

Transforming Mathematics Education

ALGEBRA II

An Integrated Approach

MODULE 3

Numbers & Operations

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

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(A.APR.1, F.BF.1b)

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3.1 It All Adds Up

A Develop Understanding Task

Whenever we're thinking about algebra and working with variables, it is useful to consider how it relates to the number system and operations on numbers. Let's see if we can make some useful comparisons between whole numbers and polynomials.



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Let's start by looking at the structure of numbers and polynomials. Consider the number 132. The way we write numbers is really a shortcut because:

$$132 = 100 + 30 + 2$$

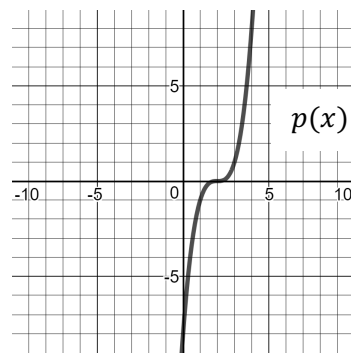
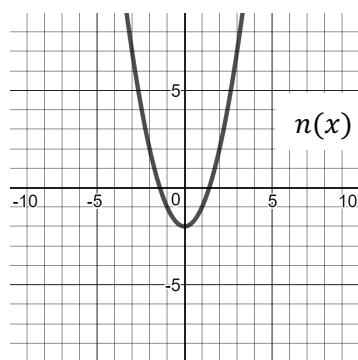
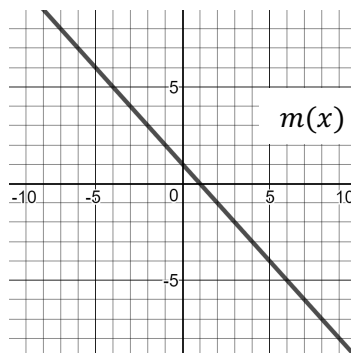
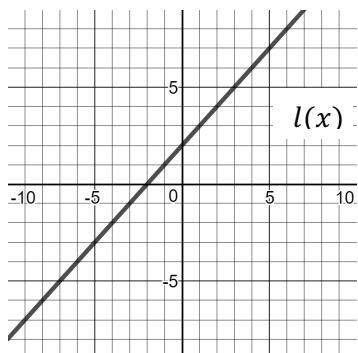
1. Compare 132 to the polynomial $x^2 + 3x + 2$. How are they alike? How are they different?
2. Write a polynomial that is analogous to the number 2,675.

When two numbers are to be added together, many people use a procedure like this:

$$\begin{array}{r} 132 \\ + 451 \\ \hline 583 \end{array}$$

3. Write an analogous addition problem for polynomials and find the sum of the two polynomials.
4. How does adding polynomials compare to adding whole numbers?
5. Use the polynomials below to find the specified sums in a-f.

$$f(x) = x^3 + 3x^2 - 2x + 10 \quad g(x) = 2x - 1 \quad h(x) = 2x^2 + 5x - 12 \quad k(x) = -x^2 - 3x + 4$$



a) $h(x) + k(x)$

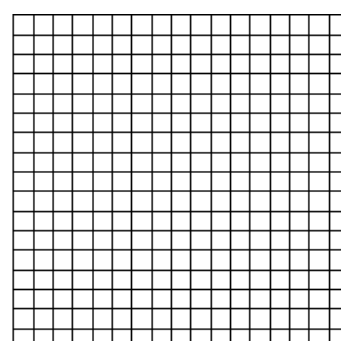
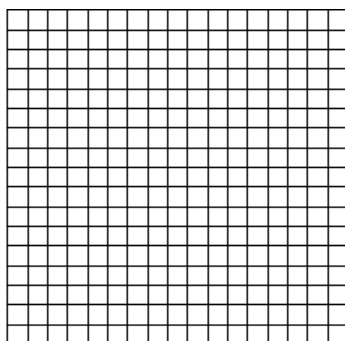
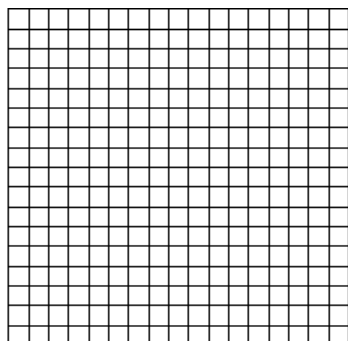
b) $g(x) + f(x)$

c) $f(x) + k(x)$

d) $l(x) + m(x)$

e) $m(x) + n(x)$

f) $l(x) + p(x)$



6. What patterns do you see when polynomials are added?

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Subtraction of whole numbers works similarly to addition. Some people line up subtraction vertically and subtract the bottom number from the top, like this:

$$\begin{array}{r} 368 \\ -157 \\ \hline 211 \end{array}$$

7. Write the analogous polynomials and subtract them.

8. Is your answer to #7 analogous to the whole number answer? If not, why not?

9. Subtracting polynomials can easily lead to errors if you don't carefully keep track of your positive and negative signs. One way that people avoid this problem is to simply change all the signs of the polynomial being subtracted and then add the two polynomials together. There are two common ways of writing this:

$$(x^3 + x^2 - 3x - 5) - (2x^3 - x^2 + 6x + 8)$$

Step 1: $= (x^3 + x^2 - 3x - 5) + (-2x^3 + x^2 - 6x - 8)$

Step 2: $= (-x^3 + 2x^2 - 9x - 13)$

Or, you can line up the polynomials vertically so that Step 1 looks like this:

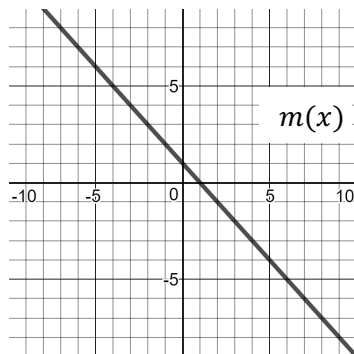
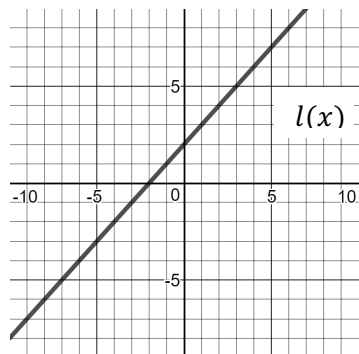
Step 1:
$$\begin{array}{r} x^3 + x^2 - 3x - 5 \\ + (-2x^3 + x^2 - 6x - 8) \\ \hline \end{array}$$

Step 2:
$$\begin{array}{r} x^3 + x^2 - 3x - 5 \\ + (-2x^3 + x^2 - 6x - 8) \\ \hline -x^3 + 2x^2 - 9x - 13 \end{array}$$

The question for you is: Is it correct to change all the signs and add when subtracting? What mathematical property or relationship can justify this action?

10. Use the given polynomials to find the specified differences in a-d.

$$f(x) = x^3 + 2x^2 - 7x - 8 \quad g(x) = -4x - 7 \quad h(x) = 4x^2 - x - 15 \quad k(x) = -x^2 + 7x + 4$$



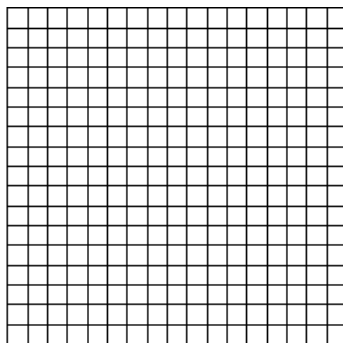
a) $h(x) - k(x)$

b) $f(x) - h(x)$

c) $f(x) - g(x)$

d) $k(x) - f(x)$

e) $l(x) - m(x)$



11. List three important things to remember when subtracting polynomials.

.

READY, SET, GO!

Name

Period

Date

READY

Topic: Using the distributive property

Multiply.

1. $2x(5x^2 + 7)$

2. $9x(-x^2 - 3)$

3. $5x^2(x^4 + 6x^3)$

4. $-x(x^2 - x + 1)$

5. $-3x^3(-2x^2 + x - 1)$

6. $-1(x^2 - 4x + 8)$

SET

Topic: Adding and subtracting polynomials

Add. Write your answers in descending order of the exponents. (Standard form)

7. $(3x^4 + 5x^2 - 1) + (2x^3 + x)$

8. $(4x^2 + 7x - 4) + (x^2 - 7x + 14)$

9. $(2x^3 + 6x^2 - 5x) + (x^5 + 3x^2 + 8x + 4)$

10. $(-6x^5 - 2x + 13) + (4x^5 + 3x^2 + x - 9)$

Subtract. Write your answers in descending order of the exponents. (Standard form)

11. $(5x^2 + 7x + 2) - (3x^2 + 6x + 2)$

12. $(10x^4 + 2x^2 + 1) - (3x^4 + 3x + 11)$

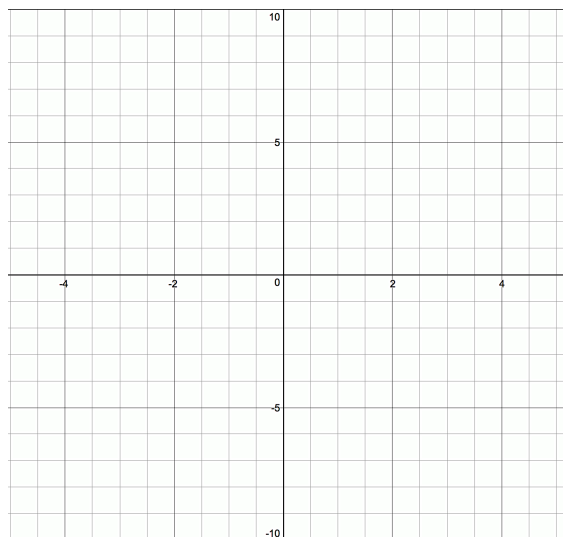
13. $(7x^3 - 3x + 7) - (4x^2 - 3x - 11)$

14. $(x^4 - 1) - (x^4 + 1)$

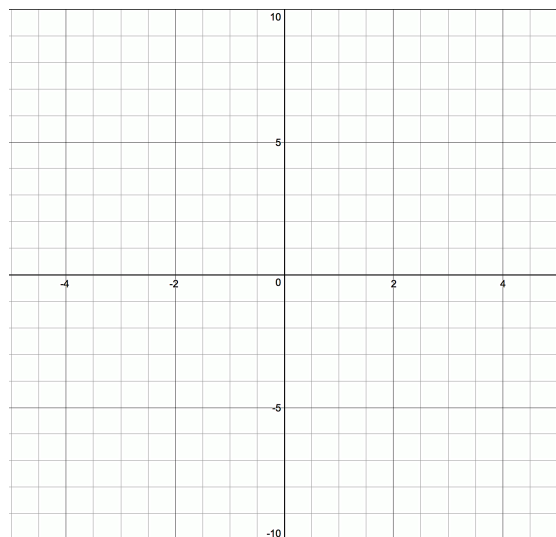
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Graph.

