mathematics
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Transforming Mathematics Education

ALGEBRA II

An Integrated Approach

MODULE 3 HONORS

Numbers & Operations

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The Mathematics Vision Project

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3.1 It All Adds Up

A Develop Understanding Task

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Whenever we're thinking about algebra and working with variables, it is useful to consider how it relates to the number system and operations on numbers. Let's see if we can make some useful comparisons between whole numbers and polynomials.

Let's start by looking at the structure of numbers and polynomials. Consider the number 132. The way we write numbers is really a shortcut because:

$$132 = 100 + 30 + 2$$

1. Compare 132 to the polynomial $x^2 + 3x + 2$. How are they alike? How are they different?

2. Write a polynomial that is analogous to the number 2,675.

When two numbers are to be added together, many people use a procedure like this:

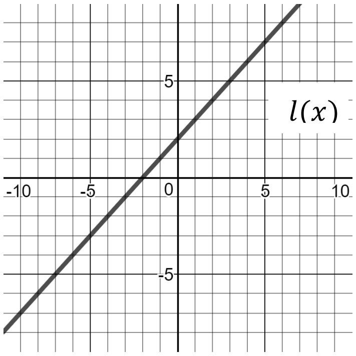
$$\begin{array}{r} 132 \\ + \underline{451} \\ 583 \end{array}$$

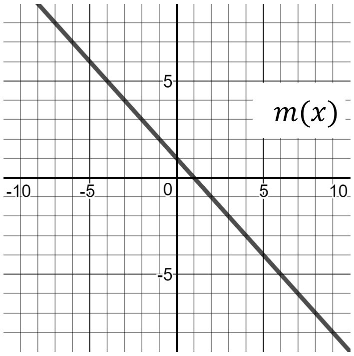
3. Write an analogous addition problem for polynomials and find the sum of the two polynomials.

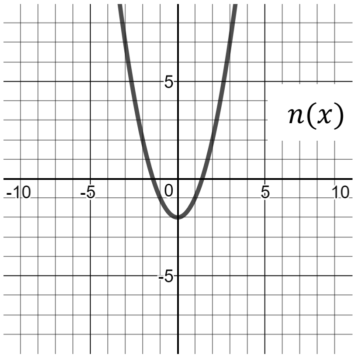
4. How does adding polynomials compare to adding whole numbers?

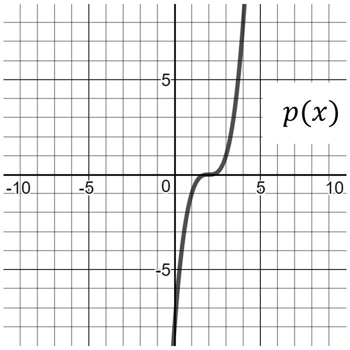
5. Use the polynomials below to find the specified sums in a-f.

$f(x) = x^3 + 3x^2 - 2x + 10$ $g(x) = 2x - 1$ $h(x) = 2x^2 + 5x - 12$ $k(x) = -x^2 - 3x + 4$









a) $h(x) + k(x)$

b) $g(x) + f(x)$

c) $f(x) + k(x)$

____________________ ____________________ ____________________

d) $l(x) + m(x)$

e) $m(x) + n(x)$

f) $l(x) + p(x)$

6. What patterns do you see when polynomials are added?

Subtraction of whole numbers works similarly to addition. Some people line up subtraction vertically and subtract the bottom number from the top, like this:

$$\begin{array}{r} 368 \\ -\underline{157} \\ 211 \end{array}$$

7. Write the analogous polynomials and subtract them.

8. Is your answer to #7 analogous to the whole number answer? If not, why not?

9. Subtracting polynomials can easily lead to errors if you don't carefully keep track of your positive and negative signs. One way that people avoid this problem is to simply change all the signs of the polynomial being subtracted and then add the two polynomials together. There are two common ways of writing this:

$$(x^3 + x^2 - 3x - 5) - (2x^3 - x^2 + 6x + 8)$$

Step 1: $= (x^3 + x^2 - 3x - 5) + (-2x^3 + x^2 - 6x - 8)$

Step 2: $= (-x^3 + 2x^2 - 9x - 13)$

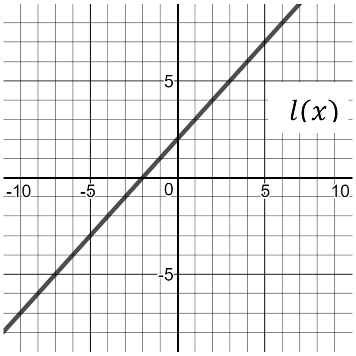
Or, you can line up the polynomials vertically so that Step 1 looks like this:

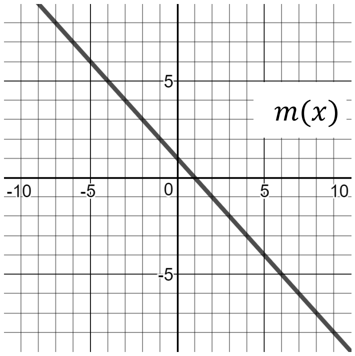
$$\begin{array}{lr} \text{Step 1:} & x^3 + x^2 - 3x - 5 \\ & +(-2x^3 + x^2 - 6x - 8) \\ \cline{2-2} \text{Step 2:} & -x^3 + 2x^2 - 9x - 13 \end{array}$$

The question for you is: Is it correct to change all the signs and add when subtracting? What mathematical property or relationship can justify this action?

10. Use the given polynomials to find the specified differences in a-d.

$f(x) = x^3 + 2x^2 - 7x - 8$ $\quad g(x) = -4x - 7$ $\quad h(x) = 4x^2 - x - 15$ $\quad k(x) = -x^2 + 7x + 4$





a) $h(x) - k(x)$

b) $f(x) - h(x)$

c) $f(x) - g(x)$

d) $k(x) - f(x)$

e) $l(x) - m(x)$

11. List three important things to remember when subtracting polynomials.

.

3.1 It All Adds Up – Teacher Notes

A Develop Understanding Task

Purpose: The purpose of this task is for students to surface comparisons between polynomials and whole numbers and use these comparisons to add and subtract polynomials algebraically. Students will also add and subtract polynomials given only graphically, adding corresponding points on the two graphs to obtain a sum. Students will make and test conjectures about the sum and differences of polynomials, such as "the sum of two quadratics is quadratic."

Core Standards Focus:

A.APR.1 Understand that polynomials for a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

F.BF.1: Write a function that describes a relationship between two quantities.
b. Combine standard function types using arithmetic operations.

F.IF.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Standards for Mathematical Practice:

SMP 7 – Look for and make use of structure

SMP 8 – Look for express regularity in repeated reasoning

The Teaching Cycle:

Launch (Whole Group):

Begin the discussion with the introduction to the task and problem 1. Help students to see that numbers are structured as the sum of powers of 10 and polynomials are structured as powers of x. Because of that, operations on polynomials and whole numbers will work similarly. Ask students to do problem 2 individually, and then check to be sure that the class understands how to write an "analogous" polynomial. Then ask students to do problem #3 and verify that they have the idea of adding like terms. They may choose to line up the polynomials as shown with whole numbers or to add them horizontally, but either way, only like terms are added. They should also notice that the exponents don't change when the terms are added. Ask if this is true of whole numbers. After discussing the first page of the task, students should be ready to work on questions 5-6.

Explore (Small Groups):

Monitor students as they are working to be sure that they are appropriately adding like terms and working with the exponents. When students are asked to add the graphs, encourage them to use the graphs without finding algebraic expressions for the graphs. It is important for students to have an understanding of how function addition works with representations other than algebraic equations.

Discuss (Whole Group):

Begin the discussion with $f(x) + k(x)$. Ask a student to share their work and explain their strategy. This is a straightforward problem which can be used to ensure that all students have the basic idea. If students are having difficulty, then ask another student to share $h(x) + k(x)$. Then, shift the discussion to adding functions graphically, using $m(x) + n(x)$. Ask a student to share how they obtained the sum by adding corresponding y-values on the two functions. Ask students why only the y-values are added, not the x-values. Reinforce the idea that $m(x) + n(x)$ means that the output values for the two functions are to be added. Ask students why the sum of the two graphs was quadratic. Would it always be true that the sum of a linear function and a quadratic function is a quadratic function?

Continue by asking a student to share his/her work on $l(x) + m(x)$, again demonstrating how to add corresponding outputs on the two graphs. Ask students why the sum turned out to be a horizontal line. Would the sum of two linear functions always be a horizontal line? Would the sum of two linear functions always be a linear function?

Ask a few students to share a conjecture on problem 6 and to explain why they believe it to be true. Ask the class to respond with support for the conjecture or counter-examples that disprove it.

This completes the discussion for the addition part of the task, so that students are ready to consider subtraction.

Re-launch (Whole Group):

Ask students to work individually to do problems #7 and 8. Briefly discuss the narrative in #9 and ask student why it is mathematically correct to change the signs of the subtrahend and then to add the two polynomials together rather than subtracting. Again, students may choose to work horizontally or vertically. Tell students to complete the task.

Explore (Small Groups):

Monitor students as they work to be sure that they have a strategy that helps them to keep the signs straight in a subtraction problem, since this is the most common error. Watch for students that are making sense of the graphs and subtracting outputs to find the difference between the two functions so that they can share during the class discussion. Encourage students that are trying to write equations to add together the graphs to try to do it without equations so that they can consider a different representation.

Discuss (Whole Group):

Ask students to demonstrate their strategies for subtracting on as many problems as time allows. Include students that have lined up the polynomials vertically and others that have worked the problems horizontally and compare the relative merits of each. Ask a student to share his/her work with subtracting the graphs and ask how this is similar to the work done with addition. End the discussion with students sharing responses to the last question about the procedure for subtracting polynomials. Highlight responses that include the following ideas:

- All of the terms in the second polynomial must be subtracted.
- Only subtract like terms.
- Subtraction can be rewritten as addition to avoid sign errors.

Aligned Ready, Set, Go: ***Numbers and Operations 3.1***

READY, SET, GO! Name Period Date

READY

Topic: Using the distributive property

Multiply.

1. $2x(5x^2+7)$
2. $9x(-x^2-3)$
3. $5x^2(x^4+6x^3)$
4. $-x(x^2-x+1)$
5. $-3x^3(-2x^2+x-1)$
6. $-1(x^2-4x+8)$

SET

Topic: Adding and subtracting polynomials

Add. Write your answers in descending order of the exponents. (Standard form)

7. $(3x^4+5x^2-1)+(2x^3+x)$
8. $(4x^2+7x-4)+(x^2-7x+14)$
9. $(2x^3+6x^2-5x)+(x^5+3x^2+8x+4)$
10. $(-6x^5-2x+13)+(4x^5+3x^2+x-9)$

Subtract. Write your answers in descending order of the exponents. (Standard form)

11. $(5x^2+7x+2)-(3x^2+6x+2)$
12. $(10x^4+2x^2+1)-(3x^4+3x+11)$
13. $(7x^3-3x+7)-(4x^2-3x-11)$
14. $(x^4-1)-(x^4+1)$

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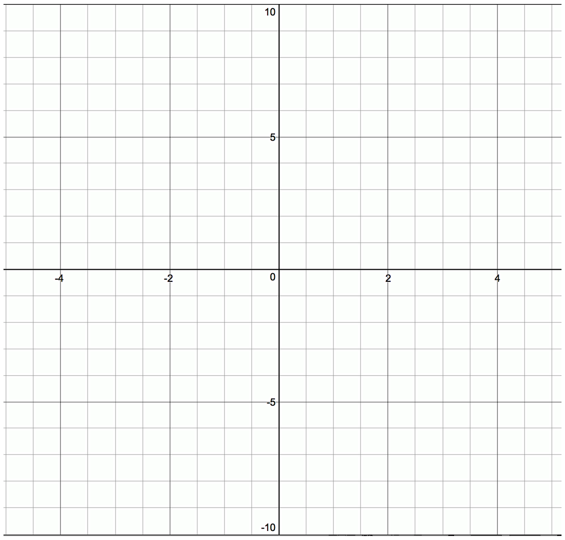
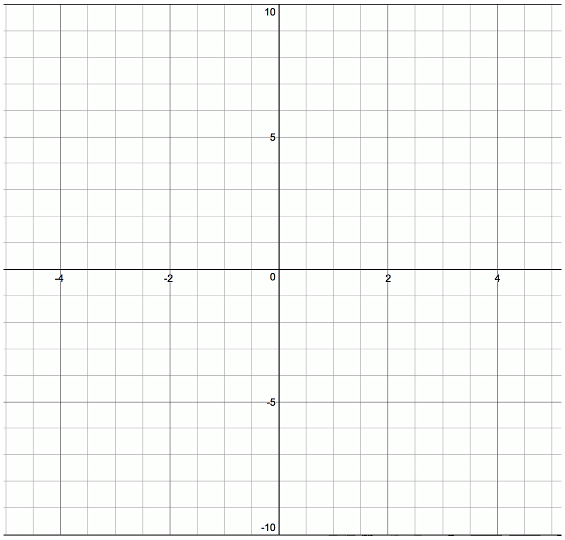
Graph.

