mathematics vision project

Transforming Mathematics Education

ALGEBRA I

A Learning Cycle Approach

MODULE 4

Equations & Inequalities

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

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4.1 Cafeteria Actions and Reactions

A Develop Understanding Task

Elvira, the cafeteria manager, has just received a shipment of new trays with the school logo prominently displayed in the middle of the tray. After unloading 4 cartons of trays in the pizza line, she realizes that students are arriving for lunch and she will have to wait until lunch is over before unloading the remaining cartons. The new trays are very popular and in just a couple of minutes 24 students have passed through the pizza line and are showing off the school logo on the trays. At this time, Elvira decides to divide the remaining trays in the pizza line into 3 equal groups so she can also place some in the salad line and the sandwich line, hoping to attract students to the other lines. After doing so, she realizes that each of the three serving lines has only 12 of the new trays.

"That's not many trays for each line. I wonder how many trays there were in each of the cartons I unloaded?"

1. Help the cafeteria manager answer her question using the data in the story about each of the actions she took. Explain how you arrive at your solution.

Elvira is interested in collecting data about how many students use each of the tables during each lunch period. She has recorded some data on Post-It Notes to analyze later. Here are the notes she has recorded:

- Some students are sitting at the front table. (I got distracted by an incident in the back of the lunchroom, and forgot to record how many students.)
- Each of the students at the front table has been joined by a friend, doubling the number of students at the table.
- Four more students have just taken seats with the students at the front table.

- The students at the front table separated into three equal-sized groups and then two groups left, leaving only one-third of the students at the table.

- As the lunch period ends, there are still 12 students seated at the front table.

Elvira is wondering how many students were sitting at the front table when she wrote her first note. Unfortunately, she is not sure what order the middle three Post-It Notes were recorded in since they got stuck together in random order. She is wondering if it matters.

2. Does it matter which order the notes were recorded in? Determine how many students were originally sitting at the front table based on the sequence of notes that appears above. Then rearrange the middle three notes in a different order and determine what the new order implies about the number of students seated at the front table at the beginning.

3. Here are three different equations that could be written based on a particular sequence of notes. Examine each equation, and then list the order of the five notes that is represented by each equation. Find the solution for each equation.

- $\frac{2(x+4)}{3}=12$

- $2\left(\frac{x}{3}+4\right)=12$

- $\frac{2x+4}{3}=12$

4. 1 Cafeteria Actions and Reactions – Teacher Notes

A Develop Understanding Task

Purpose: In this task students will develop insights into how to extend the process of solving equations—which they have previously examined for one- or two-step equations—so that the process works with multistep equations. They will observe that the process of solving an equation consists of writing a sequence of equivalent equations until the value(s) that will make each of the equations in the sequence true becomes evident. Each equation in the sequence of equivalent equations is obtained by operating on the expressions on each side of the previous equation in the same way, such as multiplying both sides of the equation by the same amount, or adding the same amount to both sides of the equation. This property of equality is often referred to as "keeping the equation in balance." Our goal in each step of the equation solving process is to make the next equivalent equation contain fewer operations than the previous one by "un-doing" one operation at a time. When there are multiple operations involved in an equation, the order in which to "un-do" the operations can be somewhat problematic. This task examines ways to determine the sequence of "un-do-it" steps by using the structure of the equation.

Core Standards Focus:

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Related Standards: A.REI.3

Standards for Mathematical Practice:

SMP 2 – Reason abstractly and quantitatively

SMP 3 – Construct viable arguments and critique the reasoning of others

SMP 7 – Look for and make use of structure

The Teaching Cycle:

Launch (Whole Class):

Read the initial story context about Elvira, the cafeteria manager, and the sequence of events that led to "twelve new trays in each of three serving lines" and challenge students to "un-do" the actions that got her to this situation to see if they can answer Elvira's question of "how many trays were in each carton?" Give students a few minutes to work individually or with a partner to analyze this situation, then have a student share his or her thinking.

Record the steps of the explanation of how to find the number of trays in a carton for use in a few minutes:

- Since there are now twelve trays in each of three serving lines, there must have been 36 trays before Elvira divided them up.
- Since 24 students removed trays after lunch period started, there must have been 36+24=60 trays that were unloaded.
- Since the 60 trays came from 4 cartons, there must have been 15 trays in each carton.

Now propose that the story of Elvira's "actions" could have been represented by the following sequence of equivalent equations, and have students connect each equation in the sequence to the appropriate action:

Let $x = 15$	This represents the number of trays in each carton.
$4x = 60$	This would represent the number of trays unloaded from four cartons.
$4x - 24 = 36$	This would represent the remaining trays after 24 students passed through the line.
$\frac{4x-24}{3} = 12$	This would represent the number of trays in each of the three lines.

This last equation represents a multi-step equation. Have students pretend that we do not know the solution for *x*. Have them discuss with a partner how they could look at this equation and "see" the steps listed on the board for "un-doing" the story, as represented by the numbers and operations in this equation. Have students share what they have noticed, then have them work on the second situation with Elvira and the Post-It Notes.

Explore (Small Group):
Encourage students to do the same thing with the 5 Post-It Notes that they did with the serving trays scenario. That is, they should write out the steps of reasoning that would help them work backwards to the solution. Have them then change the order of the middle three Post-It Notes and solve the situation again. Ask, did the order matter?

Listen for how students are making decisions about which order the notes were arranged in for each of the three different equations listed in problem 3.

Discuss (Whole Class):
The discussion should focus on question 3 and how students can recognize the order of events that got to "still 12 students seated at the front table." Once they can see how the sequence of events unfolded, they should be able to determine how to reverse the sequence of events. It may be necessary to write out the list of events and "un-do-it" explanations for each problem, similar to what was done in the launch.

Aligned Ready, Set, Go: Equations and Inequalities 4.1

Some students are sitting at the front table. (I got distracted by an incident in the back of the lunchroom, and forgot to record how many students.)

Each of the students at the front table has been joined by a friend, doubling the number of students at the table.

Four more students have just taken seats with the students at the front table.

The students at the front table separated into three equal-sized groups and then two groups left, leaving only one-third of the students at the table.

As the lunch period ends, there are still 12 students seated at the front table.

Some students are sitting at the front table. (I got distracted by an incident in the back of the lunchroom, and forgot to record how many students.)

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Four more students have just taken seats with the students at the front table.

The students at the front table separated into three equal-sized groups and then two groups left, leaving only one-third of the students at the table.

As the lunch period ends, there are still 12 students seated at the front table.

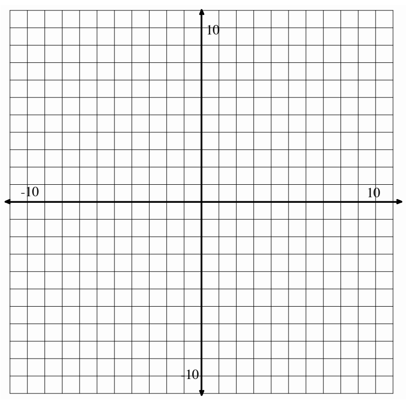
READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Solutions to an equation.

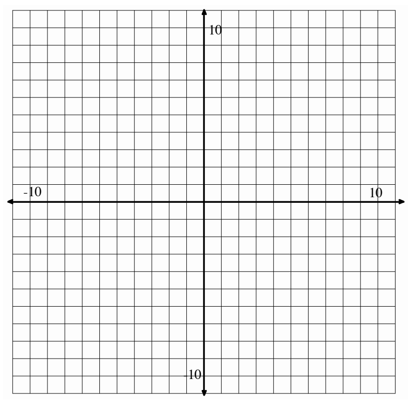
Graph the following equations on the coordinate grid. Determine if the given point is a solution to the equation?

1. $y = 5x - 2$



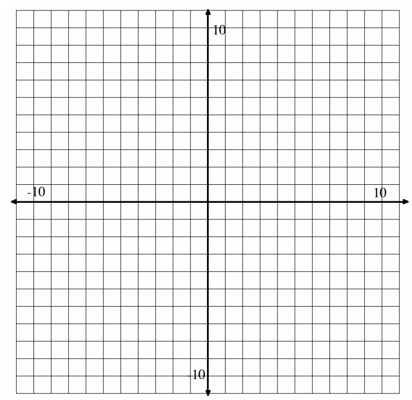
Point: (1 , 3) Yes? / No?

2. $y = -\frac{1}{2}x + 8$



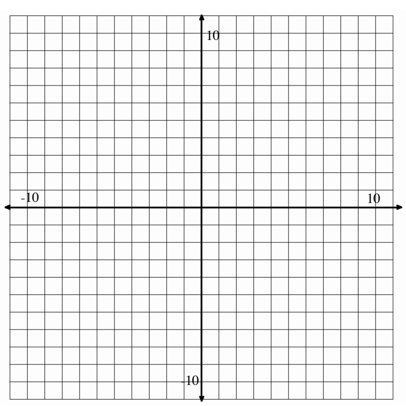
Point: (0 , 7) Yes? / No?

3. $y = -x + 4$



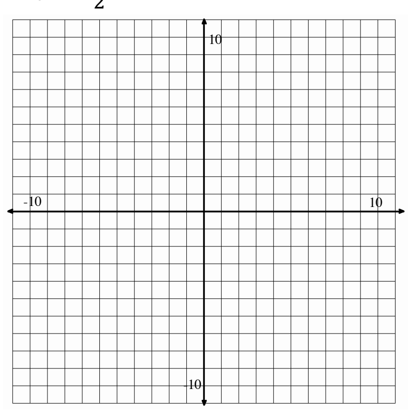
Point: (2 , 2) Yes? / No?

4. $y = x + 2$



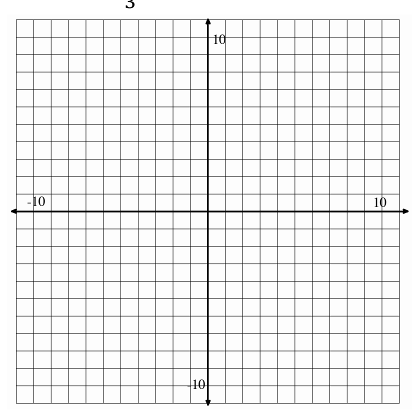
Point: (1 , 3) Yes? / No?

5. $y = \frac{5}{2}x - 7$



Point: (2 , -2) Yes? / No?

6. $y = -\frac{4}{3}x$



Point: (2 , -5) Yes? / No?

SET

Topic: Solve linear equations using parentheses.

Determine if the two expressions listed are equivalent. Explain your reasoning.

7. $14-(3a+2)$ $14-3a-2$	**8.** $4b-10$ $2(2b-5)$
9. $\frac{(x-7)}{4}$ $\frac{x}{4}-\frac{7}{4}$	**10.** $\frac{3(w-9)}{5}$ $\frac{3w}{5}-27$

11. Without solving, determine if the two equations below have the same solution. Explain why or why not?

$$3(x-5)=35 \text{ and } 3x-5=35.$$

12. Circle the expressions that are equivalent.

$\frac{4t-10}{2}$	$\frac{4t}{2}-10$	$2t-10$	$4t-5$

Solve for x.

13. $\frac{4(x-2)}{5}=20$ **14.** $4\left(\frac{x}{5}-2\right)=20$ **15.** $\frac{4x-2}{5}=20$

GO

Topic: Determine if a number is a solution to an equation.

Indicate whether the given value is a solution to the corresponding equation. Show your work.

16. $a=-3;$ $4a+3=-9$

Yes?/No?

17. $x=\frac{4}{3};$ $\frac{3}{4}x+\frac{1}{2}=\frac{3}{2}$

Yes?/No?

