mathematics vision project

Transforming Mathematics Education

ALGEBRA I

A Learning Cycle Approach

MODULE 7

Structure of Expressions

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The Mathematics Vision Project

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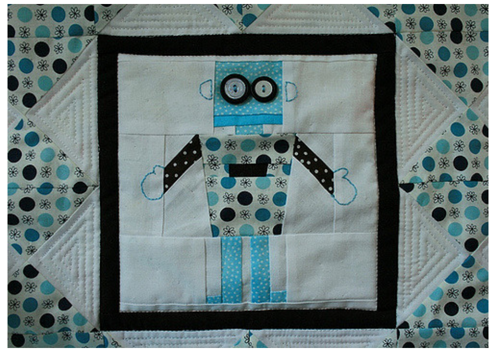
7.1 Transformers: Shifty y's

A Develop Understanding Task



Optima Prime is designing a robot quilt for her new grandson. She plans for the robot to have a square face. The amount of fabric that she needs for the face will depend on the area of the face, so Optima decides to model the area of the robot's face mathematically. She knows that the area A of a square with side length x units (which can be inches or centimeters) is modeled by the function, $A(x) = x^2$ square units.

1. What is the domain of the function $A(x)$ in this context?

2. Match each statement about the area to the function that models it:

Matching Equation (A,B, C, or D)	**Statement**	**Function Equation**
	The length of each side is increased by 5 units.	A) $A = 5x^2$
	The length of each side is multiplied by 5 units.	B) $A = (x+5)^2$
	The area of a square is increased by 5 square units.	C) $A = (5x)^2$
	The area of a square is multiplied by 5.	D) $A = x^2 + 5$

Optima started thinking about the graph of $y = x^2$ (in the domain of all real numbers) and wondering about how changes to the equation of the function like adding 5 or multiplying by 5 affect the graph. She decided to make predictions about the effects and then check them out.

3. Predict how the graphs of each of the following equations will be the same or different from the graph of $y = x^2$.

	Similarities to the graph of $y = x^2$	**Differences from the graph of $y = x^2$**
$y = 5x^2$		
$y = (x+5)^2$		
$y = (5x)^2$		
$y = x^2 + 5$		

4. Optima decided to test her ideas using technology. She thinks that it is always a good idea to start simple, so she decides to go with $y = x^2 + 5$. She graphs it along with $y = x^2$ in the same window. Test it yourself and describe what you find.

5. Knowing that things make a lot more sense with more representations, Optima tries a few more examples like $y = x^2 + 2$ and $y = x^2 - 3$, looking at both a table and a graph for each. What conclusion would you draw about the effect of adding or subtracting a number to $y = x^2$? Carefully record the tables and graphs of these examples in your notebook and explain why your conclusion would be true for any value of k, given, $y = x^2 + k$.

6. After her amazing success with addition in the last problem, Optima decided to look at what happens with addition and subtraction inside the parentheses, or as she says it, "adding to the x before it gets squared". Using your technology, decide the effect of h in the equations: $y = (x + h)^2$ and $y = (x - h)^2$. (Choose some specific numbers for h.) Record a few examples (both tables and graphs) in your notebook and explain why this effect on the graph occurs.

7. Optima thought that #6 was very tricky and hoped that multiplication was going to be more straightforward. She decides to start simple and multiply by -1, so she begins with $y = -x^2$. Predict what the effect is on the graph and then test it. Why does it have this effect?

8. Optima is encouraged because that one was easy. She decides to end her investigation for the day by determining the effect of a multiplier, a, in the equation: $y = ax^2$. Using both positive and negative numbers, fractions and integers, create at least 4 tables and matching graphs to determine the effect of a multiplier.

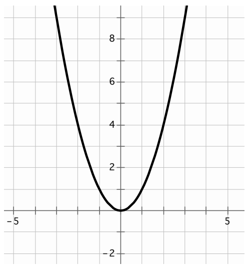
READY, SET, GO! Name Period Date

READY

Topic: Finding key features in the graph of a quadratic equation

Make a point on the vertex and draw a dotted line for the axis of symmetry. Label the coordinates of the vertex and state whether it's a maximum or a minimum. Write the equation for the axis of symmetry.

1.



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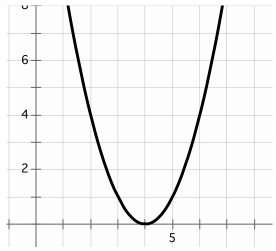

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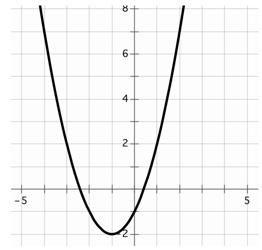

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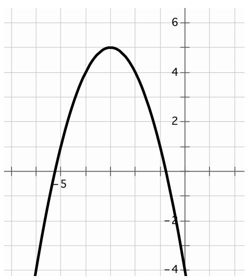

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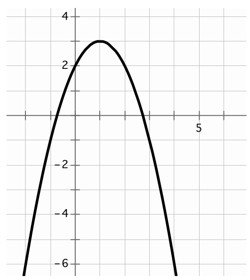

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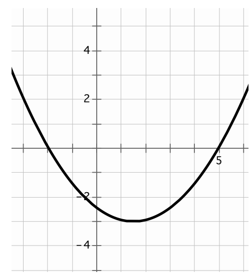
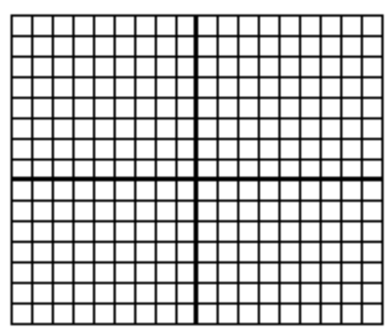


7. What connection exists between the coordinates of the vertex and the equation of the axis of symmetry?

8. Look back at #6. Try to find a way to find the exact value of the coordinates of the vertex. Test your method with each vertex in 1 - 5. Explain your conjecture.

9. How many x-intercepts can a parabola have?

10. Sketch a parabola that has no x-intercepts, then explain what has to happen for a parabola to have no x-intercepts.

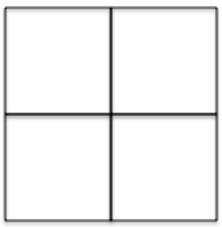
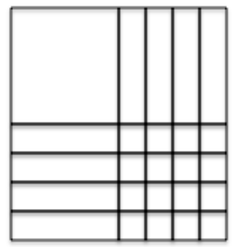
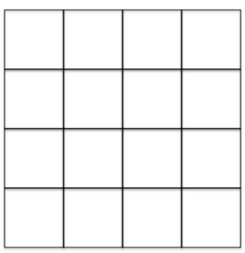
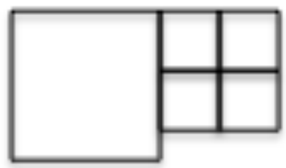


SET

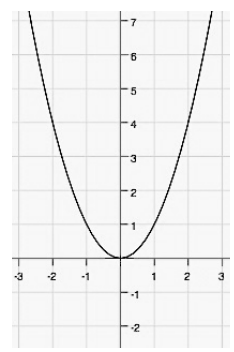
Topic: Transformations on quadratics

Matching: Choose the area model that is the best match for the equation.

__11. $x^2 + 4$ __12. $(x + 4)^2$ __13. $(4x)^2$ ___14. $4x^2$

a. 	b. 
c. 	d. 

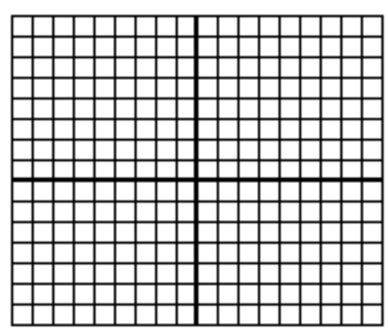
A table of values and the graph for f (x) = x^2 is given. Compare the values in the table for g(x) to those for f(x). Identify what stays the same and what changes. a) Use this information to write the vertex form of the equation of g(x). b) Graph g(x). c) Describe how the graph changed from the graph of f(x). Use words such as right, left, up, and down. d) Answer the question.



x	-3	-2	-1	0	1	2	3
f(x) = x²	9	4	1	0	1	4	9

15 a) g(x) =

x	-3	-2	-1	0	1	2	3
g(x)	2	-3	-6	-7	-6	-3	2

b) 

c) In what way did the graph move?

d) What part of the equation indicates this move?

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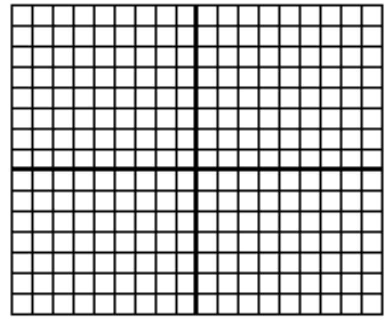
16 a) $g(x)$ =

b)



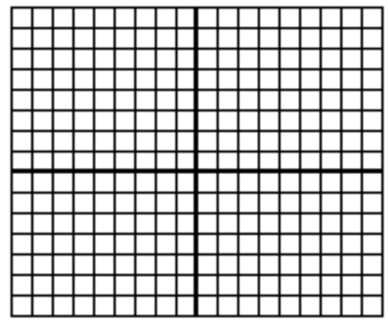
x	-3	-2	-1	0	1	2	3
$g(x)$	11	6	3	2	3	6	11

c) In what way did the graph move?

d) What part of the equation indicates this move?

17 a) $g(x)$ =

b)



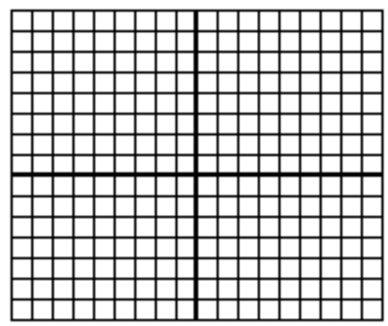
x	-4	-3	-2	-1	0	1	2
$g(x)$	9	4	1	0	1	4	9

c) In what way did the graph move?

d) What part of the equation indicates this move?

18 a) $g(x)$ =

b)



x	0	1	2	3	4	5	6
$g(x)$	9	4	1	0	1	4	9

c) In what way did the graph move?

d) What part of the equation indicates this move?

GO

Topic: Finding Square Roots

Simplify the following expressions

19. $\sqrt{49a^2b^6}$

20. $\sqrt{(x+13)^2}$

21. $\sqrt{(x-16)^2}$

22. $\sqrt{(36x+25)^2}$

23. $\sqrt{(11x-7)^2}$

24. $\sqrt{9m^2(2p^3-q)^2}$

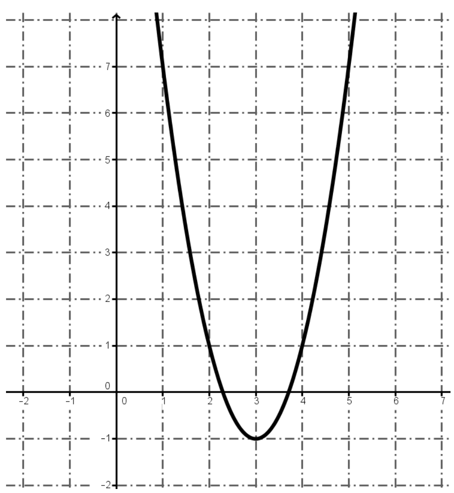
7.2 Transformers: More Than Meets the y's

A Solidify Understanding Task

Write the equation for each problem below. Use a second representation to check your equation.

1. The area of a square with side length *x*, where the side length is decreased by 3, the area is multiplied by 2 and then 4 square units are added to the area.

2.



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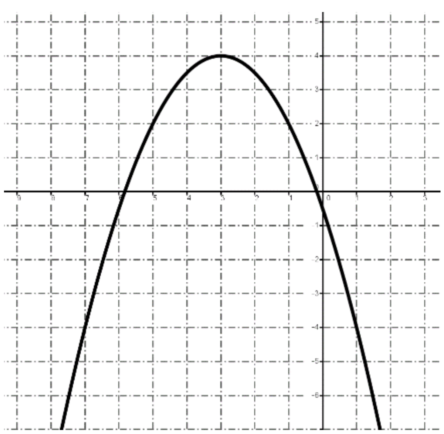
3.

x	*f(x)*
-4	7
-3	2
-2	-1
-1	-2
0	-1
1	2
2	7
3	14
4	23

4.



Graph each equation without using technology. Be sure to have the exact vertex and at least two correct points on either side of the line of symmetry.

5. $f(x) = -x^2 + 3$

6. $g(x) = (x+2)^2 - 5$

7. $h(x) = 3(x-1)^2 + 2$

8. Given: $f(x) = a(x-h)^2 + k$

   a. What point is the vertex of the parabola?

   b. What is the equation of the line of symmetry?

   c. How can you tell if the parabola opens up or down?

   d. How do you identify the dilation?

9. Does it matter in which order the transformations are done? Explain why or why not.

READY, SET, GO! Name Period Date

READY

Topic: Standard form of quadratic equations

The standard form of a quadratic equation is defined as $\mathbf{y = ax^2 + bx + c, (a \neq 0)}$.

Identify a, b, and c in the following equations.

Example: Given $\mathbf{4x^2 + 7x - 6}$**, a = 4, b = 7, and c = -6**

1. $y = 5x^2 + 3x + 6$

a = ___________
b = ___________
c = ___________

2. $y = x^2 - 7x + 3$

a = ___________
b = ___________
c = ___________

3. $y = -2x^2 + 3x$

a = ___________
b = ___________
c = ___________

4. $y = 6x^2 - 5$

a = ___________
b = ___________
c = ___________

5. $y = -3x^2 + 4x$

a = ___________
b = ___________
c = ___________

6. $y = 8x^2 - 5x - 2$

a = ___________
b = ___________
c = ___________

Multiply and write each product in the form y = ax² + bx + c. Then identify a, b, and c.

