mathematics vision project

*Transforming Mathematics Education*

# ALGEBRA I

*A Learning Cycle Approach*

MODULE 3

# Features of Functions

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**The Mathematics Vision Project**

*Scott Hendrickson, Joleigh Honey, Barbara Kuehl, Travis Lemon, Janet Sutorius*

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Identify whether or not a relation is a function given various representations ( F.IF.1, F.IF.3)

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# 3.1 Getting Ready for a Pool Party

## *A Develop Understanding Task*



Sylvia has a small pool full of water that needs to be emptied and cleaned, then refilled for a pool party. During the process of getting the pool ready, Sylvia did all of the following activities, each during a different time interval.

| | |
|---|---|
| Removed water with a single bucket | Filled the pool with a hose (same rate as emptying pool) |
| Drained water with a hose (same rate as filling pool) | Cleaned the empty pool |
| Sylvia and her two friends removed water with her three buckets | Took a break |

1. Sketch a possible graph showing the height of the water level in the pool over time. Be sure to include all of the activities Sylvia did to prepare the pool for the party. Remember that only one activity happened at a time. Think carefully about how each section of your graph will look, labeling where each activity occurs.

2. Create a story connecting Sylvia's process for emptying, cleaning, and then filling the pool to the graph you have created. Do your best to use appropriate math vocabulary.

3. Does your graph represent a function? Why or why not? Would all graphs created for this situation represent a function?

# 3 . 1 Getting Ready for a Pool Party – Teacher Notes

## *A Develop Understanding Task*

**Purpose**: This task is designed to develop the ideas of features of functions using a situation. Features of functions such as increasing/decreasing and maximum/minimum can be difficult for students to understand, even in a graphical representation, if they are not used to reading a graph from left to right. A situation using the water level of a pool over a period of time can provide opportunities for students to make connections to these features. While some parts of the graph need to come before others (emptying the pool before filling the pool), other situations can be switched around (emptying the water with buckets and emptying the water with a hose). The key features of this task include:

- The sketch of the graph is decreasing when the water is being emptied from the pool and the graph is increasing when the pool is being filled with water.
- The sketch of the graph shows a height of zero during a period of time where the pool is empty and being cleaned.
- The sketch of the graph is continuous when the hose is used (both for filling and emptying) and that the rate of change is the same both when filling and when emptying.
- The sketch of the graph looks like a "step function" when using a bucket, with the water level dropping three times faster when Sylvia has friends assisting.
- Students communicate their understanding of graphs in part 2
- Students express that this situation is a function by indicating that every input of time has exactly one output representing the depth of water.

**Core Standards Focus**:

**F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

**Related Standards:** F.IF.1, A.REI.10, F.IF.5, F.IF.7

**Standards for Mathematical Practice:**

**SMP 1 – Make sense of problems and persevere in solving them**

**SMP 3 – Construct viable arguments and critique the reasoning of others**

**The Teaching Cycle:**

**Launch (Whole Class):**

Read the initial situation and the first question. Make sure students understand they are to graph a situation where all methods for emptying, cleaning, and then filling the pool are used in the problem. When they have completed their graph, they are to write a story connecting Sylvia's process for emptying, cleaning, and then filling the pool to the graph they have created.

**Explore (Small Group):**

Your students may already be familiar with strategies for creating graphs given a situation. They are also familiar with slope and rate of change as well as graphing continuous and non-continuous situations (it can be argued that the graph is continuous even during the bucket removal as there is not an instantaneous jump). The context of this problem focuses on decreasing and increasing intervals of the graph, rate of change, and the idea of a function being a continuous linear relationship versus a situation whose rate changes in a 'step function' fashion. During the monitoring phase, press students who are not being specific enough by asking questions such as:

- What is happening during each interval of time on their graph?
- Compare and contrast the different activities. How should the graph for these situations look similar? Different?

This will help bring out the features of functions described in this lesson (see purpose statement above). Other than the features of functions listed above, this task also surfaces the idea of domain and step functions. If you do not have any students create a step function representation during the bucket situation, but they do show a discrete representation, plan to use this during the discussion part of the lesson to get at how the domain is continuous, even if the graph is not.

Have students share their story with a partner, then have them discuss what they agree about with each other's graph as well as possible errors in their thinking.

**Discuss (Whole Class):**

Choose students to share who have the following as part of their graph. (Be sure students who are sharing can also show their graphs to all students while explaining the features of their graphs):

- Student 1: have a student share that has labeled their axes and has clear ideas about where the graph should be increasing/decreasing. This student is not sharing their story, but highlighting features of their function.
- Student 2: have a student share that has represented the bucket part of the graph to be 'discrete' in nature (not a continuous decreasing linear graph). Highlight the difference between a continuous constant rate of change versus the 'jump' in the amount of water in the pool when using a bucket. At this point, we are still focusing on increasing/decreasing, comparing rates and the idea that a function can have different components within the function (i.e. continuous and discrete). If the conversation naturally comes up at this time about how the discrete portion of the graph should look different (more like a step function), then discuss this now and where appropriate discuss discontinuous versus discrete. Otherwise wait until the next student shares and then bring it all together. Again, this student is not sharing their story, but highlighting features of their function.
- Student 3: select a student to share their story while showing each part on the graph. Be sure to choose someone with an accurate story to highlight the features that are focused upon in this task. Be sure the discussion includes these features: increasing/decreasing, the y-intercept, labeling the axes and interpreting what this means, and the rates of change. At this time, if the 'step function' conversation has not occurred, use a graph that shows the bucket situation as being discrete, then ask students questions such as:
    - Does the graph tell a complete story?
    - Pointing to an interval of time that is continuous, ask students to describe what is happening at each moment.
    - Point to the discrete part of the graph and ask how much water is in the pool between the two discrete points.

While students do not need to know how to graph step functions in Secondary I, the purpose of this conversation is to have students connect that every point on a graph is a solution (A.REI.10 ) and that since time is continuous, every input value (the domain) exists from the beginning to the end of emptying, cleaning, then filling the pool (F.IF.5).

**Aligned Ready, Set, Go: Features 3.1**

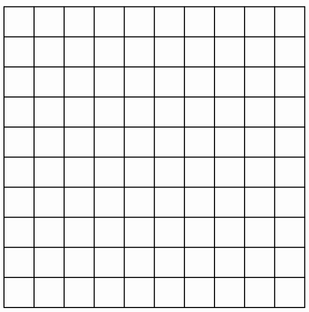
READY, SET, GO! Name Period Date

## *READY*

Topic: Graphing Linear and Exponential Functions

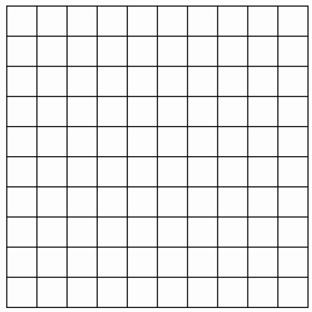
**Graph each of the functions. Name 3 points that lie on each graph. Choose a scale for your graph that will make it possible to graph your 3 chosen points.**

1. $f(x) = -2x + 5$



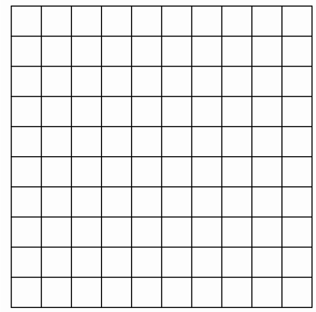
3 points:

2. $g(x) = 4 - 3x$



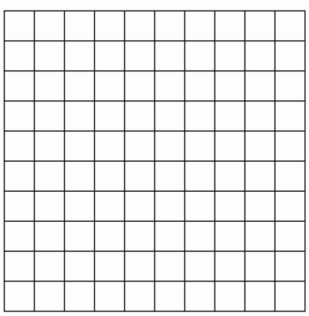
3 points:

3. $h(x) = 5(3)^x$



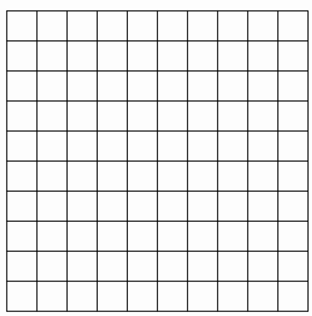
3 points:

4. $k(x) = 4(2)^x$



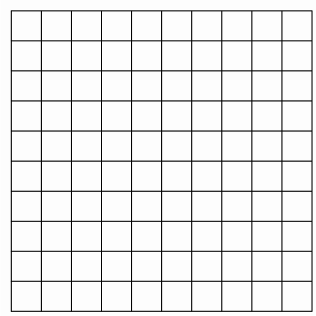
3 points:

5. $v(t) = 2.5t - 4$



3 points:

6. $f(x) = 8(3)^x$

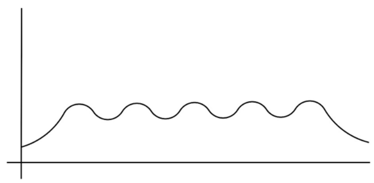
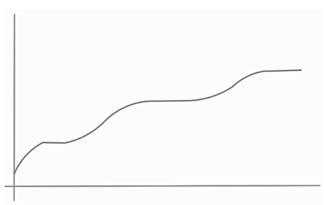
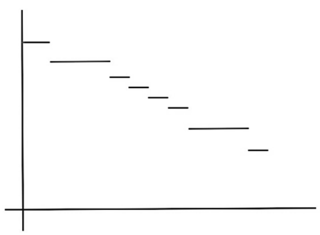
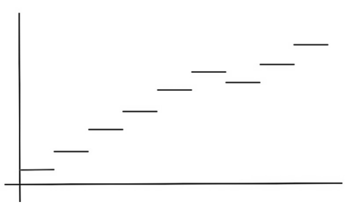
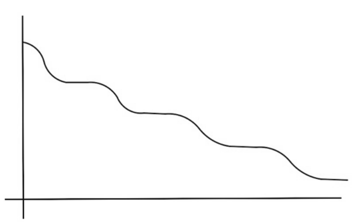


3 points:

## *SET*

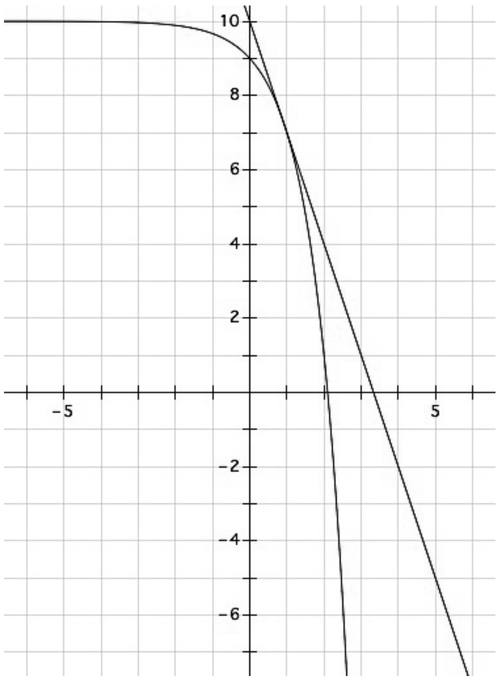
Topic: Describing attributes of a functions based on graphical representation

**For each graph given match it to the contextual description that fits best. Then label the independent and dependent axes with the proper variables.**

| Graphs | Contextual Descriptions |
|---|---|
| 7.  | a. The amount of money in a savings account where regular deposits and some withdrawals are made. |
| 8.  | b. The temperature of the oven on a day that mom bakes several batches of cookies. |
| 9.  | c. The amount of gasoline on hand at the gas station before a tanker truck delivers more. |
| 10.  | d. Watermelons are delivered to a farmer's market every Saturday morning. The number of watermelons available for sale on Thursday. |
| 11.  | e. The amount of mileage recorded on the odometer of a delivery truck over a time period. |

**Given the pair of graphs on each coordinate grid, create a list of similarities the two graphs share and a list of differences. (Consider attributes like, continuous, discrete, increasing, decreasing, linear, exponential, restrictions on domain or range, etc.)**

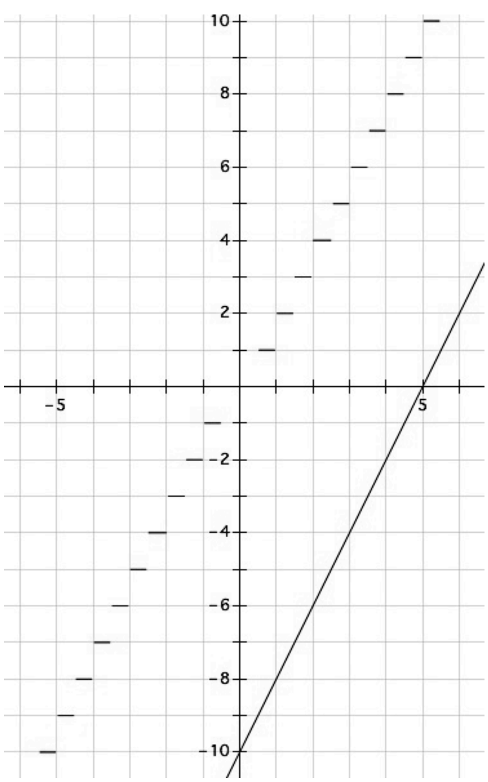
12.



