

*Transforming Mathematics Education*

# ALGEBRA II

*An Integrated Approach*

MODULE 2 HONORS

# Logarithmic Functions

MATHEMATICSVISIONPROJECT.ORG

## **The Mathematics Vision Project**

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## 2.1 Log Logic

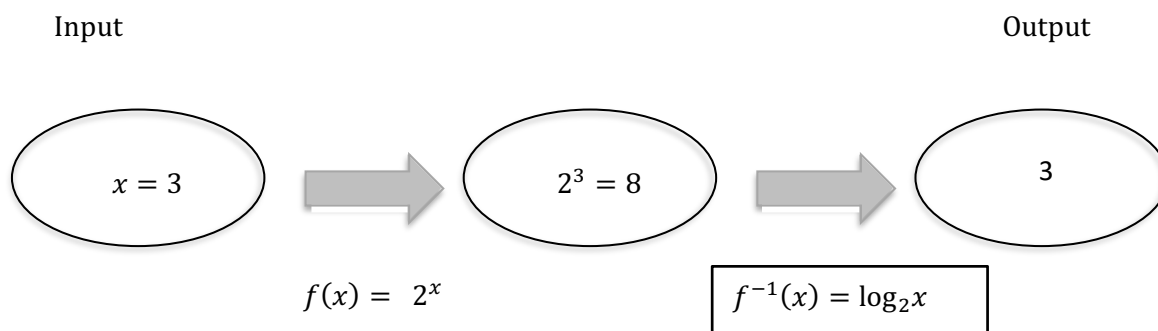
### A Develop Understanding Task

We began thinking about logarithms as inverse functions for exponentials in *Tracking the Tortoise*. Logarithmic functions are interesting and useful on their own. In the next few tasks, we will be working on understanding logarithmic expressions, logarithmic functions, and logarithmic operations on equations.



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We showed the inverse relationship between exponential and logarithmic functions using a diagram like the one below:



We could summarize this relationship by saying:

$$2^3 = 8 \quad \text{so,} \quad \log_2 8 = 3$$

Logarithms can be defined for any base used for an exponential function. Base 10 is popular. Using base 10, you can write statements like these:

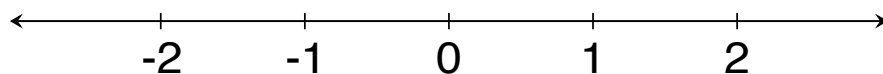
$$\begin{aligned} 10^1 &= 10 & \text{so,} & \log_{10} 10 = 1 \\ 10^2 &= 100 & \text{so,} & \log_{10} 100 = 2 \\ 10^3 &= 1000 & \text{so,} & \log_{10} 1000 = 3 \end{aligned}$$

The notation may see different, but you can see the inverse pattern where the inputs and outputs switch.

The next few problems will give you an opportunity to practice thinking about this pattern and possibly make a few conjectures about other patterns related to logarithms.

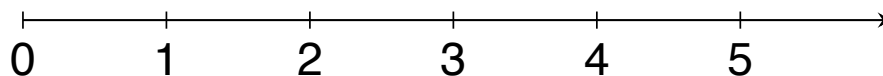
Place the following expressions on the number line. Use the space below the number line to explain how you knew where to place each expression.

1. A.  $\log_3 3$       B.  $\log_3 9$       C.  $\log_3 \left(\frac{1}{3}\right)$       D.  $\log_3 1$       E.  $\log_3 \left(\frac{1}{9}\right)$



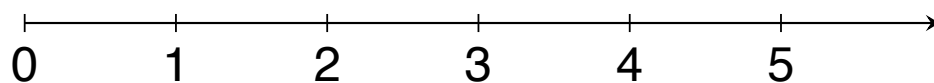
Explain: \_\_\_\_\_

2. A.  $\log_3 81$       B.  $\log_{10} 100$       C.  $\log_8 8$       D.  $\log_5 25$       E.  $\log_2 32$



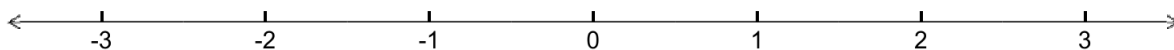
Explain: \_\_\_\_\_

3. A.  $\log_7 7$       B.  $\log_9 9$       C.  $\log_{11} 1$       D.  $\log_{10} 1$



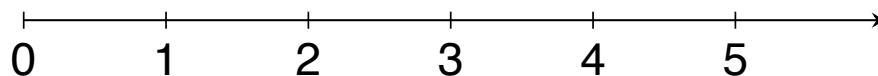
Explain: \_\_\_\_\_

4. A.  $\log_2 \left(\frac{1}{4}\right)$       B.  $\log_{10} \left(\frac{1}{1000}\right)$       C.  $\log_5 \left(\frac{1}{125}\right)$       D.  $\log_6 \left(\frac{1}{6}\right)$



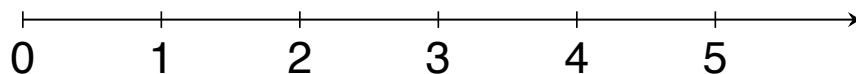
Explain: \_\_\_\_\_

5. A.  $\log_4 16$     B.  $\log_2 16$     C.  $\log_8 16$     D.  $\log_{16} 16$



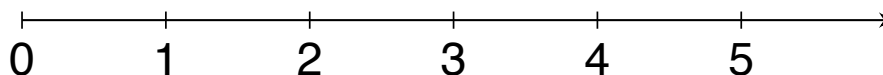
Explain: \_\_\_\_\_

6. A.  $\log_2 5$     B.  $\log_5 10$     C.  $\log_6 1$     D.  $\log_5 5$     E.  $\log_{10} 5$



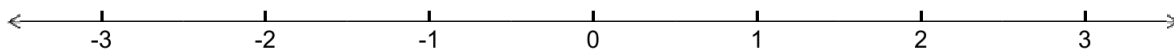
Explain: \_\_\_\_\_

7. A.  $\log_{10} 50$     B.  $\log_{10} 150$     C.  $\log_{10} 1000$     D.  $\log_{10} 500$



Explain: \_\_\_\_\_

8. A.  $\log_3 3^2$     B.  $\log_5 5^{-2}$     C.  $\log_6 6^0$     D.  $\log_4 4^{-1}$     E.  $\log_2 2^3$



Explain: \_\_\_\_\_

Based on your work with logarithmic expressions, determine whether each of these statements is always true, sometimes true, or never true. If the statement is sometimes true, describe the conditions that make it true. Explain your answers.

9. The value of  $\log_b x$  is positive.

Explain: \_\_\_\_\_

10.  $\log_b x$  is not a valid expression if  $x$  is a negative number.

Explain: \_\_\_\_\_

11.  $\log_b 1 = 0$  for any base,  $b > 0$ .

Explain: \_\_\_\_\_

12.  $\log_b b = 1$  for any  $b > 0$ .

Explain: \_\_\_\_\_

13.  $\log_2 x < \log_3 x$  for any value of  $x$ .

Explain: \_\_\_\_\_

14.  $\log_b b^n = n$  for any  $b > 0$ .

Explain: \_\_\_\_\_

READY, SET, GO!

Name \_\_\_\_\_

Period \_\_\_\_\_

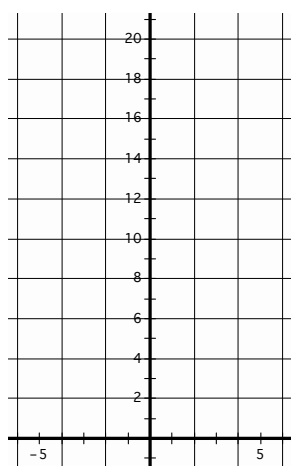
Date \_\_\_\_\_

## READY

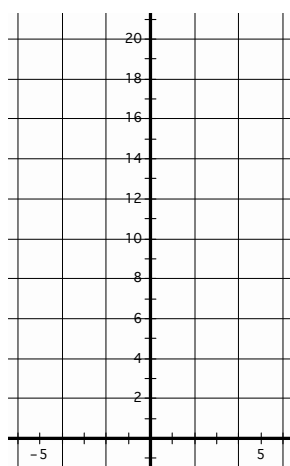
Topic: Graphing exponential equations

Graph each function over the domain  $\{-4 \leq x \leq 4\}$ .

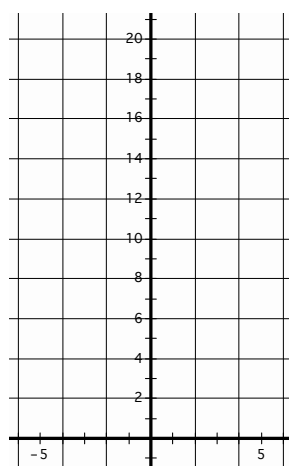
1.  $y = 2^x$



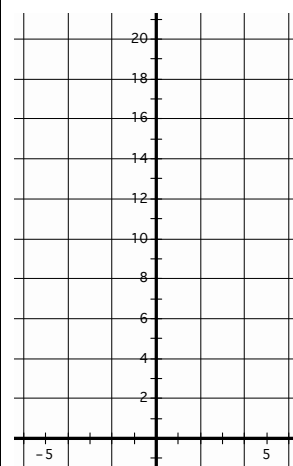
2.  $y = 2 \cdot 2^x$



3.  $y = \left(\frac{1}{2}\right)^x$



4.  $y = 2 \left(\frac{1}{2}\right)^x$



- Compare graph #1 to graph #2. Multiplying by 2 should generate a dilation of the graph, but the graph looks like it has been translated vertically. How do you explain that?
- Compare graph #3 to graph #4. Is your explanation in #5 still valid for these two graphs? Explain.

## SET

Topic: Writing the logarithmic form of an exponential equation.

**Definition of Logarithm:** For all positive numbers  $a$ , where  $a \neq 1$ , and all positive numbers  $x$ ,

$$y = \log_a x \text{ means the same as } x = a^y.$$

(Note the **base** of the exponent and the **base** of the logarithm are both  $a$ .)

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7. Why is it important that in the definition of logarithm it is stated that the base of the logarithm does not equal 1?
8. Why is it important that the definition of a logarithm states that the base of the logarithm is positive?
9. Why is it necessary that the definition states that  $x$  in the expression  $\log_a x$  is positive?

Write the following exponential equations in logarithmic form.

Exponential form	Logarithmic form	Exponential form	Logarithmic form
10. $5^4 = 625$		11. $3^2 = 9$	
12. $\left(\frac{1}{2}\right)^{-3} = 8$		13. $4^{-2} = \frac{1}{16}$	
14. $10^4 = 10000$		15. $a^y = x$	

16. Compare the exponential form of an equation to the logarithmic form of an equation. What part of the exponential equation is the **answer** to the logarithmic equation?

Topic: Considering values of logarithmic functions

**Answer the following questions. If yes, give an example of the answer. If no, explain why not.**

17. Is it possible for a logarithm to equal a negative number?
18. Is it possible for a logarithm to equal zero?
19. Does  $\log_x 0$  have an answer?
20. Does  $\log_x 1$  have an answer?
21. Does  $\log_x x^5$  have an answer?