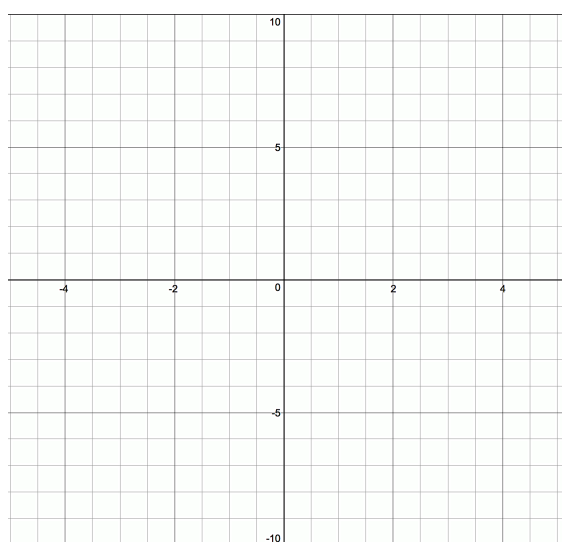
15. $y = x^3 - 2$



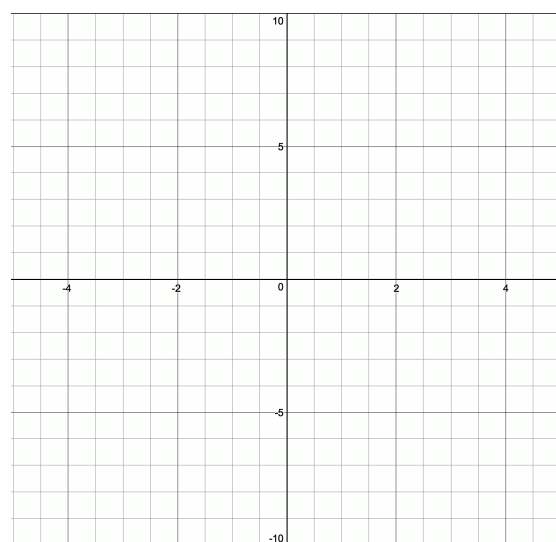
16. $y = x^3 + 1$



17. $y = (x - 3)^3$



18. $y = (x + 1)^3$

**GO**

Topic: Using exponent rules to combine expressions

Simplify.

19. $x^{7/8} \cdot x^{1/4} \cdot x^{-1/2}$

20. $x^{3/16} \cdot x^{-7/8} \cdot x^{3/4}$

21. $x^{4/7} \cdot x^{2/9} \cdot x^{-1/3}$

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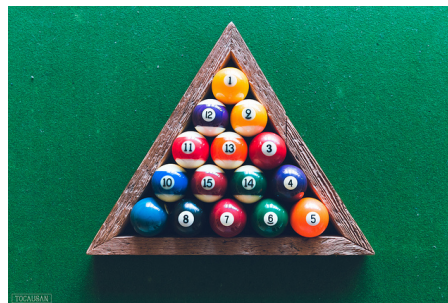
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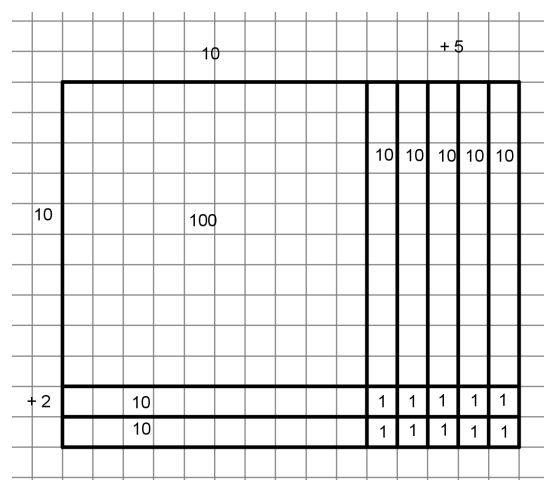
3.2 Pascal's Pride

A Solidify Understanding Task

Multiplying polynomials can require a bit of skill in the algebra department, but since polynomials are structured like numbers, multiplication works very similarly. When you learned to multiply numbers, you may have learned to use an area model. To multiply 12×15 the area model and the related procedure probably looked like this:

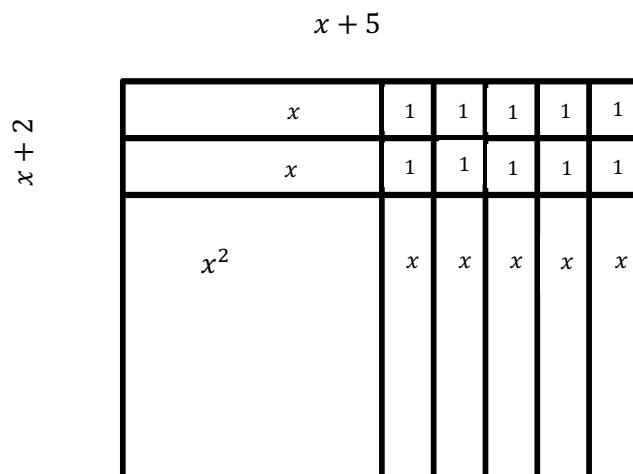


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$$\begin{array}{r}
 12 \\
 \times 15 \\
 \hline
 10 \\
 50 \\
 + 20 \\
 \hline
 100 \\
 \hline
 180
 \end{array}$$

You may have used this same idea with quadratic expressions. Area models help us think about multiplying, factoring, and completing the square to find equivalent expressions. We modeled $(x + 2)(x + 5) = x^2 + 7x + 10$ as the area of a rectangle with sides of length $x + 2$ and $x + 5$. The various parts of the rectangle are shown in the diagram below:



Some people like to shortcut the area model a little bit to just have sections of area that correspond to the lengths of the sides. In this case, they might draw the following.

	x	$+5$	
x	x^2	$5x$	
$+2$	$2x$	10	

$= x^2 + 7x + 10$

1. What is the property that all of these models are based upon?

2. Now that you've been reminded of the happy past, you are ready to use the strategy of your choice to find equivalent expressions for each of the following:

a) $(x + 3)(x + 4)$
b) $(x + 7)(x - 2)$

Maybe now you remember some of the different forms for quadratic expressions—factored form and standard form. These forms exist for all polynomials, although as the powers get higher, the algebra may get a little trickier. In standard form polynomials are written so that the terms are in order with the highest-powered term first, and then the lower-powered terms. Some examples:

Quadratic: $x^2 - 3x + 8$ or $x^2 - 9$

Cubic: $2x^3 + x^2 - 7x - 10$ or $x^3 - 2x^2 + 15$

Quartic: $x^4 + x^3 + 3x^2 - 5x + 4$

Hopefully, you also remember that you need to be sure that each term in the first factor is multiplied by each term in the second factor and the like terms are combined to get to standard form. You can use area models, boxes, or mnemonics like FOIL (first, outer, inner, last) to help you organize, or you can just check every time to be sure that you've got all the combinations. It can get more challenging with higher-powered polynomials, but the principal is the same because it is based upon the mighty Distributive Property.

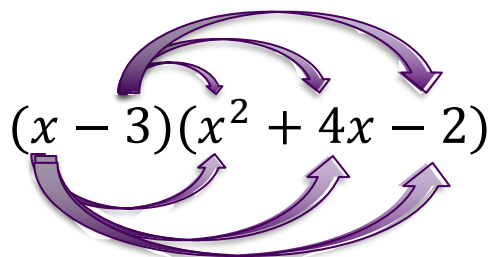
3. Tia's favorite strategy for multiplying polynomials is to make a box that fits the two factors. She sets it up like this: $(x + 2)(x^2 - 3x + 5)$

	x^2	$-3x$	$+5$
x			
$+2$			

Try using Tia's box method to multiply these two factors together and then combining like terms to get a polynomial in standard form.

4. Try checking your answer by graphing the original factored polynomial, $(x + 2)(x^2 - 3x + 5)$ and then graphing the polynomial that is your answer. If the graphs are the same, you are right because the two expressions are equivalent! If they are not the same, go back and check your work to make the corrections.

5. Tehani's favorite strategy is to connect the terms he needs to multiply in order like this:



Try multiplying using Tehani's strategy and then check your work by graphing. Make any corrections you need and figure out why they are needed so that you won't make the same mistake twice!

6. Use the strategy of your choice to multiply each of the following expressions. Check your work by graphing and make any needed corrections.

a) $(x + 5)(x^2 - x - 3)$

b) $(x - 2)(2x^2 + 6x + 1)$

c) $(x + 2)(x - 2)(x + 3)$

When graphing, it is often useful to have a perfect square quadratic or a perfect cube. Sometimes it is also useful to have these functions written in standard form. Let's try re-writing some related expressions to see if we can see some useful patterns.

7. Multiply and simplify both of the following expressions using the strategy of your choice:

a) $f(x) = (x + 1)^2$

b) $f(x) = (x + 1)^3$

Check your work by graphing and make any corrections needed.

8. Some enterprising young mathematician noticed a connection between the coefficients of the terms in the polynomial and the number pattern known as Pascal's Triangle. Put your answers from problem 5 into the table. Compare your answers to the numbers in Pascal's Triangle below and describe the relationship you see.

$(x + 1)^0$	1	1
$(x + 1)^1$	$x + 1$	1 1
$(x + 1)^2$		1 2 1
$(x + 1)^3$		1 3 3 1
$(x + 1)^4$		

9. It could save some time on multiplying the higher power polynomials if we could use Pascal's Triangle to get the coefficients. First, we would need to be able to construct our own Pascal's Triangle and add rows when we need to. Look at Pascal's Triangle and see if you can figure out how to get the next row using the terms from the previous row. Use your method to find the terms in the next row of the table above.

10. Now you can check your Pascal's Triangle by multiplying out $(x + 1)^4$ and comparing the coefficients. Hint: You might want to make your job easier by using your answers from #7 in some way. Put your answer in the table above.