Similarities:

Differences:

13.



Similarities:

Differences:

## *GO*

Topic: Solving equations

**For each equation find the value of x that makes it true. (Hint for #20 and #22: when solving a linear equation, you need to get the term containing the variable alone on one side. When solving an exponential equation, you also need to get the term containing the variable alone on one side.)**

14. $10^x = 100{,}000$

15. $3x + 7 = 5x - 21$

16. $-6x - 15 = 4x + 35$

17. $5x - 8 = 37$

18. $3^x = 81$

19. $3x - 12 = -4x + 23$

20. $10 = 2^x - 22$

21. $243 = 8x + 3$

22. $5^x - 7 = 118$

# 3.2 Floating Down the River

## *A Solidify Understanding Task*

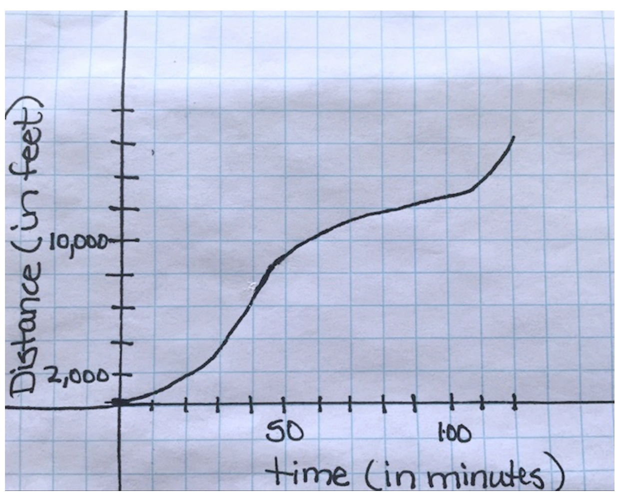


Alonzo, Maria, and Sierra were floating in inner tubes down a river, enjoying their day. Alonzo noticed that sometimes the water level was higher in some places than in others. Maria noticed there were times they seemed to be moving faster than at other times. Sierra laughed and said "Math is everywhere!" To learn more about the river, Alonzo and Maria collected data throughout the trip.

| Time (in minutes) | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Depth (in feet) | 4 | 6 | 8 | 10 | 6 | 5 | 4 | 5 | 7 | 12 | 9 | 6.5 | 5 |

1. Use the data collected by Alonzo to interpret the key features of this relationship.

2. Maria created a graph by collecting data on a GPS unit that told her the distance she had traveled over a period of time.




Using the graph created by Maria, describe the key features (increasing, decreasing, domain, range, maximum, minimum, intercepts) of this relationship.

Part II: Interpreting data

3. Sierra looked at the data collected by her two friends and made several of her own observations. Explain why you either agree or disagree with each observation made.

a) The depth of the water increases and decreases throughout the 120 minutes of floating down the river.

b) The distance traveled is always increasing.

c) The distance traveled is a function of time.

d) The distance traveled is greatest during the last ten minutes of the trip than during any other ten minute interval of time.

e) The domain of the distance/time graph is all real numbers.

f) The y-intercept of the depth of water over time function is (0,0).

g) The distance traveled increases and decreases over time.

h) The depth of the water is never 11 feet.

i) The range of the distance/time graph is from [0, 15000].

j) The domain of the depth of water with respect to time is from [0,120]

k) The range of the depth of water over time is from [4,5].

l) The distance/ time graph has no maximum value.

m) The depth of water reached a maximum at 30 minutes.

# 3 . 2 Floating Down the River – Teacher Notes

## *A Solidify Understanding Task*

**Purpose**: The purpose of this task is to further define function and to solidify key features of functions given different representations. Features include:

- domain and range
- where the function is increasing or decreasing
- *x and y* intercepts
- rates of change (informal)
- discrete versus continuous

**Core Standards Focus:**

**F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

**F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

**Related Standards: F.IF.1, F.IF.6, A.REI.10**

**Standards for Mathematical Practice:**

**SMP 2 – Reason abstractly and quantitatively**

**SMP 3 – Construct viable arguments and critique the reasoning of others**

**The Teaching Cycle:**

**Launch (Whole Class):** Read the scenario out loud and highlight the difference between the two representations (depth of water over time and distance traveled over time). Before students begin, access their background knowledge by asking "What are some of the key features of functions we look for when interpreting graphs (or tables)?" These features are still fairly new to students, with some being introduced in the prior task. After the key features are mentioned (domain, range, intervals where the function is increasing/decreasing, and intercepts), have students work on the task in small groups.

**Explore (Small Group):** As you monitor, listen for student explanations as they interpret key features of both the table and the graph. If students struggle with writing features based on the table, you may ask them to tell the story represented by the table values or prompt with a question such as "How deep is the water at 0 minutes? What do you know about the water level after this time?" Another prompt would be to ask students if using another representation would help them see the features. Look for students who make a visual connection to the table, students who use notation to identify where the water level is increasing/decreasing, and students who notice interesting features (such as we do not really know the depth of the water at 7 minutes). Encouraging students to visually connect the key feature described by the mathematical representation will help during the whole group discussion.

**Note:** If most students are struggling to name the features using the two representations (problem 1: table, problem 2: graph), bring the whole group together after all groups have had time to work on the table representation. Select a student who has visually connected the table to the context to describe where the water level is increasing and decreasing. Then ask the whole group what other features can be interpreted from the table. Include domain, range, and intercepts at this time. If a student has used interval notation, have them share. At some point, we want to bring up that while the table values are discrete, the function is continuous. If a student brings this up now, address it. Otherwise wait until the whole group discussion of the entire task.

**Discuss (Whole Class):** The goal of the whole group discussion is to highlight the connections between a given representation and the key features of that function. Be sure to use academic vocabulary throughout the whole group discussion. Before starting the whole group discussion, post the table and the graph so students can better communicate their observations about the feature they are describing. Begin the whole group discussion by going over a couple of the observation statements made by Sierra that will create an opportunity to have students communicate viable arguments. For example, the range of the depth of water conversation will bring out how it is important to look at output values when discussing the range, and not the beginning and ending point of the trip. After going over a couple of the observation statements, choose a student to share the key features of the graph (distance vs. time). Be sure this student highlights each feature on the graph while also writing the interpreted feature next to the graph. After the features are shared, ask the whole group about the other observation statements made by Sierra that relate to the graph. Then choose another student to share the key features of the table. After, go over the observation statements made by Sierra. If, at this time, the conversation has not come up about how this table of values is discrete but represents a continuous function, ask students about the domain. Be sure that at the end of this discussion, students understand that a table of values only shows some of the solution points for a continuous function.

**Aligned Ready, Set, Go: Features 3.2**

READY, SET, GO! Name Period Date

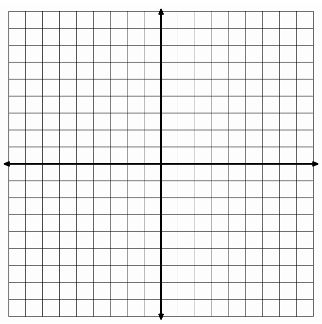
## *READY*

Topic: Solve Linear Systems by Graphing

**Graph each set of linear equations on the same set of axes. Name the coordinates of the point where the two lines intersect.**

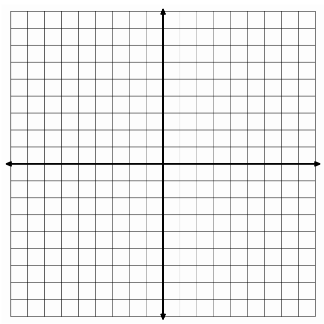
1. $\begin{cases} f(x) = 2x - 7 \\ g(x) = -4x + 5 \end{cases}$

Point of intersection:



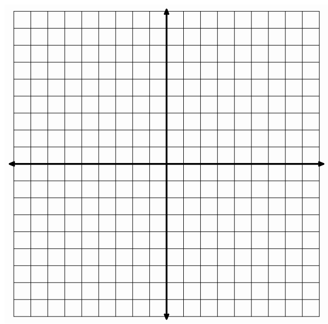
2. $\begin{cases} f(x) = -5x - 2 \\ g(x) = -2x + 1 \end{cases}$

Point of intersection:



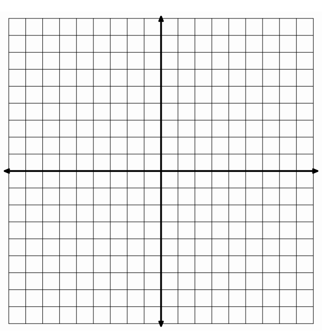
3. $\begin{cases} f(x) = -x - 2 \\ g(x) = 2x + 10 \end{cases}$

Point of intersection:



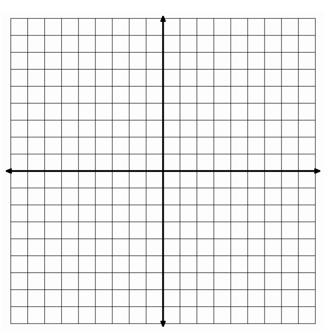
4. $\begin{cases} f(x) = x - 5 \\ g(x) = -x + 1 \end{cases}$

Point of intersection:



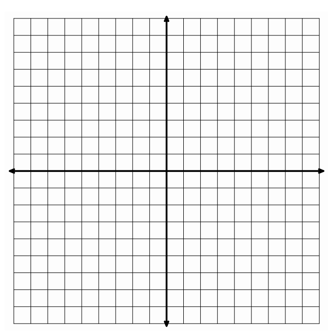
5. $\begin{cases} f(x) = \frac{2}{3}x + 4 \\ g(x) = -\frac{1}{3}x + 1 \end{cases}$

Point of intersection:



6. $\begin{cases} f(x) = x \\ g(x) = -x - 2 \end{cases}$

Point of intersection:

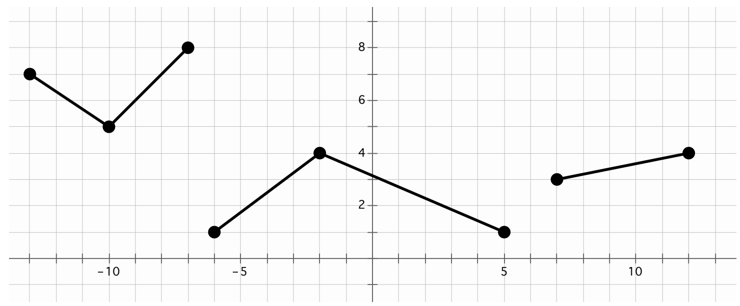


## *SET*

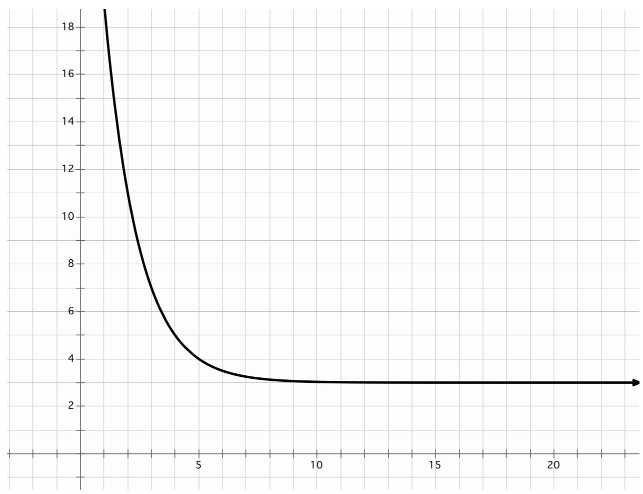
Topic: Describing attributes of a functions based on graphical representation