7. $y = x(x - 4)$

a = ___________
b = ___________
c = ___________

8. $y = (x - 1)(2x - 1)$

a = ___________
b = ___________
c = ___________

9. $y = (3x - 2)(3x + 2)$

a = ___________
b = ___________
c = ___________

10. $y = (x + 6)(x + 6)$

a = ___________
b = ___________
c = ___________

11. $y = (x - 3)^2$

a = ___________
b = ___________
c = ___________

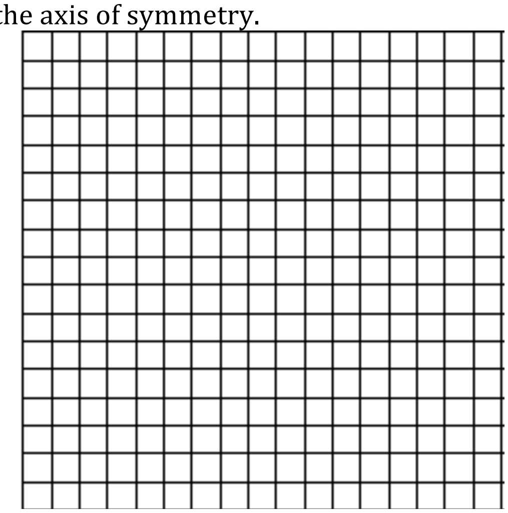
12. $y = -(x + 5)^2$

a = ___________
b = ___________
c = ___________

SET

Topic: Graphing a standard y=x² parabola

13. Graph the equation $y = x^2$.
Include at least 3 accurate points on each side of the axis of symmetry.



a. State the vertex of the parabola.

b. Complete the table of values for $y = x^2$.

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	

Topic: Writing the equation of a transformed parabola in vertex form.

Find a value for ω such that the graph will have the specified number of x-intercepts.

14. $\mathbf{y = x^2 + \omega}$
2 (x-intercepts)

15. $\mathbf{y = x^2 + \omega}$
1 (x-intercept)

16. $\mathbf{y = x^2 + \omega}$
no (x-intercepts)

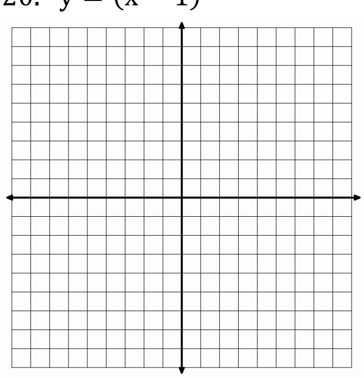
17. $y = -x^2 + \omega$
2 (x-intercepts)

18. $y = -x^2 + \omega$
1 (x-intercept)

19. $y = -x^2 + \omega$
no (x-intercepts)

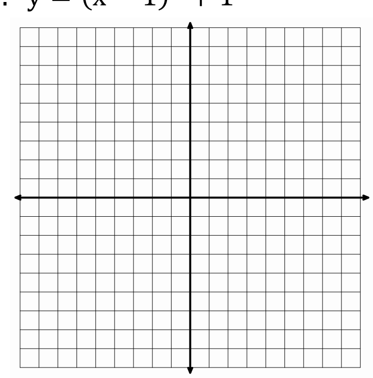
Graph the following equations. State the vertex.
(Be accurate with your key points and shape!)

20. $y = (x - 1)^2$



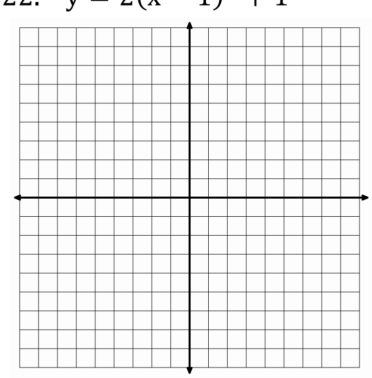
Vertex? ____________________

21. $y = (x - 1)^2 + 1$



Vertex? ____________________

22. $y = 2(x - 1)^2 + 1$



Vertex? ____________________

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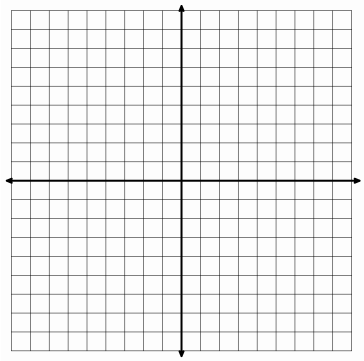
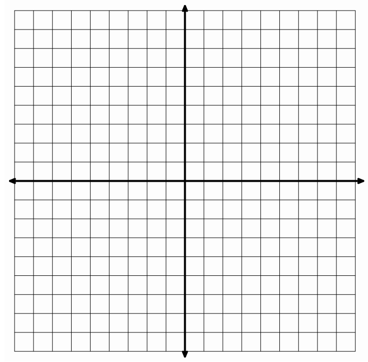
23. $y = (x+3)^2$



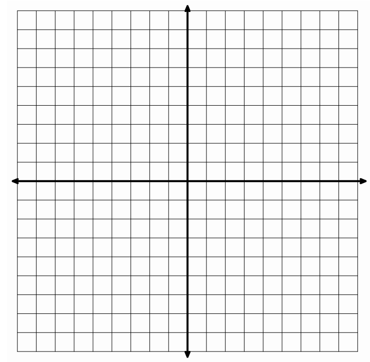
Vertex? ________________

24. $y = -(x+3)^2 - 4$



Vertex? ________________

25. $y = -0.5(x+1)^2 + 4$



Vertex? ________________

GO

Topic: Features of Parabolas

Use the table to identify the vertex, the equation for the axis of symmetry (AoS), and state the number of x-intercept(s) the parabola will have, if any. State whether the vertex will be a *minimum* or a *maximum*.

26.

x	*y*
-4	10
-3	3
-2	-2
-1	-5
0	-6
1	-5
2	-2

a. Vertex: ________

b. AoS: ___________

c. x-int(s): _______

d. MIN or MAX

27.

x	*y*
-2	49
-1	28
0	13
1	4
2	1
3	4
4	13

a. Vertex: ________

b. AoS: ___________

c. x-int(s): _______

d. MIN or MAX

28.

x	*y*
-7	-9
-6	3
-5	7
-4	3
-3	-9
-2	-29
-1	-57

a. Vertex: ________

b. AoS: ___________

c. x-int(s): _______

d. MIN or MAX

29.

x	*y*
-8	-9
-7	-8
-6	-9
-5	-12
-4	-17
-3	-24
-2	-33

a. Vertex: ________

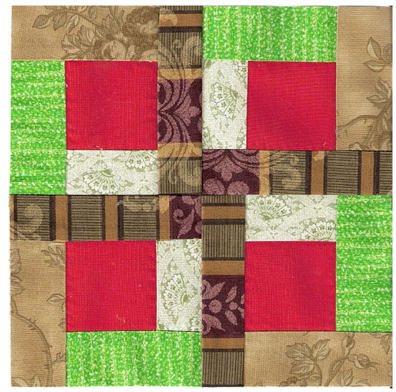
b. AoS: ___________

c. x-int(s): _______

d. MIN or MAX

7.3 Building the Perfect Square

A Develop Understanding Task

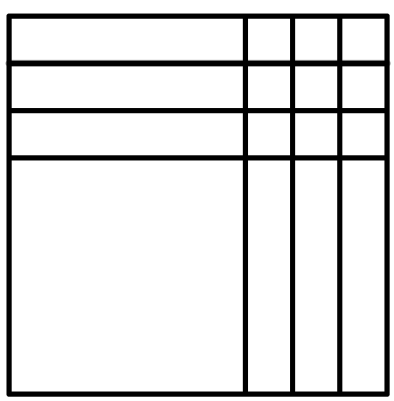


Quadratic Quilts

Optima has a quilt shop where she sells many colorful quilt blocks for people who want to make their own quilts. She has quilt designs that are made so that they can be sized to fit any bed. She bases her designs on quilt squares that can vary in size, so she calls the length of the side for the basic square x, and the area of the basic square is the function $A(x) = x^2$. In this way, she can customize the designs by making bigger squares or smaller squares.

1. If Optima adds 3 inches to the side of the square, what is the area of the square?

When Optima draws a pattern for the square in problem #1, it looks like this:



2. Use both the diagram and the equation, $A(x) = (x + 3)^2$ to explain why the area of the quilt block square, $A(x)$, is also equal to $x^2 + 6x + 9$.

The customer service representatives at Optima's shop work with customer orders and write up the orders based on the area of the fabric needed for the order. As you can see from problem #2 there are two ways that customers can call in and describe the area of the quilt block. One way describes the length of the sides of the block and the other way describes the areas of each of the four sections of the block.

For each of the following quilt blocks, draw the diagram of the block and write two equivalent equations for the area of the block.

3. Block with side length: $x + 2$.

4. Block with side length: $x + 1$.

5. What patterns do you notice when you relate the diagrams to the two expressions for the area?

6. Optima likes to have her little dog, Clementine, around the shop. One day the dog got a little hungry and started to chew up the orders. When Optima found the orders, one of them was so chewed up that there were only partial expressions for the area remaining. Help Optima by completing each of the following expressions for the area so that they describe a perfect square. Then, write the two equivalent equations for the area of the square.

   a. $x^2 + 4x$

   b. $x^2 + 6x$

c. $x^2 + 8x$

d. $x^2 + 12x$

7. If $x^2 + bx + c$ is a perfect square, what is the relationship between b and c? How do you use b to find c, like in problem 6?

Will this strategy work if b is negative? Why or why not?

Will the strategy work if b is an odd number? What happens to c if b is odd?

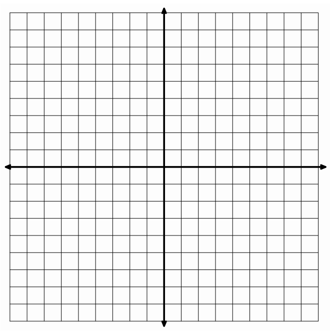
READY, SET, GO! Name Period Date

READY

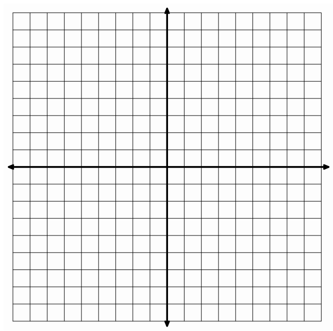
Topic: Graphing lines using the intercepts

Find the x-intercept and the y-intercept. Then graph the equation.

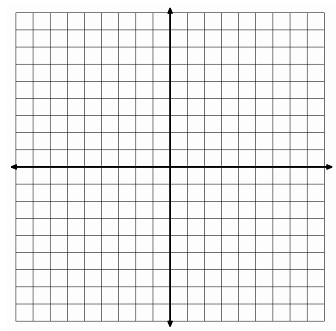
1. $3x + 2y = 12$
   a. x-intercept:
   b. y-intercept:



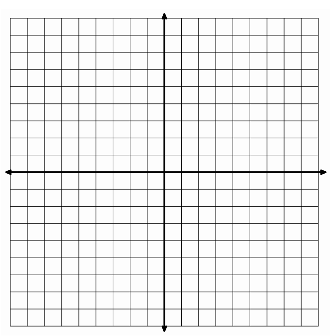
2. $8x - 12y = -24$
   a. x-intercept:
   b. y-intercept:



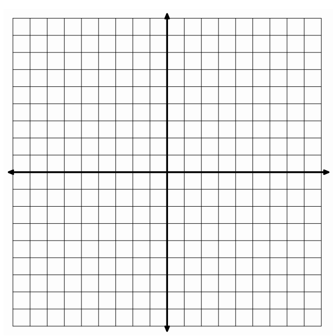
3. $3x - 7y = 21$
   a. x-intercept:
   b. y-intercept:



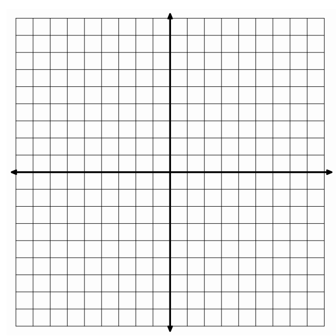
4. $5x - 10y = 20$
   a. x-intercept:
   b. y-intercept:



5. $2y = 6x - 18$
   a. x-intercept:
   b. y-intercept:



6. $y = -6x + 6$
   a. x-intercept:
   b. y-intercept:



SET

Topic: Completing the square by paying attention to the parts

Multiply. Show each step. Circle the pair of like terms before you simplify to a trinomial.

7. $(x+5)(x+5)$ 8. $(3x+7)(3x+7)$ 9. $(9x+1)^2$ 10. $(4x+11)^2$

11. Write a rule for finding the coefficient "B" of the x-term (the middle term) when multiplying and simplifying $(ax+q)^2$.

**In problems 12 – 17, (a) Fill in the number that completes the square.
(b) Then write the trinomial as the product of two factors.**

12. a) $x^2 + 8x +$ ____
b)

13. a) $x^2 + 10x +$ ____
b)

14. a) $x^2 + 16x +$ ____
b)

15. a) $x^2 + 6x +$ ____
b)

16. a) $x^2 + 22x +$ ____
b)

17. a) $x^2 + 18x +$ ____
b)

**In problems 18 – 26, (a) Find the value of "B," that will make a perfect square trinomial.
(b) Then write the trinomial as a product of two factors.**

18. $x^2 + Bx + 16$
a)
b)

19. $x^2 + Bx + 121$
a)
b)

20. $x^2 + Bx + 625$
a)
b)

21. $x^2 + Bx + 225$
a)
b)

22. $x^2 + Bx + 49$
a)
b)

23. $x^2 + Bx + 169$
a)
b)

24. $x^2 + Bx + \frac{25}{4}$
a)
b)

25. $x^2 + Bx + \frac{9}{4}$
a)
b)

26. $x^2 + Bx + \frac{49}{4}$
a)
b)

GO

Topic: Features of horizontal and vertical lines

Find the intercepts of the graph of each equation. State whether it's an x- or y- intercept.

