

*Transforming Mathematics Education*

# ALGEBRA I

*A Learning Cycle Approach*

MODULE 8

## More Functions, More Features

MATHEMATICSVISIONPROJECT.ORG

**The Mathematics Vision Project**

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## 8.1 Some of This, Some of That

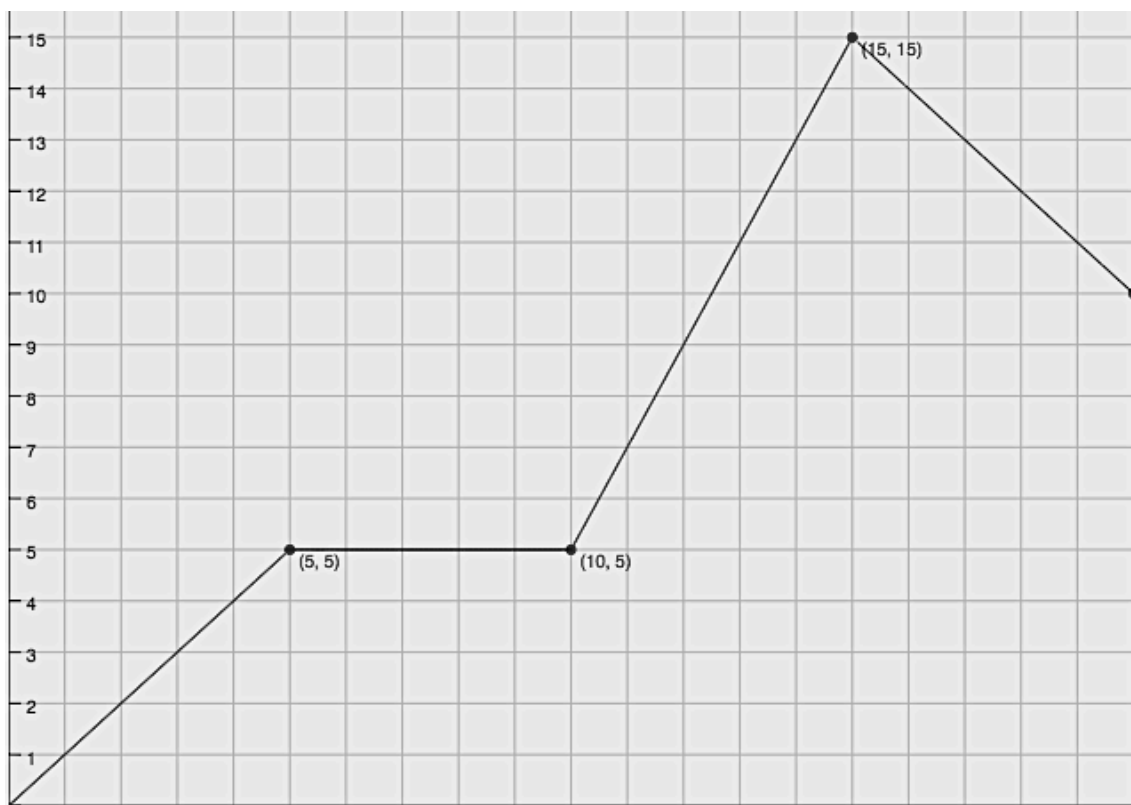
### *A Develop Understanding Task*



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#### Part I: Connect context and graphical representations

1. Create a story that matches the graph below. Label axes and be as specific as possible in describing what is happening to connect your story to the graph.



2. If you were to write a function to match each part of your story (or section of the graph), how many would you write? Explain.

3. Identify and write the function and corresponding domain for each section of the graph.

$$f(x) = \left\{ \begin{array}{ll} \text{function here} & , \quad \text{domain here} \\ & , \\ & , \\ & , \\ & , \end{array} \right\}$$

4. Make connections between the graph, functions, and context (story you created).

The function you created above is called a **piecewise function**. In mathematics, a piecewise-defined function is a function defined by more than one sub-function (or *piece* of a function), with each section only existing in a certain interval of the functions domain.

## Part II: Connecting function notation to a piecewise defined function

5. Find  $f(12)$ . Use the story you created to explain this meaning.
6. Which sub-function would you use to algebraically find the value of  $f(12)$ ?
7. Find the following:
  - a.  $f(7) =$
  - b.  $f(x) = 3$
  - c.  $f(x) = 13$
  - d.  $f(15) =$

READY, SET, GO!

Name \_\_\_\_\_

Period \_\_\_\_\_

Date \_\_\_\_\_

**READY**

Topic: Reading function values in a piece-wise defined graph.

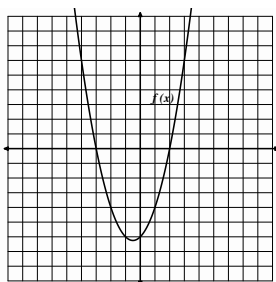
Use the graph to find the indicated function value.

1a.  $f(-3) =$

b.  $f(-2) =$

c.  $f(0) =$

d.  $f(2) =$

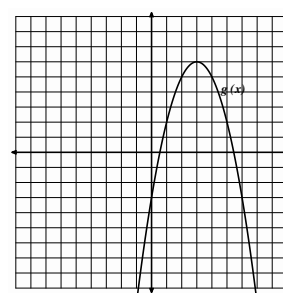


2a.  $g(0) =$

b.  $g(2) =$

c.  $g(3) =$

d.  $g(5) =$

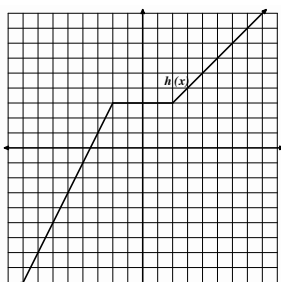


3a.  $h(-4) =$

b.  $h(0) =$

c.  $h(2) =$

d.  $h(4) =$

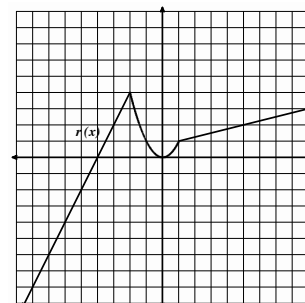


4a.  $r(-3) =$

b.  $r(-1) =$

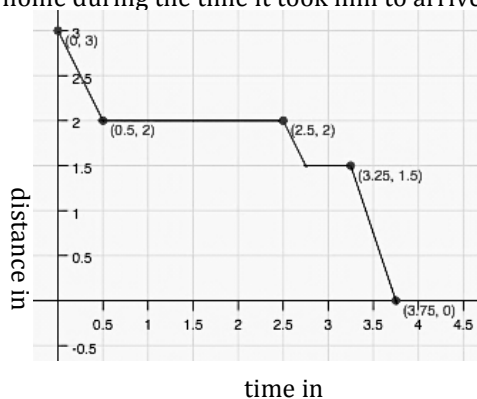
c.  $r(0) =$

d.  $r(5) =$



5. Isaac lives 3 miles away from his school. School ended at 3 pm and Isaac began his walk home with his friend Tate who lives 1 mile away from the school, in the direction of Isaac's house. Isaac stayed at Tate's house for a while and then started home. On the way he stopped at the library. Then he hurried home. The graph at the right is a **piece-wise defined function** that shows Isaac's distance from home during the time it took him to arrive home.

- How much time passed between school ending and Isaac's arrival home?
- How long did Isaac stay at Tate's house?
- How far is the library from Isaac's house?
- Where was Isaac, 3 hours after school ended?
- Use function notation to write a mathematical expression that says the same thing as question d.
- When was Isaac walking the fastest? How fast was he walking?

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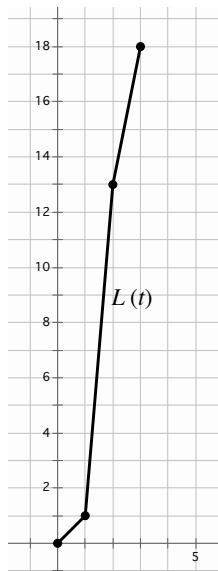
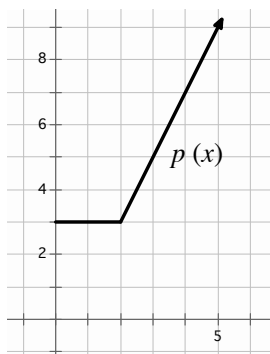
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**SET****Topic:** Writing piece-wise defined functions

6. A parking garage charges \$3 for the first two hours that a car is parked in the garage. After that, the hourly fee is \$2 per hour. Write a piece-wise function  $p(x)$  for the cost of parking a car in the garage for  $x$  hours. (The graph of  $p(x)$  is shown.)



7. Lexie completed an 18 mile triathlon. She swam 1 mile in 1 hour, bicycled 12 miles in 1 hour, and then ran 5 miles in 1 hour. The graph of Lexie's distance versus time is shown. Write a piecewise function  $L(t)$  for the graph.