**For each graph state 1)the interval(s) where it is increasing, decreasing, or constant 2)if it has a minimum or maximum, and 3)identify the domain and range. Use interval notation.**

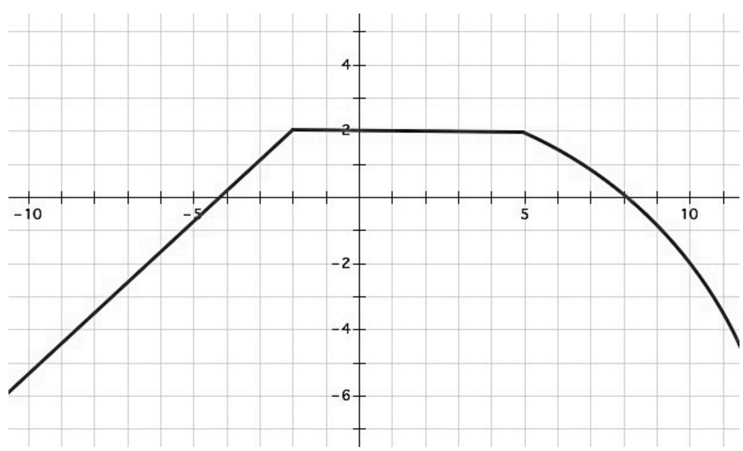
7. Description of function




8. Description of function




9. Description of function



## GO

Topic: Creating both explicit and recursive equations

**Write equations for the given tables in both recursive and explicit form.**

10.

| $n$ | $f(n)$ |
|---|---|
| 1 | 5 |
| 2 | 2 |
| 3 | -1 |

Explicit:

Recursive:

11.

| $n$ | $f(n)$ |
|---|---|
| 1 | 6 |
| 2 | 12 |
| 3 | 24 |

Explicit:

Recursive:

12.

| $n$ | $f(n)$ |
|---|---|
| 0 | -13 |
| 2 | -5 |
| 3 | -1 |

Explicit:

Recursive:

13.

| $n$ | $f(n)$ |
|---|---|
| 1 | 5 |
| 4 | 11 |
| 5 | 13 |

Explicit:

Recursive:

14.

| $n$ | $f(n)$ |
|---|---|
| 2 | 5 |
| 7 | 15,625 |
| 9 | 390,625 |

Explicit:

Recursive:

15.

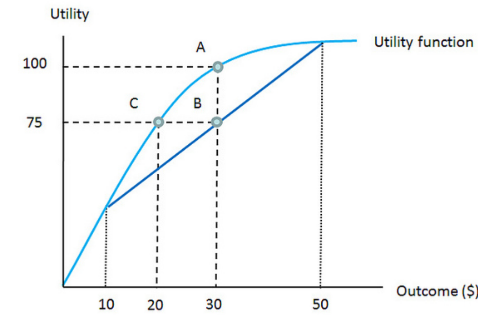
| $n$ | $f(n)$ |
|---|---|
| 0 | -4 |
| 1 | -16 |
| 2 | -64 |

Explicit:

Recursive:

# 3.3 Features of Functions

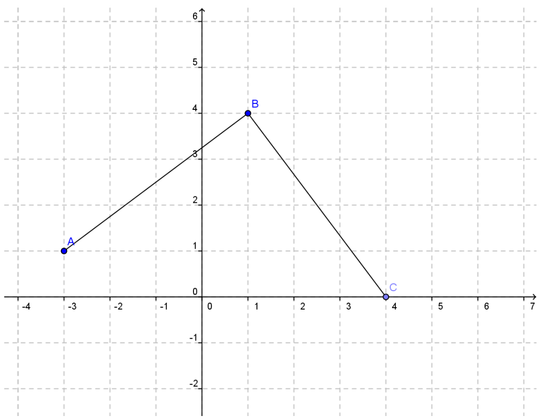
## *A Practice Understanding Task*



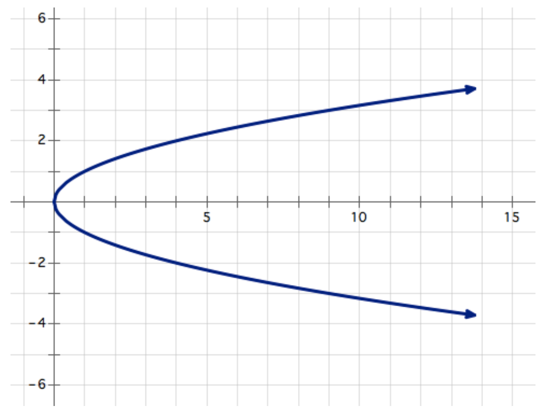



For each graph, determine if the relationship represents a function, and if so, state the key features of the function (*key features include intercepts, intervals where the function is increasing or decreasing, relative maximums and minimums, symmetries, domain and range, and end behavior).*

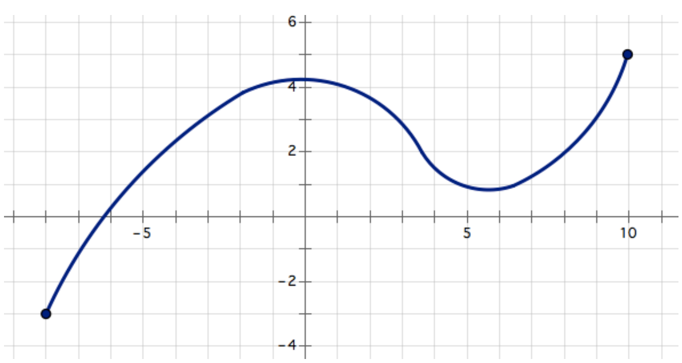
1.



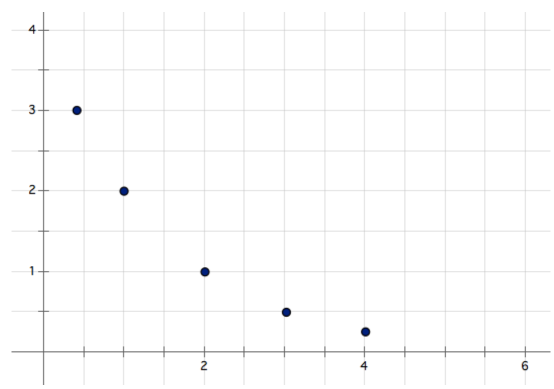

2.



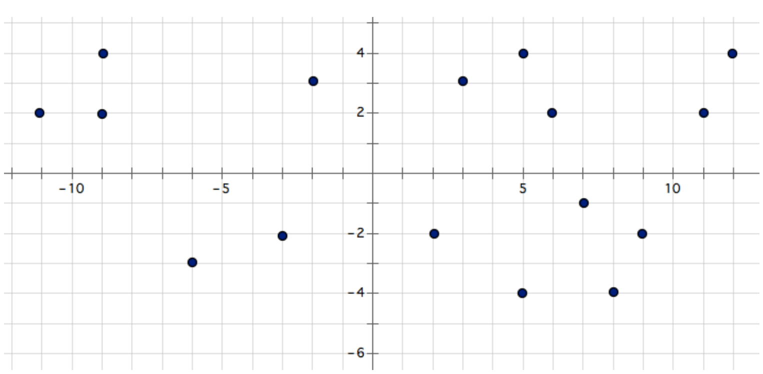
3.



4.



5.

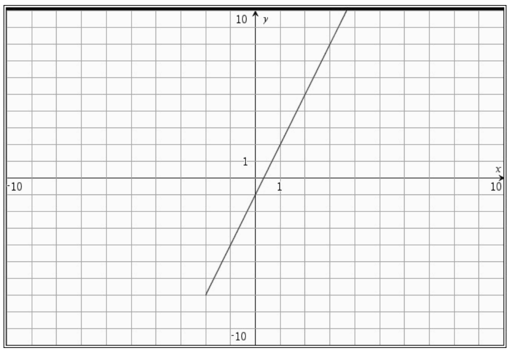






mathematics vision project

6.

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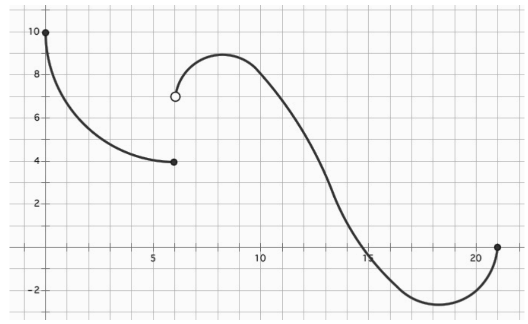



8. The table on the right represents a continuous function defined on the interval from [0, 6].

   a) Determine the domain, range, x and y intercepts.

   b) Based on the table, identify the minimum value and where it is located.

| $x$ | $f(x)$ |
|---|---|
| 0 | 2 |
| 1 | -3 |
| 2 | 0 |
| 3 | 2 |
| 4 | 6 |
| 5 | 12 |
| 6 | 20 |

9. The table represents a discrete function defined on the interval from [1,5].

   a) Determine the domain, range, x and y intercepts.

   b) Based on the table, identify the minimum value and where it is located.

| $x$ | $f(x)$ |
|---|---|
| 1 | 4 |
| 2 | 10 |
| 3 | 5 |
| 4 | 8 |
| 5 | 3 |

Describe the key features for each situation.

10. The amount of daylight (in hours) dependent on the month of the year.

11. The first term in a sequence is 36. Each consecutive term is exactly 1/2 of the previous term.

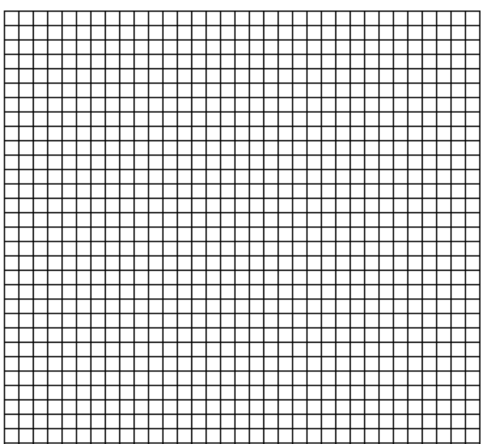
12. Marcus bought a $900 couch on a six months, interest free payment plan. He makes $50 payments to the loan each week.

13. The first term in a sequence is 36. Each consecutive term is 1/2 less than the previous term.

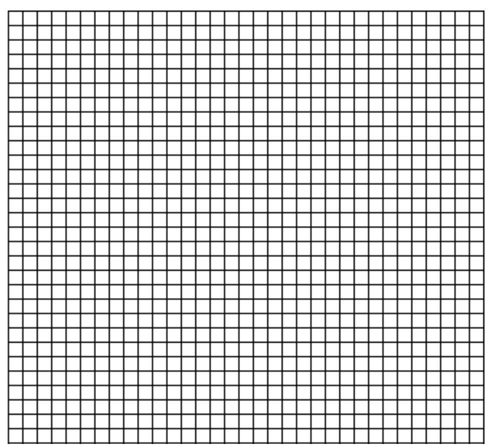
14. An empty 15 gallon tank is being filled with gasoline at a rate of 2 gallons per minute.

For each equation, sketch a graph and describe the key features of the graph.

15. $f(x) = -2x + 4, when\ \ x \geq 0$



16. $g(x) = 3^x$



# 3.3 Features of Functions– Teacher Notes

## *A Practice Understanding Task*

**Purpose**: This task is designed for students to practice interpreting key features of functions using graphs, a table of values, and situations. The key features of this task include students:

- Applying their knowledge to interpret key features of functions (domain, range, increasing, decreasing, maximum, minimum, intercepts).
- Practicing writing the domain of a function
- Comparing discrete and continuous situations
- Graphing linear and exponential equations and describing key features of the graph

**Core Standards Focus**:

**F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

**F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

**Related Standards: F.IF.1, F.IF.3**

**Standards for Mathematical Practice:**
**SMP 3 – Look for and Make Use of Structure**
**SMP 6 – Attend to Precision**
**SMP 8 – Look for and Express Regularity in Repeated Reasoning**

**The Teaching Cycle:**

**Launch (Whole Class):**
Students should be able to get started on this task without additional support, since it is similar in nature to the work they did on "Getting Ready for a Pool Party" and "Floating Down the River". This would be a good task to have students do in pairs.