27. y = -4.5 28. x = 9.5 29. x = -8.2 30. y = 112

7.4 A Square Deal

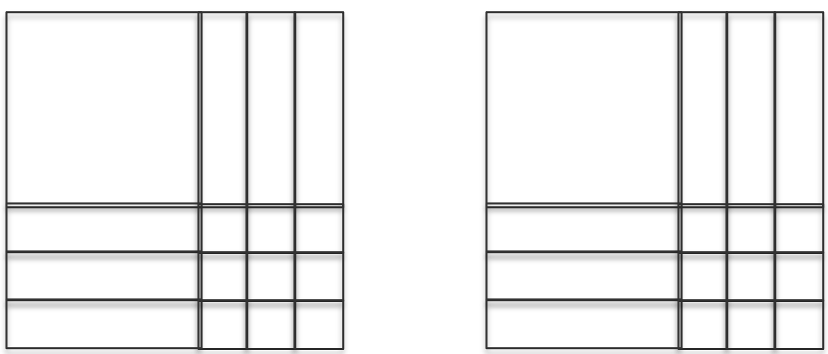
A Solidify Understanding Task

Quadratic Quilts, Revisited

Remember Optima's quilt shop? She bases her designs on quilt squares that can vary in size, so she calls the length of the side for the basic square x, and the area of the basic square is the function $A(x) = x^2$. In this way, she can customize the designs by making bigger squares or smaller squares.

1. Sometimes a customer orders more than one quilt block of a given size. For instance, when a customer orders 4 blocks of the basic size, the customer service representatives write up an order for $A(x) = 4x^2$. Model this area expression with a diagram.

2. One of the customer service representatives finds an envelope that contains the blocks pictured below. Write the order that shows two equivalent equations for the area of the blocks.



3. What equations for the area could customer service write if they received an order for 2 blocks that are squares and have both dimensions increased by 1 inch in comparison to the basic block? Write the area equations in two equivalent forms. Verify your algebra using a diagram.

4. If customer service receives an order for 3 blocks that are each squares with both dimensions increased by 2 inches in comparison to the basic block? Again, show 2 different equations for the area and verify your work with a model.

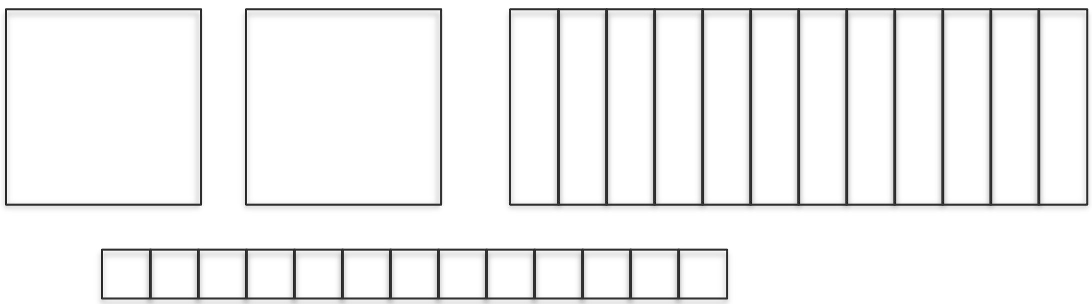
5. Clementine is at it again! When is that dog going to learn not to chew up the orders? (She also chews Optima's shoes, but that's a story for another day.) Here are some of the orders that have been chewed up so they are missing the last term. Help Optima by completing each of the following expressions for the area so that they describe a perfect square. Then, write the two equivalent equations for the area of the square.

$2x^2 + 8x$

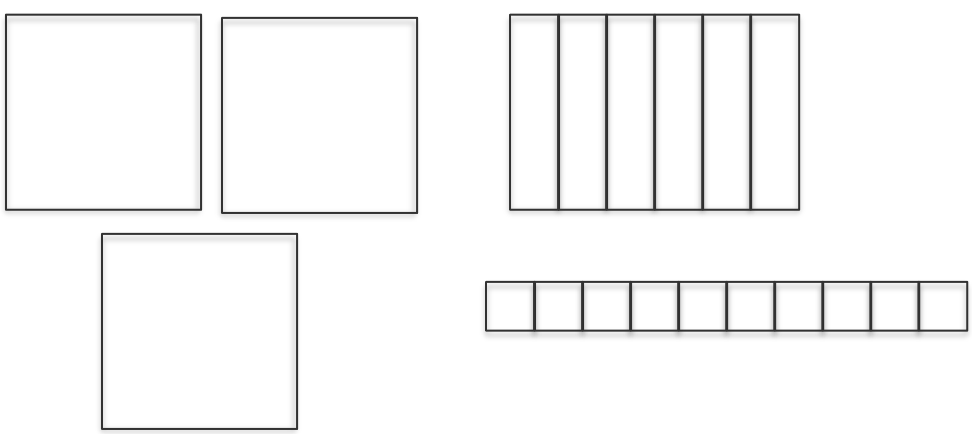
$3x^2 + 24x$

Sometimes the quilt shop gets an order that turns out not to be more or less than a perfect square. Customer service always tries to fill orders with perfect squares, or at least, they start there and then adjust as needed. They always write their equations in a way that relates the area to the closest perfect square.

6. Now here's a real mess! Customer service received an order for an area $A(x) = 2x^2 + 12x + 13$. Help them to figure out an equivalent expression for the area using the diagram.



7. Optima really needs to get organized. Here's another scrambled diagram. Write two equivalent equations for the area of this diagram:



8. Optima realized that not everyone is in need of perfect squares and not all orders are coming in as expressions that are perfect squares. Determine whether or not each expression below is a perfect square. Explain why the expression is or is not a perfect square. If it is not a perfect square, find the perfect square that seems "closest" to the given expression and show how the perfect square can be adjusted to be the given expression.

A. $A(x) = x^2 + 6x + 13$

B. $A(x) = x^2 - 8x + 16$

C. $A(x) = x^2 - 10x - 3$

D. $A(x) = 2x^2 + 8x + 14$

E. $A(x) = 3x^2 - 30x + 75$

F. $A(x) = 2x^2 - 22x + 11$

9. Now let's generalize. Given an expression in the form $ax^2 + bx + c$ $(a \neq 0)$, describe a step-by-step process for completing the square.

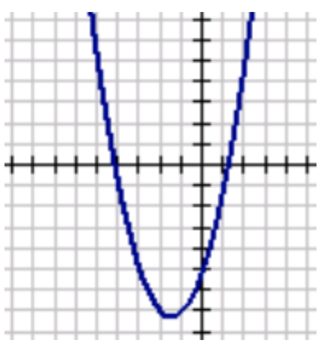
READY, SET, GO! Name Period Date

READY

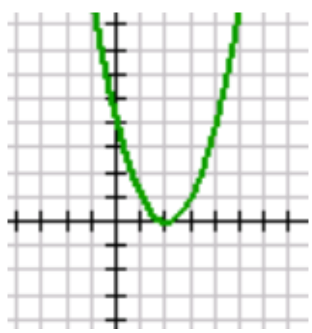
Topic: Find y-intercepts in parabolas

State the y-intercept for each of the graphs. Then match the graph with its equation.

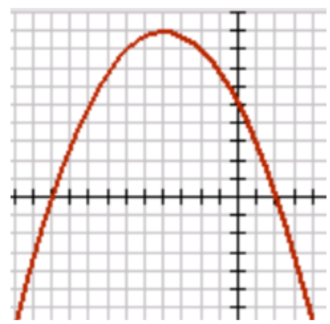
1.



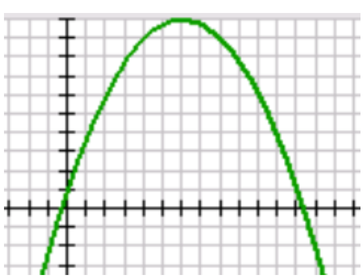
2.



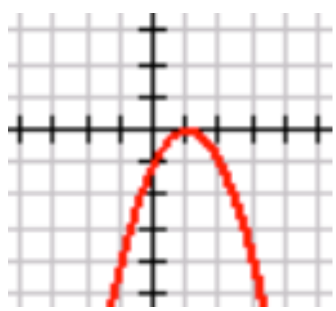
3.



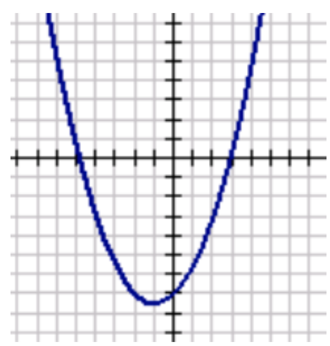
4.



5.



6.



a. $f(x) = -x^2 + 2x - 1$

b. $f(x) = -.25x^2 - 2x + 5$

c. $f(x) = x^2 + 3x - 5$

d. $f(x) = .5x^2 + x - 7$

e. $f(x) = -.25x^2 + 3x + 1$

f. $f(x) = x^2 - 4x + 4$

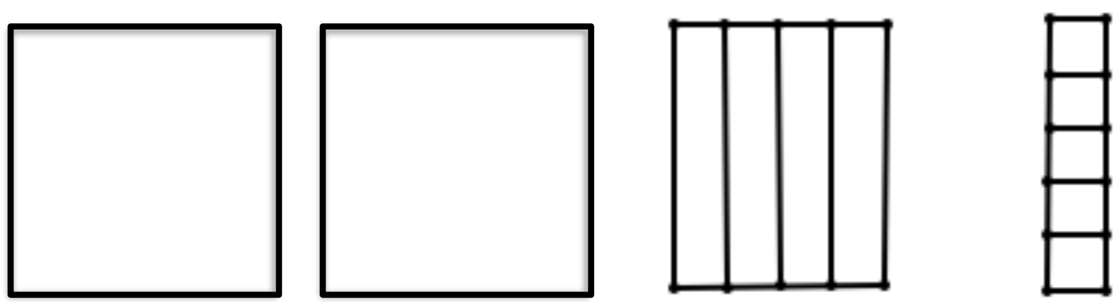
SET

Topic: Completing the square when a >1.

Fill in the missing constant so that each expression represents 5 perfect squares. Then state the dimensions of the squares in each problem.

7. $5x^2 + 30x +$ ______   8. $5x^2 - 50x +$ ______   9. $5x^2 + 10x +$ ______

10. Given the scrambled diagram below, write two equivalent equations for the area.



Determine if each expression below is a perfect square or not. If it is not a perfect square, find the perfect square that seems "closest" to the given expression and show how the perfect square can be adjusted to be the given expression.

11. $A(x) = x^2 + 10x + 14$ 12. $A(x) = 2x^2 + 16x + 6$ 13. $A(x) = 3x^2 + 18x - 12$

GO

Topic: Evaluating functions.

Find the indicated function value when $\boldsymbol{f(x) = 4x^2 - 3x - 25}$ ***and*** $\boldsymbol{g(x) = -2x^2 + x - 5}$.

14. $f(1)$ 15. $f(5)$ 16. $g(10)$ 17. $g(-5)$ 18. $f(0) + g(0)$

7.5 Be There or Be Square

A Practice Understanding Task

Quilts and Quadratic Graphs

Optima's niece, Jenny works in the shop, taking orders and drawing quilt diagrams. When the shop isn't too busy, Jenny pulls out her math homework and works on it. One day, she is working on graphing parabolas and notices that the equations she is working with looks a lot like an order for a quilt block. For instance, Jenny is supposed to graph the equation: $y = (x-3)^2 + 4$. She thinks, "That's funny. This would be an order where the length of the standard square is reduced by 3 and then we add a little piece of fabric that has as area of 4. We don't usually get orders like that, but it still makes sense. I better get back to thinking about parabolas. Hmmm…"

1. Fully describe the parabola that Jenny has been assigned to graph.

2. Jenny returns to her homework, which is about graphing quadratic functions. Much to her dismay, she finds that she has been given: $y = x^2 - 6x + 9$. "Oh dear", thinks Jenny. "I can't tell where the vertex is or identify any of the transformations of the parabola in this form. Now what am I supposed to do?"

   "Wait a minute—is this the area of a perfect square?" Use your work from *Building the Perfect Square* to answer Jenny's question and justify your answer.

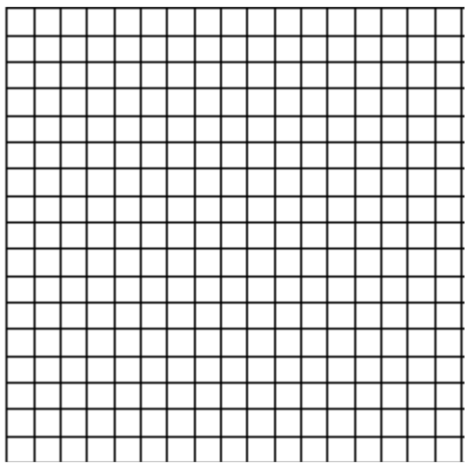
3. Jenny says, "I think I've figured out how to change the form of my quadratic equation so that I can graph the parabola. I'll check to see if I can make my equation a perfect square." Jenny's equation is: $y = x^2 - 6x + 9$.

   See if you can change the form of the equation, find the vertex, and graph the parabola.

   a. $y = x^2 - 6x + 9$ New form of the equation: ________________________________

   b. Vertex of the parabola: _____________

   c. Graph (with at least 3 accurate points on each side of the line of symmetry):

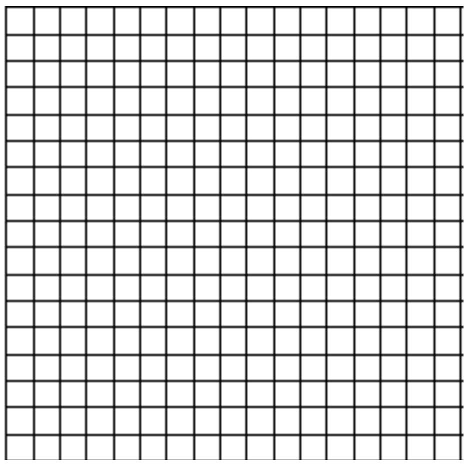


4. The next quadratic to graph on Jenny's homework is $y = x^2 + 4x + 2$. Does this expression fit the pattern for a perfect square? Why or why not?

   a. Use an area model to figure out how to complete the square so that the equation can be written in vertex form, $y = a(x - h)^2 + k$.

b. Is the equation you have written equivalent to the original equation? If not, what adjustments need to be made? Why?

c. Identify the vertex and graph the parabola with three accurate points on both sides of the line of symmetry.

