>continuous</u> function which of the domain choices is a possibility?
A. $\{x \mid x \in R, x \geq 0\}$ B. $\{x \mid x \in W\}$ C. $\{x \mid x \in Z, x \geq 0\}$ D. $\{x \mid x \in N\}$

5. Which one of the options below is the mathematical way to represent the Natural Numbers?
A. $\{x \mid x \in R, x \geq 0\}$ B. $\{x \mid x \in Q, x \geq 0\}$ C. $\{x \mid x \in Z, x \geq 0\}$ D. $\{x \mid x \in N\}$

6. Only one of the choices below would likely be used for a <u>continuous exponential</u> model, which one?
   A. $f(x) = f(x-1) \cdot 4, f(1) = 3$ B. $g(x) = 4^x(5)$
   C. $h(t) = 3t - 5$ D. $k(n) = k(n-1) - 5, k(1) = 32$

7. Only one of the choices below would likely be used for a <u>continuous linear</u> model, which one is it?
   A. $f(x) = f(x-1) \cdot 4, f(1) = 3$ B. $g(x) = 4^x(5)$
   C. $h(t) = 3t - 5$ D. $k(n) = k(n-1) - 5, k(1) = 32$

8. Which domain choice would be most appropriate for an arithmetic or geometric sequence?
   A. $\{x \mid x \in R, x \geq 0\}$ B. $\{x \mid x \in Q, x \geq 0\}$ C. $\{x \mid x \in Z, x \geq 0\}$ D. $\{x \mid x \in N\}$

9. What attributes will arithmetic or geometric sequences always have?
   (There could be more than one correct choice. Circle all that apply.)

   A. Continuous B. Discrete C. Domain: $\{x \mid x \in N\}$ D. Domain: $\{x \mid x \in R\}$

   E. Negative x-values F. Something constant G. Recursive Rule

10. What type of sequence fits with linear mathematical models?

    What is the difference between this sequence type and the overarching umbrella of linear relationships? (Use words like discrete, continuous, domain and so forth in your response.)

11. What type of sequence fits with exponential mathematical models?

    What is the difference between this sequence type and the overarching umbrella of exponential relationships? (Use words like discrete, continuous, domain and so forth in your response.)

GO

Topic: Writing explicit equations for linear and exponential models.

Write the explicit equations for the tables and graphs below. This is something you really need to know. Persevere and do all you can to figure them out. Remember the tools we have used. (#21 is bonus give it a try.)

12.

x	$f(x)$
2	-4
3	-11
4	-18
5	-25

13.

x	$f(x)$
-1	2/5
0	2
1	10
2	50

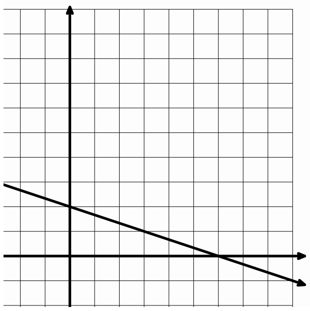
14.

x	$f(x)$
2	-24
3	-48
4	-96
5	-192

15.

x	$f(x)$
-4	81
-3	27
-2	9
-1	3

16



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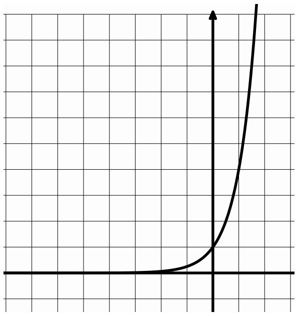

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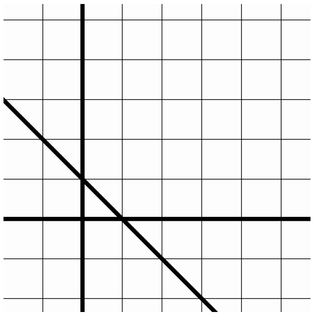

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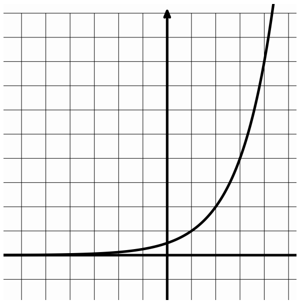

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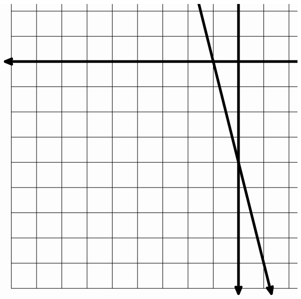
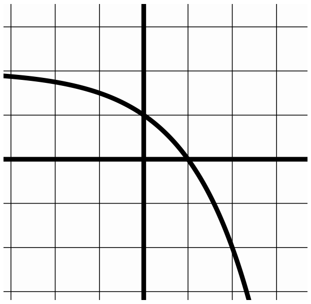


21.



2.9 Making My *Point*

A Solidify Understanding Task

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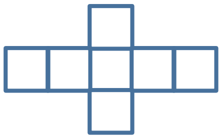
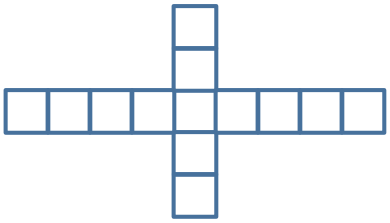
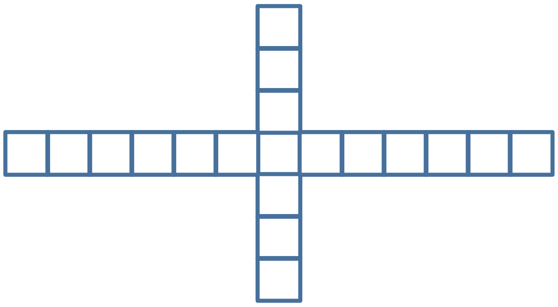
Zac and Sione were working on predicting the number of quilt blocks in this pattern:







When they compared their results, they had an interesting discussion:

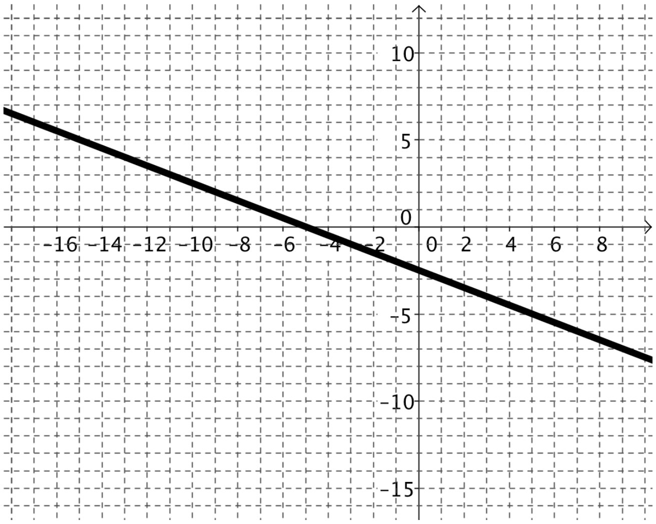
Zac: I got $y = 6n + 1$ because I noticed that 6 blocks were added each time so the pattern must have started with 1 block at $n = 0$.

Sione: I got $y = 6(n - 1) + 7$ because I noticed that at $n = 1$ there were 7 blocks and at $n = 2$ there were 13, so I used my table to see that I could get the number of blocks by taking one less than the n, multiplying by 6 (because there are 6 new blocks in each figure) and then adding 7 because that's how many blocks in the first figure. Here's my table:

1	2	3	n
7	13	19	$6(n-1)+7$

1. What do you think about the strategies that Zac and Sione used? Are either of them correct? Why or why not? Use as many representations as you can to support your answer.

The next problem Zac and Sione worked on was to write the equation of the line shown on the graph below.



When they were finished, here is the conversation they had about how they got their equations:

Sione: It was hard for me to tell where the graph crossed the y axis, so I found two points that I could read easily, (-9, 2) and (-15, 5). I figured out that the slope was $-\frac{1}{2}$ and made a table and checked it against the graph. Here's my table:

x	-15	-13	-11	-9	n
$f(x)$	5	4	3	2	$-\frac{1}{2}(n+9)+2$

I was surprised to notice that the pattern was to start with the n, add 9, multiply by the slope and then add 2.

I got the equation: $f(x) = -\frac{1}{2}(x+9)+2$.

Zac: Hey—I think I did something similar, but I used the points, (7,-6) and (9,-7).

I ended up with the equation: $f(x) = -\frac{1}{2}(x-9)-7$. One of us must be wrong because yours says that you add 9 to the n and mine says that you subtract 9. How can we both be right?

2. What do you say? Can they both be right? Show some mathematical work to support your thinking.

Zac: My equation made me wonder if there was something special about the point (9, -7) since it seemed to appear in my equation $f(x) = -\frac{1}{2}(x - 9) - 7$ when I looked at the number pattern. Now I'm noticing something interesting—the same thing seems to happen with your equation, $f(x) = -\frac{1}{2}(x + 9) + 2$ and the point (-9, 2)

3. Describe the pattern that Zac is noticing.

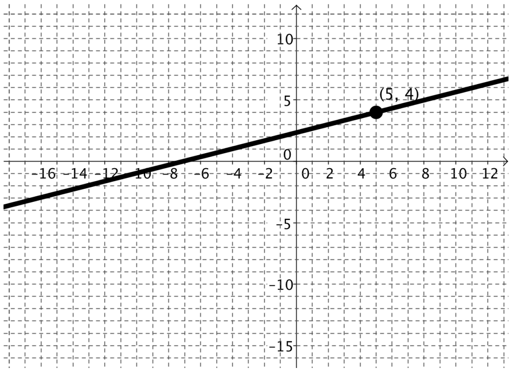
4. Find another point on the line given above and write the equation that would come from Zac's pattern.

5. What would the pattern look like with the point (a, b) if you knew that the slope of the line was m?

6. Zac challenges you to use the pattern he noticed to write the equation of line that has a slope of 3 and contains the point (2,-1). What's your answer?

   Show a way to check to see if your equation is correct.

7. Sione challenges you to use the pattern to write the equation of the line graphed below, using the point (5, 4).



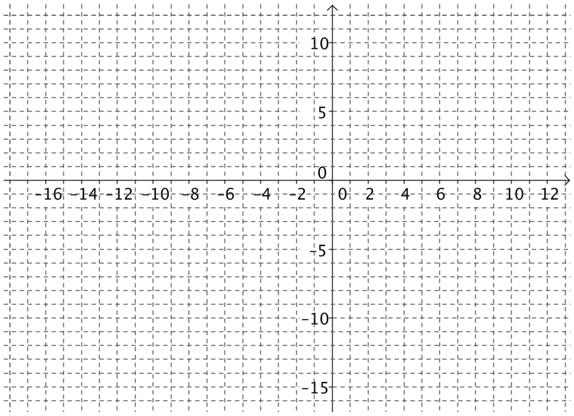
Show a way to check to see if your equation is correct.

8. **Zac:** "I'll bet you can't use the pattern to write the equation of the line through the points (1,-3) and (3,-5). Try it!"

Show a way to check to see if your equation is correct.