**GO****Topic:** Using the point-slope formula to write the equations of lines.

**Write the equation of the line (in point-slope form) that contains the given slope and point.**

8.  $p: (1, 2); m = 3$

9.  $p: (1, -2); m = -1$

10.  $p: (5, -1); m = 2$

**Write the equation of the line (in point-slope form) that contains the given points.**

11.  $K(0, 0); L(-4, 5)$

12.  $X(-1, 7); Y(3, -1)$

13.  $T(-1, -9); V(5, 18)$

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## 8.2 Bike Lovers

### *A Solidify Understanding Task*



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Michelle and Rashid love going on long bike rides.

Every Saturday, they have a particular route they bike together that takes four hours. Below is a piecewise function that estimates the distance they travel for each hour of their bike ride.

$$f(x) = \begin{cases} 16x, & 0 \leq x \leq 1 \\ 10(x - 1) + 16, & 1 < x \leq 2 \\ 14(x - 2) + 26, & 2 < x \leq 3 \\ 12(x - 3) + 40, & 3 < x \leq 4 \end{cases}$$

1. What part of the bike ride are they going the fastest? Slowest?
2. What is the domain of this function?
3. Find  $f(2)$ . Explain what this means in terms of the context.
4. How far have they traveled at 3 hours? Write the answer using function notation.
5. What is the total distance they travel on this bike ride?
6. Sketch a graph of the bike ride as a function of distance traveled over time.

Rashid also has a route he likes to do on his own and has the following continuous piecewise function to represent the average distance he travels in minutes:

$$g(x) = \begin{cases} \frac{1}{4}(x) & 0 \leq x \leq 20 \\ \frac{1}{5}(x - 20) + 5 & 20 < x \leq 50 \\ \frac{2}{7}(x - 50) + 11 & 50 < x \leq 92 \\ \frac{1}{8}(x - a) + b & 92 < x \leq 100 \end{cases}$$

7. What is the domain for this function? What does the domain tell us?
8. What is the average rate of change during the interval  $[20, 50]$ ?
9. Over which time interval is the greatest average rate of change?
10. Find the value of each, then complete each sentence frame:
  - a.  $g(30) = \underline{\hspace{2cm}}$ . This means...
  - b.  $g(64) = \underline{\hspace{2cm}}$ . This means...
  - c.  $g(10) = \underline{\hspace{2cm}}$ . When finding output values for given input values in a piecewise function, you must ...
11. Complete the last equation by finding values for  $a$  and  $b$ .
12. Sketch a graph of the bike ride as a function of distance traveled as a function of time.

Use the following continuous piecewise defined function, where  $x$  represents time in minutes and  $h(x)$  represents distance traveled in km, to answer the following questions.

$$h(x) = \begin{cases} \frac{1}{4}x^2 & 0 \leq x \leq 10 \\ \frac{1}{2}(x - 10) + c & 10 < x \leq 20 \\ 2(x - 20) + 30 & 20 < x \leq 30 \end{cases}$$

13. Find the value of  $c$ .

14. Sketch the graph (label axes).

15. What is the domain of  $h(x)$ ?

16. What is the range of  $h(x)$ ?

17. Which five minute interval of time has the greatest average rate of change?

a.  $[0, 5]$

b.  $[5, 10]$

c.  $[10, 15]$

d.  $[25, 30]$

What is the average rate of change over this interval?

18. Find  $h(8)$ .

19. Find  $h(15)$ .

**READY, SET, GO!**

Name \_\_\_\_\_

Period \_\_\_\_\_

Date \_\_\_\_\_

**READY**

Topic: Solving absolute value equations.

**Solve for x.** (You will have two answers.)

1.  $|x| = 7$

2.  $|x - 6| = 3$

3.  $|w + 4| = 11$

4.  $-9|m| = -63$

5.  $|3d| = 15$

6.  $|3x - 5| = 11$

7.  $-|m + 3| = -13$

8.  $|-4m| = 64$

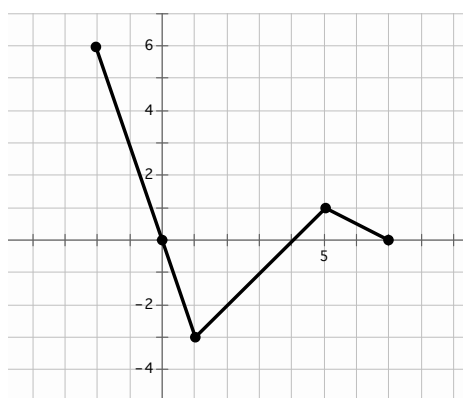
9.  $2|x + 1| - 7 = -3$

10.  $5|c + 3| - 1 = 9$

11.  $-2|2p - 3| - 1 = -11$

12. Explain why the equation  $|m| = -3$  has no solution.**SET****Topic:** Reading the domain and range from a graph**State the domain and range of the piece-wise functions in the graph. Use interval notation.**

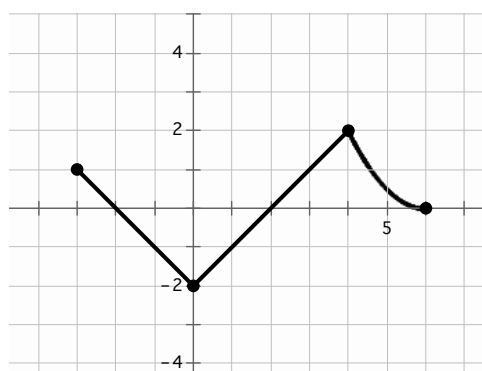
13.



a. Domain:

b. Range:

14.



a. Domain:

b. Range:

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**For each of the graphs below write the interval that defines each piece of the graph. Then write the domain of the entire piece-wise function.**

**Example:** (Look at the graph in #14. Moving left to right. Piece-wise functions use set notation.)

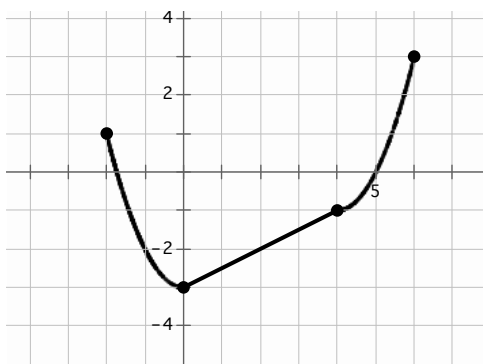
Interval 1  $-3 \leq x < 0$

Interval 2  $0 \leq x < 4$

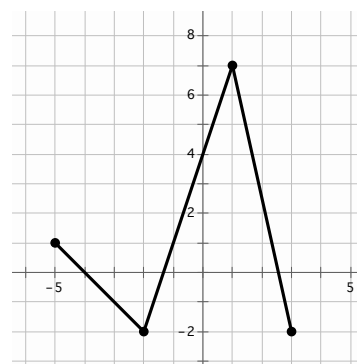
Interval 3  $4 \leq x \leq 6$

Domain:  $[-3, 6]$  (We can use interval notation on the domain, if it's continuous.)

Pay attention to your inequality symbols! You do not want the pieces of your graph to overlap. Do you know why?



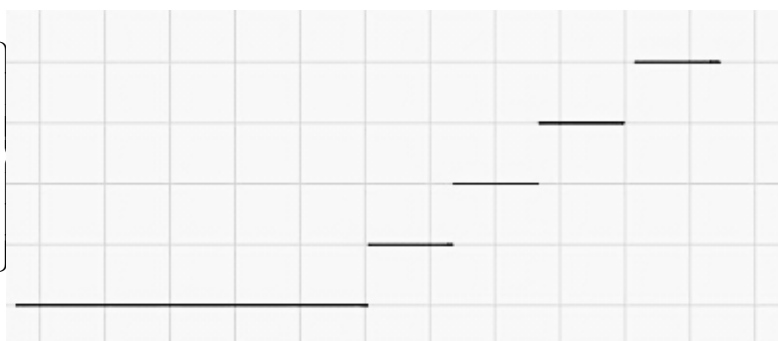
15. a. Interval 1 \_\_\_\_\_  
 b. Interval 2 \_\_\_\_\_  
 c. Interval 3 \_\_\_\_\_  
 d. Domain: \_\_\_\_\_



16. a. Interval 1 \_\_\_\_\_  
 b. Interval 2 \_\_\_\_\_  
 c. Interval 3 \_\_\_\_\_  
 d. Domain: \_\_\_\_\_

17. So far you've only seen continuous piece-wise defined functions, but piece-wise functions can also be non-continuous. In fact, you've had some real life experience with one kind of non-continuous piece-wise function. The graph below represents how some teachers calculate grades. Finish filling in the piece-wise equation. Then label the graph with the corresponding values.