**Explore (Small Group):**
Monitor students to make sure they are accurately answering the questions about the features of functions. If they are only writing down one or two features, ask them what other features they notice. This is a good task to have students justify their answer to their partner as they go through the task. Correct student misconceptions as you see them, noting problems that seem to highlight common misconceptions of the group. As you monitor, select problems to discuss as a whole group to refine student thinking and address misconceptions.

**Discuss (Whole Class):**
Since this is a Practice Task, the discussion should include going over problems that seem to be common issues as well as problems that drive home the standards. To start the whole group discussion, choose a student to go over all of the features of one of the graphs to make sure the proper vocabulary and corresponding features are shown. Use this example to then go over features that are still confusing for students. The goal of this whole group discussion is that ALL students can explain a process for interpreting key features of functions.

**Aligned Ready, Set, Go: Features of Functions 3.3**

READY, SET, GO! Name Period Date

## READY

Topic: Find the point of intersection for two lines by looking at the table.

**Fill in the table of values for each of the linear functions. Then circle the point of intersection of the two lines in each table.**

1. $f(x) = 3x - 5$ $g(x) = x + 1$

| $x$ | $f(x)$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |

| $x$ | $g(x)$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |

2. $f(x) = x + 2$ $g(x) = 2x$

| $x$ | $f(x)$ |
|---|---|
| | |
| | |
| | |
| | |
| | |

| $x$ | $g(x)$ |
|---|---|
| | |
| | |
| | |
| | |
| | |

3. $f(x) = 3x - 4$ $g(x) = -2x + 6$

| $x$ | $f(x)$ |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |

| $x$ | $g(x)$ |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |

4. $f(x) = 4x - 9$ $g(x) = 2x + 1$

| $x$ | $f(x)$ |
|---|---|
| | |
| | |
| | |
| | |
| | |

| $x$ | $g(x)$ |
|---|---|
| | |
| | |
| | |
| | |
| | |

## *SET*

Topic: Attributes of linear and exponential functions.

**Determine if the statement is true or false. If it is false, explain why.**

5. All linear functions are increasing.

6. Arithmetic sequences are an example of linear functions.

7. Exponential functions have a domain that includes all real numbers.

8. Geometric sequences have a domain that includes all integers.

9. The range for an exponential function includes all real numbers.

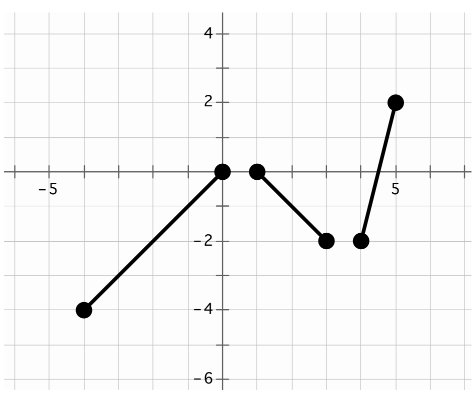
10. All linear relationships are functions with a domain and range containing all real numbers.

## *GO*

Topic: Determine the domain of a function from a graphical representation.

**For each graph state the domain of the function. Use interval notation.**

11.

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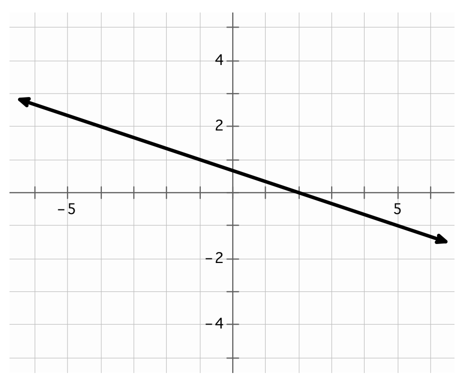


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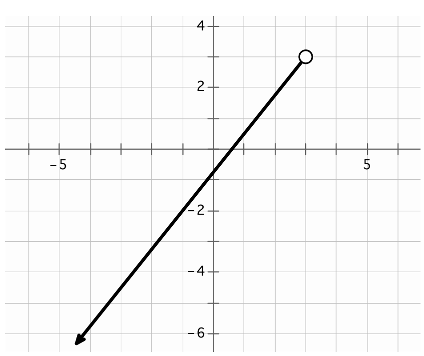


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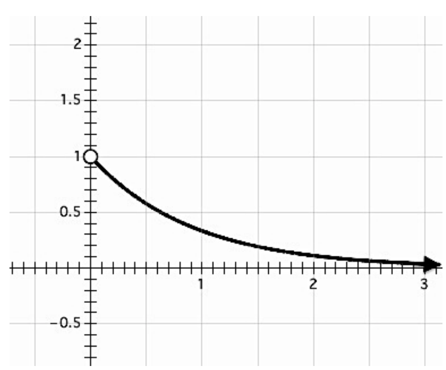
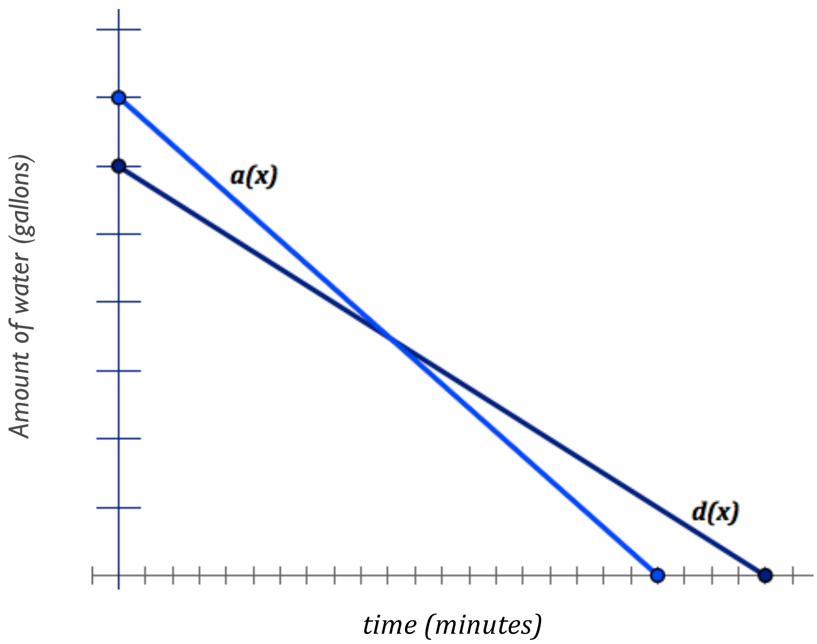


# 3.4 The Water Park

## *A Solidify Understanding Task*



Aly and Dayne work at a water park and have to drain the water at the end of each month for the ride they supervise. Each uses a pump to remove the water from the small pool at the bottom of their ride. The graph below represents the amount of water in Aly's pool, *a(x),* and Dayne's pool, *d(x)*, over time.




Part I

1. Make as many observations as possible with the information given in the graph above.

Part II

Dayne figured out that the pump he uses drains water at a rate of 1000 gallons per minute and takes 24 minutes to drain.

2. Write the equation to represent the draining of Dayne's pool, *d(x)*. What does each part of the equation mean?

3. Based on this new information, correctly label the graph above.

4. For what values of *x* make sense in this situation? (Use interval notation to write the domain of the amount of water in Dayne's pool).

5. Determine the range, or output values, that make sense in this situation. (Use interval notation to write the range of the amount of water in Dayne's pool).

6. Write the equation used to represent the draining of Aly's pool, *a(x)*. Using interval notation, state the domain and range for the function, *a(x)* as well as the domain and range of the situation. Compare the two domains by describing the constraints made by the situation.

Part III

Based on the graph and corresponding equations for each pool, answer the following questions.

7. When is $a(x) = d(x)$? What does this mean?

8. Find $a(5)$. What does this mean?

9. If $d(x)=2000$, then $x=$____. What does this mean?

10. When is $a(x) > d(x)$? What does this mean?

# 3 . 4 The Water Park – Teacher Notes

## *A Solidify Understanding Task*

**Purpose**: The purpose of this task is for students to interpret and highlight features of functions using contexts. This task provides opportunities for students to practice skills they have already learned as well as solidifying their knowledge of features of functions. This task first asks students to make observations from a graph. There are several observations to make and by having students make these observations, they are accessing their background knowledge as a way to prepare for this task. In the following sections, students solidify their understanding of domain and distinguish between the domain of a function and the domain of a situation. They also use function notation to interpret the meaning of the situation. The following mathematics should be addressed in this task:

- Interpreting x and y intercepts
- Comparing rates of change
- Finding where two functions are equivalent
- Connecting the equation to a graph and appropriately labeling a graph
- Determine the domain of a function as well as the restricted domain due to a story context
- Interpreting function notation for both input and output values

**Core Standards Focus**:

**F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

**F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

**F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes* *to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

**F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

**A.REI.11** Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

**A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

**Related Standards: F.IF.1, A.REI.10, A.REI.11, N.Q.1, A.CED.2**

**Standards for Mathematical Practice:**

**SMP 1 – Make sense of problems and persevere in solving them**

**SMP 2 – Reason abstractly and quantitatively**

**SMP 3 – Construct viable arguments and critique the reasoning of others**

**SMP 6 – Attend to precision**

**The Teaching Cycle:**

**Launch (Whole Class):**

Part 1 of this task has students make observations of a story context by interpreting two graphs on the same set of axes. The scale factor of the graph is intentionally not labeled (this will be part of what students do during the task). Students should share their observations about the story context based on their prior experience with linear functions and without having to be precise.

Begin the task with a copy of the graph displayed. After reading the prompt, have students spend a couple of minutes writing down their observations independently. Then, as a whole group, have students offer their insights to the class using appropriate academic language (i.e. decreasing, constant rate, etc). Use this opportunity to have students justify the rationale for each observation. Reinforce with visual cues (i.e. point to the intersection of the two graphs when someone shares that at a certain time, both pools will have the same amount of water). Some of the other observations to highlight include:

- Each pool is decreasing
- Each pool is decreasing at a constant rate, although not the same rate as the other
- Aly's pool is draining at a faster rate than Dayne's
- At some point in time, both pools will have the same amount of water in them
- Dayne's pool has less water in it initially
- Aly's pool will be empty before Dayne's but they will both have 0 water in them at some point in time
- At some point, both pools will be empty

**Explore (Small Group):**

After completing Part 1 as a whole group, have students continue completing Parts II and III in small groups. If students seem stuck with writing the equation for Dayne's pool, you might suggest they use a strategy they have used in the past to write the equation of a line. (Creating two points based on the information given, writing a table of values, or using the graph may be ways students determine the equation). The purpose of Part II is to have students communicate the meaning of the context and to use this information to relate the domain of the function to the situation. For Part III, listen for students to make sense of the notation. Students have background knowledge about finding intersections and missing values, and they have background knowledge about using function notation. The new twist for them here is to transfer this knowledge of making sense of the notation to answer the questions posed in this task. Encourage students to communicate this meaning in words and by showing where this shows up on the graph.

**Discuss (Whole Class):**

Most of the whole group discussion should focus on features such as domain and intercepts from Part II and making sense of notation from Part III.

For Part II, choose two different students to explain their strategies for coming up with the equations for Dayne and Aly. During this time, also make sure other highlights of Part II come out during the discussion (intercepts, domain of function versus domain of situation).

Move the conversation on to Part III. Choose students to share who have made sense of the questions. For questions 7, 8, and 9, have students explain the meaning of the question, then share their strategy for solving. Connections should be made between the graph and the equations. If some groups struggled during this portion of the task, have everyone work in their small group to solve problem 10, then go over as a whole group.

**Aligned Ready, Set, Go: Features 3.4**

READY, SET, GO! Name Period Date

## READY

Topic: Attributes of linear and exponential functions.