18. $y=2;$ $2.5y-10=-0.5$

Yes?/No?

19. $z=-5;$ $2(5-2z)=20-2(z-1)$

Yes?/No?

20. $w=\frac{1}{4};$ $4w=w+\frac{3}{4}$

Yes?/No?

21. $b=5;$ $6x-2=4(x+2)$

Yes?/No?

4. 2 Elvira's Equations

A Solidify Understanding Task

Elvira, the cafeteria manager, likes to keep track of the things she can count or measure in the cafeteria. She hopes this will help her improve the efficiency of the cafeteria. To remind herself to keep track of important quantities, she has made a table of variables and descriptions of the things she wants to record. Here is a table of things she has decided to keep track of.

Symbol	**Meaning** (description of what the symbol means in context)	**Units** (what is counted or measured)
S	Number of students that buy lunch in the salad line	
W	Number of students that buy lunch in the sandwich line	
P	Number of students that buy lunch in the pizza line	
F	Number of food servers in the cafeteria	
M_T	Number of minutes it takes to serve lunch to all students	
C	Number of classes in the school	
P_L	Price per lunch	
A		
R		
T		
D_F		
M		

Elvira has written the following equation to describe a cafeteria relationship that seems meaningful to her. She has introduced a new variable *A* to describe this relationship.

$$A = \frac{S + W + P}{C}$$

1. What does *A* represent in terms of the school and the cafeteria? Record this information in the table above.

2. Using what you know about manipulating equations, solve this equation for *S*. Your solution will be of the form S = *an expression written in terms of the variables A, C, W* and *P.*

3. Does your expression for *S* make sense in terms of the meanings of the other variables? Explain why or why not.

Here is another one of Elvira's equations.

$$R = P_L(S + W + P)$$

4. What does *R* represent in terms of the school and the cafeteria? Record this information in the table above.

5. Using what you know about manipulating equations, solve this equation for P_L.

6. Does your expression for P_L make sense in terms of the meanings of the other variables? Explain why or why not.

7. Elvira notices that she uses the expression $S + W + P$ a lot in writing other expressions. She decides to represent this expression using the variable *T*, so that $T = S + W + P$. What does *T* represent in terms of the school and the cafeteria? Record this information in the table above.

Elvira is having a meeting with the staff members who work in the lunchroom. She has created a couple of new equations for the food servers.

$$D_F = \frac{T \cdot P_L}{F} \qquad\qquad M = \frac{M_T}{T}$$

8. a. What does D_F represent in terms of the school and the cafeteria? Record this information in the table above.

   b. Solve this equation for P_L . Describe why your solution makes sense in terms of the other variables.

9. a. What does M represent in terms of the school and the cafeteria? Record this information in the table above.

   b. Solve this equation for T. Describe why your solution makes sense in terms of the other variables.

10. One of the staff members suggests that they need to write expressions for each of the following. Using the variables in the table, what would these expressions look like?

    a. The average number of students served each minute

    b. The average number of minutes students wait in the pizza line

4. 2 Elvira's Equations – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to apply the equation solving process developed in the previous task to solving literal equations and formulas. Working with literal equations solidifies the notion that operations have to be "un-done" in an appropriate order by doing the inverse operation to both sides of the equation. This task also solidifies the meaning of expressions by attending to the units associated with each of the variables.

Quantities are measured in units. When quantities are added, subtracted, multiplied or divided the units on the result of the operation may be different from the units used to measure the individual quantities. The new units formed are a consequence of the meaning of the operations. For example, if the amount of gasoline I put in my car is measured in gallons, and the distance I travel when using up that amount of gasoline is measured in miles, then a new unit of measure, miles per gallon, emerges to measure the efficiency of my car's usage of gasoline. This is a result of dividing the number of gallons of gasoline used by the number of miles driven. In this task students solidify the use of units as a tool for understanding problems and as a guide for determining what operations make sense when combining quantities using the four basic arithmetic operations.

Core Standards Focus:

N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas.

N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Related Standards: A.REI.1, A.SSE.1

Standards for Mathematical Practice:

- **SMP 2 – Reason abstractly a quantitatively**
- **SMP 3 – Construct viable arguments and critique the reasoning of others**
- **SMP 4 – Model with mathematics**
- **SMP 7 – Look for and make use of structure**

The Teaching Cycle:

Launch (Whole Class):

Give students a few minutes of individual think time to determine the units and fill out the third column of the table for each of the variables currently listed. Then have them share their proposed units for the variables as a class. Students should note that even though the descriptions on the variables may be different, the units in which the variables are measured may still be the same. For example, S, the number of students who buy lunch in the salad line, W, the number of students who buy lunch in the sandwich line, and P, the number of students who buy lunch in the pizza line, each describe different groups of students eating different lunches, but the unit "students" (or "people" or "individuals") can be used for each of these variables. Also note that some descriptions can meaningfully be assigned a variety of different units. For example, "price per lunch" could meaningfully be measured using the units "dollars per lunch" or "pennies per lunch". Since each student buys a lunch, "price per lunch" would also measure "dollars per student." This is a typical issue when modeling real-world situations—identifying the meaning of the variables in a useful, yet consistent way. Help students understand that the convention frequently used to denote units such as "dollars per student" is to write ${}^{dollars}/_{student}$.

As part of the launch, give pairs of students a few minutes to determine the meaning of the variable A in question 1 by using the descriptions and units associated with the variables in the expression that define A. With their partners, they should also work through questions 2 and 3 together, and then discussion this sequence of questions as a whole class. It is important that students understand this series of expectations for each new expression they encounter: interpret the meaning of the expression in terms of the units and descriptions of the variables involved, solve for one of the variables in the literal equation, and check the solution process by verifying that the variable solved for has the same meaning as its given description. Once variable A's description and units have been recorded in the table, set students to work with their partners on the remainder of the task.

Explore (Small Group):
Listen for how students are interpreting the meaning of new variables using the given descriptions of the variables involved in each expression. Support their work on solving each literal equation by reminding them of their work from the previous task. You might ask, How do we determine which order to un-do each operation in the expression?

It may be difficult for some students to recognize that they can multiply both sides of an equation by a variable, or subtract a variable from both sides of an equation. Some students may not even recognize the operations involved in the expressions when quantities are represented by variables instead of numbers. In such cases, it may be helpful to ask, "What if, instead of this variable, there was a number written in this position of the equation? What operation would be performed by that number?"

Note that question 10 is provided for fast-finishers, and that students who have completed the task through question 7, and are working on questions 8 and 9, should have access to the whole class discussion.

Discuss (Whole Class):
The major focus of the discussion should be on the equation solving process as applied to literal equations. Have students present their solutions to questions 5, 8b and 9b.

Once the solution strategies have been presented and discussed, turn the focus of the discussion to using units to interpret the meaning of expressions, and using units to justify the steps in the equation solving process.

If there is time, have students propose their expressions for question 10.

Aligned Ready, Set, Go: Equations and Inequalities 4.2

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Isolate a variable with inverse operations.

Isolate the indicated variable and then fill in the blank for the statement that follows.

1. Solve for *x*; $ax = 7$ I can find *1x or x* by ______________________ on both sides of the equation.
2. Solve for *p*; $8 + p = w$ I can find *1p or p* by ______________________ on both sides of the equation.
3. Solve for *m*; $e = mc^2$ I can find *1m or m* by ______________________ on both sides of the equation.
4. Solve for *t*; $d = rt$ I can find *1t or t* by ______________________ on both sides of the equation.
5. Solve for *r*; $d = rt$ I can find *r* by ______________________ on both sides of the equation.
6. Solve for *h*; $7 - h = 0$ I can find *h* by ______________________ on both sides of the equation.
7. Solve for *b*; $b - 11 = 3$ I can find *b* by ______________________ on both sides of the equation.
8. Solve for *y*; $\frac{1}{2}y = k$ I can find *y* by ______________________ on both sides of the equation.
9. Solve for *h*; $A = \frac{bh}{2}$ I can find *h* by ______________________ on both sides of the equation.
10. Solve for *x*; $y = mx + b$ I can find *x* by ______________________ on both sides of the equation.

SET

Topic: Defining and interpreting variables and units of measure.

Jaxon likes to be organized, so he made the following chart. He has decided to keep track of the miles he runs and the time he spends running. He attends P.E. class on Monday, Wednesday, and Friday, but he goes to school everyday. ***Fill in the Units column on the chart.***

Symbol	**Meaning** (Description of what the symbol means in context)	**Units** (What is counted or measured)
M	Number of miles ran in PE class on Mondays	
W	Number of miles ran PE class on Wednesdays	
F	Number of miles ran PE class on Fridays	
S	Number of miles from Jaxon's house to the school.	
H	Time (in hours) to travel to school	
t_M	Time (in minutes) spent running in PE on Monday	
t_W	Time (in minutes) spent running in PE on Wednesday	
t_F	Time (in minutes) spent running in PE on Friday	

Make meaning of the expressions below, write what they each mean!
If an expression does not make sense, say why.

11. $M + W + F$
12. $4(M + W + F)$
13. $2S$
14. $t_M + t_W + t_F$
15. $\frac{t_M + t_W + t_F}{3}$
16. $5(2H)$
17. $M + H$

GO

Topic: Set notation to interval notation. Inequalities on a number line.

Below you will find the domains of several different functions. The domains are described in either set notation or interval notation. Fill in the missing notation.

Set Notation	**Interval Notation**
18. $\{x \mid x \in \mathbb{R},\ -2 < x < 6\}$	
19.	[-4, 7]
20. $\{\mathrm{x} \mid \mathrm{x} \in \mathbb{R},\ x \geq -9\}$	
21.	(0, 13]
22. $\{\mathrm{x} \mid \mathrm{x} \in \mathbb{R},\ -15 \leq x \leq -8\}$	
23.	[-32, -15)
24.	$(-\infty, \infty)$

25. Which notation, interval or set, would be most appropriate when working with a domain of whole numbers?

For each of the inequalities provided graph the values being described on the numbers line.

26. $x < 6$	
27. $x > 5$	
28. $x \geq -9$	
29. $-7 \leq x < 0$	
30. $3 \leq x \leq 25$	
31. $-15 < x \leq 8$	

4. 3 Solving Equations Literally

A Practice Understanding Task

Solve each of the following equations for x:

1. $\frac{3x+2}{5}=7$

2. $\frac{3x+2y}{5}=7$

3. $\frac{4x}{3}-5=11$

4. $\frac{4x}{3}-5y=11$

5. $\frac{2}{5}(x+3)=6$

6. $\frac{2}{5}(x+y)=6$

7. $2(3x+4)=4x+12$

8. $2(3x+4y)=4x+12y$

Write a verbal description for each step of the equation solving process used to solve the following equations for x. Your description should include statements about how you know what to do next. For example, you might write, "*First I* ________________, *because* ____________________ . . ."

9. $\frac{ax+b}{c}-d=e$

10. $r\cdot\sqrt{\frac{mx}{n}+s}=t$

4. 3 Solving Equations Literally – Teacher Notes

A Practice Understanding Task

Purpose: This task provides practice for solving linear equations in one variable, solving linear equations in two variables for one of its variables, and solving literal equations. The process for solving multi-variable equations for one of its variables becomes more apparent when juxtaposed with similarly-formatted equations in one variable. The only difference in the solution process is the ability to carry out numerical computations to simplify the expressions in the one-variable equations.

Core Standards Focus:
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Related Standards:

Standards for Mathematical Practice:
SMP 7 – Look and make use of structure
SMP 8- Look for and express regularity in repeated reasoning

The Teaching Cycle:
Launch (Whole Class):
Encourage students to note the similarities in their work on the pairs of problems in questions 1-8. Also point out that they are to write detailed explanations of their solution strategy on problems 9 and 10. One way to facilitate this would be to have students fold a piece of paper in half lengthwise.

