mathematics vision project

Transforming Mathematics Education

ALGEBRA II

An Integrated Approach

Standard Teacher Notes

MODULE 10 HONORS

Matrices Revisited

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

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10.1H "Row by Row . . ."

A Solidify Understanding Task

Carlos likes to buy supplies for the twin's business, *Curbside Rivalry,* at the *All a Dollar Paint Store* where the price of every item is a multiple of $1. This makes it easy to keep track of the total cost of his purchases. Clarita is worried that items at *All a Dollar Paint Store* might cost more, so she is going over the records to see how much Carlos is paying for different supplies. Unfortunately, Carlos has once again forgotten to write down the cost of each item he purchased. Instead, he has only recorded what he purchased and the total cost of all of the items.

Carlos and Clarita are trying to figure out the cost of a gallon of paint, the cost of a paintbrush, and the cost of a roll of masking tape based on the following purchases:

Week 1: Carlos bought 2 gallons of paint and 1 roll of masking tape for $30.
Week 2: Carlos bought 1 gallon of paint and 4 brushes for $20.
Week 3: Carlos bought 2 brushes and 1 roll of masking tape for $10.

1. Determine the cost of each item using whatever strategy you want. Show the details of your work so that someone else can follow your strategy.

You probably recognized that this problem could be represented as a system of equations. In previous math courses you have developed several methods for solving systems: graphing, substitution, elimination, and row reduction of matrices.

2. Which of the methods you have developed previously for solving systems of equations could be applied to this system? Which methods seem more problematic? Why?

In the MVP Algebra I tasks *To Market with Matrices* and *Solving Systems with Matrices* you learned how to solve systems of equations involving two equations and two unknown quantities using row reduction of matrices. (You may want to review those two tasks before continuing.)

3. Modify the "row reduction of matrices" strategy so you can use it to solve Carlos and Clarita's system of three equations using row reduction. What modifications did you have to make, and why?

4. Decide on a reasonable context and write a story for the information given in the following matrix:

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 16 \\ 2 & 1 & 3 & 17 \\ 1 & 2 & 1 & 13 \end{array}\right]$$

5. Solve for the unknowns in your story by using row reduction on the given matrix. Check your results in the story context to make sure they are correct.

10.1H "Row by Row" – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to extend the process of solving linear systems using row reduction of matrices to linear systems that include more than two equations and more than two unknowns. The row reduction of matrices method for solving systems of equations was introduced for the 2-equation, 2-variable case in the MVP Algebra I Honors curriculum. This task reviews the idea of representing a system of equations with an augmented matrix, then row reducing the matrix using ideas that emerged from students' work with solving systems of equations by elimination:

- Replace an equation (or a row of the matrix) with a multiple of that equation (or row); *Example:* $3R_2 \rightarrow R_2$.
- Replace an equation (or a row of the matrix) with the sum or difference of that equation (or row) with a multiple of another equation (or row); *Example:* $2R_1 + R_2 \rightarrow R_2$

Core Standards Focus:

MVP Honors Standard: Solve systems of linear equations using matrices.

Related Standards: A.REI.6

Standards for Mathematical Practice:

SMP 6 – Attend to precision

SMP 7 – Look for and make use of structure

SMP 8 – Look for and express regularity in repeated reasoning

The Teaching Cycle:

Launch (Whole Class):

Prior to beginning this task review with students, as needed, the following tasks from the MVP Algebra I curriculum:

5.11H To Market with Matrices

5.12H Solving Systems with Matrices

Explore (Small Group):

Students might solve the system in question 1 using an informal method based on the similar items in each purchase. As they move through questions 2 and 3, help students formalize their work using row reduction of matrices.

Listen for students to present in the whole class discussion who create interesting contexts to fit the matrix given in question 4, and can successfully apply the strategy of row reduction to solve this system. This matrix is more complicated than the matrix students will use in question 3, since there are not a lot of 0 coefficients. Watch for strategies to emerge that involve getting 1s and 0s in the columns in ways that will not impact coefficients in a row that already has 1s or 0s in the appropriate positions when that row and a multiple of another row are added together.

Discuss (Whole Class):

Move to the whole class discussion when students have made enough progress on question 5 that they can either describe an efficient procedure for row reducing a matrix, or they are primed and ready for some hints that will demystify the process. As needed, provide a suggestion for a next possible step in the row-reduction process, then let students continue to work of developing their strategies.

Aligned Ready, Set, Go: Matrices Revisited 10.1H

READY, SET, GO! Name Period Date

READY

Topic: Solving Systems by Substitution and Elimination

Solve each system of equations using an algebraic method.

1. $\begin{aligned} x - 2y &= 6 \\ 3x - y &= 13 \end{aligned}$

2. $\begin{aligned} 2x - 3y &= 9 \\ x + 5y &= -28 \end{aligned}$

3. $\begin{aligned} x - y &= 0 \\ 4x + y &= 9 \end{aligned}$

4. $\begin{aligned} 2x + 3y &= 2 \\ -4x - 6y &= -14 \end{aligned}$

5. $\begin{aligned} 7x - y &= 14 \\ x + 7y &= -48 \end{aligned}$

6. $\begin{aligned} 8x + 5y &= 9 \\ -x - 5y &= 5 \end{aligned}$

7. Do any of the systems in problems 1 - 6 represent parallel lines?
If so, how do you know?

8. Do any of the systems in problems 1 - 6 represent perpendicular lines?
If so, how do you know?

SET

Topic: Solving matrices using row reduction

9. Create a matrix to match each step in the solving of the system of equations given. Also, write a description of what happened to the equation and the matrix between steps.

	System of Equations	**Description**	**Matrix**
Given System	$\begin{cases} 3x+2y=40 \\ x-7y=-2 \end{cases}$		$\left[\begin{array}{cc\|c} 3 & 2 & 40 \\ 1 & -7 & -2 \end{array}\right]$
	↓	$-3R_2 \rightarrow R_2$	↓
Step 1	$\begin{cases} 3x+2y=40 \\ -3x+21y=6 \end{cases}$	↓	$\left[\begin{array}{cc\|c} & 2 & 40 \\ -3 & & 6 \end{array}\right]$
	↓	__________	↓
Step 2	$\begin{cases} 3x+2y=40 \\ 0x+23y=46 \end{cases}$	↓	$\left[\begin{array}{cc\|c} & & 40 \\ 0 & & \end{array}\right]$
	↓	__________	↓
Step 3	$\begin{cases} 3x+2y=40 \\ 0x+y=2 \end{cases}$	↓	$\left[\begin{array}{cc\|c} & & \\ & & \end{array}\right]$
	↓	__________	↓
Step 4	$\begin{cases} 3x+0y=36 \\ 0x+y=2 \end{cases}$	↓	$\left[\begin{array}{cc\|c} & & \\ & & \end{array}\right]$
	↓	__________	↓
Step 5	$\begin{cases} x+0y=12 \\ 0x+y=2 \end{cases}$		$\left[\begin{array}{cc\|c} & & \\ & & \end{array}\right]$