9. **Sione:** I wonder if we could use this pattern to graph lines, thinking of the starting point and using the slope. Try it with the equation: $f(x) = -2(x + 1) - 3$.
Starting point: Slope:

Graph:



10. Zac wonders, “What is it about lines that makes this work?” How would you answer Zac?

11. Could you use this pattern to write the equation of any linear function? Why or why not?

2.9 Making My *Point* – Teacher Notes

A Solidify Understanding Task

Purpose:
This is the first task of two that focus on understanding and using various notations for linear functions. The task involves students in thinking about a context where students have selected the index in two different ways, thus getting two different, but equivalent equations. The idea is extended so that students can see the relationship expressed in point/slope form of the equation of the line.

Core Standards Focus:

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.
  a) Interpret parts of an expression, such as terms, factors, and coefficients.

A-SSE.6 Use the structure of an expression to identify ways to rewrite it.

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Standards for Mathematical Practice

SMP 2 – Reason abstractly and quantitatively.

SMP 7 – Look for and express regularity in repeated reasoning.

The Teaching Cycle

Launch (Whole Class):

In previous tasks students have worked with visual patterns such as the one in this task. Start the lesson by telling students that Zac and Sione have worked the problem and come up with two different answers, which they are trying to resolve with sound reasoning. Students need to figure out how Zac and Sione have arrived at different equations and who is right through each of the scenarios in the task.

Explore (Small Group):

Monitor students as they work through the task to see that they understand each scenario. In problem #1, watch for students that have labeled the figures to match the equations; either starting with $n = 0$ or $n = 1$. For problems 2-5, watch to see that students are noticing patterns in how the numbers are used in the equation and making sense of the tables.

Discuss (Whole Class):

Be prepared for the whole group discussion by having large versions of the figure in #1 ready to be used. Ask a student to explain the difference between Zac and Sione's equations and why they both make sense as models for the figures. Ask a student to show whether or not the two equations are equivalent.

Move to the next scenario, asking for verbal descriptions of the pattern they noticed in #3. Ask for a student to give some examples of equations that they wrote for #4 using the pattern. Ask, "Are the equations equivalent? How do you know?" Ask for students to give their answer for #4. If there are differences in equations among the groups, discuss the differences. Finally, ask students for reasons why this relationship should hold for any linear function. After discussing their reasons, offer that this pattern is often used as a formula for writing equations and graphing lines and is called point/slope form of the equation of a line. You may wish to show them that this form can be derived from the slope formula:

$$m = \frac{y-y_1}{x-x_1}$$

With a little rearranging:

$$m(x - x_1) = y - y_1$$

$$y = m(x - x_1) + y_1$$

The focus of this task is on the connections between representations and how any point can be used to create an equation. This is the task for students to think about this concept. The derivation is important but may come later. For each problem, demonstrate the connection between the strategies that students used and the slope/intercept formula. After these three problems, solicit answers for question #10; what is it about lines that cause this connection? Answers should include the idea that the constant rate of change makes it possible to start at any point on the line and find another point. In the past, students have used the y-intercept as the starting point, but any known point will work as well to write an equation or to graph the line.

Aligned Ready, Set, Go: ***Linear and Exponential Functions 2.9***

READY, SET, GO! Name Period Date

READY

Topic: Writing equations of lines.

Write the equation of a line in slope-intercept form: $y = mx + b$, using the given information.

1. m = -7, b = 4

2. m = 3/8, b = -3

3. m = 16, b = -1/5

Write the equation of the line in point-slope form: $y = m(x - x_1) + y_1$, using the given information.

4. m = 9, (0. -7)

5. m = 2/3, (-6, 1)

6. m = -5, (4, 11)

7. (2,-5) (-3, 10)

8. (0, -9) (3, 0)

9. (-4, 8) (3, 1)

SET

Topic: Graphing linear and exponential functions

Make a graph of the function based on the following information. Add your axes. Choose an appropriate scale and label your graph. Then write the equation of the function.

10. The beginning value is 5 and its value is 3 units smaller at each stage.

Equation:

11. The beginning value is 16 and its value is ¼ smaller at each stage.

Equation:

12. The beginning value is 1 and its value is 10 times as big at each stage.

Equation:

13. The beginning value is -8 and its value is 2 units larger at each stage.

Equation:

GO

Topic: Equivalent equations

Prove that the two equations are equivalent by simplifying the equation on the right side of the equal sign. The justification in the example is to help you understand the steps for simplifying. You do NOT need to justify your steps.

Example:	Justification
$2x - 4 = 8 + x - 5x + 6(x - 2)$	Add $x - 5x$ and distribute the 6 over $(x - 2)$
$= 8 - 4x + 6x - 12$	Combine like terms.
$= -4 + 2x$	
$2x - 4 = 2x - 4$	Commutative property of addition

14. $x - 5 = 5x - 7 + 2(3x + 1) - 10x$

15. $6 - 13x = 24 - 10(2x + 8) + 62 + 7x$

16. $14x + 2 = 2x - 3(-4x - 5) - 13$

17. $x + 3 = 6(x + 3) - 5(x + 3)$

18. $4 = 7(2x + 1) - 5x - 3(3x + 1)$

19. $x = 12 + 8x - 3(x + 4) - 4x$

20. Write an expression that equals $(x - 13)$. It must have at least two sets of parentheses and one minus sign. Verify that it is equal to $(x - 13)$.