11. Make sure that the answer you get from multiplying $(x + 1)^4$ and the numbers in Pascal's Triangle match, so that you're sure you've got both answers right. Then describe how to get the next row in Pascal's Triangle using the terms in the previous row.

12. Complete the next row of Pascal's Triangle and use it to find the standard form of $(x + 1)^5$. Write your answers in the table on #6.

13. Pascal's Triangle wouldn't be very handy if it only worked to expand powers of $x + 1$. There must be a way to use it for other expressions. The table below shows Pascal's Triangle and the expansion of $x + a$.

$(x + a)^0$	1	1
$(x + a)^1$	$x + a$	1 1
$(x + a)^2$	$x^2 + 2ax + a^2$	1 2 1
$(x + a)^3$	$x^3 + 3ax^2 + 3a^2x + a^3$	1 3 3 1
$(x + a)^4$	$x^4 + 4ax^3 + 6a^2x^2 + 3a^3x + a^4$	1 4 6 4 1

What do you notice about what happens to the a in each of the terms in a row?

14. Use the Pascal's Triangle method to find standard form for $(x + 2)^3$. Check your answer by multiplying.

15. Use any method to write each of the following in standard form:

a) $(x + 3)^3$

b) $(x - 2)^3$

c) $(x + 5)^4$

READY, SET, GO!

Name

Period

Date

READY

Topic: Recalling the meaning of division

1. Given: $f(x) = (x + 7)(2x - 3)$ and $g(x) = (x + 7)$. Find $g(x) \overline{)f(x)}$.
2. Given: $f(x) = (5x + 7)(-3x + 11)$ and $g(x) = (-3x + 11)$. Find $g(x) \overline{)f(x)}$.
3. Given: $f(x) = (x + 2)(x^2 + 3x + 2)$ and $g(x) = (x + 2)$. Find $g(x) \overline{)f(x)}$.
4. Given: $f(x) = (5x - 3)(x^2 - 11x - 9)$ and $g(x) = (5x - 3)$ and $h(x) = (x^2 - 11x - 9)$.
 - a.) Find $g(x) \overline{)f(x)}$
 - b.) Find $h(x) \overline{)f(x)}$
5. Given: $f(x) = (5x - 6)(2x^2 - 5x + 3)$ and $g(x) = (x - 1)$ and $h(x) = (2x - 3)$.
 - a.) Find $g(x) \overline{)f(x)}$
 - b.) Find $h(x) \overline{)f(x)}$

SET

Topic: Multiplying polynomials

Multiply. Write your answers in standard form.

6. $(a + b)(a + b)$
7. $(x - 3)(x^2 + 3x + 9)$
8. $(x - 5)(x^2 + 5x + 25)$
9. $(x + 1)(x^2 - x + 1)$
10. $(x + 7)(x^2 - 7x + 49)$
11. $(a - b)(a^2 + ab + b^2)$

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$(x + a)^0$	1	1
$(x + a)^1$	$x + a$	1 1
$(x + a)^2$	$x^2 + 2ax + a^2$	1 2 1
$(x + a)^3$	$x^3 + 3ax^2 + 3a^2x + a^3$	1 3 3 1
$(x + a)^4$	$x^4 + 4ax^3 + 6a^2x^2 + 3a^3x + a^4$	1 4 6 4 1

Use the table above to write each of the following in standard form.

12. $(x + 1)^5$

13. $(x - 5)^3$

14. $(x - 1)^4$

15. $(x + 4)^3$

16. $(x + 2)^4$

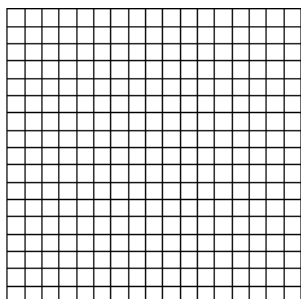
17. $(3x + 1)^3$

GO

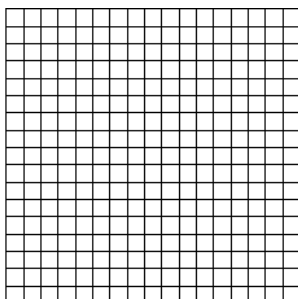
Topic: Examining transformations on different types of functions

Graph the following functions.

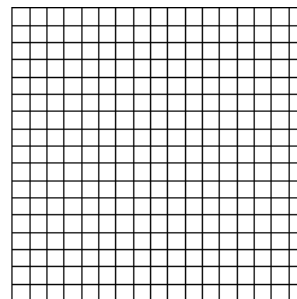
18. $g(x) = x + 2$



19. $h(x) = x^2 + 2$



20. $f(x) = 2^x + 2$



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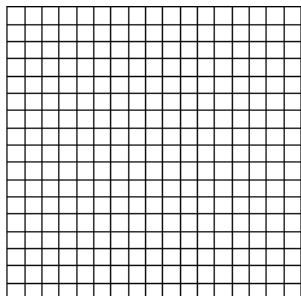
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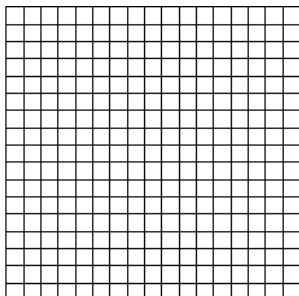
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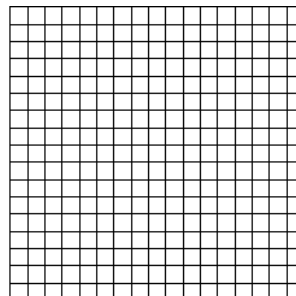
21. $g(x) = 3(x - 2)$



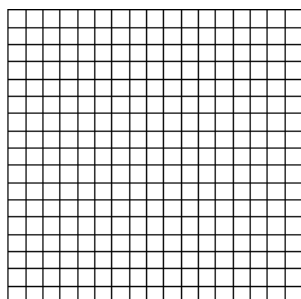
22. $h(x) = 3(x - 2)^2$



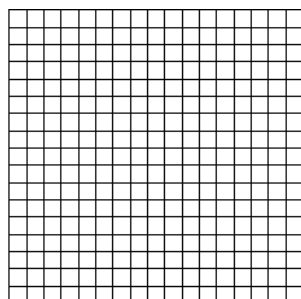
23. $f(x) = 3\sqrt{x - 2}$



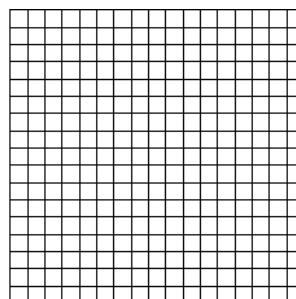
24. $g(x) = \frac{1}{2}(x - 1) - 2$



25. $h(x) = \frac{1}{2}(x - 1)^2 - 2$



26. $f(x) = |x - 1| - 2$



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3.3 Divide And Conquer

A Solidify Understanding Task

We've seen how numbers and polynomials relate in addition, subtraction, and multiplication. Now we're ready to consider division.



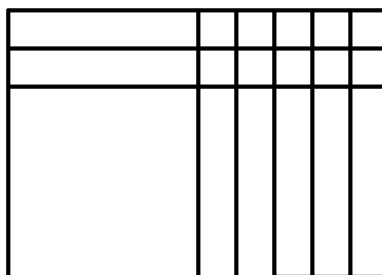
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Division, you say? Like, long division? Yup, that's what we're talking about. Hold the judgment! It's actually pretty cool.

As usual, let's start by looking at how the operation works with numbers. Since division is the inverse operation of multiplication, the same models should be useful. The area model that we used with multiplication is also used with division. When we were using area models to factor a quadratic expression, we were actually dividing.

Let's brush up on that a bit.

1. The area model for $x^2 + 7x + 10$ is shown below:



Use the area model to write $x^2 + 7x + 10$ in factored form.

2. We also used number patterns to factor without drawing the area model. Use any strategy to factor the following quadratic polynomials:

a) $x^2 + 7x + 12$	b) $x^2 + 2x - 15$

c) $x^2 - 11x + 24$	d) $x^2 - 5x - 36$
---------------------	--------------------

Factoring works great for quadratics and a few special cases of other polynomials. Let's look at a more general version of division that is a lot like what we do with numbers. Let's say we want to divide 1452 by 12. If we write the analogous polynomial division problem, it would be:

$$(x^3 + 4x^2 + 5x + 2) \div (x + 2).$$

Let's use the division process for numbers to create a division process for polynomials. (Don't panic—in many ways it's easier with polynomials than numbers!)

Step 1: Start with writing the problem as long division. The polynomial needs to have the terms written in descending order. If there are any missing powers, it's easier if you leave a little space for them.

$$12 \overline{)1452}$$

$$x + 2 \overline{)x^3 + 4x^2 + 5x + 2}$$

Step 2: Determine what you could multiply the divisor by to get the first term of the dividend.

$$\begin{array}{r} 1 \\ 12 \overline{)1452} \end{array}$$

$$\begin{array}{r} x^2 \\ x + 2 \overline{)x^3 + 4x^2 + 5x + 2} \end{array}$$

Step 3: Multiply and put the result below the dividend.

$$\begin{array}{r} 1 \\ 12 \overline{)1452} \\ -1200 \end{array}$$

$$\begin{array}{r} x^2 \\ x + 2 \overline{)x^3 + 4x^2 + 5x + 2} \\ -(x^3 + 2x^2) \end{array}$$

Step 4: Subtract. (It helps to keep the signs straight if you change the sign on each term and add on the polynomial.)

$$\begin{array}{r} 1 \\ 12 \overline{)1452} \\ \underline{-1200} \\ 252 \end{array}$$

$$\begin{array}{r} x^2 \\ x+2 \overline{)x^3+4x^2+5x+2} \\ +(-x^3-2x^2 \quad \quad) \\ \hline 2x^2+5x+2 \end{array}$$

Step 5: Repeat the process with the number or expression that remains in the dividend.

$$\begin{array}{r} 12 \\ 12 \overline{)1452} \\ \underline{-1200} \\ 252 \\ \underline{-240} \\ 12 \end{array}$$

$$\begin{array}{r} x^2+2x \\ x+2 \overline{)x^3+4x^2+5x+2} \\ +(-x^3-2x^2 \quad \quad) \\ \hline 2x^2+5x+2 \\ -(2x^2+4x \quad) \\ \hline x+2 \end{array}$$

Step 6: Keep going until the number or expression that remains is smaller than the divisor.

$$\begin{array}{r} 121 \\ 12 \overline{)1452} \\ \underline{-1200} \\ 252 \\ \underline{-240} \\ 12 \\ \underline{-12} \\ 0 \end{array}$$

$$\begin{array}{r} x^2+2x+1 \\ x+2 \overline{)x^3+4x^2+5x+2} \\ +(-x^3-2x^2 \quad \quad) \\ \hline 2x^2+5x+2 \\ -(2x^2+4x \quad) \\ \hline x+2 \\ -(x+2) \\ \hline 0 \end{array}$$

In this case, 121 divided by 12 leaves no remainder, so we would say that 12 is a factor of 121. Similarly, since $(x^3 + 4x^2 + 5x + 2)$ divided by $(x + 2)$ leaves no remainder, we would say that $(x + 2)$ is a factor of $(x^3 + 4x^2 + 5x + 2)$.