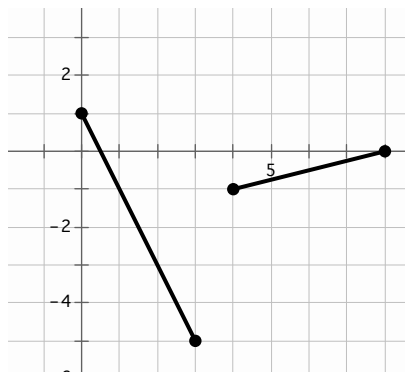
$$f(x) = \left\{ \begin{array}{ll} A, & \text{_____ } x \text{ _____} \\ B, & \text{_____ } x \text{ _____} \\ C, & \text{_____ } x \text{ _____} \\ D, & \text{_____ } x \text{ _____} \\ F, & \text{_____ } x \text{ _____} \end{array} \right\}$$



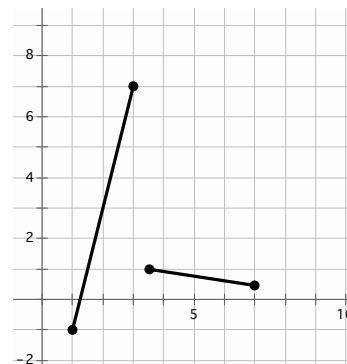
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Write the piece-wise equations for the given graphs.

18.



19.



### GO

Topic: Transformations on quadratic equations

Beginning with the parent function  $f(x) = x^2$ , write the equation of the new function  $g(x)$  that is a transformation of  $f(x)$  as described. Then graph it.

20. Shift  $f(x)$  left 3 units, stretch vertically by 2, reflect  $f(x)$  vertically, and shift down 5 units.

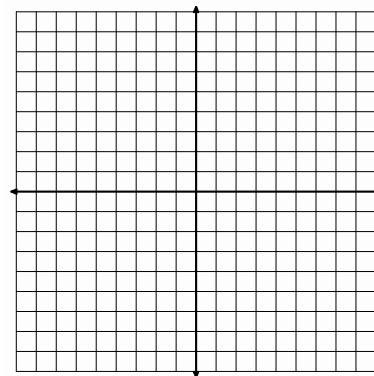
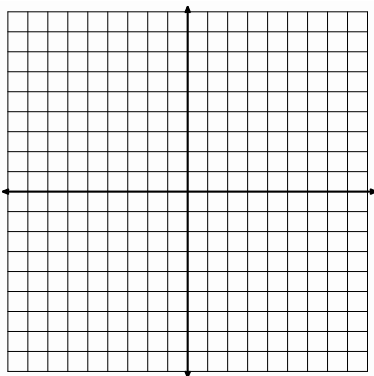
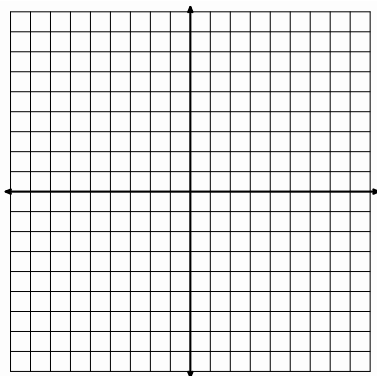
21. Shift  $f(x)$  right 1, stretch vertically by 3, and shift up 4 units.

22. Shift  $f(x)$  up 3 units, left 6, reflect vertically, and stretch by  $\frac{1}{2}$

$g(x) =$  \_\_\_\_\_

$g(x) =$  \_\_\_\_\_

$g(x) =$  \_\_\_\_\_



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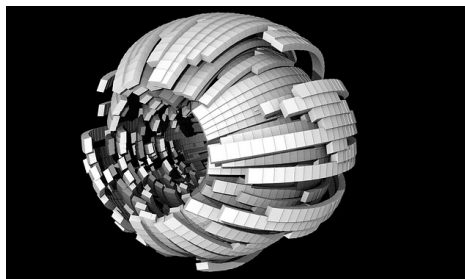
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## 8.3 More Features, More Functions

### *A Solidify Understanding Task*

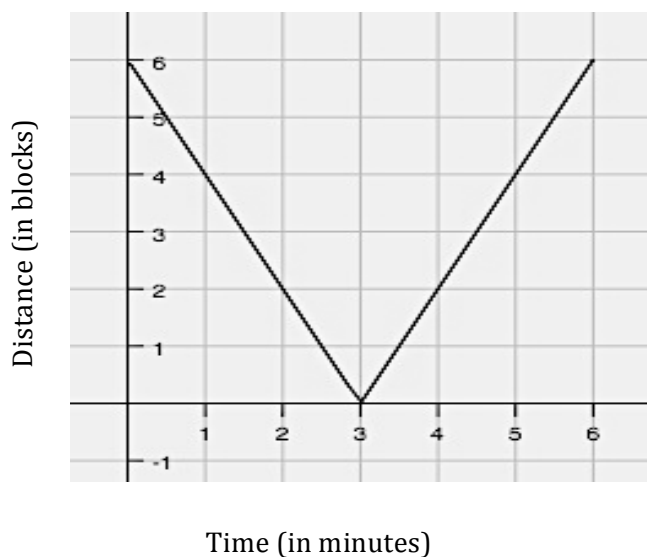


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#### Part I

Michelle likes riding her bike to and from her favorite lake on Wednesdays. She created the following graph to represent the distance she is away from the lake while biking.

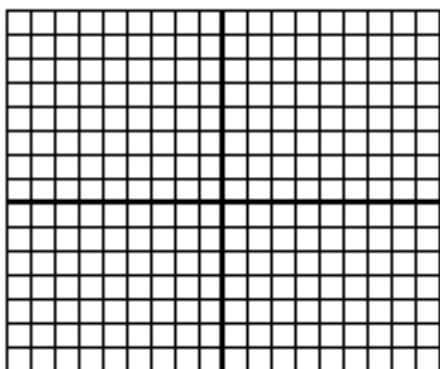


1. Interpret the graph by writing three observations about Michelle's bike ride.
2. Write a piecewise function for this situation, with each linear function being in point-slope form using the point  $(3,0)$ . What do you notice?
3. This particular piecewise function is called a linear absolute value function. What are the traits you are noticing about linear absolute value functions?

Part II

In this part of the task, you will solidify your understanding of piecewise and use your knowledge of transformations to make sense of absolute value functions. Follow the directions and answer the questions below.

4. Graph the linear function  $f(x) = x$ .



5. On the same set of axes, graph  $g(x) = |f(x)|$ .
6. Explain what happens graphically from  $f(x)$  to  $g(x)$ .
7. Write the piecewise function for  $g(x)$ . Explain your process for creating this piecewise function and how it connects to your answer in question 3.
8. Complete the table of values from  $[-4, 4]$  for  $f(x)$  and  $g(x)$ . Explain how this connects to your answer in questions 3 and 4.

$x$	$f(x)$	$g(x)$
-4		
-3		
-2		
-1		
0		
1		
2		
3		
4		

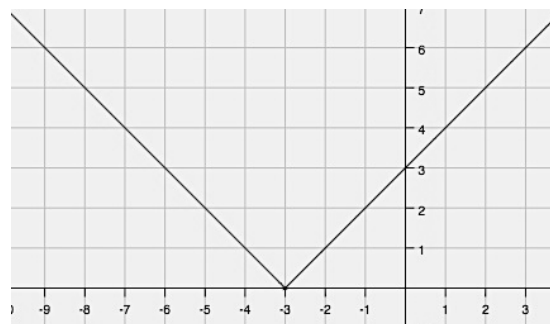
Part III

9. The graph below is another example of an absolute value function. The equation of this function can be written two ways:

as an absolute value function:  $f(x) = |x + 3|$

or as a piecewise:

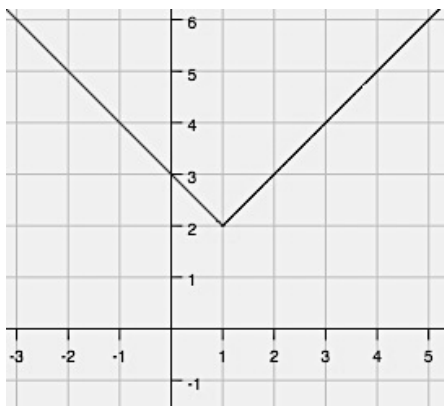
$$f(x) = \begin{cases} -(x + 3), & x < -3 \\ (x + 3), & x \geq -3 \end{cases}$$



How do these two equations relate to each other?

Below are graphs and equations of more linear absolute value functions. Write the piecewise function for each. See if you can create a strategy for writing these equations.