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GO

Topic: Reviewing properties of Exponents

**Write each expression as an integer or a simple fraction.**

22.  $27^0$

23.  $11(-6)^0$

24.  $-3^{-2}$

25.  $4^{-3}$

26.  $\frac{9}{2^{-1}}$

27.  $\frac{4^3}{8^0}$

28.  $3\left(\frac{29^3}{11^5}\right)^0$

29.  $\frac{3}{6^{-1}}$

30.  $\frac{32^{-1}}{4^{-1}}$

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## 2.2 Falling Off a Log

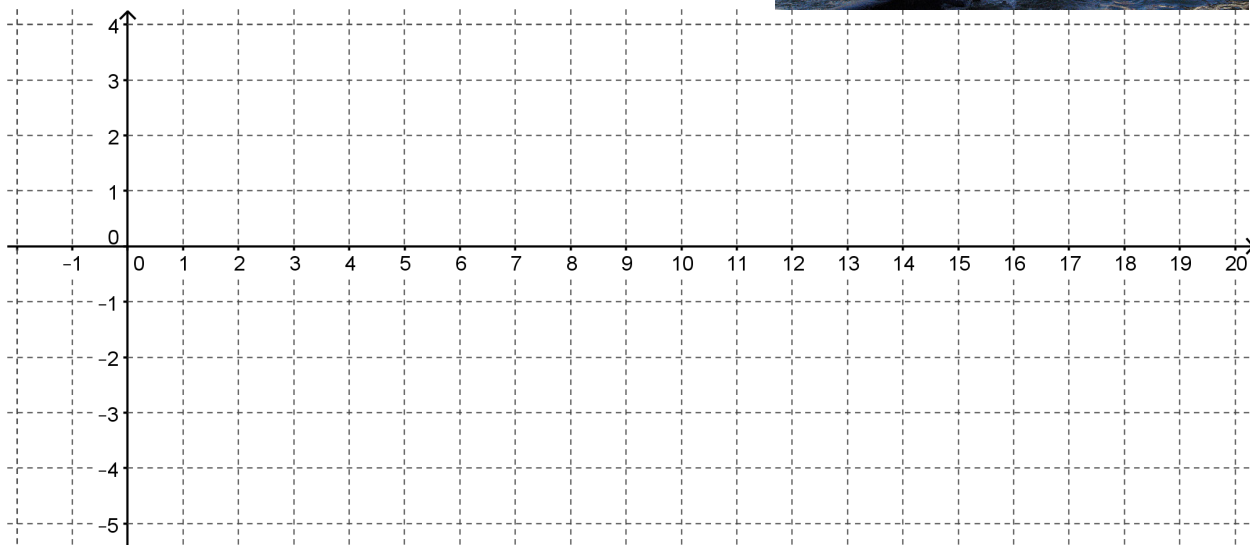
### *A Solidify Understanding Task*

- Construct a table of values and a graph for each of the following functions. Be sure to select at least two values in the interval  $0 < x < 1$ .

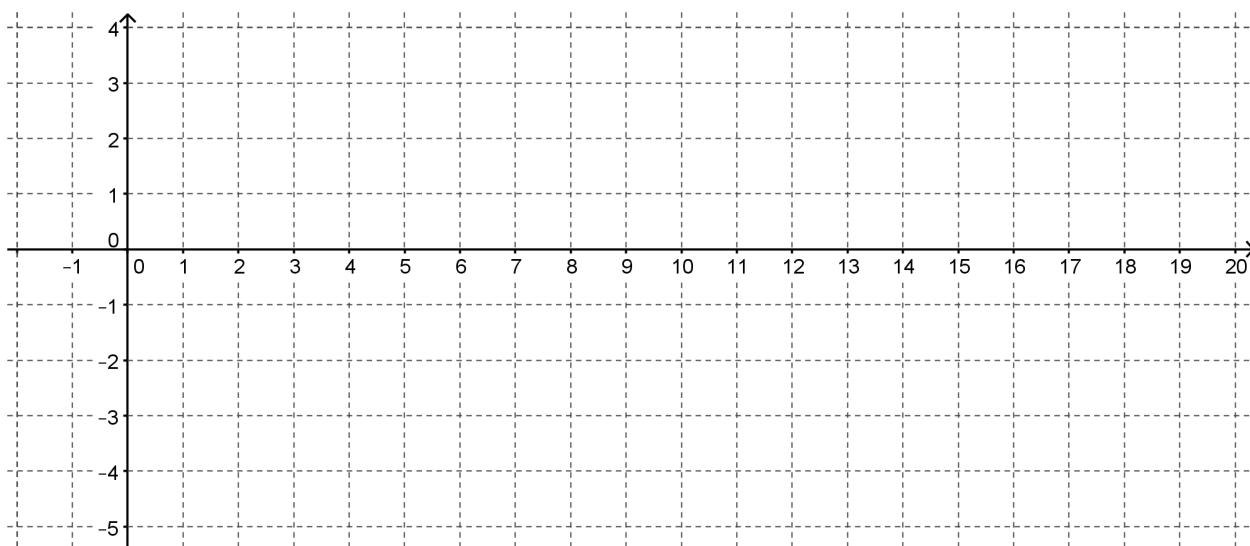
a)  $f(x) = \log_2 x$



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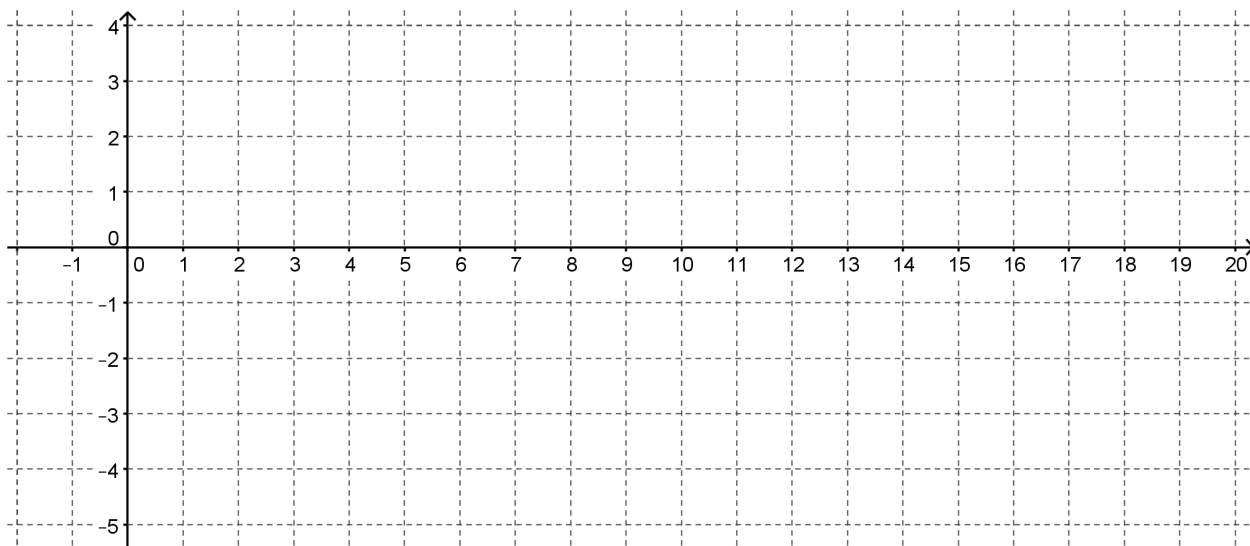


b)  $g(x) = \log_3 x$

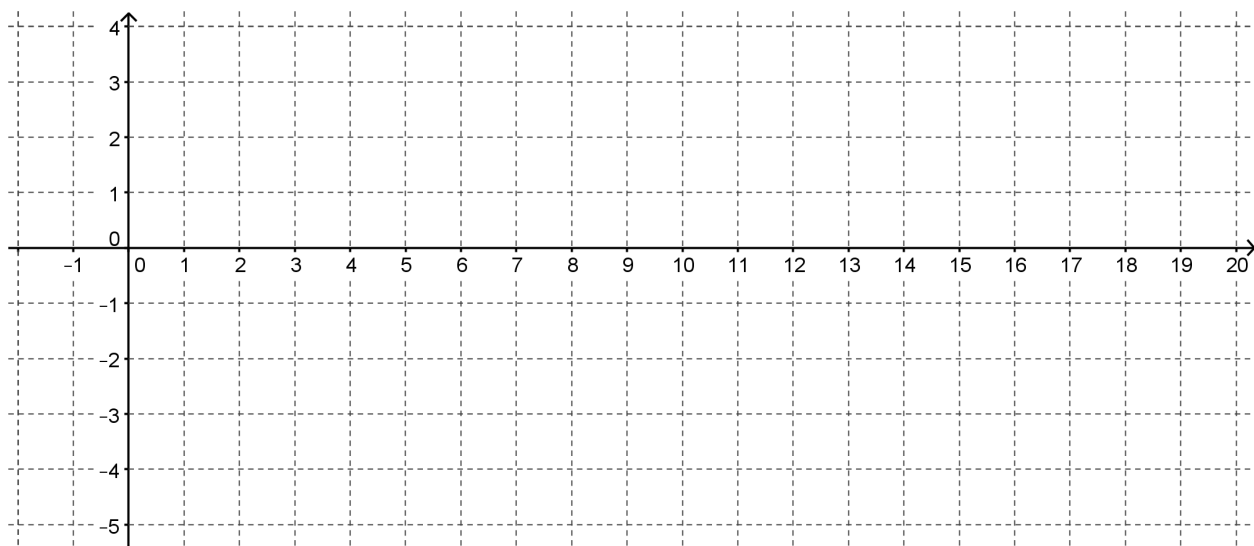


c)  $h(x) = \log_4 x$





d)  $k(x) = \log_{10} x$



2. How did you decide what values to use for  $x$  in the table?
3. How did you use the  $x$  values to find the  $y$  values in the table?
4. What similarities do you see in the graphs?

5. What differences do you observe in the graphs?
6. What is the effect of changing the base on the graph of a logarithmic function?

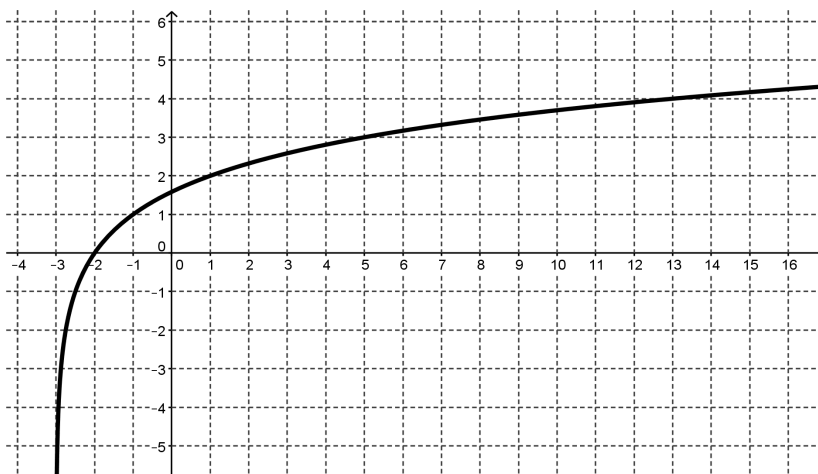
Let's focus now on  $k(x) = \log_{10}x$  so that we can use technology to observe the effects of changing parameters on the function. **Because base 10 is a commonly used base for exponential and logarithmic functions, it is often abbreviated and written without the base, like this:  $k(x) = \log x$ .**

7. Use technology to graph  $y = \log x$ . How does the graph compare to the graph that you constructed?
8. How do you predict that the graph of  $y = a + \log x$  will be different from the graph of  $y = \log x$ ?
9. Test your prediction by graphing  $y = a + \log x$  for various values of  $a$ . What is the effect of  $a$  on the graph? Make a general argument for why this would be true for all logarithmic functions.
10. How do you predict that the graph of  $y = \log(x + b)$  will be different from the graph of  $y = \log x$ ?
11. Test your prediction by graphing  $y = \log(x + b)$  for various values of  $b$ .
  - What is the effect of adding  $b$ ?
  - What will be the effect of subtracting  $b$  (or adding a negative number)?

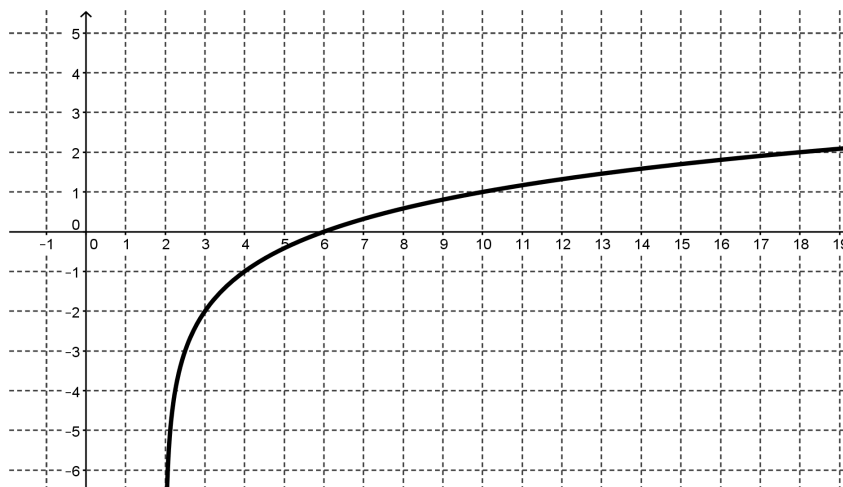
- Make a general argument for why this is true for all logarithmic functions.