2.10 Form Follows Function

A Practice Understanding Task

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In our work so far, we have worked with linear and exponential equations in many forms. Some of the forms of equations and their names are:

Linear Functions	
Equation	**Name**
$y = \frac{1}{2}x + 1$	Slope Intercept Form $y = mx + b$, where m is the slope and b is the y-intercept
$y = \frac{1}{2}(x - 4) + 3$	Point Slope Form $y = m(x - x_1) + y_1$, where m is the slope and (x_1, y_1) the coordinates of a point on the line
$f(0) = 1, f(n) = f(n-1) + \frac{1}{2}$	Recursion Formula $f(n) = f(n-1) + \text{D}$, Given an initial value *f(a)* D = constant difference in consecutive terms (used only for discrete functions)

Exponential Functions	
Equation	**Name**
$y = 10(3)^x$	Explicit Form $y = a(b)^x$
$f(0) = 10, f(n+1) = 3f(n)$	Recursion Formula $f(n+1) = Rf(n)$ Given an initial value *f(a)* *R* = constant ratio between consecutive terms (used only for discrete functions)

Knowing a number of different forms for writing and graphing equations is like having a mathematical toolbox. You can select the tool you need for the job, or in this case, the form of the equation that makes the job easier. Any master builder will tell you that the more tools you have the better. In this task, we'll work with our mathematical tools to be sure that we know how to use them all efficiently. As you model the situations in the following problems, think about the important information in the problem and the conclusions that can be drawn from it. Is the function linear or exponential? Does the problem give you the slope, a point, a ratio, a y-intercept? Is the function discrete or continuous? This information helps you to identify the best tools and get to work!

1. In his job selling vacuums, Joe makes \$500 each month plus \$20 for each vacuum he sells. Write the equation that describes Joe's monthly income I as a function of the n, the number of vacuums sold.

   Name the form of the equation you wrote and why you chose to use that form.

   This function is: linear exponential neither (choose one)

   This function is: continuous discrete neither (choose one)

2. Write the equation of the line with a slope of -1 through the point (-2, 5)

   Name the form of the equation you wrote and why you chose to use that form.

   This function is: linear exponential neither (choose one)

   This function is: continuous discrete neither (choose one)

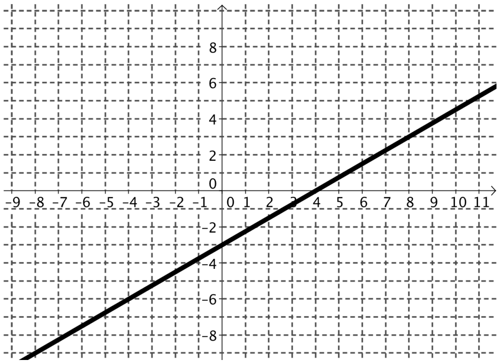
3. Write the equation of the geometric sequence with a constant ratio of 5 and a first term of -3.

Name the form of the equation you wrote and why you chose to use that form.

This function is: linear exponential neither (choose one)

This function is: continuous discrete neither (choose one)

3. Write the equation of the function graphed below:



Name the form of the equation you wrote and why you chose to use that form.

This function is: linear exponential neither (choose one)

This function is: continuous discrete neither (choose one)

4. The population of the resort town of Java Hot Springs in 2003 was estimated to be 35,000 people with an annual rate of increase of about 2.4%. Write the equation that models the number of people in Java Hot Springs, with t = the number of years from 2003?

Name the form of the equation you wrote and why you chose to use that form.

This function is: linear exponential neither (choose one)

This function is: continuous discrete neither (choose one)

5. Yessica's science fair project involved growing some seeds to see what fertilizer made the seeds grow fastest. One idea she had was to use an energy drink to fertilize the plant. (She thought that if they make people perky, they might have the same effect on plants.) This is the data that shows the growth of the seed each week of the project.

Week	1	2	3	4	5
Height (cm)	1.7	2.9	4.1	5.3	6.5

Write the equation that models the growth of the plant over time.

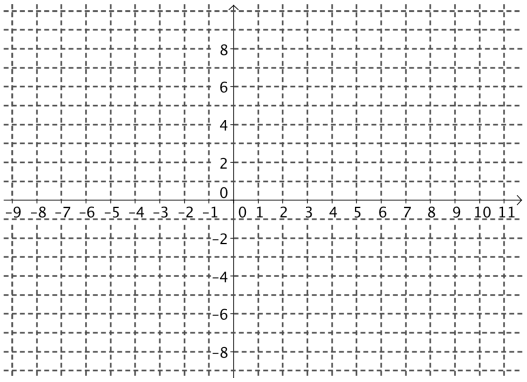
Name the form of the equation you wrote and why you chose to use that form.

This function is: linear exponential neither (choose one)

This function is: continuous discrete neither (choose one)

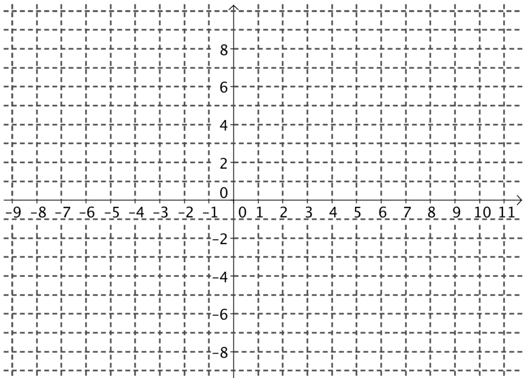
An equation gives us information that we can use to graph the function. Pick out the important information given in each of the following equations and use the information to graph the function.

6. $y = \frac{1}{2}x - 5$



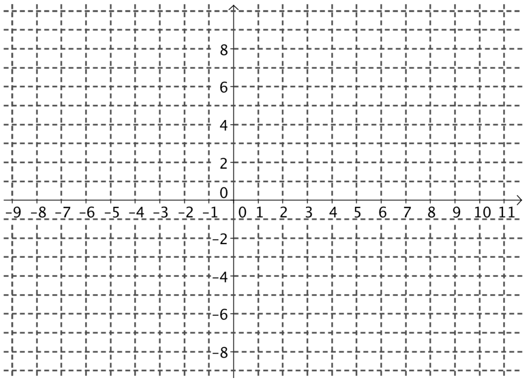
What do you know from the equation that helps you to graph the function?