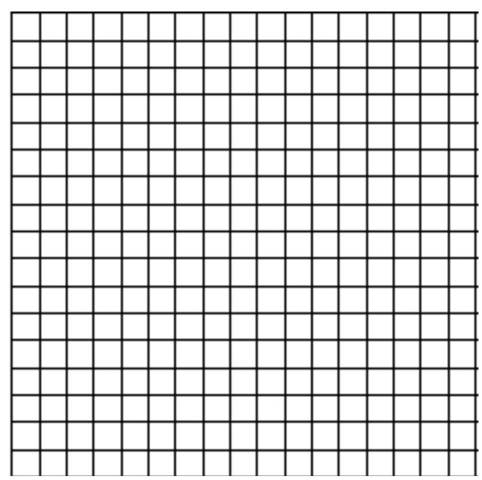


5. Jenny hoped that she wasn't going to need to figure out how to complete the square on an equation where b is an odd number. Of course, that was the next problem. Help Jenny to find the vertex of the parabola for this quadratic function:

$$g(x) = x^2 + 7x + 10$$

6. Don't worry if you had to think hard about #5. Jenny has to do a couple more:
   a. $g(x) = x^2 - 5x + 3$
   b. $g(x) = x^2 - x - 5$

7. It just gets better! Help Jenny find the vertex and graph the parabola for the quadratic function: $h(x) = 2x^2 - 12x + 17$



8. This one is just too cute—you've got to try it! Find the vertex and describe the parabola that is the graph of: $f(x) = \frac{1}{2}x^2 + 2x - 3$

READY, SET, GO! Name Period Date

READY

Topic: Recognizing Quadratic Equations

Identify whether or not each equation represents a quadratic function. Explain how you knew it was a quadratic.

1. $x^2 + 13x - 4 = 0$

Quadratic or no?

Justification:

2. $3x^2 + x = 3x^2 - 2$

Quadratic or no?

Justification:

3. $x(4x - 5) = 0$

Quadratic or no?

Justification:

4. $(2x - 7) + 6x = 10$

Quadratic or no?

Justification:

5. $2^x + 6 = 0$

Quadratic or no?

Justification:

6. $32 = 4x^2$

Quadratic or no?

Justification:

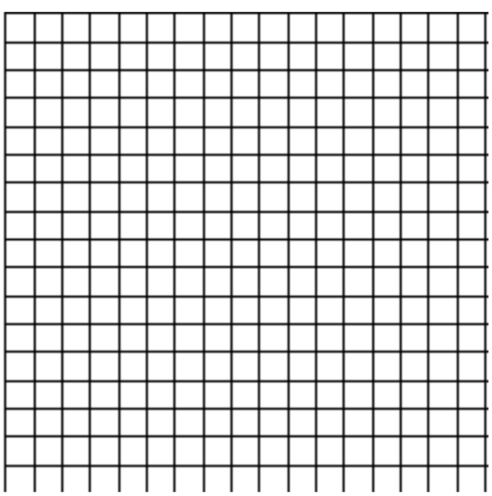
SET

Topic: Changing from standard form of a quadratic to vertex form.

Change the form of each equation to vertex form: $y = a(x - h)^2 + k$. State the vertex and graph the parabola. Show at least 3 accurate points on each side of the line of symmetry.

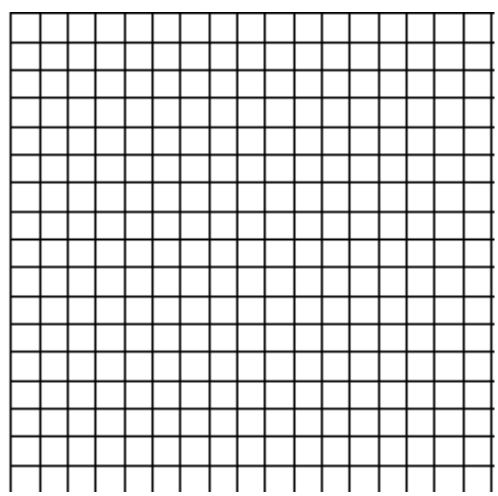
7. $y = x^2 - 4x + 1$

vertex:



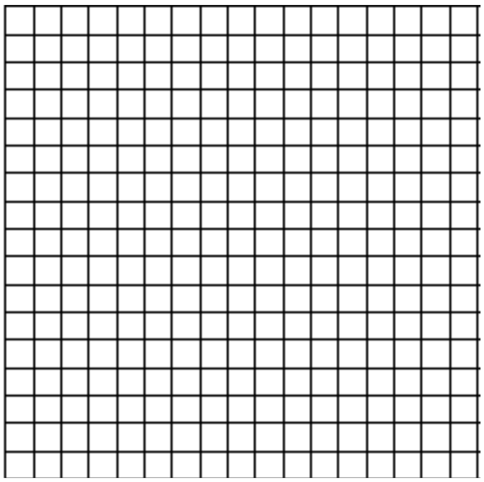
8. $y = x^2 + 2x + 5$

vertex:



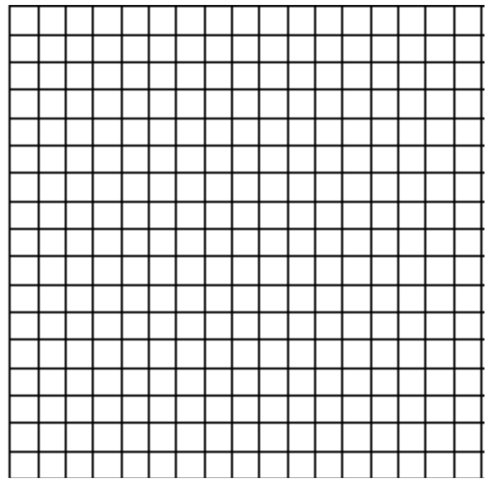
9. $y = x^2 + 3x + \frac{13}{4}$

vertex:



10. $y = \frac{1}{2}x^2 - x + 5$

vertex:



11. One of the parabolas in problems 9 – 10 should look "wider" than the others. Identify the parabola. Explain why this parabola looks different.

Fill in the blank by completing the square. Leave the number that completes the square as an improper fraction. Then write the trinomial in factored form.

12. $x^2 - 11x +$ ___

13. $x^2 + 7x +$ ___

14. $x^2 + 15x +$ ___

15. $x^2 + \frac{2}{3}x +$ ___

16. $x^2 - \frac{1}{5}x +$ ___

17. $x^2 - \frac{3}{4}x +$ ___

GO

Topic: Writing recursive equations for quadratic functions.

Identify whether the table represents a linear or quadratic function. If the function is linear, <u>write both the explicit and recursive equations.</u> If the function is quadratic, write only the recursive equation.

18.

x	$f(x)$
1	0
2	3
3	6
4	9
5	12

Type of function:

Equation(s):

19.

x	$f(x)$
1	7
2	10
3	16
4	25
5	37

Type of function:

Equation(s):

20.

x	$f(x)$
1	8
2	10
3	12
4	14
5	16

Type of function:

Equation(s):

21.

x	$f(x)$
1	28
2	40
3	54
4	70
5	88

Type of function:

Equation(s):

7.6 Factor Fixin'

A Develop Understanding Task

At first, *Optima's Quilts* only made square blocks for quilters and Optima spent her time making perfect squares. Customer service representatives were trained to ask for the length of the side of the block, x, that was being ordered, and they would let the customer know the area of the block to be quilted using the formula $A(x) = x^2$.

Optima found that many customers that came into the store were making designs that required a combination of squares and rectangles. So, *Optima's Quilts* has decided to produce several new lines of rectangular quilt blocks. Each new line is described in terms of how the rectangular block has been modified from the original square block. For example, one line of quilt blocks consists of starting with a square block and extending one side length by 5 inches and the other side length by 2 inches to form a new rectangular block. The design department knows that the area of this new block can be represented by the expression: $A(x) = (x + 5)(x + 2)$, but they do not feel that this expression gives the customer a real sense of how much bigger this new block is (e.g., how much more area it has) when compared to the original square blocks.

1. Can you find a different expression to represent the area of this new rectangular block? You will need to convince your customers that your formula is correct using a diagram.

Here are some additional new lines of blocks that *Optima's Quilts* has introduced. Find two different algebraic expressions to represent each rectangle, and illustrate with a diagram why your representations are correct.

2. The original square block was extended 3 inches on one side and 4 inches on the other.

3. The original square block was extended 4 inches on only one side.

4. The original square block was extended 5 inches on each side.

5. The original square block was extended 2 inches on one side and 6 inches on the other.

Customers start ordering custom-made block designs by requesting how much additional area they want beyond the original area of x^2. Once an order is taken for a certain type of block, customer service needs to have specific instructions on how to make the new design for the manufacturing team. The instructions need to explain how to extend the sides of a square block to create the new line of rectangular blocks.

The customer service department has placed the following orders on your desk. For each, describe how to make the new blocks by extending the sides of a square block with an initial side length of x. Your instructions should include diagrams, written descriptions and algebraic descriptions of the area of the rectangles in using expressions representing the lengths of the sides.

6. $x^2 + 5x + 3x + 15$

7. $x^2 + 4x + 6x + 24$

8. $x^2 + 9x + 2x + 18$

9. $x^2 + 5x + x + 5$

Some of the orders are written in an even more simplified algebraic code. Figure out what these entries mean by finding the sides of the rectangles that have this area. Use the sides of the rectangle to write equivalent expressions for the area.

10. $x^2 + 11x + 10$

11. $x^2 + 7x + 10$

12. $x^2 + 9x + 8$

13. $x^2 + 6x + 8$

14. $x^2 + 8x + 12$

15. $x^2 + 7x + 12$

16. $x^2 + 13x + 12$

17. What relationships or patterns do you notice when you find the sides of the rectangles for a given area of this type?

18. A customer called and asked for a rectangle with area given by: $x^2 + 7x + 9$. The customer service representative said that the shop couldn't make that rectangle. Do you agree or disagree? How can you tell if a rectangle can be constructed from a given area?

READY, SET, GO! Name Period Date

READY

Topic: Creating Binomial Quadratics
Multiply. (Use the distributive property, write in standard form.)

1. x(4x − 7) 2. 5x(3x + 8) 3. 3x(3x − 2)

4. The answers to problems 1, 2, & 3 are quadratics that can be represented in standard form $ax^2 + bx + c$. Which coefficient, **a**, **b**, or **c** equals 0 for all of the exercises above?

Factor the following. (Write the expressions as the product of two linear factors.)

5. $x^2 + 4x$ 6. $7x^2 - 21x$ 7. $12x^2 + 60x$ 8. $8x^2 + 20x$

Multiply
9. $(x + 9)(x - 9)$ 10. $(x + 2)(x - 2)$ 11. $(6x + 5)(6x - 5)$ 12. $(7x + 1)(7x - 1)$

13. The answers to problems 9, 10, 11, &12 are quadratics that can be represented in standard form $ax^2 + bx + c$. Which coefficient, **a**, **b**, or **c** equals 0 for all of the exercises above?

SET

Topic: Factoring Trinomials
Factor the following quadratic expressions into two binomials.

14. $x^2 + 14x + 45$ 15. $x^2 + 18x + 45$ 16. $x^2 + 46x + 45$

17. $x^2 + 11x + 24$ 18. $x^2 + 10x + 24$ 19. $x^2 + 14x + 24$

20. $x^2 + 12x + 36$ 21. $x^2 + 13x + 36$ 22. $x^2 + 20x + 36$

23. $x^2 - 15x - 100$ 24. $x^2 + 20x + 100$ 25. $x^2 + 29x + 100$

26. Look back at each "row" of factored expressions in problems 14 to 25 above. Explain how it is possible that the coefficient ***(b)*** of the middle term can be different numbers in each problem when the "outside" coefficients ***(a)*** and ($\boldsymbol{c}$) are the same. (Recall the standard form of a quadratic is $\boldsymbol{ax^2} + \boldsymbol{bx} + \boldsymbol{c}$.)

GO

Topic: Taking the square root of perfect squares.

Only some of the expressions inside the radical sign are perfect squares. Identify which ones are perfect squares and take the square root. Leave the ones that are not perfect squares under the radical sign. Do not attempt to simplify them. (Hint: Check your answers by squaring them. You should be able to get what you started with, if you are right.)

27. $\sqrt{(17xyz)^2}$

28. $\sqrt{(3x-7)^2}$

29. $\sqrt{121a^2b^6}$

30. $\sqrt{x^2+8x+16}$

31. $\sqrt{x^2+14x+49}$

32. $\sqrt{x^2+14x-49}$

33. $\sqrt{x^2+10x+100}$

34. $\sqrt{x^2+20x+100}$

35. $\sqrt{x^2-20x+100}$

7.7 The x Factor

A Solidify Understanding Task

Now that *Optima's Quilts* is accepting orders for rectangular blocks, their business in growing by leaps and bounds. Many customers want rectangular blocks that are bigger than the standard square block on one side. Sometimes they want one side of the block to be the standard length, x, with the other side of the block 2 inches bigger.

1. Draw and label this block. Write two different expressions for the area of the block.

Sometimes they want blocks with one side that is the standard length, x, and one side that is 2 inches less than the standard size.

2. Draw and label this block. Write two different expressions for the area of the block. Use your diagram and verify algebraically that the two expressions are equivalent.

There are many other size blocks requested, with the side lengths all based on the standard length, x. Draw and label each of the following blocks. Use your diagrams to write two equivalent expressions for the area. Verify algebraically that the expressions are equal.

3. One side is 1" less than the standard size and the other side is 2" more than the standard size.

4. One side is 2” less than the standard size and the other side is 3” more than the standard size.

5. One side is 2” more than the standard size and the other side is 3” less than the standard size.

6. One side is 3” more than the standard size and the other side is 4” less than the standard size.

7. One side is 4” more than the standard size and the other side is 3” less than the standard size.

8. An expression that has 3 terms in the form: $ax^2 + bx + c$ is called a trinomial. Look back at the trinomials you wrote in questions 3-7. How can you tell if the middle term (bx) is going to be positive or negative?

9. One customer had an unusual request. She wanted a block that is extended 2 inches on one side and decreased by 2 inches on the other. One of the employees thinks that this rectangle will have the same area as the original square since one side was decreased by the same amount as the other side was increased. What do you think? Use a diagram to find two expressions for the area of this block.

