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Transforming Mathematics Education

ALGEBRA II

An Integrated Approach

MODULE 10 HONORS

Matrices Revisited

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The Mathematics Vision Project

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10.1H "Row by Row . . ."

A Solidify Understanding Task

Carlos likes to buy supplies for the twin's business, *Curbside Rivalry,* at the *All a Dollar Paint Store* where the price of every item is a multiple of $1. This makes it easy to keep track of the total cost of his purchases. Clarita is worried that items at *All a Dollar Paint Store* might cost more, so she is going over the records to see how much Carlos is paying for different supplies. Unfortunately, Carlos has once again forgotten to write down the cost of each item he purchased. Instead, he has only recorded what he purchased and the total cost of all of the items.

Carlos and Clarita are trying to figure out the cost of a gallon of paint, the cost of a paintbrush, and the cost of a roll of masking tape based on the following purchases:

Week 1: Carlos bought 2 gallons of paint and 1 roll of masking tape for $30.
Week 2: Carlos bought 1 gallon of paint and 4 brushes for $20.
Week 3: Carlos bought 2 brushes and 1 roll of masking tape for $10.

1. Determine the cost of each item using whatever strategy you want. Show the details of your work so that someone else can follow your strategy.

You probably recognized that this problem could be represented as a system of equations. In previous math courses you have developed several methods for solving systems: graphing, substitution, elimination, and row reduction of matrices.

2. Which of the methods you have developed previously for solving systems of equations could be applied to this system? Which methods seem more problematic? Why?

In the MVP Algebra I tasks *To Market with Matrices* and *Solving Systems with Matrices* you learned how to solve systems of equations involving two equations and two unknown quantities using row reduction of matrices. (You may want to review those two tasks before continuing.)

3. Modify the "row reduction of matrices" strategy so you can use it to solve Carlos and Clarita's system of three equations using row reduction. What modifications did you have to make, and why?

4. Decide on a reasonable context and write a story for the information given in the following matrix:

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 16 \\ 2 & 1 & 3 & 17 \\ 1 & 2 & 1 & 13 \end{array}\right]$$

5. Solve for the unknowns in your story by using row reduction on the given matrix. Check your results in the story context to make sure they are correct.

READY, SET, GO! Name Period Date

READY

Topic: Solving Systems by Substitution and Elimination

Solve each system of equations using an algebraic method.

1. $$\begin{aligned} x - 2y &= 6 \\ 3x - y &= 13 \end{aligned}$$

2. $$\begin{aligned} 2x - 3y &= 9 \\ x + 5y &= -28 \end{aligned}$$

3. $$\begin{aligned} x - y &= 0 \\ 4x + y &= 9 \end{aligned}$$

4. $$\begin{aligned} 2x + 3y &= 2 \\ -4x - 6y &= -14 \end{aligned}$$

5. $$\begin{aligned} 7x - y &= 14 \\ x + 7y &= -48 \end{aligned}$$

6. $$\begin{aligned} 8x + 5y &= 9 \\ -x - 5y &= 5 \end{aligned}$$

7. Do any of the systems in problems 1 - 6 represent parallel lines?
   If so, how do you know?

8. Do any of the systems in problems 1 - 6 represent perpendicular lines?
   If so, how do you know?

SET

Topic: Solving matrices using row reduction

9. Create a matrix to match each step in the solving of the system of equations given. Also, write a description of what happened to the equation and the matrix between steps.

	System of Equations	**Description**	**Matrix**
Given System	$\begin{cases} 3x+2y=40 \\ x-7y=-2 \end{cases}$		$\left[\begin{array}{cc\|c} 3 & 2 & 40 \\ 1 & -7 & -2 \end{array}\right]$
	↓	$-3R_2 \to R_2$	↓
Step 1	$\begin{cases} 3x+2y=40 \\ -3x+21y=6 \end{cases}$	↓	$\left[\begin{array}{cc\|c} & 2 & 40 \\ -3 & & 6 \end{array}\right]$
	↓	______	↓
Step 2	$\begin{cases} 3x+2y=40 \\ 0x+23y=46 \end{cases}$	↓	$\left[\begin{array}{cc\|c} & & 40 \\ 0 & & \end{array}\right]$
	↓	______	↓
Step 3	$\begin{cases} 3x+2y=40 \\ 0x+y=2 \end{cases}$	↓	$\left[\begin{array}{cc\|c} & & \\ & & \end{array}\right]$
	↓	______	↓
Step 4	$\begin{cases} 3x+0y=36 \\ 0x+y=2 \end{cases}$	↓	$\left[\begin{array}{cc\|c} & & \\ & & \end{array}\right]$
	↓	______	↓
Step 5	$\begin{cases} x+0y=12 \\ 0x+y=2 \end{cases}$		$\left[\begin{array}{cc\|c} & & \\ & & \end{array}\right]$