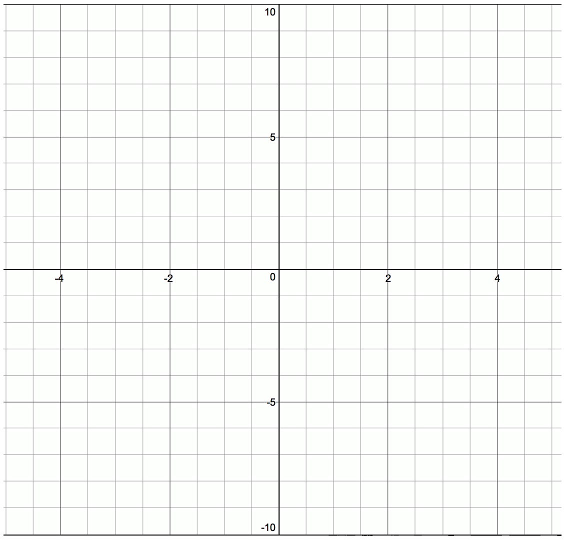
15. $y = x^3 - 2$



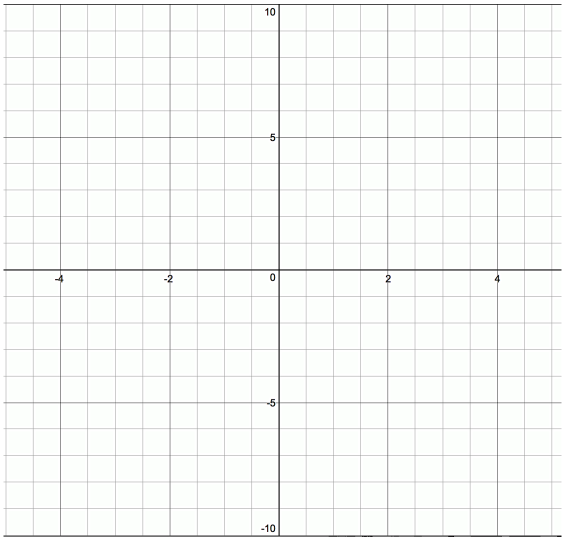
16. $y = x^3 + 1$



17. $y = (x - 3)^3$



18. $y = (x + 1)^3$



GO

Topic: Using exponent rules to combine expressions

Simplify.

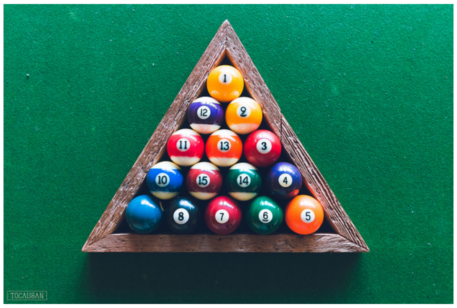
19. $x^{7/8} \cdot x^{1/4} \cdot x^{-1/2}$

20. $x^{3/16} \cdot x^{-7/8} \cdot x^{3/4}$

21. $x^{4/7} \cdot x^{2/9} \cdot x^{-1/3}$

3.2 Pascal's Pride

A Solidify Understanding Task



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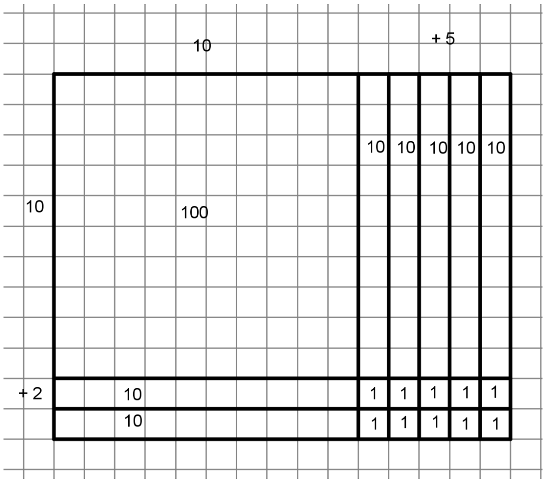
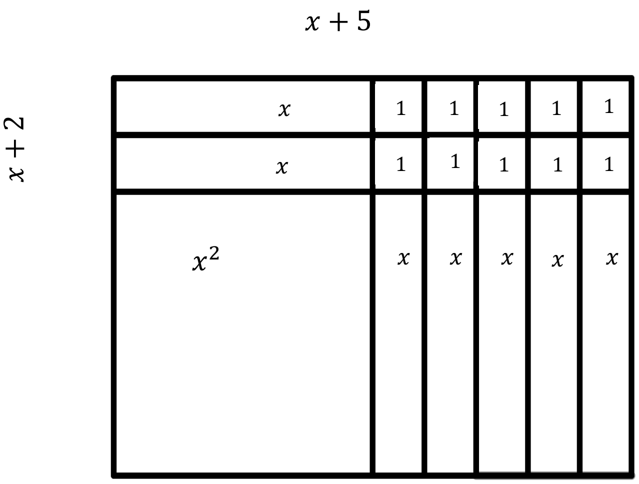
Multiplying polynomials can require a bit of skill in the algebra department, but since polynomials are structured like numbers, multiplication works very similarly. When you learned to multiply numbers, you may have learned to use an area model. To multiply 12 × 15 the area model and the related procedure probably looked like this:



$$\begin{array}{r} 12\\ \times\ \underline{15}\\ 10\\ 50\\ +\ \ 20\\ \underline{\ 100}\\ \mathbf{180}\end{array}$$

You may have used this same idea with quadratic expressions. Area models help us think about multiplying, factoring, and completing the square to find equivalent expressions. We modeled $(x+2)(x+5) = x^2+7x+10$ as the area of a rectangle with sides of length $x+2$ and $x+5$. The various parts of the rectangle are shown in the diagram below:



Some people like to shortcut the area model a little bit to just have sections of area that correspond to the lengths of the sides. In this case, they might draw the following.

	x	$+5$
x	x^2	$5x$
$+2$	$2x$	10

$$= x^2 + 7x + 10$$

1. What is the property that all of these models are based upon?

2. Now that you've been reminded of the happy past, you are ready to use the strategy of your choice to find equivalent expressions for each of the following:

a) $(x + 3)(x + 4)$

b) $(x + 7)(x - 2)$

Maybe now you remember some of the different forms for quadratic expressions—factored form and standard form. These forms exist for all polynomials, although as the powers get higher, the algebra may get a little trickier. In standard form polynomials are written so that the terms are in order with the highest-powered term first, and then the lower-powered terms. Some examples:

Quadratic: $x^2 - 3x + 8$ or $x^2 - 9$

Cubic: $2x^3 + x^2 - 7x - 10$ or $x^3 - 2x^2 + 15$

Quartic: $x^4 + x^3 + 3x^2 - 5x + 4$

Hopefully, you also remember that you need to be sure that each term in the first factor is multiplied by each term in the second factor and the like terms are combined to get to standard form. You can use area models, boxes, or mnemonics like FOIL (first, outer, inner, last) to help you organize, or you can just check every time to be sure that you've got all the combinations. It can get more challenging with higher-powered polynomials, but the principal is the same because it is based upon the mighty Distributive Property.

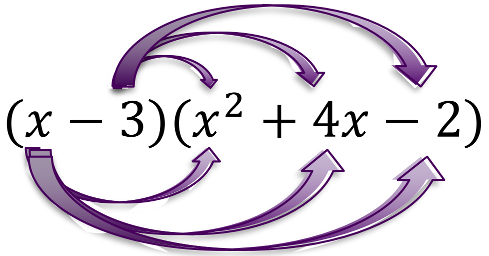
3. Tia's favorite strategy for multiplying polynomials is to make a box that fits the two factors. She sets it up like this: $(x+2)(x^2-3x+5)$

	x^2	$-3x$	$+5$
x			
$+2$			

Try using Tia's box method to multiply these two factors together and then combining like terms to get a polynomial in standard form.

4. Try checking your answer by graphing the original factored polynomial, $(x+2)(x^2-3x+5)$ and then graphing the polynomial that is your answer. If the graphs are the same, you are right because the two expressions are equivalent! If they are not the same, go back and check your work to make the corrections.

5. Tehani's favorite strategy is to connect the terms he needs to multiply in order like this:



$$(x-3)(x^2+4x-2)$$

Try multiplying using Tehani's strategy and then check your work by graphing. Make any corrections you need and figure out why they are needed so that you won't make the same mistake twice!

6. Use the strategy of your choice to multiply each of the following expressions. Check your work by graphing and make any needed corrections.

a) $(x+5)(x^2-x-3)$

b) $(x-2)(2x^2+6x+1)$

c) $(x+2)(x-2)(x+3)$

When graphing, it is often useful to have a perfect square quadratic or a perfect cube. Sometimes it is also useful to have these functions written in standard form. Let's try re-writing some related expressions to see if we can see some useful patterns.

7. Multiply and simplify both of the following expressions using the strategy of your choice:

a) $f(x) = (x+1)^2$ b) $f\,(x) = (x+1)^3$

Check your work by graphing and make any corrections needed.

8. Some enterprising young mathematician noticed a connection between the coefficients of the terms in the polynomial and the number pattern known as Pascal's Triangle. Put your answers from problem 5 into the table. Compare your answers to the numbers in Pascal's Triangle below and describe the relationship you see.

$(x+1)^0$	1	1
$(x+1)^1$	$x+1$	1 1
$(x+1)^2$		1 2 1
$(x+1)^3$		1 3 3 1
$(x+1)^4$		

9. It could save some time on multiplying the higher power polynomials if we could use Pascal's Triangle to get the coefficients. First, we would need to be able to construct our own Pascal's Triangle and add rows when we need to. Look at Pascal's Triangle and see if you can figure out how to get the next row using the terms from the previous row. Use your method to find the terms in the next row of the table above.

10. Now you can check your Pascal's Triangle by multiplying out $(x+1)^4$ and comparing the coefficients. Hint: You might want to make your job easier by using your answers from #7 in some way. Put your answer in the table above.

11. Make sure that the answer you get from multiplying $(x+1)^4$ and the numbers in Pascal's Triangle match, so that you're sure you've got both answers right. Then describe how to get the next row in Pascal's Triangle using the terms in the previous row.

12. Complete the next row of Pascal's Triangle and use it to find the standard form of $(x+1)^5$. Write your answers in the table on #6.

13. Pascal's Triangle wouldn't be very handy if it only worked to expand powers of $x+1$. There must be a way to use it for other expressions. The table below shows Pascal's Triangle and the expansion of $x+a$.

$(x+a)^0$	1	1
$(x+a)^1$	$x+a$	1 1
$(x+a)^2$	$x^2+2ax+a^2$	1 2 1
$(x+a)^3$	$x^3+\ 3ax^2+3a^2x+a^3$	1 3 3 1
$(x+a)^4$	$x^4+4ax^3+\ 6a^2x^2+3a^3x+a^4$	1 4 6 4 1

What do you notice about what happens to the a in each of the terms in a row?

14. Use the Pascal's Triangle method to find standard form for $(x+2)^3$. Check your answer by multiplying.

15. Use any method to write each of the following in standard form:

a) $(x+3)^3$

b) $(x-2)^3$

c) $(x+5)^4$

3.2 Pascal's Pride – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to sharpen students' multiplication skills for polynomials and to introduce the binomial theorem to expand polynomials using Pascal's Triangle. Students are reminded of the connections between polynomials and whole numbers, along with the area model for multiplying both polynomials and whole numbers. Standard form for polynomials is introduced and students work with polynomials in factored form and write them in standard form.

Core Standards Focus:

A-APR.5: Know and apply the Binomial Theorem for the expansion of $(x+y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Standards for Mathematical Practice:

SMP 6 – Attend to precision

SMP 8 – Look for express regularity in repeated reasoning

Vocabulary: Standard form of a polynomial

The Teaching Cycle:

Launch (Whole Group):

Begin the task by telling students that quadratics are one kind of polynomial that they have worked with previously. In this task we will be extending our work with quadratics to higher-powered polynomials, taking them from factored form to standard form. Focus students on the area models presented at the beginning of the task and ask how the large area model and the shortcut area model are related. Ask students question 1, and make sure that they know that all multiplication strategies are based upon the Distributive Property. Have students work problem 2 individually and then briefly discuss how to use the shortcut area model to do the problems. Then, ask students to work on the rest of the task.

Explore (Small Group):

Monitor students as they work to be sure that they understand both Tia and Tehani's strategy. Listen for students that have noticed that when using Tia's strategy (the box strategy), that the like terms are often lined up on the diagonal. This is a useful idea to share during the discussion to help students simplify their expressions once they have all the terms. Help students to check their work using technology as described in the task. Encourage them to not only make corrections, but to see where they are making errors so that they can avoid them in the future. As students are working, look for a student to present problem 6c that has noticed that two of the factors can be combined quickly because they are the factors that make a difference of squares.

As students work on the second part of the task with the binomial expansions, listen for students that can articulate the patterns in Pascal's Triangle that help them to construct and expand the triangle. If students are having trouble with constructing the next row of the triangle, ask them what relationships they see between numbers on one row with numbers on the next. Are there ways that they could combine numbers on one row to get the next row of numbers? Look for students to share during the discussion that can describe the pattern in the coefficients and relate it to Pascal's triangle.

Discuss (Whole Group):

Begin the discussion by having a student share their work on question #3. The student selected should be able to share the idea that the polynomial can be simplified and that the like terms are along the diagonals of the box. Ask another student to show how they used technology to check his/her answers and be sure that this strategy is understood by the class.

Next, have a student share their work with Tehani's strategy in #5. Ask the class to compare the two strategies. They should recognize that they produce the same terms and are simply two different ways to keep track of all the terms. Tell students that they may use either strategy (or both) as they choose. Ask a student to present their work in problem 6c demonstrating that two of the factors make a difference of squares, and then that result can be multiplied by the remaining factor. Point out that that the three linear factors multiplied together make a cubic. Ask students to notice what kind of polynomial is produced when a linear factor is multiplied by a quadratic factor.

Transition from problem 6c to problem 7b by asking how student used their work from #7a to help them in #7b. Ask a previously-selected student to share how they found $(x+1)^2$ and multiplied it by $(x+1)$ to find the answer. Ask students to describe how the coefficients of the terms in #7a and #7b compare to the numbers in Pascal's triangle.

Have the previously-selected student share how to construct the next row of Pascal's triangle and test their strategy by constructing the next row (problem 9). Then have a student show how to use Pascal's triangle to find the solution for problem #10. Ask a student to share their work on problem 14 and help students to see how the coefficients obtained from Pascal's triangle are multiplied by the increasing powers of 2 to get the expanded polynomial. Reinforce this idea by sharing as many of the remaining problems as time allows.