7. $y = 2^n$



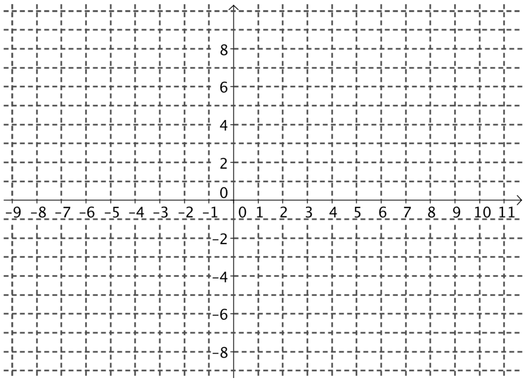
What do you know from the equation that helps you to graph the function?

8. $y = -2(x + 6) + 8$



What do you know from the equation that helps you to graph the function?

9. $f(1) = -5, f(n) = f(n - 1) + 1$



What do you know from the equation that helps you to graph the function?

2.10 Form Follows Function – Teacher Notes

A Practice Understanding Task

Purpose:
The purpose of the task is to build fluency with the procedural work of linear and exponential functions. This task is designed to help students recognize the information given in a problem and use it efficiently. In the task, students will work with both linear and exponential functions given in tables, graphs, equations, and story contexts. They will construct various representations, with an emphasis on writing equations using various forms and using equations to graph the functions.

Core Standards Focus:

F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

- Graph linear and quadratic functions and show intercepts, maxima, and minima.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior

★For F.IF.7a, 7e, and 9 focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y=3^n$ and $y=100\cdot2^n$.

A-SSE.6 Use the structure of an expression to identify ways to rewrite it.

Standards for Mathematical Practice

SMP 3 - Construct viable arguments and critique the reasoning of others.

SMP 6 – Attend to precision.

The Teaching Cycle

Launch (Whole Class): Begin by telling students that various forms of equations are tools for our work. When we solve problems, different information is available every time, so having a different ways of using the information is often handy. Briefly review the names for the various forms of the equations given at the beginning of the task. Tell students that in this task they should be working on becoming fluent with writing and using equations for both linear and exponential functions.

Explore (Small Group): Monitor students as they work, looking for students recognizing when one form may be more efficient than another. What are the features of the problem that make one form more useful than another? Look for students that have used different forms for writing equations on the same problem to share during the discussion. Also watch for problems that are more difficult for students so that they can be discussed.

Discuss (Whole Class): Begin the discussion with one of the linear problems (1-4) that two students have used different strategies to model. Ask them to present their reasoning about the strategy they chose and then ask the class to compare which strategy is the most efficient. Share as many problems as time permits, each time asking students why they selected a particular form and why it was the most efficient form to use in the circumstance.

As students share how they graphed the functions given, emphasize the use of the information given in each form. When we recognize the information and know how to use it, it often makes it unnecessary to construct a table of values, saving time. It also helps us to have a quick image to help in visualizing equations when we are trying to think about a particular function.

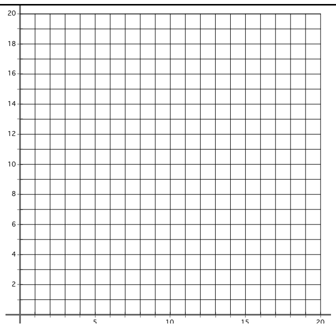
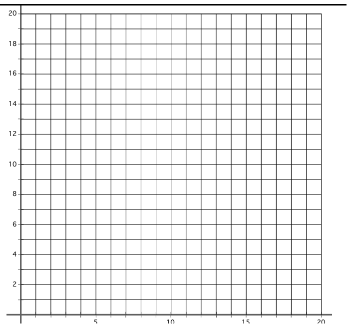
Aligned Ready, Set, Go: ***Linear and Exponential Functions 2.10***

READY, SET, GO! Name Period Date

READY

Topic: Comparing linear and exponential models.

Comparing different characteristics of each type of function by filling in the cells of each table as completely as possible.

	$y = 4 + 3x$	$y = 4(3^x)$
1. Type of growth		
2. What kind of sequence corresponds to each model?		
3. Make a table of values	x y	x y
4. Find the rate of change		
5. Graph each equation. Compare the graphs. What is the same? What is different?		
6. Find the y-intercept for each function.		

7. Find the y-intercepts for the following equations

a) $y = 3x$

b) $y = 3^x$

8. Explain how you can find the y-intercept of a linear equation and how that is different from finding the y-intercept of a geometric equation.

SET

Topic: Efficiency with different forms of linear and exponential functions.

For each exercise or problem below use the given information to determine which of the forms would be the most efficient to use for what is needed. (See task 2.6, Linear: slope-intercept, point-slope form, recursive, Exponential: explicit and recursive forms)

9. Jasmine has been working to save money and wants to have an equation to model the amount of money in her bank account. She has been depositing $175 a month consistently, she doesn't remember how much money she deposited initially, however on her last statement she saw that her account has been open for 10 months and currently has $2475 in it. Create an equation for Jasmine.

Which equation form do you chose? **Write the equation.**

10.

The table below shows the number of rectangles created every time there is a fold made through the center of a paper. Use this table for each question.