10. The result of the unusual request made the employee curious. Is there a pattern or a way to predict the two expressions for area when one side is increased and the other side is decreased by the same number? Try modeling these two problems, look at your answer to #8, and see if you can find a pattern in the result.

a. $(x+1)(x-1)$

b. $(x+3)(x-3)$

11. What pattern did you notice? What is the result of $(x+a)(x-a)$?

12. Some customers want both sides of the block reduced. Draw the diagram for the following blocks and find a trinomial expression for the area of each block. Use algebra to verify the trinomial expression that you found from the diagram.

a. $(x-2)(x-3)$

b. $(x-1)(x-4)$

13. Look back over all the equivalent expressions that you have written so far, and explain how to tell if the third term in the trinomial expression ax^2+bx+c will be positive or negative.

14. Optima's quilt shop has received a number of orders that are given as rectangular areas using a trinomial expression. Find the equivalent expression that shows the lengths of the two sides of the rectangles.

a. $x^2 + 9x + 18$

b. $x^2 + 3\text{x} - 18$

c. $x^2 - 3x - 18$

d. $x^2 - 9x + 18$

e. $x^2 - 5x + 4$

f. $x^2 - 3x + 4$

g. $x^2 + 2x - 15$

15. Write an explanation of how to factor a trinomial in the form: $x^2 + bx + c$.

READY, SET, GO! Name Period Date

READY

Topic: Exploring the density of the number line.

Find three numbers that are between the two given numbers.

1. $5\frac{3}{4}$ *and* $6\frac{1}{3}$
2. $-2\frac{1}{4}$ *and* $-1\frac{1}{2}$
3. $\frac{1}{4}$ *and* $\frac{5}{8}$
4. $\sqrt{3}$ *and* $\sqrt{5}$
5. 4 *and* $\sqrt{23}$
6. $-9\frac{3}{4}$ *and* -8.5
7. $\sqrt{\frac{1}{4}}$ *and* $\sqrt{\frac{4}{9}}$
8. $\sqrt{13}$ *and* $\sqrt{14}$

SET

Topic: Factoring Quadratics

The area of a rectangle is given in the form of a trinomial expression. Find the equivalent expression that shows the lengths of the two sides of the rectangle.

9. x^2+9x+8
10. x^2-6x+8
11. x^2-2x-8
12. x^2+7x-8
13. $x^2-11x+24$
14. $x^2-14x+24$
15. $x^2-25x+24$
16. $x^2-10x+24$
17. $x^2-2x-24$
18. $x^2-5x-24$
19. $x^2+5x-24$
20. $x^2-10x+25$
21. x^2-25
22. $x^2-2x-15$
23. $x^2+10x-75$
24. $x^2-20x+51$
25. $x^2+14x-32$
26. x^2-1
27. x^2-2x+1
28. $x^2+12x-45$

GO

Topic: Graphing Parabolas

Graph each parabola. Include the vertex and at least 3 accurate points on each side of the axis of symmetry. Then describe the transformation in words.

29. $f(x) = x^2$

Description:

30. $g(x) = x^2 - 3$

Description:

31. $h(x) = (x-2)^2$

Description:

32. $b(x) = -(x+1)^2 + 4$

Description:

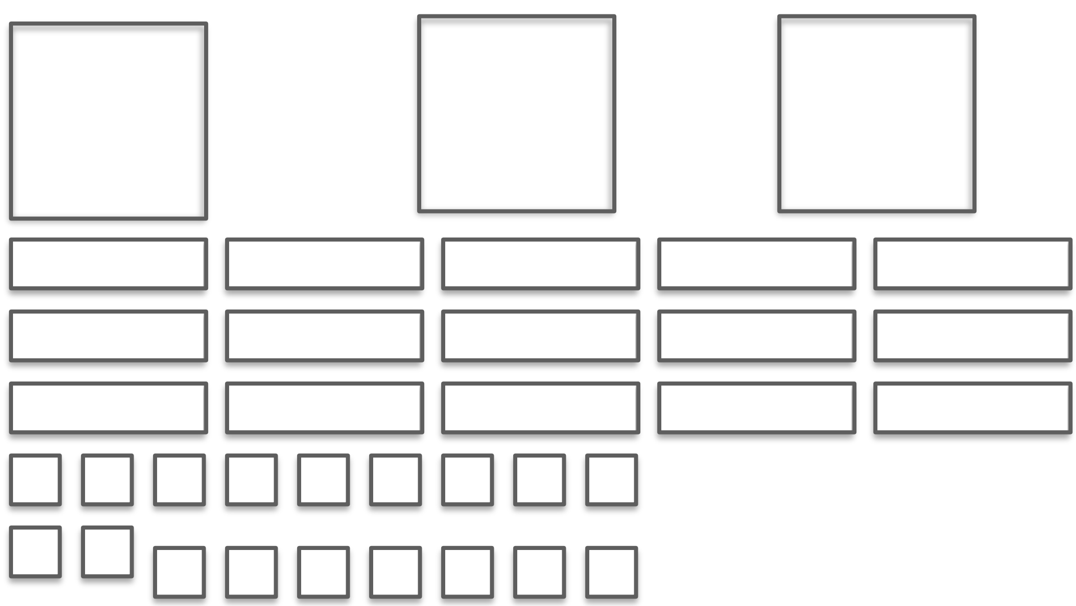
7.8H The Wow Factor

A Solidify Understanding Task

Optima's Quilts sometimes gets orders for blocks that are multiples of a given block. For instance, Optima got an order for a block that was exactly twice as big as the rectangular block that has a side that is 1" longer than the basic size, x, and one side that is 3" longer than the basic size.

1. Draw and label this block. Write two equivalent expressions for the area of the block.

2. Oh dear! This order was scrambled. The pieces are show here. Put the pieces together to make a rectangular block and write two equivalent expressions for the area of the block.



mathematics vision project

3. What do you notice when you compare the two equivalent expressions in problems #1 and #2?

4. Optima has a lot of new orders. Use diagrams to help you find equivalent expressions for each of the following:

a. $5x^2 + 10x$

b. $3x^2 + 21x + 36$

c. $2x^2 + 2x - 4$

d. $2x^2 - 10x + 12$

e. $3x^2 - 27$

Because she is a great business manager, Optima offers her customers lots of options. One option is to have rectangles that have side lengths that are more than one x. For instance, Optima made this cool block:

5. Write two equivalent expressions for this block. Use the distributive property to verify that your answer is correct.

6. Here we have some partial orders. We have one of the expressions for the area of the block and we know the length of one of the sides. Use a diagram to find the length of the other side and write a second expression for the area of the block. Verify your two expressions for the area of the block are equivalent using algebra.

a. Area: $2x^2 + 7x + 3$ Side: $(x + 3)$

Equivalent expression for area:

b. Area: $5x^2 + 8x + 3$ Side: $(x + 1)$

Equivalent expression for area:

c. Area: $2x^2 + 7x + 3$ Side: $(2x + 1)$

Equivalent expression for area:

7. What are some patterns you see in the two equivalent expressions for area that might help you to factor?

8. Business is booming! More and more orders are coming in! Use diagrams or number patterns (or both) to write each of the following orders in factored form:

a. $3x^2 + 16x + 5$

b. $2x^2 - 13x + 15$

c. $3x^2 + x - 10$

d. $2x^2 + 9x - 5$

9. In *The x Factor*, you wrote some rules for deciding about the signs inside the factors. Do those rules still work in factoring these types of expressions? Explain your answer.

10. Explain how Optima can tell if the block is a multiple of another block or if one side has a multiple of x in the side length.

11. There's one more twist on the kind of blocks that Optima makes. These are the trickiest of all because they have more than one x in the length of both sides of the rectangle! Here's an example:

Write two equivalent expressions for this block. Use the distributive property to verify that your answer is correct.

12. All right, let's try the tricky ones. They may take a little messing around to get the factored expression to match the given expression. Make sure you check your answers to be sure that you've got them right. Factor each of the following:

a. $6x^2 + 7x + 2$

b. $10x^2 + 17x + 3$

c. $4x^2 - 10x + 3$

d. $4x^2 + 4x - 3$

e. $9x^2 - 9x - 10$

12. Write a "recipe" for how to factor trinomials in the form, $ax^2 + bx + c$.

READY, SET, GO! Name Period Date

READY

Topic: Comparing arithmetic and geometric sequences

The first and fifth terms of each sequence are given. Fill in the missing numbers.

Example:

		+80		+80		+80		+80	
Arithmetic	4		84		164		244		324
Geometric	4		12		36		108		324
		× 3		× 3		× 3		× 3	

1.

Arithmetic	3				1875
Geometric	3				1875

2.

Arithmetic	-1458				-18
Geometric	-1458				-18

3.

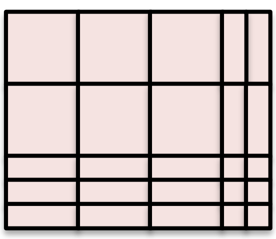
Arithmetic	1024				4
Geometric	1024				4

SET

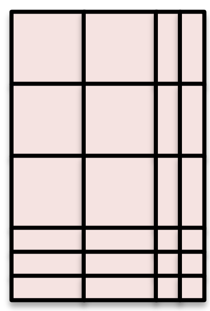
Topic: Writing an area model as a quadratic expression

Write two equivalent expressions for the area of each block. Let x be the side length of each of the large squares.

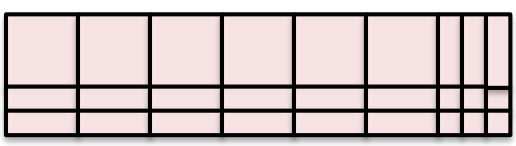
4.



5.



6.



7. Problems 4, 5, and 6 all contain the same number of squares measuring x^2 and 1^2.

   A. What is different about them?

   B. How does this difference affect the quadratic expression that represents them?

   C. Describe how the arrangement of the squares and rectangles affects the factored form.

Topic: Factoring quadratic expressions when $\boldsymbol{a > 1}$

Factor the following quadratic expressions.

8. $4x^2 + 7x - 2$
9. $2x^2 - 7x - 15$
10. $6x^2 + 7x - 3$
11. $4x^2 - x - 3$
12. $4x^2 + 19x - 5$
13. $3x^2 - 10x + 8$
14. $6x^2 + x - 2$
15. $3x^2 - 14x - 24$
16. $2x^2 + 9x + 10$
17. $5x^2 + 31x + 6$
18. $5x^2 + 7x - 6$
19. $4x^2 + 8x - 5$
20. $3x^2 - 75$
21. $3x^2 + 7x + 2$
22. $4x^2 + 8x - 5$
23. $2x^2 + x - 6$

GO

Topic: Finding the equation of the line of symmetry of a parabola

Given the x-intercepts of a parabola, write the equation of the line of symmetry.

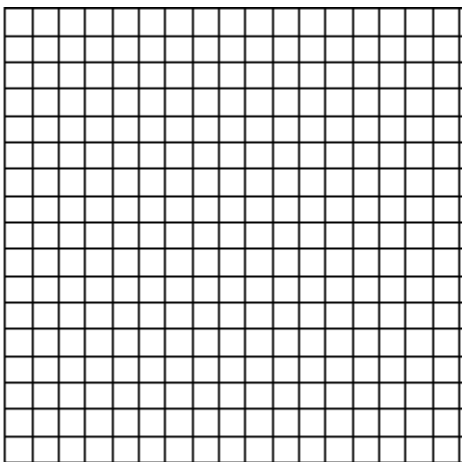
24. x-intercepts: (-3, 0) and (3, 0)
25. x-intercepts: (-4, 0) and (16, 0)
26. x-intercepts: (-2, 0) and (5, 0)
27. x-intercepts: (-14, 0) and (-3, 0)
28. x-intercepts: (17, 0) and (33, 0)
29. x-intercepts: $(-0.75, 0)$and $(2.25, 0)$

7.9 Lining Up Quadratics

A Practice Understanding Task

Graph each function and find the vertex, the y-intercept and the x-intercepts. Be sure to properly write the intercepts as points.

1. $y = (x-1)(x+3)$



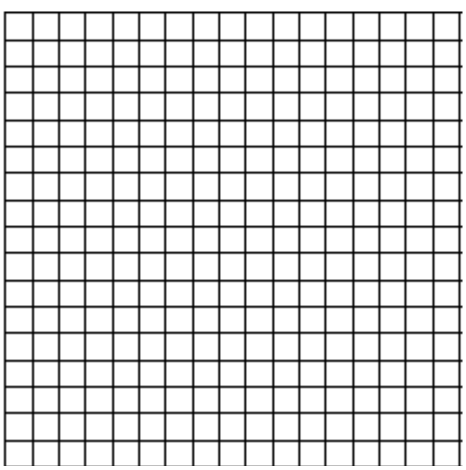
Line of Symmetry ________

Vertex _________

x -intercepts ________ ________

y-intercept _______

2. $f(x) = 2(x-2)(x-6)$



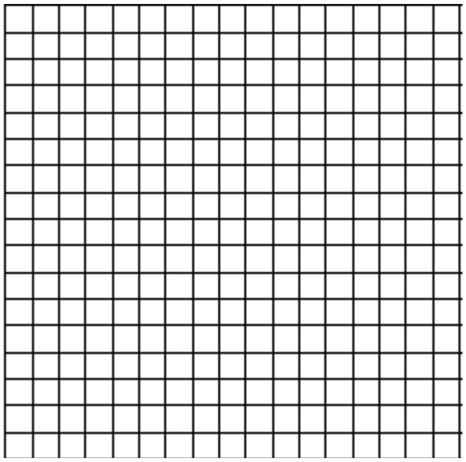
Line of Symmetry ________

Vertex _________

x -intercepts ________ ________

y-intercept _______

3. $g(x) = -x(x+4)$



Line of Symmetry __________

Vertex _________

x -intercepts ________ ________

y-intercept _______

4. Based on these examples, how can you use a quadratic function in factored form to:

a. Find the line of symmetry of the parabola?

b. Find the vertex of the parabola?

c. Find the x -intercepts of the parabola?