Aligned Ready, Set, Go: ***Numbers and Operations 3.2***

READY, SET, GO! Name Period Date

READY

Topic: Recalling the meaning of division

1. Given: $f(x) = (x+7)(2x-3)$ *and* $g(x) = (x+7)$. Find $g(x)\overline{)f(x)}$.

2. Given: $f(x) = (5x+7)(-3x+11)$ *and* $g(x) = (-3x+11)$. Find $g(x)\overline{)f(x)}$

3. Given: $f(x) = (x+2)(x^2+3x+2)$ *and* $g(x) = (x+2)$ Find $g(x)\overline{)f(x)}$

4. Given: $f(x) = (5x-3)(x^2-11x-9)$ *and* $g(x) = (5x-3)$ *and* $h(x) = (x^2-11x-9)$.

   a.) Find $g(x)\overline{)f(x)}$ b.) Find $h(x)\overline{)f(x)}$

5. Given: $f(x) = (5x-6)(2x^2-5x+3)$ *and* $g(x) = (x-1)$ *and* $h(x) = (2x-3)$.

   a.) Find $g(x)\overline{)f(x)}$ b.) Find $h(x)\overline{)f(x)}$

SET

Topic: Multiplying polynomials

Multiply. Write your answers in standard form.

6. $(a+b)(a+b)$

7. $(x-3)(x^2+3x+9)$

8. $(x-5)(x^2+5x+25)$

9. $(x+1)(x^2-x+1)$

10. $(x+7)(x^2-7x+49)$

11. $(a-b)(a^2+ab+b^2)$

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$(x+a)^0$	1	1
$(x+a)^1$	$x+a$	1 1
$(x+a)^2$	$x^2+2ax+a^2$	1 2 1
$(x+a)^3$	$x^3+\ 3ax^2+3a^2x+a^3$	1 3 3 1
$(x+a)^4$	$x^4+4ax^3+\ 6a^2x^2+3a^3x+a^4$	1 4 6 4 1

Use the table above to write each of the following in standard form.

12. $(x+1)^5$

13. $(x-5)^3$

14. $(x-1)^4$

15. $(x+4)^3$

16. $(x+2)^4$

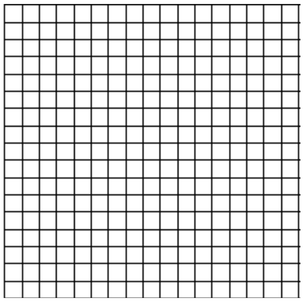
17. $(3x+1)^3$

GO

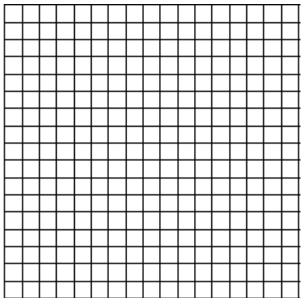
Topic: Examining transformations on different types of functions

Graph the following functions.

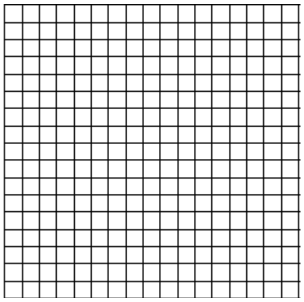
18. $g(x) = x+2$



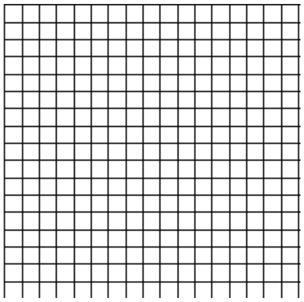
19. $h(x) = x^2+2$



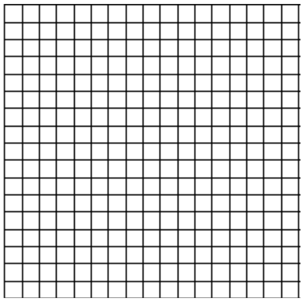
20. $f(x) = 2^x+2$



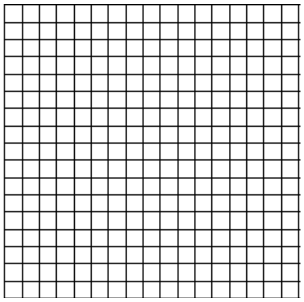
21. $g(x) = 3(x - 2)$



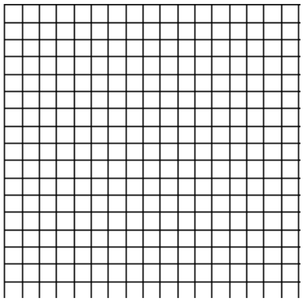
22. $h(x) = 3(x - 2)^2$



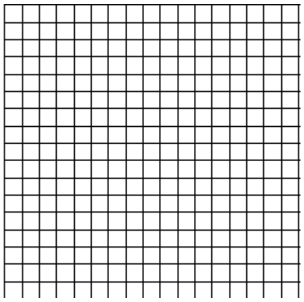
23. $f(x) = 3\sqrt{x - 2}$



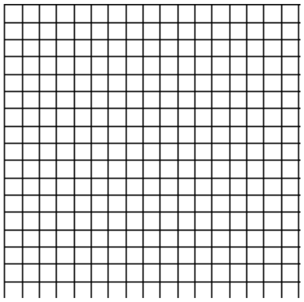
24. $g(x) = \frac{1}{2}(x - 1) - 2$



25. $h(x) = \frac{1}{2}(x - 1)^2 - 2$



26. $f(x) = |x - 1| - 2$



3.3 Divide And Conquer

A Solidify Understanding Task

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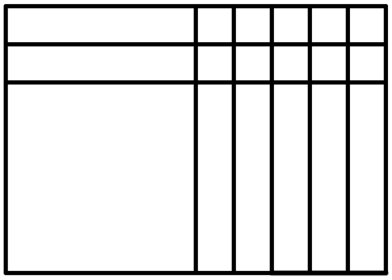
We've seen how numbers and polynomials relate in addition, subtraction, and multiplication. Now we're ready to consider division.

Division, you say? Like, long division? Yup, that's what we're talking about. Hold the judgment! It's actually pretty cool.

As usual, let's start by looking at how the operation works with numbers. Since division is the inverse operation of multiplication, the same models should be useful. The area model that we used with multiplication is also used with division. When we were using area models to factor a quadratic expression, we were actually dividing.

Let's brush up on that a bit.

1. The area model for $x^2 + 7x + 10$ is shown below:



Use the area model to write $x^2 + 7x + 10$ in factored form.

2. We also used number patterns to factor without drawing the area model. Use any strategy to factor the following quadratic polynomials:

a) $x^2 + 7x + 12$	b) $x^2 + 2x - 15$

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c) $x^2 - 11x + 24$	d) $x^2 - 5x - 36$

Factoring works great for quadratics and a few special cases of other polynomials. Let's look at a more general version of division that is a lot like what we do with numbers. Let's say we want to divide 1452 by 12. If we write the analogous polynomial division problem, it would be: $(x^3 + 4x^2 + 5x + 2) \div (x + 2)$.

Let's use the division process for numbers to create a division process for polynomials. (Don't panic—in many ways it's easier with polynomials than numbers!)

Step 1: Start with writing the problem as long division. The polynomial needs to have the terms written in descending order. If there are any missing powers, it's easier if you leave a little space for them.

$$12\overline{)1452} \qquad\qquad x+2\overline{)x^3+4x^2+5x+2}$$

Step 2: Determine what you could multiply the divisor by to get the first term of the dividend.

$$\begin{array}{r} 1 \\ 12\overline{)1452} \end{array} \qquad\qquad \begin{array}{r} x^2 \\ x+2\overline{)x^3+4x^2+5x+2} \end{array}$$

Step 3: Multiply and put the result below the dividend.

$$\begin{array}{r} 1 \\ 12\overline{)1452} \\ \underline{-1200} \end{array} \qquad\qquad \begin{array}{r} x^2 \\ x+2\overline{)x^3+4x^2+5x+2} \\ -(\underline{x^3+2x^2}) \end{array}$$

Step 4: Subtract. (It helps to keep the signs straight if you change the sign on each term and add on the polynomial.)

$$\begin{array}{r} 1 \\ 12\overline{)1452} \\ \underline{-1200} \\ 252 \end{array} \qquad\qquad \begin{array}{r} x^2 \\ x+2\overline{)x^3+4x^2+5x+2} \\ +(\underline{-x^3-2x^2}) \\ 2x^2+5x+2 \end{array}$$

Step 5: Repeat the process with the number or expression that remains in the dividend.

$$\begin{array}{r} 12 \\ 12\overline{)1452} \\ \underline{-1200} \\ 252 \\ -\underline{240} \\ 12 \end{array} \qquad\qquad \begin{array}{r} x^2+2x \\ x+2\overline{)x^3+4x^2+5x+2} \\ +(\underline{-x^3-2x^2}) \\ 2x^2+5x+2 \\ -(\underline{2x^2+4x}) \\ x+2 \end{array}$$

Step 6: Keep going until the number or expression that remains is smaller than the divisor.

$$\begin{array}{r} 121 \\ 12\overline{)1452} \\ \underline{-1200} \\ 252 \\ -\underline{240} \\ 12 \\ -\underline{12} \\ 0 \end{array} \qquad\qquad \begin{array}{r} x^2+2x+1 \\ x+2\overline{)x^3+4x^2+5x+2} \\ +(\underline{-x^3-2x^2}) \\ 2x^2+5x+2 \\ -(\underline{2x^2+4x}) \\ x+2 \\ -\underline{(x+2)} \\ 0 \end{array}$$

In this case, 121 divided by 12 leaves no remainder, so we would say that 12 is a factor of 121. Similarly, since (x^3+4x^2+5x+2) divided by $(x+2)$ leaves no remainder, we would say that $(x+2)$ is a factor of (x^3+4x^2+5x+2).

Polynomial division doesn't always match up perfectly to an analogous whole number problem, but the process is always the same. Let's try it.

3. Use long division to determine if $(x-1)$ a factor of $(x^3-3x^2-13x+15)$. Don't worry: the steps for the division process are below:

a) Write the problem as long division.
b) What do you have to multiply x by to get x^3? Write your answer above the bar.
c) Multiply your answer from step b by $(x-1)$ and write your answer below the dividend.
d) Subtract. Be careful to subtract each term. (You might want to change the signs and add.)
e) Repeat steps a-d until the expression that remains is less than $(x-1)$.

We hope you survived the division process. Is $(x-1)$ a factor of $(x^3-3x^2-13x+15)$? ________

4. Try it again. Use long division to determine if $(2x+3)$ is a factor of $2x^3+7x^2+2x+9$. No hints this time. You can do it!

When dividing numbers, there are several ways to deal with the remainder. Sometimes, we just write it as the remainder, like this:

$$3\overset{8r.1}{\overline{\big)\ 25}}$$ because $3(8)+1=25$

You may remember also writing the remainder as a fraction like this:

$$\begin{array}{r} 8\,{}^{1}\!/_{3} \\ 3\overline{)\;\;25} \end{array}$$ because $3\left(8\frac{1}{3}\right) = 25$

We do the same things with polynomials.

Maybe you found that $(2x^3 + 7x^2 + 2x + 9) \div (2x + 3) = \ (x^2 + 2x - 2)\ r.15$. (We sure hope so.) You can use it to write two multiplication statements:

$$(2x + 3)(x^2 + 2x - 2) + 15 = (2x^3 + 7x^2 + 2x + 9)$$

and

$$(2x + 3)(x^2 + 2x - 2 + \frac{15}{2x + 3}) = (2x^3 + 7x^2 + 2x + 9)$$

5. Divide each of the following polynomials. Write the two multiplication statements that go with your answers if there is a remainder. Write only one multiplication statement if the divisor is a factor. Use graphing technology to check your work and make the necessary corrections.

	a) $(x^3 + 6x^2 + 13x + 12) \div (x + 3)$	b) $(x^3 - 4x^2 + 2x + 5) \div (x - 2)$
Multiplication statements:		

	c) $(6x^3 - 11x^2 - 4x + 5) \div (2x - 1)$	d) $(x^4 - 23x^3 + 49x + 4) \div (x^2 + x + 2)$
Multiplication statements:		

3.3 Divide And Conquer – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to reinforce students' prior knowledge of factoring and to introduce polynomial long division. The task draws connections between polynomial long division and division of whole numbers to support students in understanding the procedure. Students will write remainders in two ways to create equivalent polynomial expressions and determine whether a given expression is a factor according to the Polynomial Remainder Theorem. Both factoring and long division will be used in upcoming tasks to write polynomials in factored form and to find their roots.

Core Standards Focus:

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A.APR.2: Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

Standards for Mathematical Practice:

SMP 8 – Look for and express regularity in repeated reasoning

Vocabulary: Remainder Theorem

The Teaching Cycle:

Launch (Whole Group):

Introduce the task by asking students how they determine factors of numbers like 243. Without actually going through the process, students should recognize that they would divide to determine factors. Remind students that they have factored quadratic trinomials and this was a way to see what expressions had been multiplied together to give the trinomial, which is essentially a division process. Tell students that in upcoming work, they will need to find factors of higher-powered polynomials, which cannot usually be done with the same strategies as quadratics. In this task, we will be learning how to divide polynomials so that we can determine if an expression is a factor. Tell students that the task begins with brushing up on factoring. Remind students how they used area models to factor by working problem 1 together. Then ask students to work problem 2 individually. After giving students a few minutes to work, ask two students to share their work on the two problems.

Next, briefly explain how 1452 ÷12 is analogous to $(x^3 + 4x^2 + 5x + 2) \div (x + 2)$. Demonstrate each of the steps of the long division process shown in the task, with the number problem parallel to the polynomial problem so that students can see that the two processes are the same. Then, ask students to work on the remainder of the task.

Explore (Small Groups):

Support students as they are working by drawing them back to the steps in the division process for whole numbers, which they already know. Many of the problems that students typically have in long division are a result of making mistakes when subtracting, so you may wish to encourage students to subtract by consistently changing the signs and adding, as suggested in *3.3 It All Adds Up*.