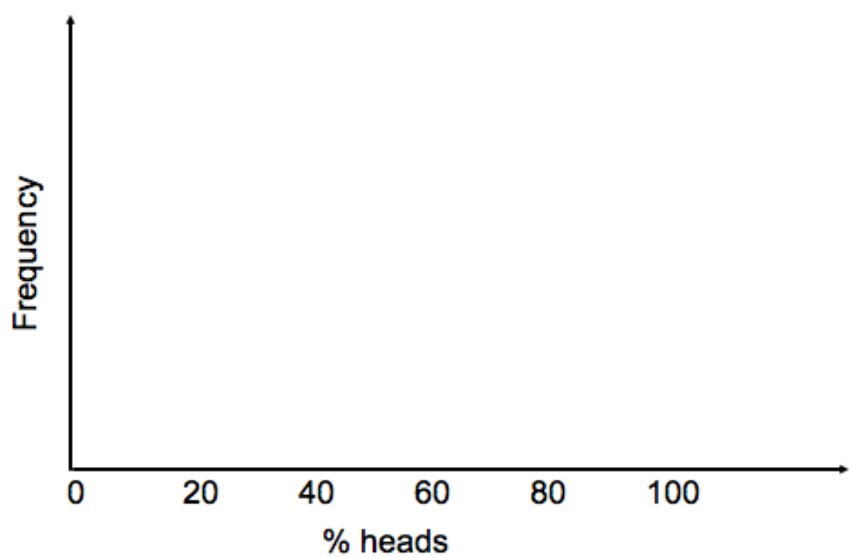
GO

Topic: Reviewing histograms

10. Flip a coin 5 times. Record the number of times the coin lands with heads up. Repeat this process 20 times, either by hand or by simulation using technology, each time recording your results in the table below. *(Every time you perform the simulation, count the number of heads you have and record the result in the Tally column below. For example, if you flip the coin 5 times and get 3 heads, put a tally mark by the 3 heads or 60% row.)*
http://www.rossmanchance.com/applets/CoinTossing/CoinToss.html

# Heads	**% Heads**	**Tally**
0	0%	
1	20%	
2	40%	
3	60%	
4	80%	
5	100%	

11. Create a histogram of your results. Describe the shape of the histogram (Shape, Center, Spread)

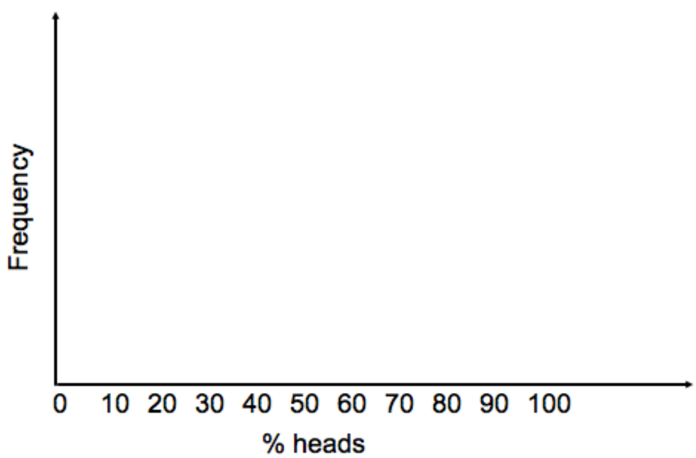


12. Flip a coin 20 times. Record the number of times heads lands side up. Repeat this process 20 times either by hand or by simulation using technology.
http://www.rossmanchance.com/applets/CoinTossing/CoinToss.html

Record your results in the table below.

# Heads	% Heads	Frequency	# Heads	% Heads	Frequency
0	0%		11	55%	
1	5%		12	60%	
2	10%		13	65%	
3	15%		14	70%	
4	20%		15	75%	
5	25%		16	80%	
6	30%		17	85%	
7	35%		18	90%	
8	40%		19	95%	
9	45%		20	100%	
10	50%				

13. Create a histogram of your results below. Describe the shape of the histogram (Shape, Center, Spread)



14. Compare the shape center and spread of each distribution. What do you notice?

15. If you repeated this process with 500 flips instead of 5 or 20, predict what would happen to the shape, spread, and center of the new histogram.