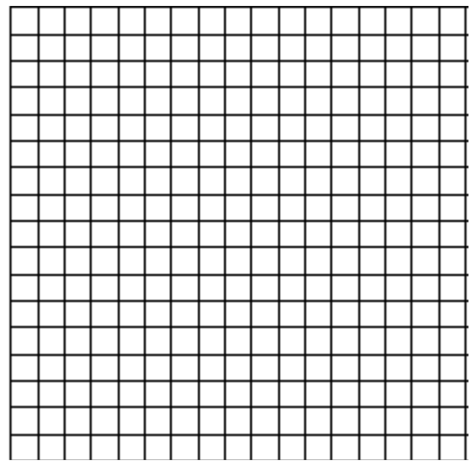
d. Find the y-intercept of the parabola?

e. Find the vertical stretch?

5. Choose any two **linear** functions and write them in the form: $f(x) = m(x - c)$, where m is the slope of the line. Graph the two functions.

Linear function 1:

Linear function 2:



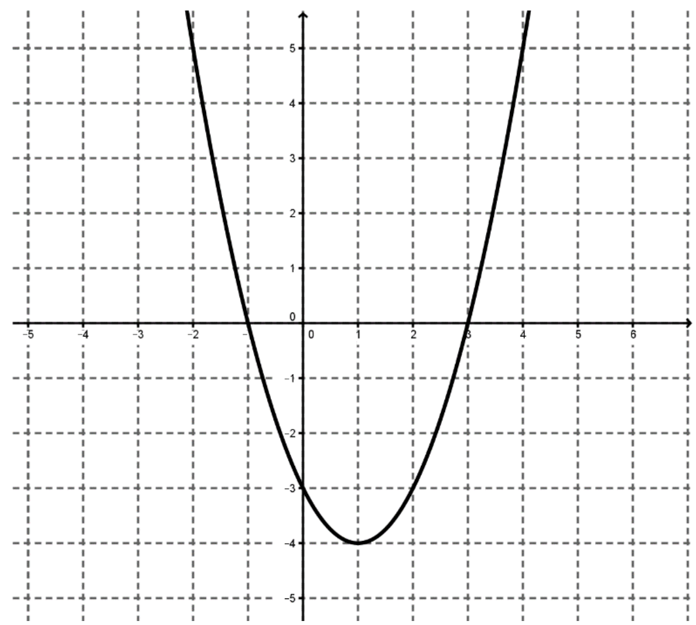
6. On the same graph as #5, graph the function *P(x)* that is the product of the two linear functions that you have chosen. What shape is created?

7. Describe the relationship between x-intercepts of the linear functions and the x-intercepts of the function *P(x).* Why does this relationship exist?

8. Describe the relationship between *y*-intercepts of the linear functions and the *y*-intercepts of the function *P(x).* Why does this relationship exist?

9. Given the parabola to the right, sketch two lines that could represent its linear factors.

10. Write an equation for each of these two lines.



11. How did you use the *x* and *y* intercepts of the parabola to select the two lines?

12. Are these the only two lines that could represent the linear factors of the parabola? If so, explain why. If not, describe the other possible lines.

13. Use your two lines to write the equation of the parabola. Is this the only possible equation of the parabola?

READY, SET, GO! Name Period Date

READY

Topic: Multiplying Binomials Using a Two-Way Table

Multiply the following binomials using the given two-way table to assist you.

Example: $(2x + 3)(5x - 7)$

	$(5x - 7)$	
$(2x + 3)$	$10x^2$	$-14x$
	$+15x$	-21

$= 10x^2 + x - 21$

1. $(3x - 4)(7x - 5)$
2. $(9x + 2)(x + 6)$
3. $(4x - 3)(3x + 11)$
4. $(7x + 3)(7x - 3)$
5. $(3x - 10)(3x + 10)$
6. $(11x + 5)(11x - 5)$
7. $(4x + 5)^2$
8. $(x + 9)^2$
9. $(10x - 7)^2$

10. The "like-term" boxes in #'s 7, 8, and 9 reveal a special pattern. Describe the relationship between the middle coefficient ***(b)*** and the coefficients ***(a)*** and ***(c).***

SET

Topic: Factored Form of a Quadratic Function

Given the *factored form* of a quadratic function, identify the vertex, intercepts, and vertical stretch of the parabola.

11. $y = 4(x-2)(x+6)$
    a. Vertex:_____________
    b. x-inter(s) __________
    c. y-inter _____________
    d. Stretch _____________

12. $y = -3(x+2)(x-6)$
    a. Vertex:_____________
    b. x-inter(s) __________
    c. y-inter: _____________
    d. Stretch _____________

13. $y = (x+5)(x+7)$
    a. Vertex:_____________
    b. x-inter(s) __________
    c. y-inter _____________
    d. Stretch _____________

14. $y = \frac{1}{2}(x-7)(x-7)$
    a. Vertex:_____________
    b. x-inter(s) __________
    c. y-inter _____________
    d. Stretch _____________

15. $y = -\frac{1}{2}(x-8)(x+4)$
    a. Vertex:_____________
    b. x-inter(s) __________
    c. y-inter: _____________
    d. Stretch _____________

16. $y = \frac{3}{5}(x-25)(x-9)$
    a. Vertex:_____________
    b. x-inter(s) __________
    c. y-inter _____________
    d. Stretch _____________

17. $y = \frac{3}{4}(x-3)(x+3)$
    a. Vertex:_____________
    b. x-inter(s) __________
    c. y-inter _____________
    d. Stretch _____________

18. $y = -(x-5)(x+5)$
    a. Vertex:_____________
    b. x-inter(s) __________
    c. y-inter: _____________
    d. Stretch _____________

19. $y = \frac{2}{3}(x+10)(x+10)$
    a. Vertex:_____________
    b. x-inter(s) __________
    c. y-inter _____________
    d. Stretch _____________

GO

Topic: Vertex Form of a Quadratic Equation

Given the *vertex form* of a quadratic function, identify the vertex, intercepts, and vertical stretch of the parabola.

20. $y = (x + 2)^2 - 4$
    a. Vertex:____________
    b. x-inter(s) __________
    c. y-inter ____________
    d. Stretch ____________

21. $y = -3(x + 6)^2 + 3$
    a. Vertex:____________
    b. x-inter(s) __________
    c. y-inter: ____________
    d. Stretch ____________

22. $y = 2(x - 1)^2 - 8$
    a. Vertex:____________
    b. x-inter(s) __________
    c. y-inter ____________
    d. Stretch ____________

23. $y = 4(x + 2)^2 - 64$
    a. Vertex:____________
    b. x-inter(s) __________
    c. y-inter ____________
    d. Stretch ____________

24. $y = -3(x - 2)^2 + 48$
    a. Vertex:____________
    b. x-inter(s) __________
    c. y-inter: ____________
    d. Stretch ____________

25. $y = (x + 6)^2 - 1$
    a. Vertex:____________
    b. x-inter(s) __________
    c. y-inter ____________
    d. Stretch ____________

26. Did you notice that the parabolas in problems 11, 12, & 13 are the same as the ones in problems 23, 24, & 25 respectively? If you didn't, go back and compare the answers in problems 11, 12, & 13 and problems 23, 24, & 25.

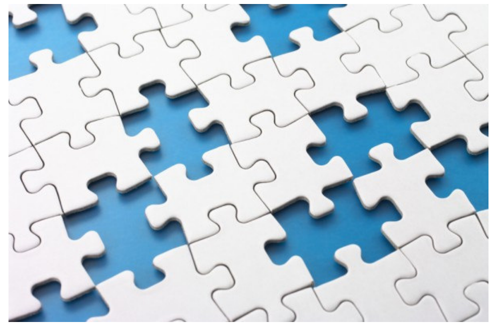
Prove that a. $4(x - 2)(x + 6) = 4(x + 2)^2 - 64$

b. $-3(x + 2)(x - 6) = -3(x - 2)^2 + 48$

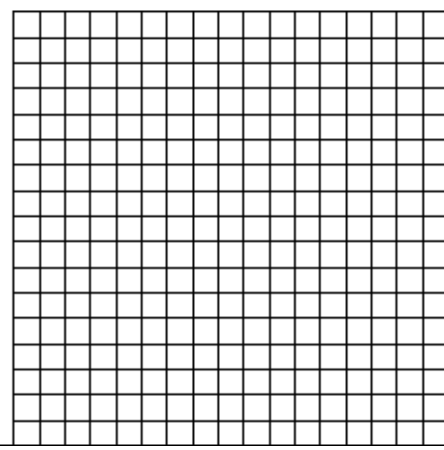
c. $(x + 5)(x + 7) = (x + 6)^2 - 1$

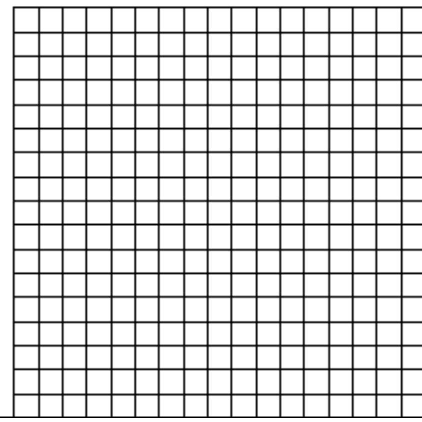
7.10 I've Got a Fill-in

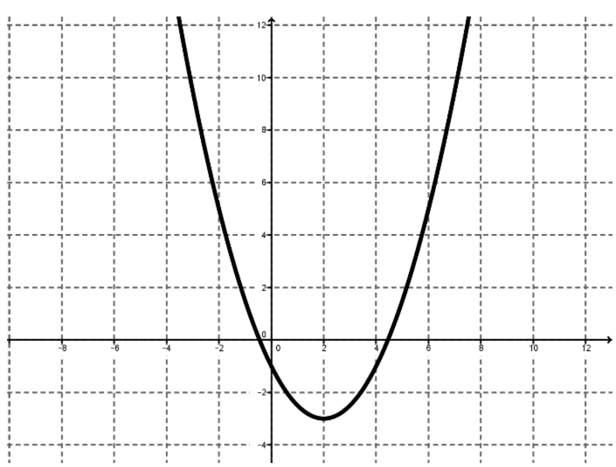
A Practice Understanding Task

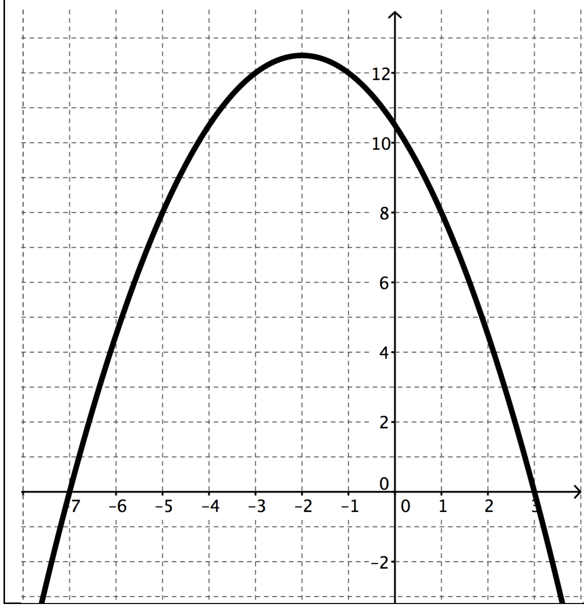


For each problem below, you are given a piece of information that tells you a lot. Use what you know about that information to fill in the rest.

1. You get this:	Fill in this:
$y = x^2 - x - 12$	Factored form of the equation:
	Graph of the equation: 

2. You get this:	Fill in this:
$y = x^2 - 6x + 3$	Vertex form of the equation:
	Graph of the equation: 

3. You get this:	Fill in this:
	Vertex form of the equation:
	Standard form of the equation:

4.	You get this:	Fill in this:
		Factored form of the equation:
		Standard form of the equation:

5.	You get this:	Fill in this:
	$y = -x^2 - 6x + 16$	Either form of the equation other than standard form.
		Vertex of the parabola
		x-intercepts and y-intercept

6. You get this:	Fill in this:
$y = 2x^2 + 12x + 13$	Either form of the equation other than standard form.
	Vertex of the parabola
	x-intercepts and y-intercept

7. You get this:	Fill in this:
$y = -2x^2 + 14x + 60$	Either form of the equation other than standard form.
	Vertex of the parabola
	x-intercepts and y-intercept

READY, SET, GO! Name Period Date

READY

A golf-pro practices his swing by driving golf balls off the edge of a cliff into a lake. The height of the ball above the lake (measured in meters) as a function of time (measured in seconds and represented by the variable t) from the instant of impact with the golf club is

$$58.8 + 19.6t - 4.9t^2.$$

<u>The expressions below are equivalent:</u>

$-4.9t^2 + 19.6t + 58.8$	standard form
$-4.9(t-6)(t+2)$	factored form
$-4.9(t-2)^2 + 78.4$	vertex form

1. Which expression is the most useful for finding how many seconds it takes for the ball to hit the water? Why?

2. Which expression is the most useful for finding the maximum height of the ball? Justify your answer.

3. If you wanted to know the height of the ball at exactly 3.5 seconds, which expression would help the most to find the answer? Why?

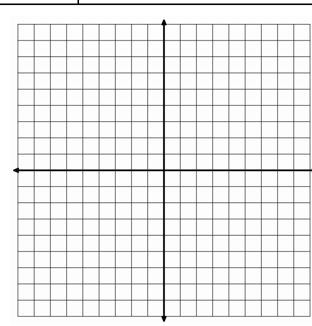
4. If you wanted to know the height of the cliff above the lake, which expression would you use? Why?

SET

Topic: Finding multiple representations of a quadratic

One form of a quadratic function is given. Fill-in the missing forms.

5 a. *Standard Form*	b. *Vertex Form*	c. *Factored Form* $y = (x+5)(x-3)$

d. *Table* (Include the vertex and at least 2 points on each side of the vertex.)	e. *Graph*
x | y Show the first differences and the second differences.	