12. Write an equation for each of the following functions that are transformations of  $f(x) = \log_2 x$ .

a)

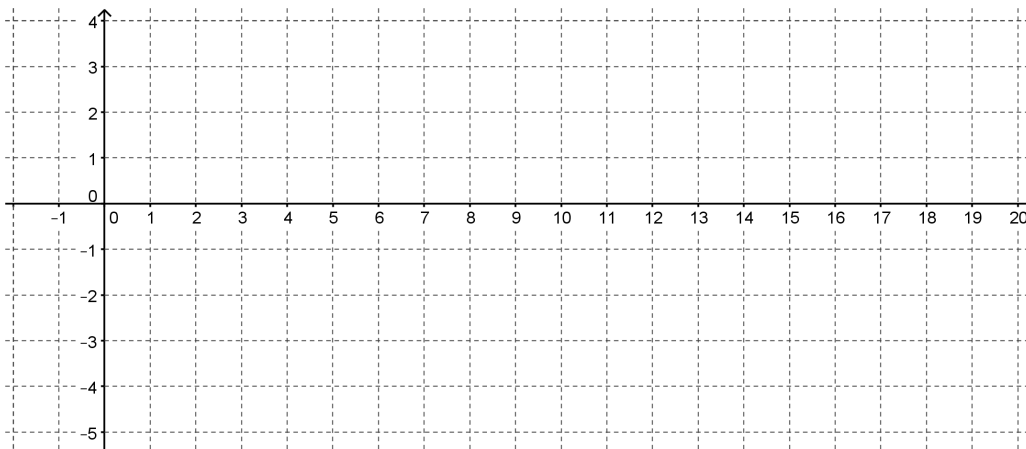


b)

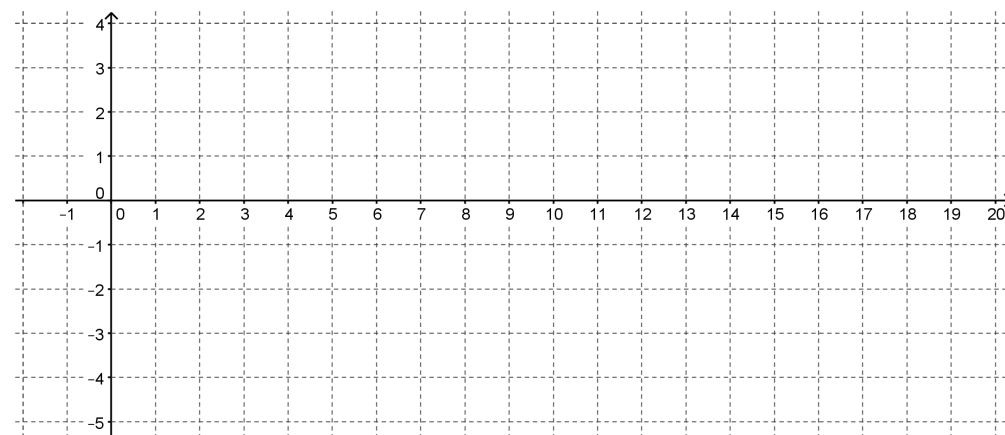


13. Graph and label each of the following functions:

a)  $f(x) = 2 + \log_2(x - 1)$



b)  $g(x) = -1 + \log_2(x + 2)$



14. Compare the transformation of the graphs of logarithmic functions with the transformation of the graphs of quadratic functions.

READY, SET, GO!

Name \_\_\_\_\_

Period \_\_\_\_\_

Date \_\_\_\_\_

**READY**

Topic: Solving simple logarithmic equations

**Find the answer to each logarithmic equation. Then explain how each equation supports the statement, “The answer to a logarithmic equation is always the exponent.”**

1.  $\log_5 625 =$

2.  $\log_3 243 =$

3.  $\log_5 0.2 =$

4.  $\log_9 81 =$

5.  $\log 1,000,000 =$

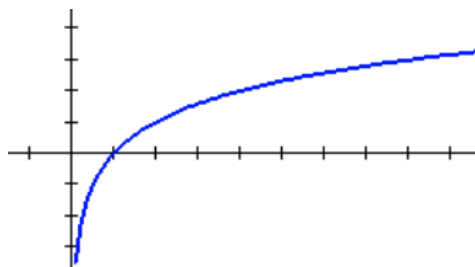
6.  $\log_x x^7 =$

**SET**

Topic: Exploring transformations on logarithmic functions

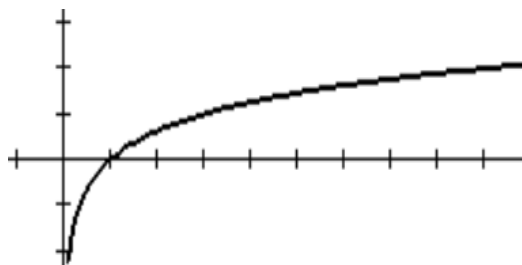
**Answer the questions about each graph.**

7.



- What is the value of  $x$  when  $f(x) = 0$ ?
- What is the value of  $x$  when  $f(x) = 1$ ?
- What is the value of  $f(x)$  when  $x = 2$ ?
- What will be the value of  $x$  when  $f(x) = 4$ ?
- What is the equation of this graph?

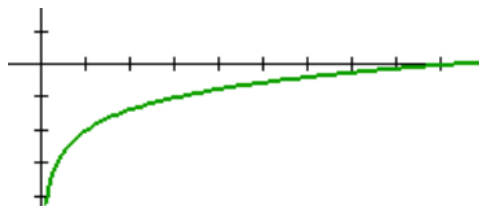
8.



- What is the value of  $x$  when  $f(x) = 0$ ?
- What is the value of  $x$  when  $f(x) = 1$ ?
- What is the value of  $f(x)$  when  $x = 9$ ?
- What will be the value of  $x$  when  $f(x) = 4$ ?
- What is the equation of this graph?

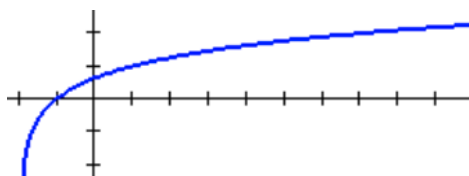
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9. Use the graph and the table of values for the graph to write the equation of the graph. Explain which numbers in the table helped you the most to write the equation.



X	Y <sub>1</sub>
1	-2
2	-1.369
3	-1
5	-.535
7	-.2288
9	0

10. Use the graph and the table of values for the graph to write the equation of the graph. Explain which numbers in the table helped you the most to write the equation.



X	Y <sub>1</sub>
-2	ERROR
-1	0
0	.63093
1	1
5	1.7712
6	1.8928
7	2

### GO

Topic: Using the power to a power rule with exponents

**Simplify each expression. Answers should have only positive exponents.**

11.  $(2^3)^4$
12.  $(x^3)^2$
13.  $(a^3)^{-2}$
14.  $(2^3w)^4$
15.  $(b^{-7})^3$
16.  $(d^{-3})^{-2}$
17.  $x^2 \cdot (x^5)^2$
18.  $m^{-3} \cdot (m^2)^3$
19.  $(x^5)^{-4} \cdot x^{25}$
20.  $(5a^3)^2$
21.  $(6^{-3})^2$
22.  $(2a^3b^2)^2$

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## 2.3 Chopping Logs

### *A Solidify Understanding Task*

Abe and Mary were working on their math homework together when Abe has a brilliant idea!

**Abe:** I was just looking at this *log* function that we graphed in *Falling Off A Log*:

$$y = \log_2(x + b).$$

I started to think that maybe I could just “distribute” the *log* so that I get:

$$y = \log_2 x + \log_2 b.$$

I guess I’m saying that I think these are equivalent expressions, so I could write it this way:

$$\log_2(x + b) = \log_2 x + \log_2 b$$

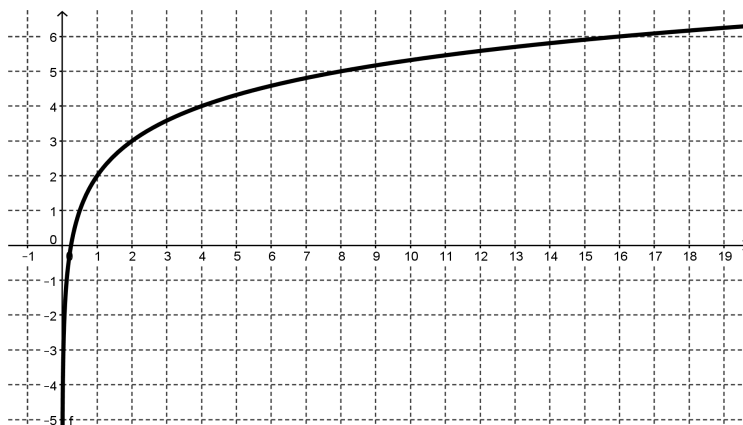
**Mary:** I don’t know about that. *Logs* are tricky and I don’t think that you’re really doing the same thing here as when you distribute a number.

1. What do you think? How can you verify if Abe’s idea works?
2. If Abe’s idea works, give some examples that illustrate why it works. If Abe’s idea doesn’t work, give a counter-example.



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**Abe:** I just know there is something going on with these *logs*. I just graphed  $f(x) = \log_2(4x)$ . Here it is:



It's weird because I think this graph is just a translation of  $y = \log_2 x$ . Is it possible the equation of this graph could be written more than one way?

3. How would you answer Abe's question? Are there conditions that could allow the same graph to have different equations?

**Mary:** When you say, "a translation of  $y = \log_2 x$ " do you mean that it is just a vertical or horizontal shift? What could that equation be?

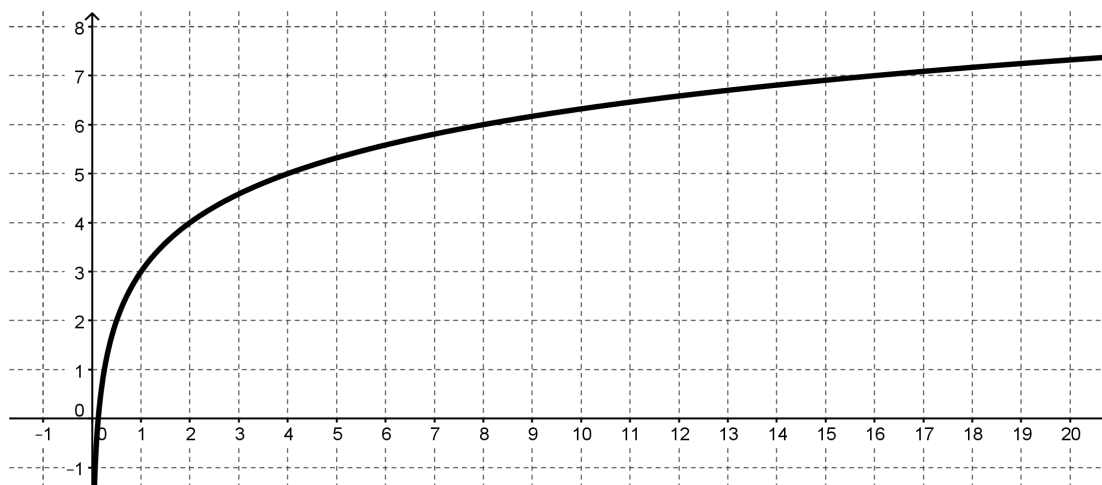
4. Find an equation for  $f(x)$  that shows it to be a horizontal or vertical shift of  $y = \log_2 x$ .



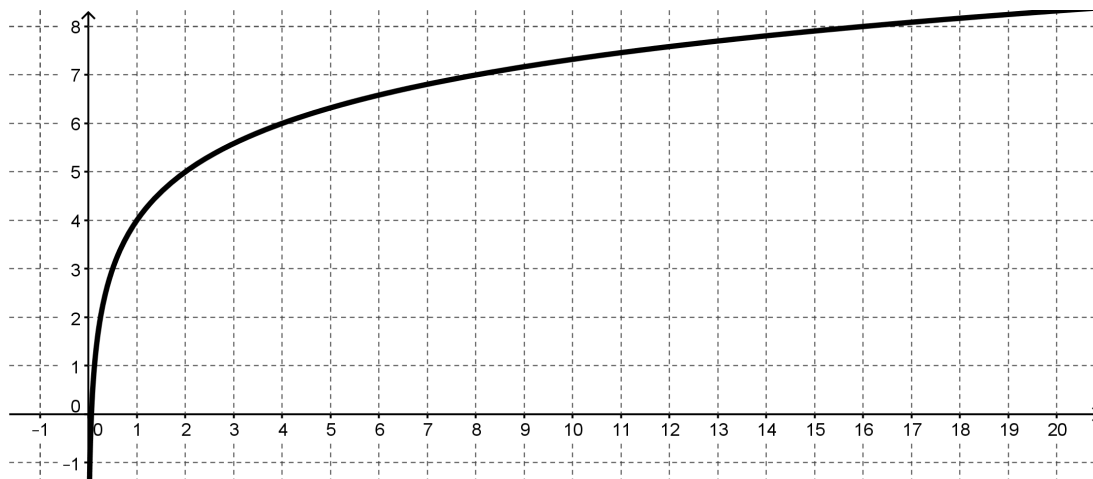
**Mary:** I wonder why the vertical shift turned out to be up 2 when the  $x$  was multiplied by 4. I wonder if it has something to do with the power that the base is raised to, since this is a *log* function. Let's try to see what happens with  $y = \log_2(8x)$  and  $y = \log_2(16x)$ .