10.2H "... and Row by Column"

A Solidify Understanding Task

In the previous task you chose a context and wrote a story for the information given in the following matrix:

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 16 \\ 2 & 1 & 3 & 17 \\ 1 & 2 & 1 & 13 \end{array}\right]$$

Your story could also have been represented by the following system of equations:

$$x + 2y + 2z = 16$$
$$2x + y + 3z = 17$$
$$x + 2y + z = 13$$

Using row reduction on the matrix, you found that the solution to this system of equations is: $x = 2,\ y = 4, z = 3$. In a later task in this module you will learn another method for solving linear systems using matrices. This new method will use matrix multiplication, so let's review that operation.

We can verify that $x = 2, y = 4, z = 3$ is a solution to the system of equations given above using matrix multiplication.

1. Use the following to review how matrix multiplication works. Explain how the numbers in the matrix on the right side of the equation were obtained as the product of the two matrices on the left side of the equation.

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 16 \\ 17 \\ 13 \end{bmatrix}$$

My explanation:

2. In a context, each entry in a matrix represents two pieces of information, depending on the row and column in which it is located. Organize the following information into a matrix, and label the rows and columns.

   - Week 1: Clarita painted 6 curbside logos and Carlos painted 4 driveway mascots
   - Week 2: Clarita painted 8 curbside logos and Carlos painted 3 driveway mascots
   - Week 3: Clarita painted 5 curbside logos and Carlos painted 6 driveway mascots

3. Carlos and Clarita charge \$8 for a curbside logo and \$20 for a driveway mascot. Using this additional information, show how you can use matrix multiplication to determine how much revenue Carlos and Clarita collected during weeks 1, 2 and 3.

READY, SET, GO! Name Period Date

READY

Topic: Adding matrices

Add the given matrices. If the matrices can't be added, give a reason why not.

1. $\begin{bmatrix} 5 & -2 \\ 23 & 11 \end{bmatrix} + \begin{bmatrix} 10 & 8 \\ -4 & 19 \end{bmatrix} =$

2. $\begin{bmatrix} 99 & 55 \\ 98 & 64 \\ 78 & 12 \end{bmatrix} + \begin{bmatrix} 1 & 45 & 2 \\ 22 & 88 & 0 \end{bmatrix} =$

3. $\begin{bmatrix} 70 & 14 & 62 \\ -31 & 39 & 85 \\ 66 & 13 & -7 \end{bmatrix} + \begin{bmatrix} -70 & -14 & -62 \\ 31 & -39 & -85 \\ -66 & -13 & 7 \end{bmatrix} =$

4. $\begin{bmatrix} 6 \\ -8 \\ 10 \end{bmatrix} + \begin{bmatrix} -10 \\ 8 \\ -6 \end{bmatrix} =$

SET

Topic: Multiplying matrices

5. Recall from today's lesson that in a context, each entry in a matrix represents two pieces of information, depending on the row and column in which it is located. Organize the following information into a matrix, and label the rows and columns.

The following number of items were sold during the lunch rush at ***Fried Freddy's Café***:

- Day 1 – 65 orders of fried chicken, 62 orders of fish, and 145 orders of french fries
- Day 2 – 53 orders of fried chicken, 60 orders of fish, and 125 orders of french fries
- Day 3 – 76 orders of fried chicken, 82 orders of fish, and 198 orders of french fries
- Day 4 – 84 orders of fried chicken, 68 orders of fish, and 147 orders of french fries
- Day 5 – 91 orders of fried chicken, 88 orders of fish, and 203 orders of french fries

5. $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 15 \\ 5 \\ 10 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$

6. $\begin{bmatrix} 10 & 20 & 20 \\ 20 & 14 & 5 \\ 11 & 15 & 2 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 2 \\ 8 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$

7. $\begin{bmatrix} 4 & 3 & 0.5 \\ 2 & 1 & 3 \\ 5 & 4 & 0.25 \end{bmatrix} \cdot \begin{bmatrix} 25 \\ 30 \\ 20 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$

8. $\begin{bmatrix} 5 & 2 & 4 \\ 4 & 4 & 4 \\ 8 & 2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 15 \\ 25 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$

GO

Topic: Finding probabilities from a two-way table

The following data represents a random sample of boys and girls and how many prefer cats or dogs. Use the information to answer the questions below.

	Cats	**Dogs**	**Total**
Boys	32	68	100
Girls	41	11	52
Total	73	79	152

9. $P(B) =$

10. $P(G) =$

11. $P(C) =$

12. $P(D) =$

13. $P(C|G) =$

14. $P(C \text{ or } B) =$

15. $P(D|B) =$

16. $P(B \cap D) =$

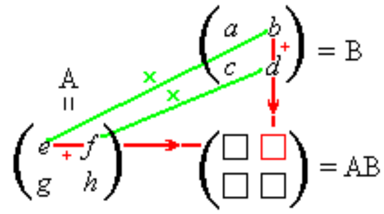
17. If this is a random sample from a school, what total percent of boys in this school do you think would prefer dogs?