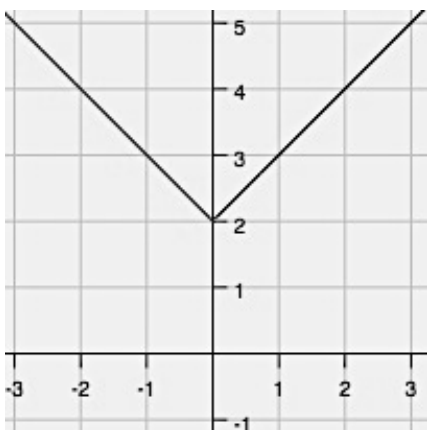
10.



Absolute value:  $f(x) = |x - 1| + 2$

Piecewise:  $f(x) =$

11.

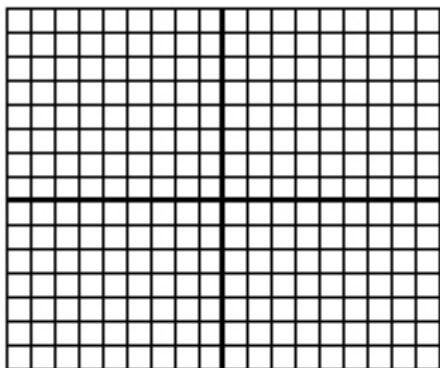


Absolute value:  $f(x) = |x| + 2$

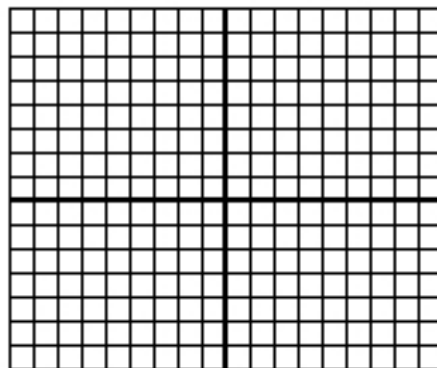
Piecewise:  $f(x) =$

Graph the following linear absolute value piecewise functions.

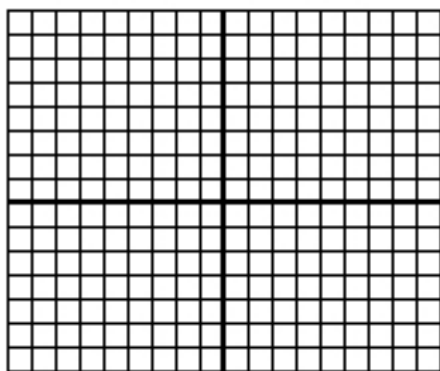
12.  $f(x) = |x - 4| = \begin{cases} -(x - 4), & x < 4 \\ (x - 4), & x \geq 4 \end{cases}$



13.  $f(x) = |x| + 1 = \begin{cases} -(x) + 1, & x < 0 \\ (x) + 1, & x \geq 0 \end{cases}$



14.



Piecewise:  $f(x) = \begin{cases} -3(x + 2) + 1, & x < -2 \\ 3(x + 2) + 1, & x \geq -2 \end{cases}$

Absolute Value:  $f(x) =$

15. Explain your method for doing the following:

- Writing piecewise linear absolute value functions from a graph.
- Writing piecewise linear absolute value functions from an absolute value function.
- Graphing absolute value functions (from either a piecewise or an absolute value equation).

## READY, SET, GO!

Name \_\_\_\_\_

Period \_\_\_\_\_

Date \_\_\_\_\_

**READY**

Topic: Finding the x-intercept(s) for a quadratic function

Find the x-intercepts of the following quadratic functions.

1.  $y = x^2 + 3x - 10$

2.  $y = x^2 + 8x + 7$

3.  $y = 6x^2 + 7x - 20$

4.  $y = (x - 2)^2 - 9$

5.  $y = -(x + 3)^2 + 9$

6.  $y = \frac{1}{2}(x - 1)^2 - 2$

**SET**

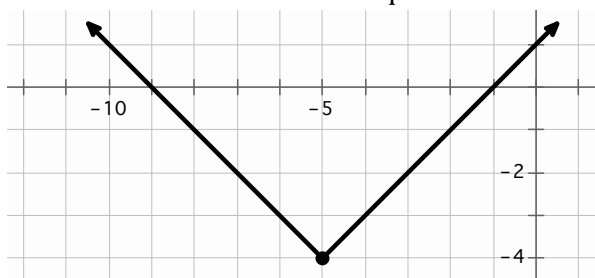
Topic: Absolute value equations

Use the given information to write the indicated form of the function.

7. Piecewise equation

$x$	$f(x)$
-1	9
0	6
1	3
2	0
3	3
4	6

8. Absolute value equation

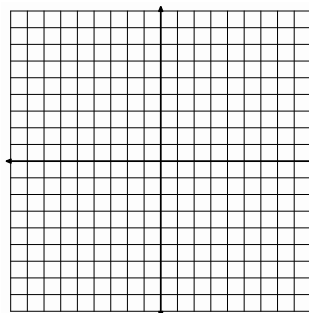


9. Make a table of values. Be sure to include the vertex in the table.

$$h(x) = 5|x - 6| - 8$$

$x$	$h(x)$

10. Graph  $f(x) = \begin{cases} -\frac{2}{3}(x - 6) + 4, & x < 6 \\ \frac{2}{3}(x - 6) + 4, & x \geq 6 \end{cases}$

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**GO**

Topic: Interpreting absolute value

**Evaluate each expression for the given value of the variable.**

11.  $-s$ ;  $s = 4$

12.  $-t$ ;  $t = -7$

13.  $-x$ ;  $x = 0$

14.  $-w$ ;  $w = -11$

15.  $|v|$ ;  $v = -25$

16.  $-(a)$ ;  $a = -25$

17.  $-(-n)$ ;  $n = -2$

18.  $-(-p)$ ;  $p = -6$

19.  $-(-q)$ ;  $q = 8$

20.  $-| -(-r) |$ ;  $r = -9$

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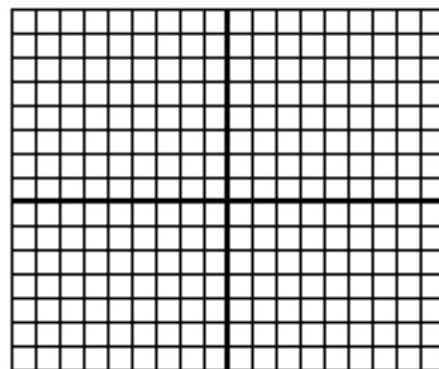
## 8.4 Reflections of a Bike Lover

### *A Practice Understanding Task*

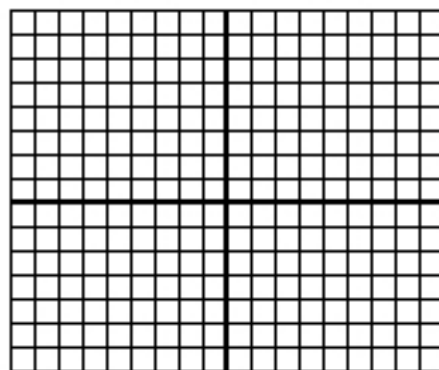


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1. Graph the function  $f(x) = x^2 - 4$
2. Graph  $g(x) = |f(x)|$  on the same set of axes as  $f(x)$ .
3. Explain what happens to the graph.
4. Write the piecewise function for  $g(x)$ .



5. Graph the function  $f(x) = (x + 1)^2 - 9$
6. Graph  $g(x) = |f(x)|$ .
7. Explain what happens to the graph.
8. Write the piecewise function for  $g(x)$ .

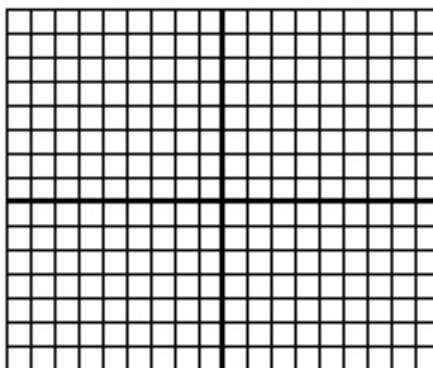


9. What do you have to think about when writing any absolute value piecewise function?

Graph the following absolute value functions and write the corresponding piecewise functions for each.