Polynomial division doesn't always match up perfectly to an analogous whole number problem, but the process is always the same. Let's try it.

3. Use long division to determine if $(x - 1)$ a factor of $(x^3 - 3x^2 - 13x + 15)$. Don't worry: the steps for the division process are below:

- Write the problem as long division.
- What do you have to multiply x by to get x^3 ? Write your answer above the bar.
- Multiply your answer from step b by $(x - 1)$ and write your answer below the dividend.
- Subtract. Be careful to subtract each term. (You might want to change the signs and add.)
- Repeat steps a-d until the expression that remains is less than $(x - 1)$.

We hope you survived the division process. Is $(x - 1)$ a factor of $(x^3 - 3x^2 - 13x + 15)$? _____

4. Try it again. Use long division to determine if $(2x + 3)$ is a factor of $2x^3 + 7x^2 + 2x + 9$. No hints this time. You can do it!

When dividing numbers, there are several ways to deal with the remainder. Sometimes, we just write it as the remainder, like this:

$$\begin{array}{r} 8r.1 \\ 3 \overline{) 25} \end{array} \text{ because } 3(8) + 1 = 25$$

$$3 \overline{) 25} \text{ because } 3 \left(8\frac{1}{3} \right) = 25$$

Maybe you found that $(2x^3 + 7x^2 + 2x + 9) \div (2x + 3) = (x^2 + 2x - 2) r. 15$. (We sure hope so.) You can use it to write two multiplication statements:

and

$$(2x + 3)(x^2 + 2x - 2 + \frac{15}{2x + 3}) = (2x^3 + 7x^2 + 2x + 9)$$

	a) $(x^3 + 6x^2 + 13x + 12) \div (x + 3)$	b) $(x^3 - 4x^2 + 2x + 5) \div (x - 2)$
Multiplication statements:		

	c) $(6x^3 - 11x^2 - 4x + 5) \div (2x - 1)$	d) $(x^4 - 23x^3 + 49x + 4) \div (x^2 + x + 2)$
Multiplication statements:		

READY, SET, GO!

Name

Period

Date

READY

Topic: Solving linear equations

Solve for x.

1. $5x + 13 = 48$

2. $\frac{1}{3}x - 8 = 0$

3. $-4 - 9x = 0$

4. $x^2 - 16 = 0$

5. $x^2 + 4x + 3 = 0$

6. $x^2 - 5x + 6 = 0$

7. $(x + 8)(x + 11) = 0$

8. $(x - 5)(x - 7) = 0$

9. $(3x - 18)(5x - 10) = 0$

SET

Topic: Dividing polynomials

Divide each of the following polynomials. Write only one multiplication statement if the divisor is a factor. Write the two multiplication statements that go with your answers if there is a remainder.

10. $(x + 1) \overline{)x^3 - 3x^2 + 6x + 11}$

11. $(x - 5) \overline{)x^3 - 9x^2 + 23x - 15}$

Multiplication statement(s)

Multiplication statement(s)

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12. $(2x-1)\overline{)2x^3+15x^2-34x+13}$

13. $(x+4)\overline{)x^3+13x^2+26x-25}$

Multiplication statement(s)

Multiplication statement(s)

14. $(x+7)\overline{)x^3-8x^2-111x+10}$

15. $(3x-4)\overline{)3x^3+23x^2+6x-28}$

Multiplication statement(s)

Multiplication statement(s)

GO

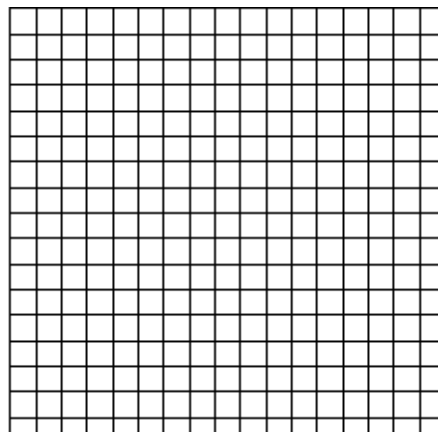
Topic: Describing the features of a variety of functions

Graph the following functions. Then identify the key features of the functions. Include domain, range, intervals where the function is increasing/decreasing, intercepts, maximum/minimum, and end behavior.

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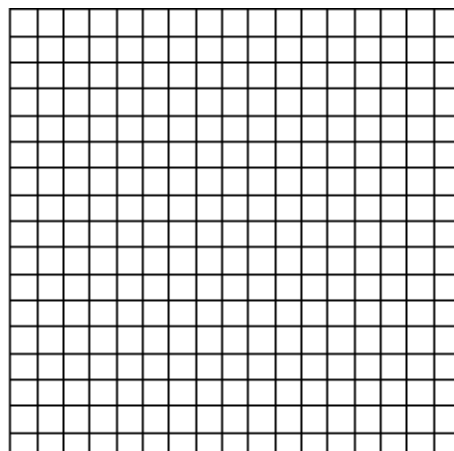
16. $f(x) = x^2 - 9$

domain: range:
 increasing: decreasing:
 y-intercept: x-intercept(s):



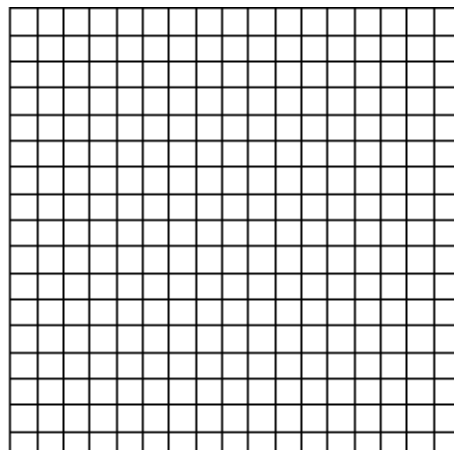
17. $f(n - 1) = f(n) + 3; f(1) = 4$

domain: range:
 increasing: decreasing:
 y-intercept: x-intercept(s):



18. $f(x) = \sqrt{x - 3} + 1$

domain: range:
 increasing: decreasing:
 y-intercept: x-intercept(s):



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19. $f(x) = \log_2 x - 1$

domain:

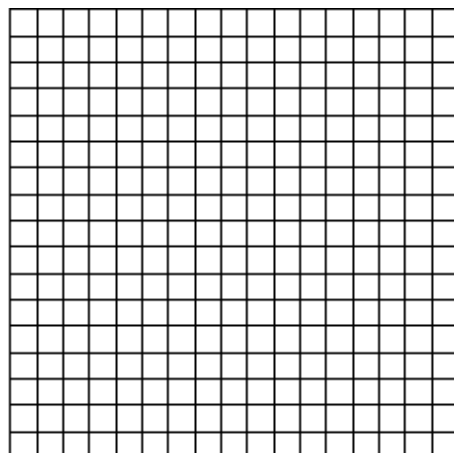
range:

increasing:

decreasing:

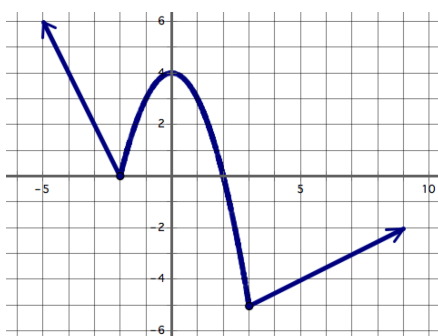
y-intercept:

x-intercept(s):



Identify the key features of the graphed functions.

20.



domain:

range:

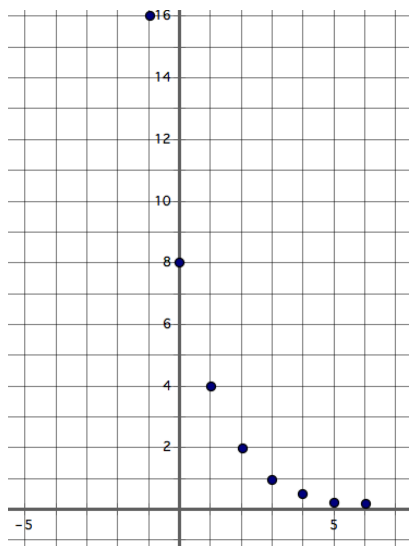
increasing:

decreasing:

y-intercept:

x-intercept(s):

21.



domain:

range:

increasing:

decreasing:

y-intercept:

x-intercept(s):

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3.4 To Be Determined . . .

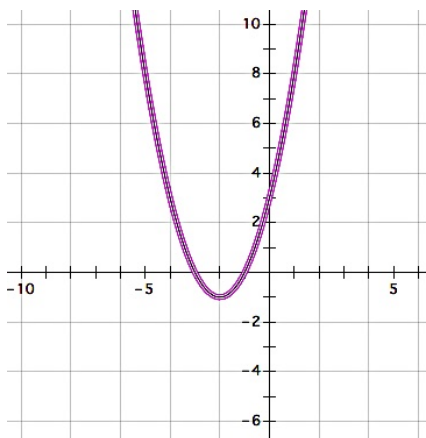
A Develop Understanding Task

Israel and Miriam are working together on a homework assignment. They need to write the equations of quadratic functions from the information given in a table or a graph. At first, this work seemed really easy. However, as they continued to work on the assignment, the algebra got more challenging and raised some interesting questions that they can't wait to ask their teacher.

Work through the following problems from Israel and Miriam's homework. Use the information in the table or the graph to write the equation of the quadratic function in all three forms. You may start with any form you choose, but you need to find all three equivalent forms. (If you get stuck, your teacher has some hints from Israel and Miriam that might help you.)