18. What percent of students at the school would prefer cats?

19. If you sampled a different 152 students, would you get the same percentages? Explain.

20. What would happen to your percentages if you used a larger sample size?

10.3H More Arithmetic of Matrices



http://commons.wikimedia.org/wiki/File:Matriz_A_por_B.png

A Solidify Understanding Task

In this task you will have an opportunity to examine some of the properties of matrix addition and matrix multiplication. We will restrict this work to square 2×2 matrices.

The table below defines and illustrates several properties of addition and multiplication for real numbers and asks you to determine if these same properties hold for matrix addition and matrix multiplication. While the chart asks for a single example for each property, you should experiment with matrices until you are convinced that the property holds or you have found a counter-example to show that the property does not hold. Can you base your justification on more that just trying out several examples?

Property	**Example with Real Numbers**	**Example with Matrices**
Associative Property of Addition $(a + b) + c = a + (b + c)$		
Associative Property of Multiplication $(ab)c = a(bc)$		
Commutative Property of Addition $a + b = b + a$		

Commutative Property of Multiplication $ab = ba$		
Distributive Property of Multiplication Over Addition $a(b + c) = ab + ac$		

In addition to the properties listed in the table above, addition and multiplication of real numbers include properties related to the numbers 0 and 1. For example, the number 0 is referred to as the *additive identity* because $a + 0 = 0 + a = a$, and the number 1 is referred to as the *multiplicative identity* since $a \cdot 1 = 1 \cdot a = a$. Once the additive and multiplicative identities have been identified, we can then define additive inverses a and $-a$ since $a + -a = 0$, and multiplicative inverses a and $1/a$ since $a \cdot \frac{1}{a} = 1$. To decide if these properties hold for matrix operations, we will need to determine if there is a matrix that plays the role of 0 for matrix addition, and if there is a matrix that plays the role of 1 for matrix multiplication.

The Additive Identity Matrix

Find values for *a, b, c* and *d* so that the matrix below that contains these variables plays the role of 0, or the additive identity matrix, for the following matrix addition. Will this same matrix work as the additive identity for all 2×2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

The Multiplicative Identity Matrix

Find values for *a, b, c* and *d* so that the matrix below that contains these variables plays the role of 1, or the multiplicative identity matrix, for the following matrix multiplication. Will this same matrix work as the multiplicative identity for all 2×2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

Now that we have identified the additive identity and multiplicative identity for 2×2 matrices, we can search for the additive inverses and multiplicative inverses of matrices.

Finding an Additive Inverse Matrix

Find values for *a, b, c* and *d* so that the matrix below that contains these variables plays the role of the additive inverse of the first matrix. Will this same process work for finding the additive inverse of all 2×2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Finding a Multiplicative Inverse Matrix

Find values for *a, b, c* and *d* so that the matrix below that contains these variables plays the role of the multiplicative inverse of the first matrix. Will this same process work for finding the multiplicative inverse of all 2×2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

READY, SET, GO! Name Period Date

READY

Topic:

SET

Topic: : Finding the additive and multiplicative inverse of a matrix

2. Given: Matrix $A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$

a. Find the additive inverse of matrix A

b. Find the multiplicative inverse of matrix A

3. Given: Matrix $B = \begin{bmatrix} 4 & 2 \\ 3 & 2 \end{bmatrix}$

a. Find the additive inverse of matrix B.

b. Find the multiplicative inverse of matrix B

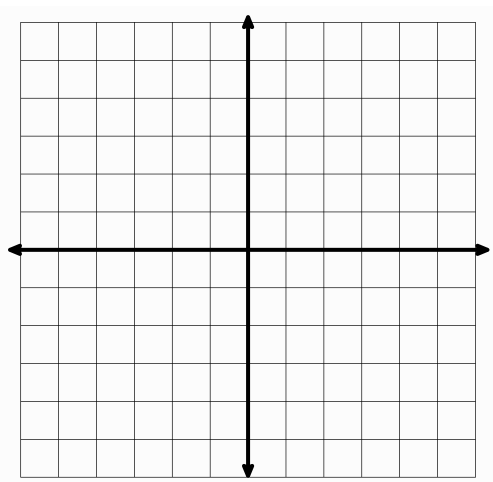
GO

Topic: Parallel lines, perpendicular lines, and length from a coordinate geometry perspective

Given the four points: *A* (2, 1), *B* (5, 2), *C* (4, 5), and *D* (1, 4)

4. Is *ABCD* a parallelogram?

Provide convincing evidence for your answer.