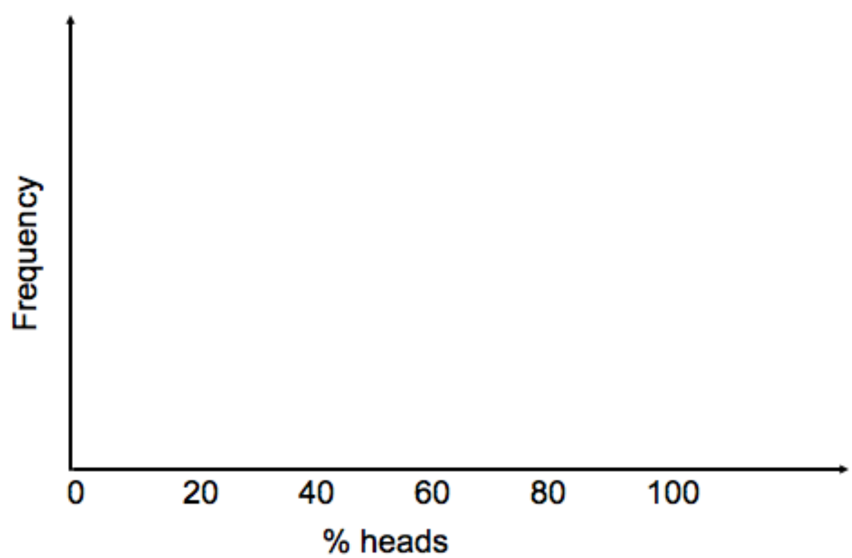
GO

Topic: Reviewing histograms

10. Flip a coin 5 times. Record the number of times the coin lands with heads up. Repeat this process 20 times, either by hand or by simulation using technology, each time recording your results in the table below. *(Every time you perform the simulation, count the number of heads you have and record the result in the Tally column below. For example, if you flip the coin 5 times and get 3 heads, put a tally mark by the 3 heads or 60% row.)*
http://www.rossmanchance.com/applets/CoinTossing/CoinToss.html

# Heads	% Heads	Tally
0	0%	
1	20%	
2	40%	
3	60%	
4	80%	
5	100%	

11. Create a histogram of your results. Describe the shape of the histogram (Shape, Center, Spread)

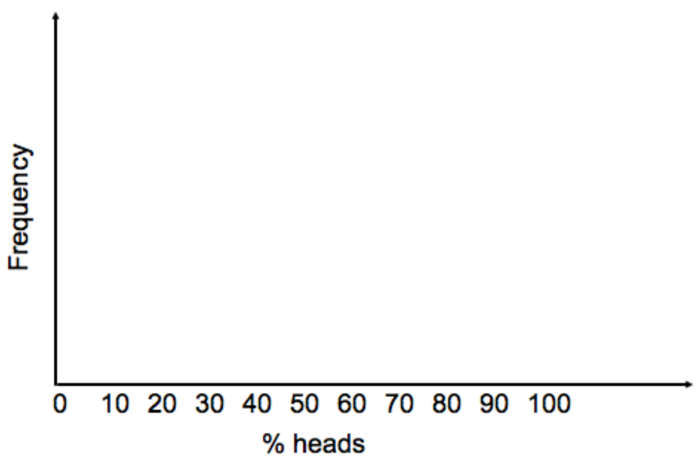


12. Flip a coin 20 times. Record the number of times heads lands side up. Repeat this process 20 times either by hand or by simulation using technology.
http://www.rossmanchance.com/applets/CoinTossing/CoinToss.html

Record your results in the table below.

# Heads	**% Heads**	**Frequency**	**# Heads**	**% Heads**	**Frequency**
0	0%		11	55%	
1	5%		12	60%	
2	10%		13	65%	
3	15%		14	70%	
4	20%		15	75%	
5	25%		16	80%	
6	30%		17	85%	
7	35%		18	90%	
8	40%		19	95%	
9	45%		20	100%	
10	50%				

13. Create a histogram of your results below. Describe the shape of the histogram (Shape, Center, Spread)



14. Compare the shape center and spread of each distribution. What do you notice?

15. If you repeated this process with 500 flips instead of 5 or 20, predict what would happen to the shape, spread, and center of the new histogram.

10.2H "… and Row by Column"

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A Solidify Understanding Task

In the previous task you chose a context and wrote a story for the information given in the following matrix:

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 16 \\ 2 & 1 & 3 & 17 \\ 1 & 2 & 1 & 13 \end{array}\right]$$

Your story could also have been represented by the following system of equations:

$$x + 2y + 2z = 16$$
$$2x + y + 3z = 17$$
$$x + 2y + z = 13$$

Using row reduction on the matrix, you found that the solution to this system of equations is: $x = 2,\ y = 4, z = 3$. In a later task in this module you will learn another method for solving linear systems using matrices. This new method will use matrix multiplication, so let's review that operation.

We can verify that $x = 2, y = 4, z = 3$ is a solution to the system of equations given above using matrix multiplication.

1. Use the following to review how matrix multiplication works. Explain how the numbers in the matrix on the right side of the equation were obtained as the product of the two matrices on the left side of the equation.

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 16 \\ 17 \\ 13 \end{bmatrix}$$

My explanation:

2. In a context, each entry in a matrix represents two pieces of information, depending on the row and column in which it is located. Organize the following information into a matrix, and label the rows and columns.

   - Week 1: Clarita painted 6 curbside logos and Carlos painted 4 driveway mascots
   - Week 2: Clarita painted 8 curbside logos and Carlos painted 3 driveway mascots
   - Week 3: Clarita painted 5 curbside logos and Carlos painted 6 driveway mascots

3. Carlos and Clarita charge \$8 for a curbside logo and \$20 for a driveway mascot. Using this additional information, show how you can use matrix multiplication to determine how much revenue Carlos and Clarita collected during weeks 1, 2 and 3.

10.2H ". . . and Row by Column" – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to review matrix multiplication, in preparation for solving systems of equations using matrix multiplication and inverse matrices, as developed in the subsequent task. Students have been introduced to matrix multiplication in the following Algebra I tasks: *4.7H Cafeteria Consumptions and Cost, 4.8H Eating Up the Lunchroom Budget*, and *4.9H Arithmetic of Matrices.*

Core Standards Focus:

N.VM.6 Use matrices to represent and manipulate data.

N.VM.8 Add, subtract, and multiply matrices of appropriate dimensions.

Related Standards: N.Q.1

Standards for Mathematical Practice:

SMP 2 – Reason Abstractly and Quantitatively

SMP 4 – Model with mathematics

SMP 7 – Look for and make use of structure

The Teaching Cycle:

Launch (Whole Class):

Prior to beginning this task review with students, as needed, the following tasks from the MVP Algebra I curriculum:

4.7H Cafeteria Consumptions and Cost

4.8H Eating Up the Lunchroom Budget

4.9H Arithmetic of Matrices

Explore (Small Group):

Verify that students can recall the row by column strategy for multiplying two matrices by listening to their conversations on question 1. This will provide formative assessment as to how much work with the previous tasks listed in the *launch* you will need to do with students.

Once students have re-familiarized themselves with the process of matrix multiplication, their work should focus on how to organize data into matrices so that the row and column multiplication procedure provides meaningful information when trying to interpret the results in the product matrix. Allow students time to grapple with this idea, perhaps by thinking about the partial multiplications that need to be summed up to find the revenue for each week. Watch for students who might want to keep track of Carlos and Clarita's work independently, since that is not the issue the twins are trying to answer. Labeling the rows and columns of each factor matrix reveals useful information for interpreting the results of the multiplication.