5. Try to write an equivalent equation for each of these graphs that is a vertical shift of  $y = \log_2 x$ .

a)  $y = \log_2(8x)$       Equivalent equation: \_\_\_\_\_



b)  $y = \log_2(16x)$       Equivalent equation: \_\_\_\_\_



**Mary:** Oh my gosh! I think I know what is happening here! Here's what we see from the graphs:

$$\log_2(4x) = 2 + \log_2 x$$

$$\log_2(8x) = 3 + \log_2 x$$

$$\log_2(16x) = 4 + \log_2 x$$

Here's the brilliant part: We know that  $\log_2 4 = 2$ ,  $\log_2 8 = 3$ , and  $\log_2 16 = 4$ . So:

$$\log_2(4x) = \log_2 4 + \log_2 x$$

$$\log_2(8x) = \log_2 8 + \log_2 x$$

$$\log_2(16x) = \log_2 16 + \log_2 x$$

I think it looks like the “distributive” thing that you were trying to do, but since you can't really distribute a function, it's really just a *log* multiplication rule. I guess my rule would be:

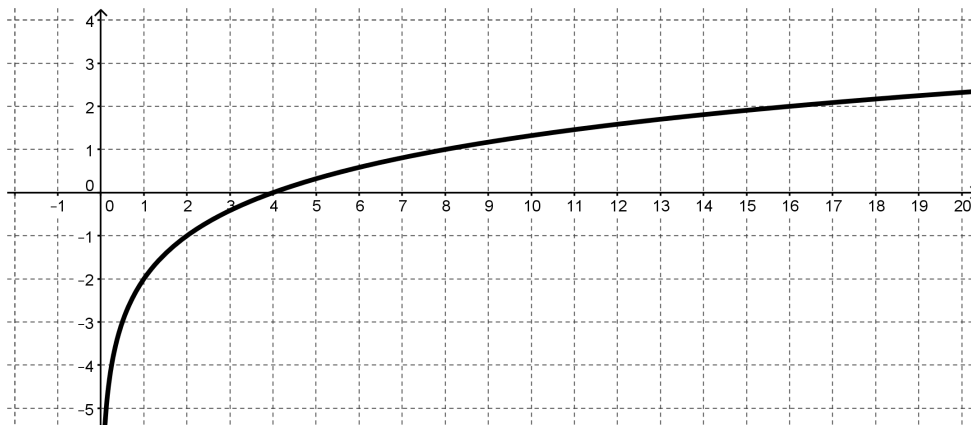
$$\log_2(ab) = \log_2 a + \log_2 b$$

6. How can you express Mary's rule in words?
  
  
  
  
  
  
  
  
  
  
7. Is this statement true? If it is, give some examples that illustrate why it works. If it is not true provide a counter-example.

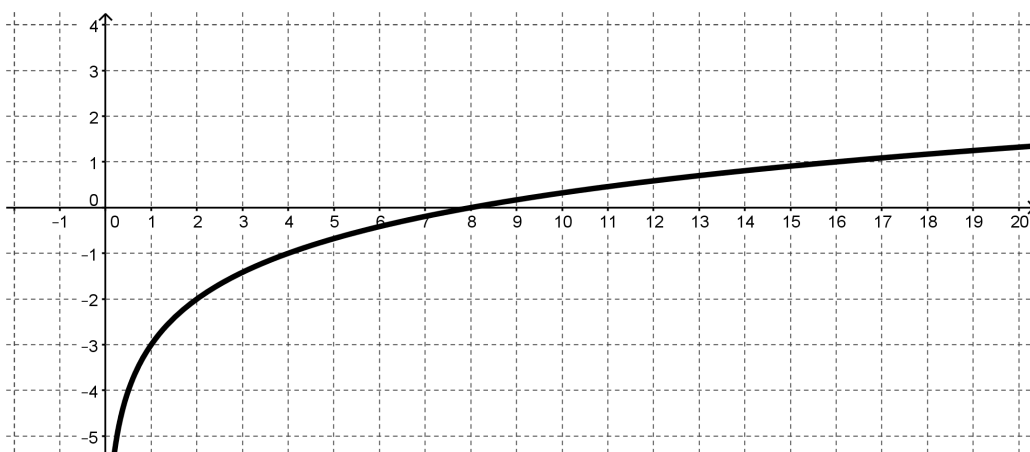
**Mary:** So, I wonder if a similar thing happens if you have division inside the argument of a *log* function. I'm going to try some examples. If my theory works, then all of these graphs will just be vertical shifts of  $y = \log_2 x$ .

8. Here are Abe's examples and their graphs. Test Abe's theory by trying to write an equivalent equation for each of these graphs that is a vertical shift of  $y = \log_2 x$ .

a)  $y = \log_2 \left( \frac{x}{4} \right)$       Equivalent equation: \_\_\_\_\_



b)  $y = \log_2 \left( \frac{x}{8} \right)$       Equivalent equation: \_\_\_\_\_



9. Use these examples to write a rule for division inside the argument of a *log* that is like the rule that Mary wrote for multiplication inside a *log*.

10. Is this statement true? If it is, give some examples that illustrate why it works. If it is not true provide a counter-example.

**Abe:** You're definitely brilliant for thinking of that multiplication rule. But I'm a genius because I've used your multiplication rule to come up with a power rule. Let's say you start with:

$$\log_2(x^3)$$

Really that's the same as having:

$$\log_2(x \cdot x \cdot x)$$

So, I could use your multiplying rule and write:

$$\log_2 x + \log_2 x + \log_2 x$$

I notice there are 3 terms that are all the same. That makes it:

$$3 \log_2 x$$

So my rule is:

$$\log_2(x^3) = 3 \log_2 x$$

If your rule is true, then I have proven my power rule.

**Mary:** I don't think it's really a power rule unless it works for any power. You only showed how it might work for 3.

**Abe:** Oh, good grief! Ok, I'm going to say it can be any number  $x$ , raised to any power,  $k$ . My power rule is:

$$\log_2(x^k) = k \log_2 x$$

Are you satisfied?

11. Provide an argument about Abe's power rule. Is it true or not?

**Abe:** Before we win the Nobel Prize for mathematics I suppose that we need to think about whether or not these rules work for any base.

12. The three rules, written for any base  $b > 1$  are:

**Log of a Product Rule:**  $\log_b(xy) = \log_b x + \log_b y$

**Log of a Quotient Rule:**  $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

**Log of a Power Rule:**  $\log_b(x^k) = k \log_b x$

Make an argument for why these rules will work in any base  $b > 1$  if they work for base 2.

13. How are these rules similar to the rules for exponents? Why might exponents and *logs* have similar rules?

READY, SET, GO!

Name

Period

Date

**READY**

Topic: Recalling fractional exponents

**Write the following with an exponent. Simplify when possible.**

1.  $\sqrt[5]{x}$

2.  $\sqrt[7]{s^2}$

3.  $\sqrt[3]{w^8}$

4.  $\sqrt[3]{8r^6}$

5.  $\sqrt[5]{125m^5}$

6.  $\sqrt[3]{(8x)^2}$

7.  $\sqrt[3]{9b^8}$

8.  $\sqrt{75x^6}$

**Rewrite with a fractional exponent. Then find the answer.**

9.  $\log_3 \sqrt[5]{3} =$

10.  $\log_2 \sqrt[3]{4} =$

11.  $\log_7 \sqrt[5]{343} =$

12.  $\log_5 \sqrt[5]{3125} =$

**SET**

Topic: Using the properties of logarithms to expand logarithmic expressions

**Use the properties of logarithms to expand the expression as a sum or difference, and/or constant multiple of logarithms.** (Assume all variables are positive.)

13.  $\log_5 7x$

14.  $\log_5 10a$

15.  $\log_5 \frac{5}{b}$

16.  $\log_5 \frac{d}{4}$

17.  $\log_6 x^3$

18.  $\log_5 9x^2$

19.  $\log_2 (7x)^4$

20.  $\log_3 \sqrt{w}$

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21.  $\log_5 \frac{xyz}{w}$

22.  $\log_5 \frac{9\sqrt{x}}{y^3}$

23.  $\log_2 \left( \frac{x^2-4}{x^3} \right)$

24.  $\log_2 \left( \frac{x^2}{y^5 w^3} \right)$

**GO**

Topic: Writing expressions in exponential form and logarithmic form

**Convert to logarithmic form.**

25.  $2^9 = 512$

26.  $10^{-2} = 0.01$

27.  $\left( \frac{2}{3} \right)^{-1} = \frac{3}{2}$

**Write in exponential form.**

28.  $\log_4 2 = \frac{1}{2}$

29.  $\log_{\frac{1}{3}} 3 = -1$

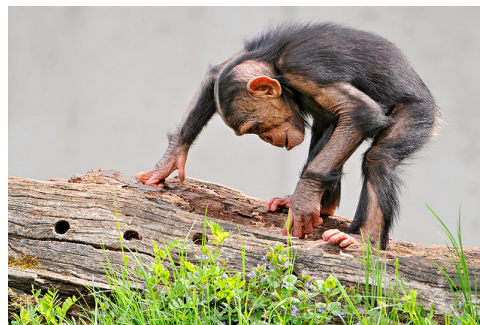
30.  $\log_{\frac{2}{5}} \frac{8}{125} = 3$

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## 2.4 Log-Arithm-etic

### *A Practice Understanding Task*

Abe and Mary are feeling good about their *log* rules and bragging about their mathematical prowess to all their friends when this exchange occurs:



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**Stephen:** I guess you think you're pretty smart because you figured out some *log* rules, but I want to know what they're good for.

**Abe:** Well, we've seen a lot of times when equivalent expressions are handy. Sometimes when you write an expression with a variable in it in a different way it means something different.

1. What are some examples from your previous experience where equivalent expressions were useful?

**Mary:** I was thinking about the *Log Logic* task where we were trying to estimate and order certain *log* values. I was wondering if we could use our log rules to take values we know and use them to find values that we don't know.

For instance: Let's say you want to calculate  $\log_2 6$ . If you know what  $\log_2 2$  and  $\log_2 3$  are then you can use the product rule and say:

$$\log_2(2 \cdot 3) = \log_2 2 + \log_2 3$$

**Stephen:** That's great. Everyone knows that  $\log_2 2 = 1$ , but what is  $\log_2 3$ ?

**Abe:** Oh, I saw this somewhere. Uh,  $\log_2 3 = 1.585$ . So Mary's idea really works. You say:

$$\log_2(2 \cdot 3) = \log_2 2 + \log_2 3$$

$$= 1 + 1.585$$

$$= 2.585$$

$$\log_2 6 = 2.585$$

2. Based on what you know about logarithms, explain why 2.585 is a reasonable value for  $\log_2 6$ ?



**Stephen:** Oh, oh! I've got one. I can figure out  $\log_2 5$  like this:

$$\begin{aligned}\log_2(2 + 3) &= \log_2 2 + \log_2 3 \\ &= 1 + 1.585 \\ &= 2.585 \\ \log_2 5 &= 2.585\end{aligned}$$

3. Can Stephen and Mary both be correct? Explain who's right, who's wrong (if anyone) and why.

Now you can try applying the *log* rules yourself. Use the values that are given and the ones that you know by definition, like  $\log_2 2 = 1$ , to figure out each of the following values. Explain what you did in the space below each question.

$$\log_2 3 = 1.585 \qquad \log_2 5 = 2.322 \qquad \log_2 7 = 2.807$$

The three rules, written for any base  $b > 1$  are:

**Log of a Product Rule:**  $\log_b(xy) = \log_b x + \log_b y$

**Log of a Quotient Rule:**  $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

**Log of a Power Rule:**  $\log_b(x^k) = k \log_b x$

4.  $\log_2 9 =$  \_\_\_\_\_

5.  $\log_2 10 =$  \_\_\_\_\_

6.  $\log_2 12 =$  \_\_\_\_\_

7.  $\log_2 \left( \frac{7}{3} \right) =$  \_\_\_\_\_

8.  $\log_2 \left( \frac{30}{7} \right) =$  \_\_\_\_\_

9.  $\log_2 486 =$  \_\_\_\_\_

10. Given the work you have just done, what other values would you need to figure out the value of the base 2 log for any number?