6 a. *Standard Form*

b. *Vertex Form*
$y = -3(x-1)^2 + 3$

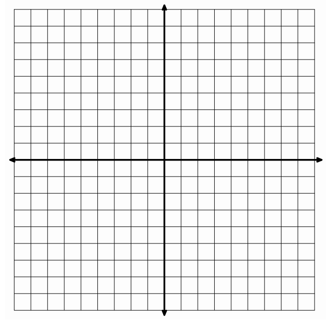
c. *Factored Form*

d. *Table* (Include the vertex and at least 2 points on each side of the vertex.)

x	y

Show the first differences and the second differences.

e. *Graph*



7 a. *Standard Form*
$y = -x^2 + 10x - 25$

b. *Vertex Form*

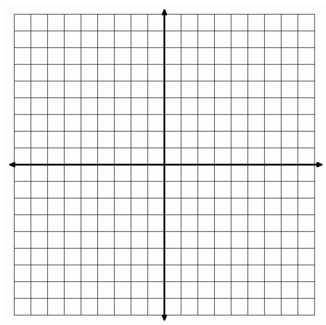
c. *Factored Form*

d. *Table* (Include the vertex and at least 2 points on each side of the vertex.)

x	y

Show the first differences and the second differences.

e. *Graph*



8 a. *Standard Form*

b. *Vertex Form*

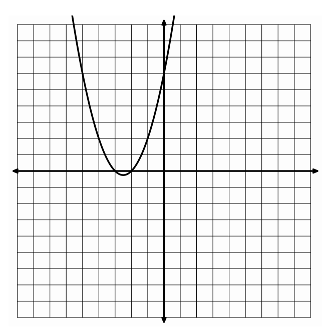
c. *Factored Form*

d. *Table* (Include the vertex and at least 2 points on each side of the vertex.)

x	y

Show the first differences and the second differences.

e. *Graph*



9 a. *Standard Form*

b. *Vertex Form*

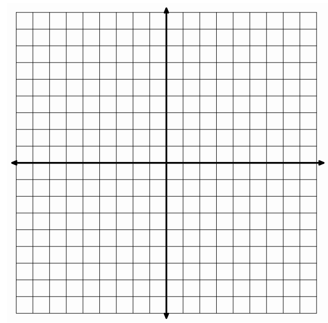
c. *Factored Form*
Skip this for now

d. *Table*

x	y
0	12
1	2
2	-4
3	-6
4	-4
5	2
6	12

Show the first differences and the second differences.

e. *Graph*



GO

Topic: Factoring Quadratics

Verify each factorization by multiplying.

10. $x^2 + 12x - 64 = (x + 16)(x - 4)$

11. $x^2 - 64 = (x + 8)(x - 8)$

12. $x^2 + 20x + 64 = (x + 16)(x + 4)$

13. $x^2 - 16x + 64 = (x - 8)(x - 8)$

Factor the following quadratic expressions, if possible. (Some will not factor.)

14. $x^2 - 5x + 6$

15. $x^2 - 7x + 6$

16. $x^2 - 5x - 36$

17. $m^2 + 16m + 63$

18. $s^2 - 3s - 1$

19. $x^2 + 7x + 2$

20. $x^2 + 14x + 49$

21. $x^2 - 9$

22. $c^2 + 11c + 3$

23. Which quadratic expression above could represent the area of a square? Explain.

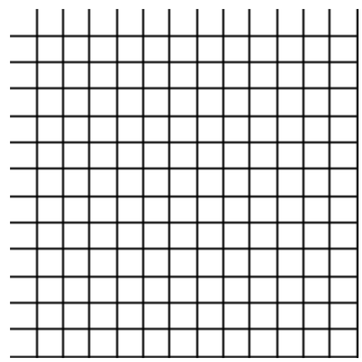
24. Would any of the expressions above NOT be the side-lengths for a rectangle? Explain.

7.11 Throwing an Interception

A Develop Understanding Task

The x-intercept(s) of the graph of a function $f(x)$ are often very important because they are the solution to the equation $f(x)=0$. In past tasks, we learned how to find the x-intercepts of the function by factoring, which works great for some functions, but not for others. In this task we are going to work on a process to find the x-intercepts of any quadratic function that has them. We'll start by thinking about what we already know about a few specific quadratic functions and then use what we know to generalize to all quadratic functions with x-intercepts.

1. What can you say about the graph of the function $f(x)=x^2-2x-3$?
   a. Graph the function



   b. What is the equation of the line of symmetry?

   c. What is the vertex of the function?

2. Now let's think specifically about the x-intercepts.
   a. What are the x-intercepts of $f(x)=x^2-2x-3$?

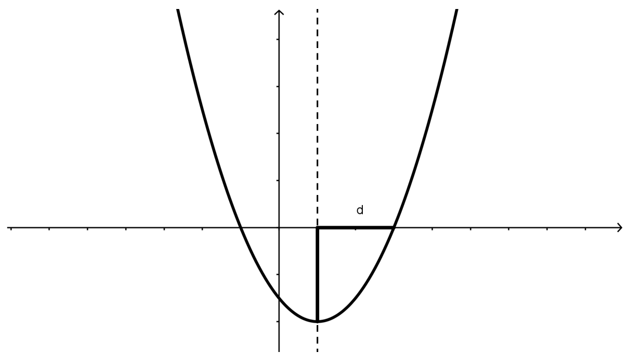
   b. How far are the x-intercepts from the line of symmetry?

   c. If you knew the line of symmetry was the line $x = h$, and you know how far the x-intercepts are from the line of symmetry, how would you find the actual x-intercepts?

   d. How far above the vertex are the x-intercepts?

   e. What is the value of $f(x)$ at the x-intercepts?

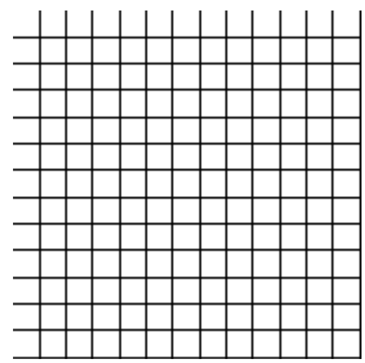
Just to make it a little easier to talk about some of the features that relate to the intercepts, let's name them with variables. From now on, when we talk about the distance from the line of symmetry to either of the x intercepts, we'll call it d. The diagram below shows this feature.



We will always refer to the line of symmetry as the line $x = h$, so the two x-intercepts will be at the points
$(h - d, 0)$ and $(h + d, 0)$.

3. So, let's think about another function: $f(x) = x^2 - 6x + 4$

   a. Graph the function by putting the equation into vertex form.



   b. What is the vertex of the function?

   c. What is the equation of the line of symmetry?

   d. What do you estimate the x-intercepts of the function to be?

   e. What do you estimate d to be?

   f. What is the value of $f(x)$ at the x-intercepts?

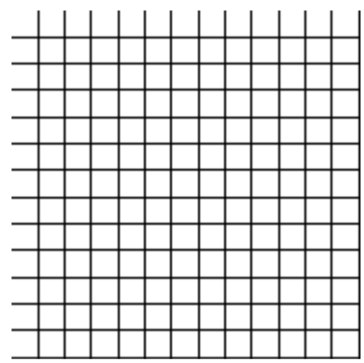
g. Using the vertex form of the equation and your answer to "f " above, write an equation and solve it to find the exact values of the *x* intercepts.

h. What is the exact value of *d*?

i. Use a calculator to find approximations for the *x*-intercepts. How do they compare with your estimates?

4. What about a function with a vertical stretch? Can we find exact values for the *x*-intercepts the same way? Let's try it with: $f(x) = 2x^2 - 8x + 5$.

a. Graph the function by putting the equation into vertex form.



b. What is the vertex of the function?

c. What is the equation of the line of symmetry?

d. What do you estimate the *x*-intercepts of the function to be?

e. What do you estimate *d* to be?

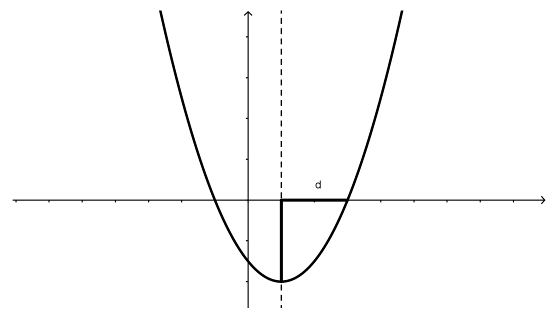
f. What is the value of $f(x)$ at the *x*-intercepts?

g. Using the vertex form of the equation and your answer to "f " above, write an equation and solve it to find the exact values of the x-intercepts.

h. What is the exact value of d?

i. Compare your solution to your estimate of the roots. How did you do?

5. Finally, let's try to generalize this process by using: $f(x) = ax^2 + bx + c$ to represent any quadratic function that has x-intercepts. Here's a possible graph of $f(x)$.



a. Start the process the usual way by putting the equation into vertex form. It's a little tricky, but just do the same thing with a, b, and c as what you did in the last problem with the numbers.

b. What is the vertex of the parabola?

c. What is the line of symmetry of the parabola?

d. Write and solve the equation for the x-intercepts just as you did previously.

6. How could you use the solutions you just found to tell what the x-intercepts are for the function $f(x) = x^2 - 3x - 1$?

7. You may have found the algebra for writing the general quadratic function $f(x) = ax^2 + bx + c$ in vertex form a bit difficult. Here is another way we can work with the general quadratic function leading to the same results you should have arrived at in 5d.

   a. Since the two x-intercepts are d units from the line of symmetry $x = h$, if the quadratic crosses the x-axis its x-intercepts are at $(h - d, 0)$ and $(h + d, 0)$. We can always write the factored form of a quadratic if we know its x-intercepts. The factored form will look like $f(x) = a(x - p)(x - q)$ where p and q are the two x-intercepts. So, using this information, write the factored form of the general quadratic $f(x) = ax^2 + bx + c$ using the fact that its x-intercepts are at h-d and h+d.

   b. Multiply out the factored form (you will be multiplying two **trinomial** expressions together) to get the quadratic in standard form. Simplify your result as much as possible by combining like terms.

c. You now have the same general quadratic function written in standard form in two different ways, one where the **coefficients** of the terms are a, b and c and one where the coefficients of the terms are expressions involving a, h and d. Match up the coefficients; that is, b, the coefficient of x in one version of the standard form is equivalent to ________ in the other version of the standard form. Likewise c, the constant term in one version of the standard form is equivalent to _________ in the other.

d. Solve the equations b = ________ and c = ________ for h and d. Work with your equations until you can express h and d with expressions that only involve a, b and c.

e. Based on this work, how can you find the x-intercepts of any quadratic using only the values for a, b and c?

f. How does your answer to "e" compare to your result in 5d?

8. All of the functions that we have worked with on this task have had graphs that open upward. Would the formula work for parabolas that open downward? Tell why or why not using an example that you create using your own values for the coefficients a, b, and c.

READY, SET, GO! Name Period Date

READY

Topic: Converting measurement of area, area and perimeter.

While working with areas is sometimes essential to convert between units of measure, for example changing from square yards to square feet and so forth. Convert the areas below to the desired measure. (Hint: area is two dimensional, for example 1 yd² = 9 ft² because 3 ft along each side of a square yard equals 9 square feet.)

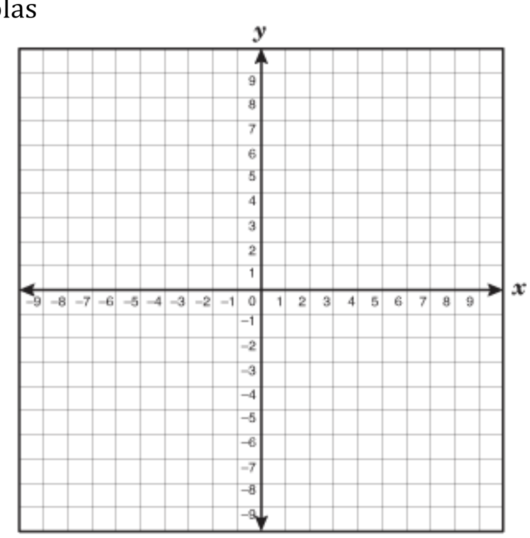
1. 7 yd² = ? ft²
2. 5 ft² = ? in²
3. 1 mile² = ? ft²
4. 100 m² = ? cm²
5. 300 ft² = ? yd²
6. 96 in² = ? ft²

SET

Topic: Transformations and parabolas, symmetry and parabolas

7a. Graph each of the quadratic functions.

$$f(x) = x^2$$
$$g(x) = x^2 - 9$$
$$h(x) = (x+2)^2 - 9$$



b. How do the functions compare to each other?

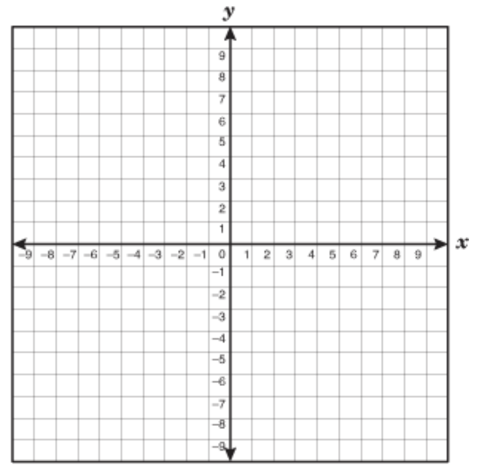
c. How do *g(x)* and *h(x)* compare to *f(x)*?

d. Look back at the functions above and identify the x-intercepts of *g(x).* What are they?

e. What are the coordinates of the points corresponding to the x-intercepts in *g(x)* in each of the other functions*?* How do these coordinates compare to one another?

8a. Graph each of the quadratic functions.

$$f(x) = x^2$$
$$g(x) = x^2 - 4$$
$$h(x) = (x-1)^2 - 4$$



b. How do the functions compare to each other?

c. How do *g(x)* and *h(x)* compare to *f(x)*?

d. Look back at the functions above and identify the x-intercepts of *g(x).* What are they?

e. What are the coordinates of the points corresponding to the x-intercepts in *g(x)* in each of the other functions*?* How do these coordinates compare to one another?

9. How can the transformations that occur to the function $f(x) = x^2$ be used to determine where the x-intercepts of the function's image will be?

GO

Topic: Function Notation and Evaluating Functions

Use the given functions to find the missing values. (Check your work using a graph.)