Discuss (Whole Group):

Begin the discussion by asking a student to share problem 3, walking through each step carefully. Continue the discussion with as many of the rest of the problems as time allows, each time having a student share work and reinforcing the steps. At the end of each problem, ask whether the divisor is a factor. Discuss the two forms for writing polynomials that are equivalent to the dividend. Demonstrate how to use the two multiplication statements to check answers by graphing a multiplication statement and the dividend and checking to see that the two graphs coincide. Be sure to include problem 5d, which does not contain a squared term in the dividend. Most students will not anticipate leaving a space for the term, so this problem provides an opportunity to discuss why it is helpful to have all the terms in the dividend in decreasing order and to leave space for any of the terms that are missing. This problem also has a remainder that contains two terms.

Emphasize that the remainder is still handled the same way, and then ask a student to write the two multiplication statements that are equivalent to the dividend.

Aligned Ready, Set, Go: ***Numbers and Operations 3.3***

READY, SET, GO! Name Period Date

READY

Topic: Solving linear equations

Solve for x.

1. $5x + 13 = 48$
2. $\frac{1}{3}x - 8 = 0$
3. $-4 - 9x = 0$
4. $x^2 - 16 = 0$
5. $x^2 + 4x + 3 = 0$
6. $x^2 - 5x + 6 = 0$
7. $(x + 8)(x + 11) = 0$
8. $(x - 5)(x - 7) = 0$
9. $(3x - 18)(5x - 10) = 0$

SET

Topic: Dividing polynomials

Divide each of the following polynomials. Write only one multiplication statement if the divisor is a factor. Write the two multiplication statements that go with your answers if there is a remainder.

10. $(x+1)\overline{\smash{)}x^3 - 3x^2 + 6x + 11}$

11. $(x-5)\overline{\smash{)}x^3 - 9x^2 + 23x - 15}$

Multiplication statement(s)

Multiplication statement(s)

12. $(2x-1)\overline{)2x^3+15x^2-34x+13}$

Multiplication statement(s)

13. $(x+4)\overline{)x^3+13x^2+26x-25}$

Multiplication statement(s)

14. $(x+7)\overline{)x^3-8x^2-111x+10}$

Multiplication statement(s)

15. $(3x-4)\overline{)3x^3+23x^2+6x-28}$

Multiplication statement(s)

GO

Topic: Describing the features of a variety of functions

Graph the following functions. Then identify the key features of the functions. Include domain, range, intervals where the function is increasing/decreasing, intercepts, maximum/minimum, and end behavior.

16. $f(x) = x^2 - 9$

domain: range:

increasing: decreasing:

y-intercept: x-intercept(s):

17. $f(n-1) = f(n) + 3;\ f(1) = 4$

domain: range:

increasing: decreasing:

y-intercept: x-intercept(s):

18. $f(x) = \sqrt{x-3} + 1$

domain: range:

increasing: decreasing:

y-intercept: x-intercept(s):

19. $f(x) = log_2 x - 1$

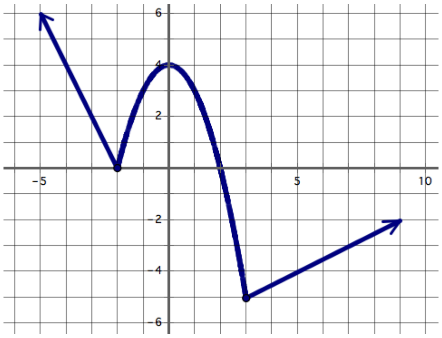
domain: range:

increasing: decreasing:

y-intercept: x-intercept(s):

Identify the key features of the graphed functions.

20.

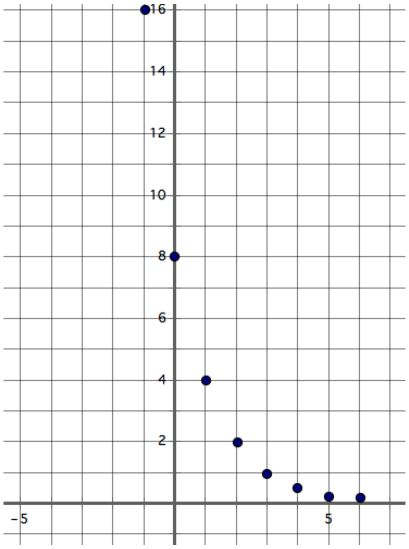


domain: range:

increasing: decreasing:

y-intercept: x-intercept(s):

21.



domain: range:

increasing: decreasing:

y-intercept: x-intercept(s):

3.4 To Be Determined . . .

A Develop Understanding Task

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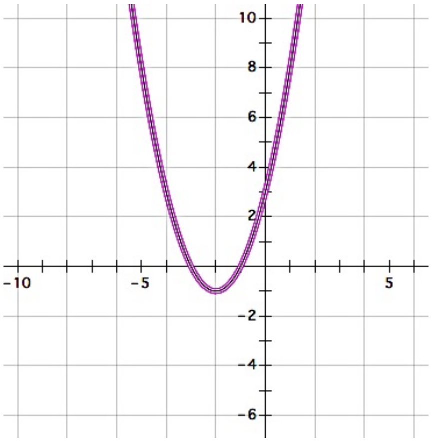
Israel and Miriam are working together on a homework assignment. They need to write the equations of quadratic functions from the information given in a table or a graph. At first, this work seemed really easy. However, as they continued to work on the assignment, the algebra got more challenging and raised some interesting questions that they can't wait to ask their teacher.

Work through the following problems from Israel and Miriam's homework. Use the information in the table or the graph to write the equation of the quadratic function in all three forms. You may start with any form you choose, but you need to find all three equivalent forms. (If you get stuck, your teacher has some hints from Israel and Miriam that might help you.)

1.

x	y
-5	8
-4	3
-3	0
-2	-1
-1	0
0	3
1	8
2	15
3	24
4	35



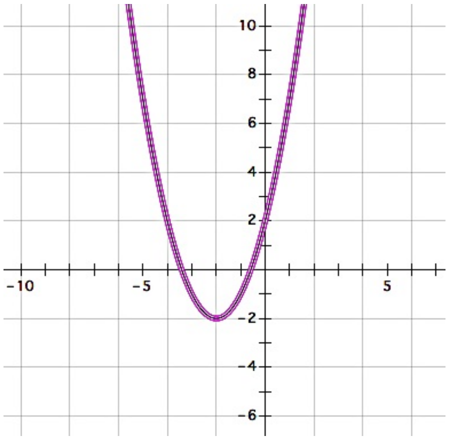
Standard form:

Factored form:

Vertex form:

2.

x	y
-5	7
-4	2
-3	-1
-2	-2
-1	-1
0	2
1	7
2	14
3	23
4	34



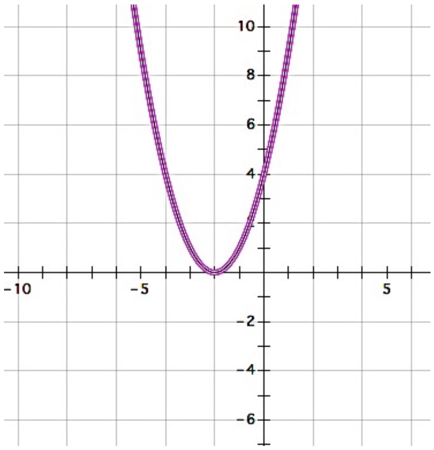
Standard form:

Factored form:

Vertex form:

3.

x	y
-5	9
-4	4
-3	1
-2	0
-1	1
0	4
1	9
2	16
3	25
4	36



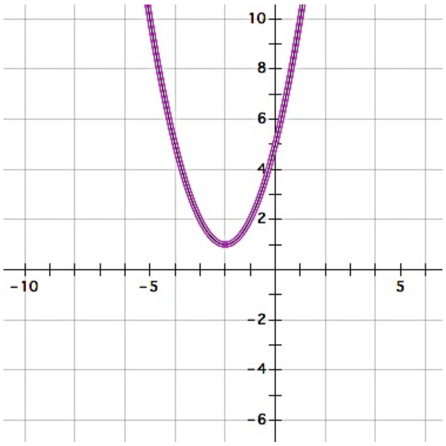
Standard form:

Factored form:

Vertex form:

4.

x	y
-5	10
-4	5
-3	2
-2	1
-1	2
0	5
1	10
2	17
3	26
4	37



Standard form:

Factored form:

Vertex form:

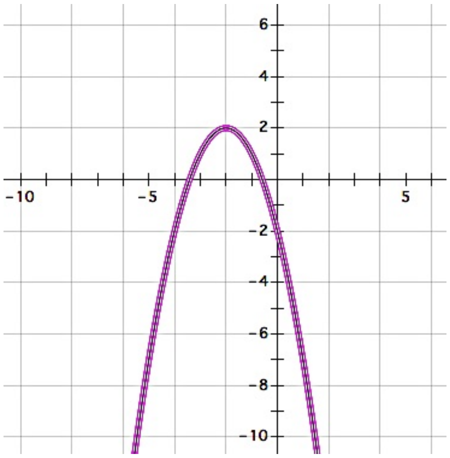
5. Israel was concerned that his factored form for the function in question 4 didn't look quite right. Miriam suggested that he test it out by plugging in some values for *x* to see if he gets the same points as those in the table. Test *your* factored form. Do you get the same values as those in the table?

6. Why might Israel be concerned about writing the factored form of the function in question 4?

Here are some more from Israel and Miriam's homework.

7.

x	y
-5	-7
-4	-2
-3	1
-2	2
-1	1
0	-2
1	-7
2	-14
3	-23
4	-34



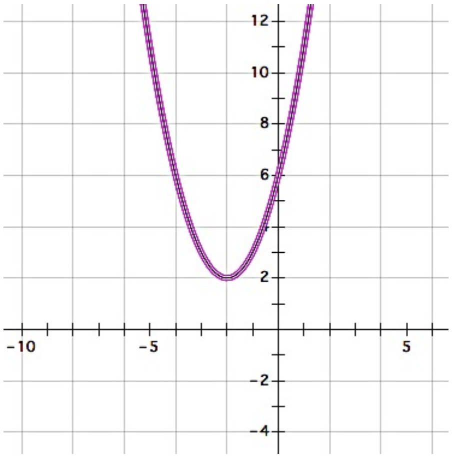
Standard form:

Factored form:

Vertex form:

8.

x	y
-5	11
-4	6
-3	3
-2	2
-1	3
0	6
1	11
2	18
3	27
4	38



Standard form:

Factored form:

Vertex form:

9. Miriam notices that the graphs of function 7 and function 8 have the same vertex point. Israel notices that the graphs of function 2 and function 7 are mirror images across the *x*-axis. What do you notice about the roots of these three quadratic functions?

The Fundamental Theorem of Algebra

A polynomial function is a function of the form:

$$y = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots a_{n-3}x^3 + a_{n-2}x^2 + a_{n-1}x + a_n$$

where all of the exponents are positive integers and all of the coefficients $a_0 \ldots a_n$ are constants.

As the theory of finding roots of polynomial functions evolved, a 17th century mathematician, Girard (1595-1632) made the following claim which has come to be known as the Fundamental Theorem of Algebra: *An nth degree polynomial function has n roots*.

10. In the next module you will study polynomial functions that contain higher-ordered terms such as x^3 or x^5. Based on you work in this task, do you believe this theorem holds for quadratic functions? That is, do all functions of the form $y = ax^2 + bx + c$ always have two roots? [Examine the graphs of each of the quadratic functions you have written equations for in this task. Do they all have two roots? Why or why not?]

3.4 To Be Determined . . . – Teacher Notes

A Develop Understanding Task

Purpose: In the context of using procedures students have developed previously for writing equations for quadratic functions from the information given in a table or a graph, students will examine the nature of the roots of quadratic functions and surface the need for non-real roots when the quadratic function does not intersect the x-axis. This task follows the approach of the historical development of these non-real numbers. As mathematicians developed formulas for solving quadratic and cubic polynomials, the square root of a negative number would sometimes occur in their work. Although such expressions seemed problematic and undefined, when mathematicians persisted in working with these expressions using the same algebraic rules that applied to real-valued radical expressions, the work would lead to correct results. In this task, students will be able to write the equation of quadratic #4 in both vertex and standard form, but attempting to use the quadratic formula to find the roots, and therefore the factored form, will produce expressions that contain the square root of a negative number. However, if students persist in expanding out this factored form using the usual rules of arithmetic, the non-real-valued radical expressions will go away, leaving the same standard form as that obtained by expanding the vertex form. This should give some validity to these non-real-valued radical expressions. It is suggested that these numbers not be referred to as "imaginary" numbers in this task, but only that they are noted to be problematic in the sense of not representing a real value.

(Note: In the early history of mathematics even negative real numbers were considered "fictitious" or "false" solutions to quadratic and cubic equations, although Cardano (1501-1576) and Bombelli (1526-1572) also used square roots of negative numbers in their work. By the 17th century negative numbers were recognized as legitimate solutions to polynomial equations, but complex numbers remained controversial through the 18th century, even though they were useful in the theory of equations. Descartes (1596-1650) called all complex numbers "imaginary" and it was Euler (1707-1783) that introduced the symbol *i* for the square root of -1. Although expanding the number system to include complex numbers was sufficient to solve quadratic equations, it was not known if complex numbers were sufficient to solve cubic and higher-degree polynomial equations until 1799 when Gauss published a proof that all polynomial equations of degree *n* have *n* roots of the form *a* + *bi*. See a brief history of complex numbers at http://www.clarku.edu/~djoyce/complex/)

Core Standards Focus:

N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.

N.CN.8 Extend polynomial identities to the complex numbers.

N.CN.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

A.REI.4 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

Note for Mathematics II A.REI.4a, A.REI.4b

Extend to solving any quadratic equation with real coefficients, including those with complex solutions.

Standards for Mathematical Practice:

SMP 5 – Use appropriate tools strategically

SMP 7 – Look for and make use of structure

SMP 8 – Look for and express regularity in repeated reasoning

The Teaching Cycle:

Launch (Whole Class):

Ask students to describe how the quadratic graphs shown in the questions on this task are similar and different. They should notice that all of the graphs are the same shape; that in many cases they have been translated up or down the line $x = -2$; they all are the same shape as the parent graph

$y = x^2$, so the coefficient a = 1; graphs 2 and 7 are mirror images across the x-axis, and graphs 7 and 8 have the same vertex. Remind students of the three forms we have used for writing quadratic expressions (Note that a = 1): standard form, $x^2 + bx + c$; vertex form, $(x - h)^2 + k$; and factored-form, $(x + p)(x + q)$.