Sometimes thinking about equivalent expressions with logarithms can get tricky. Consider each of the following expressions and decide if they are always true for the numbers in the domain of the logarithmic function, sometimes true, or never true. Explain your answers. If you answer “sometimes true”, describe the conditions that must be in place to make the statement true.

11.  $\log_4 5 - \log_4 x = \log_4 \left(\frac{5}{x}\right)$  \_\_\_\_\_

12.  $\log 3 - \log x - \log x = \log \left(\frac{3}{x^2}\right)$  \_\_\_\_\_

13.  $\log x - \log 5 = \frac{\log x}{\log 5}$  \_\_\_\_\_

14.  $5 \log x = \log x^5$  \_\_\_\_\_

15.  $2 \log x + \log 5 = \log(x^2 + 5)$  \_\_\_\_\_

16.  $\frac{1}{2} \log x = \log \sqrt{x}$  \_\_\_\_\_

17.  $\log(x - 5) = \frac{\log x}{\log 5}$  \_\_\_\_\_

READY, SET, GO!

Name

Period

Date

**READY**

Topic: Solving simple exponential and logarithmic equations

You have solved exponential equations before based on the idea that  $a^x = a^y$ , **if and only if**  $x = y$ .

You can use the same logic on logarithmic equations.  $\log_a x = \log_a y$ , **if and only if**  $x = y$

**Rewrite each equation to set up a one-to-one correspondence between all of the parts. Then solve for  $x$ .**

Example: <i>Original equation</i>	<i>Rewritten equation:</i>	<i>Solution:</i>
a.) $3^x = 81$	$3^x = 3^4$	$x = 4$
b.) $\log_2 x - \log_2 5 = 0$	$\log_2 x = \log_2 5$	$x = 5$

1.  $3^{x+4} = 243$

2.  $\left(\frac{1}{2}\right)^x = 8$

3.  $\left(\frac{3}{4}\right)^x = \frac{27}{64}$

4.  $\log_2 x - \log_2 13 = 0$

5.  $\log_2(2x - 4) - \log_2 8 = 0$

6.  $\log_2(x + 2) - \log_2 9x = 0$

7.  $\frac{\log 2x}{\log 14} = 1$

8.  $\frac{\log(5x-1)}{\log 29} = 1$

9.  $\frac{\log 5^{(x-2)}}{\log 625} = 1$

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**SET**

Topic: Rewriting logs in terms of known logs

**Use properties of logarithms to rewrite the indicated logarithms in terms of the given values, then find the indicated logarithm with using a calculator.**

Do not use a calculator to evaluate the logarithms.

**Given:**  $\log 16 \approx 1.2$   
 $\log 5 \approx 0.7$   
 $\log 8 \approx 0.9$

10. Find  $\log \frac{5}{8}$

11. Find  $\log 25$

12. Find  $\log \frac{1}{2}$

13. Find  $\log 80$

14. Find  $\log \frac{1}{64}$

**Given**  $\log_3 2 \approx 0.6$   
 $\log_3 5 \approx 1.5$

15. Find  $\log_3 16$

16. Find  $\log_3 108$

17. Find  $\log_3 \frac{3}{50}$

18. Find  $\log_3 \frac{8}{15}$

19. Find  $\log_3 486$

20. Find  $\log_3 18$

21. Find  $\log_3 120$

22. Find  $\log_3 \frac{32}{45}$

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GO

Topic: Using the definition of logarithm to solve for x

**Use your calculator and the definition of  $\log x$  (recall the base is 10) to find the value of x.**  
(Round your answers to 4 decimals.)

23.  $\log x = -3$

24.  $\log x = 1$

25.  $\log x = 0$

26.  $\log x = \frac{1}{2}$

27.  $\log x = 1.75$

28.  $\log x = -2.2$

29.  $\log x = 3.67$

30.  $\log x = \frac{3}{4}$

31.  $\log x = 6$

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## 2.5 Powerful Tens

### A Practice Understanding Task

### Table Puzzles

1. Find the missing values of  $x$  in the tables:

a.

$x$	$y = 10^x$
-2	$\frac{1}{100}$
1	10
	50
	100
3	1000



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b.

$x$	$y = 3(10^x)$
	0.3
0	3
	94.87
2	300
	1503.56

c. What equations could be written, in terms of  $x$  only, for each of the rows that are missing the  $x$  in the two tables above?

d.

$x$	$y = \log x$
0.01	-2
	-1
10	1
	1.6
100	2

e.

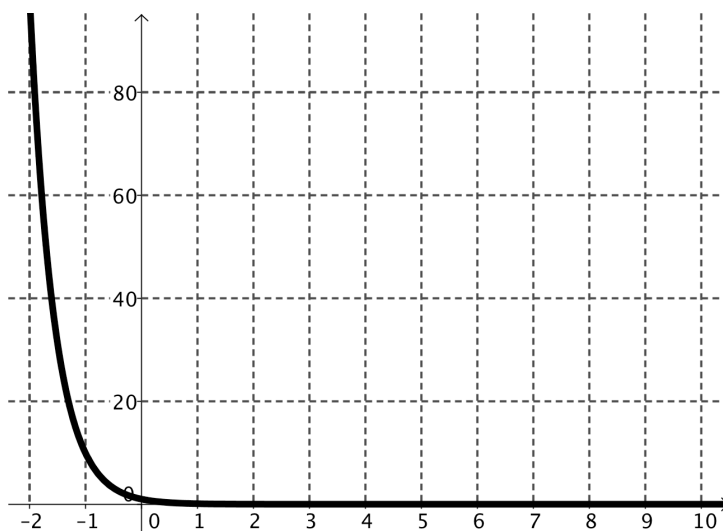
$x$	$y = \log(x + 3)$
	-2
-2.9	-1
	0.3
7	1
	3

- f. What equations could be written, in terms of  $x$  only, for each of the rows that are missing the  $x$  in the two tables above?
2. What strategy did you use to find the solutions to equations generated by the tables that contained exponential functions?
3. What strategy did you use to find the solutions to equations generated by the tables that contained logarithmic functions?

## Graph Puzzles

4. The graph of  $y = 10^{-x}$  is given below. Use the graph to solve the equations for  $x$  and label the solutions.

- a.  $40 = 10^{-x}$   
 $x = \underline{\hspace{2cm}}$   
Label the solution with an A on the graph.
- b.  $10^{-x} = 10$   
 $x = \underline{\hspace{2cm}}$   
Label the solution with a B on the graph.
- c.  $10^{-x} = 0.1$   
 $x = \underline{\hspace{2cm}}$   
Label the solution with a C on the graph.





5. The graph of  $y = -2 + \log x$  is given below. Use the graph to solve the equations for  $x$  and label the solutions.

a.  $-2 + \log x = -2$

$x = \underline{\hspace{2cm}}$

Label the solution with an A on the graph.

b.  $-2 + \log x = 0$

$x = \underline{\hspace{2cm}}$

Label the solution with a B on the graph.

c.  $-4 = -2 + \log x$

$x = \underline{\hspace{2cm}}$

Label the solution with a C on the graph.

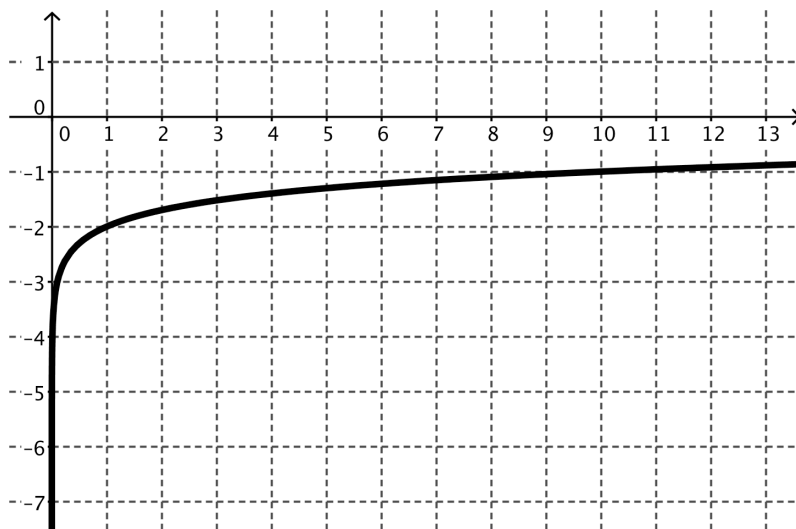
d.  $-1.3 = -2 + \log x$

$x = \underline{\hspace{2cm}}$

Label the solution with a D on the graph.

e.  $1 = -2 + \log x$

$x = \underline{\hspace{2cm}}$



6. Are the solutions that you found in #5 exact or approximate? Why?

## Equation Puzzles

Solve each equation for  $x$

7.  $10^x = 10,000$

8.  $125 = 10^x$

9.  $10^{x+2} = 347$

10.  $5(10^{x+2}) = 0.25$

11.  $10^{-x-1} = \frac{1}{36}$

12.  $-(10^{x+2}) = 16$

READY, SET, GO!

Name \_\_\_\_\_

Period \_\_\_\_\_

Date \_\_\_\_\_

**READY**

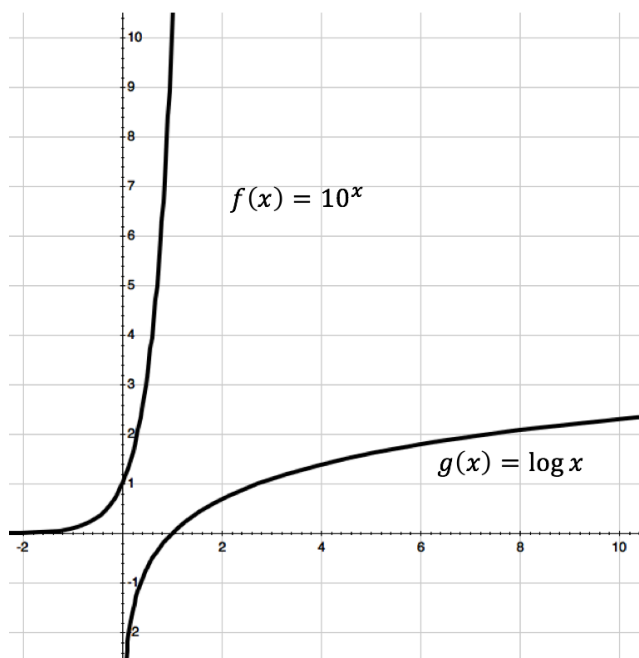
Topic: Comparing the graphs of the exponential and logarithmic functions

The graphs of  $f(x) = 10^x$  and  $g(x) = \log x$  are shown in the same coordinate plane.

Make a list of the characteristics of each function.

1.  $f(x) = 10^x$

2.  $g(x) = \log x$



Each question below refers to the graphs of the functions  $f(x) = 10^x$  and  $g(x) = \log x$ . State whether they are true or false. If they are false, correct the statement so that it is true.

- \_\_\_\_\_ 3. Every graph of the form  $g(x) = \log x$  will contain the point  $(1, 0)$ .
- \_\_\_\_\_ 4. Both graphs have vertical asymptotes.
- \_\_\_\_\_ 5. The graphs of  $f(x)$  and  $g(x)$  have the same rate of change.
- \_\_\_\_\_ 6. The functions are inverses of each other.
- \_\_\_\_\_ 7. The range of  $f(x)$  is the domain of  $g(x)$ .
- \_\_\_\_\_ 8. The graph of  $g(x)$  will never reach 3.

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### SET

Topic: Solving logarithmic equations (*base 10*) by taking the log of each side

**Evaluate the following logarithms**

- |                  |                     |                         |                  |
|------------------|---------------------|-------------------------|------------------|
| 9. $\log 10$     | 10. $\log 10^{-7}$  | 11. $\log 10^{75}$      | 12. $\log 10^x$  |
| 13. $\log_3 3^5$ | 14. $\log_8 8^{-3}$ | 15. $\log_{11} 11^{37}$ | 16. $\log_m m^n$ |

You can use this property of logarithms to help you solve logarithmic equations.