10. $f(x) = x^2 + 4x - 12$

a. $f(0) =$ ______

b. $f(2) =$ _____

c. $f(x) = 0, \quad x =$ _____

d. $f(x) = 20, \quad x =$ _____

11. $g(x) = (x-5)^2 + 2$

a. $g(0) =$ _____

b. $g(5) =$ ______

c. $g(x) = 0, \quad x =$ ______

d. $g(x) = 16, \quad x =$ ______

12. $f(x) = x^2 - 6x + 9$

a. $f(0) =$ ______

b. $f(-3) =$ _____

c. $f(x) = 0$, $x =$ ______

d. $f(x) = 16$, $x =$ ______

13. $g(x) = (x - 2)^2 - 3$

a. $g(0) =$ _____

b. $g(5) =$ ______

c. $g(x) = 0$, $x =$ ______

d. $g(x) = -3$, $x =$ ______

14. $f(x) = (x + 5)^2$

a. $f(0) =$ ______

b. $f(-2) =$ _____

c. $f(x) = 0$, $x =$ ______

d. $f(x) = 9$, $x =$ ______

15. $g(x) = -(x + 1)^2 + 8$

a. $g(0) =$ _____

b. $g(2) =$ ______

c. $g(x) = 0$, $x =$ ______

d. $g(x) = 4$, $x =$ ______

7.12 Curbside Rivalry

A Solidify Understanding Task

Carlos and Clarita have a brilliant idea for how they will earn money this summer. Since the community in which they live includes many high schools, a couple of universities, and even some professional sports teams, it seems that everyone has a favorite team they like to root for. In Carlos' and Clarita's neighborhood these rivalries take on special meaning, since many of the neighbors support different teams. They have observed that their neighbors often display handmade posters and other items to make their support of their favorite team known. The twins believe they can get people in the neighborhood to buy into their new project: painting team logos on curbs or driveways.

For a small fee, Carlos and Clarita will paint the logo of a team on a neighbor's curb, next to their house number. For a larger fee, the twins will paint a mascot on the driveway. Carlos and Clarita have designed stencils to make the painting easier and they have priced the cost of supplies. They have also surveyed neighbors to get a sense of how many people in the community might be interested in purchasing their service. Here is what they have decided, based on their research.

- *A curbside logo will require 48 in^2 of paint*
- *A driveway mascot will require 16 ft^2 of paint*
- *Surveys show the twins can sell 100 driveway mascots at a cost of \$20, and they will sell 10 fewer mascots for each additional \$5 they charge*

1. If a curbside logo is designed in the shape of a square, what will its dimensions be?

A square logo will not fit nicely on a curb, so Carlos and Clarita are experimenting with different types of rectangles. They are using a software application that allows them to stretch or shrink their logo designs to fit different rectangular dimensions.

2. Carlos likes the look of the logo when the rectangle in which it fits is 8 inches longer than it is wide. What would the dimensions of the curbside logo need to be to fit in this type of rectangle? As part of your work, write a quadratic equation that represents these requirements.

3. Clarita prefers the look of the logo when the rectangle in which it fits is 13 inches longer than it is wide. What would the dimensions of the curbside logo need to be to fit in this type of rectangle? As part of your work, write a quadratic equation that represents these requirements.

Your quadratic equations on the previous two problems probably started out looking like this: $x(x+n)=48$ where *n* represents the number of inches the rectangle is longer than it is wide. The expression on the left of the equation could be multiplied out to get and equation of the form $x^2+nx=48$. If we subtract 48 from both sides of this equation we get $x^2+nx-48=0$. In this form, the expression on the left looks more like the quadratic functions you have been working with in previous tasks, $y=x^2+nx-48$.

4. Consider Carlos' quadratic equation where n = 8, so $x^2 + 8x - 48 = 0$. How can we use our work with quadratic functions like $y = x^2 + 8x - 48$ to help us solve the quadratic equation $x^2 + 8x - 48 = 0$? Describe at least two different strategies you might use, and then carry them out. Your strategies should give you solutions to the equation as well as a solution to the question Carlos is trying to answer in #2.

5. Now consider Clarita's quadratic equation where n = 13, so $x^2 + 13x - 48 = 0$. Describe at least two different strategies you might use to solve this equation, and then carry them out. Your strategies should give you solutions to the equation as well as a solution to the question Clarita is trying to answer in #3.

6. After much disagreement, Carlos and Clarita agree to design the curbside logo to fit in a rectangle that is 6 inches longer than it is wide. What would the dimensions of the curbside logo need to be to fit in this type of rectangle? As part of your work, write and solve a quadratic equation that represents these requirements.

7. What are the dimensions of a driveway mascot if it is designed to fit in a rectangle that is 6 feet longer than it is wide? (See the requirements for a driveway mascot given in the bulleted list above.) As part of your work, write and solve a quadratic equation that represents these requirements.

8. What are the dimensions of a driveway mascot if it is designed to fit in a rectangle that is 10 feet longer than it is wide? (See the requirements for a driveway mascot given in the bulleted list above.) As part of your work, write and solve a quadratic equation that represents these requirements.

Carlos and Clarita are also examining the results of their neighborhood survey, trying to determine how much they should charge for a driveway mascot. Remember, this is what they have found from the survey: *They can sell 100 driveway mascots at a cost of $20, and they will sell 10 fewer mascots for each additional $5 they charge.*

9. Make a table, sketch a graph, and write an equation for the number of driveway mascots the twins can sell for each \$5 increment, x, in the price of the mascot.

number of \$5 increments in the price	number of mascots purchased
0	100
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

10. Make a table, sketch a graph (on the same set of axes), and write an equation for the price of a driveway mascot for each \$5 increment, x, in the price.

number of \$5 increments in the price	price of a mascot
0	20
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

11. Make a table, sketch a graph, and write an equation for the revenue the twins will collect for each \$5 increment in the price of the mascot.

number of \$5 increments in the price	Revenue = price × number of mascots sold
0	2000
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

12. The twins estimate that the cost of supplies will be \$250 and they would like to make \$2000 in profit from selling driveway mascots. Therefore, they will need to collect \$2250 in revenue. Write and solve a quadratic equation that represents collecting \$2250 in revenue. Be sure to clearly show your strategy for solving this quadratic equation.

READY, SET, GO! Name Period Date

READY

Topic: Finding x-intercepts for linear equations

1. Find the x-intercept of each equation below. Write your answer as an ordered pair. Consider how the format of the given equation either facilitates or inhibits your work.

a. $3x + 4y = 12$	b. $y = 5x - 3$	c. $y - 5 = -4(x + 1)$
d. $y = -4x + 1$	e. $y - 6 = 2(x + 7)$	f. $5x - 2y = 10$

2. Which of the linear equation formats above facilitates your work in finding x-intercepts? Why?

3. Using the same equations from question 1, find the y-intercepts. Write your answers as ordered pairs

a. $3x + 4y = 12$

b. $y = 5x - 3$

c. $y - 5 = -4(x + 1)$

d. $y = -4x + 1$

e. $y - 6 = 2(x + 7)$

f. $5x - 2y = 10$

4. Which of the formats above facilitate finding the y-intercept? Why?

SET

Topic: Solve Quadratic Equations, Connecting Quadratics with Area

For each of the given quadratic equations, (a) describe the rectangle the equation fits with. (b) What constraints have been placed on the dimensions of the rectangle?

5. $x^2 + 7x - 170 = 0$

6. $x^2 + 15x - 16 = 0$

7. $x^2 + 2x - 35 = 0$

8. $x^2 + 10x - 80 = 0$

Solve the quadratic equations below.

9. $x^2 + 7x - 170 = 0$

10. $x^2 + 15x - 16 = 0$

11. $x^2 + 2x - 35 = 0$

12. $x^2 + 10x - 80 = 0$

GO

Topic: Factoring Expressions

Write each of the expressions below in factored form.

13. $x^2 - x - 132$

14. $x^2 - 5x - 36$

15. $x^2 + 5x + 6$

16. $x^2 + 13x + 42$

17. $x^2 + x - 56$

18. $x^2 - x$

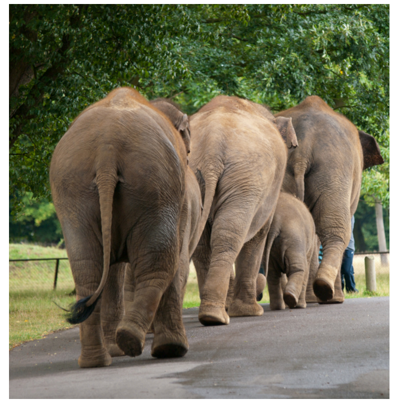
19. $x^2 - 8x + 12$

20. $x^2 - 10x + 25$

21. $x^2 + 5x$

7.13 Perfecting My Quads

A Practice Understanding Task



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Carlos and Clarita, Tia and Tehani, and their best friend Zac are all discussing their favorite methods for solving quadratic equations of the form $ax^2 + bx + c = 0$. Each student thinks about the related quadratic function $y = ax^2 + bx + c$ as part of his or her strategy.

Carlos: "I like to make a table of values for *x* and find the solutions by inspecting the table."

Zac: "I like to graph the related quadratic function and use my graph to find the solutions."

Clarita: "I like to write the equation in factored form, and then use the factors to find the solutions."

Tia: "I like to treat it like a quadratic function that I put in vertex form by completing the square. I can then use a square root to undo the squared expression."

Tehani: "I also like to use the quadratic formula to find the solutions."

Demonstrate how each student might solve each of the following quadratic equations.

Solve: $x^2 - 2x - 15 = 0$	Carlos' Strategy	Zac's Strategy
Clarita's Strategy	Tia's Strategy	Tehani's Strategy

Solve: $2x^2 + 3x + 1 = 0$	Carlos' Strategy	Zac's Strategy
Clarita's Strategy	Tia's Strategy	Tehani's Strategy

Solve: $x^2 + 4x - 8 = 0$	Carlos' Strategy	Zac's Strategy
Clarita's Strategy	Tia's Strategy	Tehani's Strategy

Describe why each strategy works.

As the students continue to try out their strategies, they notice that sometimes one strategy works better than another. Explain how you would decide when to use each strategy.

Here is an extra challenge. How might each student solve the following system of equations?

Solve the system: $y_1 = x^2 - 4x + 1$ $y_2 = x - 3$	Carlos' Strategy	Zac's Strategy
Clarita's Strategy	Tia's Strategy	Tehani's Strategy

READY, SET, GO! Name Period Date

READY

Topic: Symmetry and Distance

The given functions provide the connection between possible areas, *A(x)* , that can be created by a rectangle for a given side length, *x*, and a set amount of perimeter. You could think of it as the different amounts of area you can close in with a given amount of fencing as long as you always create a rectangular enclosure.

1. $A(x) = x\,(10 - x\,)$

Find the following:

a. $A(3) =$ b. $A(4) =$

c. $A(6) =$ d. $A(x) = 0$

e. When is A(x) at its maximum? Explain or show how you know.

2. $A(x) = x\,(50 - x)$

Find the following:

a. $A(10) =$ b. $A(20) =$

c. $A(30) =$ d. $A(x) = 0$

e. When is A(x) at its maximum? Explain or show how you know.

3. $A(x) = x(\,75 - x)$

Find the following:

a. $A(20) =$ *b.* $A(35) =$

c. $A(40) =$ *d.* $A(x) = 0$

e. When is A(x) at its maximum? Explain or show how you know.

4. $A(x) = x(\,48 - x)$

Find the following:

a. $A(10) =$ b. $A(20) =$

c. $A(28) =$ d. $A(x) = 0$

e. When is A(x) at its maximum? Explain or show how you know.

SET

Topic: Solve Quadratic Equations Efficiently

For each of the given quadratic equations find the solutions using an efficient method. State the method you are using as well as the solutions. You must use at least three different methods.

5. $x^2 + 17x + 60 = 0$

6. $x^2 + 16x + 39 = 0$

7. $x^2 + 7x - 5 = 0$

8. $3x^2 + 14x - 5 = 0$

9. $x^2 - 12x = -8$

10. $x^2 + 6x = 7$

Summarize the process for solving a quadratic by the indicated strategy. Give examples along with written explanation, also indicate when it is best to use this strategy.

11. Completing the Square

12. Factoring

13. Quadratic Formula

GO

Topic: Graphing Quadratics and finding essential features of the graph. Solving systems of equations.

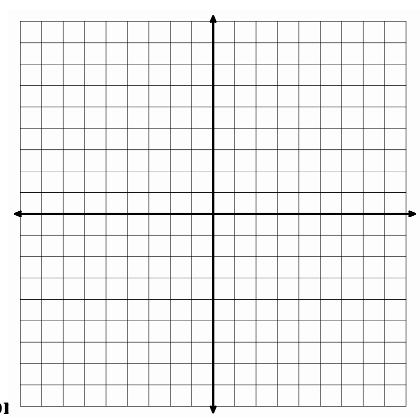
Graph the quadratic function and supply the desired information about the graph.

14. $f(x) = x^2 + 8x + 13$

a. Line of symmetry:

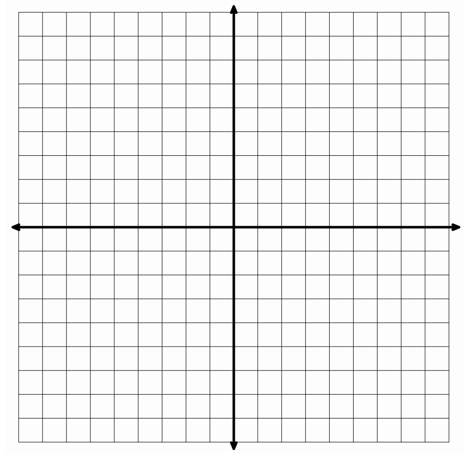
b. x-intercepts:

c. y-intercept:



d. vertex:

15. $f(x) = x^2 - 4x - 1$



a. Line of symmetry:

b. x-intercepts:

c. y-intercept:

d. vertex:

Solve each system of equations using an algebraic method and check your work!

16.

$$\begin{cases} 3x + 5y = 15 \\ 3x - 2y = 6 \end{cases}$$

17.

$$\begin{cases} y = -7x + 12 \\ y = 5x - 36 \end{cases}$$

18.

$$\begin{cases} y = 2x + 12 \\ y = 10x - x^2 \end{cases}$$

19.

$$\begin{cases} y = 24x - x^2 \\ y = 8x + 48 \end{cases}$$