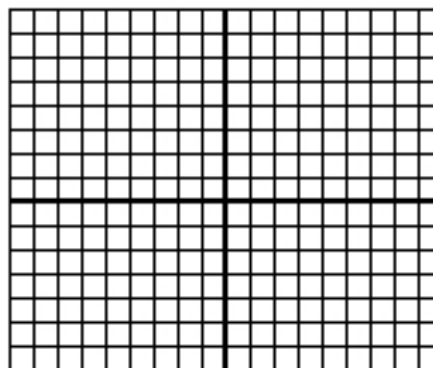
10.  $g(x) = |x^2 - 4| + 1$

Piecewise:



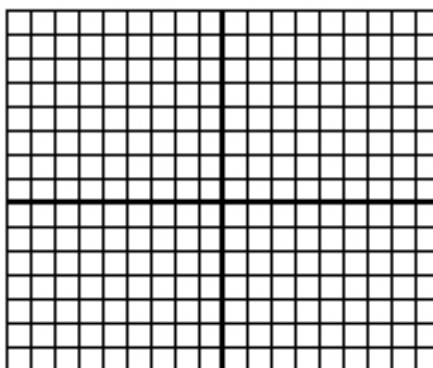
11.  $g(x) = |(x + 2)^2 - 4| + 3$

Piecewise:



12.  $g(x) = |2^x - 4|$

Piecewise:



## READY, SET, GO!

Name \_\_\_\_\_

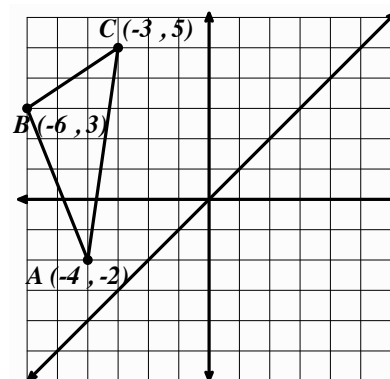
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Date \_\_\_\_\_

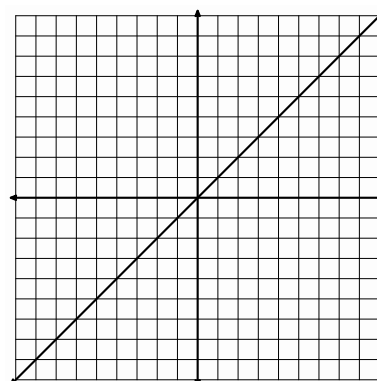
**READY**

Topic: Reflecting Images

1. Reflect  $\triangle ABC$  across the line  $y = x$ . Label the new image as  $\triangle A'B'C'$ . Label the coordinates of *points*  $A'B'C'$ . Connect segments  $AA'$ ,  $BB'$ , and  $CC'$ . Describe how these segments are related to each other and to the line  $y = x$ .



2. On the graph provided to the right, draw a 5-sided figure in the 4<sup>th</sup> quadrant. Label the vertices of the pre-image. Include the coordinates of the vertices. Reflect the pre-image across the line  $y = x$ . Label the image, including the coordinates of the vertices.



3. A table of values for a four-sided figure is given in the first two columns. Reflect the image across the line  $y = x$ , and write the coordinates of the reflected image in the space provided.

<i>A</i>	$(-6, 2)$	<i>A'</i>	
<i>B</i>	$(-4, 5)$	<i>B'</i>	
<i>C</i>	$(-2, 3)$	<i>C'</i>	
<i>D</i>	$(-3, -1)$	<i>D'</i>	

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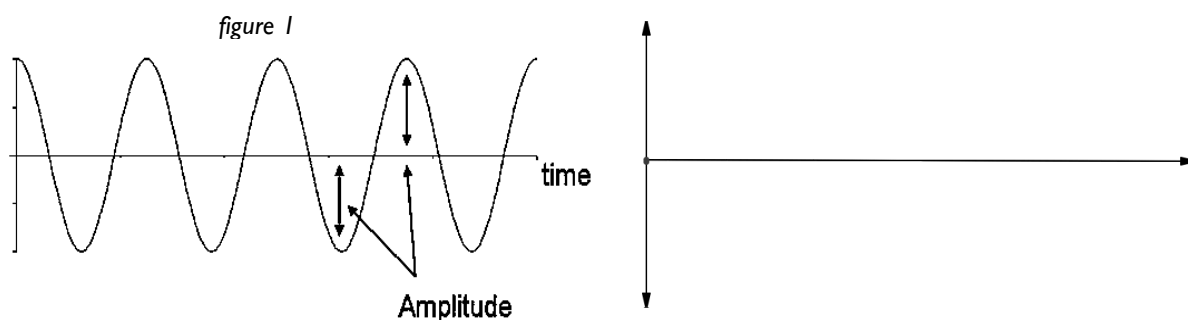
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**SET****Topic:** Absolute value and non-linear functions

4. Figure 1 is the graph of a sound wave. The height (or depth) of the graph indicates the magnitude and direction  $f(x)$  reaches from the norm or the undisturbed value. In this case that would be the x-axis.

When we are only concerned with the distance from the x-axis, we refer to this distance as the **amplitude**. Since distance alone is always positive, **amplitude** can be described as the absolute value of  $f(x)$ . Use the graph of a sound wave to sketch a graph of the absolute value of the amplitude or  $y = |f(x)|$ .



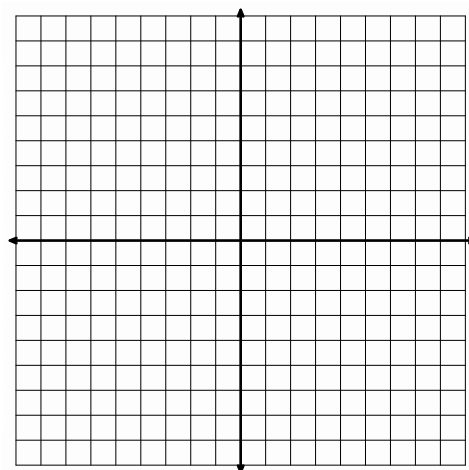
5. Figure 2 is a table of values for  $g(x) = (x + 3)^2 - 9$

What values in the table would need to change if the function were redefined as  $h(x) = |g(x)|$ ?

6. Graph  $h(x) = |g(x)|$ .

figure 2

$x$	$g(x)$
-8	16
-7	7
-6	0
-4	-5
-3	-8
-2	-9
-1	-8
0	0
1	7
2	16



7. Write the piece-wise equation for  $h(x) = |g(x)|$ , as defined in question 6. Let the domain be all real numbers in the interval  $[-8, 2]$ .

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**GO**

Topic: Simplifying Radical Expressions

**Simplify. Write the answers in simplest radical form. Some answers may consist of numbers with no radical sign.**

8.  $(-7 - 2\sqrt{5}) + (6 + 8\sqrt{5})$

9.  $(-10 - \sqrt{13}) - (-11 + 5\sqrt{13})$

10.  $(4 - \sqrt{50}) + (7 + 3\sqrt{18}) - (12 - 2\sqrt{72})$

11.  $\sqrt{98} + \sqrt{8}$

12.  $(-2 - 7\sqrt{5}) + (2\sqrt{125}) - 3\sqrt{625}$

13.  $(3r^2 - 8\sqrt{3b^2}) - (2r^2 - 3\sqrt{27b^2})$

14. Assume that  $x \geq 0$ .Simplify using rational exponents  $\sqrt{x} + \sqrt{x^3} + \sqrt{x^5} + \sqrt{x^7} + \sqrt{x^9} + \sqrt{x^{11}} + \sqrt{x^{13}} + \sqrt{x^{15}}$ .Need help? Visit [www.rsgsupport.org](http://www.rsgsupport.org)

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## 8.5 What is Your Pace?

### *A Develop Understanding Task*



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Chandler and Isaac both like to ride bikes for exercise. They were discussing whether or not they have a similar pace so that they could plan a time to bike together. Chandler said she bikes about 12 miles per hour (or 12 miles in 60 minutes). Isaac looked confused and said he does not know how many miles he bikes in an hour because he calculates his pace (or rate) differently.

1. Use multiple representations (table, graph, equation, diagram) to model Chandler's information. Be sure to label each representation.
2. Explain the connections between the representations created.
3. How many miles will Chandler travel at 30 minutes?

Isaac says he calculates his pace differently. He explains that he bikes a five minute mile, meaning that for every five minutes he bikes, he travels one mile.

4. Use multiple representations (table, graph, equation, diagram) to model Isaac's information. Be sure to label each representation.

5. How is this different than how Chandler describes her rate? Who goes at a faster rate?
6. How many miles will Isaac travel at 30 minutes?
7. Using the equations, tables, and graphs for Isaac and Chandler, make a list of observations about how these situations relate to each other.
8. The two situations in this task are considered **inverse functions** of each other. If Chandler's function is written as  $f(x)$  then we can say the inverse of this function is written as  $f^{-1}(x)$ . What do you think are characteristics of inverse functions?

**READY, SET, GO!**

Name \_\_\_\_\_

Period \_\_\_\_\_

Date \_\_\_\_\_

**READY**

Topic: Square Roots

**The area of a square is given. Find the length of the side.**

1.  $16 \text{ in}^2$



2.  $(x - 11)^2 \text{ ft}^2$



3.  $(25a^2 + 60a + 36) \text{ cm}^2$

4. If the length of the side of a square is  $(x - 24) \text{ cm}$ , what do we know about the value of  $x$ ?**Complete the table of values for  $f(x) = \sqrt{x}$ . Write answers in simplest radical form.**

5.