Explore (Small Group):

Watch for students who start question 1 by locating the x-intercepts of the graph (the zeroes in the table) and then using these values to write the two factors for the factored form. Once the factors are written they might multiply out the expression to get standard form, and then complete the square to get vertex form. This procedural approach works well for question 1, but will not work for questions 2 and 4, since the roots of the quadratic are not readily apparent from the table or the graph. For these students, suggest that Israel and Miriam noticed that all of the parabolas are symmetric about the same line x = -2, and so the vertex always lies on this line. Encourage these students to consider how they might use this fact to write the vertex form of each equation.

Watch for how students approach question 3, where the vertex lies on the x-axis. It may be difficult for students to recognize that the x-intercept at x = -2 is a root of multiplicity 2 and that the vertex form is $y = (x+2)^2$. For these students, suggest that Israel and Miriam noticed that as this sequence of parabolas are shifted up along the axis of symmetry the two x-intercepts get closer and closer together until they merge at x = -2. This would suggest that we have a "double root" at x = -2.

The most useful strategy students might use for question 2 is to write the vertex form by locating the minimum point in either the table or the graph. Once students have written the vertex form they can expand it to get standard form. They can then use the quadratic formula to find the zeros of the function, and then use these zeroes to write the corresponding factors. The irrational roots in question 2 can be found in this way. Have students verify that the radical values found using the quadratic formula fit in the intervals between -4 and -3 and between -1 and 0 by calculating approximate values for these roots using a calculator. In a similar way, students can find the roots of the quadratic in question 4 using the quadratic formula. At this point in time, allow students to write these roots as $-2-\sqrt{-1}$ and $-2+\sqrt{-1}$; you do not need to introduce the notation for complex numbers using i to represent the square root of -1. This will be the focus of the next task.

Students may try to find approximate values for these roots using a calculator or recognize the dilemma of taking the square root of a negative number as being undefined in terms of real numbers. Acknowledge this dilemma, but also ask students to test their factored form for a few points (see question 5) and to multiply out their factored form in the usual way to verify that it yields the same standard form of the equation that they got when they expanded their vertex form. The goal here is to help students see that these numbers—while undefined in the real number system—yield correct results when manipulated with the familiar rules of algebra.

It is anticipated that students will get bogged down with this algebraic work. You can move to a whole class discussion during problems 4-6 to resolve these algebraic issues.

Discuss (Whole Class):
Begin the whole class discussion by examining question 4. First, have a student present the vertex form of this equation $y=(x+2)^2+1$. Then have a student present the standard form, which can be obtained by multiplying out the vertex form: $y=x^2+4x+5$. Finally ask how we might write the factored form of this function, since it does not cross the *x*-axis and therefore has no *x*-intercepts. If you have identified students who used the quadratic formula to find the "zeros" or "roots" of the quadratic, have them present their work.

Students may have questions about how to write the roots of this quadratic after substituting values for *a, b* and *c* into the quadratic formula, or how to write the factored form since the roots contain two terms. Help support this algebraic work, including simplifying the radical expression $\sqrt{-4}=\sqrt{4}\cdot\sqrt{-1}=2\sqrt{-1}$, and the rational expression $\frac{-4\pm2\sqrt{-1}}{2}=\frac{-4}{2}\pm\frac{2\sqrt{-1}}{2}=-2\pm\sqrt{-1}$. The factored form is $y=\left[x-\left(-2+\sqrt{-1}\right)\right]\cdot\left[x-\left(-2-\sqrt{-1}\right)\right]=\left(x+2-\sqrt{-1}\right)\left(x+2+\sqrt{-1}\right)$. Once students have written the factored form, ask them to multiply out the two trinomial factors to obtain the standard form. Help them observe that $\sqrt{-1}\cdot\sqrt{-1}=-1$ is consistent with the properties of radicals we have defined previously, and that this interpretation leads to the same standard form we started with. This consistency of properties will lead us in the next task to define the set of complex numbers.

Once the algebra of working with these negative radical expressions has been demonstrated, have students continue to work on the remainder of the task. Questions 7-9 point out that the algebraic work for irrational roots is similar to the algebraic work for these non-real roots.

Be sure to have a whole class discussion about the Fundamental Theorem of Algebra (question 10). Students may not feel like the theorem is true for quadratics, since #3 has only one real root and #4 and #8 have no real roots at all. Point out that we need to count these non-real roots, as well as multiple roots (such as x = -2 being a root of multiplicity 2 for question 3) to account for two roots for every quadratic. This will lead to the definition of complex numbers as roots in the next task.

Aligned Ready, Set, Go: *Numbers and Operations 3.4*

READY, SET, GO! Name Period Date

READY

Topic: Simplifying Radicals

Simplify each of the radicals below.

1. $\sqrt{8}$

2. $\sqrt{18}$

3. $\sqrt{32}$

4. $\sqrt{20}$

5. $\sqrt{45}$

6. $\sqrt{80}$

7. What is the connection between the radicals above? Explain.

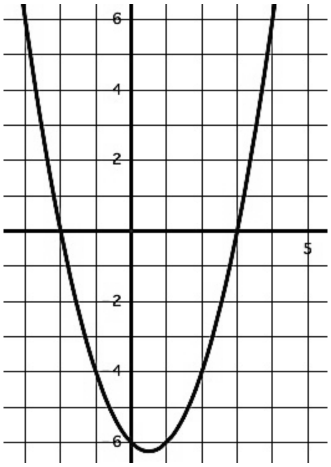
SET

Topic: Determine the nature of the x-intercepts for each quadratic below.

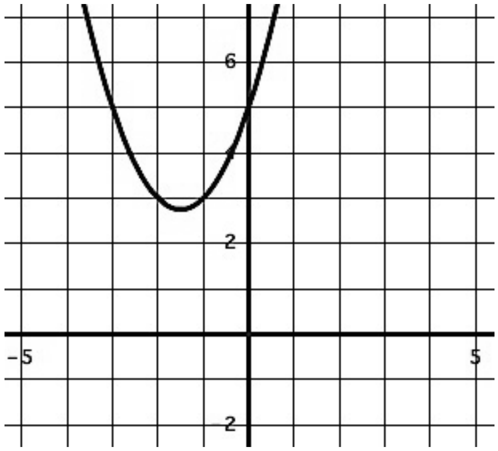
Given the quadratic function, its graph or other information, below determine the nature of the x-intercepts (what type of number it is). Explain or show how you know.

(Whole numbers "$\mathbb{W}$", Integers "$\mathbb{Z}$", Rational "$\mathbb{Q}$", Irrational "$\overline{\mathbb{Q}}$", or finally, "not Real")

8. Determine the nature of the x-intercepts.



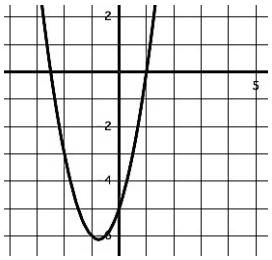
9. Determine the nature of the x-intercepts



10. Determine the nature of the x-intercepts.
$f(x) = x^2 + 4x - 24$

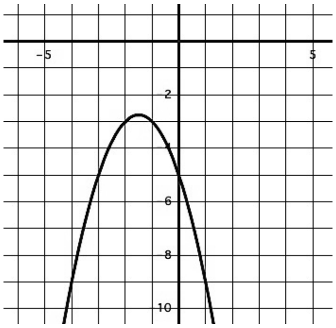
11. Determine the nature of the x-intercepts.
$g(x) = (2x - 1)(5x + 2)$

12. Determine the nature of the x-intercepts.



$f(x) = 2x^2 + 3x - 5$

13. Determine the nature of the x-intercepts.



14. Determine the nature of the x-intercepts.
$r(t) = t^2 - 8t + 16$

15. Determine the nature of the x-intercepts.
$h(x) = 3x^2 - 5x + 9$

Determine the number of roots that each polynomial will have.

16. $x^5 + 7x^3 - x^2 + 4x - 21$

17. $4x^3 + 2x^2 - 3x - 9$

18. $2x^7 + 4x^5 - 5x^2 + 16x + 3$

GO

Topic: Finding x-intercepts for quadratics using factoring and quadratic formula.

If the given quadratic function can be factored then factor and provide the x-intercepts. If you cannot factor the function then use the quadratic formula to find the x-intercepts.

19. $A(x) = x^2 + 4x - 21$

20. $B(x) = 5x^2 + 16x + 3$

21. $C(x) = x^2 - 4x + 1$

22. $D(x) = x^2 - 16x + 4$

23. $E(x) = x^2 + 3x - 40$

24. $F(x) = 2x^2 - 3x - 9$

25. $G(x) = x^2 - 3x$

26. $H(x) = x^2 + 6x + 8$

27. $K(x) = 3x^2 - 11$

3.5 My Irrational and Imaginary Friends

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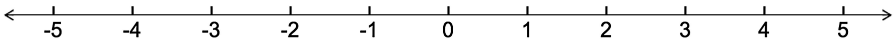
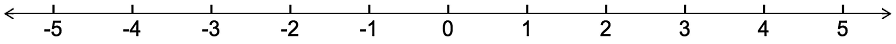
A Solidify Understanding Task

Part 1: Irrational numbers

1. Verify that $4\left(x-\frac{5}{2}\right)\left(x+\frac{3}{2}\right)=0$ and $4x^2-4x-15=0$ are equivalent equations (show your work), and plot the solutions to the quadratic equations on the following number line:

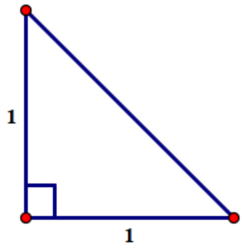


2. Verify that $(x-2+\sqrt{2})(x-2-\sqrt{2})=0$ and $4x^2-4x+2=0$ are equivalent equations (show your work), and plot the solutions to the quadratic equations on the following number line:



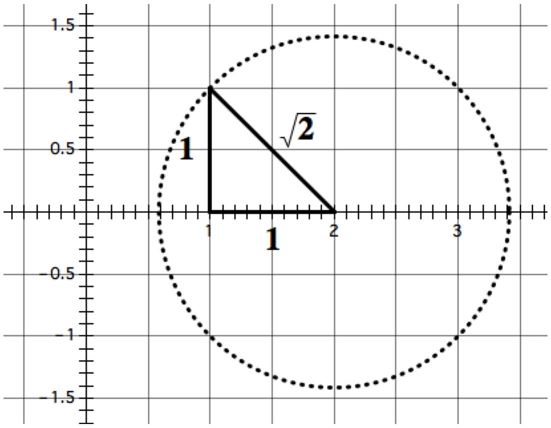
You may have found it difficult to locate the exact points on the number line that represent the two solutions to the 2nd pair of quadratic equations given above. The following diagrams might help.

3. Find the perimeter of this isosceles triangle. Express your answer as simply as possible.



We might approximate the perimeter of this triangle with a decimal number, but the exact perimeter is $2+\sqrt{2}$, which cannot be simplified any farther. Note that this notation represents a single number—the distance around the perimeter of the triangle—even though it is written as the sum of two terms.

4. Explain how you could use this diagram to locate the two solutions to the quadratic equations given in the 2nd problem above: $2 + \sqrt{2}$ and $2 - \sqrt{2}$.



5. Are the numbers we have located on the number line in this way rational numbers or irrational numbers? Explain your answer.

Both sets of quadratic equations given in problems 1 and 2 above have solutions that can be plotted on a number line. The solutions to the first set of quadratic equations are rational numbers. The solutions to the 2nd set of quadratic equations are irrational numbers.

Big Idea #1: The set of numbers that contains all of the *rational numbers* and all of the *irrational numbers* is called the set of *real numbers*. The location of all points on a number line can be represented by real numbers.

Part 2: Imaginary and Complex Numbers

In the previous task, *To Be Determined . . .*, you found that the quadratic formula gives the solutions to the quadratic equation $x^2 + 4x + 5 = 0$ as $-2+\sqrt{-1}$ and $-2-\sqrt{-1}$. Because the square root of a negative number has no defined value as either a rational or an irrational number, Euler proposed that a new number $i = \sqrt{-1}$ be included in what came to be known as the complex number system.

6. Based on Euler's definition of i, what would the value of i^2 be?

With the introduction of the number *i*, the square root of *any* negative number can be represented. For example, $\sqrt{-2}=\sqrt{2}\cdot\sqrt{-1}=\sqrt{2}\cdot i$ and $\sqrt{-9}=\sqrt{9}\cdot\sqrt{-1}=3i$.

7. Find the values of the following expressions. Show the details of your work.

   (a) $\left(\sqrt{2}\cdot i\right)^2$

   (b) $3i\times 3i$

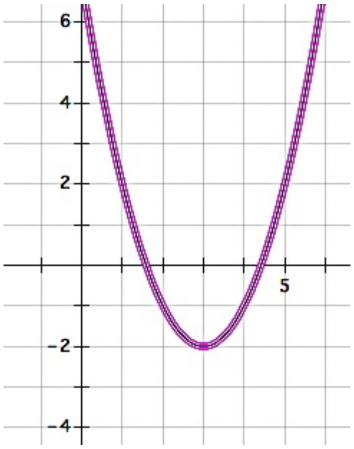
Using this new notation, the solutions to the equation $x^2+4x+5=0$ can be written as $-2+i$ and $-2-i$, and the factored form of x^2+4x+5 can be written as $(x+2-i)(x+2+i)$.

8. Verify that x^2+4x+5 and $(x+2-i)(x+2+i)$ are equivalent by expanding and simplifying the factored form. Show the details of your work.

Big Idea #2: Numbers like $3i$ and $\sqrt{2}\cdot i$ are called *pure imaginary numbers*. Numbers like $-2-i$ and $-2+i$ that include a real term and an imaginary term are called *complex numbers*.