1.

x	y
-5	8
-4	3
-3	0
-2	-1
-1	0
0	3
1	8
2	15
3	24
4	35



Standard form:

Factored form:

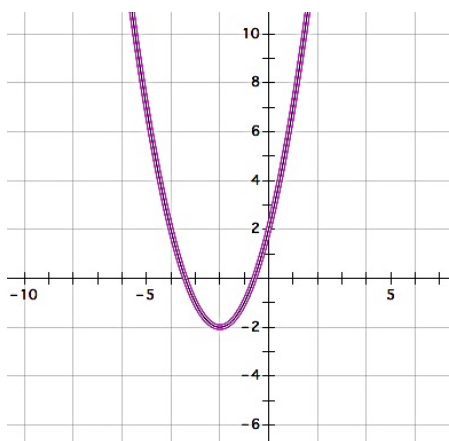
Vertex form:



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<https://flic.kr/p/5xVGij>

2.

x	y
-5	7
-4	2
-3	-1
-2	-2
-1	-1
0	2
1	7
2	14
3	23
4	34



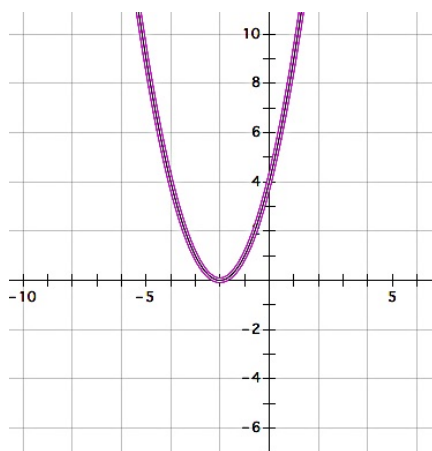
Standard form:

Factored form:

Vertex form:

3.

x	y
-5	9
-4	4
-3	1
-2	0
-1	1
0	4
1	9
2	16
3	25
4	36



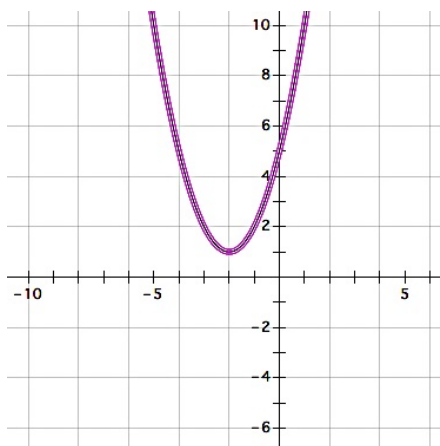
Standard form:

Factored form:

Vertex form:

4.

x	y
-5	10
-4	5
-3	2
-2	1
-1	2
0	5
1	10
2	17
3	26
4	37



Standard form:

Factored form:

Vertex form:

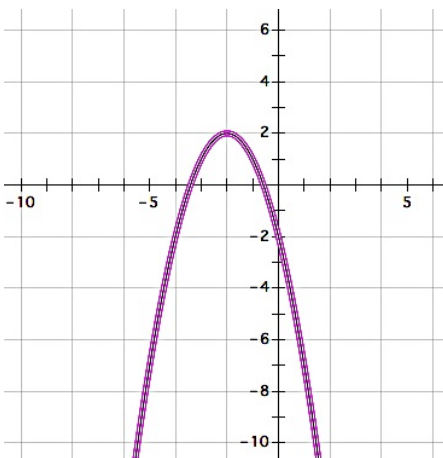
5. Israel was concerned that his factored form for the function in question 4 didn't look quite right. Miriam suggested that he test it out by plugging in some values for x to see if he gets the same points as those in the table. Test your factored form. Do you get the same values as those in the table?

6. Why might Israel be concerned about writing the factored form of the function in question 4?

Here are some more from Israel and Miriam's homework.

7.

x	y
-5	-7
-4	-2
-3	1
-2	2
-1	1
0	-2
1	-7
2	-14
3	-23
4	-34



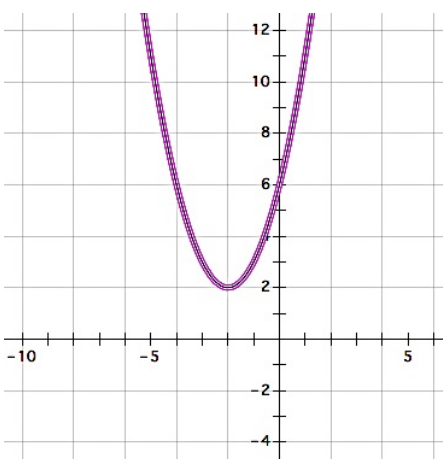
Standard form:

Factored form:

Vertex form:

8.

x	y
-5	11
-4	6
-3	3
-2	2
-1	3
0	6
1	11
2	18
3	27
4	38



Standard form:

Factored form:

Vertex form:

9. Miriam notices that the graphs of function 7 and function 8 have the same vertex point. Israel notices that the graphs of function 2 and function 7 are mirror images across the x -axis. What do you notice about the roots of these three quadratic functions?

The Fundamental Theorem of Algebra

A polynomial function is a function of the form:

$$y = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-3}x^3 + a_{n-2}x^2 + a_{n-1}x + a_n$$

where all of the exponents are positive integers and all of the coefficients $a_0 \dots a_n$ are constants.

As the theory of finding roots of polynomial functions evolved, a 17th century mathematician, Girard (1595-1632) made the following claim which has come to be known as the Fundamental Theorem of Algebra: *An n^{th} degree polynomial function has n roots.*

10. In the next module you will study polynomial functions that contain higher-ordered terms such as x^3 or x^5 . Based on your work in this task, do you believe this theorem holds for quadratic functions? That is, do all functions of the form $y = ax^2 + bx + c$ always have two roots? [Examine the graphs of each of the quadratic functions you have written equations for in this task. Do they all have two roots? Why or why not?]

READY, SET, GO!

Name _____

Period _____

Date _____

READY

Topic: Simplifying Radicals

Simplify each of the radicals below.

1. $\sqrt{8}$

2. $\sqrt{18}$

3. $\sqrt{32}$

4. $\sqrt{20}$

5. $\sqrt{45}$

6. $\sqrt{80}$

7. What is the connection between the radicals above? Explain.

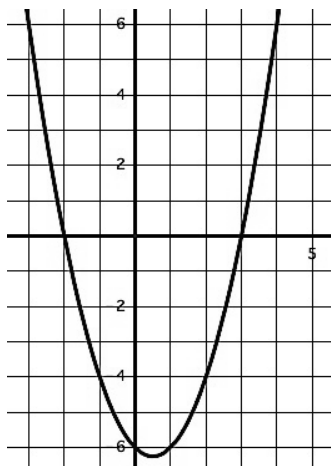
SET

Topic: Determine the nature of the x-intercepts for each quadratic below.

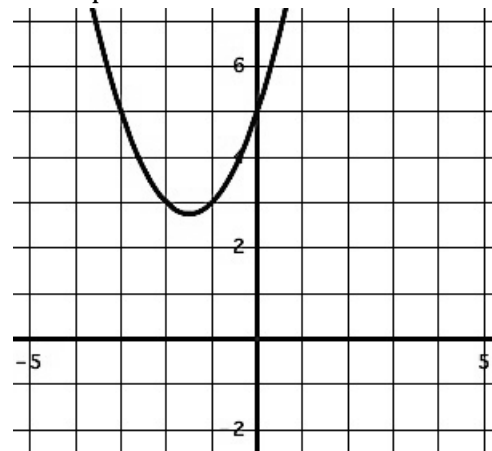
Given the quadratic function, its graph or other information, below determine the nature of the x-intercepts (what type of number it is). Explain or show how you know.

(Whole numbers “W”, Integers “Z”, Rational “Q”, Irrational “ \bar{Q} ”, or finally, “not Real”)

8. Determine the nature of the x-intercepts.



9. Determine the nature of the x-intercepts



10. Determine the nature of the x-intercepts.

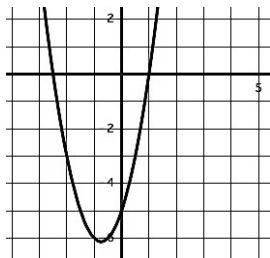
$$f(x) = x^2 + 4x - 24$$

11. Determine the nature of the x-intercepts.

$$g(x) = (2x - 1)(5x + 2)$$

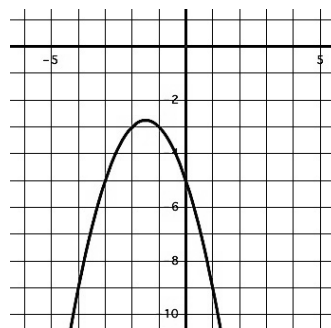
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12. Determine the nature of the x-intercepts.



$$f(x) = 2x^2 + 3x - 5$$

13. Determine the nature of the x-intercepts.



14. Determine the nature of the x-intercepts.

$$r(t) = t^2 - 8t + 16$$

15. Determine the nature of the x-intercepts.

$$h(x) = 3x^2 - 5x + 9$$

Determine the number of roots that each polynomial will have.

16. $x^5 + 7x^3 - x^2 + 4x - 21$

17. $4x^3 + 2x^2 - 3x - 9$

18. $2x^7 + 4x^5 - 5x^2 + 16x + 3$

GO

Topic: Finding x-intercepts for quadratics using factoring and quadratic formula.

If the given quadratic function can be factored then factor and provide the x-intercepts. If you cannot factor the function then use the quadratic formula to find the x-intercepts.