On the left side of the paper they write out their algebra steps, and on the right side they write out their justifications.

Explore (Small Group):

Monitor students while working on these problems and offer appropriate feedback, as necessary. Some of the problems have alternative strategies, such as #5 where you can distribute the 2/5 first, or multiply both sides of the equations by 5/2 first.

Help students recognize the difference between changing the form of an expression on one side of an equation versus writing an equivalent equation by applying the same operation to both sides.

If students are having difficulties with #9 or #10, have them write a related equation in which they replace all letters with numbers except for the x. See if they can solve the related equation for x and if that work can help them solve the original literal equation. Problem 10 involves a square root in order to emphasize that one of the key issues in solving an equation is to "un-do" an operation by applying the inverse operation to both sides. Help students think about how that would play out in problem 10. That is, how might they "un-do" a square root?

Discuss (Whole Class):

Have students present their solution process for any problems that may have been difficult for a number of students. You might also want to have students critique each other's explanations on problems 9 and 10 by having students exchange papers. They should fold their partner's paper in half, so that only the right side with the written explanation is showing. On a separate sheet of paper they should write-out the algebra steps they would take to solve each problem, based only on the wording of their partners' explanation. They should discuss any explanations that are unclear with their partner.

Aligned Ready, Set, Go: Equations and Inequalities 4.3

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Solving Inequalities.

Use the inequality $-9 < 2$ to complete each row in the table.

Apply each operation to the original inequality $-9 < 2$	Result	Is the resulting inequality true or false?
Example: Add 3 to both sides	-9+3 < 2+3 → -6 < 5	True
1. Subtract 7 from both sides.		
2. Add 15 to both sides.		
3. Add -10 to both sides.		
4. Multiply both sides by 10.		
5. Divide both sides by 5.		
6. Multiply both sides by -6.		
7. Divide both sides by -3.		

8. What operations when performed on an inequality, reverse the inequality? (Be very specific!)

SET

Topic: Solve literal equations that require more than one step.

Solve for the indicated variable. Show your work!!!

9. Solve for h. $Q = 25\pi h$

10. Solve for h. $Q = \pi r^2 h$

11. Solve for m. $y = 7m + 6$

12. Solve for m. $y = mx + b$

13. Solve for z. $A = (z + 7)3$

14. Solve for z. $A = (z + 7)w$

15. Solve for x. $\frac{x+2}{7} = 4$

16. Solve for x. $\frac{x+2y}{7} = 4$

17. Solve for x. $\frac{2x}{5} - 9 = 6$

18. Solve for x. $\frac{2x}{5} - 9y = 6$

19. Solve for x. $\frac{3}{4}(x - 2) = 12$

20. Solve for x. $\frac{3}{4}(x - 2y) = 12$

GO

Topic: Identifying x-intercepts and y-intercepts
Locate the x-intercept and y-intercept in the table. Write each as an ordered pair.

21.

x	y
-4	12
-3	10
-2	8
-1	6
0	4
1	2
2	0

x – intercept:

y – intercept:

22.

x	y
0	-6
3	-5
6	-4
9	-3
12	-2
15	-1
18	0

x – intercept:

y – intercept:

23.

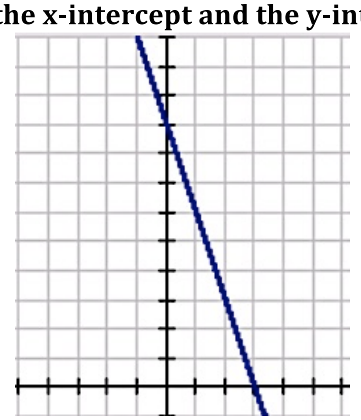
x	y
-3	10
-2	8
-1	6
0	4
1	2
2	0
3	-2

x – intercept:

y – intercept:

Locate the x-intercept and the y-intercept in the graph. Write each as an ordered pair.

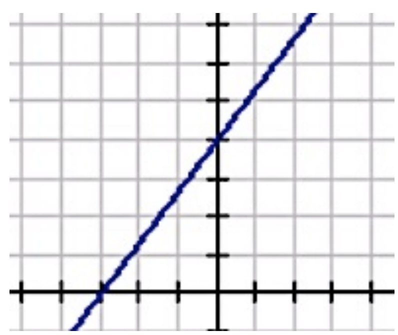
24.



x – intercept:

y – intercept:

25.



x – intercept:

y – intercept:

Solve each equation for x. Provide the justifications for each step. See the first example as a reminder for the types of justifications that might be used.

Example:

$3x - 6 = 15$	Justification
$+6 \quad +6$	Addition Property of equality
$\frac{3x}{3} = \frac{21}{3}$	Division Property of equality
$x = 7$	

26.

$4x - 10 = 2$	Justification

27.

$-16 = 3x + 11$	Justification

28.

$6 - 2x = 10$	Justification

29.

$6x + 3 = x + 18$	Justification

30.

$3x - 10 = 2x + 12$	Justification

31.

$12x + 3y = 15$	Justification

32.

$X(B + 7) = 9$	Justification

4.4 Greater Than?

A Develop Understanding Task

For each situation you are given a mathematical statement and two expressions beneath it.

1. Decide which of the two expressions is greater, if the expressions are equal, or if the relationship cannot be determined from the statement.
2. Write an equation or inequality that shows your answer.
3. Explain why your answer is correct.

Watch out—this gets tricky!

Example:
Statement: $x = 8$
Which is greater? $x + 5$ or $3x + 2$
Answer: $3x + 2 > x + 5$ because if $x = 8$, $3x + 2 = 26$, $x + 5 = 13$ and $26 > 13$.

Try it yourself:

1. Statement: $y < x$
   Which is greater? $x - y$ or $y - x$

2. Statement: $2x - 3 > 7$
   Which is greater? 5 or x

3. Statement: $10 - 2x < 6$
   Which is greater? x or 2

4. Statement: $4x \leq 0$
   Which is greater? 1 or x

5. Statement: n is an integer
Which is greater? n or $-n$

6. Statement $x > y$
Which is greater? $x + a$ or $y + a$

7. Statement: $x > y$
Which is greater? $x - a$ or $y - a$

8. Statement: $5 > 4$
Which is greater? $5x$ or $4x$

9. Statement: $5 > 4$
Which is greater? $\frac{5}{x}$ or $\frac{4}{x}$

10. Statement: $0 < x < 10$ and $0 < y < 12$
Which is greater? x or y

11. Statement: $3^{n+2} \geq 27$
Which is greater? n or 1

4.4 Greater Than? – Teacher Notes

A Develop Understanding Task

Purpose: The purpose of this task is to challenge students to reason about inequality relationships and to develop an understanding of the properties of inequalities. Each problem requires reasoning about numbers, including negative numbers and fractions, and thinking mathematically about the various possibilities in the given problem situation.

Core Standards Focus:

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution.
a. Construct a viable argument to justify a solution method.
b. Solve equations and inequalities in one variable.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematics I Note: *Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for.*

Standards for Mathematical Practice:

SMP 1 – Make sense of problems and persevere in solving them

SMP 2 – Reason abstractly and quantitatively

SMP 8 – Look for and express regularity in repeated reasoning

The Teaching Cycle:

Launch (Whole Class):

Explain to students that this task is a big logic puzzle. All of the problems require thinking about all the different possibilities to decide which expression is greater. There are some that cannot be determined from the information given. When that happens, students should write down what information they would need in order to have a definite answer for the question. (You may choose not to tell students this in advance so that they have an opportunity to wrestle with the ideas and to justify their position.) Start by asking students to read the example given. To confirm the instructions, ask how they see the three required parts of the explanation in the answer. Next, refer students to problem #1. Give them a few minutes to answer and write their own explanation. Ask the class for their answers and explanations and model how to write an answer with a complete explanation. You may also want to model thinking about possible values for x and y, like: "If x is a negative number, then y must also be a negative number because it is less than x."

Explore (Small Group or Pairs): Monitor students as they work. Encourage them to think about the various possibilities for x and y in each case. Be sure that their written explanations adequately communicate their logic. Watch for problems that generate disagreement or difficulty for the class discussion. Also look for students' explanations that demonstrate sound mathematical logic or good communication to be highlighted in the discussion. If you notice a common misconception occurring during the exploration, plan to raise it as an issue in the discussion.

Discuss (Whole Class): Start the discussion with problems 6-9. Ask previously-selected students to give their explanations for each of these problems. Be sure that the explanations include example of both positive and negative numbers. Highlight for the class that these three problems are asking them to justify the properties of inequalities. Generally, students will have tested specific numbers and made generalizations about all numbers based upon the selected examples. Ask the class if they can create an argument as to why each property can be generalized to all real numbers. Write each of the properties of inequalities (addition, subtraction, multiplication, and division), and ask students to state them in their own words. After going through each of these, turn the discussion to any misconceptions or provocative problems that were selected during the exploration phase.

Aligned Ready, Set, Go Homework: Getting Ready 4.4

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Write an equation from a context. Interpret notation for inequalities.

Write an equation that describes the story. Then answer the question asked by the story.

1. Virginia's Painting Service charges $10 per job and $0.20 per square foot. If Virginia earned $50 for painting one job, how many square feet did she paint at the job?

2. Renting the ice-skating rink for a party costs $200 plus $4 per person. If the final charge for Dane's birthday party was $324, how many people attended his birthday party?

Indicate if the following statements are <u>true</u> or <u>false</u>. Explain your thinking.

3. The notation $12 < x$ means the same thing as $x < 12$. It works just like $12 = x$ *and* $x = 12$.

4. The inequality $-2(x + 10) \geq 75$ says the same thing as $-2x - 20 \geq 75$. I can multiply by -2 on the left side without reversing the inequality symbol.

5. When solving the inequality $10x + 22 < 2$, the second step should say $10x > -20$ because I added -22 to both sides and I got a negative number on the right.

6. When solving the inequality $-5x \geq 45$, the answer is $x \leq -9$ because I divided both sides of the inequality by a negative number.

7. The words that describe the inequality $x \geq 100$ are "*x is greater than or equal to 100*."

SET

Topic: Solve inequalities. Verify that given numbers are elements of the solution set.

Solve for x. (Show your work.) Indicate if the given value of x is an element of the solution set.

8. $2x - 9 < 3$

Is this value part of the solution set? $x = 6$; *yes*? *no*?

9. $4x + 25 > 13$

Is this value part of the solution set? $x = -5$; *yes*? *no*?

10. $6x - 4 \leq -28$

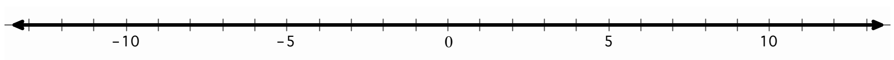
Is this value part of the solution set? $x = -10;$ $yes?$ $no?$

11. $3x - 5 \geq -5$

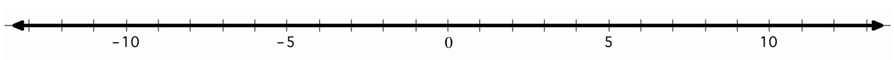
Is this value part of the solution set? $x = 1;$ $yes?$ $no?$

Solve each inequality and graph the solution on the number line.