The quadratic formula is usually written in the form $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. An equivalent form is $\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$. If *a*, *b* and *c* are rational coefficients, then $\frac{-b}{2a}$ is a rational term, and $\frac{\sqrt{b^2-4ac}}{2a}$ may be a rational term, an irrational term or an imaginary term, depending on the value of the expression under the square root sign.

9. Examine the roots of the quadratic $y = x^2 - 6x + 7$ shown in the graph at the right. How do the terms $\frac{-b}{2a}$ and $\frac{\sqrt{b^2-4ac}}{2a}$ show up in this graph?

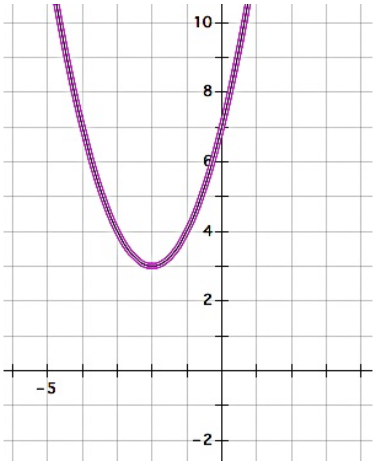


Look back at the work you did in the task *To Be Determined . . .*

10. Which quadratics in that task had complex roots? (List them here.)

11. How can you determine if a quadratic has complex roots from its graph?

12. Find the complex roots of the following quadratic function represented by its graph.



13. Reflect the graph of the quadratic function given in question 12 over the horizontal line $y = 3$. Find the irrational roots of the reflected quadratic function.

14. How is the work you did to find the roots of the quadratic functions in questions 12 and 13 similar?

Big Idea #3: Complex numbers are not real numbers—they do not lie on the real number line that includes all of the rational and irrational numbers; also note that the real numbers are a subset of the complex numbers since a real number results when the imaginary part of *a* + *bi* is 0, that is, *a* + 0*i*.

The Fundamental Theorem of Algebra, Revisited

Remember the following information given in the previous task:

A polynomial function is a function of the form:

$$y = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots a_{n-3}x^3 + a_{n-2}x^2 + a_{n-1}x + a_n$$

where all of the exponents are positive integers and all of the coefficients $a_0 \dots a_n$ are constants.

As the theory of finding roots of polynomial functions evolved, a 17th century mathematician, Girard (1595-1632) made the following claim which has come to be known as the Fundamental Theorem of Algebra: *An nth degree polynomial function has n roots.*

15. Based on you work in this task, do you believe this theorem holds for quadratic functions? That is, do all functions of the form $y = ax^2 + bx + c$ always have two roots?

3.5 My Irrational and Imaginary Friends – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to examine the meaning and the arithmetic of irrational numbers, mainly non-rational radical numbers, as well as the meaning and arithmetic of complex numbers. Students will note similarities and differences in the rules of simplifying such expressions.

Unlike rational numbers, where we can always find a common rational unit of measure that fits into two different rational lengths, irrational numbers are said to be incommensurate, since no rational unit of measure can be found that fits into an irrational length evenly. For example, a length of $\frac{2}{3}+\frac{3}{4}$ can be accurately measured using a $\frac{1}{12}$ unit of length. On the other hand, a length of $2+\sqrt{2}$ cannot be measured with any rational unit of length. Decimal units—tenths, hundredths, thousandths, etc.—cannot be used to measure an irrational length exactly. Consequently, there is no exact decimal representation for irrational lengths. We can only approximate irrational lengths with a finite decimal number. Thinking about rational and irrational numbers from a geometric perspective will give students a sense of what it means to add, subtract and multiply when one addend or factor is rational and the other is irrational. This discussion is then extended to consider what happens when the numbers to be added or multiplied are imaginary or complex numbers.

Core Standards Focus:

N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Note for Algebra II: *Connect N.RN.3 to physical situations, e.g., finding the perimeter of a square of area 2.*

N.CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.

N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Note for Mathematics II*:* *Limit to multiplications that involve i^2 as the highest power of i.*

N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.

N.CN.8 Extend polynomial identities to the complex numbers.

N.CN.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials**.**

Standards for Mathematical Practice:

- **SMP 6 – Attend to precision**
- **SMP 7 – Look for and make use of structure**
- **SMP 8 – Look for and express regularity in repeated reasoning**

The Teaching Cycle:

Launch (Whole Class):

Part 1 of this task, locating the solutions of quadratic functions whose solutions are rational or irrational numbers, serves as the launch for part 2 of this task. Work through each of the five problems in part 1 together. Give students a few minutes to work on each question, then have students who have correct solutions present, and finally discuss the key ideas of each question before moving onto the next. Clarify any algebraic work that seemed difficult or problematic for students. Help students become fluent in the arithmetic work represented by these problems, particularly the distributive property when factors have been decomposed into multiple terms.

The key idea of question 1 is that rational solutions of quadratic equations can be plotted on a number line by identifying an appropriate fractional unit, in this case ½ units. Ask students if a single fractional unit will always work for plotting both of the solutions to a particular quadratic equation. That is, could one of the solutions be a multiple of one fractional unit, such as ½, and the other root be a multiple of a different fractional unit, such as 1/3? This question is intended to get students thinking about how the fractional unit is related to the quadratic formula, and it need not be resolved at this time.

The key idea of question 2 is that irrational solutions of quadratic equations can also be plotted on a number line. It is anticipated that students will use a decimal approximation to plot the solutions to this quadratic. Pose the question, "Is there an exact location on the number line where these two solutions $2+\sqrt{2}$ and $2-\sqrt{2}$ should be plotted?" Pose the additional question, "Do you think there is a small unit fraction that could be used to divide up the number line in problem 2 so that we could plot these solutions exactly, like we did for problem 1?" Students may initially believe this might be possible, perhaps by using a very small unit fraction like $^1/_{100}$ or $^1/_{1000}$, but an examination of a decimal approximation of $\sqrt{2}$ would suggest we would have to go out to several decimal places to find such a unit. The ancient Greeks, prior to Hippasus and Zeno, believed that all lengths were discrete and composed of a finite number of units of a given size. Hippasus proved that there was no common unit of measure for the right triangle given in question 2 using a proof by contradiction: If one assumes there is a common unit of measure for both the hypotenuse and the legs of the right triangle, then it can be shown that a leg must contain both an even and an odd number of those units; this contradiction implies that the assumption that there is such a unit is false. The details of such a proof could be found here http://en.wikipedia.org/wiki/Irrational_number at the time of this writing. You may choose to share the details of such a proof with your students, or just share a description of the work and the results, as we have done here.

Have students work on questions 3-5 before stopping to discuss their work. The goal of this discussion is to point out that the sum of the lengths of the sides (i.e., the perimeter) of the right triangle in question 3 has to be written as a sum of two terms—a rational term and an irrational term—since these two terms are "incommensurable magnitudes." However, this single irrational number can be plotted on the number line given the strategy outlined and illustrated by the diagram in problem 4. Conclude this launch activity by discussing Big Idea #1.

Launch (Whole Class): [part 2: Imaginary and complex numbers]

Introduce Euler's notation for representing $\sqrt{-1}$ with *i*, and then discuss question 6 and the example that precedes question 7. Assign students to work on the rest of the task.

Explore (Small Group): [part 2: Imaginary and complex numbers]

Monitor students' work on simplifying sums and products of complex numbers and identify any algebraic work that needs to be discussed as a whole class.

Discuss (Whole Class): [part 2: Imaginary and complex numbers]

Clarify any algebraic work that seemed difficult or problematic for students. Discuss how one can tell if the roots of a quadratic function are complex from a graph. Have students share how they found the complex roots for the quadratic whose graph is shown in question 12. One strategy would be to write the vertex form of the quadratic function and set it equal to 0 and then solve the resulting quadratic equation, either by isolating the squared-binomial and taking the square root of both sides of the equation, or by expanding out the binomial to get standard form and then using the quadratic function. Do the same for the quadratic function described in question 13. Ask students to point out the similarities in the results obtained for the solutions to both of these related quadratics.

Revisit the discussion about the Fundamental Theorem of Algebra from the previous task, and point out that a quadratic function has two roots when real, complex and multiple roots are considered.

Aligned Ready, Set, Go: ***Numbers and Operations 3.5***

READY, SET, GO! Name Period Date

READY

Topic: Classifying numbers according to set.

Classify each of the numbers represented below according to the sets to which they belong. If a number fits in more than one set then list all that apply.

(Whole numbers “$\mathbb{W}$”, Integers “$\mathbb{Z}$”, Rational “$\mathbb{Q}$”, Irrational “$\overline{\mathbb{Q}}$”, Real “$\mathbb{R}$”, Complex “$\mathbb{C}$”)

1. π
2. -13
3. $\sqrt{-16}$
4. 0
5. $\sqrt{75}$
6. $\frac{9}{3}$
7. $\sqrt{\frac{4}{9}}$
8. $5 + \sqrt{2}$
9. $\sqrt{-40}$

SET

Topic: Simplifying radicals, imaginary numbers

Simplify each radical expression below.

10. $3 + \sqrt{2} - 7 + 3\sqrt{2}$
11. $\sqrt{5} - 9 + 8\sqrt{5} + 11 - \sqrt{5}$
12. $\sqrt{12} + \sqrt{48}$
13. $\sqrt{8} - \sqrt{18} + \sqrt{32}$
14. $11\sqrt{7} - 5\sqrt{7}$
15. $7\sqrt{7} + 5\sqrt{3} - 3\sqrt{7} + \sqrt{3}$

Simplify. Express as a complex number using “ *i* ” if necessary.

16. $\sqrt{-2} \cdot \sqrt{-2}$
17. $7 + \sqrt{-25}$
18. $(4i)^2$
19. $i^2 \cdot i^3 \cdot i^4$
20. $\left(\sqrt{-4}\right)^3$
21. $(2i)(5i)^2$

Solve each quadratic equation over the set of complex numbers.

22. $x^2 + 100 = 0$

23. $t^2 + 24 = 0$

24. $x^2 - 6x + 13 = 0$

25. $r^2 - 2r + 5 = 0$

GO

Topic: Solve Quadratic Equations

Use the discriminant to determine the nature of the roots to the quadratic equation.

26. $x^2 - 5x + 7 = 0$

27. $x^2 - 5x + 6 = 0$

28. $2x^2 - 5x + 5 = 0$

29. $x^2 + 7x + 2 = 0$

30. $2x^2 + 7x + 6 = 0$

31. $2x^2 + 7x + 7 = 0$

32. $2x^2 - 7x + 6 = 0$

33. $2x^2 + 7x - 6 = 0$

34. $x^2 + 6x + 9 = 0$

Solve the quadratic equations below using an appropriate method.

35. $m^2 + 15m + 56 = 0$

36. $5x^2 - 3x + 7 = 0$

37. $x^2 - 10x + 21 = 0$

38. $6x^2 + 7x - 5 = 0$

Need help? Visit **www.rsgsupport.org**

3.6 Sorry, We're Closed

A Practice Understanding Task

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Now that we have compared operations on polynomials with operations on whole numbers and started thinking about new number sets including irrational and complex numbers, it's time to generalize about the results

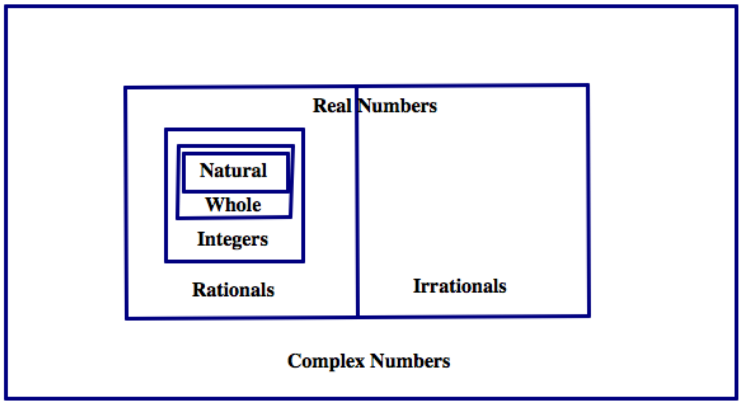
Maybe you have noticed in the past that when you add two even numbers, the answer you get is always an even number. Mathematically, we say that the set of even numbers is **closed** under addition. Mathematicians are interested in results like this because it helps us to understand how numbers or functions of a particular type behave with the various operations.

1. You can try it yourself: Is the set of odd numbers closed under multiplication? In other words, if you multiply two odd numbers together will you get an odd number? Explain.

If you find any two odd numbers that have an even product, then you would say that odd numbers are not closed under multiplication. Even if you have several examples that support the claim, if you can find one **counterexample** that contradicts the claim, then the claim is false.

Consider the following claims and determine whether they are true or false. If a claim is true, give a reason with at least two examples that illustrate the claim. If the claim is false, give a reason with one counterexample that proves the claim to be false.

This graphic will help you to think about the relationship between different sets of numbers, including the complex numbers that we have found as solutions to quadratic equations.



Do the following for each of the following claims:

- Determine if the claim is true or false.
- If you decide that the claim is true, create at least three examples to support the claim.
- If you decide that the claim is false, find a counter-example to prove the claim to be false.

1. The set of integers is closed under addition.

2. The set of irrational numbers is closed under addition.

3. The set of complex numbers is closed under addition.

4. The set of whole numbers is closed under subtraction.

5. The set of rational numbers is closed under subtraction.

6. The set of integers is closed under multiplication.

7. The set of integers is closed under division.

8. The set of rational numbers is closed under multiplication.

9. The set of irrational numbers is closed under multiplication.

10. The set of complex numbers is closed under multiplication.

The Arithmetic of Polynomials

To evaluate similar claims about polynomials, we must be very clear on the definition of a polynomial. The definition of a polynomial is:

A polynomial function has the form:

$$\boldsymbol{f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0}$$

where $a_n, a_{n-1}, \ldots a_1, a_0$ are real numbers and n is a nonnegative integer. In other words, a polynomial is the sum of one or more monomials with real coefficients and nonnegative integer exponents. The degree of the polynomial function is the highest value for n where a_n is not equal to 0.