19. $A(x) = x^2 + 4x - 21$

20. $B(x) = 5x^2 + 16x + 3$

21. $C(x) = x^2 - 4x + 1$

22. $D(x) = x^2 - 16x + 4$

23. $E(x) = x^2 + 3x - 40$

24. $F(x) = 2x^2 - 3x - 9$

25. $G(x) = x^2 - 3x$

26. $H(x) = x^2 + 6x + 8$

27. $K(x) = 3x^2 - 11$

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3.5 My Irrational and Imaginary Friends

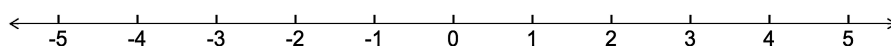
A Solidify Understanding Task



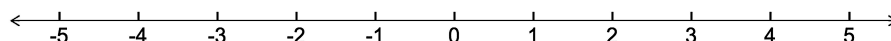
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<https://flic.kr/p/f7YDU9>

Part 1: Irrational numbers

1. Verify that $4(x - \frac{5}{2})(x + \frac{3}{2}) = 0$ and $4x^2 - 4x - 15 = 0$ are equivalent equations (show your work), and plot the solutions to the quadratic equations on the following number line:

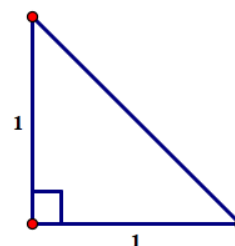


2. Verify that $(x - 2 + \sqrt{2})(x - 2 - \sqrt{2}) = 0$ and $4x^2 - 4x + 2 = 0$ are equivalent equations (show your work), and plot the solutions to the quadratic equations on the following number line:



You may have found it difficult to locate the exact points on the number line that represent the two solutions to the 2nd pair of quadratic equations given above. The following diagrams might help.

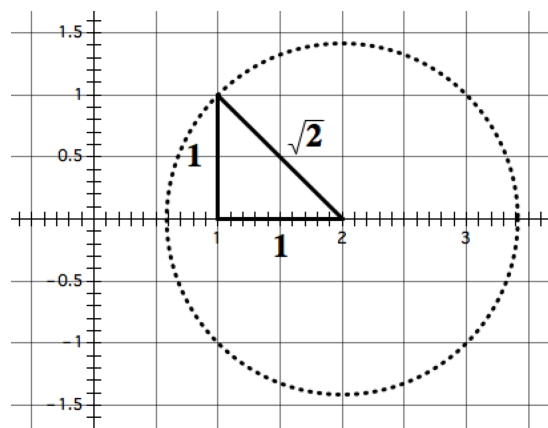
3. Find the perimeter of this isosceles triangle. Express your answer as simply as possible.



We might approximate the perimeter of this triangle with a decimal number, but the exact perimeter is $2 + \sqrt{2}$, which cannot be simplified any farther. Note that this notation represents a single number—the distance around the perimeter of the triangle—even though it is written as the sum of two terms.

4. Explain how you could use this diagram to locate the two solutions to the quadratic equations given in the 2nd problem above: $2 + \sqrt{2}$ and $2 - \sqrt{2}$.

5. Are the numbers we have located on the number line in this way rational numbers or irrational numbers? Explain your answer.



Both sets of quadratic equations given in problems 1 and 2 above have solutions that can be plotted on a number line. The solutions to the first set of quadratic equations are rational numbers. The solutions to the 2nd set of quadratic equations are irrational numbers.

Big Idea #1: The set of numbers that contains all of the *rational numbers* and all of the *irrational numbers* is called the *set of real numbers*. The location of all points on a number line can be represented by real numbers.

Part 2: Imaginary and Complex Numbers

In the previous task, *To Be Determined . . .*, you found that the quadratic formula gives the solutions to the quadratic equation $x^2 + 4x + 5 = 0$ as $-2 + \sqrt{-1}$ and $-2 - \sqrt{-1}$. Because the square root of a negative number has no defined value as either a rational or an irrational number, Euler proposed that a new number $i = \sqrt{-1}$ be included in what came to be known as the complex number system.

6. Based on Euler's definition of i , what would the value of i^2 be?

With the introduction of the number i , the square root of *any* negative number can be represented. For example, $\sqrt{-2} = \sqrt{2} \cdot \sqrt{-1} = \sqrt{2} \cdot i$ and $\sqrt{-9} = \sqrt{9} \cdot \sqrt{-1} = 3i$.

7. Find the values of the following expressions. Show the details of your work.

(a) $(\sqrt{2} \cdot i)^2$

(b) $3i \times 3i$

Using this new notation, the solutions to the equation $x^2 + 4x + 5 = 0$ can be written as $-2 + i$ and $-2 - i$, and the factored form of $x^2 + 4x + 5$ can be written as $(x + 2 - i)(x + 2 + i)$.

8. Verify that $x^2 + 4x + 5$ and $(x + 2 - i)(x + 2 + i)$ are equivalent by expanding and simplifying the factored form. Show the details of your work.

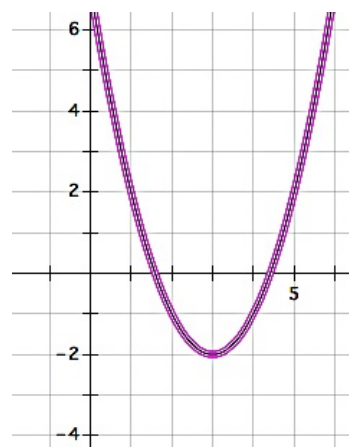
Big Idea #2: Numbers like $3i$ and $\sqrt{2} \cdot i$ are called *pure imaginary numbers*. Numbers like $-2 - i$ and $-2 + i$ that include a real term and an imaginary term are called *complex numbers*.

The quadratic formula is usually written in the form $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. An equivalent form is

$$\frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}. \text{ If } a, b \text{ and } c \text{ are rational coefficients, then } \frac{-b}{2a} \text{ is a rational term, and } \frac{\sqrt{b^2 - 4ac}}{2a}$$

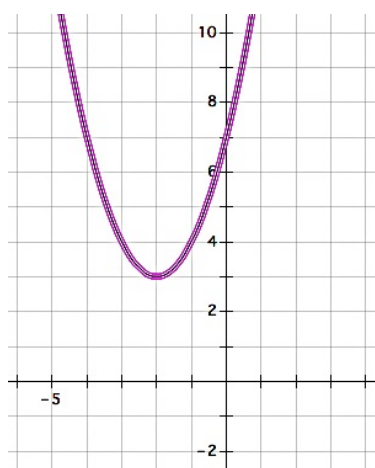
may be a rational term, an irrational term or an imaginary term, depending on the value of the expression under the square root sign.

9. Examine the roots of the quadratic $y = x^2 - 6x + 7$ shown in the graph at the right. How do the terms $\frac{-b}{2a}$ and $\frac{\sqrt{b^2 - 4ac}}{2a}$ show up in this graph?



Look back at the work you did in the task *To Be Determined...*

10. Which quadratics in that task had complex roots? (List them here.)
11. How can you determine if a quadratic has complex roots from its graph?
12. Find the complex roots of the following quadratic function represented by its graph.



13. Reflect the graph of the quadratic function given in question 12 over the horizontal line $y = 3$. Find the irrational roots of the reflected quadratic function.

14. How is the work you did to find the roots of the quadratic functions in questions 12 and 13 similar?

Big Idea #3: Complex numbers are not real numbers—they do not lie on the real number line that includes all of the rational and irrational numbers; also note that the real numbers are a subset of the complex numbers since a real number results when the imaginary part of $a + bi$ is 0, that is, $a + 0i$.

The Fundamental Theorem of Algebra. Revisited

Remember the following information given in the previous task:

A polynomial function is a function of the form:

$$y = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-3}x^3 + a_{n-2}x^2 + a_{n-1}x + a_n$$

where all of the exponents are positive integers and all of the coefficients $a_0 \dots a_n$ are constants.

As the theory of finding roots of polynomial functions evolved, a 17th century mathematician, Girard (1595-1632) made the following claim which has come to be known as the Fundamental Theorem of Algebra: *An n^{th} degree polynomial function has n roots.*

15. Based on your work in this task, do you believe this theorem holds for quadratic functions? That is, do all functions of the form $y = ax^2 + bx + c$ always have two roots?

READY, SET, GO!

Name

Period

Date

READY

Topic: Classifying numbers according to set.

Classify each of the numbers represented below according to the sets to which they belong. If a number fits in more than one set then list all that apply.

(Whole numbers “W”, Integers “Z”, Rational “Q”, Irrational “Q̄”, Real “R”, Complex “C”)

1. π

2. -13

3. $\sqrt{-16}$

4. 0

5. $\sqrt{75}$

6. $\frac{9}{3}$

7. $\sqrt{\frac{4}{9}}$

8. $5 + \sqrt{2}$

9. $\sqrt{-40}$

SET

Topic: Simplifying radicals, imaginary numbers

Simplify each radical expression below.

10. $3 + \sqrt{2} - 7 + 3\sqrt{2}$

11. $\sqrt{5} - 9 + 8\sqrt{5} + 11 - \sqrt{5}$

12. $\sqrt{12} + \sqrt{48}$

13. $\sqrt{8} - \sqrt{18} + \sqrt{32}$

14. $11\sqrt{7} - 5\sqrt{7}$

15. $7\sqrt{7} + 5\sqrt{3} - 3\sqrt{7} + \sqrt{3}$

Simplify. Express as a complex number using “i” if necessary.

16. $\sqrt{-2} \cdot \sqrt{-2}$

17. $7 + \sqrt{-25}$

18. $(4i)^2$

19. $i^2 \cdot i^3 \cdot i^4$

20. $(\sqrt{-4})^3$

21. $(2i)(5i)^2$

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Solve each quadratic equation over the set of complex numbers.

22. $x^2 + 100 = 0$

23. $t^2 + 24 = 0$

24. $x^2 - 6x + 13 = 0$

25. $r^2 - 2r + 5 = 0$

GO

Topic: Solve Quadratic Equations

Use the discriminant to determine the nature of the roots to the quadratic equation.

26. $x^2 - 5x + 7 = 0$

27. $x^2 - 5x + 6 = 0$

28. $2x^2 - 5x + 5 = 0$

29. $x^2 + 7x + 2 = 0$

30. $2x^2 + 7x + 6 = 0$

31. $2x^2 + 7x + 7 = 0$

32. $2x^2 - 7x + 6 = 0$

33. $2x^2 + 7x - 6 = 0$

34. $x^2 + 6x + 9 = 0$

Solve the quadratic equations below using an appropriate method.