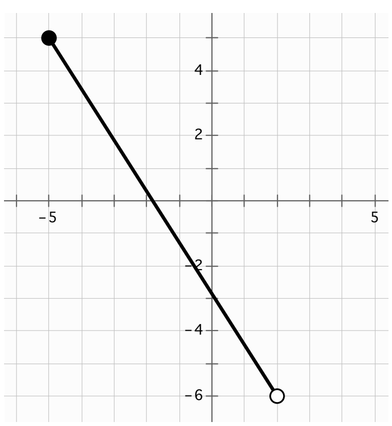
1. Compare and contrast linear and exponential functions. Be sure to include as many characteristics of each function as possible and be clear about the similarities and differences between these functions.

## SET

Topic: Identifying attributes of functions from their graphs.

**For each graph, identify the domain, range and whether or not the function is increasing or decreasing. Use interval notation when you state the domain and range.**

2.

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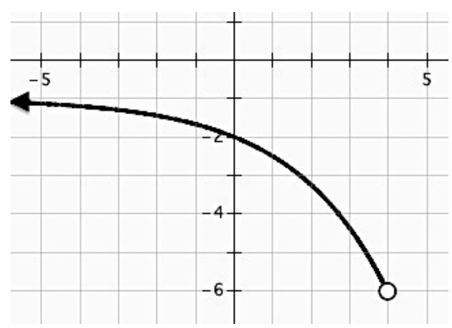

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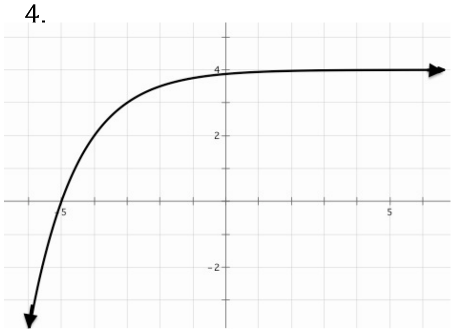

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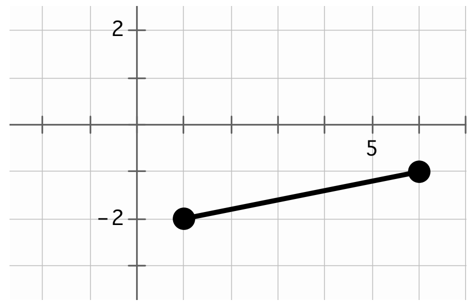


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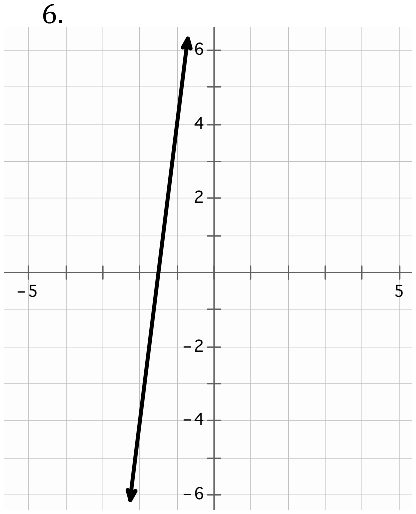


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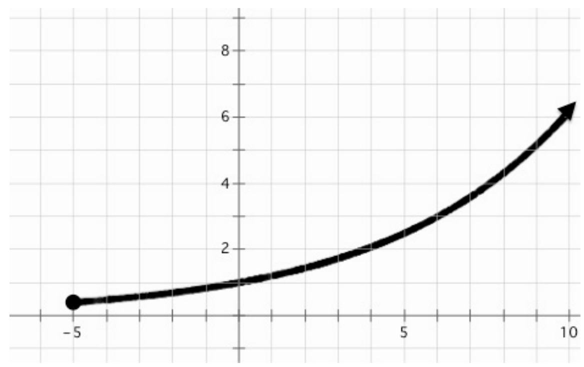


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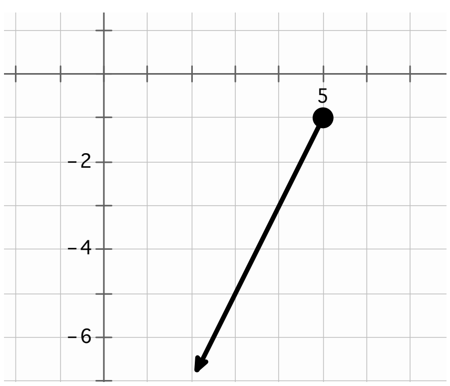


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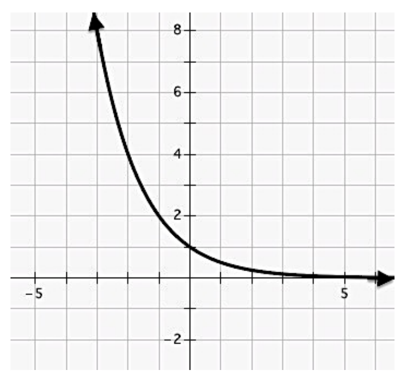


## GO

Topic: Finding equations for functions.

**Find both the explicit and recursive equations for the tables below.**

10.

| $n$ | $f(n)$ |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 7 |
| 4 | 9 |

Explicit:

Recursive:

11.

| $n$ | $f(n)$ |
|---|---|
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |
| 5 | 32 |

Explicit:

Recursive:

12.

| $n$ | $f(n)$ |
|---|---|
| 6 | 23 |
| 7 | 19 |
| 8 | 15 |
| 9 | 11 |

Explicit:

Recursive:

13.

| $n$ | $f(n)$ |
|---|---|
| 1 | 1 |
| 2 | 3 |
| 3 | 9 |

Explicit:

Recursive:

14.

| $n$ | $f(n)$ |
|---|---|
| 3 | 8 |
| 4 | 4 |
| 5 | 2 |

Explicit:

Recursive:

15.

| $n$ | $f(n)$ |
|---|---|
| 6 | 7 |
| 9 | 13 |
| 12 | 19 |

Explicit:

Recursive:

16.

| $n$ | $f(n)$ |
|---|---|
| 2 | 40 |
| 4 | 32 |
| 8 | 16 |

Explicit:

Recursive:

17.

| $n$ | $f(n)$ |
|---|---|
| 2 | 16 |
| 3 | 4 |
| 4 | 1 |

Explicit:

Recursive:

18.

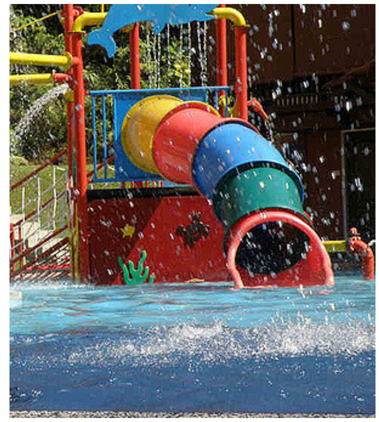
| $n$ | $f(n)$ |
|---|---|
| 17 | 5 |
| 20 | 10 |
| 26 | 20 |

Explicit:

Recursive:

# 3.5 Pooling it Together

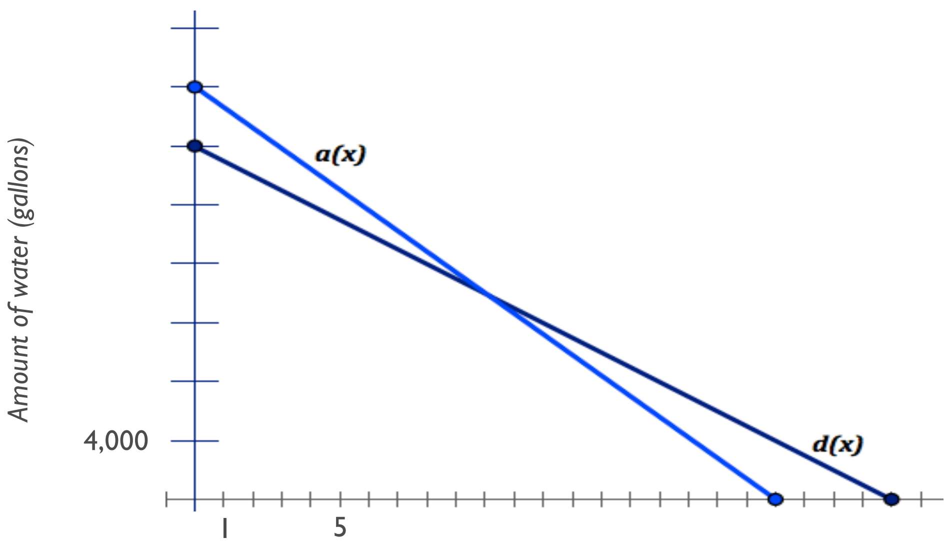
## *A Solidify Understanding Task*



CC BY Hanumann
https://flic.kr/p/7tZHkq

Aly and Dayne work at a water park and have to drain the water at the end of each month for the ride they supervise. Each uses a pump to remove the water from the small pool at the bottom of their ride. The graph below represents the amount of water in Aly's pool, $a(x)$, and Dayne's pool, $d(x)$, over time. In this scenario, they decided to work together to drain their pools and created the equation:

$$g(x) = a(x) + d(x).$$




Answer the following questions about $g(x)$.

1. What does $g(x)$ represent?

2. Create the graph of $g(x)$ on a new set of axes using the graphs of $a(x)$ and $d(x)$. Identify $g(x)$ and label (scale, axes).

3. Write the equation for the function *g(x)* using the graph you created. Compare this equation to the algebraic representation of finding the sum of the equations for *a(x)* and *d(x). (*The equations were created in the last task, "The Water Park" task).

4. Should the algebraic equation of *g(x)* be the same as the algebraic function created from the graph? Why or why not?

5. Use both the graphical as well as the algebraic representation to describe characteristics of *g(x)* and explain what each characteristic means (each intercept, domain and range for this situation and for the equation, maxima and minima, whether or not *g(x)* is a function, etc.)

6. Explain why adding the two values of the y-intercepts together in *a(x) and d(x)* can be used to find the y-intercept in *g(x).*

7. Can a similar method be used to find the x-intercepts? Explain.

# 3 . 5 Pooling it Together – Teacher Notes

## *A Solidify Understanding Task*

**Purpose**: The purpose of this task is for students to combine functions, make sense of function notation, and connect multiple representations (context, equations, and graphs). Students will also address features of functions as they solve problems that arise from this context.

**Core Standards Focus**:

**F.BF.1** Write a function that describes a relationship between two quantities.

b. Combine standard function types using arithmetic operations.

*For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

**F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

**F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

**F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

**F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

**A.REI.11** Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

**A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

**Related Standards: F.IF.1, A.REI.10, A.REI.11, N.Q.1, A.CED.2**

**Standards for Mathematical Practice:**
**SMP 1 – Make sense of problems and persevere in solving them**
**SMP 2 – Reason abstractly and quantitatively**
**SMP 3 – Construct viable arguments and critique the reasoning of others**
**SMP 4 – Model with mathematics**
**SMP 7 – Look and make use of structure**

**The Teaching Cycle:**
**Launch (Whole Class):**
Read the introduction and remind students of the task "The Water Park" where Aly and Dayne drained the water from the pools they supervise in a water park. Ask the whole group "What does $g(x) = a(x)+d(x)$ mean in context?" After a short discussion that $g(x)$ represents the combined

efforts of draining the two pools, have students move to question 2, which has them graph *g(x)* by graphically adding the two individual functions.

**Explore (Small Group):**

For 'stuck' students, prompt them with questions such as "What information do you know about Aly and Dayne's pools?" and "How can you represent or organize the information you know about *a(x) and d(x)* so that you can make sense of *g(x)?"* (Students can find solutions to solve for *g(x)* by creating a table, a graph, or looking at equations, although the goal is that they add the graphs together at this point to see visually that you are adding outputs. Equations often 'hide' this observation).

Look for students who use different representations to answer the questions from the task. Make note of this as for the whole group discussion, you may wish to select students who use different methods for solving the first two questions. A common misconception will be that students who use intercepts will either add or find the average of the x-intercepts to find the 'new' x-intercept. This is a great opportunity to distinguish why it is appropriate to add the y-intercepts to find the new y-intercept (they are the output values hence are the values of *a(x) + d(x)* and represent the amount of water in the pool) but why you do not add the x-intercepts to find the new x-intercept (they are the input values and represent the amount of time it takes for each pool to drain separately).