*\*Note: This property only works when the base of the logarithm matches the base of the exponent.*

**Solve the equations by inserting  $\log$  on both sides of the equation.** (You will need a calculator.)

- |                    |                    |                     |                      |
|--------------------|--------------------|---------------------|----------------------|
| 17. $10^n = 4.305$ | 18. $10^n = 0.316$ | 19. $10^n = 14,521$ | 20. $10^n = 483.059$ |
|--------------------|--------------------|---------------------|----------------------|

### GO

Topic: Solving equations involving compound interest

**Formula for compound interest:** If  $P$  dollars is deposited in an account paying an annual rate of interest  $r$  compounded (paid)  $n$  times per year, the account will contain  $A = P \left(1 + \frac{r}{n}\right)^{nt}$  dollars after  $t$  years.

21. How much money will there be in an account at the end of 10 years if \$3000 is deposited at 6% annual interest compounded as follows: (Assume no withdrawals are made.)
- a.) annually
  - b.) semiannually
  - c.) quarterly
  - d.) daily (Use  $n = 365$ .)
22. Find the amount of money in an account after 12 years if \$5,000 is deposited at 7.5% annual interest compounded as follows: (Assume no withdrawals are made.)
- a.) annually
  - b.) semiannually
  - c.) quarterly
  - d.) daily (Use  $n = 365$ .)

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## 2.6H Compounding the Problem

### *A Develop Understanding Task*



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Part I: As an enterprising young mathematician, you know that your superior knowledge of mathematics will help you make better decisions about all kinds of things in your life. One important area is money \$\$\$\$. So, you've been contemplating the world and wondering how you could maximize the money that you make in your savings account.

You're young and you haven't saved much money yet. As a matter of fact, you only have \$100, but you really want to make the best of it. You like the idea of compound interest, meaning that the bank pays you interest on all the money in your savings account, including whatever interest that they had previously paid you. This sounds like a very good deal. You even remember that the formula for compound interest is exponential. Let's see, it is:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Where  $A$  = the amount of money in the account at any time  $t$

$P$  = the principal, or the original amount invested in the account

$r$  = the annual interest rate

$n$  = the number of compounding periods each year

$t$  = the number of years

1. If your saving account pays a generous 5% per year and is compounded only once each year, how much money would be in the account at the end of one year?
2. How much money would be in the account at the end of 20 years?

It seems like the more compounding periods in the year, the more money that you should make. The question is, does it make a big difference?

3. Compare the amount of money that you would have after 20 years if it is compounded twice each year (semi-annually), 4 times per year (quarterly), 12 times per year (monthly), 365 times per year (daily), and then hourly. Find a way to organize, display and explain your results to your class.

It turns out that the value you found in your compounding problem is 100 times a very famous irrational number, named  $e$ . Because  $e$  is irrational it is a non-terminal, non-repeating decimal number, like  $\pi$ . The first few digits of  $e$  are 2.7182818284590452353602874713527. Like  $\pi$ ,  $e$  is a number that occurs in the mathematics of many real world situations, including exponential growth. One of the formulas using  $e$  results from the thinking that you just did about compound interest. It can be shown that the amount of money  $A$  in a savings account where money is compounded continuously is given by:

$$A = Pe^{rt}$$

$P$  = the principal, or the original amount invested in the account

$r$  = the annual interest rate

$t$  = the number of years

4. It is fairly typical for savings accounts to be compounded monthly. Compare the amount of money in two savings accounts after 10 years with the same initial investment of \$500 and interest rate of 3% in each account if the first account is compounded monthly and the second account is compounded continuously.

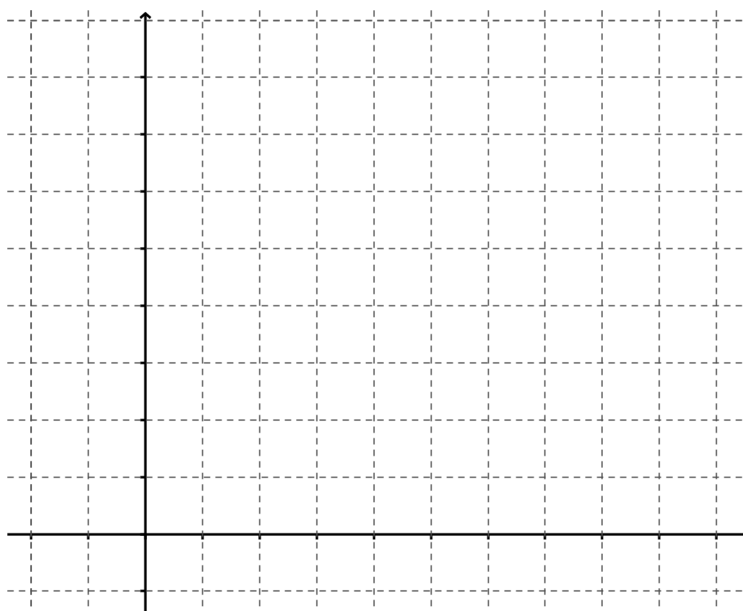
5. Use technology to compare the graphs of the two accounts. What conclusions would you draw about the effect of changing the number of compounding periods on a savings account?

## Part II

Since  $e$  is widely used to model exponential growth and decay in many contexts, let's get a little more familiar with the base  $e$  exponential function:

$$f(x) = e^x$$

1. Make a prediction about the graph of  $f(x)$ . Explain what knowledge you used to make your prediction.
2. Create a table and a graph and describe the mathematical features of  $f(x)$ .



READY, SET, GO!

Name

Period

Date

## READY

Topic: Identifying linear and quadratic patterns by examining the rates of growth

**Use first and second differences to identify the pattern in the tables as *linear*, *quadratic*, or *neither*. Write the recursive equation for the patterns that are *linear* or *quadratic*.**

1.

x	y
-3	-23
-2	-17
-1	-11
0	-5
1	1
2	7
3	13

a. pattern

b. recursive equation

2.

x	y
-3	4
-2	0
-1	-2
0	-2
1	0
2	4
3	10

a. pattern

b. recursive equation

3.

x	y
-3	-15
-2	-10
-1	-5
0	0
1	5
2	10
3	15

a. pattern

b. recursive equation

4.

x	y
-3	24
-2	22
-1	20
0	18
1	16
2	14
3	12

a. pattern

b. recursive equation

5.

x	y
-3	48
-2	22
-1	6
0	0
1	4
2	18
3	42

a. pattern

b. recursive equation

6.

x	y
-3	4
-2	1
-1	0
0	1
1	4
2	9
3	16

a. pattern

b. recursive equation

## SET

Topic: Compounding interest continuously and base  $e$

**Recall the equations for compound interest you used in class today:**

$$A = P \left( 1 + \frac{r}{n} \right)^{nt} \text{ and } A = Pe^{rt}$$

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7. Calculate the amount of money in an account after 15 years if \$7000 is deposited at 6% annual interest compounded as follows:
- a) annually    b) semiannually    c) quarterly    d) daily (Use  $n = 365$ .)    e) continuously
8. How much money will be in an account at the end of 34 years if \$17,000 is deposited at 12% annual interest compounded as follows?
- a) annually    b) semiannually    c) quarterly    d) daily (Use  $n = 365$ .)    e) continuously
9. Fill in the table for each of the given functions. Then graph each function on the same axes.

a)

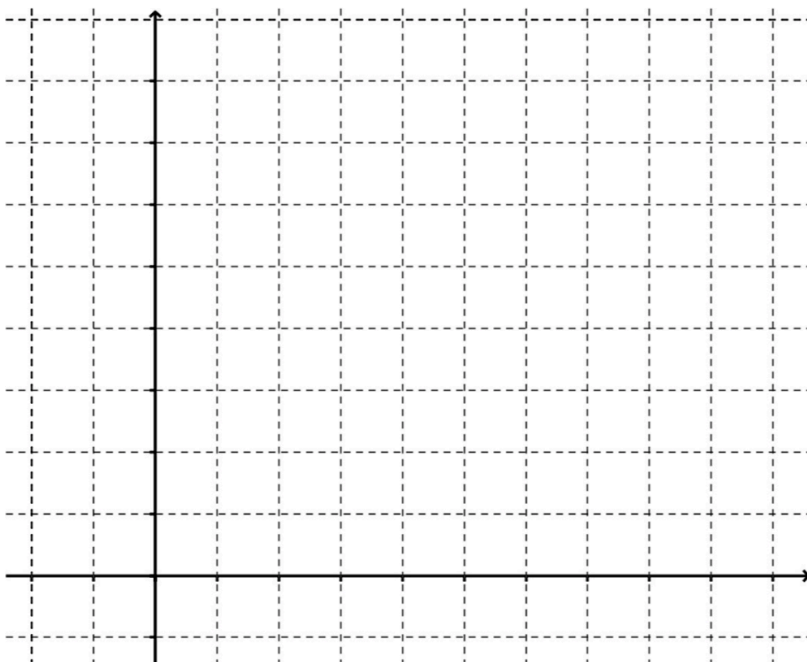
x	$f(x) = 2^x$
-2	
-1	
0	
1	
2	

b)

x	$g(x) = 4^x$
-2	
-1	
0	
1	
2	

c)

x	$h(x) = e^x$
-2	
-1	
0	
1	
2	



10. What point do all 3 functions share?

Why?

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11. Given that  $f(x) = 2^x$ ,  $g(x) = 4^x$ , and  $h(x) = e^x$ , create a true inequality by filling in the spaces on the inequalities with  $f(x)$ ,  $g(x)$ , or  $h(x)$ .

a) \_\_\_\_\_ < \_\_\_\_\_ < \_\_\_\_\_ when  $x > 0$

b) \_\_\_\_\_ < \_\_\_\_\_ < \_\_\_\_\_ when  $x < 0$

c) Write an expression that describes the relationship between  $f(x)$ ,  $g(x)$ , and  $h(x)$  when  $x = 0$ .

### GO

Topic: Exploring properties of logarithms

**Replace the question marks and fill in the blanks using your own logic and what you know about logarithms.**

12.  $\log_7 1 = 0$  because  $7^? =$  \_\_\_\_\_. 13.  $\log_a 1 = 0$  because  $a^? =$  \_\_\_\_\_.

14.  $\log_6 6 = 1$  because  $6^? =$  \_\_\_\_\_. 15.  $\log_a a = 1$  because  $a^? =$  \_\_\_\_\_.

16.  $\log_4 4^x =$  \_\_\_\_\_ because  $4^? = 4^x$ . 17.  $\log_a a^x =$  \_\_\_\_\_ because  $a^? = a^x$ .

18. If  $\log_a x = \log_a y$ , then \_\_\_\_\_ = \_\_\_\_\_.

19.  $3^{\log_3 81} =$  \_\_\_\_\_ because  $\log_3 81$  equals the exponent \_\_\_\_\_ that makes  $3^? = 81$ .

20.  $a^{\log_a x} =$  \_\_\_\_\_ because  $\log_a x$  equals the exponent that makes  $a^? = x$ .

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## 2.7H Logs Go Viral

### *A Solidify Understanding Task*

As we learned in *Compounding the Problem*, when money is compounded continuously, it turns out that the base of the exponential growth function is  $e \approx 2.71828$ . It is true that basically all situations that grow or decay continuously can be modeled with a base  $e$  exponential function. The basic formulas are:

Continuous growth:

$$A = Pe^{rt}$$

Continuous Decay:

$$A = Pe^{-rt}$$

In both cases,  $A$  is the amount of “stuff” at time  $t$ .  $P$  is the amount of “stuff” to start with, or at  $t = 0$ , and  $r$  is the rate of growth or decay. So, if we were talking about money,  $A$  and  $P$  are dollar amounts. If we’re talking about a population,  $A$  and  $P$  are the number of people in the population. If we’re talking about radioactive decay, then  $A$  and  $P$  are the mass of the radioactive substance. In other words,  $P$  is what we started with and  $A$  is what we have after it grows or decays.