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5. Is $ABCD$ a rectangle?

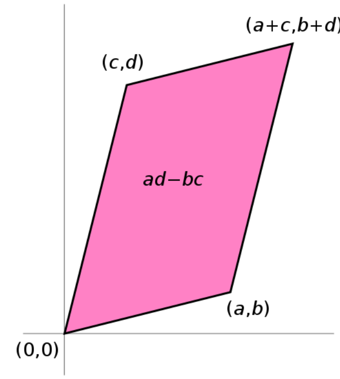
Provide convincing evidence for your answer.

6. Is $ABCD$ a rhombus? Provide convincing evidence for your answer.

7. Is $ABCD$ a square? Provide convincing evidence for your answer.

10.4H The Determinant of a Matrix

A Solidify Understanding Task



In the previous task we learned how to find the multiplicative inverse of a matrix. Use that process to find the multiplicative inverse of the following two matrices.

1. $\begin{bmatrix} 5 & 1 \\ 6 & 2 \end{bmatrix}$

2. $\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$

3. Were you able to find the multiplicative inverse for both matrices?

There is a number associated with every square matrix called the **determinant**. If the determinant is not equal to zero, then the matrix has a multiplicative inverse.

For a 2 × 2 matrix the determinant can be found using the following rule: (note: the vertical lines, rather than the square brackets, which are used to indicate that we are finding the determinant of the matrix)

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

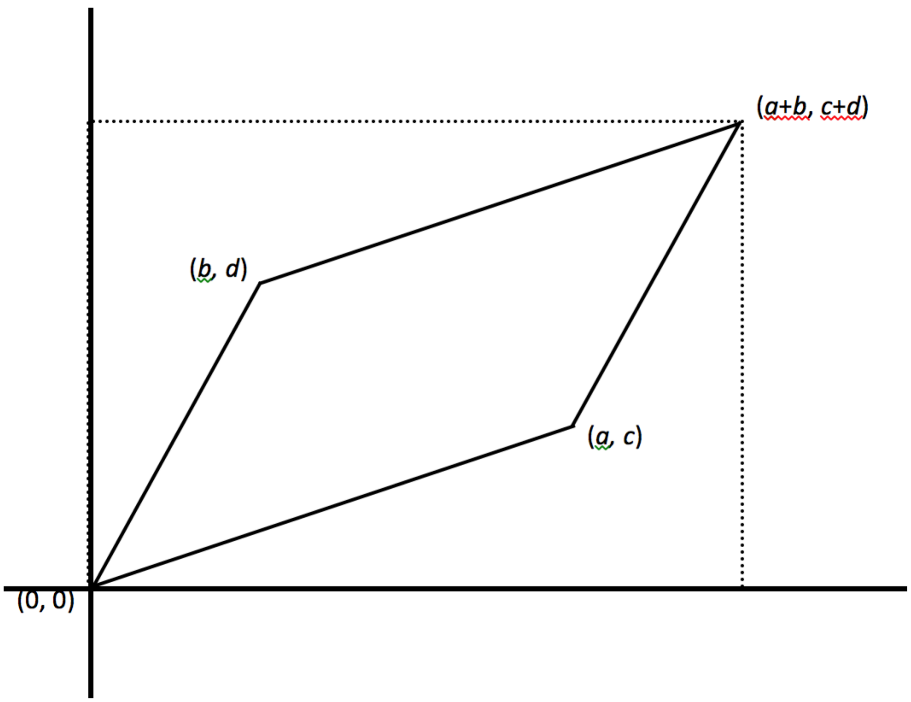
4 Using this rule, find the determinant of the two matrices given in problems 1 and 2 above.

The absolute value of the determinant of a 2×2 matrix can be visualized as the area of a parallelogram, constructed as follows:

- Draw one side of the parallelogram with endpoints at (0, 0) and (a, c).
- Draw a second side of the parallelogram with endpoints at (0, 0) and (b, d).
- Locate the fourth vertex that completes the parallelogram.

(Note that the elements in the columns of the matrix are used to define the endpoints of the vectors that form two sides of the parallelogram.)

5. Use the following diagram to show that the area of the parallelogram is given by $ad - bc$.



6. Draw the parallelograms whose areas represent the determinants of the two matrices listed in questions 1 and 2 above. How does a zero determinant show up in these diagrams?