**Discuss (Whole Class):**

The goal of this task is to make sure students have a deeper understanding of key features of functions, to surface ideas about building functions, and to deepen understanding about function notation. The whole group discussion should cover what each part of a function represents and how this plays out when using function notation. It is most effective when students see this graphically, numerically, and with equations and make connections with the features of the function. There are many ways the whole group discussion can accomplish these goals. Below is a suggestion for how to facilitate the whole group discussion using student error that is also a common misconception. You may wish to start the whole group discussion by having two students post their graphs of *g(x)*, one being correct and the other being the common misconception (only do this if you feel your class has a safe environment and students believe that part of the learning process is to learn from mistakes). Start the conversation with how these are the two most common graphs throughout the room and that many people have either one or the other on their paper. Ask the whole group what is similar and what is different (both groups will have the same y-intercept). Then choose a student who has created a table showing the sum of the output values who agrees with the correct graph (choose this student in advance). Also have a student show how the equation of $a(x) + d(x)$ shows up in the correct graph. Be sure that students who share are explicit in the connections showing how the equations $a(x)$, $d(x)$, and $g(x)$ relate to the graphs of $a(x)$, $d(x)$, and $g(x)$. After all students see the connections between the correct graph and other representations, ask what the common misconception was in the 'incorrect graph'. In the end, students should leave with how $x$ is the input value and that $g(x)$ is the solution to the value at $x$.

**Aligned Ready, Set, Go: Features 3.5**

READY, SET, GO! Name Period Date

## READY

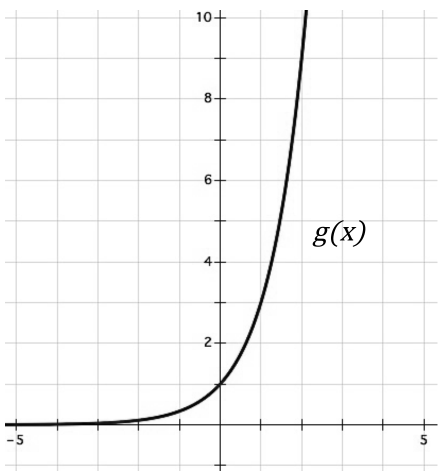
Topic: Interpreting function notation to find the output or input based on what is given

**For each function, find the indicated values.**

1. $Given: h(t) = 2t - 5$

a. $h(-4) =$ ____ b. $h(t) = 23, \ t =$ _____ c. $h(13) =$ _____ d. $h(t) = -33, \ t =$ _____

2.



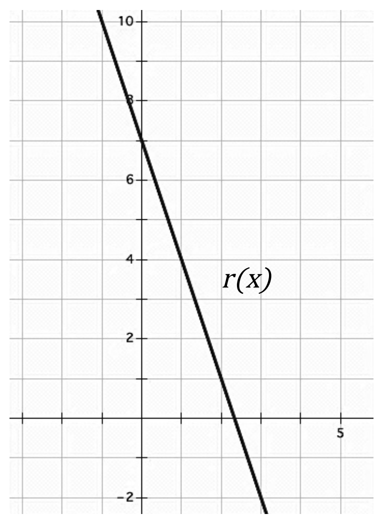

a. $g(2) =$ _____

b. $g(x) = 3, \ x =$ _____

c. $g(0) =$ ______

d. Write the explicit rule for $g(x)$.

3.




a. $r(-1) =$ _____

b. $r(x) = 4, \ x =$ _____

c. $r(2) =$ _____

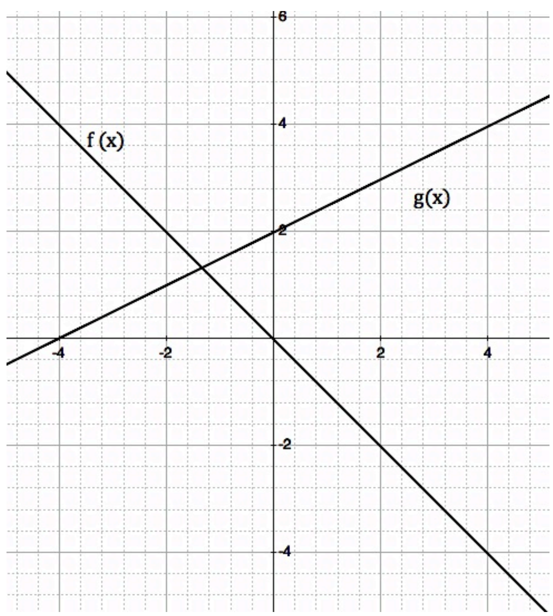
d. Write the explicit rule for $r(x)$.

## SET

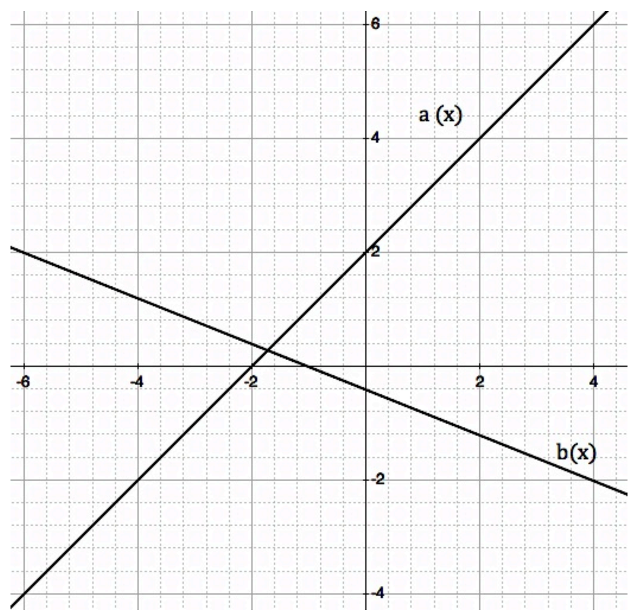
Topic: Adding functions

**Two functions are graphed. Graph a new function on the same grid by adding the two given functions.**

4. $h(x) = f(x) + g(x)$




5. $s(x) = a(x) + b(x)$



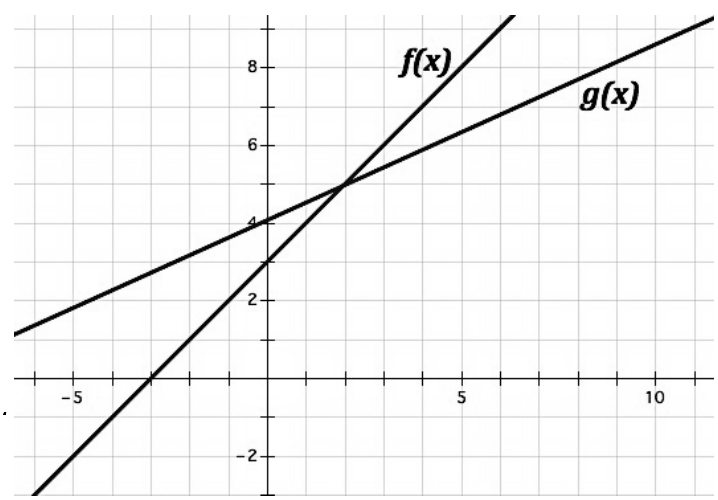

6. Use the graph to answer the following questions.

a. Where does $f(x) = g(x)$?

b. What is $f(4) + g(4)$?

c. What is $g(-2) - f(-2)$?

d. State the interval where $g(x) > f(x)$.



7. Use the graph to answer the following questions.

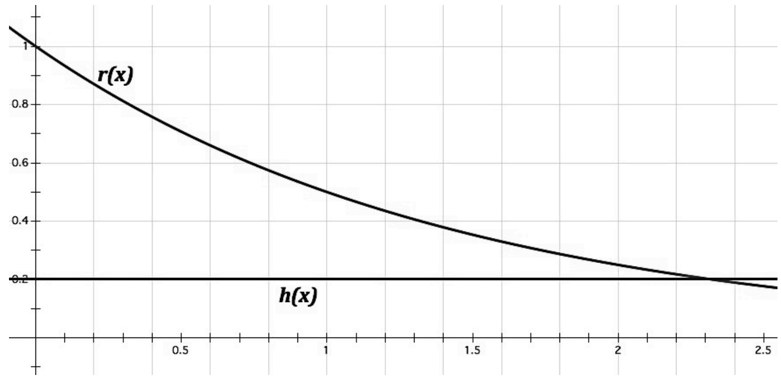
a. Where is $r(x) > h(x)$?

b. What is $r(1) - h(1)$?

c. What is $r(0) + h(0)$?

d. Write an explicit rule for $r(x)$ and for $h(x)$.

e. Sketch $\boldsymbol{r(x) - h(x)}$ on the graph.



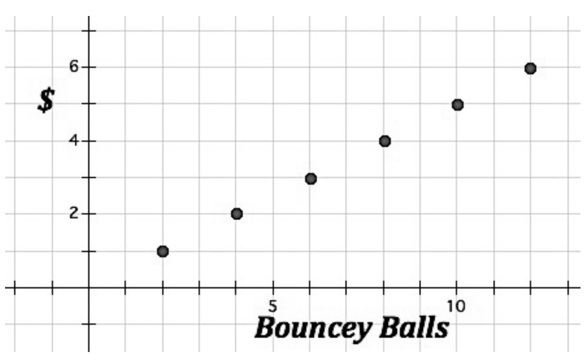

## GO

Topic: Distinguishing between discrete and continuous functions

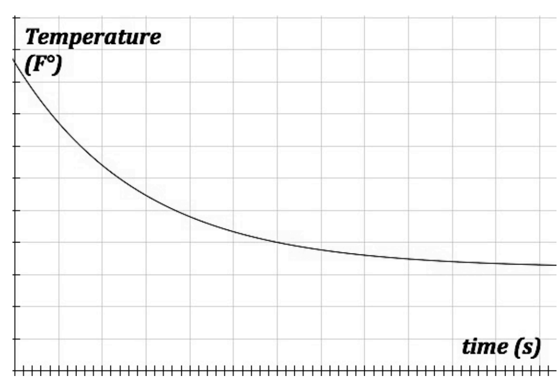
**For each context or representation determine whether it is discrete or continuous or could be modeled best in a discrete or continuous way. Justify your answer.**

8. Susan puts exactly $5 a week in her piggy bank.

9.




10.








mathematics vision project

11. Marshal tracks the number of hits he gets each baseball game and is recording his total number of hits for the season in a table.

12. The distance you have traveled since the day began.

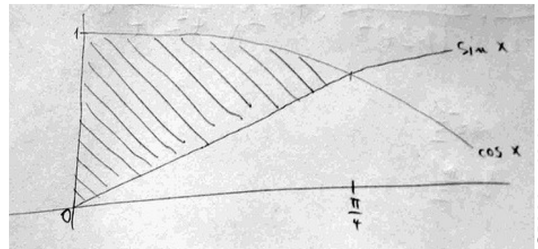
13.

| Number of gumballs | Cost |
|---|---|
| 5 | 10¢ |
| 10 | 20¢ |
| 15 | 30¢ |
| 20 | 40¢ |

14. Stephen deposited $1,000 in a savings account at the bank when he turned 21. He deposits $100 each month. He plans to never withdraw any money until the balance is $150,000.