Discuss (Whole Class):

Move to the whole class discussion when most students have successfully set up a product of matrices that answer the question of weekly revenue collected. Have students present how they thought about organizing the two factor matrices so that the partial products found by multiplying numbers is the rows of the first matrix by the numbers in the column of the second matrix would produce meaningful results in the product matrix. If they did not label the rows and columns of each matrix, ask them to do so, and to use those labels in their explanations of their work.

Conclude the discussion by asking why it is possible to multiply matrices of different sizes, and how they can tell when matrix multiplication is possible.

Aligned Ready, Set, Go: Matrices Revisited 10.2H

READY, SET, GO! Name Period Date

READY

Topic: Adding matrices

Add the given matrices. If the matrices can't be added, give a reason why not.

1. $\begin{bmatrix} 5 & -2 \\ 23 & 11 \end{bmatrix} + \begin{bmatrix} 10 & 8 \\ -4 & 19 \end{bmatrix} =$

2. $\begin{bmatrix} 99 & 55 \\ 98 & 64 \\ 78 & 12 \end{bmatrix} + \begin{bmatrix} 1 & 45 & 2 \\ 22 & 88 & 0 \end{bmatrix} =$

3. $\begin{bmatrix} 70 & 14 & 62 \\ -31 & 39 & 85 \\ 66 & 13 & -7 \end{bmatrix} + \begin{bmatrix} -70 & -14 & -62 \\ 31 & -39 & -85 \\ -66 & -13 & 7 \end{bmatrix} =$

4. $\begin{bmatrix} 6 \\ -8 \\ 10 \end{bmatrix} + \begin{bmatrix} -10 \\ 8 \\ -6 \end{bmatrix} =$

SET

Topic: Multiplying matrices

5. Recall from today's lesson that in a context, each entry in a matrix represents two pieces of information, depending on the row and column in which it is located. Organize the following information into a matrix, and label the rows and columns.

The following number of items were sold during the lunch rush at ***Fried Freddy's Café***:

- Day 1 – 65 orders of fried chicken, 62 orders of fish, and 145 orders of french fries
- Day 2 – 53 orders of fried chicken, 60 orders of fish, and 125 orders of french fries
- Day 3 – 76 orders of fried chicken, 82 orders of fish, and 198 orders of french fries
- Day 4 – 84 orders of fried chicken, 68 orders of fish, and 147 orders of french fries
- Day 5 – 91 orders of fried chicken, 88 orders of fish, and 203 orders of french fries

5. $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 15 \\ 5 \\ 10 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$

6. $\begin{bmatrix} 10 & 20 & 20 \\ 20 & 14 & 5 \\ 11 & 15 & 2 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 2 \\ 8 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$

7. $\begin{bmatrix} 4 & 3 & 0.5 \\ 2 & 1 & 3 \\ 5 & 4 & 0.25 \end{bmatrix} \cdot \begin{bmatrix} 25 \\ 30 \\ 20 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$

8. $\begin{bmatrix} 5 & 2 & 4 \\ 4 & 4 & 4 \\ 8 & 2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 15 \\ 25 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$

GO

Topic: Finding probabilities from a two-way table

The following data represents a random sample of boys and girls and how many prefer cats or dogs. Use the information to answer the questions below.

	Cats	**Dogs**	**Total**
Boys	32	68	100
Girls	41	11	52
Total	73	79	152

9. $P(B) =$

10. $P(G) =$

11. $P(C) =$

12. $P(D) =$

13. $P(C|G) =$

14. $P(C \text{ or } B) =$

15. $P(D|B) =$

16. $P(B \cap D) =$

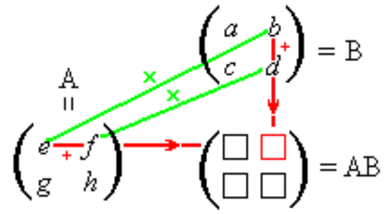
17. If this is a random sample from a school, what total percent of boys in this school do you think would prefer dogs?

18. What percent of students at the school would prefer cats?

19. If you sampled a different 152 students, would you get the same percentages? Explain.

20. What would happen to your percentages if you used a larger sample size?

10.3H More Arithmetic of Matrices



A Solidify Understanding Task

http://commons.wikimedia.org/wiki/File:Matriz_A_por_B.png

In this task you will have an opportunity to examine some of the properties of matrix addition and matrix multiplication. We will restrict this work to square 2×2 matrices.

The table below defines and illustrates several properties of addition and multiplication for real numbers and asks you to determine if these same properties hold for matrix addition and matrix multiplication. While the chart asks for a single example for each property, you should experiment with matrices until you are convinced that the property holds or you have found a counter-example to show that the property does not hold. Can you base your justification on more that just trying out several examples?

Property	**Example with Real Numbers**	**Example with Matrices**
Associative Property of Addition $(a + b) + c = a + (b + c)$		
Associative Property of Multiplication $(ab)c = a(bc)$		
Commutative Property of Addition $a + b = b + a$		

Commutative Property of Multiplication $ab = ba$		
Distributive Property of Multiplication Over Addition $a(b + c) = ab + ac$		

In addition to the properties listed in the table above, addition and multiplication of real numbers include properties related to the numbers 0 and 1. For example, the number 0 is referred to as the *additive identity* because $a + 0 = 0 + a = a$, and the number 1 is referred to as the *multiplicative identity* since $a \cdot 1 = 1 \cdot a = a$. Once the additive and multiplicative identities have been identified, we can then define additive inverses a and $-a$ since $a + -a = 0$, and multiplicative inverses a and $1/a$ since $a \cdot \frac{1}{a} = 1$. To decide if these properties hold for matrix operations, we will need to determine if there is a matrix that plays the role of 0 for matrix addition, and if there is a matrix that plays the role of 1 for matrix multiplication.

The Additive Identity Matrix

Find values for *a, b, c* and *d* so that the matrix below that contains these variables plays the role of 0, or the additive identity matrix, for the following matrix addition. Will this same matrix work as the additive identity for all 2 × 2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

The Multiplicative Identity Matrix

Find values for *a, b, c* and *d* so that the matrix below that contains these variables plays the role of 1, or the multiplicative identity matrix, for the following matrix multiplication. Will this same matrix work as the multiplicative identity for all 2×2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

Now that we have identified the additive identity and multiplicative identity for 2×2 matrices, we can search for the additive inverses and multiplicative inverses of matrices.

Finding an Additive Inverse Matrix

Find values for *a, b, c* and *d* so that the matrix below that contains these variables plays the role of the additive inverse of the first matrix. Will this same process work for finding the additive inverse of all 2×2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Finding a Multiplicative Inverse Matrix

Find values for *a, b, c* and *d* so that the matrix below that contains these variables plays the role of the multiplicative inverse of the first matrix. Will this same process work for finding the multiplicative inverse of all 2×2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

10.3H More Arithmetic of Matrices – Teacher Notes

A Solidify Understanding Task

Purpose: In previous modules students have learned how to add, subtract and multiply matrices. The purpose of this task is to examine the similarities between the properties of operations with real numbers and the properties of operations with matrices. Students will examine the associative, distributive and commutative properties to determine if they hold for matrix addition and multiplication. They will also examine the properties of additive and multiplicative identities and inverses for matrix operations. As part of this task they will develop a strategy for finding the multiplicative inverse of a square matrix. The inverse of a matrix will be used in a subsequent task to give students an alternative strategy for solving systems of linear equations.