35. $m^2 + 15m + 56 = 0$

36. $5x^2 - 3x + 7 = 0$

37. $x^2 - 10x + 21 = 0$

38. $6x^2 + 7x - 5 = 0$

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3.6 Sorry, We're Closed

A Practice Understanding Task

Now that we have compared operations on polynomials with operations on whole numbers and started thinking about new number sets including irrational and complex numbers, it's time to generalize about the results



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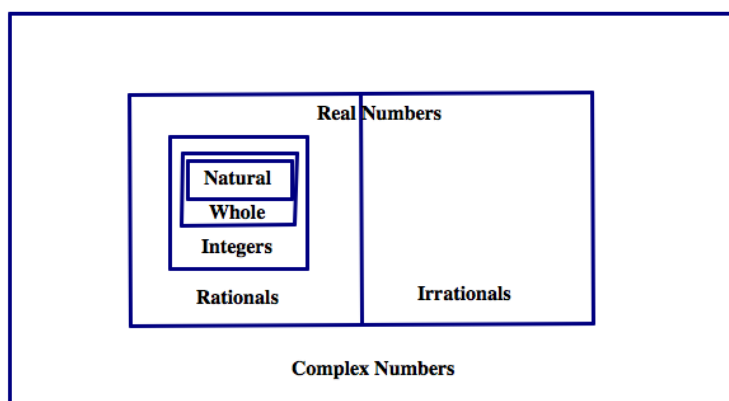
Maybe you have noticed in the past that when you add two even numbers, the answer you get is always an even number. Mathematically, we say that the set of even numbers is **closed** under addition. Mathematicians are interested in results like this because it helps us to understand how numbers or functions of a particular type behave with the various operations.

1. You can try it yourself: Is the set of odd numbers closed under multiplication? In other words, if you multiply two odd numbers together will you get an odd number? Explain.

If you find any two odd numbers that have an even product, then you would say that odd numbers are not closed under multiplication. Even if you have several examples that support the claim, if you can find one **counterexample** that contradicts the claim, then the claim is false.

Consider the following claims and determine whether they are true or false. If a claim is true, give a reason with at least two examples that illustrate the claim. If the claim is false, give a reason with one counterexample that proves the claim to be false.

This graphic will help you to think about the relationship between different sets of numbers, including the complex numbers that we have found as solutions to quadratic equations.



Do the following for each of the following claims:

- Determine if the claim is true or false.
- If you decide that the claim is true, create at least three examples to support the claim.
- If you decide that the claim is false, find a counter-example to prove the claim to be false.

1. The set of integers is closed under addition.
2. The set of irrational numbers is closed under addition.
3. The set of complex numbers is closed under addition.
4. The set of whole numbers is closed under subtraction.
5. The set of rational numbers is closed under subtraction.
6. The set of integers is closed under multiplication.
7. The set of integers is closed under division.
8. The set of rational numbers is closed under multiplication.

9. The set of irrational numbers is closed under multiplication.

10. The set of complex numbers is closed under multiplication.

The Arithmetic of Polynomials

To evaluate similar claims about polynomials, we must be very clear on the definition of a polynomial. The definition of a polynomial is:

A polynomial function has the form:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

where $a_n, a_{n-1}, \dots, a_1, a_0$ are real numbers and n is a nonnegative integer. In other words, a polynomial is the sum of one or more monomials with real coefficients and nonnegative integer exponents. The degree of the polynomial function is the highest value for n where a_n is not equal to 0.

1. The following examples and non-examples will help you to see the important implications of the definition of a polynomial function. For each pair, determine what is different between the example of a polynomial and the non-example that is not a polynomial.

These are polynomials:	These are not polynomials:
a) $f(x) = x^3$	b) $g(x) = 3^x$
How are a and b different?	
c) $f(x) = 2x^2 + 5x - 12$	d) $g(x) = \frac{2x^2}{x^2 - 3x + 2}$
How are c and d different?	
e) $f(x) = -x^3 + 3x^2 - 2x - 7$	f) $g(x) = x^3 + 3x^2 - 2x + 10x^{-1} - 7$
How are e and f different?	
h) $f(x) = \frac{1}{2}x$	i) $g(x) = \frac{1}{2x}$
How are h and i different?	

j) $f(x) = x^2$	k) $g(x) = x^{\frac{1}{2}}$
How are j and k different?	

2. Based on the definition and the examples above, how can you tell if a function is a polynomial function?

Now we will consider claims about polynomials. Do the following for each of the following claims:

- Determine if the claim is true or false.
- If you decide that the claim is true, create at least three examples to support the claim.
- If you decide that the claim is false, find a counter-example to prove the claim to be false.

3. The sum of a quadratic polynomial and a linear polynomial is a cubic polynomial.

4. The sum of a linear polynomial and an exponential expression is a polynomial.

5. A cubic polynomial subtracted from a cubic polynomial is a cubic polynomial.

6. A cubic polynomial divided by a linear polynomial is a quadratic polynomial.

7. The set of polynomials is closed under addition.

8. The set of polynomial functions is closed under subtraction.
9. The set of polynomials is closed under multiplication.
10. The set of polynomial functions is closed under division.
11. Write two claims of your own about polynomials and use examples to demonstrate that they are true.

Claim #1:

Claim #2:

READY, SET, GO!

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Period _____

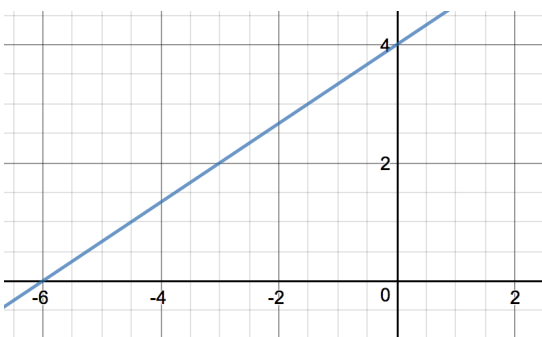
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READY

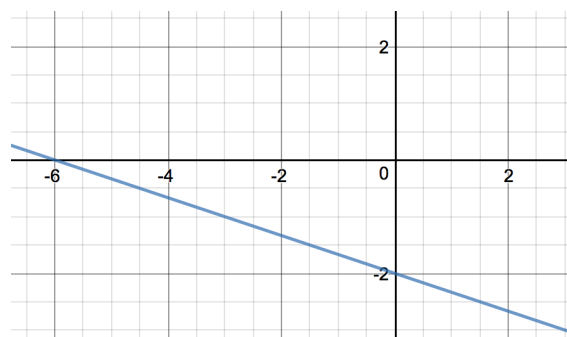
Topic: Connecting the zeros of a function to the solution of the equation

When we solve equations, we often set the equation equal to zero and then find the value of x . Another way to say this is “find when $f(x) = 0$.” That’s why we call solutions to equations the zeros of an equation. Find the zeros for the given equations. Then mark the solution(s) as a point on the graph of the equation.

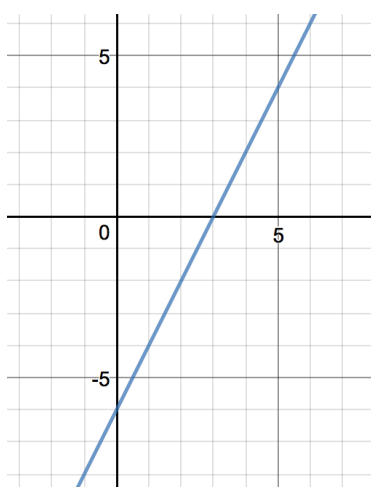
1. $f(x) = \frac{2}{3}x + 4$



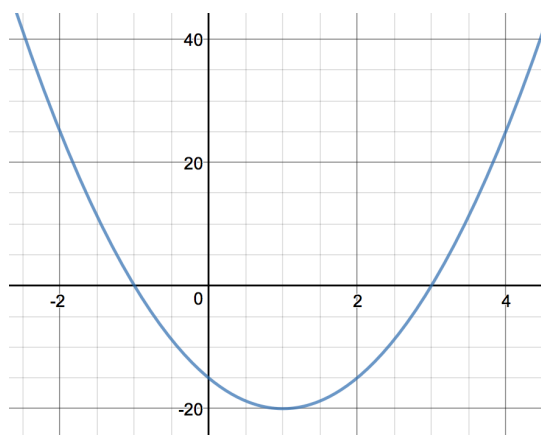
2. $g(x) = -\frac{1}{3}x - 2$



3. $h(x) = 2x - 6$



4. $p(x) = 5x^2 - 10x - 15$

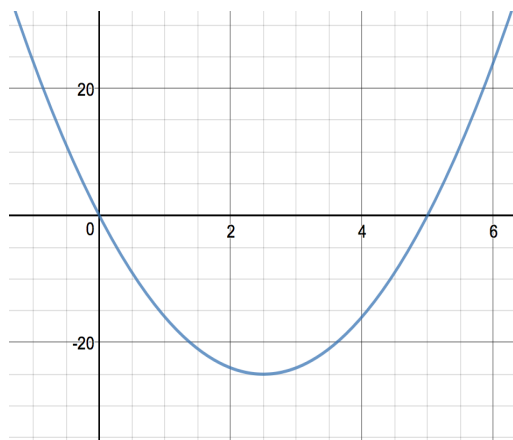
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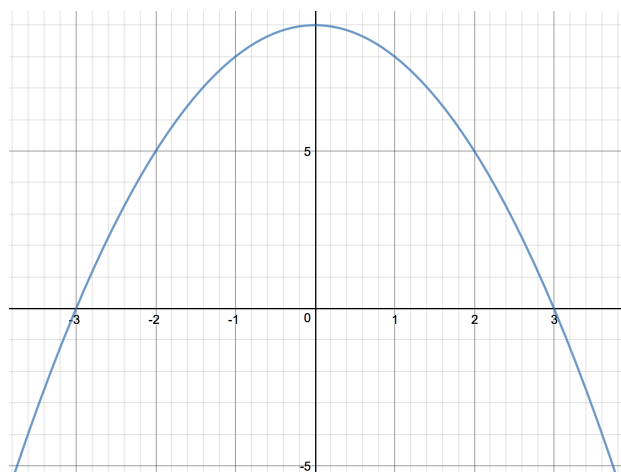
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5. $q(x) = 4x^2 - 20x$



6. $d(x) = -x^2 + 9$

**SET**

Topic: Exploring closed mathematical number sets

Identify the following statements as *sometimes* true, *always* true, or *never* true. If your answer is *sometimes* true, give an example of when it's true and an example of when it's not true. If it's *never* true, give a counter-example.

7. The product of a whole number and a whole number is an integer.
8. The quotient of a whole number divided by a whole number is a whole number.
9. The set of integers is **closed** under division.
10. The difference of a linear function and a linear function is an integer.
11. The difference of a linear function and a quadratic function is a linear function.

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12. The product of a linear function and a linear function is a quadratic function.
13. The sum of a quadratic function and a quadratic function is a polynomial function.
14. The product of a linear function and a quadratic function is a cubic function.
15. The product of three linear functions is a cubic function.
16. The set of polynomial functions is **closed** under addition.