$x$	$f(x)$
1	
4	
9	
16	
25	
36	
49	
64	
81	
100	

6.

$x$	$f(x)$
25	
50	
75	
100	
125	
150	
175	
200	
225	
250	

7.

$x$	$f(x)$
$x^2 - 2x + 1$	
$x^2 - 4x + 4$	
$x^2 - 6x + 9$	
$x^2 - 8x + 16$	
$x^2 - 10x + 25$	
$x^2 - 12x + 36$	
$x^2 - 14x + 49$	
$x^2 - 16x + 64$	
$x^2 - 18x + 81$	
$x^2 - 20x + 100$	

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**SET**

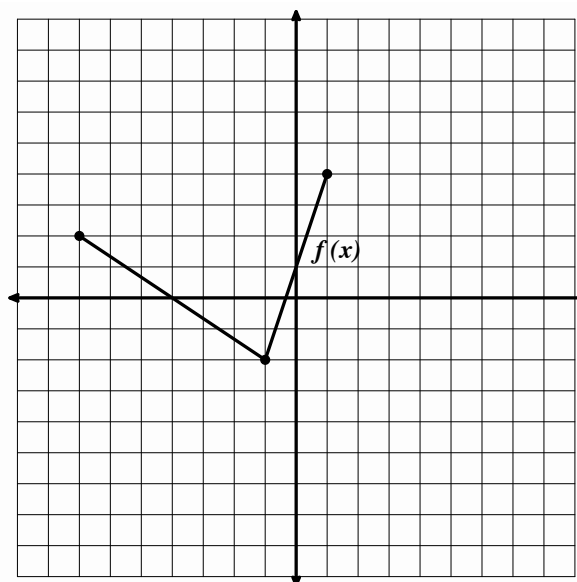
**Topic:** Inverse functions

8. **Given:**  $f(x) = \{(-13, 5)(-9, -9)(-5, -2)(-1, -5)(0, -4)(4, 6)(9, 10)(14, 32)\}$

Find  $f^{-1}(x) = \{( , )( , )( , )( , )( , )( , )( , )( , )\}$

9. The function  $f(x)$  is shown on the graph. Graph  $f^{-1}(x)$  on the same set of axes.

10. Is the graph of  $f^{-1}(x)$  also a function?  
 Justify your answer.



11. I am going on a long trip to Barcelona, Spain. I am only taking one suitcase and it is packed very full. I plan to arrive completely exhausted at my hotel in the middle of the night. The only thing I will want to take out of my suitcase is a pair of pajamas. So when I packed my suitcase at home, did I want to put my pajamas in first, somewhere in the middle, or last? Explain.

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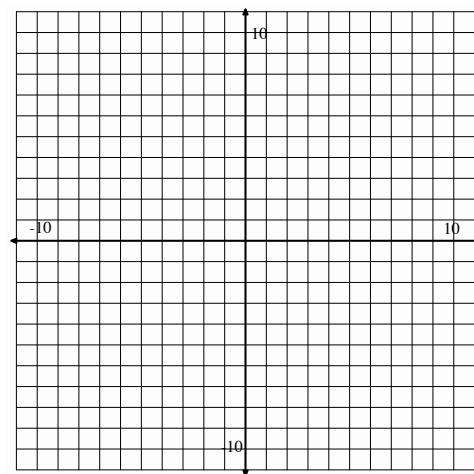
12. Write the inverse function for the table of values.

Input $x$	-10	-6	-2	2	6
Output $g(x)$	-2	-1	0	1	2

Input $x$					
Output $g^{-1}(x)$					

13. Use the points in problem 12. Graph  $g(x)$  in black and  $g^{-1}(x)$  in a different color on the coordinate grid at the right. Graph the line of reflection for the corresponding points.

14. Is  $g^{-1}(x)$  also a function? Justify your answer.



## GO

Topic: Multiplying square roots

**Multiply. Write your answers in simplest radical form.**

15.  $\sqrt{3}(4 + 5\sqrt{3})$

16.  $6\sqrt{11}(2 - \sqrt{11})$

17.  $(1 - 7\sqrt{2})(1 - \sqrt{2})$

18.  $(3 + 2\sqrt{13})(3 - 2\sqrt{13})$

19.  $(4 + 3\sqrt{5})(4 - 3\sqrt{5})$

20.  $(1 - 3\sqrt{6})(5 - 2\sqrt{6})$

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## 8.6 Bernie's Bikes

### *A Solidify Understanding Task*



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Bernie owns *Bernie's Bike Shop* and is advertising his company by taking his logo and placing it around town on different-sized signs. After creating a few signs, he noticed a relationship between the amount of ink he needs for his logo and the size of the sign.

- The table below represents some of the signs Bernie has created and the relationship between the amount of ink needed versus the size of the sign. Complete the information below to help Bernie see this relationship (don't forget to label your graph).

Length of sign (in feet)	Ink needed (in ounces)
3	9
4	16
2	4
15	225
$x$	

**Function:**

**Domain:**

**Range:**

**Graph:**

- Using question 1, complete the information below for the *inverse* of this function (don't forget to label your graph).


**Function:**

**Domain:**

**Range:**

**Graph:**

- Explain in words what the inverse function represents.

Bernie likes the look of his signs when the vertical height is 5 inches more than twice the horizontal length.

4. Complete the table and write a function for the vertical height of signs with a given horizontal length. Use Bernie's verbal rule to help you decide what numbers go in the table.

Length of sign (in feet)	Vertical height (in feet)
3	
4	
2	
15	
$x$	

**Verbal rule:** *The vertical height is 5 inches more than twice the horizontal length.*

**Function:**  $f(x) =$

**Domain:**

**Range:**

5. Create a table for the inverse function,  $f^{-1}(x)$ . Write a verbal rule that will help Bernie understand what your inverse function does. Label the table to clearly indicate what the input variable  $x$  represents for your inverse function.


**Verbal rule:**

**Function:**  $f^{-1}(x) =$

**Domain:**

**Range:**

6. Bernie's rule contained two operations, multiplying by 2 and adding 5, performed in that order. List the operations in the correct order that appear in your inverse rule:
7. In general, describe how you can find the inverse rule for a function by listing the order of operations in the original function.

Part II

Determine the inverse rule for each function, then sketch the graphs and state the domain and range for both the original function and it's inverse.

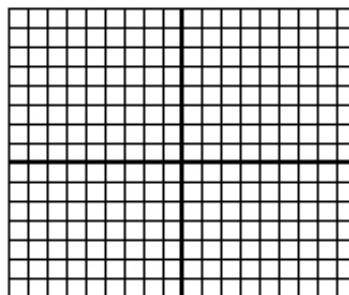
8.  $f(x) = x^2 - 1$  ;  $f^{-1}(x) =$

Domain:

Domain:

Range:

Range:



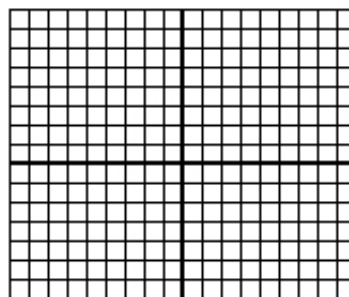
9.  $g(x) = 3x + 2$  ;  $g^{-1}(x) =$

Domain:

Domain:

Range:

Range:



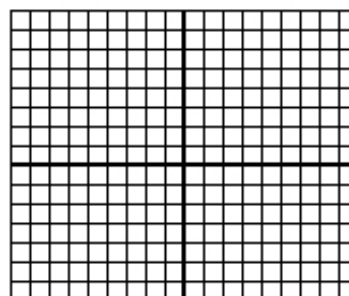
10.  $f(x) = (x + 3)^2$  ;  $f^{-1}(x) =$

Domain:

Domain:

Range:

Range:



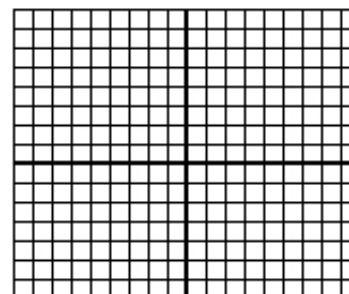
11.  $f(x) = x^3$  ;  $f^{-1}(x) =$

Domain:

Domain:

Range:

Range:



**READY, SET, GO!**

Name \_\_\_\_\_

Period \_\_\_\_\_

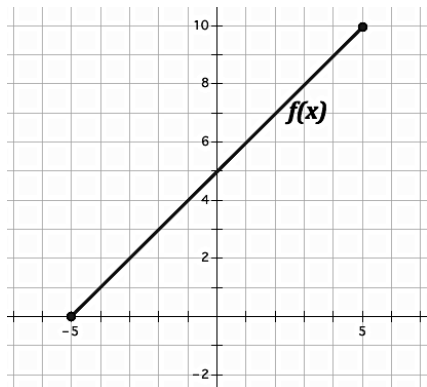
Date \_\_\_\_\_

**READY**

Topic: Identifying Features of Functions

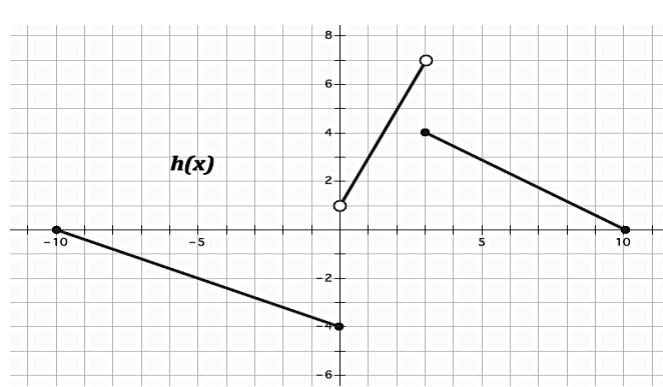
**Given each representation of a function, determine the domain and range. Then indicate whether the function is discrete, continuous, or discontinuous and increasing, decreasing, or constant.**

1.