# 3.6 Interpreting Functions

## *A Practice Understanding Task*

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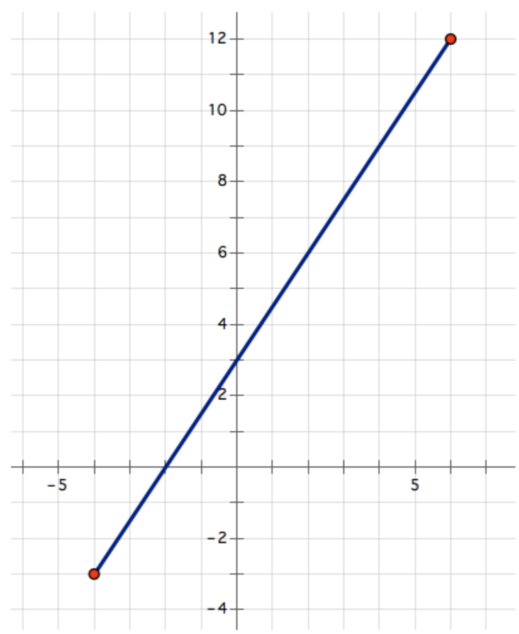
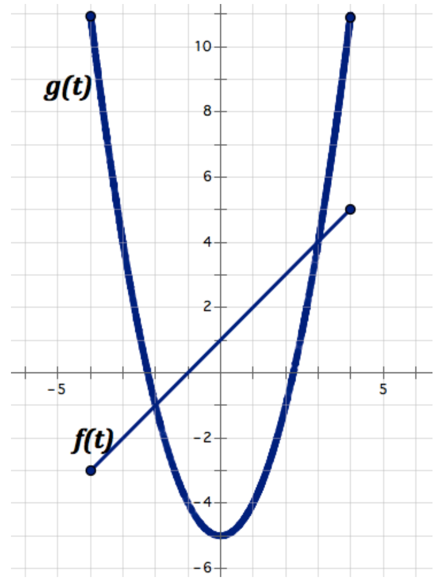
https://flic.kr/p/EKgAa

Given the graph of *f(x),* answer the following questions. Unless otherwise specified, restrict the domain of the function to what you see in the graph below. Approximations are appropriate answers.

1. What is *f*(2)?
2. For what values, if any, does *f(x)* = 3?
3. What is the x-intercept?
4. What is the domain of *f(x)*?
5. On what intervals is *f(x)* > 0?
6. On what intervals is *f(x)* increasing?
7. On what intervals is *f(x)* decreasing?
8. For what values, if any, is *f(x)* > 3?



Consider the linear graph of *f(t)* and the nonlinear graph of *g(t)* to answer questions 9-14. Approximations are appropriate answers.




9. Where is *f(t)* = *g(t)*?

10. Where is *f(t)* > *g(t)* ?

11. What is *f(0)* + *g(0)* ?

12. What is *f(-1)* + *g(-1)* ?

13. Which is greater: *f(0) or g(-3)*?

14. Graph: *f(t)* + *g(t)* from [-1, 3]

The following table of values represents two continuous functions, *f(x) and g(x)*. Use the table to answer the following questions:

| *x* | *f(x)* | *g(x)* |
|---|---|---|
| -5 | 44 | -13 |
| -4 | 30 | -9 |
| -3 | 20 | -5 |
| -2 | 12 | -1 |
| -1 | 6 | 3 |
| 0 | 2 | 7 |
| 1 | 0 | 11 |
| 2 | 0 | 15 |
| 3 | 2 | 19 |
| 4 | 6 | 23 |
| 5 | 12 | 27 |
| 6 | 20 | 31 |

15. What is *g*(-3)?

16. For what value(s) is *f(x)* = 0?

17. For what values does *f(x)* seem to be increasing?

18. On what interval is *g(x)* > *f(x)*

19. Which function is changing faster in the interval [-5, -1]? Why?

Use the following relationships to answer the questions below.

$h(x) = 2^x$ $\qquad f(x) = 3x - 2$ $\qquad g(x) = 8$ $\qquad x = 4$ $\qquad y = 5x + 1$

20. Which of the above relations are functions? Explain.
21. Find *f(2), g(2), and h(2).*
22. Write the equation for *g(x) + h(x).*
23. Where is *g(x)< h(x)* ?
24. Where is *f(x)* increasing?
25. Which of the above functions has the fastest growth rate?

Create a graph for each of the following functions, using the given conditions

26. This function has the following features: *f(2)* is positive; *f(-2) = 0, f(x) is always Increasing and has a domain of All Real Numbers.*
27. This function has the following features: *f(3)> f(6); f(1) = 0; f(2) = 4; f(x) is increasing from [-5, 3); has a domain from [-5, 10]*
28. This function has the following features: *f(x) has a constant rate of change; f(5) = 0*
29. Create your own conditions- have at least three and then create examples where the solution could be different graphs.

# 3 . 6 Interpreting Functions – Teacher Notes

## *A Practice Understanding Task*

**Purpose**: Students have been using function notation in various forms and have become more comfortable with features of functions. In this task, the purpose is for students to practice their understanding of the following:

- Distinguish between input and output values when using notation
- Evaluate functions for inputs in their domains
- Determine the solution where the graphs of *f(x)* *and* *g(x)* intersect based on tables of values and by interpreting graphs
- Combine standard function types using arithmetic operations (finding values of *f(x)+ g(x))*
- Create graphs of functions given conditions.

**Core Standards Focus**:

**F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

**F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.

**F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

**F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

**F.BF.1b** Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations.

**A.REI.11** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

**A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

**Related Standards: F.IF.1, A.REI.10, A.REI.11, N.Q.1, A.CED.2**

**Standards for Mathematical Practice:**

**SMP 7 – Look and make use of structure**

**SMP 8 – Look for and express regularity in repeated reasoning**

**The Teaching Cycle:**

**Launch (Whole Class):**

Students should be able to get started on this task without additional support, since it is similar in nature to the work they did on "The Water Park" and "Pooling It Together". If preferred, you may wish to sketch a graph on the board and ask students a couple of questions using function notation before having them begin the task. This would be a good task to have students start on their own, then have them pair up after most have completed the first set of questions.

**Explore (Small Group):**

Watch for students who confuse input/output values. Without context, keeping track of this is a common mistake. Encourage students to explain their reasoning to each other while working through solutions to problems. If students are incorrect in their thinking, be sure to redirect their thinking. As you monitor, make note of the areas where students are struggling and select students who explain/clarify details.

**Discuss (Whole Class):**

Go over problems that seem to be common issues that students are still grappling with first. After this, choose students to share their method for graphing number 14. Compare students who used point by point to those who added on from one graph to the next. Last, choose students who have correct but different graphs for one of the following problems (either 26 or 27). Have students compare/contrast graphs and explain why both are appropriate given the conditions.

The goal of this whole group discussion is that ALL students can evaluate functions using notation, can interpret features of functions using a graph or table of values, can create a graph given conditions, and can combine two functions to make another function.

**Aligned Ready, Set, Go: Features 3.6**

READY, SET, GO! Name Period Date

## READY

Topic: Solving Systems by Substitution

**In prior work the meaning of $f(x) = g(x)$ was discussed. This means to find the point where the two equations are equal and where the two graphs intersect. It is possible to find the point of intersection algebraically instead of graphing the two lines. Since $f(x) = g(x)$, it's possible to set each equation equal to the other and solve for x.**

**Example:** Find the point of intersection of function $f(x) = 3x + 4$ and function $g(x) = 4x + 1$.

Since, $f(\mathrm{x}) = \mathrm{g}(\mathrm{x})$, *let* $3x + 4 = 4x + 1$. Then solve for $x$.

$$\begin{array}{r} 3x + 4 = 4x + 1 \\ \underline{-3x - 1 = -3x - 1} \\ 0x + 3 = 1x + 0 \\ 3 = 1x \end{array}$$

Subtract $3x$ and 1 from both sides of the equation.

Now let x = 3 in each equation to find $f(x)$ *and* $g(x)$ *when* $x = 3$.

$f(3) = 3(3) + 4 \rightarrow 9 + 4 = 13$ *and* $g(3) = 4(3) + 1 \rightarrow 12 + 1 = 13$

When $x = 3, f(3)$ *and* $g(3)$ *both equal* 13. *The point of intersection is* $(3, 13)$.

**Find the point of intersection for $f(x)$ and $g(x)$ using the algebraic method in the example above.**

1. $f(x) = -5x + 12$ and $g(x) = -2x - 3$

2. $f(x) = \frac{1}{2}x + 2$ and $g(x) = 2x - 7$

3. $f(x) = -\frac{2}{3}x + 5$ and $g(x) = -x + 7$

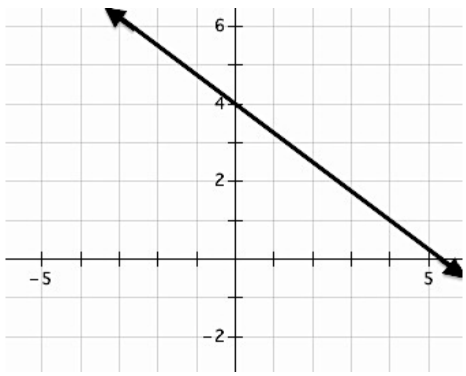
4. $f(x) = x - 6$ and $g(x) = -x - 6$

## *SET*

Topic: Describing attributes of a functions based on graphical representation

**Use the graph of each function provided to find the indicated values.**

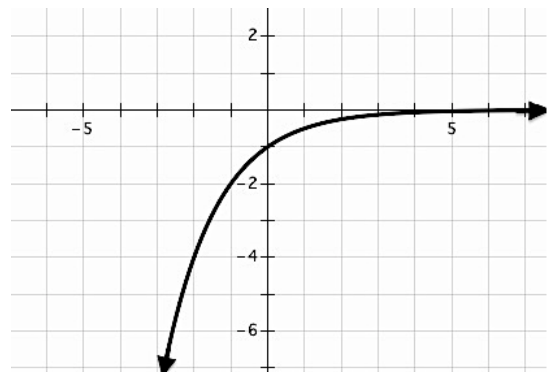
5. $f(x)$




a. $f(4)$ = _______ b. $f(-4)$ = _________

c. $f(x) = 4$, $x$ = ________ d. $f(x) = 7$, $x$ = _______

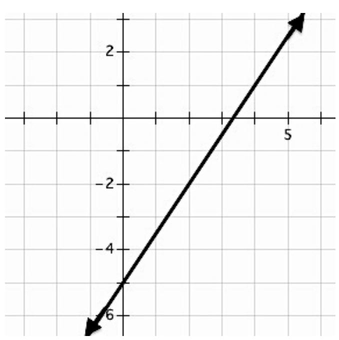
6. $g(x)$




a. $g(-1)$= _________ b. $g(-3)$ = __________

c. $g(x) = -4$ $x$ = ______ d. $g(x) = -1$, $x$ = ________

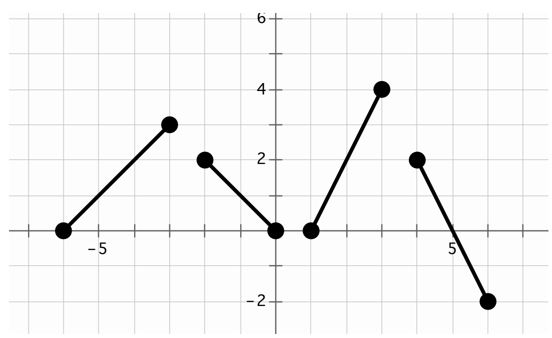
7. $h(x)$




a. $h(0)$ = _______ b. $h(3)$ = _________

c. $h(x) = 1$, $x$ = ________ d. $h(x) = -2$, $x$ = _____

8. $d(x)$




a. $d(-5)$ = _______ b. $d(4)$ = _________

c. $d(x) = 4$, $x$ = ________ d. $d(x) = 0$, $x$ = _____

**For each situation either create a function or use the given function to find and interpret solutions.**

9. Fran collected data on the number of feet she could walk each second and wrote the following rule to model her walking rate $d(t) = 4t$.

   a. What is Fran looking for if she writes $d(12) =$ ______?

   b. In this situation what does $d(t) = 100$ tell you?

   c. How can the function rule be used to indicate a time of 16 seconds was walked?

   d. How can the function rule be used to indicate that a distance of 200 feet was walked?

10. Mr. Multbank has developed a population growth model for the rodents in the field by his house. He believes that starting each spring the population can be modeled based on the number of weeks with the function $p(t) = 8(2^t)$.

   Find $p(t) = 128$.  Find $p(4)$.  Find $p(10)$.

   d. Find the number of weeks it will take for the population to be over 20,000.

   e. In a year with 16 weeks of summer, how many rodents would he expect by the end of the summer using Mr. Multbank's model?