Core Standards Focus:

N.VM.9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

N.VM.10 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers.

Related Standards: N.VM.8

Standards for Mathematical Practice:

SMP 8 – Look for and express regularity in repeated reasoning

The Teaching Cycle:

Launch (Whole Class):

In this task students will be using language like *additive identity* or *multiplicative inverse* or *the associative property of multiplication*. You might begin this task by reviewing what these properties mean for addition and multiplication of real numbers. Use the first column of the chart in the task to review and illustrate these properties. Then ask students if vector addition, as defined in the previous task has some of these same properties. That is, ask questions such as the following and allow students time to express why they think these properties hold. Accept generic diagrams of vectors with non-specific lengths as "representation-based proof" of their claims about these properties.

- Is vector addition associative?
- Is vector addition commutative?
- Is there a vector that plays the role that 0 plays in the addition of real numbers? That is, is there an additive identity for vector addition?
- Do vectors have additive inverses?
- Is scalar multiplication distributive over vector addition?

Once these properties have been reviewed in terms of operations with real numbers, and explored in terms of operations with matrices, inform students that they will be considering these properties in terms of adding and subtracting square matrices.

Explore (Small Group):
Students should experiment with square matrices of their own choosing as they explore each of the properties listed in the chart for operations with square matrices. Encourage them to find counter-examples, or to convince themselves why counter-examples may not exist. Listen for "proof-like" arguments, such as "the associative property of addition holds because we are just adding the three elements in the same position of the three matrices which is the same as the associative property for three real numbers", rather than justification based solely on trying out specific cases.

The remainder of the task asks students to find the elements of identity and inverse matrices. This work should be fairly easy based on what students already know about matrix addition and multiplication, with the exception of finding the multiplicative inverse of a matrix. All other cases can be resolved by guess and check. In the case of the multiplicative inverse, suggest that students multiply out the matrices on the left side and set the resulting expressions equal to the appropriate elements of the matrix on the right side of the equation. This should lead to the following four equations:

$3a + c = 1$

$3b + d = 0$

$4a + 2c = 0$

$4b + 2d = 1$

Students can then solve for *a, b, c* and *d* by pairing equations that contain the same two variables into systems of equations.

Discuss (Whole Class):

Have students share their justifications that matrix addition and matrix multiplication share the same properties of operations as the corresponding operations with real numbers, with the exception of the commutative property of multiplication. Have students present a counter-example for this exception. Make sure students can find the multiplicative inverse of a matrix by setting up the matrix equation with variables as the elements of the inverse matrix, turning the matrix equation into two systems of linear equations, and solving for the variables that represent the elements of the inverse matrix. If technology is available, you might show students how to obtain the multiplicative inverse matrix using technology. We will use the multiplicative inverse matrix in a later task to provide an alternative way for solving systems using matrices.

Aligned Ready, Set, Go: Matrices Revisited 10.3H

READY, SET, GO! Name Period Date

READY

Topic:

SET

Topic: : Finding the additive and multiplicative inverse of a matrix

2. Given: Matrix $A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$

a. Find the additive inverse of matrix *A*

b. Find the multiplicative inverse of matrix *A*

3. Given: Matrix $B = \begin{bmatrix} 4 & 2 \\ 3 & 2 \end{bmatrix}$

a. Find the additive inverse of matrix *B*.

b. Find the multiplicative inverse of matrix *B*

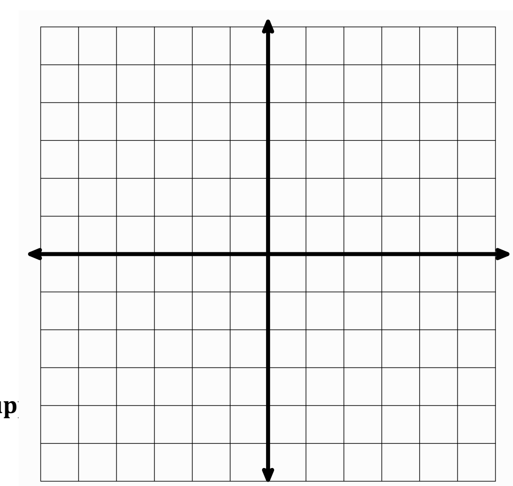
GO

Topic: Parallel lines, perpendicular lines, and length from a coordinate geometry perspective

Given the four points: *A* (2, 1), *B* (5, 2), *C* (4, 5), and *D* (1, 4)

4. Is *ABCD* a parallelogram?

Provide convincing evidence for your answer.



5. Is $ABCD$ a rectangle?

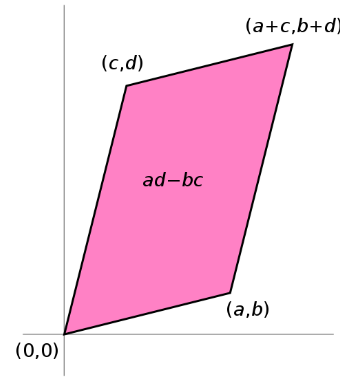
Provide convincing evidence for your answer.

6. Is $ABCD$ a rhombus? Provide convincing evidence for your answer.

7. Is $ABCD$ a square? Provide convincing evidence for your answer.

10.4H The Determinant of a Matrix

A Solidify Understanding Task



In the previous task we learned how to find the multiplicative inverse of a matrix. Use that process to find the multiplicative inverse of the following two matrices.

1. $\begin{bmatrix} 5 & 1 \\ 6 & 2 \end{bmatrix}$

2. $\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$

3. Were you able to find the multiplicative inverse for both matrices?

There is a number associated with every square matrix called the **determinant**. If the determinant is not equal to zero, then the matrix has a multiplicative inverse.

For a 2×2 matrix the determinant can be found using the following rule: (note: the vertical lines, rather than the square brackets, which are used to indicate that we are finding the determinant of the matrix)