GO

Topic: Identifying conjugate pairs

A **conjugate pair** is simply a pair of binomials that have the same numbers but differ by having opposite signs between them. For example $(a + b)$ and $(a - b)$ are conjugate pairs. You've probably noticed them when you've factored a quadratic expression that is the difference of two squares. **Example:** $x^2 - 25 = (x + 5)(x - 5)$. The two factors $(x + 5)(x - 5)$ are conjugate pairs.

The quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ can generate both solutions to a quadratic equation because of the \pm located in the numerator of the formula. When the $\sqrt{b^2 - 4ac}$ part of the formula generates an irrational number (e.g. $\sqrt{2}$) or an imaginary number (e.g. $2i$), the formula produces a pair of numbers that are conjugates. This is important because this type of solution to a quadratic always comes in pairs. **Example:** The conjugate of $(3 + \sqrt{2})$ is $(3 - \sqrt{2})$. The conjugate of $(-2i)$ is $(+2i)$. Think of it as $(0 - 2i)$ and $(0 + 2i)$. Change only the sign between the two numbers.

Write the conjugate of the given value.

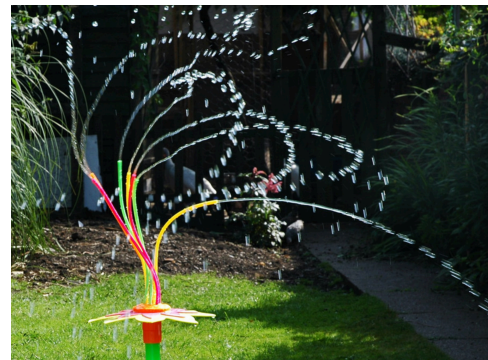
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|----------------------|-----------------|------------------------|------------------|
| 17. $(8 + \sqrt{5})$ | 18. $(11 + 4i)$ | 19. $9i$ | 20. $-5\sqrt{7}$ |
| 21. $(2 - 13i)$ | 22. $(-1 - 2i)$ | 23. $(-3 + 5\sqrt{2})$ | 24. $-4i$ |

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3.7 Quadratic Quandaries

A Develop & Solidify Understanding Task

In the task *Curbside Rivalry* Carlos and Clarita were trying to decide how much they should charge for a driveway mascot. Here are the important details of what they had to consider.



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- *Surveys show the twins can sell 100 driveway mascots at a cost of \$20, and they will sell 10 fewer mascots for each additional \$5 they charge.*
- *The twins estimate that the cost of supplies will be \$250 and they would like to make \$2000 in profit from selling driveway mascots. Therefore, they will need to collect \$2250 in revenue.*

This information led Carlos and Clarita to write and solve the quadratic equation

$$(100 - 10x)(20 + 5x) = 2250.$$

1. Either review your work from *Curbside Rivalry* or solve this quadratic equation for x again.
2. What do your solutions for x mean in terms of the story context?
3. How would your solution change if this had been the question Carlos and Clarita had asked: “How much should we charge if we want to collect at least \$2250 in revenue?”

4. What about this question: “How much should we charge if we want to maximize our revenue?”

As you probably observed, the situation represented in question 3 didn’t have a single solution, since there are many different prices the twins can charge to collect more than \$2250 in revenue. Sometimes our questions lead to quadratic inequalities rather than quadratic equations.

Here is another quadratic inequality based on your work on *Curbside Rivalry*.

5. Carlos and Clarita want to design a logo that requires less than 48 in^2 of paint, and fits inside a rectangle that is 8 inches longer than it is wide. What are the possible dimensions of the rectangular logo?

Again question 5 has multiple answers, and those answers are restricted by the context. Let’s examine the inequality you wrote for question 5, but not restricted by the context.

6. What are the solutions to the inequality $x(x + 8) < 48$?
7. How might you support your answer to question 6 with a graph or a table?

Here are some more quadratic inequalities without contexts. Show how you might use a graph, along with algebra, to solve each of them.

8. $x^2 + 3x - 10 \geq 0$

9. $2x^2 - 5x < 12$

10. $x^2 - 4 \leq 4x + 1$

Carlos and Clarita both used algebra and a graph to solve question 10, but they both did so in different ways. Illustrate each of their methods with a graph and with algebra.

11. Carlos: “I rewrote the inequality to get 0 on one side and a factored form on the other. I found the zeroes for each of my factors. To decide what values of x made sense in the inequality I also sketched a graph of the quadratic function that was related to the quadratic expression in my inequality. I shaded solutions for x based on the information from my graph.”
12. Clarita: “I graphed a linear function and a quadratic function related to the linear and quadratic expressions in the inequality. From the graph I could estimate the points of intersection, but to be more exact I solved the quadratic equation $x^2 - 4 = 4x + 1$ by writing an equivalent equation that had 0 on one side. Once I knew the x -values for the points of intersection in the graph, I could shade solutions for x that made the inequality true.”

READY, SET, GO!

Name _____

Period _____

Date _____

READY

Topic: Factoring Polynomials

Factor each of the polynomials completely.

1.

$$x^2 + x - 12$$

2.

$$x^2 - 2x - 8$$

3.

$$x^2 + 5x - 14$$

4.

$$x^2 - x - 6$$

5.

$$x^2 + 6x + 9$$

6.

$$x^2 - 7x + 10$$

7.

$$2x^2 - 9x - 5$$

8.

$$3x^2 - 3x - 18$$

9.

$$2x^2 + 8x - 42$$

10. How is the factored form of a quadratic helpful when graphing the parabola?

SET

Topic: Solving Quadratic Inequalities

Solve each of the quadratic inequalities.

11.

$$x^2 + x - 12 > 0$$

12.

$$x^2 - 2x - 8 \leq 0$$

13.

$$x^2 + 5x - 14 \geq 0$$

14.

$$2x^2 - 9x - 5 \geq 0$$

15.

$$3x^2 - 3x - 18 < 0$$

16.

$$x^2 + 4x - 21 < 0$$

17.

$$x^2 - 4x \leq 0$$

18.

$$x^2 \leq 25$$

19.

$$x^2 - 4x \leq 5$$

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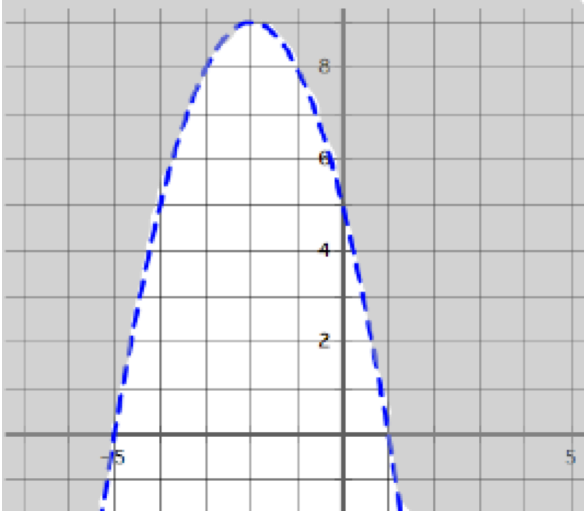
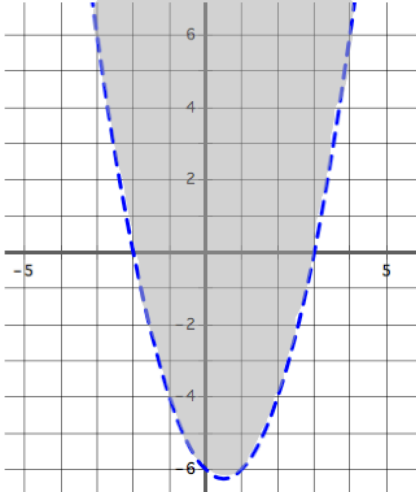
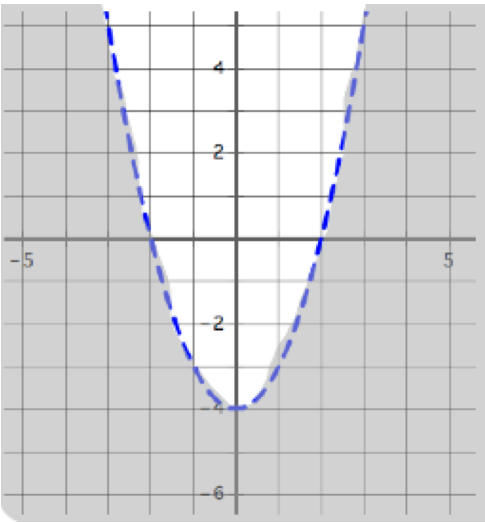
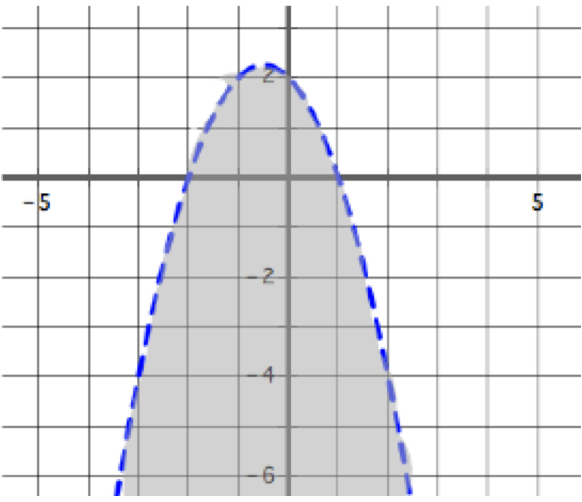
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Match each graph with its inequality.

<p>a.</p> 	<p>b.</p> 
<p>c.</p> 	<p>d.</p> 
<p>20. $y > x^2 - x - 6$</p>	<p>21. $y < x^2 - 4$</p>
<p>22. $y < (x + 2)(1 - x)$</p>	<p>23. $y > 5 - 4x - x^2$</p>

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GO

Topic: Vertex form of quadratic equations

Write each quadratic function below in vertex form.

24.

$$f(x) = x^2 + 6x + 5$$

25.

$$f(x) = (x + 3)(x - 5)$$

26.

$$f(x) = (x - 2)(x + 6)$$

27.

$$f(x) = x^2 - 12x + 20$$

28.

$$f(x) = 2x^2 + 16x + 8$$

29.

$$f(x) = x^2 - 2x - 8$$

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