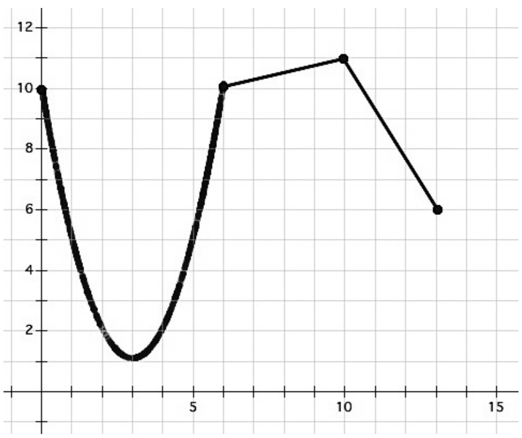
   What are some factors that could change the actual result from your estimate?

## GO

Topic: Describe features of functions from the graphical representation.

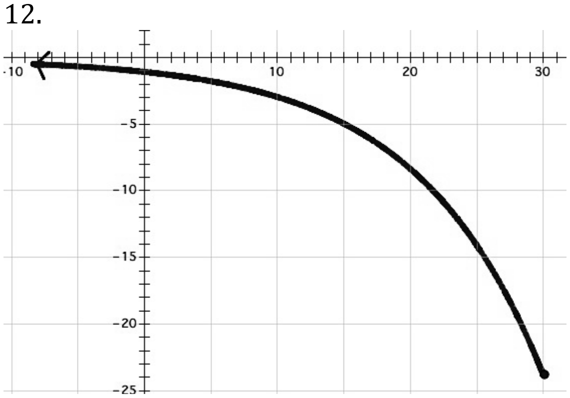
**For each graph given provide a description of the function. Be sure to consider the following: decreasing/increasing, min/max, domain/range.**

11.



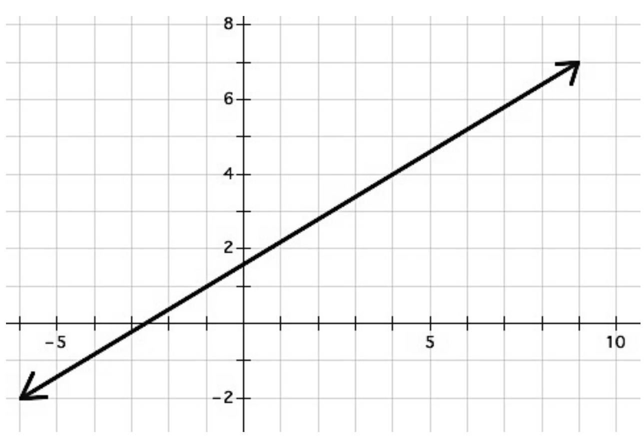

Description of function:

12.




Description of function:

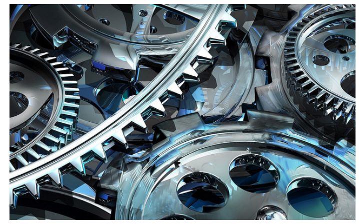
13.




Description of function:

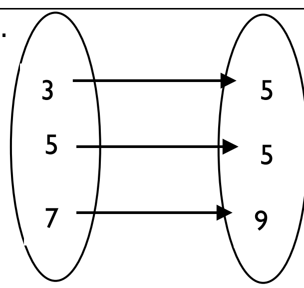
# 3.7 To Function or Not to Function

## *A Practice Understanding Task*





**Identify the two variables for each situation and determine which is independent and which is dependent. Then, determine if the relationship is a function and justify your reasoning.**

<table>
<tr>
<td>1. A person's name versus their social security number.</td>
<td>2. A person's social security number versus their name.</td>
<td>3. The cost of gas versus the amount of gas pumped.</td>
</tr>
<tr>
<td>4.<br>{ (3,6), (4, 10), (8,12) }</td>
<td>5. The temperature in degrees Fahrenheit with respect to the time of day.</td>
<td>6.<br><table><tr><td>distance</td><td>days</td></tr><tr><td>6</td><td>2</td></tr><tr><td>10</td><td>4</td></tr><tr><td>6</td><td>5</td></tr><tr><td>9</td><td>8</td></tr></table></td>
</tr>
<tr>
<td>7. The area of a circle as it relates to the radius.</td>
<td>8.<br><br></td>
<td>9. The volume of water in a given cylinder is dependent on the height of water in cylinder.</td>
</tr>
</table>

| | | |
|---|---|---|
| 10. The size of the radius of a circle dependent on the area. | 11. Students letter grade dependent on the percent earned. | 12. The length of fence needed with respect to the amount of rectangular area to be enclosed. |
| 13. The explicit formula for the recursive situation below: $f(1) = 3 \; and$ $f(n+1) = f(n) + 4$ | 14. If $x$ is a rational number, then $f(x) = 1$<br><br>If $x$ is an irrational number, then $f(x) = 0$ | 15. The national debt with respect to time. |

# 3 . 7 To Function or Not to Function – Teacher Notes

## *A Practice Understanding Task*

**Purpose**: This task is designed for students to practice their understanding of function. After reviewing various naming conventions of function ("versus", "with respect to", "over", "dependent on"), students are to determine whether or not each situation is a function, then justify their answer.

**Core Standards Focus**:

**F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

**F.IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

**Related Standards: A.REI.10 and interpreting function (IF) standards**

**Standards for Mathematical Practice:**

**SMP 1 – Make sense of problems and persevere in solving them**

**SMP 3 – Construct viable arguments and critique the reasoning of others**

**SMP 6 – Attend to precision**

**SMP 8 – Look for and express regularity in repeated reasoning**

**The Teaching Cycle:**

**Launch (Whole Class):**

Begin this task by going over naming conventions for determining the input and output of relationships and reading the directions. The focus of the task is about understanding the definition of function at another level. There is no need to let the wording of "vs" or "with respect to" get in the way. For example, "distance with respect to time" is the same as "distance vs time", with distance being dependent on time. Students should first determine the variables within the problem, then determine which is independent and which is dependent, then determine whether or not the relationship is a function. The goal is that students are deepening their understanding of function and refining what it means to be a function. This would be a good task to have students do in pairs while making sure each student is responsible for communicating their explanations for each problem in writing.

**Explore (Small Group):**

As you monitor, be sure students are using appropriate academic vocabulary as they explain to their partner whether or not each relationship is a function. Make note of the areas where students are struggling to highlight these misconceptions in the whole group discussion. Many students only see functions that are continuous and have a domain of all real numbers during eighth grade. This task helps focus on the standard F.IF.1 and has examples of relationships with various domain situations.

**Discuss (Whole Class):**

Since this is a Practice Task, the discussion should include going over problems that seem to be common issues as well as problems that drive home the standards (in this case, refining the definition of function beyond what was learned in eighth grade mathematics). To start the whole group discussion, have a student share their understanding of function by comparing questions 1 and 2. Highlight that two people can have the same name, but not the same social security number. So, if a name is given, there is a possibility that more than one social security number could correspond to the given name (David Smith, for example). This would not be a function. On the other hand, a specific social security number will correspond to one specific person (and is a function). For the remaining problems, select students to share for problems that highlight misconceptions. The goal of this whole group discussion is that ALL students can determine whether or not a relationship is a function based on the standard:

**F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

**Aligned Ready, Set, Go: Features 3.7**

READY, SET, GO! Name Period Date

## *READY*

Topic: Determine domain and range and whether the relation is a function or not.
**Determine if each set of ordered pairs is a function or not and then state the domain and range.**

Determine if each set of ordered pairs is a function, then state the domain and range.

1. { (-7, 2), (3, 5), (8, 4), (-6, 5), (-2, 3)} Function: Yes / No

   Domain: Range:

2. { (9, 2), (0, 4), (4, 0), (5, 3), (2, 7) (0, -3), (3, -1)} Function: Yes / No

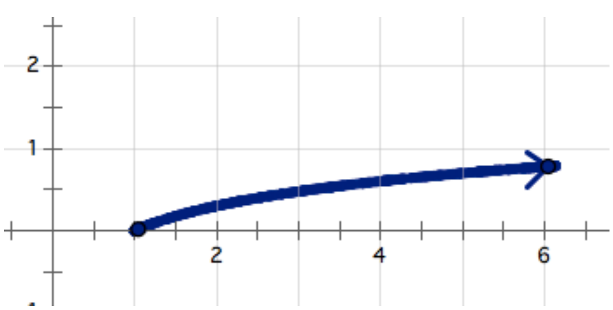
   Domain: Range:

3. { (1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (8, 9)} Function: Yes / No

   Domain: Range:

Determine the domain and range for each of the given functions.

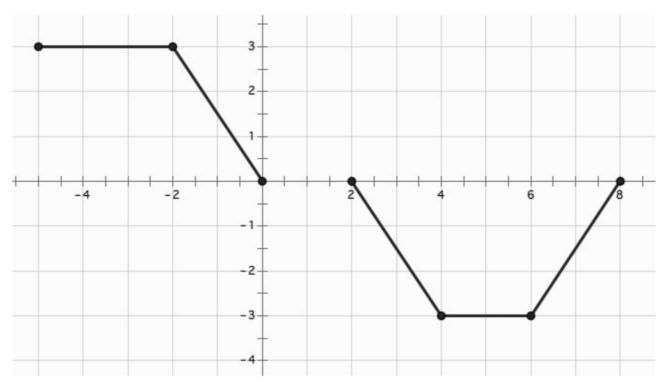
4.



Domain:

Range:

5.



Domain:

Range:

6. $f(x) = -2x + 7$

Domain:

Range:

7. $g(x) = 3\,(5)^x$

Domain:

Range:

8. The elements in the table define the entire function.

Domain:

Range:

| x | h(x) |
|---|---|
| 1 | 9 |
| 2 | 98 |
| 3 | 987 |
| 4 | 9876 |

## *SET*

Topic: Determine whether or not the relationship is a function.

**Determine the domain and range then determine whether or not the relationship is a function.**

9. The distance a person is from the ground related to time as they ride a Ferris Wheel.

10. The amount of daylight during a day throughout the calendar year.

11. The value of a Volkswagen Bug convertible from time of first purchase in 1978 to now.

12. A person's name versus their phone number.

13. The stadium in which a football player is playing related to the outcome of the game.

## *GO*

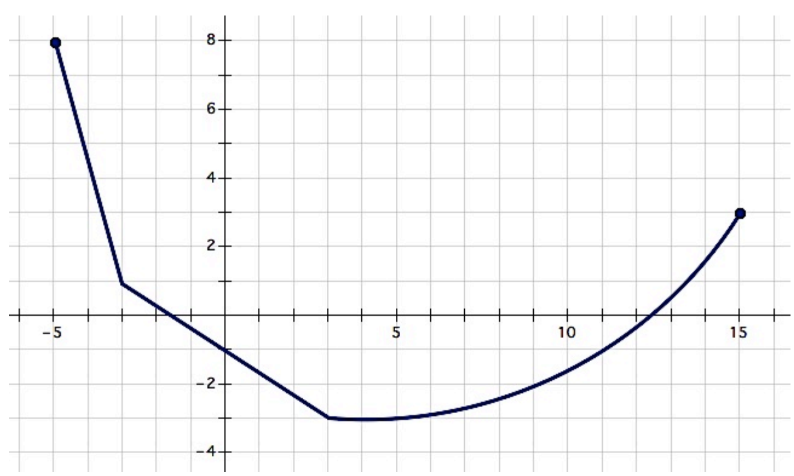
Topic: Determine the features of functions.

14. Describe the function in the graph.

Write the intervals where it is decreasing and/or increasing.

Identify the min and/or max.

State the domain and range.



15. For each situation use the given function to find and interpret solutions.

*Hope has been tracking the progress of her family as they travel across the country, she knows they are driving 78 miles per hour, during their vacation and she has created a function, d(t)= 78t to model the progress they are making.*

a. What would Hope be attempting to find if she writes $d(4) = 78(4)$?

b. What would the expression $d(t) = 450$ mean in this situation?

c. What would the expression $d(3.5)$ mean in this situation?

d. How could Hope use the function to find the time it would take to travel 800 miles?

16. **Use the given representation of the functions to answer the questions.**

a. Where does $f(x) = g(x)$?

b. What is $g(0) + f(0)$?

c. On what interval(s) is $g(x) > f(x)$?

d. What is $g(-8) + f(-8)$?