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

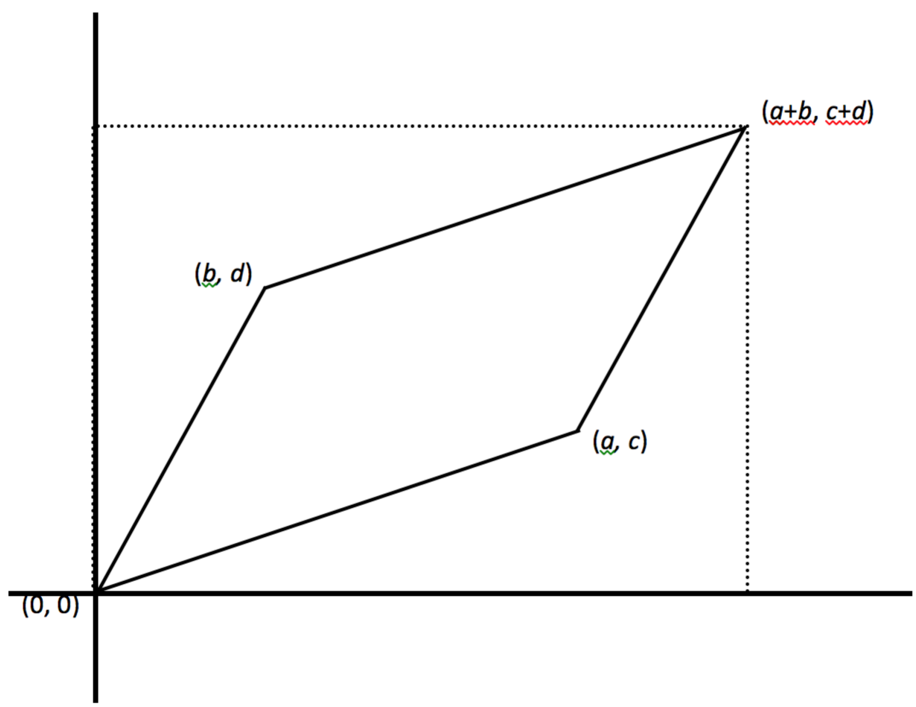
4 Using this rule, find the determinant of the two matrices given in problems 1 and 2 above.

The absolute value of the determinant of a 2×2 matrix can be visualized as the area of a parallelogram, constructed as follows:

- Draw one side of the parallelogram with endpoints at $(0, 0)$ and (a, c).
- Draw a second side of the parallelogram with endpoints at $(0, 0)$ and (b, d).
- Locate the fourth vertex that completes the parallelogram.

(Note that the elements in the columns of the matrix are used to define the endpoints of the vectors that form two sides of the parallelogram.)

5. Use the following diagram to show that the area of the parallelogram is given by $ad - bc$.



6. Draw the parallelograms whose areas represent the determinants of the two matrices listed in questions 1 and 2 above. How does a zero determinant show up in these diagrams?

7. Create a matrix for which the determinant will be negative. Draw the parallelogram associated with the determinant of your matrix and find the area of the parallelogram.

The determinant can be used to provide an alternative method for finding the inverse of 2×2 matrix.

8. Use the process you used previously to find the inverse of a generic 2×2 matrix whose elements are given by the variables *a, b, c* and *d.* For now, we will refer to the elements of the inverse matrix as M_1, M_2, M_3 and M_4 as illustrated in the following matrix equation. Find expressions for M_1, M_2, M_3 and M_4 in terms of the elements of the first matrix, *a, b, c* and *d*.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$M_1 =$

$M_2 =$

$M_3 =$

$M_4 =$

Use your work above to explain this strategy for finding the inverse of a 2×2 matrix: (note: the $^{-1}$ superscript is used to indicate that we are finding the multiplicative inverse of the matrix)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$ where $ad - bc$ is the determinant of the matrix

10.4H The Determinant of a Matrix – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to introduce the determinant of a square matrix, and to connect the determinant of a 2×2 matrix to the area of a parallelogram. The "area of a parallelogram" representation will be used to illustrate why some square matrices have a determinant of 0. Students will observe that if the determinant of a matrix is equal to zero, the matrix will not have a multiplicative inverse. In the next task, *Solving Systems with Matrices, Revisited*, non-zero determinants will be used as an indicator that a system of linear equations has a unique solution.

In the last part of the task students are introduced to an alternative way for finding the inverse of a 2×2 matrix using the determinant. This will support the work of the next task where students will solve systems of linear equations using inverse matrices. Once students have a clear understanding of what an inverse matrix means and have found a few inverses of 2×2 matrices using both of the methods of this task, you may also want to introduce them to the use of technology to find inverse matrices. (A third algebraic method for finding the inverse of a matrix by using row reduction is not introduced in this sequence of tasks, but could be explored with more advanced students.)

Core Standards Focus:

N.VM.10 Understand that the determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

N.VM.12 Work with 2×2 matrices and interpret the absolute value of the determinant in terms of area.

Standards for Mathematical Practice:

SMP 7 – Look for and make use of structure

The Teaching Cycle:

Launch (Whole Class):

Give students a few minutes individually to work on finding the inverse of the matrices in questions 1 and 2 using the strategy developed in the previous task. That is, students should look for the values of *a, b, c* and *d* that make the following matrix equations true.

For question 1: $\begin{bmatrix} 5 & 1 \\ 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Students can multiply the two matrices on the left side of this equation and set the resulting expressions equal to the corresponding elements in the matrix on the right side of the equation. This will lead to the following two systems that can be solved for *a, b, c,* and *d*.

$$\begin{cases} 5a + c = 1 \\ 6a + 2c = 0 \end{cases} \text{ and } \begin{cases} 5b + d = 0 \\ 6b + 2d = 1 \end{cases}.$$

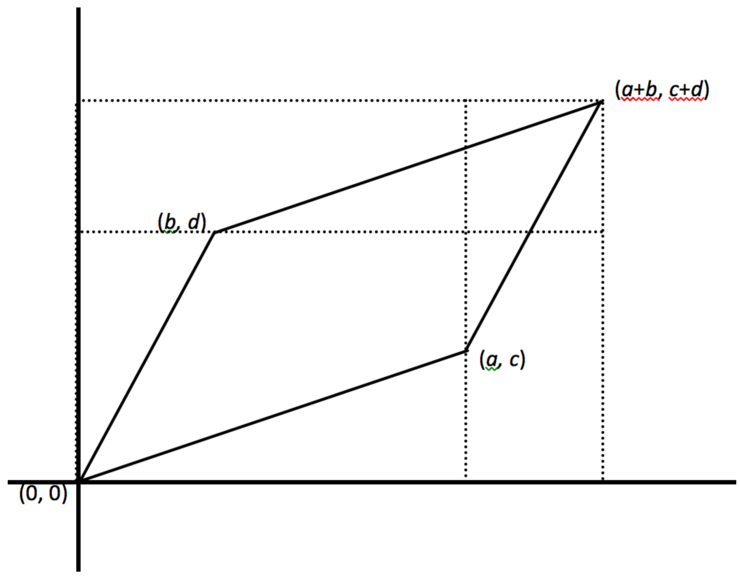
For question 2: $\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

As students examine the related systems, such as $\begin{cases} 6a + 2b = 1 \\ 3a + b = 0 \end{cases}$, they will find that the systems have no solutions and therefore, the given matrix does not have a multiplicative inverse. Introduce the determinant as a number that will tell us if a square matrix has a multiplicative inverse before going through the work of trying to calculate what the inverse matrix is, and have students apply the notation given prior to question 4 to find the determinant of the two given matrices. Then assign students to work on the remainder of the task.