Since there are many things that grow or decay continuously in nature,  $e$  is called the natural exponential function and the base  $e$  logarithm is called the natural logarithm. Think about the growth of bacteria. One cell splits into two cells. The two cells begin to split, and the resulting cells split and so on. In a group of many cells, there would be a cell splitting nearly continuously, thus it is a base  $e$  exponential function. So, to use this base  $e$  exponential and its inverse, imagine the following scenarios:

You are an epidemiologist, a person who studies the outbreak and spread of diseases. Part of your job is to help avoid a pandemic—a worldwide outbreak of a disease. You know some of the most difficult diseases to deal with are viruses because they don’t respond to many of the medicines we have available and because viruses are able to mutate and change quickly, making it more difficult to contain them.

You have been studying a new virus that causes people to break out in spots. Suddenly, a colleague rushes into your office to inform you there is a confirmed outbreak of the virus in Europe. The growth of the virus through a population is continuous (until it is somehow contained) at a rate around 3% per day. The current outbreak has 5 confirmed victims.



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1. Using this information, create a model of the spread of the spotted virus in this region if it is not contained. To simplify your model slightly, consider the 5 victims as the number of victims on day 0.

Table

Graph

Equation

2. Based on your model, how many people will be infected on:
  - a) Day 30?
  - b) Day 60?
3. Based upon your model, on what day will there be 50 victims? Show how you arrived at your answer.

4. Will the number of days it will take the virus to claim 100 victims be double the number of days that it took to claim 50 victims? Why or why not?

5. Calculate the number of days it will take for the virus to claim 100 victims.

6. On what day will there be 150 victims?

Now you have received a report of a mysterious illness that seems to turn the infected humans into mindless zombies has broken out in a major American city. Since the hungry zombies prey upon innocent people, the outbreak grows continuously at a rate of 12% per day. The outbreak begins with 80 people.

7. How many zombies will there be after 5 days?

8. How many days will it take for the zombie population to reach 3,700,000 (about the population of Los Angeles, CA)?

9. At what rate would the zombie population be growing if it reached 190,000 people (about the population of Salt Lake City, Utah) in 20 days?

Now we're going to get a little more far-fetched in the scenario. Let's say that zombies produce radioactive goo that decays continuously with a half-life of 3 years. (That's one more danger of having zombies around.) The half-life tells us that after 3 years, only half of the amount of goo we started with is remaining.

10. If we start with 10 pounds of zombie goo, how much will be remaining after 5 years?
11. How long will it take for the amount of zombie goo to decay to an amount less than 0.5 pounds?
12. When will there be no zombie goo left?

READY, SET, GO!

Name

Period

Date

## READY

Topic: Identifying polynomial patterns by examining the rates of growth

Fill in the table for each of the given functions.

1.

$x$	$y = x^1$
-3	
-2	
-1	
0	
1	
2	
3	

2.

$x$	$y = x^2$
-3	
-2	
-1	
0	
1	
2	
3	

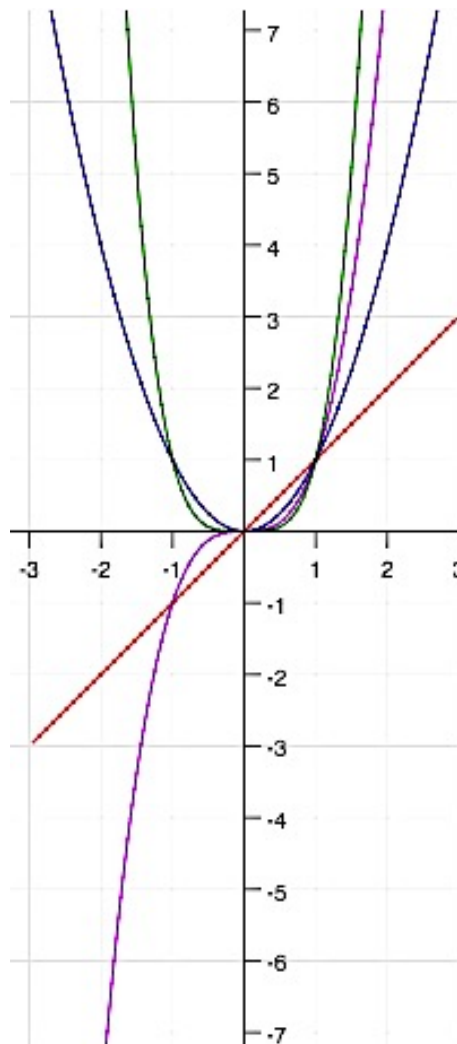
3.

$x$	$y = x^3$
-3	
-2	
-1	
0	
1	
2	
3	

4.

$x$	$y = x^4$
-3	
-2	
-1	
0	
1	
2	
3	

5. Label each graph with the function that describes it.



6. Identify the point(s) that all of the functions share.  
Explain why this is logical.

7. Contrast the graphs of the even functions with the graphs of the odd functions.

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**SET**

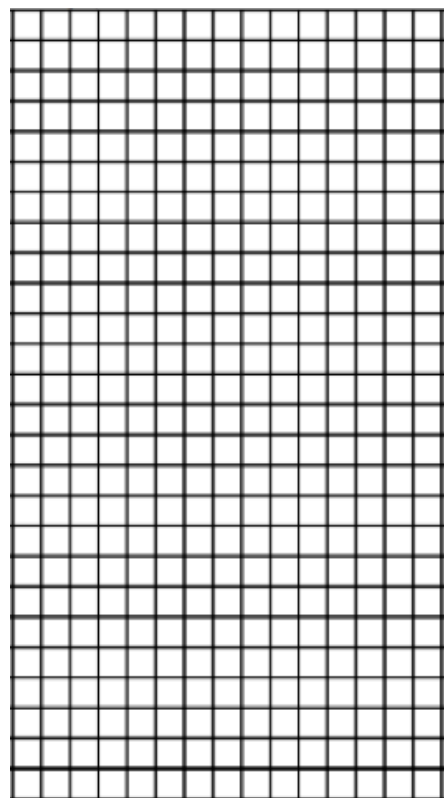
Topic: Using logarithms to solve exponential equations

8. A certain bacteria population is known to double every 15 minutes. An experiment is being conducted in a microbiology lab. Suppose there are initially 7 bacteria in a petri dish. Make a table, graph, and an equation that will predict the number of bacteria in  $t$  hours. Label the scale on both the x and y axes. The increments on the x-axis should be  $\frac{1}{4}$  of an hour or less. Make sure you can fit at least 3 points on your graph.

Time in hours (4 periods of doubling per hour)	Number of bacteria
0	
1	
2	
3	
4	
5	

Equation:

9. Between what times to the nearest  $\frac{1}{4}$  of an hour will the number of bacteria exceed 10,000? 1,000,000?
10. Predict the number of bacteria after a 24 hour period. (Write your answer in scientific notation)
11. Write a logarithmic equation that would allow you to find the time  $t$  when there are 700 bacteria.
12. Calculate the time when there are 700 bacteria. (Round your answer to 3 decimals.)



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GO

Topic: Applying the properties of logarithms

**Use the properties of logarithms and the values below to find the value of the indicated logarithm. Do not use a calculator to evaluate the logarithms.**

13.  $\log 12 \approx 1.1$  Find  $\log \frac{2}{3}$ .

$$\log 8 \approx 0.9$$

$$\log 7 \approx 0.8$$

14.  $\log 12 \approx 1.1$  Find  $\log \frac{1}{7}$ .

$$\log 8 \approx 0.9$$

$$\log 7 \approx 0.8$$

15.  $\log 12 \approx 1.1$  Find  $\log \frac{7}{8}$ .

$$\log 8 \approx 0.9$$

$$\log 7 \approx 0.8$$

16.  $\log 12 \approx 1.1$  Find  $\log \frac{3}{14}$ .

$$\log 8 \approx 0.9$$

$$\log 7 \approx 0.8$$

17.  $\log_8 6 \approx 0.86$  Find  $\log_8 729$ .

$$\log_8 9 \approx 1.06$$

$$\log_8 7 \approx 0.94$$

18.  $\log_8 6 = A$  Find  $\log_8 729$ .

$$\log_8 9 = B$$

$$\log_8 7 = C$$

19.  $\log_8 6 \approx 0.86$  Find  $\log_8 \frac{2}{3}$ .

$$\log_8 9 \approx 1.06$$

$$\log_8 7 \approx 0.94$$

20.  $\log_8 6 = A$  Find  $\log_8 \frac{14}{3}$ .

$$\log_8 9 = B$$

$$\log_8 7 = C$$

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## 2.8H Choose This, Not That

### *A Solidify Understanding Task*

In each of the following equations you are given two options for the next step. Your job is to pick the most productive of the two options, solve the equation and check your solution to be sure you made the right choice. When you are finished, go back and explain why the option you did not choose was either wrong or unproductive.

1.  $\log 2x = 3$

Option 1:  $2x = \log 3$

Option 2:  $10^3 = 2x$

Solution:

Check:

Why I didn't select Option \_\_\_\_:

2.  $\ln(x + 3) = 2$

Option 1:  $\ln x + \ln 3 = 2$

Option 2:  $e^2 = x + 3$

Solution:

Check:

Why I didn't select Option \_\_\_\_:



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3.  $\log_3(2x + 1) = 2$

Option 1:  $3^{2x+1} = 3^2$

Option 2:  $2x + 1 = 3^2$

Solution:

Check:

Why I didn't select Option \_\_\_\_:

4.  $\log_5(2x - 7) = \log_5 3$

Option 1:  $2x - 7 = 3$

Option 2:  $5^3 = 2x - 7$

Solution:

Check:

Why I didn't select Option \_\_\_\_:

5.  $2 \log_3 x = \log_3 4$

Option 1:  $2x = 4$

Option 2:  $\log_3 x^2 = \log_3 4$

Solution:

Check:

Why I didn't select Option \_\_\_\_:

6.  $3 \ln x = \ln 16 + \ln 4$

Option 1:  $\ln x^3 = \ln(16 \cdot 4)$

Option 2:  $3x = 16 + 4$

Solution:

Check:

Why I didn't select Option \_\_\_\_:

7.  $\log_2 2x - \log_2(x - 2) = \log_2 3$

Option 1:  $\log_2 \left( \frac{2x}{x-2} \right) = \log_2 3$

Option 2:  $\frac{\log_2 2x}{\log_2(x-2)} = \log_2 3$

Solution:

Check:

Why I didn't select Option \_\_\_\_:

8.  $-2 = \log_x \frac{1}{9}$

Option 1:  $x^{-2} = \frac{1}{9}$

Option 2:  $-2 = \log_x 1 - \log_x 9$

Solution:

Check:

Why I didn't select Option \_\_\_\_:

9.  $x = \log_3 10$

Option 1:  $x^3 = 10$

Option 2:  $3^x = 10$

Solution:

Check:

Why I didn't select Option \_\_\_\_:

10.  $\log_a(x^2 + 1) + 2\log_a 4 = \log_a 40x$

Option 1:  $\log_a 16(x^2 + 1) = \log_a 40x$

Option 2:  $\log_a 8(x^2 + 1) = \log_a 40x$

Solution:

Check:

Why I didn't select Option \_\_\_\_:

READY, SET, GO!

Name \_\_\_\_\_

Period \_\_\_\_\_

Date \_\_\_\_\_

**READY**

Topic: Evaluating functions

1. Find  $h(-11)$  given that  $h(x) = 2x^2 + 9x - 43$
2. Find  $r(-1)$  given that  $r(x) = -5x^2 - 3x + 9$
3. Find  $f(4)$ , given that  $f(x) = x^2 + 11$ .
4. Find  $m(3)$  given that  $m(x) = \log_x 81$ .
5. Find  $g(-3)$  given that  $g(x) = x^2 + 2x + 4$ .
6. Find  $p(3)$  given that  $p(x) = 5^x + 2x$ .
7. Find  $q(2)$  given that  $q(x) = 7^x + 11x$ .
8. Find  $s\left(\frac{1}{2}\right)$  given that  $s(x) = 12x^2$ .