Folds	Rectangles
1	2
2	4
3	8
4	16

A. Find the number or rectangles created with 5 folds.

Which equation form do you chose? **Write the equation.**

B. Find the number of rectangles created with 14 folds.

Which equation form do you chose? **Write the equation.**

11. Using a new app that I just downloaded I want to cut back on my calorie intake so that I can lose weight. I currently weigh 90 kilograms, my plan is to lose 1.2 kilograms a week until I reach my goal. How can I make an equation to model my weight loss for the next several weeks.

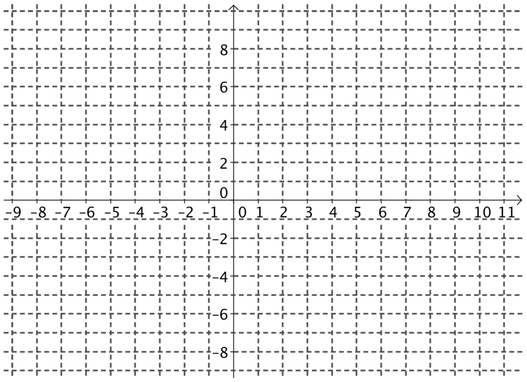
Which equation form do you chose? **Write the equation.**

12. Since Scott started doing his work out plan Janet has been inspired to set her self a goal to do more exercise and walk a little more each day. She has decided to walk 10 meters more every day. On the day 20 she walked 800 meters. How many meters will she walk on day 21? On day 60?

Which equation form do you chose? **Write the equation.**

For each equation provided state what information you see in the equation that will help you graph it, then graph it. Also, use the equation to fill in any four coordinates on the table.

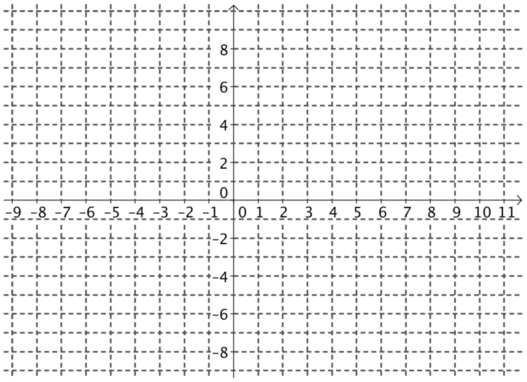
13. $y = \left(\frac{1}{2}\right)^n 8$



What do you know from the equation that helps you to graph the function?

14. $y = 5(x - 2) - 6$

What do you know from the equation that helps you to graph the function?



GO

Topic: Solving one-step equations with justification.

Recall the two properties that help us solve equations.

The Additive property of equality states:
You can add any number to both sides of an equation and the equation will still be true.

The Multiplicative **property of equality states:**
You can multiply any number to both sides of an equation and the equation will still be true.

Solve each equation. Justify your answer by identifying the property(s) you used to get it.

Example 1: $x - 13 = 7$	Justification
$+13 \quad +13$	additive property of equality
$x + 0 = 20$	addition
$x = 20$	additive identity (You added 0 and got x.)

Example 2: $5x = 35$	Justification
$\frac{5}{5}x = \frac{35}{5}$	multiplicative property of equality (multiplied by $\frac{1}{5}$)
$1x = 7$	multiplicative identity (A number multiplied by its reciprocal = 1)

15. $3x = 15$ Justification	16. $x - 10 = 2$ Justification
17. $-16 = x + 11$ Justification	18. $6 + x = 10$ Justification
19. $6x = 18$ Justification	20. $-3x = 2$ Justification

2.11H I Can See—Can't You?

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A Solidify Understanding Task

Kwan's parents bought a home for $50,000 in 1997 just as real estate values in the area started to rise quickly. Each year, their house was worth more until they sold the home in 2007 for $309,587.

1. Model the growth of the home's value from 1997 to 2007 with both a linear and an exponential equation. Graph the two models below.

Linear model:

Exponential model:

2. What was the change in the home's value from 1997 to 2007?

The average rate of change is defined as the change in y (or *f(x)*) divided by the change in x.

3. What was the average rate of change of the linear function from 1997 to 2007?

4. What is the average rate of change of the exponential function in the interval from 1997 to 2007?

5. How do the average rates of change from 1997 to 2007 compare for the two functions? Explain.

6. What was the average rate of change of the linear function from 1997 to 2002?

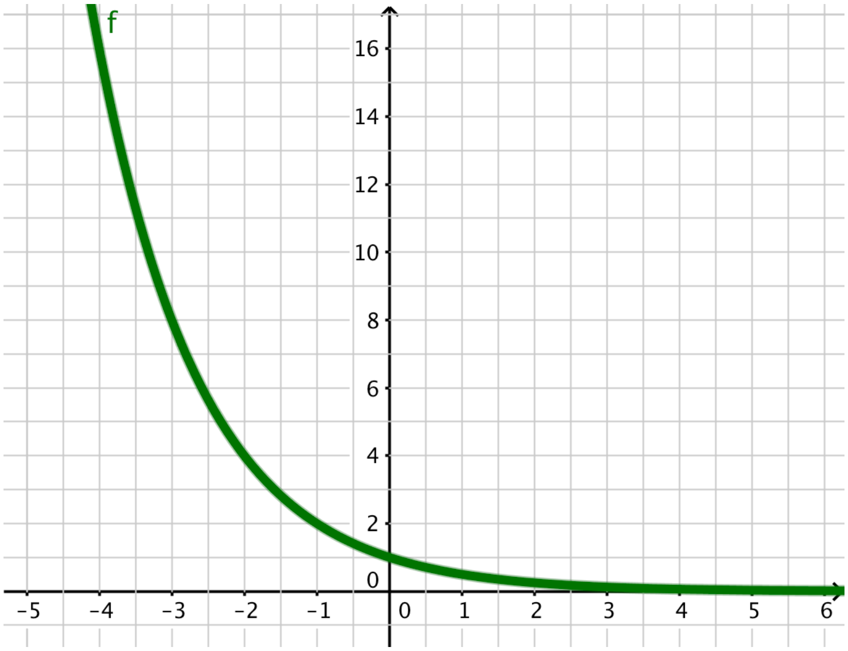
7. What is the average rate of change of the exponential function in the interval from 1997 to 2002?