1. The following examples and non-examples will help you to see the important implications of the definition of a polynomial function. For each pair, determine what is different between the example of a polynomial and the non-example that is not a polynomial.

These **are** polynomials:	These **are not** polynomials:
a) $f(x) = x^3$	b) $g(x) = 3^x$
How are a and b different?	
c) $f(x) = 2x^2 + 5x - 12$	d) $g(x) = \frac{2x^2}{x^2-3x+2}$
How are c and d different?	
e) $f(x) = -x^3 + 3x^2 - 2x - 7$	$f)$ $g(x) = x^3 + 3x^2 - 2x + 10x^{-1} - 7$
How are e and f different?	
h) $f(x) = \frac{1}{2}x$	i) $\mathrm{g}(x) = \frac{1}{2x}$
How are h and i different?	

j) $f(x) = x^2$	k) $g(x) = x^{\frac{1}{2}}$
How are j and k different?	

2. Based on the definition and the examples above, how can you tell if a function is a polynomial function?

Now we will consider claims about polynomials. Do the following for each of the following claims:

- Determine if the claim is true or false.
- If you decide that the claim is true, create at least three examples to support the claim.
- If you decide that the claim is false, find a counter-example to prove the claim to be false.

.

3. The sum of a quadratic polynomial and a linear polynomial is a cubic polynomial.

4. The sum of a linear polynomial and an exponential expression is a polynomial.

5. A cubic polynomial subtracted from a cubic polynomial is a cubic polynomial.

6. A cubic polynomial divided by a linear polynomial is a quadratic polynomial.

7. The set of polynomials is closed under addition.

8. The set of polynomial functions is closed under subtraction.

9. The set of polynomials is closed under multiplication.

10. The set of polynomial functions is closed under division.

11. Write two claims of your own about polynomials and use examples to demonstrate that they are true.
Claim #1:

Claim #2:

3.6 Sorry, We're Closed – Teacher Notes

A Practice Understanding Task

Purpose: This task is designed to serve several purposes: 1) to formalize students' understanding of the relationships among number sets 2) to motivate deeper thinking and develop fluency with operations on polynomials, and 2) to formalize the definition of a polynomial and understand that polynomials are closed under addition, subtraction, and multiplication. In the task, students are asked to consider various conjectures about numbers and polynomials and use examples to demonstrate that a conjecture is true or provide a counterexample to prove that the conjecture is false. To support students with this work, the task contains the definition of a polynomial and carefully contrasts the features of polynomials with other functions.

Core Standards Focus:

N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Note for Mathematics II: *Connect N.RN.3 to physical situations, e.g., finding the perimeter of a square of area 2.*

N.CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.

N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Standards for Mathematical Practice:

SMP 3 – Construct viable arguments and critique the reasoning of others

SMP 6 – Attend to precision

Vocabulary: Closed

The Teaching Cycle:

Part 1 (Operations with Numbers):

Launch (Whole Group):

Examine the Venn diagram of the complex number system given at the beginning of the task. Ask students to suggest a quadratic equation that would not have a solution if we limited the set of acceptable solutions to only natural numbers (for example, $x^2 + 3x = 0$ would not be solvable). What if we limited the set of acceptable solutions to only integers? (Then $(2x+1)(3x-2)=0$ would not be solvable.) What if we limited the set of acceptable solutions to rational numbers? Real numbers? Point out to students that we have expanded our number system to allow us to solve a greater variety of equations.

Let students know that their work today is to make conjectures about operations within these sets of numbers. Can we always do arithmetic within in a set of numbers without having to go outside the set to get an answer? Mathematicians refer to this property of a set of numbers as *closure*.

Explore (Small Group):

As students explore the conjectures, make sure they are trying out a range of possibilities, and trying to search for counter-examples to the conjectures. Remind students of the work of the previous task as they consider operations with irrational and complex numbers. Observe how students' work with the arithmetic of complex numbers, since this class of numbers will still be relatively new to them. Do they draw upon their previous knowledge of adding like terms, subtracting by adding the opposite, and distributing binomial factors that represent complex numbers using the same strategies they would use for algebraic expressions?

Discuss (Whole Class):

As needed, share conjectures and supporting evidence with the whole class. Press students to justify their conjectures with more than just their examples. For example, students might conclude that the product of two rational numbers is another rational number (i.e., a ratio of two integers), since both the numerator and denominator are obtained by multiplying two integers together, and will therefore yield another ratio of integers. Or they might refer to the meaning of the operation of multiplying fractions by saying something like, “I know that $\frac{1}{2} \times \frac{3}{4}$ means that we need to find $\frac{1}{2}$ of a portion that is $\frac{3}{4}$ of a whole. This would describe a portion of the whole of size $\frac{3}{8}$, another rational number.” While this description uses specific numbers, it can be generalized: A portion of a portion of a whole would be another portion of the whole, which would be represented by some rational number.

If some students used arguments based on the number line, have them share their work. For example, using a diagram similar to the one shown in task 3.9, students could represent adding irrational numbers as combining lengths of line segments on a number line. Since the sum of the lengths of any two real numbers can be found on the number line, the sum would also be a real number.

Part 2 (Operations with Polynomials):

Launch (Whole Group):

Present the definition of a polynomial function. Tell students that this definition describes some important features of polynomials that students may not have noticed from simply reading the definition. Tell students that the examples in #1 are designed to help them further differentiate between functions that are polynomials and those that are not. Go through each of the comparisons in problem 1. For each one, give students a few minutes to think about the differences and then have a short discussion about what is different about the function that is a polynomial compared to the one that is not. After completing problem 1, ask students to respond to question 2 individually. Record student responses highlighting the following ideas:

- A polynomial is a sum of terms.
- The exponents on the variables must be whole numbers.
- If the variable is in the exponent, the function is exponential, not polynomial.
- If there is division by a variable in one of the terms, then the function is not a polynomial. (It is a rational function, a function type that will be explored in Module 4.)

Then, give students instructions for the rest of the task and let them begin work.

Explore (Small Group):

Begin by giving students time to respond to the conjectures and come up with some examples before sharing. This will help to enhance the discussion when it is time to share. As students are working, watch for students that are thinking about counterexamples for statements that appear to be true. These students should be selected to share their ideas in the class discussion. If students need help in creating examples, some are given in the Teacher Resources section. You will probably want to keep these examples handy while you are monitoring students, so that you can offer these examples when they are stuck. Since it is sometimes hard for students to create good examples for division, some of the examples were made for this purpose. Specifically, $l(x)$ divides evenly by $f(x)$ and $j(x)$ divides evenly by $g(x)$.

Discuss:

Begin the discussion with a previously-selected student that can share a counter-example to show that #3 is false. Discuss briefly the idea that one counterexample proves a statement is false, but examples are not enough to prove a statement true. Students have encountered this idea in geometry.

Move the discussion to #7, then #8 and #9. In each case, ask selected students to share their examples, and then ask the class to use the examples to help make a general argument that might convince us that that each statement is always true.

Ask a previously-selected student to present their thinking on #10. Students may think that the statement is true because they don't recognize that when one polynomial is divided by another and there is a remainder that the quotient is not a polynomial. Remind students how to write the quotient in the form where the remainder is a fraction so that they can see that the variable in the denominator means that the quotient is not a polynomial function. Refer back to the work done in problem #1 and the definition of polynomial.

Aligned Ready, Set, Go: ***Polynomial Functions 3.6***

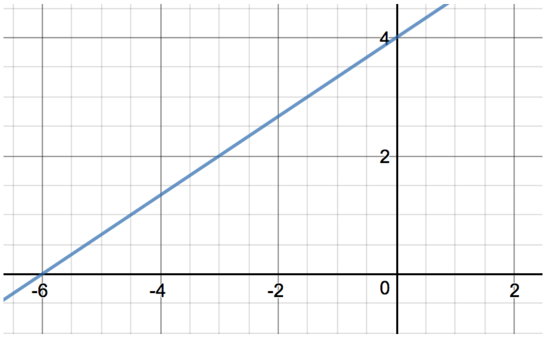
READY, SET, GO! Name Period Date

READY

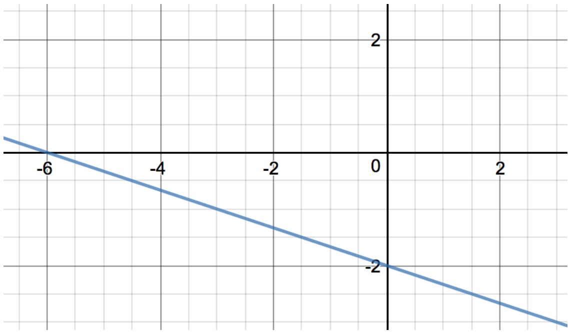
Topic: Connecting the zeros of a function to the solution of the equation

When we solve equations, we often set the equation equal to zero and then find the value of x. Another way to say this is "find when $f(x) = 0$." That's why we call solutions to equations the zeros of an equation. Find the zeros for the given equations. Then mark the solution(s) as a point on the graph of the equation.

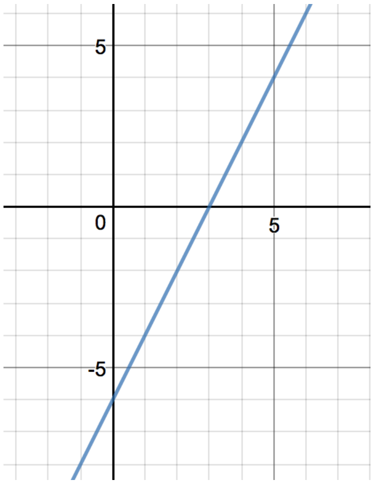
1. $f(x) = \frac{2}{3}x + 4$



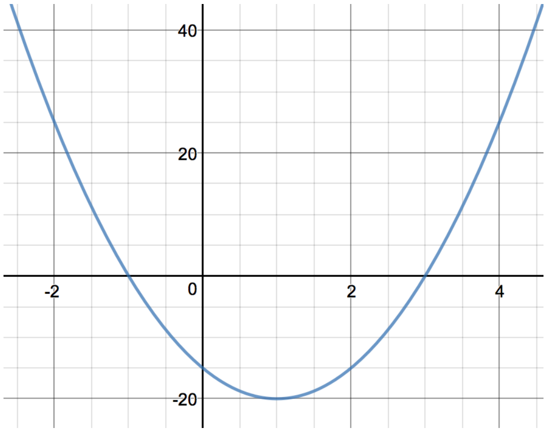
2. $g(x) = -\frac{1}{3}x - 2$



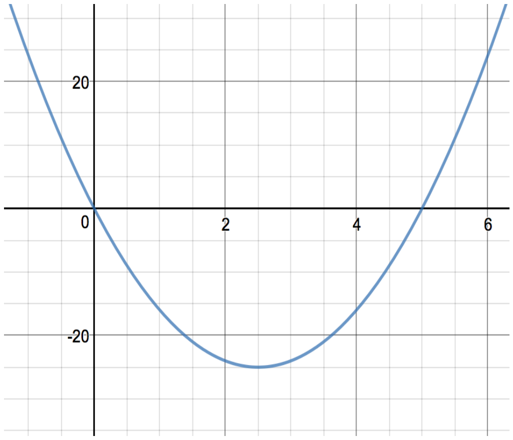
3. $h(x) = 2x - 6$



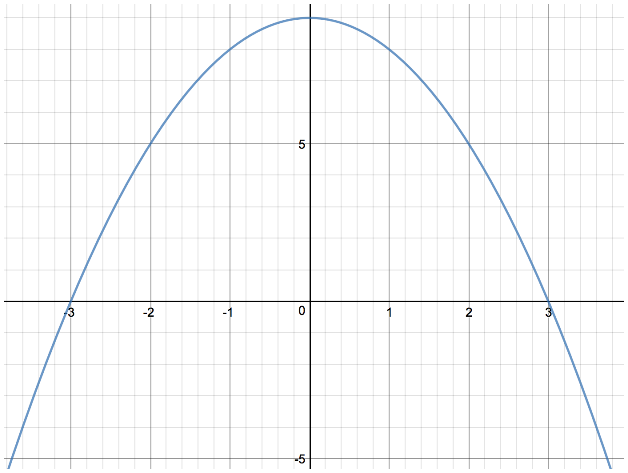
4. $p(x) = 5x^2 - 10x - 15$



5. $q(x) = 4x^2 - 20x$



6. $d(x) = -x^2 + 9$



SET

Topic: Exploring closed mathematical number sets

Identify the following statements as *sometimes* true, *always* true, or *never* true. If your answer is *sometimes* true, give an example of when it's true and an example of when it's not true. If it's *never* true, give a counter-example.

7. The product of a whole number and a whole number is an integer.

8. The quotient of a whole number divided by a whole number is a whole number.

9. The set of integers is **closed** under division.

10. The difference of a linear function and a linear function is an integer.

11. The difference of a linear function and a quadratic function is a linear function.

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12. The product of a linear function and a linear function is a quadratic function.

13. The sum of a quadratic function and a quadratic function is a polynomial function.

14. The product of a linear function and a quadratic function is a cubic function.

15. The product of three linear functions is a cubic function.

16. The set of polynomial functions is **closed** under addition.

GO

Topic: Identifying conjugate pairs

A **conjugate pair** is simply a pair of binomials that have the same numbers but differ by having opposite signs between them. For example $(a + b)$ and $(a - b)$ are conjugate pairs. You've probably noticed them when you've factored a quadratic expression that is the difference of two squares. **Example:** $x^2 - 25 = (x+5)(x-5)$. The two factors $(x+5)(x-5)$ are conjugate pairs.