7. Create a matrix for which the determinant will be negative. Draw the parallelogram associated with the determinant of your matrix and find the area of the parallelogram.

The determinant can be used to provide an alternative method for finding the inverse of 2×2 matrix.

8. Use the process you used previously to find the inverse of a generic 2×2 matrix whose elements are given by the variables *a, b, c* and *d.* For now, we will refer to the elements of the inverse matrix as M_1, M_2, M_3 and M_4 as illustrated in the following matrix equation. Find expressions for M_1, M_2, M_3 and M_4 in terms of the elements of the first matrix, *a, b, c* and *d*.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$M_1 =$

$M_2 =$

$M_3 =$

$M_4 =$

Use your work above to explain this strategy for finding the inverse of a 2×2 matrix: (note: the $^{-1}$ superscript is used to indicate that we are finding the multiplicative inverse of the matrix)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$ where $ad - bc$ is the determinant of the matrix

READY, SET, GO! Name Period Date

READY

Topic: Solving systems of equations using row reduction

Given the system of equations $\begin{cases} 5x - 3y = 3 \\ 2x + y = 10 \end{cases}$

1. Zac started solving this problem by writing $\begin{bmatrix} 5 & -3 & 3 \\ 2 & 1 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -5 & -17 \\ 2 & 1 & 10 \end{bmatrix}$

Describe what Zac did to get from the matrix on the left to the matrix on the right.

2. Lea started solving this problem by writing $\begin{bmatrix} 5 & -3 & 3 \\ 2 & 1 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & -3 & 3 \\ 1 & \frac{1}{2} & 5 \end{bmatrix}$

Describe what Lea did to get from the matrix on the left to the matrix on the right.

3. Using either Zac's or Lea's first step, continue solving the system using row reduction. Show each matrix along with notation indicating how you got from one matrix to another. Be sure to check your solution.

SET

Topic: Finding the determinant of a 2 X 2 matrix

4. Use the determinant of each 2×2 matrix to decide which matrices have multiplicative inverses, and which do not.

a. $\begin{bmatrix} 8 & -2 \\ 4 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$ c. $\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$

5. Find the multiplicative inverse of each of the matrices in 4, provided the inverse matrix exists.

a. b. c.

6. Generally matrix multiplication is not commutative. That is, if A and B are matrices, typically $A \cdot B \neq B \cdot A$. However, multiplication of inverse matrices is commutative. Test this out by showing that the pairs of inverse matrices you found in question 7 give the same result when multiplied in either order.

GO

Topic: Parallel and perpendicular lines

Determine if the following pairs of lines are parallel, perpendicular or neither. Explain how you arrived at your answer.

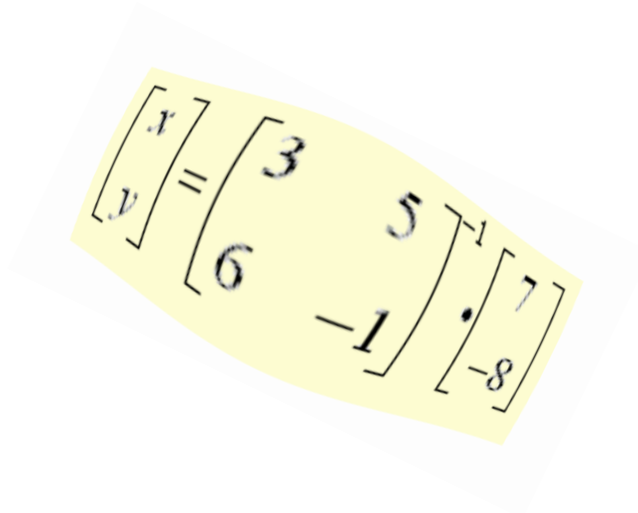
7. $3x + 2y = 7$ and $6x + 4y = 9$

8. $y = \frac{2}{3}x - 5$ and $y = -\frac{2}{3}x + 7$

9. $y = \frac{3}{4}x - 2$ and $4x + 3y = 3$

10. Write the equation of a line that is parallel to $y = \frac{4}{5}x - 2$ and has a y-intercept at $(0, 4)$.

10.5H Solving Systems with Matrices, Revisited



A Solidify Understanding Task

When you solve linear equations, you use many of the properties of operations that were revisited in the task *More Arithmetic of Matrices*.

1. Solve the following equation for x and list the properties of operations that you use during the equation solving process.

$$\frac{2}{3}x = 8$$

The list of properties you used to solve this equation probably included the use of a multiplicative inverse and the multiplicative identity property. If you didn't specifically list those properties, go back and identify where they might show up in the equation solving process for this particular equation.