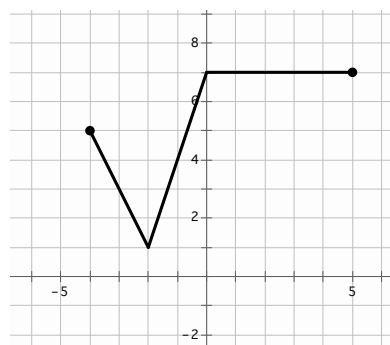
Description of Function:

2.



Description of Function:

3.



Description of Function:

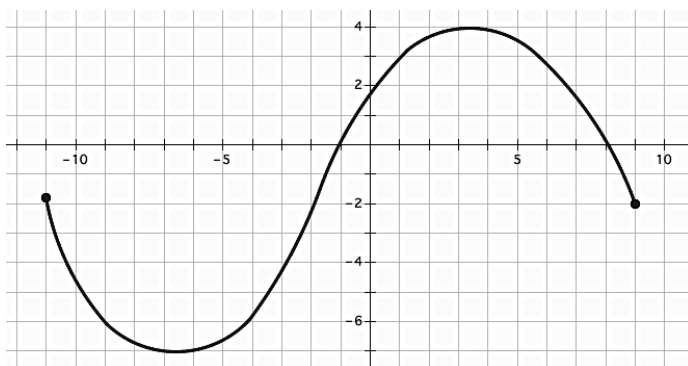
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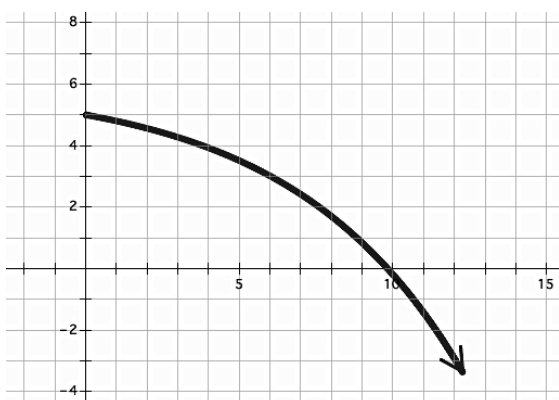
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4.



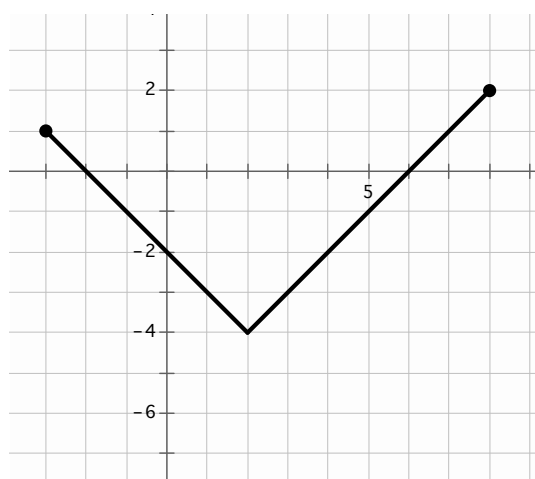
Description of Function:

5.



Description of Function:

6.



Description of Function:

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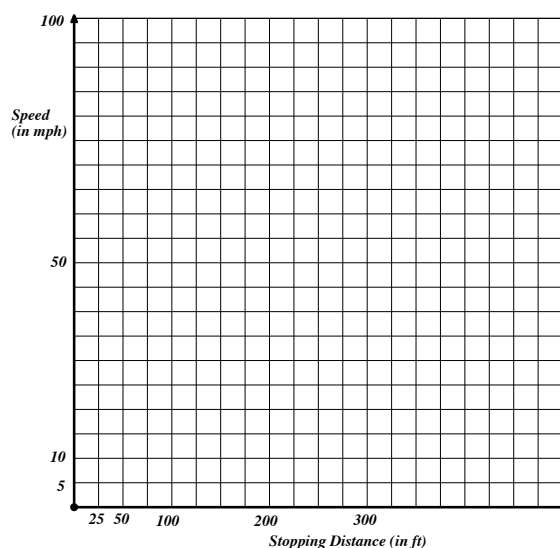
**SET****Topic:** Square root functions

The speed limit for driving in a school zone is 20mph. That seems so slow if you're riding in a car. But have you ever wondered how quickly you could come to a complete stop going that speed (even if you had super quick reflexes)? It would take you over 13 feet! The **speed of a vehicle**  $s$  and the **stopping distance**  $d$  are related by the function  $s(d) = \sqrt{30d}$ .

Fill in the table of values for  $s(d)$ . (Round to nearest whole number.) Then graph  $s(d)$  and answer the questions.

7.

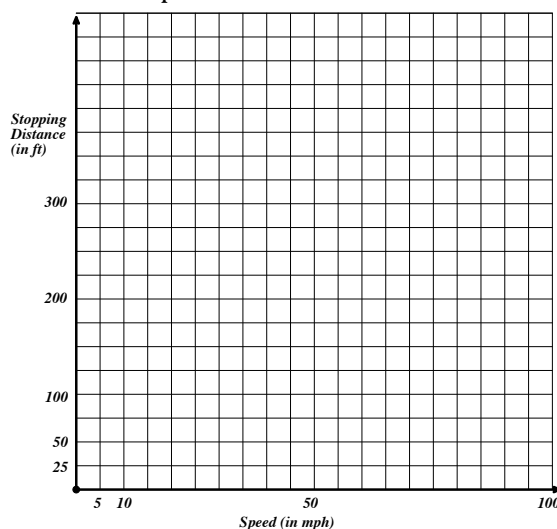
$d$ ft	$s(d)$ mph
25	
50	
100	
200	
300	



8. If you were a police officer investigating the site of an accident, you would be able to measure the length of the skid marks on the road and then approximate the speed of the driver. The driver swears he was sure he was going under 60 mph. The tire marks show a pattern for 150 feet. Is the driver's sense of his speed accurate? Justify your answer.

9. Use your answers in problem 8 to make a graph of stopping distance as a function of speed.

10. How are the two graphs related?



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**GO**

Topic: Solving literal equations for a given variable

11.  $C = 2\pi r$ ; Solve for  $r$ .

12.  $A = \pi r^2$ ; Solve for  $r$ .

13.  $V = \pi r^2 h$ ; Solve for  $h$ .

14.  $V = \pi r^2 h$ ; Solve for  $r$ .

15.  $V = e^3$ ; Solve for  $e$ .