8. How do the average rates of change from 1997 to 2002 compare for the two functions? Explain.

9. How can you use the equation of the exponential function to find the average rate of change over a given interval?

   How does this process compare to finding the slope of the line through the endpoints of the interval?

Consider the following graph:



10. What is the equation of the graph shown?

11. What is the average rate of change of this function on the interval from x = -3 to x = 1?

12. What is the average rate of change of this function in the interval from x = -3 to x = 0?

13. What is the average rate of change of this function in the interval from x = -3 to x = -1?

14. What is the average rate of change of this function in the interval from x = -3 to x = -1.5?

15. Draw the line through the point at the beginning and end of each of the intervals in 11, 12, 13 and 14. What is the slope of each of these lines?
16. Which of these average rates of change best represents the change at the point (-2, 4)? Explain your answer.

17. How does the average rate of change compare to the change factor for an exponential function? What is described by each of these quantities?

2.11H I Can See—Can't You? – Teacher Notes

A Solidify Understanding Task

Purpose:

The purpose of this task is to introduce students to the idea of the average rate of change of a function in a given interval. Students will compare the average rate of change in a given interval for both a linear and an exponential function. This will develop the idea of the slope of the secant line through the endpoints of the interval is equal to the average rate of change of the function in the interval. Students will calculate average rates of change in an increasingly smaller interval and be asked to consider how they could model the instantaneous rate of change at a given point. This task introduces many ideas that are typically taught in a Calculus course, in the context of linear and exponential functions.

Core Standards Focus:

F.IF Interpret functions that arise in applications in terms of the context

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

SMP 7 – Look for and make use of structure

The Teaching Cycle:

Launch 1 (Whole Group)

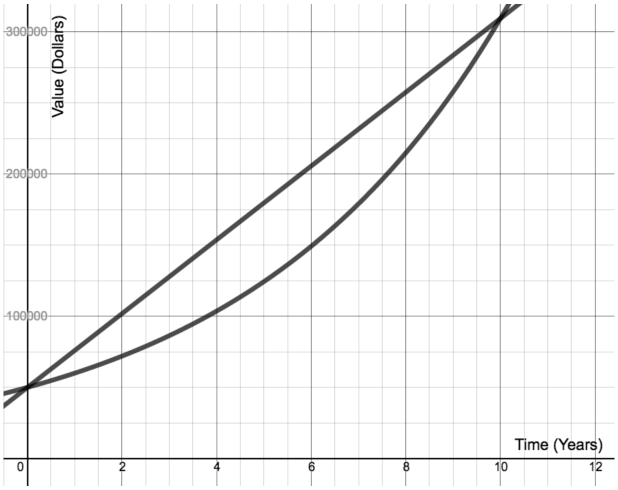
Before starting, be sure that students have graphing calculators or other technology tools available. Begin this lesson by providing necessary background information about home prices and reading the problem situation. Ask students to develop the linear and exponential models for the situation and calculate the change, problems 1 and 2. It will be useful to recommend using 1997 as year 0 for the functions.

Explore 1 (Small Group)

Most students will probably create their exponential model using a guess and check strategy. Many will guess at a change factor and create a table like this one, which shows 20% growth each year (the right change factor for this data).

	Year	Value
1997	0	50,000
1998	1	60000
1999	2	72000
2000	3	86400
2001	4	103680
2002	5	124416
2003	6	149299.2
2004	7	179159
2005	8	214990.8
2006	9	257989
2007	10	309586.8

As students are working on the graph, you may need to help them to consider a useful viewing window so that they can see all the graphs that relate to this context. An example would be:



Discuss (Whole Class):

Once students have created equations and graphs, project a graph of the equations. Ask students how they calculated the change in the interval and how it appears on the graph. Students should describe subtracting the y-values at the beginning and end of the interval. Next, ask students how they could find the average rate of change of each of the linear functions over the interval from $x = 0$ to $x = 10$. The word "average" may throw them off a bit, but remind them how they have found the rate of change on a linear function previously. Then ask how they might calculate average rate of change for the exponential function. Students will probably draw upon their previous experience with averages and think about ways to add up numbers and divide, like when they have calculated means. Remind them that in this case they don't have any rates to add up, so this method will not work. Help students to think about the idea that the average rate of change will again be the change in y values, divided by the change in *x*.

Explore (Small Group): Let students work on the remaining problems. Monitor students as they work, especially in the beginning as they discuss how to find the change in *y*, divided by the change in *x*. Listen for students describe using the graph and the equation. If students are struggling, the graph will probably be the easiest way to get them started, but they will need to consider how to use the equations by problem 9. After they have used both the equation and the graph to find the average rate of change, then encourage them to use both strategies in problems 11-14. In #14 the graph will not be accurate enough, so they should use the equation and a calculator.

Watch for students that have articulated a process for using the equation to find average rate of change and for students that have created a formula (even if the notation isn't quite right) for using the equation so that these strategies can be shared in the whole group discussion. Also listen for students who are recognizing that the formula or process that they are working on is the same as finding the slope of the secant line (this term will be defined in the whole group discussion.)