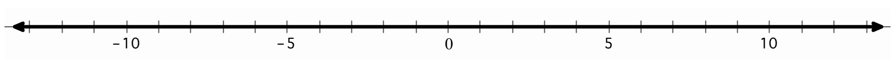
12. $x + 9 \leq 7$



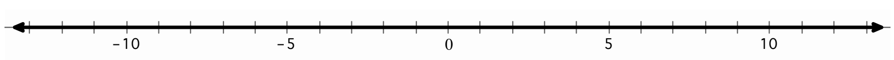
13. $-3x - 4 > 2$



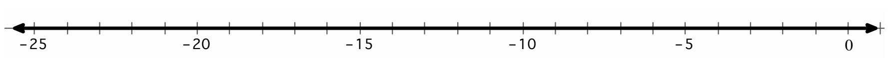
14. $3x < -6$



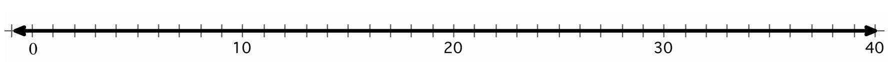
15. $\frac{x}{5} > -\frac{3}{10}$



16. $-10x > 150$



17. $\frac{x}{-7} \geq -5$



Solve each multi-step inequality.

18. $x - 5 > 2x + 3$

19. $\frac{3(x-4)}{12} \leq \frac{2x}{3}$

20. $2(x - 3) \leq 3x - 2$

GO

Topic: Use substitution to solve linear systems

Solve each system of equations by using substitution.

Example: $\begin{cases} y = 12 \\ 2x - y = 14 \end{cases}$

The first equation states that $y = 12$. *That information can be used in the second equation to find the value of x by replacing y with 12. The second equation now says* $\mathbf{2x - (12) = 14}$. *Solve this new equation by adding 12 to both sides and then dividing by 2. The result is x = 13.*

21. $\begin{cases} y = 5 \\ -x + y = 1 \end{cases}$

22. $\begin{cases} x = 8 \\ 5x + 2y = 0 \end{cases}$

23. $\begin{cases} 2y = 10 \\ 4x - 2y = 50 \end{cases}$

24. $\begin{cases} 3x = 12 \\ 4x - y = 5 \end{cases}$

25. $\begin{cases} y = 2x - 5 \\ y = x + 8 \end{cases}$

26. $\begin{cases} 3x = 9 \\ 5x + y = -5 \end{cases}$

4.5 May I Have More, Please?

A Solidify Understanding Task

Elvira, the cafeteria manager, has to be careful with her spending and manages the cafeteria so that they can serve the best food at the lowest cost. To do this, Elvira keeps good records and analyzes all of her budgets.

1. Elvira's cafeteria has those cute little cartons of milk that are typical of school lunch. The milk supplier charges $0.35 per carton of milk, in addition to a delivery charge of $75. What is the maximum number of milk cartons that Elvira can buy if she has budgeted $500 for milk?
    a. Write and solve an inequality that models this situation.

    b. Describe in words the quantities that would work in this situation.

    c. Write your answer using appropriate notation.

2. Students love to put ranch dressing on everything, so Elvira needs to keep plenty in stock. The students eat about 2.25 gallons of ranch each day! Elvira started the school year with 130 gallon of ranch dressing. She needs to have at least 20 gallons left when she reorders to have enough in stock until the new order comes. For how many days will her ranch dressing supply last before she needs to reorder?
    a. Write and solve an inequality that models this situation.

b. Describe in words the quantities that would work in this situation.

c. Write your answer in both interval and set notation.

3. The prices on many of the cafeteria foods change during the year. Elvira finds that she has ordered veggie burgers four times and paid $78, $72, $87, and $90 on the orders. To stay within her budget, Elvira needs to be sure that the average order of veggie burgers is not more than $82. How much can she spend on the fifth order to keep the average order within her budget?

   a. Write and solve an inequality that models this situation.

   b. Describe in words the quantities that would work in this situation.

   c. Write your answer in both interval and set notation.

4. Elvira can purchase ready-made pizzas for $14.50 each. If she makes them in the cafeteria, she can spend $44.20 on ingredients and $6.25 per pizza on labor. For how many pizzas is it cheaper for the cafeteria to make the pizzas themselves rather than buy them ready-made?

   a. Write and solve an inequality that models this situation.

   b. Describe in words the quantities that would work in this situation.

   c. Write your answer in both interval and set notation.

5. Elvira is comparing prices between two different suppliers of fresh lettuce. Val's Veggies charges $250 for delivery plus $1.50 per bag of lettuce. Sally's Salads charges $100 for delivery plus $4.00 per bag of lettuce. How many bags of lettuce must be purchased for Val's Veggies to be the cheaper option?
   a. Write and solve an inequality that models this situation.

   b. Describe in words the quantities that would work in this situation.

   c. Write your answer in both interval and set notation.

6. Each student that buys school lunch pays $2.75. The cafeteria typically brings in between $1168.75 and $1438.25. How many students does the cafeteria usually serve?
   a. Model this situation using an inequality.

   b. Describe in words the quantities that would work in this situation.

   c. Write your answer in both interval and set notation.

4. 5 May I Have More, Please? – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to for students to solidify their understanding of how to use and solve inequalities. The task gives students a number of situations and asks them to write inequalities to model the situations. Students will use the properties of inequalities, along with applying their experience in solving equations to solve the inequalities and answer the questions. Students will also work with both interval and set notation to write their solutions. They have previously used these two notations to write the domain and range for functions. The notation should be familiar, but this is a slightly different purpose for using it than students have seen. In each case, students are asked to state their solution in words before writing it using both set builder and interval notation so that they will relate the notation to the range of solutions.

Core Standards Focus:

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution.
a. Construct a viable argument to justify a solution method.
b. Solve equations and inequalities in one variable.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematics I Note: *Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for.*

Standards for Mathematical Practice

SMP 2 – Reason abstractly and quantitatively

SMP 4 – Model with mathematics

Teaching Cycle

Launch (Whole Class):

Read the initial story context about Elvira, the cafeteria manager, and the problem situation given in #1. Ask students to write a model for the amount paid for milk, which will be: $0.35x + 75$. Ask students how they can use the inequality symbols to show that the amount paid for milk cannot be more than $500. Discuss why the inequality would be written as: $0.35x + 75 \leq 500$. Walk students through the solution of the inequality, asking them to justify each step using one of the properties of inequalities.

$$0.35x + 75 \leq 500$$
$$0.35x \leq 425$$
$$x \leq 1214.29$$

This context requires interpreting the algebraic solution. First, since Elvira can't purchase a fraction of a milk carton, the maximum number she could purchase would be 1214. Next, discuss with the students that the range of numbers implied by the solution $x \leq 1214.29$ includes negative numbers, which don't make sense in this context. Ask students to describe the actual solution in words, which should be something like: from 0 to 1214 cartons of milk. Then, model using and reading both notations:

Interval notation: $[0,1214]$

Set builder notation: $\{x|\ 0 \leq x \leq 1214\}$

Explore (Small Group):

Encourage students to continue with the rest of the situations. Listen as they are working to see that they understand the contexts and are interpreting their solutions to match the situation given. Watch students as they make sense of problem #6, which could be written as a compound inequality or as two inequalities. Most students will not have experience in solving compound inequalities, but look for productive approaches that can be leveraged during the class discussion.

Discuss (Whole Class):

Begin the discussion with problem #2. Select a student that has written the equation:

$130 - 2.25x \geq 20$. Ask students to explain how they decided to use $\geq$ or $\leq$. Ask the students to show how they solved the inequality and justify their steps. Be sure to highlight the fact that if they divide by -2.25, they need to reverse the inequality.

If time permits, you may ask students to present problems #3 and #4, but be sure to leave time to discuss problems #5 and #6. Problem #5 leads to variables on both sides of the inequality, which adds a bit of algebraic complexity to work through. Problem 6 presents a compound inequality or two inequalities. If both approaches are available in the class, have them both presented and ask the class to compare the approaches so that students see that a compound inequality of this form combines two conditions that must both be met.

Aligned Ready, Set, Go: Equations and Inequalities 4.5

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Interpret phrases that imply an inequality.
Rewrite the given "word sentence" as a "math sentence." Each math sentence will use one of the following symbols: $>$, $<$, $\leq$, $\geq$. Use "x" in place of the number.

	Word Sentence	Math Sentence
Example:	I am thinking of a number that is greater than 13.	$x > 13$
1.	I am thinking of a number that is at least 13.	
2.	I am thinking of a number that is no fewer than 13.	
3.	I am thinking of a number that does not exceed 13.	
4.	I am thinking of a number that is at most 13.	
5.	I am thinking of a number that is no more than 13.	
6.	I am thinking of a number that is fewer than 13.	
7.	I am thinking of a number that is not above 13.	
8.	I am thinking of a number that is less than 13.	
9.	I am thinking of a number that is not under 13.	
10.	I am thinking of a number that is not greater than 13.	

SET

Topic: Write and solve inequalities from a context.

11. To take sweepstakes for the largest pumpkin crop at the Riverside County Fair, the average weight of Ethan's two pumpkins must be greater than 875 lbs. One of his pumpkins weighs 903 lbs. What is the least amount of pounds the second pumpkin could weigh in order for Ethan to win the prize?

    a) Write an inequality that models this situation. Be sure to define your variables.

    b) Describe in words the quantities that would work in this situation.

    c) Write your answer in both interval and set notation.

12. The average of Aaron's three test scores must be at least 93 to earn an A in the class. Aaron scored 89 on the first test and 94 on the second test. What scores can Aaron get on his third test to guarantee an A in the class? (The highest possible score is 100.)

    a) Write and solve an inequality that models this situation. Be sure to define your variables.

    b) Describe in words the quantities that would work in this situation.

    c) Write your answer in both interval and set notation.

13. A cell phone company offers a plan that costs \$35.99 and includes unlimited texting. Another company offers a plan that costs \$19.99 and charges \$0.25 per text. For what number of texts does the second company's plan cost more than the first company's plan?

   a) Write and solve an inequality that models this situation. Be sure to define your variables.

   b) Describe in words the quantities that would work in this situation.

   c) Write your answer in both interval and set notation.

GO

Topic: Use substitution to solve linear systems

Solve each system of equations by using substitution.

Example: $\begin{cases} y = x + 3 \\ 2x - y = 14 \end{cases}$

The first equation states that $y = x + 3$. *That information can be used in the second equation to find the value of x by replacing y with* $x + 3$. *The second equation now says* $\mathbf{2x - (x + 3) = 14}$. *Solve this new equation by first distributing the negative over* $(x + 3)$. *The new equation will be* $\mathbf{2x - x - 3 = 14.}$ *Combine like terms. You will get the equivalent equation* $\mathbf{x - 3 = 14}$. *Add 3 to both sides. You should get* $\mathbf{x = 17}$. *But you still don't know the value of* y. *Now that you know the value of x, you can use either equation to figure out the value of* y. *Since the first equation is simpler, you may want to substitute the known value of x (recall that* $x = 17$*) into it. It should be easy to see what y equals.* $\mathbf{y = (17) + 3 = 20}$.

21. $\begin{cases} y = x + 5 \\ 2x + y = -1 \end{cases}$

22. $\begin{cases} x = y - 1 \\ 5x + 2y = 9 \end{cases}$

23. $\begin{cases} y = 10 - x \\ 4x - 2y = 40 \end{cases}$

24. $\begin{cases} x = 1 + y \\ 4x - y = 7 \end{cases}$

4.6 Taking Sides

A Practice Understanding Task

Joaquin and Serena work together productively in their math class. They both contribute their thinking and when they disagree, they both give their reasons and decide together who is right. In their math class right now, they are working on inequalities. Recently they had a discussion that went something like this:

> Joaquin: The problem says that "6 less than a number is greater than 4." I think that we should just follow the words and write: $6 - n > 4$.
>
> Serena: I don't think that works because if n is 20 and you do 6 less than that you get 20 – 6 = 14. I think we should write $n - 6 > 4$
>
> Joaquin: Oh, you're right. Then it makes sense that the solution will be $n > 10$, which means we can choose any number greater than 10.