Systems of linear equations can be represented with matrix equations that can be solved using the same properties that are used to solve the above equation. First, we need to recognize how a matrix equation can represent a system of linear equations.

2. Write the linear system of equations that is represented by the following matrix equation. (Think about the procedure for multiplying matrices you developed in previous tasks.)

$$\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

3. Using the relationships you noticed in question 3, write the matrix equation that represents the following system of equations.

$$\begin{cases} 2x + 3y = 14 \\ 3x + 4y = 20 \end{cases}$$

4. The rational numbers $\frac{2}{3}$ and $\frac{3}{2}$ are multiplicative inverses. What is the multiplicative inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$? Note: The inverse matrix is usually denoted by $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}^{-1}$.

5. The following table lists the steps you may have used to solve $\frac{2}{3}x = 8$ and asks you to apply those same steps to the matrix equation you wrote in question 4. Complete the table using these same steps.

Original equation	$\frac{2}{3}x = 8$	$\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ 20 \end{bmatrix}$
Multiply both sides of the equation by the multiplicative inverse	$\frac{3}{2} \cdot \frac{2}{3}x = \frac{3}{2} \cdot 8$	
The product of multiplicative inverses is the multiplicative identity on the left side of the equation	$1 \cdot x = \frac{3}{2} \cdot 8$	
Perform the indicated multiplication on the right side of the equation	$1 \cdot x = 12$	

Apply the property of the multiplicative identity on the left side of the equation	$x = 12$	

6. What does the last line in the table in question 5 tell you about the system of equations in question 3?

7. Use the process you have just examined to solve the following system of linear equations.

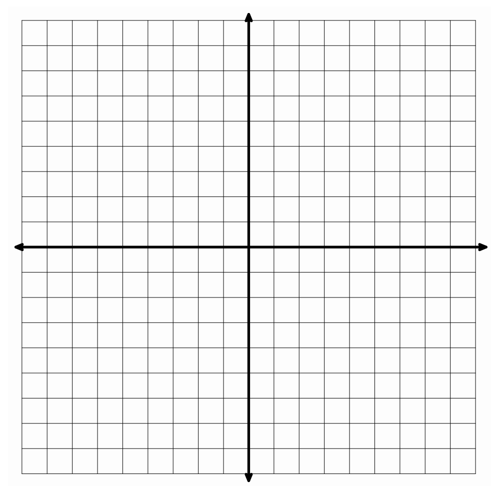
$$\begin{cases} 3x + 5y = -1 \\ 2x + 4y = 4 \end{cases}$$

READY, SET, GO! Name Period Date

READY

Topic: Reflections and Rotations

1. The following three points form the vertices of a triangle: (3, 2), (6, 1), (4, 3)



a. Plot these three points on the coordinate grid and then connect them to form a triangle.

b. Reflect the original triangle over the y-axis and record the coordinates of the vertices here:

c. Reflect the original triangle over the x-axis and record the coordinates of the vertices here:

d. Rotate the original triangle 90° counter-clockwise about the origin and record the coordinates of the vertices here:

e. Rotate the original triangle 180° about the origin and record the coordinates of the vertices here.

SET

Topic: Solving Systems Using Inverse Matrices

Two of the following systems have unique solutions (that is, the lines intersect at a single point).

2. Use the determinant of a 2×2 matrix to decide which systems have unique solutions, and which one does not.

a. $\begin{cases} 8x - 2y = -2 \\ 4x + y = 5 \end{cases}$

b. $\begin{cases} 3x + 2y = 7 \\ 6x + 4y = -5 \end{cases}$

c. $\begin{cases} 4x + 2y = 0 \\ 3x + y = 2 \end{cases}$

3. For each of the systems in #2 which have a unique solution, find the solution to the system by solving a matrix equation using an *inverse matrix*.
   a. b. c.

GO

Topic: Reviewing the Properties of Arithmetic

Match each example on the left with the name of a property of arithmetic on the right. Not all answers will be used.

____ 4. $2(x + 3y) = 2x + 6y$

____ 5. $(2x + 3y) + 4y = 2x + (3y + 4y)$

____ 6. $2x + 3y = 3y + 2x$

____ 7. $2(3y) = (2 \cdot 3)y = 6y$

____ 8. $\frac{2}{3} \cdot \frac{3}{2} x = 1x$

____ 9. $x + -x = 0$

____ 10. $xy = yx$

a. multiplicative inverses

b. additive inverses

c. multiplicative identity

d. additive identity

e. commutative property of addition

f. commutative property of multiplication

g. associative property of addition

h. associative property of multiplication

i. distributive property of addition over multiplication

10.6H All Systems Go!