Discuss (Whole Group): Start the whole group discussion with questions 3 and 4. Have students show both how they used the graph and the equation for each. Students should be noticing that these processes are exactly the same. If this is not coming up, quickly go through questions 6 and 7. Turn attention to question 9. Have students give a step-by-step process for using the equation to find the average rate of change. Then have a student write a formula for the process. If no student has a formula, walk them through the steps of developing:

Average rate of change of a function over the interval [a,b]: $\frac{f(b)-f(a)}{b-a}$

After presenting the formula, ask students about question 10. Tell them that the line that intersects a function through the endpoints of the interval is called a **secant line**. The average rate of change on an interval is the same as the slope of the secant line in that interval.

Finally turn to the remaining problems. Project the given graph for the discussion and graph the secant lines for questions 11, 12, 13, and 14. Ask students to give the slopes of the secant lines, the average rates of change over each interval. Finally ask how they might find the rate of change right at the point (-2, 4). This would be called the instantaneous rate of change because there is no interval. The four secant lines that they have drawn will suggest that diminishing the size of the interval brings us closer to the instantaneous rate of change. Ask what the line might look like at that point, helping students to visualize the tangent line at that point. Finding the slope of the tangent line and the instantaneous rate of change will be a Calculus topic. This task is designed as an early introduction to the idea.

Aligned Ready, Set, Go: ***Linear and Exponential Functions 2.7H***

2.11H

READY, SET, GO! Name Period Date

READY

Topic: Finding an appropriate viewing window.

When viewing the secant line of an exponential function on a calculator, you want a window that shows the two points on the curve that are being connected. Since exponential functions get very large or small in just a few steps, you may want to change the scale as well as the dimensions of the window. Don't be afraid to experiment until you are satisfied with what you see.

The graphs below depict an exponential function and a secant line. The equations are given. Identify the dimensions of the viewing window. Include the scale for both the x and y values. Check your answer by matching your calculator screen to the one displayed here.

1. $\boldsymbol{Y_1 = 4(0.2)^x}$ ***and*** $\boldsymbol{Y_2 = -1.92x + 4}$

WINDOW

a. X min = ____________________

b. X max = ____________________

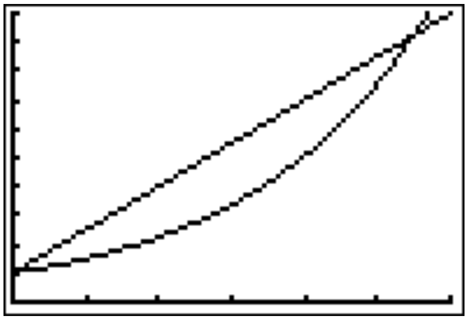
c. X scl = ____________________

d. Y min = ____________________

e. Y max = ____________________

f. Y scl = ____________________

2. $Y_1 = 1.5^X$ and $Y_2 = 1.5x + 1$



WINDOW

a. X min = ____________________

b. X max = ____________________

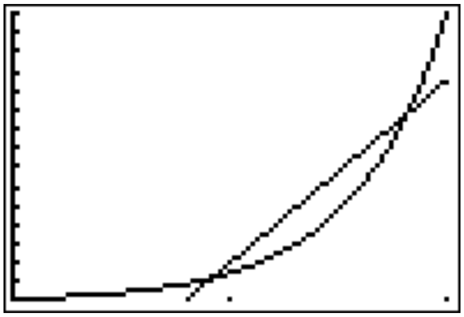
c. X scl = ____________________

d. Y min = ____________________

e. Y max = ____________________

f. Y scl = ____________________

3. $Y_1 = 150\,(10)^X$ and $Y_2 = 9500x - 7500$



WINDOW

a. X min = ______________________

b. X max = ______________________

c. X scl = ______________________

d. Y min = ______________________

e. Y max = ______________________

f. Y scl = ______________________

SET

Topic: Using slope to compare change in linear and exponential models.

The tables below show the values for a linear model and an exponential model. Use the slope formula between *each set of 2 points* to calculate the rate of change.

Example: Find the slope between the points (30 , 1) and (630 , 2) then between (630 , 2) and (1230 , 3). Do the same between each pair of points in the table for the exponential model.

Linear Model

x	*y*
1	30
2	630
3	1230
4	1830
5	2430

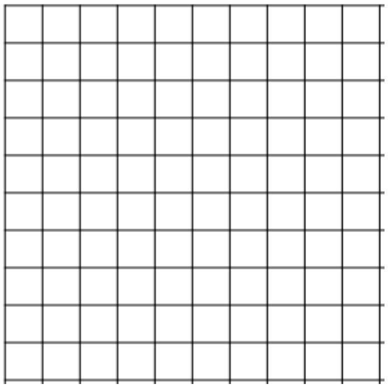
Exponential Model

x	*y*
1	30
2	90
3	270
4	810
5	2430

4. Compare the change between each pair of points in the linear model to the change between each pair of points in the exponential model. Describe your observations and conclusions.

5. Find the average of the 4 rates of change of the exponential model. How does the average of the rates of change of the exponential model compare to the rates of change of the linear model?

6. Without using a graphing calculator, make a rough sketch on the same set of axes of what you think the linear model and the exponential model would look like.



7. How did your observations in #5 influence your sketch?

8. Explain how a table of 5 consecutive values can begin and end with the same y-values and be so different in the middle 3 values. How does this idea connect to the meaning of a secant line?

GO

Topic: Finding slope between points.

Use your calculator and the slope formula to find the slope of the line that passes through the two points.

9. A (-10, 17) , B (10, 97)

10. P (57, 5287) , Q (170, 4948)

11. R (6.055, 23.1825) , S (5.275, 12.0675)

12. G (0.0012, 0.125) , H (2.5012, 6.375)