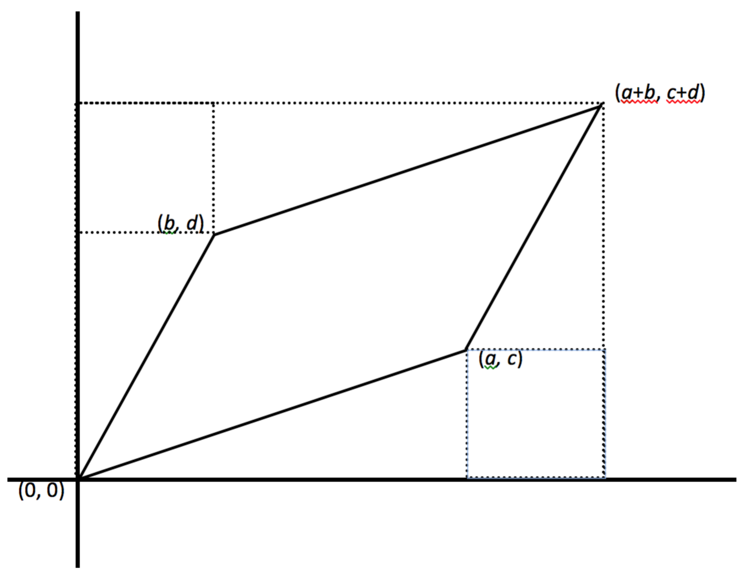
Explore (Small Group):

Associating the determinant of a 2 × 2 matrix with the area of a parallelogram will help students see what happens when the determinant of a matrix is zero. This occurs when the parallelogram degenerates into two overlapping line segments. In this drawing of the parallelogram, the columns of the matrix are treated as defining two vectors that form two sides of a parallelogram—the same parallelogram formed in the task *The Arithmetic of Vectors* to add vectors using the parallelogram rule. To show that the area of this parallelogram is given by $ad - bc$, students may choose to find the area of the surrounding rectangle, and then subtract the area of the extra dart-shaped pieces, leaving the area of the parallelogram.

The area of the surrounding rectangle is given by $(a + b)(c + d)$. Since students may not know how to multiply these two binomials together, they may need to decompose the larger rectangle into smaller rectangles, as shown in the following diagram.



The area of the extra pieces that need to be subtracted from the surrounding rectangle can be found by decomposing the extra dart-shaped pieces into rectangles and triangles, as in the following diagram.



In question 8 students develop an alternative method for finding the inverse of a 2×2 matrix, if the inverse exists. Students will first set up two systems of equations to solve for M_1, M_2, M_3 and M_4. Each of these values will consist of a fraction whose denominator can be represented by $ad - bc$, the value of the determinant. Consequently, the value $\frac{1}{ad - bc}$ can be factored out of the matrix as a scalar multiple, leading to the rule given in the task for the inverse matrix.

Discuss (Whole Class):

Have selected students present their work showing that the area of the parallelogram is given by the value of the determinant $ad - bc$. In the cases where the determinant is negative, the area of the parallelogram is $|ad - bc|$. This will come out as students share their work on question 7, and the issue with a 0 determinant will come out of the work on question 6. Students may need assistance with the algebra in question 8. If so, work through this derivation as a whole class, rather than with each individual group. The derivation starts by forming the following two systems:

$$\begin{cases} aM_1 + bM_3 = 1 \\ cM_1 + bM_3 = 0 \end{cases} \quad \text{and} \quad \begin{cases} aM_2 + bM_4 = 0 \\ cM_2 + dM_4 = 1 \end{cases}$$

Solving these systems for M_1, M_2, M_3 and M_4 leads to:

$$M_1 = \frac{d}{ad - bc}$$

$$M_2 = \frac{-b}{ad - bc}$$

$$M_3 = \frac{-c}{ad - bc}$$

$$M_4 = \frac{a}{ad - bc}$$

Therefore, the inverse matrix is $\begin{bmatrix} \dfrac{d}{ad-bc} & \dfrac{-b}{ad-bc} \\ \dfrac{-c}{ad-bc} & \dfrac{a}{ad-bc} \end{bmatrix}$ or, after factoring out the scalar factor $\dfrac{1}{ad-bc}$, the inverse matrix is given by $\dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Aligned Ready, Set, Go: Matrices Revisited 10.4H

READY, SET, GO! Name Period Date

READY

Topic: Solving systems of equations using row reduction

Given the system of equations $\begin{cases} 5x - 3y = 3 \\ 2x + y = 10 \end{cases}$

1. Zac started solving this problem by writing $\begin{bmatrix} 5 & -3 & 3 \\ 2 & 1 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -5 & -17 \\ 2 & 1 & 10 \end{bmatrix}$

Describe what Zac did to get from the matrix on the left to the matrix on the right.

2. Lea started solving this problem by writing $\begin{bmatrix} 5 & -3 & 3 \\ 2 & 1 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & -3 & 3 \\ 1 & \frac{1}{2} & 5 \end{bmatrix}$

Describe what Lea did to get from the matrix on the left to the matrix on the right.

3. Using either Zac's or Lea's first step, continue solving the system using row reduction. Show each matrix along with notation indicating how you got from one matrix to another. Be sure to check your solution.

SET

Topic: Finding the determinant of a 2 X 2 matrix

4. Use the determinant of each 2×2 matrix to decide which matrices have multiplicative inverses, and which do not.

   a. $\begin{bmatrix} 8 & -2 \\ 4 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$ c. $\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$

5. Find the multiplicative inverse of each of the matrices in 4, provided the inverse matrix exists.

   a. b. c.

6. Generally matrix multiplication is not commutative. That is, if A and B are matrices, typically $A \cdot B \neq B \cdot A$. However, multiplication of inverse matrices is commutative. Test this out by showing that the pairs of inverse matrices you found in question 7 give the same result when multiplied in either order.

GO

Topic: Parallel and perpendicular lines

Determine if the following pairs of lines are parallel, perpendicular or neither. Explain how you arrived at your answer.

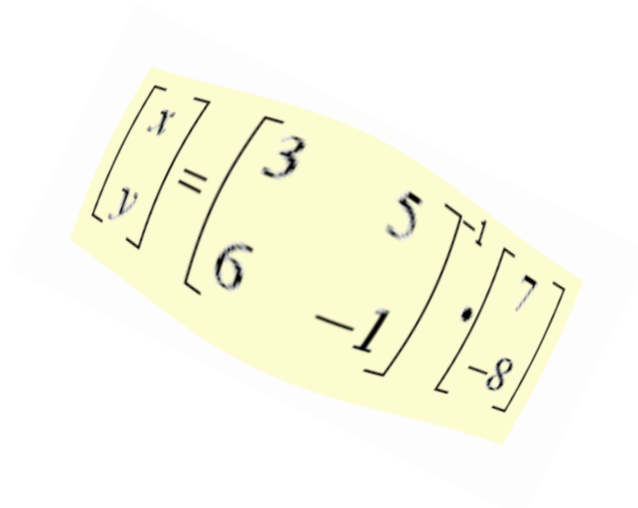
7. $3x + 2y = 7$ and $6x + 4y = 9$

8. $y = \frac{2}{3}x - 5$ and $y = -\frac{2}{3}x + 7$

9. $y = \frac{3}{4}x - 2$ and $4x + 3y = 3$

10. Write the equation of a line that is parallel to $y = \frac{4}{5}x - 2$ and has a y-intercept at (0, 4).

10.5H Solving Systems with Matrices, Revisited



A Solidify Understanding Task

When you solve linear equations, you use many of the properties of operations that were revisited in the task *More Arithmetic of Matrices*.

1. Solve the following equation for x and list the properties of operations that you use during the equation solving process.

$$\frac{2}{3}x = 8$$

The list of properties you used to solve this equation probably included the use of a multiplicative inverse and the multiplicative identity property. If you didn't specifically list those properties, go back and identify where they might show up in the equation solving process for this particular equation.