**SET**

Topic: Finding solutions to logarithmic equations

**Three possible solutions are given for each equation. Determine which solution is correct.**

**Justify your answers.**

- |  |  |   |
|--|--|---|
| <p>9. <math>\log 5x = 3</math></p> <p>a. <math>x = 3 - \log 5</math></p> <p>b. <math>x = 200</math></p> <p>c. <math>x = \frac{3}{5}</math></p>                           | <p>10. <math>\log(x + 28) = 2</math></p> <p>a. <math>x = 72</math></p> <p>b. <math>x = 2 - \log 28</math></p> <p>c. <math>x = \frac{100}{28}</math></p>              | <p>11. <math>\log_3(2x + 1) = 2</math></p> <p>a. <math>x = \frac{1}{2}</math></p> <p>b. <math>x = \frac{5}{2}</math></p> <p>c. <math>x = 4</math></p> |
| <p>12. <math>\log_5(3x - 8) = \log_5 13</math></p> <p>a. <math>x = 7</math></p> <p>b. <math>x = \frac{5^{13} + 8}{3}</math></p> <p>c. <math>x = \frac{104}{3}</math></p> | <p>13. <math>3\log x = \log 16 + \log 4</math></p> <p>a. <math>x = \frac{20}{3}</math></p> <p>b. <math>x = 4</math></p> <p>c. <math>x = \sqrt[3]{10^{20}}</math></p> | <p>14. <math>\log_2 2x - \log_2(x - 2) = \log_2 3</math></p> <p>a. <math>x = 6</math></p> <p>b. <math>x = 3</math></p> <p>c. <math>x = -6</math></p>  |
| <p>15. <math>-3 = \log_x\left(\frac{1}{8}\right)</math></p> <p>a. <math>x = -2</math></p> <p>b. <math>x = 2</math></p> <p>c. <math>x = 4</math></p>                      | <p>16. <math>x = \log_3 15</math></p> <p>a. <math>x = \sqrt[3]{15}</math></p> <p>b. <math>x = 5</math></p> <p>c. <math>x \approx 2.465</math></p>                    | <p>17. <math>\log_a(x - 7) = 0</math></p> <p>a. <math>x = 7</math></p> <p>b. <math>x = 8</math></p> <p>c. no solution</p>                             |

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**Circle the expressions that are equal. Explain why they are equal.**

18.  $\log_5 \sqrt{50}$ ,  $\log_5 25$ ,  $1 + \log_5 \sqrt{2}$

19.  $\frac{\log_2 32}{\log_2 4}$ ,  $\log_2 \frac{32}{4}$ ,  $\log_2 32 - \log_2 4$

20.  $\log \sqrt{90}$ ,  $\log 3 + \frac{1}{2}$ ,  $\frac{1}{2} \log 2 + \log 45$

21.  $\log_7 \left( \frac{1}{49} \right)$ ,  $\log_7 1 - \log_7 49$ ,  $-2(\log_7 7)$

**GO**

Topic: Solving exponential equations

**Solve for x.**

22.  $4^{(2x-7)} = 64$

23.  $5^x = \frac{1}{125}$

24.  $3^{(2x+8)} = 729$

25.  $\left( \frac{1}{2} \right)^x = 128$

26.  $36^{(x+5)} = 216^{(x-3)}$

27.  $\left( \frac{2}{3} \right)^x = \frac{16}{81}$

28.  $3^{-x} = 27$

29.  $\left( \frac{3}{4} \right)^x = \frac{16}{9}$

30.  $125^{(3x-4)} = 625^{(x+1)}$

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## 2.9H Don't Forget Your Login

### *A Practice Understanding Task*

Solve each of the following equations. When you have finished, sort the equations into categories based upon the strategy you used to solve them. Name each category and then describe how to solve equations in this category.

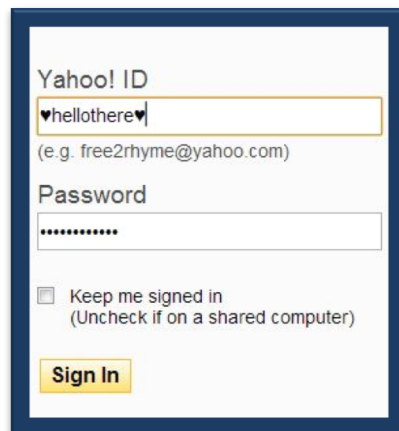
1.  $\log 3x = 2$

2.  $-3 = \log_x \frac{1}{125}$

3. The rate at which caffeine is eliminated from the bloodstream of an average adult is about 15% per hour. If the peak level of caffeine in the bloodstream is 30 milligrams, the amount of caffeine left in the bloodstream  $t$  hours after the caffeine reaches its peak level can be modeled by the function:  $C(t) = 30(0.85)^t$ . After how many hours will there be 15 mg left in the bloodstream?

4.  $x = \log_5 100$

5.  $\ln(5x - 3) + \ln 2 = \ln(24 - 2x)$



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6.  $\log_5(4x - 3) = \log_5 29$
7. The Richter scale, which measures the magnitude of earthquakes is a logarithmic scale, where the magnitude of the earthquake,  $M$  depends on the energy released by the earthquake  $E$  (in Joules). In 1994, an earthquake of magnitude 6.6 on the Richter scale injured thousands of people and cost billions of dollars in damages. That earthquake could be modeled with the equation:  $6.6 = \frac{2}{3} \log \left( \frac{E}{10^{11.8}} \right)$ . Find the energy released by the earthquake.
8.  $\log_5(3x + 1) = 2$
9.  $\log_b x^3 = \log_b 27$
10. Ever wonder why suddenly your kitchen is full of fruit flies? Given good conditions, fruit fly populations can grow at the amazing rate of 28% per day. If 25 fruit flies enter your house to hang out on a piece of ripe fruit, the fly population after  $t$  days can be modeled as:  $P(t) = 25(1.28)^t$ . How long will it take for you to have 100 little fruit flies buzzing around?
11.  $\log_x 5 = \frac{1}{4}$
12.  $3^x = 5^{2.3}$



13.  $\log_2 2x - \log_2(x - 2) = \log_2 3$

14.  $\log_3 2x = \log_3(x - 1)$

15.  $\ln(x - 1) = 3$

16.  $\log(x^2 - 2) + 2 \log 6 = \log 6x$

17.  $x = \log_3 10$

18.  $2\log_a x + \log_a 2 = \log_a(5x + 3)$

19.  $3 + 7^{3x+1} = 346$

Category Name and Description:	Category Name and Description:
How to solve equations in this category:	How to solve equations in this category:
Category Name and Description:	Category Name and Description:
How to solve equations in this category:	How to solve equations in this category:
Category Name and Description:	Category Name and Description:
How to solve equations in this category:	How to solve equations in this category:

Note: You may not need all these categories.

READY, SET, GO!

Name \_\_\_\_\_

Period \_\_\_\_\_

Date \_\_\_\_\_

## READY

Topic: I Comparing solutions and x-intercepts

**Solve for x in each equation.**

1.  $x^2 + x - 2 = 0$

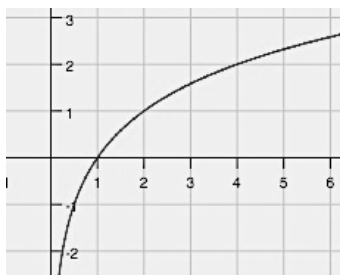
2.  $2x - 6 = 0$

3.  $0 = \log_2 x$

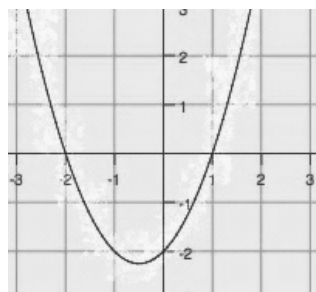
4.  $x^3 - 4x = 0$

5. Match each of the graphs below with one of the above equations. Explain the criteria you used to decide which graph was related to each equation.

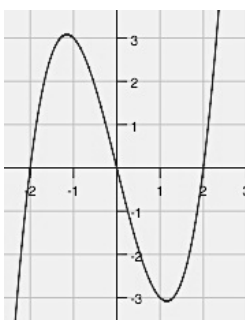
a) Equation? How did you know?



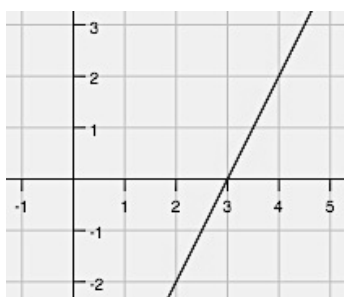
b) Equation? How did you know?



c) Equation? How did you know?



d) Equation? How did you know?



6. Compare the solutions to each equation with the x-intercepts in the graphs. Make a statement about your observations.

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**SET**

Topic: Solving logarithmic equations

**Solve each equation.**

7.  $3 \log_2(x + 4) = 9$

8.  $\log_3(x^2 + 2) = 3$

9.  $\log_5(x^2 + x + 5) = 2$

10.  $2 \log_5 x = 3 \log_5 4$

11.  $\log_7(3x) = \log_7(5x - 8)$

12.  $\log(7x - 12) = 2 \log x$

13.  $4^{1-2x} = 2$

14.  $9^{2x} = 27^{3x-4}$

15.  $5^{x-3} = \sqrt{5}$

16.  $10^{2x} = 80$

17.  $\log_2 4x - \log_2(x - 5) = \log_2 8$

18.  $-3 \log_5 \left( \frac{7x}{10} \right) = 3$

19. On the Richter scale, the magnitude  $R$  of an earthquake of intensity  $I$  is  $R = \log_{10} \frac{I}{I_0}$ , where  $I_0 = 1$  is the minimum intensity used for comparison. If we substitute 1 for  $I_0$ , the formula for measuring the intensity per unit area for an earthquake becomes  $R = \log_{10} I$ .

Find the intensity  $I$  for each of the following earthquakes.

a) San Francisco, California 1906:  $R = 7.7$

b) The World Series Quake, San Francisco, California 1989:  $R = 6.9$

c) The 1906 quake was “only” 0.8 units more on the Richter scale. That doesn’t seem like much. Create a ratio of the intensity of the 1906 quake to the 1989 quake. How many times greater in intensity was the 1906 quake?

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GO

Topic: Reviewing logarithmic functions

Arrange the following expressions in numerical order from smallest to largest. Do not use a calculator. Be prepared to explain your logic.

- |     | 1           | 2            | 3           | 4                     | 5            |
|-----|-------------|--------------|-------------|-----------------------|--------------|
| 20. | $\log_3 81$ | $\log_5 125$ | $\log_8 8$  | $\log_4 1$            | $\log 100$   |
| 21. | $\log 200$  | $\log 0.02$  | $\log_2 10$ | $\log_2 \frac{1}{10}$ | $\log_2 200$ |

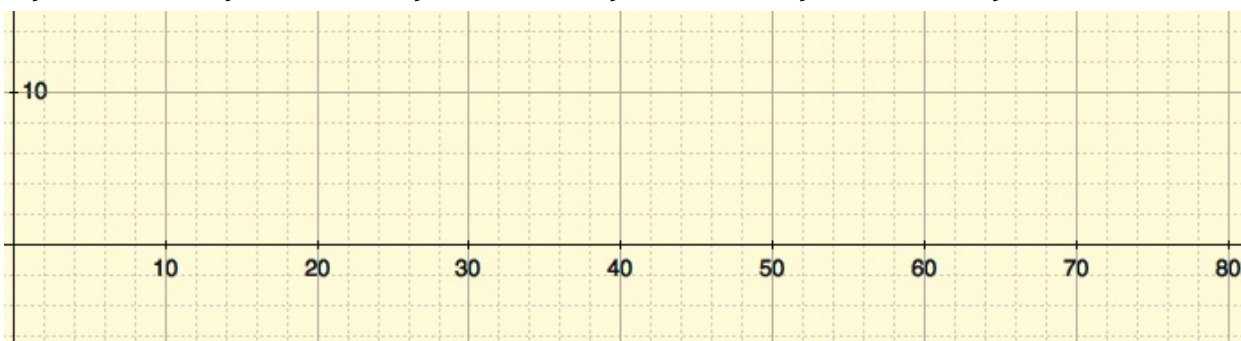
22. Convert to Logarithmic Form.    a)  $3^6 = 729$     b)  $5^{-2} = 0.04$     c)  $\left(\frac{4}{7}\right)^{-1} = \frac{7}{4}$
23. Convert to Exponential Form.    a)  $\log_6 216 = 3$     b)  $\log_9 1 = 0$     c)  $\log_2 0.5 = -1$

Use the properties of logarithms to rewrite each expression in expanded form. Assume all variables represent positive real numbers.

24.  $\log_4 4x^3$     25.  $\log_5 \sqrt{\frac{m}{n}}$     26.  $\log_3 \frac{9w}{xyz}$

27. The number of fish in an aquarium is given by  $f(t) = 4\log(5t + 10)$ , where  $t$  is time in months. Find the number of fish present given the following times. Then graph  $f(t)$ .

- a)  $t = 0$     b)  $t = 12$     c)  $t = 24$     d)  $t = 36$     e)  $t = 60$     f)  $t = 72$



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