The situations below are a few more of the disagreements and questions that Joaquin and Serena have. Your job is to decide how to answer their questions, decide who is right, and give a mathematical explanation of your reasoning.

1. Joaquin and Serena are assigned to graph the inequality $x \geq -7$.
Joaquin thinks the graph should have an open dot -7.
Serena thinks the graph should have a closed dot at -7.
Explain who is correct and why.

2. Joaquin and Serena are looking at the problem $3x + 1 > 0$.
Serena says that the inequality is always true because multiplying a number by three and then adding one to it makes the number greater than zero.
Is she right? Explain why or why not.

3. The word problem that Joaquin and Serena are working on says, "4 greater than x".
Joaquin says that they should write: $4 > x$.
Serena says they should write: $4 + x$
Explain who is correct and why.

4. Joaquin is thinking hard about equations and inequalities and comes up with this idea:
If $45 + 47 = t$, then $t = 45 + 47$.
So, if $45 + 47 < t$, then $t < 45 + 47$.
Is he right? Explain why or why not.

5. Joaquin's question in #4 made Serena think about other similarities and differences in equations and inequalities. Serena wonders about the equation $-\frac{x}{3} = 4$ and the inequality $-\frac{x}{3} > 4$. Explain to Serena ways that solving these two problems are alike and ways that they are different. How are the solutions to the problems alike and different?

6. Joaquin solved $-15q \leq 135$ by adding 15 to each side of the inequality. Serena said that he was wrong. Who do you think is right and why?

Joaquin's solution was $q \leq 150$. He checked his work by substituting 150 for q in the original inequality. Does this prove that Joaquin is right? Explain why or why not.

Joaquin is still skeptical and believes that he is right. Find a number that satisfies his solution but does not satisfy the original inequality.

7. Serena is checking her work with Joaquin and finds that they disagree on a problem. Here's what Serena wrote:

$$3x + 3 \leq -2x + 5$$
$$3x \leq -2x + 2$$
$$x \leq 2$$

Is she right? Explain why or why not?

8. Joaquin and Serena are having trouble solving $-4(3m - 1) \geq 2(m + 3)$
Explain how they should solve the inequality, showing all the necessary steps and identifying the properties you would use.

9. Joaquin and Serena know that some equations are true for any value of the variable and some equations are never true, no matter what value is chosen for the variable. They are wondering about inequalities. What could you tell them about the following inequalities? Do they have solutions? What are they? How would you graph their solutions on a number line?

a. $4s + 6 \geq 6 + 4s$

b. $3r + 5 > 3r - 2$

c. $4(n + 1) < 4n - 3$

10. The partners are given the literal inequality $ax + b > c$ to solve for *x*. Joaquin says that he will solve it just like an equation. Serena says that he needs to be careful because if a is a negative number, the solution will be different. What do you say? What are the solutions for the inequality?

4.6 Taking Sides – Teacher Notes
A Practice Understanding Task

Purpose: The purpose of this task is to practice the reasoning and solving of inequalities. Many common misconceptions and mistakes made when solving inequalities are addressed in the task. Students are asked to think about the differences and similarities in solving inequalities versus solving equations, including that most inequalities produce a range of solutions, that the inequality sign must be turned when multiplying or dividing by a negative number, and that the reflexive property holds only for equations.

Core Standards:

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution.
a. Construct a viable argument to justify a solution method.
b. Solve equations and inequalities in one variable.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematics I Note: *Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for.*

Standards for Mathematical Practice:

SMP 3 - Construct viable arguments and critique the reasoning of others.

SMP6 - Attend to precision.

The Teaching Cycle:

Launch (Whole Class):

Hand out the worksheet and go over the scenario given about Joaquin and Serena. Ask a student to demonstrate Serena's argument that changed Joaquin's mind. Start with #1, giving students about 3 minutes to answer the question individually, writing their best explanation in complete sentences. Ask a student to read their explanation and then model a thorough answer on the board. The explanation should be something like:

The inequality $x \geq -7$ *should be graphed with a closed dot on -7 and all points to the right of -7 filled in. This is because the symbol ≥ means greater than or equal to, so -7 is a solution to the inequality. The closed dot on -7 shows that the number is included in the solution set. An open dot would indicate that -7 is not a solution.*

Next, ask the class to think about #2. Give students a few minutes to talk to their neighbor about their reasoning and then ask them to write individually for a few minutes. While they are working, circulate through the room to find an exemplar to use with the class. It is important that students are first considering the logic of why Serena believes the inequality to always be true. Then, students should recognize that she is only considering nonnegative numbers. Be sure that both parts of the argument are discussed to model the thinking process that will be required throughout the task and also what a complete answer will look like.

Explore (Small Group):

Let students work on the remainder of the task as you monitor their thinking. Throughout the task, students will be challenged to consider whether the strategies that they have learned for equations will apply to inequalities. If you find that some groups are spending excessive time on a problem, redirect their focus to problems 4, 5, 7, and 8, which will be the focus of the whole group discussion. Identify a group to present one of each of these problems.

Discuss (Whole Group):

Start the discussion with the presentation of problem #4. Be sure that the group identifies that if the two sides of an equation are switched, the equation remains true. Their explanation should also include the idea that if the sides of an inequality are switched, then the inequality sign must be turned. Ask the group to demonstrate this thinking by substituting numbers into the inequality for x. Remind students that this often comes up because the solutions to inequalities are conventionally written with the variable on the left side of the inequality. So, an expression like $4 < x$ is routinely converted to $x > 4$.

Next, move the discussion to #5. Again, students are asked to consider the difference between solving an equation and solving an inequality. The explanation from the group should include the idea that the steps in solving the equation and the inequality are the same. The only difference is that the inequality sign must be turned when multiplying by -3. Press students to explain why that rule holds, rather than simply stating the rule.

The explanation of #7 should include showing all the proper steps in the solution of the inequality. As students demonstrate the problem, they will see that Serena did not properly add $2x$ to both sides of the inequality. Ask students why this inequality seemed to be solved exactly like an equation, but other inequalities do not.

Close the discussion with question #8. Emphasize the reasons for each step and compare the properties of inequalities with the properties of equations.

Aligned Ready, Set, Go: Equations and Inequalities 4.6

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Solving equations and inequalities from a context.

Write the given situation as an equation or inequality and then solve it.

1. The local amusement park sells summer memberships for $50 each. Normal admission to the park costs $25; admission for members costs $15.

   a. If Darren wants to spend no more than $100 on trips to the amusement park this summer, how many visits can he make if he buys a membership with part of that money?

   b. How many visits can he make if he pays normal admission instead?

   c. If he increases his budget to $160, how many visits can he make as a member?

   d. How many can he make as a non-member with the increased budget of $160?

2. Jade just took a math test with 20 questions, each question is worth an equal number of points. The test is worth 100 points total.

   a. Write an equation that can be used to calculate Jade's score based on the number of questions she got right on the test.

   b. If a score of 70 points earns a grade of *C*-, how many questions would Jade need to get right to get at least a *C*- on the test?

   c. If a score of 83 points earns a grade of *B*, how many questions would Jade need to get right to get at least a *B* on the test?

   d. Suppose Jade got a score of 60% and then was allowed to retake the test. On the retake, she got all the questions right that she got right the first time, and also got half the questions right that she got wrong the first time. What percent of the questions did Jade get right, in total, on the retake?

SET

Topic: Solve and justify one variable inequalities

Solve each inequality, justifying each step you use.

3.

$-5x < 35$	Justification

4.

$x + 68 \geq 75$	Justification

5.

$2x - 4 \leq 10$	Justification

6.

$5 - 4x \leq 17$	Justification

7.

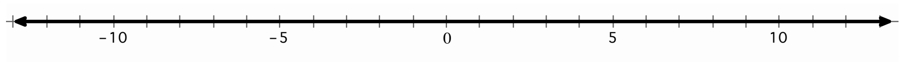
$\frac{x}{-3} > -\frac{10}{9}$	Justification

8.

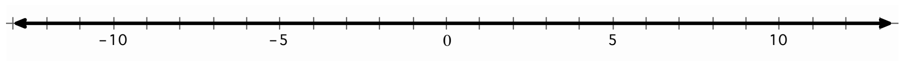
$2(x - 3) \leq 3x - 2$	Justification

Solve each inequality and graph the solution on the number line.

9. $x - 8 > -20$



10. $x + 11 > 13$



Solve each multi-step inequality.

11. $4x + 3 < -1$

12. $4 - 6x \leq 2(2x + 3)$

13. $5(4x + 3) \geq 9(x - 2) - x$

14. $\frac{2}{3}x - \frac{1}{2}(4x - 1) \geq x + 2(x - 3)$

Topic: Solve literal equations

15. Solve the following equation for C: $F = \frac{9}{5}C + 32$

16. Given $V = \frac{1}{3}\pi r^2 h$, rewrite the formula to isolate the variable r.

17. The area formula of a regular polygon is $A = \frac{1}{2}Pa$. The variable a represents the apothem and P represents the perimeter of the polygon. Solve the equation for the apothem, a.

18. The equation $y = mx + b$ is the equation of a line. Isolate the variable *b.*

19. The equation for the circumference c of a circle with radius r is $c = 2\pi r$.
Solve the equation for the radius, r.

20. The equation for the area of a circle A based on diameter d is $A = \pi \frac{d^2}{4}$.
Solve the equation to isolate the diameter, d.

GO

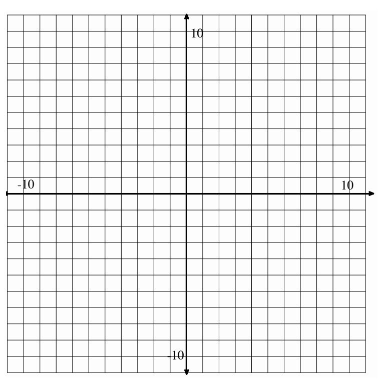
Topic: Solve systems of equations by graphing

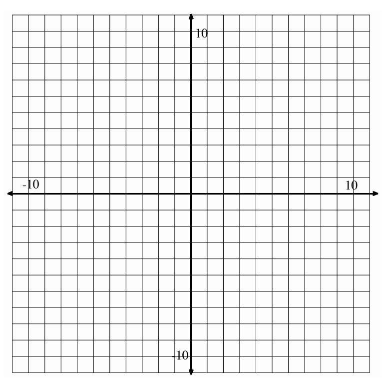
Graph both lines on the same coordinate grid. Identify the point of intersection. Then test the x and y values of the point of intersection in the two equations.

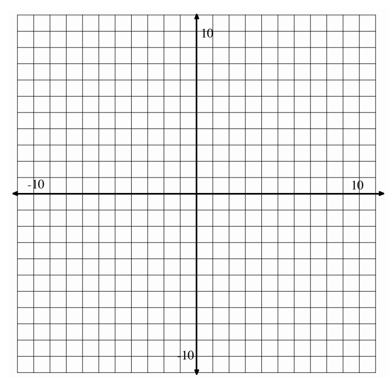
21. $\begin{cases} y = 2x + 5 \\ -x + y = 1 \end{cases}$

22. $\begin{cases} 10 + y = 3x \\ 2x + y = 0 \end{cases}$

23. $\begin{cases} x + y = 9 \\ x - y = -7 \end{cases}$







4.7H Cafeteria Consumption and Costs

A Develop Understanding Task

Sometimes Elvira hosts after school events in the cafeteria as clubs and teams celebrate their accomplishments. Frequently she orders too much food for such events—and occasionally not enough. For example, she has noticed that the chess club eats less than the football team, but more than the cheerleaders.