A Practice Understanding Task

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This module began with the following problem:

Carlos and Clarita are trying to figure out the cost of a gallon of paint, the cost of a paintbrush, and the cost of a roll of masking tape based on the following purchases:

Week 1: Carlos bought 2 gallons of paint and 1 roll of masking tape for $30.
Week 2: Carlos bought 1 gallon of paint and 4 brushes for $20.
Week 3: Carlos bought 2 brushes and 1 roll of masking tape for $10.

In the previous sequence of tasks *More Arithmetic of Matrices*, *Solving Systems with Matrices, Revisited* and *The Determinant of a Matrix* you learned how to solve systems using multiplication of matrices. In this task, we are going to extend this strategy to include systems with more that two equations and two variables.

1. Multiply the follow pairs of matrices:

   a. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 1 \\ 1 & 4 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 0.4 & 0.2 & -0.4 \\ -0.1 & 0.2 & 0.1 \\ 0.2 & -0.4 & 0.8 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 1 \\ 1 & 4 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

2. What property is illustrated by the multiplication in question 1a?

3. What property is illustrated by the multiplication in question 1b?

4. Rewrite the following system of equations, which represents Carlos and Clarita's problem, as a matrix equation in the form **AX = B** where **A**, **X** and **B** are all matrices.

$$\begin{aligned} 2g + 0b + 1t &= 30 \\ 1g + 4b + 0t &= 20 \\ 0g + 2b + 1t &= 10 \end{aligned}$$

5. Solve your matrix equation by using multiplication of matrices. Show the details of your work so that someone else can follow it.

You were able to solve this equation using matrix multiplication because you were given the inverse of matrix **A.** Unlike 2×2 matrices, where the inverse matrix can easily be found by hand using the methods described in *More Arithmetic of Matrices*, the inverses of an $n \times n$ matrix in general can be difficult to find by hand. In such cases, we will use technology to find the inverse matrix so that this method can be applied to all linear systems involving n equations and n unknown quantities. Here is one online resource you might use: https://matrixcalc.org/en/

6. Solve the following system using a matrix equation and inverse matrices. Although you may use technology to find the inverse matrix, make sure you record all of your work in the space below, including your inverse matrix.

$$\begin{gathered} x + 2y + 2z = 16 \\ 2x + y + 3z = 17 \\ x + 2y + z = 13 \end{gathered}$$

READY, SET, GO! Name Period Date

READY

Topic: Reviewing rational exponents and methods for solving quadratics

Write each exponential expression in radical form.

1. $10^{\frac{3}{2}}$

2. $x^{\frac{1}{5}}$

3. $3n^{\frac{1}{3}}$

4. $6^{\frac{2}{7}}$

5. $7^{\frac{5}{3}}$

6. $t^{\frac{4}{5}}$

Write each radical expression in exponential form.

7. $\sqrt[5]{3}$

8. $\left(\sqrt[6]{7a}\right)^5$

9. $\sqrt{x^3}$

10. $\sqrt[3]{n^5}$

11. $\left(\sqrt[y]{n}\right)^x$

12. $\sqrt[p]{n^q}$

Explain each strategy for solving quadratic equations and explain the circumstances in which the strategy is most efficient.

13. Graphing

14. Factoring

15. Completing the square

16. What other strategies do you know for solving quadratic equations? When would you use them?

SET

Topic: Solving systems with three unknowns.

Solve the system of equations using matrices. Create a matrix equation for the system of equations that can be used to find the solution. Then find the inverse matrix and use it to solve the system.

17. $\begin{cases} 2x - 4y + z = 0 \\ 5x - 4y - 5z = 12 \\ 4x + 4y + z = 24 \end{cases}$

18. $\begin{cases} x + 2y + 5z = -15 \\ x + y - 4z = 12 \\ x - 6y + 4z = -12 \end{cases}$

19. $\begin{cases} 4p + q - 2r = 5 \\ -3p - 3q - 4r = -16 \\ 4p - 4q + 4r = -4 \end{cases}$

20. $\begin{cases} -6x - 4y + z = -20 \\ -3x - y - 3z = -8 \\ -5x + 3y + 6z = -4 \end{cases}$

GO

Topic: Solving quadratic equations

Solve each of the equations below using an appropriate and efficient method.

21. $x^2 - 5x = -6$

22. $3x^2 - 5 = 0$

23. $5x^2 - 10 = 0$

24. $x^2 + 1x - 30 = 0$

25. $x^2 + 2x = 48$

26. $x^2 - 3x = 0$