Systems of linear equations can be represented with matrix equations that can be solved using the same properties that are used to solve the above equation. First, we need to recognize how a matrix equation can represent a system of linear equations.

2. Write the linear system of equations that is represented by the following matrix equation. (Think about the procedure for multiplying matrices you developed in previous tasks.)

$$\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

3. Using the relationships you noticed in question 3, write the matrix equation that represents the following system of equations.

$$\begin{cases} 2x + 3y = 14 \\ 3x + 4y = 20 \end{cases}$$

4. The rational numbers $\frac{2}{3}$ and $\frac{3}{2}$ are multiplicative inverses. What is the multiplicative inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$? Note: The inverse matrix is usually denoted by $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}^{-1}$.

5. The following table lists the steps you may have used to solve $\frac{2}{3}x = 8$ and asks you to apply those same steps to the matrix equation you wrote in question 4. Complete the table using these same steps.

Original equation	$\frac{2}{3}x = 8$	$\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ 20 \end{bmatrix}$
Multiply both sides of the equation by the multiplicative inverse	$\frac{3}{2} \cdot \frac{2}{3}x = \frac{3}{2} \cdot 8$	
The product of multiplicative inverses is the multiplicative identity on the left side of the equation	$1 \cdot x = \frac{3}{2} \cdot 8$	
Perform the indicated multiplication on the right side of the equation	$1 \cdot x = 12$	

Apply the property of the multiplicative identity on the left side of the equation	$x = 12$	

6. What does the last line in the table in question 5 tell you about the system of equations in question 3?

7. Use the process you have just examined to solve the following system of linear equations.

$$\begin{cases} 3x + 5y = -1 \\ 2x + 4y = 4 \end{cases}$$

10.5H Solving Systems with Matrices, Revisited – Teacher Notes

A Solidify Understanding Task

Purpose: Students have previously solved systems using matrices and row reduction—a process associated with the elimination method for solving systems. The purpose of this task is to give students an alternative method for solving systems using the inverse of the coefficient matrix. This method can be generalized to solving larger systems of linear equations using technology to produce the inverse matrix. You may wish to augment this task with a few additional problems in which students solve a larger system of linear equations using inverse matrices produced by technology. This will help them gain an appreciation for the strategy being developed in this task.

Core Standards Focus:
MVP Honors Standard: Solve systems of linear equations using matrices.

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Related Standards: A.REI.6

Standards for Mathematical Practice:
SMP 8 – Look for and express regularity in repeated reasoning

The Teaching Cycle:
Launch (Whole Class):
Give students a few minutes individually to work on questions 1 and 2. While students may solve the equation in question 1 by multiplying both sides of the equation by 3 and then dividing both sides of the equation by 2, question 2 encourages them to multiply both sides of the equation by the

multiplicative inverse of $\frac{2}{3}$ to get *1x* on the left side of the equation and then to simplify 1*x* to *x* using the multiplicative identity property. Look for a student who has approached the problem in this way to present.

Work with students on using a matrix equation to represent a system of equations, as suggested in questions 3 and 4. Contrast this matrix representation with the augmented matrix representation of a system used in the *Systems* module. You may want to review solving a system by row reduction and then point out that in this task students will develop an alternative method for solving systems based on the arithmetic of matrices. Set students to work on the remainder of the task.

Explore (Small Group):

Since matrix multiplication is not commutative, when students multiply both sides of the matrix equation by the inverse matrix, they will have to place the inverse matrix to the left of the matrices given on each side of the equation. Watch for students who are having difficulty multiplying matrices on the right side of the equation because of this issue. Ask students why this is an issue with the matrix equation, but not with the equation that contains the rational number coefficient and the multiplication by $\frac{3}{2}$ on both sides of the equation. Likewise, rewriting 1*x* as *x* is a subtle and over-looked idea when solving the equation containing rational numbers, but it becomes a more apparent issue when working with the matrix equation and recognizing that $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$ can be re-written simply as $\begin{bmatrix} x \\ y \end{bmatrix}$ because of the multiplicative identity property.

Discuss (Whole Class):

Have a student present their work on question 8, showing how they solved the system using a matrix equation and an inverse matrix. While we have not developed a process for finding the inverse of a 3 × 3 matrix, it may help students appreciate the value of solving systems using inverse matrices to consider solving a larger system using this strategy versus using row reduction, or substitution or elimination. Here is a 3 × 3 system to consider.

$$\begin{cases} 4x - 2y + z = 3 \\ 2x + y - z = 1 \\ 3x - y + 2z = 7 \end{cases}$$

The inverse of the coefficient matrix can be obtained using technology. Have students confirm that the matrix produced by technology is indeed the inverse matrix by having them carry out the multiplication of the matrix and its inverse by hand. Once the system has been solved using the inverse matrix, have students solve the system using another method, such as row reduction of the augmented matrix, or substitution or elimination.

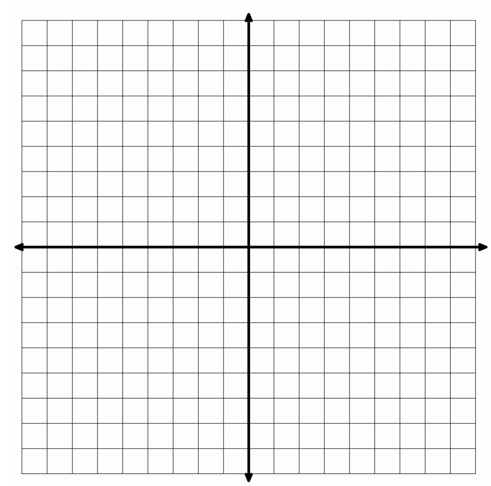
Aligned Ready, Set, Go: Matrices Revisited 10.5H

READY, SET, GO! Name Period Date

READY

Topic: Reflections and Rotations

1. The following three points form the vertices of a triangle: (3, 2), (6, 1), (4, 3)



a. Plot these three points on the coordinate grid and then connect them to form a triangle.

b. Reflect the original triangle over the y-axis and record the coordinates of the vertices here:

c. Reflect the original triangle over the x-axis and record the coordinates of the vertices here:

d. Rotate the original triangle 90° counter-clockwise about the origin and record the coordinates of the vertices here:

e. Rotate the original triangle 180° about the origin and record the coordinates of the vertices here.

SET

Topic: Solving Systems Using Inverse Matrices

Two of the following systems have unique solutions (that is, the lines intersect at a single point).

2. Use the determinant of a 2×2 matrix to decide which systems have unique solutions, and which one does not.

a. $\begin{cases} 8x - 2y = -2 \\ 4x + y = 5 \end{cases}$

b. $\begin{cases} 3x + 2y = 7 \\ 6x + 4y = -5 \end{cases}$

c. $\begin{cases} 4x + 2y = 0 \\ 3x + y = 2 \end{cases}$

3. For each of the systems in #2 which have a unique solution, find the solution to the system by solving a matrix equation using an *inverse matrix*.
   a. b. c.

GO

Topic: Reviewing the Properties of Arithmetic

Match each example on the left with the name of a property of arithmetic on the right. Not all answers will be used.