Elvira has asked you to help her sort through her records for the past few years, hoping she can better plan on how much food to order for the upcoming soccer team and drama club events. Unfortunately, Elvira kept most of her records on Post-It Notes, and now everything is out of order. Fortunately, she used a different color of Post-It Notes each year, so you at least have a place to start.

1. Here is the information you have identified from the past three years for the soccer team and drama club events. The blue Post-It Notes are from three years ago, the yellow from two years ago, and the pink from last year's events. Organize the data for each year in such a way that it can be combined with similar data from other years.

Blue Post-It Note Ordered 10 packages of chips for the soccer team— Way too much!	**Yellow Post-It Note** Ordered 6 packages of chips for the soccer team— Definitely not enough!	**Blue Post-It Note** Ordered 3 dozen cookies for the drama club— Should have ordered more
Blue Post-It Note Ordered 4 gallons of drinks for the soccer team. They poured some on their coach! (big mess)	**Pink Post-It Note** Ordered 8 packages of chips for the soccer team—My neighbor is on the team!	**Yellow Post-It Note** Ordered 5 dozen cookies for the drama club— I really like these kids!
Pink Post-It Note Ordered 10 packages of chips for the drama club—They talked a lot with fake accents	**Blue Post-It Note** Ordered 5 gallons of drinks for the drama club (they talk a lot and seem to get thirsty!)	**Pink Post-It Note** Ordered 4 dozen cookies for the drama club—Too much drama, too little character!

Blue Post-It Note Ordered 8 packages of chips for the drama club— Needed more!	**Pink Post-It Note** Ordered 8 dozen cookies for the soccer team—slipped a few extra to my neighbor.	**Pink Post-It Note** Ordered 4 gallons of drinks for the soccer team—Watched the players like a hawk!
Pink Post-It Note Ordered 4 gallons of drinks for the drama club— Seemed about right	**Yellow Post-It Note** Ordered 4 gallons of drinks for the soccer team—Warned them not to repeat last year's prank!	**Yellow Post-It Note** Ordered 3 gallons of drinks for the drama club—drinks were gone long before the chips
Yellow Post-It Note Ordered 7 dozen cookies for the soccer team— Should I have ordered more?	**Yellow Post-It Note** Ordered 12 packages of chips for the drama club— Sent extra home with kids	**Blue Post-It Note** Ordered 6 dozen cookies for the soccer team— Could have ordered more

2. You suggest to Elvira that for each event she should order the average amount of each item based on what she has ordered over the past three years. How might you represent this year's order in a concise, organized way? Describe in detail how you calculated the amount of each item to be ordered for each event so Elvira can follow your description when planning for future events.

3. Elvira just informed you that the soccer team won the state championship and the drama club took major awards at the Shakespearean Festival competition. Consequently, both groups have decided to allow each member of the team or club to invite three guests to accompany them to their celebration events. Elvira assumes that each of the guests will consume about the same amount of food as the team or club members they accompany. Explain to Elvira how to use your representation of the original amount of food to order from question 2 to determine the new amount of food to order.

4. Elvira can order food from either *Mainstreet Market* or *Grandpa's Grocery*, and she has given you a list of the prices at each store for each item to be purchased. She would like you to find the total cost of purchasing the recommended amount of food from question 2 for each event from each store. Elvira knows that for some events it might be best to purchase the food from *Mainstreet Market* and for other events it may be better to purchase the food from *Grandpa's Grocery*. She also realizes that it is too time consuming to purchase some items from one store and some from another. You will need to keep track of the details of your computations for the total cost so Elvira can use your strategy for future events.

	Mainstreet Market	***Grandpa's Grocery***
Cost per package of chips	\$2.50	\$2.00
Cost per dozen cookies	\$3.00	\$4.00
Cost per gallon of drink	\$2.00	\$1.50

4.7H Cafeteria Consumption and Costs – Teacher Notes

A Develop Understanding Task

Purpose: This task provides opportunities for students to develop a reason for organizing data into rectangular arrays or matrices. Each element in a matrix represents two characteristics or quantities, one by virtue of the row it is located in and one by virtue of the column it is in. Consequently, each element has units associated with it that describe both the characteristics of the row and column in which it is located. Paying attention to these units guides ways that we can combine matrices by addition, subtraction, and scalar or matrix multiplication. In this task, students will notice that in order to add data represented by matrices, the corresponding rows and columns of each matrix need to contain similar information. The data in a matrix can be scaled up or scaled down by multiplying each element in each row and column by the same scale factor (e.g., scalar multiplication). This task also surfaces thinking about multiplying corresponding factors and adding corresponding terms to find desired linear combinations—an essential idea for matrix multiplication that will be examined and solidified in the following task.

Core Standards Focus:

N.VM.6 Use matrices to represent and manipulate data.

N.VM.7 Multiply matrices by scalars to produce new matrices.

N.VM.8 Add, subtract, and multiply matrices of appropriate dimensions.

Related Standards: N.Q.1

Standards for Mathematical Practice:

SMP 4 – Model with mathematics

SMP 7 – Look for and make use of structure

SMP 8 – Look for and express regularity in repeated reasoning

The Teaching Cycle:

Launch (Whole Class):

Read through the introductory paragraphs and the scenario in question 1. Distribute the blue, yellow and pink notes (see attached handouts) for students to rearrange as they consider different ways to organize the information on the notes. (Mix up the notes similar to the task handout.)

Explore (Small Group):

As students are working on the task you may need to remind them that their goal is to organize each year's data in such a way that "it can be combined with similar data from other years." You might need to remind students that the data was collected over a period of three years, so the pink and yellow notes didn't exist when the blue notes were created. Consequently, it would not make sense to mix the colored Post-it notes together.

Watch for useful organizational schemes such as putting all of the data for the soccer team in the same row (or column) and all of the data for the drama club in a separate row (or column) and then organizing the food items in the same order throughout both rows (or columns). This same organizational structure should be repeated for each year's set of data. Once the data is organized in this way, it makes sense that pieces of data that appear in corresponding rows and columns from each of the three years can be added together. Note that not every group will organize their data in the same way—rows and columns might be interchanged, as well as the order of food items within a row or column. The essential idea is to get corresponding pieces of data to appear in the same row and column each year.

Once groups have organized their data, have them move on to discuss questions 2, 3 and 4. Questions two and three introduce the idea of scalar multiplication. For question 2, look for students who first create a matrix (or rectangular arrangement of the data) that represents the total of the three years for each food item for each event, and then multiplies each item in their matrix by 1/3 (or divides by 3) to get a new matrix representing the average amount of each item for each event. For question 3, look for students who multiply each element in the matrix obtained in question 2 by a factor of 3.

Question 4 is designed to lay a foundation for thinking about matrix multiplication. While students do not yet know how to organize and manipulate matrices to solve this problem, they can think through the required multiplication of factors and sum of terms that will determine the total cost of purchasing all of the items for each event at either store. Make sure they are keeping track of their computations by recording equations that represent their work. For example, to find the total cost of buying food for the soccer event at *Mainstreet Market* would carry out the following computation: (8 packages of chips $\times$ \$2.50/package of chips) + (7 dozen cookies $\times$ \$3.00/dozen cookies) + (4 gallons of drink $\times$ \$2.00/gallon of drink) = \$49. Since the main focus of the whole class discussion will be on questions 1-3, if students do not finish question 4 assign it as homework. A discussion of question 4 will serve as the launch of the next task.

Select 2 or 3 groups who have organized the three years of data differently to present to the class. Give the selected groups time to glue their organizational structures of the data to chart paper, before beginning the discussion. Choose groups who have finished more quickly than others to prepare for this presentation, while other groups continue to work on the task.

Discuss (Whole Class):

Have each of the selected groups present the ways they organized their data. Ask the class to identify similarities and differences between the organizational schemes, and to account for the similarities (e.g., it is easier to add like terms if the same types of data appears in corresponding rows and columns each year), and explain why the differences don't matter (e.g., one group may have used rows to represent teams and clubs and columns to represent food items, while another group might have exchanged the roles of the rows and columns; it doesn't matter the order in which the food items are listed, as long as the order is consistent from year to year).
As students discuss questions 2 and 3, introduce the notation of scalar multiplication of a matrix. Depending on time and the progress of the students, you may begin a discussion of question 4 or give students additional time to work on question 4 in preparation for the next task.

Aligned Ready, Set, Go: Equations and Inequalities 4.7H

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Creating tables from graphs

For each of the given functions, either explicit or recursive, find the missing values in the table. Use the explicit rules and equations as a tool to find the values. If you are not given the explicit rule you might consider creating it to help you with your work.

1. $f(x) = -2x + 7$

x	$f(x)$
-3	
0	
	43
21	

2. $g(x) = 3x - 25$

x	$g(x)$
-8	
	26
12	
	-37

3. $h(x) = h(x-1) + 6;\ h(1) = -13$

x	$h(x)$
3	
15	
	5
	23

4. $d(t) = d(t-1) - 2; d(1) = 34$

t	$d(t)$
2	
	-2
10	
	10

5. $y = 7x - 35$

x	y
-2	
0	
	21
	0

6. $x + y = 24$

x	y
10	
	0
	-20
0	

SET

Topic: Organizing information in matrices

Elvira has been running a private catering business to make extra money. She needs some help organizing the information in problems 7 through 10 below so that she can better predict amounts to purchase and improve her profits. Assist her by organizing the information in a meaningful way so that she can average the years and do better for the coming year.

7. The last three years Elvira has catered family gatherings and city events. Last year she provided the following at family gatherings she catered: 5 bags of chips, 6 dozen cookies and 4 gallons of drink. Last year at city events she provided the following: 16 bags of chips, 20 gallons or drink and 24 dozen cookies. Organize this information.

8. Two year ago Elvira provided the following at family events: 5 gallons of drink, 4 bags of chips and 5 dozen cookies. While she provided the following at city events: 20 dozen cookies, 18 gallons or drink and 12 bags of chips.

9. Three years ago Elvira provided the following at city events: 14 bags of chips, 20 gallons of drink and 19 dozen cookies. She also provided the following at family gatherings: 6 bags of chips, 7 dozen cookies and 9 gallons or drink.

10. If you provide Elvira with an average amount to be ordered for the gatherings and events she caters in the coming year, how much of each item would she need? Present the average in an organized way.

GO

Topic: Arithmetic and Geometric Sequences

Remember sequences from the beginning of the year. For each sequence below, determine whether it is either arithmetic or geometric and find both the recursive and the explicit rules.

11. 1, 3, 5, 7, …

Arithmetic or geometric?

Recursive: ____________________________

Explicit: ____________________________

12. 3, 6, 12, 24, …

Arithmetic or geometric?

Recursive: ____________________________

Explicit: ____________________________

13.

Time (in days)	Number of people
1	3
2	7
3	11
4	15

Arithmetic or geometric?

Recursive: ____________________________

Explicit: ____________________________

14.

Time (in days)	Number of bacteria
1	5
2	8
3	12.8
4	20.48

Arithmetic or geometric?

Recursive: ____________________________

Explicit: ____________________________

15. Elvira likes to exercise in her spare time. She has been running. The first days she went running she did 1000 yards. She has been running an additional 50 yards each day she works out.

Arithmetic or geometric?

Recursive: ____________________________

Explicit: ____________________________

16. Evan has started a business and, following Bill's example, is going to send out a chain email to get customers. He sends the email to 5 people the first day and they each are going to send it to 7 people. Then those will each send it to 7 more and so on.