16.  $A = \frac{b_1 + b_2}{2} h$ ; Solve for  $h$

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## 8.7 More Features, More Functions

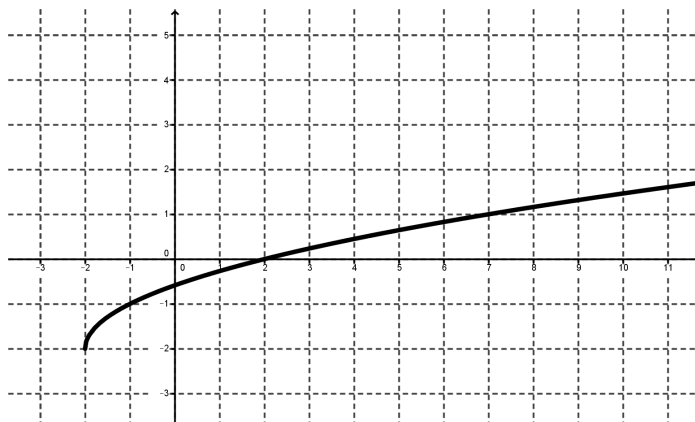
### *A Practice Understanding Task*

#### Part I: Features of Functions

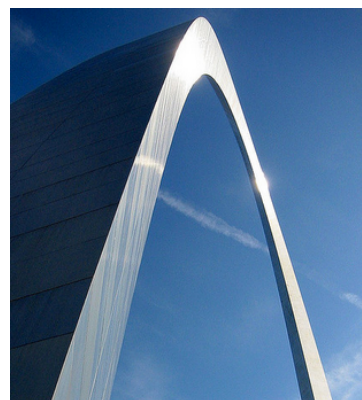
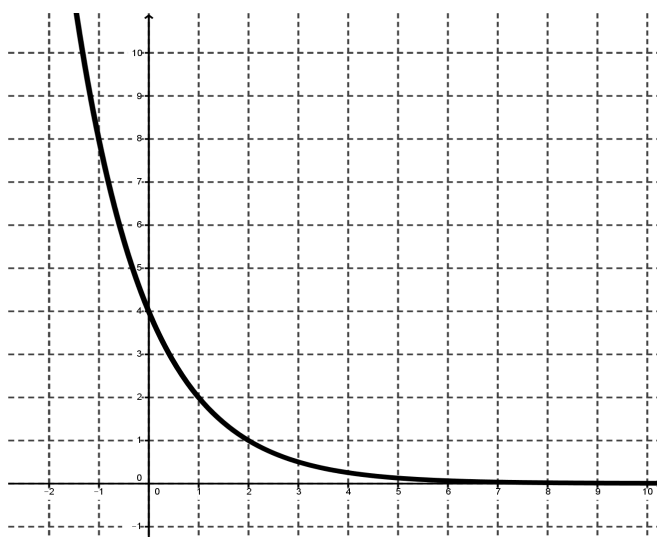
Find the following for each function (all graphs have a scale value of one on both the x-axis and y-axis)

- Equation of the function
- Domain and range
- Intercepts
- Location and value of maxima/minima
- Intervals where function is increasing or decreasing
- Sketch the inverse of the function (on a new set of axes or overlay on the given graph)

1.

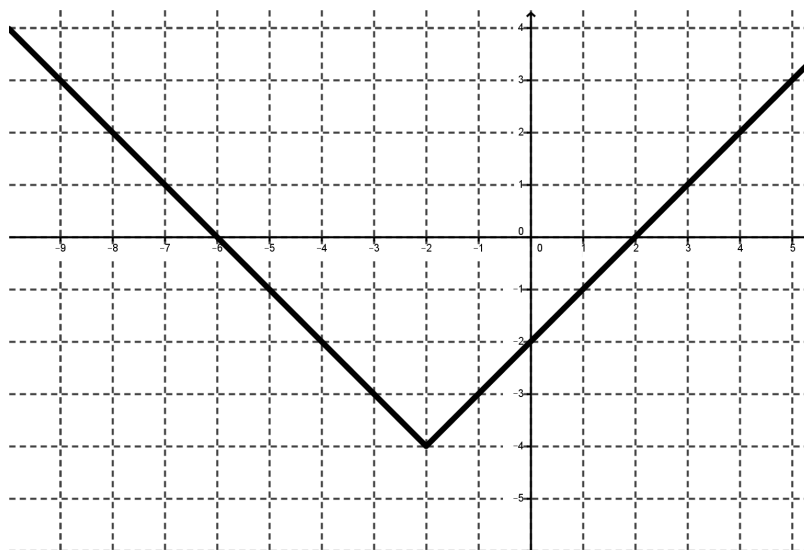


2.

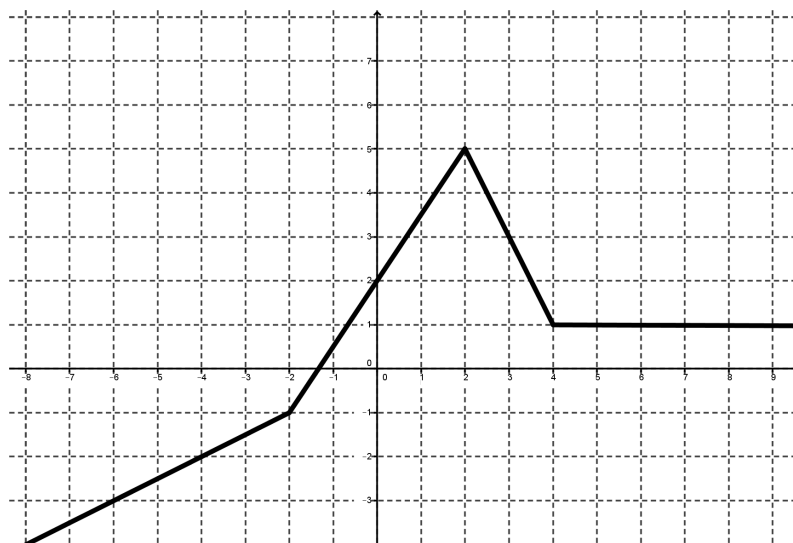


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3.



4.



## Part II: Creating Functions

**Directions:** Write **two** different functions that meet the given requirements.

5. A function that is always increasing
6. A function that is symmetrical about the  $y$ -axis

7. A function with a minimum of  $-2$  at  $x = 5$
8. A function that is decreasing from  $(-\infty, -3)$  then increasing from  $(-3, \infty)$
9. A function with zero real roots
10. A function that has a domain from  $[3, \infty)$  and a range from  $[0, \infty)$
11. A function with a constant rate of change
12. A function whose second difference is a constant rate of change
13. A function whose inverse is also a function

## READY, SET, GO!

Name \_\_\_\_\_

Period \_\_\_\_\_

Date \_\_\_\_\_

**READY**

Topic: Geometric Symbols

**Make a sketch that matches the geometric symbols. Label your sketch appropriately.**

1.  $\triangle RST$

2.  $\overleftrightarrow{AB}$

3.  $\angle XYZ$

4.  $\overleftrightarrow{GH}$

5.  $\overline{JK} \perp \overline{PQ}$

6. Point S bisects  $\overline{MN}$ .

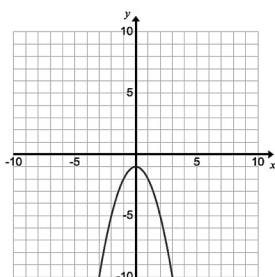
7.  $\overline{AB}$  bisects  $\angle XYZ$

**SET**

Topic: Features or Functions

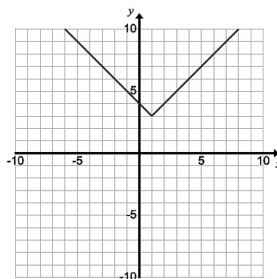
**Find the following key features for each function:**

8.



- Domain and range
- Intercepts
- Location and value of maxima/minima
- Intervals where function is increasing or decreasing

9.



- Domain and range
- Intercepts
- Location and value of maxima/minima
- Intervals where function is increasing or decreasing

10.

$$f(x) = \begin{cases} -(x + 3), & x < -3 \\ (x + 3), & x \geq -3 \end{cases}$$

- Domain and range
- Intercepts
- Location and value of maxima/minima
- Intervals where function is increasing or decreasing

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**Write a function that meet the given requirements.**

11. A function that is always decreasing
12. A function that is symmetrical about the line  $x=3$
13. A function with a minimum of 5 at  $x = 1$
14. A function that is increasing from  $(-\infty, 2)$  then decreasing from  $[2, \infty)$
15. A function with one real root
16. A function that has a domain from  $[-2, \infty)$
17. A function with a range from  $[0, \infty)$
18. A function with a common factor of 2
19. A function that is also a geometric sequence
20. A function with  $x$ -intercepts at  $(-1, 0)$  and  $(1, 0)$

**GO**

Topic: Inverse Function

**Find the inverse of each function. If the inverse is not a function, restrict the domain.**

21.  $f(x) = x^2$ ;  $f^{-1}(x) =$

22.  $g(x) = 2x + 4$ ;  $g^{-1}(x) =$

23.  $f(x) = (x + 1)^2$ ;  $f^{-1}(x) =$

24.  $h(x) = \frac{1}{3}x + 6$ ;  $h^{-1}(x) =$

25.  $f(x) = \{(-3, 5)(-2, -9)(-1, -2)(0, -5)(1, -4)(2, 6)(3, 10)(4, 8)\};$

$$f^{-1}(x) = \{( , )( , )( , )( , )( , )( , )( , )( , )\}$$

**Write the piecewise-defined function for the following absolute value functions**

26.  $h(x) = |x + 3|$

27.  $f(x) = |x^2 - 4| + 1$

28.  $g(x) = 5|x + 3|$

29.  $f(x) = |x^2 - 16|$

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