____ 4. $2(x + 3y) = 2x + 6y$

____ 5. $(2x + 3y) + 4y = 2x + (3y + 4y)$

____ 6. $2x + 3y = 3y + 2x$

____ 7. $2(3y) = (2 \cdot 3)y = 6y$

____ 8. $\frac{2}{3} \cdot \frac{3}{2} x = 1x$

____ 9. $x + -x = 0$

____ 10. $xy = yx$

a. multiplicative inverses

b. additive inverses

c. multiplicative identity

d. additive identity

e. commutative property of addition

f. commutative property of multiplication

g. associative property of addition

h. associative property of multiplication

i. distributive property of addition over multiplication

10.6H All Systems Go!

A Practice Understanding Task

This module began with the following problem:

Carlos and Clarita are trying to figure out the cost of a gallon of paint, the cost of a paintbrush, and the cost of a roll of masking tape based on the following purchases:

Week 1: Carlos bought 2 gallons of paint and 1 roll of masking tape for \$30.
Week 2: Carlos bought 1 gallon of paint and 4 brushes for \$20.
Week 3: Carlos bought 2 brushes and 1 roll of masking tape for \$10.

In the previous sequence of tasks *More Arithmetic of Matrices*, *Solving Systems with Matrices, Revisited* and *The Determinant of a Matrix* you learned how to solve systems using multiplication of matrices. In this task, we are going to extend this strategy to include systems with more that two equations and two variables.

1. Multiply the follow pairs of matrices:

   a. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 1 \\ 1 & 4 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 0.4 & 0.2 & -0.4 \\ -0.1 & 0.2 & 0.1 \\ 0.2 & -0.4 & 0.8 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 1 \\ 1 & 4 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

2. What property is illustrated by the multiplication in question 1a?

3. What property is illustrated by the multiplication in question 1b?

4. Rewrite the following system of equations, which represents Carlos and Clarita's problem, as a matrix equation in the form **AX = B** where **A**, **X** and **B** are all matrices.

$$\begin{aligned} 2g + 0b + 1t &= 30 \\ 1g + 4b + 0t &= 20 \\ 0g + 2b + 1t &= 10 \end{aligned}$$

5. Solve your matrix equation by using multiplication of matrices. Show the details of your work so that someone else can follow it.

You were able to solve this equation using matrix multiplication because you were given the inverse of matrix **A.** Unlike 2×2 matrices, where the inverse matrix can easily be found by hand using the methods described in *More Arithmetic of Matrices*, the inverses of an $n \times n$ matrix in general can be difficult to find by hand. In such cases, we will use technology to find the inverse matrix so that this method can be applied to all linear systems involving n equations and n unknown quantities. Here is one online resource you might use: https://matrixcalc.org/en/

6. Solve the following system using a matrix equation and inverse matrices. Although you may use technology to find the inverse matrix, make sure you record all of your work in the space below, including your inverse matrix.

$$x + 2y + 2z = 16$$
$$2x + y + 3z = 17$$
$$x + 2y + z = 13$$

10.6H All Systems Go! – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is to extend the process of solving linear systems using matrix equations and inverse matrices to linear systems that include more than two equations and more than two unknowns. This task extends the idea of representing a system with a matrix equation that includes a vector variable, and then solving the matrix equation for the vector variable by multiplying by an inverse matrix to higher-ordered $n \times n$ matrices. The inverses for $n \times n$ matrices are found using technology.

Core Standards Focus:

A.REI.8 Represent a system of linear equations as a single matrix equation in a vector variable.

A.REI.9 Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Standards for Mathematical Practice:

SMP 8 – Look for and express regularity in repeated reasoning

The Teaching Cycle:

Launch (Whole Class):

Pose the essential question listed above to students, and have them consider the different ways they have learned to solve systems of equations, and when each method might be useful: graphing, substitution, elimination, row reduction of matrices, using inverse matrices. Students might note the limitations of graphing to situations that involve 2 equations and 2 unknown, and that substitution, elimination and reduction of matrices gets more complicated or tedious as the number of equations and unknowns increases. At this point, students will probably state that using inverse matrices seems problematic, since they only know how to find the inverse of a 2×2 matrix. Once this issue has surfaced, let students start on the task.

Explore (Small Group):
Based on the previous sequences of tasks, students should be able to work through problems 1-5 independently, although they may need to have a partner (or the teacher) point out that one of the matrices in question 1b is the matrix that represents Carlos and Clarita's system of equations, and that they have demonstrated that the other matrix is its inverse.

You will need to decide what technological resource students might use to find the inverse of a matrix, a graphing calculator or the online matrix calculator referenced in the task. Provide ample support for students as they are learning how to use the technology.

Listen for students who are successfully applying the strategy of using a matrix equation multiplied by an inverse matrix to present during the whole class discussion.

Discuss (Whole Class):
Move to the whole class discussion when you have students who can present how they used technology to find the inverse matrix, and how they applied this to solve the given system of equations. If necessary, show students how to find inverse matrices using available technology, then let students work through the problem of solving the matrix equation using technology.

Aligned Ready, Set, Go: Matrices Revisited 10.6H

READY, SET, GO! Name Period Date

READY

Topic: Reviewing rational exponents and methods for solving quadratics

Write each exponential expression in radical form.

1. $10^{\frac{3}{2}}$

2. $x^{\frac{1}{5}}$

3. $3n^{\frac{1}{3}}$

4. $6^{\frac{2}{7}}$

5. $7^{\frac{5}{3}}$

6. $t^{\frac{4}{5}}$

Write each radical expression in exponential form.

7. $\sqrt[5]{3}$

8. $\left(\sqrt[6]{7a}\right)^5$

9. $\sqrt{x^3}$

10. $\sqrt[3]{n^5}$

11. $\left(\sqrt[y]{n}\right)^x$

12. $\sqrt[p]{n^q}$

Explain each strategy for solving quadratic equations and explain the circumstances in which the strategy is most efficient.

13. Graphing

14. Factoring

15. Completing the square

16. What other strategies do you know for solving quadratic equations? When would you use them?

SET

Topic: Solving systems with three unknowns.

Solve the system of equations using matrices. Create a matrix equation for the system of equations that can be used to find the solution. Then find the inverse matrix and use it to solve the system.

17. $\begin{cases} 2x - 4y + z = 0 \\ 5x - 4y - 5z = 12 \\ 4x + 4y + z = 24 \end{cases}$

18. $\begin{cases} x + 2y + 5z = -15 \\ x + y - 4z = 12 \\ x - 6y + 4z = -12 \end{cases}$

19. $\begin{cases} 4p + q - 2r = 5 \\ -3p - 3q - 4r = -16 \\ 4p - 4q + 4r = -4 \end{cases}$

20. $\begin{cases} -6x - 4y + z = -20 \\ -3x - y - 3z = -8 \\ -5x + 3y + 6z = -4 \end{cases}$

GO

Topic: Solving quadratic equations

Solve each of the equations below using an appropriate and efficient method.

21. $x^2 - 5x = -6$

22. $3x^2 - 5 = 0$

23. $5x^2 - 10 = 0$

24. $x^2 + 1x - 30 = 0$

25. $x^2 + 2x = 48$

26. $x^2 - 3x = 0$