Arithmetic or geometric?

Recursive: ____________________________

Explicit: ____________________________

4.8H Eating Up the Lunchroom Budget

A Solidify Understanding Task

In *Cafeteria Consumption and Costs* you created a matrix to represent the number of food items Elvira planned to order this year for the soccer team and drama club celebrations. Your matrix probably looked something like this: (Note: labels have been added to keep track of the meaning of the rows and columns.)

$$\begin{array}{cc} & \begin{array}{ccc} \text{chips} & \text{cookies} & \text{drinks} \end{array} \\ \begin{array}{c} \text{Soccer} \\ \text{Drama} \end{array} & \begin{bmatrix} 8 & 7 & 4 \\ 10 & 4 & 4 \end{bmatrix} \end{array}$$

You were also given information about the cost of purchasing each food item at two different stores, *Mainstreet Market* and *Grandpa's Grocery*. That information could also be represented in a matrix like this:

$$\begin{array}{cc} & \begin{array}{cc} \text{Mainstreet Market} & \text{Grandpa's Grocery} \end{array} \\ \begin{array}{l} \text{Cost per package of chips} \\ \text{Cost per dozen cookies} \\ \text{Cost per gallon of drink} \end{array} & \begin{bmatrix} 2.50 & 2.00 \\ 3.00 & 4.00 \\ 2.00 & 1.50 \end{bmatrix} \end{array}$$

In question 4 of the previous task you were asked to determine how much each event would cost if all of the food for the event was purchased at *Mainstreet Market* or *Grandpa's Grocery.* These total amounts could be recorded in a matrix that looks like this:

$$\begin{array}{cc} & \begin{array}{cc} \text{Mainstreet Market} & \text{Grandpa's Grocery} \end{array} \\ \begin{array}{c} \text{Soccer} \\ \text{Drama} \end{array} & \begin{bmatrix} a & b \\ c & d \end{bmatrix} \end{array}$$

1. Calculate the values of *a, b, c,* and *d* in the matrix above.

2. Explain, in detail, how you would use the numbers in the first two matrices above to obtain the values for the third matrix.

3. In addition to the soccer team and drama club, Elvira plans to host events for the chess club, the cheerleaders and the football team. She gives you the following matrix to represent food items that need to be ordered for each of the events. Can you use matrix multiplication with the cost matrix given above to determine the total cost of each event if items are purchased at each store? If yes, show how. If no, explain why not.

$$\begin{array}{c c} & \begin{array}{ccc} \textit{chips} & \textit{cookies} & \textit{drinks} \end{array} \\ \begin{array}{c} \textit{Soccer} \\ \textit{Drama} \\ \textit{Chess} \\ \textit{Cheerleaders} \\ \textit{Football} \end{array} & \begin{bmatrix} 8 & 7 & 4 \\ 10 & 4 & 4 \\ 3 & 4 & 2 \\ 2 & 3 & 2 \\ 14 & 12 & 8 \end{bmatrix} \end{array}$$

4. In addition to chips, cookies and drinks, Elvira plans to add rolls and cold cuts to the events' menu. She gives you the following matrix to represent all of the food items that need to be ordered for each of the events. Can you use matrix multiplication with the cost matrix given above to determine the total cost of each event if items are purchased at each store? If yes, show how. If no, explain why not.

$$\begin{array}{c c} & \begin{array}{ccccc} \textit{chips} & \textit{cookies} & \textit{drinks} & \textit{rolls} & \textit{cold cuts} \end{array} \\ \begin{array}{c} \textit{Soccer} \\ \textit{Drama} \\ \textit{Chess} \\ \textit{Cheerleaders} \\ \textit{Football} \end{array} & \begin{bmatrix} 8 & 7 & 4 & 6 & 4 \\ 10 & 4 & 4 & 8 & 5 \\ 3 & 4 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 & 2 \\ 14 & 12 & 8 & 12 & 10 \end{bmatrix} \end{array}$$

4. 8H Eating Up the Lunchroom Budget – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to examine the row by column procedure for multiplying matrices. Unlike addition, where both matrix addends had to have the same number of rows and the same number of columns, the factor matrices in matrix multiplication do not need to be the same size. However, they do need to be compatible sizes. This task will help students notice that the number of columns in the first factor matrix needs to be the same as the number of rows of the second factor matrix for multiplication to be possible. Students will also notice that the product of matrix multiplication is a matrix with the same number of rows as the first factor matrix and the same number of columns as the second factor matrix. Each of these last two statements implies that matrix multiplication, in general, is not commutative.

Core Standards Focus:
N.VM.6 Use matrices to represent and manipulate data.
N.VM.8 Add, subtract, and multiply matrices of appropriate dimensions.
Related Standards: N.Q.1

Standards for Mathematical Practice:

SMP 2 – Reason Abstractly and Quantitatively
SMP 4 – Model with mathematics
SMP 7 – Look for and make use of structure

The Teaching Cycle:

Launch (Whole Class):

Students should have already worked on the computations involved in the scenario represented in the introductory paragraph during their work on question 4 of *Cafeteria Consumption and Costs*. If you have not had an opportunity to discuss this problem, do so now. Ask students to describe how they would compute the cost of buying all of the food items for a particular event at a particular store. For example, how would they compute the cost of purchasing the food for the drama club event at *Mainstreet Market*? In doing so, point out that students have calculated the value of the variable c in the third matrix, as requested in question 1 of this task. Then set students to work on the remainder of the task.

Explore (Small Group):

While students may be able to calculate the values of a, b, c and d based on the context, they may not immediately notice how the sum of partial products involved in these computations are related to the arrangement of the numbers in the matrices. Listen for students who can articulate that the numbers along the rows of the first matrix are being multiplied by the corresponding numbers down the columns of the second matrix (for example, while moving across the numbers in the second row of the first matrix, you move down the numbers in the second column of the second matrix, multiply corresponding factors as you move across the row and down the column), and that the partial products are added together and recorded in a corresponding row and column in the product matrix (e.g., row two, column two for our example).

Questions 3 and 4 are intended to help students think about conditions that need to be met in order for matrices to be multiplied. These conditions include the compatible size issues described in the purpose statement and the observation that, in general, matrix multiplication is not commutative. In addition, the quantities and units used in the context must be compatible for matrix multiplication to make sense (see discussion below).

Discuss (Whole Class):

Work towards a good articulation of the row and column structure of matrix multiplication, by sharing students' descriptions of the process as written in response to question 2. Write out each of the problems as a matrix multiplication equation, without the row and column labels.
Highlight the size of the factors and product matrices, and discuss the size conditions that need to be met in order for matrices to be multiplied. In our initial example, a 2-row, 3-column matrix is multiplied by a 3-row, 2-column matrix to obtain a 2-row, 2-column matrix. In question 3, a 5-row,

3-column matrix is multiplied by a 3-row, 2-column matrix to obtain a 5-row, 2-column matrix. In question 4, we attempt to multiply a 5-row, 5-column matrix by a 3-row, 2-column matrix, but realize that this is not possible. Make sure that the impossibility of this last multiplication is discussed both in terms of the context (we need to know the cost of rolls and cold cuts at each of the markets) as well as in terms of a matrix equation (as we move across the numbers in the rows of the first matrix while multiplying by the numbers in the columns of the second matrix we run out of factors from our second matrix since it does not have the same number of rows as the number of columns in the first matrix). These examples should help students articulate a compatibility rule for multiplying matrices.

Highlight the quantities and units being used in the factors and product of the matrix multiplication by adding the row and column labels to the matrix multiplication equations. For example, in the initial problem an "event by food item matrix" is multiplied by a "cost per food item by store" matrix to obtain an "event by store" matrix which represents the cost of purchasing all of the food items for each event at each store. Relate this idea to the size compatibility rule for multiplying matrices to indicate how the row and column arrangement of the quantities and units represented by the matrices must also be compatible for matrix multiplication to make sense.

In the next task, students will practice arranging data from a context into matrices that can reasonably be multiplied, both by their size and by the quantities and units they represent.

Aligned Ready, Set, Go: Equations and Inequalities 4.8H

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Equivalent Equations

The pairs of equations below are equivalent. Determine what was done to the first equation in order to obtain the second equation. (For example, everything multiplied by 5 or Multiplicative Property of Equality) If more than one operation was performed please indicate the operations and the order they were performed.

1. $x + y = 5$
   $3x + 3y = 15$

2. $4x + 3y = 12$
   $x + \frac{3}{4}y = 3$

3. $6x + 4y = 20$
   $y = -\frac{3}{2}x + 5$

Determine whether or not the pairs of equations below are equivalent. If equivalent state the operations used to create the second from the first. If not equivalent show why not.

4. $54x - 42y = 90$
   $9x + 7y = 15$

5. $12x + 9y = 21$
   $4x + 3y = 7$

6. $2x + 5y = 10$
   $y = \frac{2}{5}x + 10$

Literal Equations: Solve each of the equations below for y, put the equation into slope intercept form (y=mx+b). Show your work and justifications.

7.

$-5x + y = 12$	Justification

8.

$6x + 2y = 12$	Justification

9.

$-12x + 4y = -16$	Justification

10.

$5x - 2y = 10$	Justification

SET

Topic: Matrix Multiplication

The equipment manager for the school athletics department is attempting to restock some of the needed uniform and equipment items for the upcoming seasons of baseball and football. It has been determined based on current levels of inventory and the number of players that will be returning that more socks, pants and helmets will be needed. The equipment manager has organized the information in the matrix below.

$$\begin{array}{r c} & \begin{array}{ccc} \text{Socks} & \text{Pants} & \text{Helmets} \end{array} \\ \begin{array}{r} \text{Baseball} \\ \text{Football} \end{array} & \begin{bmatrix} 13 & 15 & 7 \\ 24 & 45 & 20 \end{bmatrix} \end{array}$$

The school has contracted with two supply stores in the past for equipment needs. The matrix below shows how much each store charges for the needed items.

$$\begin{array}{r c} & \begin{array}{cc} \text{Big Sky Sportingoods} & \text{Play It Forever} \end{array} \\ \begin{array}{r} \textbf{Cost per pair of socks} \\ \textbf{Cost per pair of pants} \\ \textbf{Cost per helmet} \end{array} & \begin{bmatrix} 3.50 & 3.00 \\ 35.00 & 40.00 \\ 22.00 & 45.50 \end{bmatrix} \end{array}$$

11. Calculate the values of ***a, b, c*** and ***d*** in the "Total Cost Matrix".

Total Cost Matrix

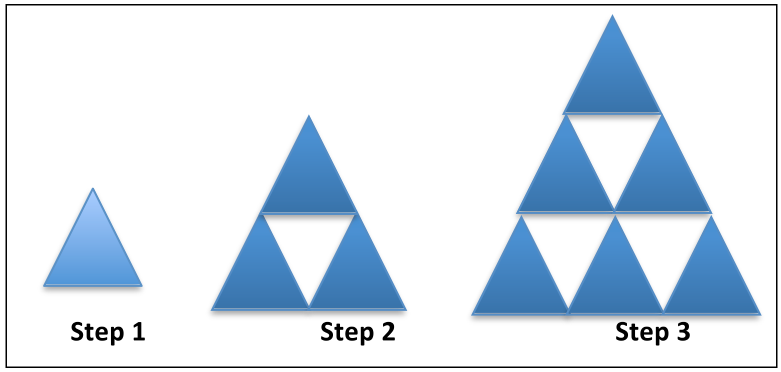
$$\begin{array}{r c} & \begin{array}{cc} \text{Big Sky Sportingoods} & \text{Play it Forever} \end{array} \\ \begin{array}{r} \text{Baseball} \\ \text{Football} \end{array} & \begin{bmatrix} a & b \\ c & d \end{bmatrix} \end{array}$$

12. Show the detailed calculations for finding the value of ***a*** and ***b***. How do you use the numbers in the first two matrices above to obtain the values for the "Total Costs Matrix" ?

GO

Topic: Representing visual patterns of change with equations, finding patterns

Create a table and equation for the visual pattern to the right. If you are unable to create an equation for the attribute identified then state the pattern you notice. (All triangles are equilateral and the side length of the triangle in step 1 is one unit in length.)



13. The width of the large triangle with respect to the Step number.

14. The number of small triangles with side length of one in the large triangle with respect to the Step number.

15. The perimeter of the large triangle with respect to the Step number.

16. The number of 60-degree angles in the figure with respect to the Step number.

17. The number of white triangles in the large triangle with respect to the Step number.

4.9H The Arithmetic of Matrices

A Practice Understanding Task

Many clubs do not have the funds to pay for parties and events in the cafeteria. Therefore, Elvira lets club members, and their parents, volunteer for service hours and gives each club credit for the amount of volunteer hours they provide. There are three types of chores the volunteers can do: setting up tables, mopping the floors, and washing dishes. They can volunteer for weekday hours or for weekend hours, which earn more credits.

Elvira has recorded volunteer hours for the month of September for the drama club and the chess club in the following matrices:

Drama Club

$$\begin{array}{r c} & \begin{array}{cc} \textit{weekdays} & \textit{weekends} \end{array} \\ \begin{array}{r} \textit{tables} \\ \textit{floors} \\ \textit{dishes} \end{array} & \begin{bmatrix} 12 & 4 \\ 8 & 3 \\ 6 & 3 \end{bmatrix} \end{array}$$

Chess Club

$$\begin{array}{r c} & \begin{array}{cc} \textit{weekdays} & \textit{weekends} \end{array} \\ \begin{array}{r} \textit{tables} \\ \textit{floors} \\ \textit{dishes} \end{array} & \begin{bmatrix} 5 & 2 \\ 3 & 2 \\ 2 & 1 \end{bmatrix} \end{array}$$

1. Write a matrix equation to show how you can combine these two matrices to find the total weekday and weekend volunteer hours for each type of chore.

The drama club has committed to provide the same number of volunteer hours each month for all 9 months of the school year.

2. Write a matrix equation to show how you can create a matrix that gives the total weekday and weekend volunteer hours for each type of chore that will be provided during the school year by the drama club.

Because it is harder to get volunteers for weekends than for weekdays, Elvira gives more credit for weekend hours than for weekday hours. She uses the following matrix to keep track of the dollars per hour earned on weekdays and weekends.

$$\begin{array}{rc} & {}^{\$}\!/_{hr} \\ \textit{weekdays} & \left[\begin{array}{c} 4 \\ 6 \end{array}\right] \\ \textit{weekends} & \end{array}$$

3. Write a matrix equation to show how you can combine two matrices to find the total credit earned by the Chess club during September for each type of chore.

Elvira is getting good at manipulating matrices, but realizes that sometimes she only needs one element in the sum or product matrix (for example, the cost of buying ingredients at *Grandpa's Grocery* on a specific day) and so she would like to be able to calculate a single result without completing the rest of the matrix operation. For the following matrix operations, calculate the indicated missing elements in the sum or product, without calculating the rest of the individual elements in the sum or product matrix.

4. $$\begin{bmatrix} 5 & -2 & 3 & 6 \\ 7 & 1 & -4 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 5 & -7 \\ 4 & -3 & 2 & 5 \end{bmatrix} = \begin{bmatrix} - & - & \boxed{} & - \\ - & \boxed{} & - & - \end{bmatrix}$$

5. $$\begin{bmatrix} -2 & 3 \\ 4 & -1 \\ 2 & 5 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & -3 & 4 \\ -1 & 5 & -2 \end{bmatrix} = \begin{bmatrix} - & \boxed{} & - \\ \boxed{} & - & - \\ - & - & \boxed{} \\ - & - & - \end{bmatrix}$$

6. $$3 \cdot \begin{bmatrix} 2 & 4 \\ -1 & 5 \end{bmatrix} - 4 \cdot \begin{bmatrix} 2 & -3 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} -\boxed{} \\ \boxed{} & - \end{bmatrix}$$

7. $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \boxed{} & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & \boxed{} \\ 1 & 2 & 4 \\ \boxed{} & 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & \boxed{} & 5 \\ 1 & 2 & \boxed{} \\ 4 & 3 & \boxed{} \end{bmatrix}$$

4.9H The Arithmetic of Matrices – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is to practice using matrices to organize information in a way that allows several similar computations to be represented simultaneously. A particular focus is on setting up matrices in such a way that the row-by-column procedure for multiplying matrices will make sense. Students practice carrying out the operations of matrix addition and multiplication, as well as interpreting the resulting sum or product matrix in terms of the context. The last problems of the task remind students of the differences in the way individual elements of matrices are combined during matrix addition and matrix multiplication. Scalar multiplication and subtraction of matrices are also addressed.

Core Standards Focus:

N.VM.6 Use matrices to represent and manipulate data.

N.VM.8 Add, subtract, and multiply matrices of appropriate dimensions.

Related Standards: N.Q.1, N.VM.6, N.VM.7

Standards for Mathematical Practice:

SMP 6 – Attend to Precision

SMP 7 – Look for and make use of structure

SMP 8 – Look for and express regularity in repeated reasoning

The Teaching Cycle:

Launch (Whole Class):

Present the scenario at the beginning of the task and make sure students are comfortable with the expectations of the first few questions on the task, then set them to work.

Explore (Small Group):

In the first three questions students need to distinguish between the operations of matrix addition, scalar multiplication, and matrix multiplication, and apply each operation appropriately. Writing their own matrix equations should be easy for the matrix addition problem (question 1) and the scalar multiplication problem (question 2), but may prove problematic for the matrix multiplication problem (question 3) since students will need to decide which order the write the factor matrices in. Watch for students who do this by trial and error—one order doesn't work with the row-by-column multiplication procedure we have developed in the previous task, but the other order does—versus the students who reason through the meanings of the rows and columns to decide which order makes sense.

On the last three problems, encourage students to only find the target elements in the sum, product or difference matrices without computing the other outcomes. This will require students to think about where the numbers in the answer matrix come from, rather than just carrying out a procedure by rote memorization.

Discuss (Whole Class):

Begin the discussion by having students describe how they decided to write the matrix equation in question 3. You might want to have a student present the trial and error approach to highlight that matrix multiplication is not commutative. Be sure to have a student present how to reason from the meaning of the rows and columns (or provide this explanation yourself, if necessary). Students should be able to use units to reason that a "chore-by-day type" matrix multiplied by a "day type-by-dollars per hour" matrix yields a "chore-by-dollars per hour" matrix. Relate this to the observation that a "3-row, 2-column" matrix can be multiplied by a "2-row, 1-column" matrix since the numbers across the rows of the first matrix will correspond with the numbers down the columns of the second, and the result is a "3-row, 1-column" matrix. Generalize this idea to note that an m-by-n matrix can be multiplied by an n-by-p matrix, and the result is an m-by-p matrix.

A short discussion about what students needed to attend to when finding a specific element in the sum, difference or product matrix may be helpful, particularly contrasting how the element in row 2 column 2 was found for the sum matrix in question 5 and the product matrix in question 6. It may be helpful to introduce notation such as $a_{2,2}$ to represent this element.

Aligned Ready, Set, Go: Equations and Inequalities 4.9H

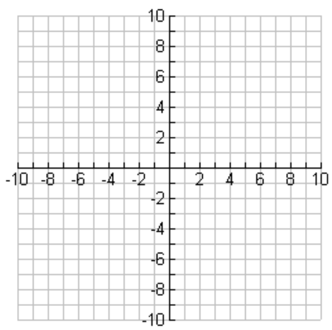
READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Graph each relationship that is given by a table or a graph.

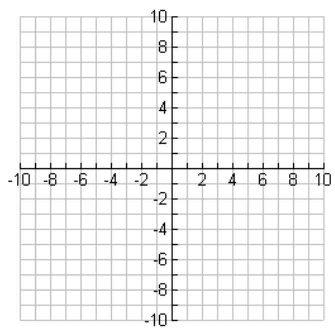
Graph each relationship that is given in the table or equation on the grid provided.

1. $y = 3x - 5$

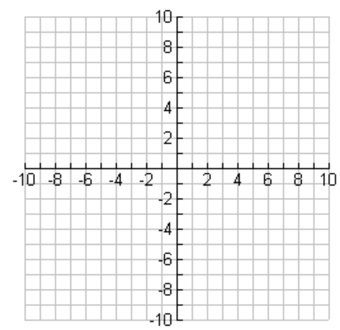


2.

x	y
-1	5
0	7
1	9
2	11

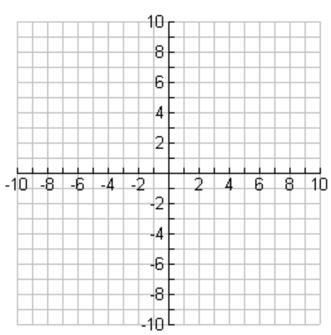


3. $y = -2x$

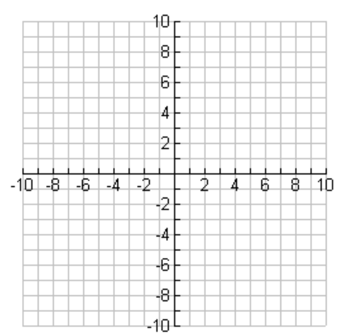


4.

x	y
-5	-1
10	2
-30	-6
25	5



5. $y = 6x - 7$



SET

Topic: Matrix Arithmetic

Perform each of the operations indicated on the matrices below.

6. $\begin{bmatrix} -3 & 5 \\ 4 & -7 \end{bmatrix} + \begin{bmatrix} 8 & 9 \\ -6 & -5 \end{bmatrix}$

7. $\begin{bmatrix} 11 & -12 \\ -4 & 6 \\ 5 & 8 \end{bmatrix} - \begin{bmatrix} 1 & -20 \\ -15 & 9 \\ 2 & -2 \end{bmatrix}$

8. $5\times\begin{bmatrix} 4 & -2 & 9 \\ 5 & 7 & -8 \end{bmatrix}$

9. $\begin{bmatrix} 6 & 7 & 8 \\ -3 & 5 & -2 \end{bmatrix} + 4\begin{bmatrix} -7 & 2 & 1 \\ 1 & -2 & -5 \end{bmatrix}$

10. $4 \cdot \begin{bmatrix} 1 & 5 \\ 7 & 6 \\ 5 & 4 \end{bmatrix}$

11. $3\times\begin{bmatrix} 4 & 8 \\ 0 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 7 & 5 \end{bmatrix} =$

12. $\begin{bmatrix} 3 & 5 \\ 4 & -7 \end{bmatrix} \cdot \begin{bmatrix} 8 & 9 \\ 6 & 0 \end{bmatrix}$

13. $\begin{bmatrix} 2 & 0 \\ 1 & -2 \\ 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 7 \\ 4 & 0 \end{bmatrix}$

GO

Topic: Evaluating Expressions

Evaluate each expression below given ***x* = 7, *y* = -3 and *z* = 5**

14. $\frac{xy-z}{2}$

15. $5x - 2^3 + (2y+z)^4$

16. $\frac{(z-3)^6}{6y-2x}$

17. $(6x - 5y + 4z)^2$

18. $\frac{2(x-z)}{6}$

19. $5(y-6) - (y-6) + 2y - 12$