The quadratic formula $\boldsymbol{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$ can generate both solutions to a quadratic equation because of the $\pm$ located in the numerator of the formula. When the $\sqrt{b^2-4ac}$ part of the formula generates an irrational number (e.g. $\sqrt{2}$) or an imaginary number (e.g. $2i$), the formula produces a pair of numbers that are conjugates. This is important because <u>this type of solution to a quadratic always comes in pairs.</u> **Example:** The conjugate of $(3+\sqrt{2})\ is\ (3-\sqrt{2})$. The conjugate of $(-2i)$ is $(+2i)$. Think of it as $(0-2i)\ and\ (0+2i)$. <u>Change only the sign between the two numbers.</u>

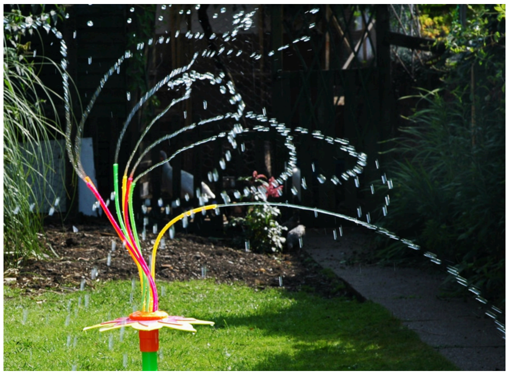
Write the conjugate of the given value.

17. $(8+\sqrt{5})$
18. $(11+4i)$
19. $9i$
20. $-5\sqrt{7}$

21. $(2-13i)$
22. $(-1-2i)$
23. $(-3+5\sqrt{2})$
24. $-4i$

3.7H Quadratic Quandaries

A Develop & Solidify Understanding Task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In the task *Curbside Rivalry* Carlos and Clarita were trying to decide how much they should charge for a driveway mascot. Here are the important details of what they had to consider.

- *Surveys show the twins can sell 100 driveway mascots at a cost of $20, and they will sell 10 fewer mascots for each additional $5 they charge.*
- *The twins estimate that the cost of supplies will be $250 and they would like to make $2000 in profit from selling driveway mascots. Therefore, they will need to collect $2250 in revenue.*

This information led Carlos and Clarita to write and solve the quadratic equation

$$(100-10x)(20+5x)=2250.$$

1. Either review your work from *Curbside Rivalry* or solve this quadratic equation for x again.

2. What do your solutions for x mean in terms of the story context?

3. How would your solution change if this had been the question Carlos and Clarita had asked: "How much should we charge if we want to collect at least $2250 in revenue?"

4. What about this question: "How much should we charge if we want to maximize our revenue?"

As you probably observed, the situation represented in question 3 didn't have a single solution, since there are many different prices the twins can charge to collect more than $2250 in revenue. Sometimes our questions lead to quadratic inequalities rather than quadratic equations.

Here is another quadratic inequality based on your work on *Curbside Rivalry.*

5. Carlos and Clarita want to design a logo that requires less than 48 in^2 of paint, and fits inside a rectangle that is 8 inches longer than it is wide. What are the possible dimensions of the rectangular logo?

Again question 5 has multiple answers, and those answers are restricted by the context. Let's examine the inequality you wrote for question 5, but not restricted by the context.

6. What are the solutions to the inequality $x(x+8) < 48$?

7. How might you support your answer to question 6 with a graph or a table?

Here are some more quadratic inequalities without contexts. Show how you might use a graph, along with algebra, to solve each of them.

8. $x^2 + 3x - 10 \geq 0$

9. $2x^2 - 5x < 12$

10. $x^2 - 4 \leq 4x + 1$

Carlos and Clarita both used algebra and a graph to solve question 10, but they both did so in different ways. Illustrate each of their methods with a graph and with algebra.

11. Carlos: "I rewrote the inequality to get 0 on one side and a factored form on the other. I found the zeroes for each of my factors. To decide what values of x made sense in the inequality I also sketched a graph of the quadratic function that was related to the quadratic expression in my inequality. I shaded solutions for x based on the information from my graph."

12. Clarita: "I graphed a linear function and a quadratic function related to the linear and quadratic expressions in the inequality. From the graph I could estimate the points of intersection, but to be more exact I solved the quadratic equation $x^2 - 4 = 4x + 1$ by writing an equivalent equation that had 0 on one side. Once I knew the x-values for the points of intersection in the graph, I could shade solutions for x that made the inequality true."

Carlos and Clarita have decided to create 3-D mascots out of clay for their customers who want them. They want the mascot to fit within a rectangular box with a volume that is no more than 96 in^3 and whose width is 2 inches shorter than its length, and whose height is 8 inches more than its length.

Carlos writes this inequality to represent the box's description: $x(x-2)(x+8) \leq 96$

With the help of his cousin who is in advanced mathematics he is able to rewrite this inequality in an equivalent factored form that has 0 on one side of the inequality:
$(x-4)(x+4)(x+8) \leq 0$

Because Carlos doesn't know how to graph cubic polynomials any better than he can factor them, he is wondering how his work with quadratic inequalities might help him solve this cubic inequality.

13. Devise a strategy based on your work with quadratic inequalities that could be used to solve this cubic inequality with three factors: $(x-4)(x+4)(x+8) \leq 0$

14. Use the solutions to this cubic inequality to determine the dimensions of rectangular boxes that meet their criteria.

15. Here is the algebra work produced by Carlos' cousin. Explain each step in the process that led from Carlos' inequality to his cousin's.

$$x(x-2)(x+8) \le 96$$
$$x(x^2+6x-16) \le 96$$
$$x^3+6x^2-16x \le 96$$
$$x^3+6x^2-16x-96 \le 0$$
$$x^2(x+6)-16(x+6) \le 0$$
$$(x^2-16)(x+6) \le 0$$
$$(x-4)(x+4)(x+6) \le 0$$

3.7H Quadratic Quandaries – Teacher Notes

A Develop and Solidify Understanding Task

Purpose: The purpose of this task is to develop a strategy for solving quadratic inequalities and extend this strategy to higher-degree polynomials when the factors are known. The context of the task gives students an opportunity to engage in mathematical modeling: students will use mathematical models, in this case quadratic and cubic inequalities, to model various contextualized situations. The solutions to the inequalities then have to be interpreted in terms of what they mean in the situations. That is, the solutions for x in the inequalities are not the answers to the questions being asked in the situations—rather they provide information from which those questions can be answered. Students will have to keep track of the meaning of the variables as they work through these problems.

Core Standards Focus:

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Note for Mathematics II A.CED.1

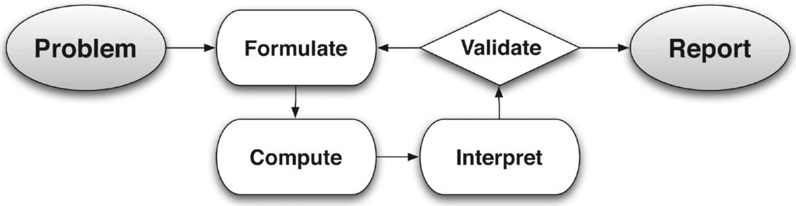
Extend work on linear and exponential equations in Mathematics I to quadratic equations.

Secondary II Honors Standard: Solve polynomial and rational inequalities in one variable.

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Related Standards: High School Modeling Standard



Standards for Mathematical Practice:

SMP 2 – Reason abstractly and quantitatively

SMP 8 – Look for and express regularity in repeated reasoning

The Teaching Cycle:

Launch (Whole Class):

Use question 1 to remind students of the *Curbside Rivalry* context and the work they did to solve the quadratic equation $(100-10x)(20+5x)=2250$ by first multiplying out the binomials, subtracting to get 0 on one side of the equation and then factoring the resulting quadratic into new factors whose product equals 0. Remind them that this meant the solutions to the equation would result when either factor equaled 0. Once this strategy for solving quadratics is firmly in place, set students to work on the task questions, which will involve solving quadratic and cubic inequalities.

Explore (Small Group):

For question 2, make sure students understand that x represents the number of $5 increments in the price, and therefore, once we have solved for x we have to substitute the solutions back into the expression for price to determine the amount we should charge if we want to collect $2250 in revenue. On question 3 students should note that there is an interval of prices for which the revenue is greater than $2250. Again, x will not give us the prices, but will help us to find the prices we should charge. Make sure students are making sense of how to use the algebraic model to answer questions from the context. Question 4 cannot be answered by solving an equation or inequality. Rather, it requires changing the form of the quadratic to vertex form so the maximum value can be identified.

The table and graph requested in question 6 should lend support to how to interpret the algebraic inequality written in question 5. Listen for students who are beginning to articulate a strategy for solving quadratic inequalities:

> Get 0 on one side of the inequality and factor the quadratic expression on the other side. Find the zeroes of the factors. Determine the sign of the values of the quadratic expression between the zeroes by either testing specific points or referring to the graph. Select the

intervals that yield the appropriate signs for the values—positive values for expressions that are greater than 0 or negative values for expressions that are less than 0.

Questions 5 and 6 are a good place to start the modeling conversation. Make sure the students understand what the factors represent in the inequality $x(x+8) \leq 48$ relative to the story context. While this inequality represents the story perfectly, it is not easy to solve the inequality while written in this form. The preferred form for solving the inequality algebraically, $x^2 + 8x - 48 \leq 0$, obscures the context, but allows us to work algebraically towards a solution. The solution obtained algebraically, $-12 \leq x \leq 4$, solves the inequality, but does not answer the question asked by the scenario, since in the scenario x represents a length, and therefore, cannot be negative. So the solution to the question asked by the story context is $0 \leq x \leq 4$. Make sure that students understand that the work of this task requires both the algebraic work of solving polynomial inequalities, as well as the work of interpreting those solutions within the context for which the inequality was written.

Questions 8-10 give students opportunities to solidify their strategy for solving polynomial inequalities. Watch for alternative strategies in addition to the one described above. You might want to discuss the strategies that have emerged during student work on questions 2-10 before assigning the remaining portion of the task.

Questions 11 and 12 illustrate two alternative strategies for solving quadratic inequalities. Questions 13 and 14 ask students to adapt one of these strategies (Carlos' strategy in question 11) to a situation involving a cubic polynomial. Make sure students focus on both the strategy and the modeling context. Ask probing questions such as, "How does your solution to the cubic inequality help you determine the dimensions of boxes that fit the required conditions?"

Discuss (Whole Class):

The whole class discussion should focus on student strategies for solving the quadratic inequalities in questions 8-10, and which strategy they used for the cubic inequality in question 13. Once all issues with the methods for solving polynomial inequalities have been resolved, turn the discussion to the modeling issues. Make sure that students have interpreted their solutions to the inequalities to get appropriate answers for the story contexts in questions 5 and 14.

Question 15 introduces some interesting algebra: factoring a cubic polynomial by grouping. While this algebra should be accessible to your students, it is not an expected procedure for this course.

Aligned Ready, Set, Go: *Numbers and Operations 3.7H*

3.7H

READY, SET, GO! Name Period Date

READY

Topic: Factoring Polynomials

Factor each of the polynomials completely.

1. $x^2 + x - 12$
2. $x^2 - 2x - 8$
3. $x^2 + 5x - 14$
4. $x^2 - x - 6$
5. $x^2 + 6x + 9$
6. $x^2 - 7x{+}10$
7. $2x^2 - 9x - 5$
8. $3x^2 - 3x - 18$
9. $2x^2 + 8x - 42$

10. How is the factored form of a quadratic helpful when graphing the parabola?

SET

Topic: Solving Quadratic Inequalities

Solve each of the quadratic inequalities.

11. $x^2 + x - 12 > 0$
12. $x^2 - 2x - 8 \leq 0$
13. $x^2 + 5x - 14 \geq 0$
14. $2x^2 - 9x - 5 \geq 0$
15. $3x^2 - 3x - 18 < 0$
16. $x^2 + 4x - 21 < 0$
17. $x^2 - 4x \leq 0$
18. $x^2 \leq 25$
19. $x^2 - 4x \leq 5$

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Match each graph with its inequality.

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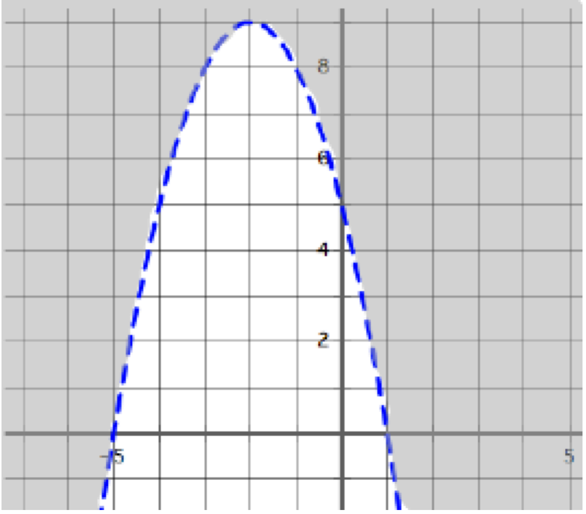
a.



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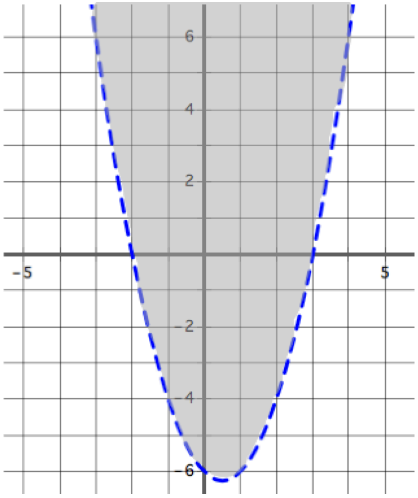

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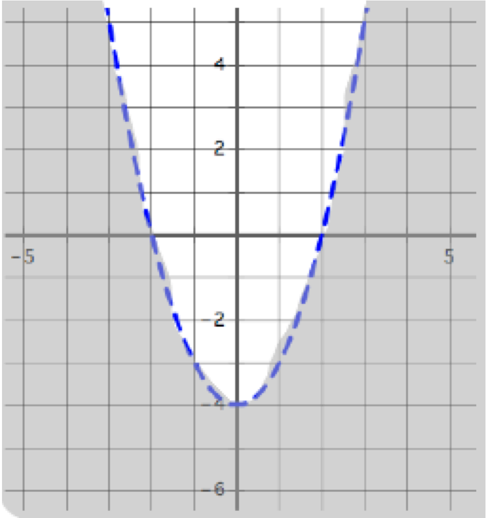

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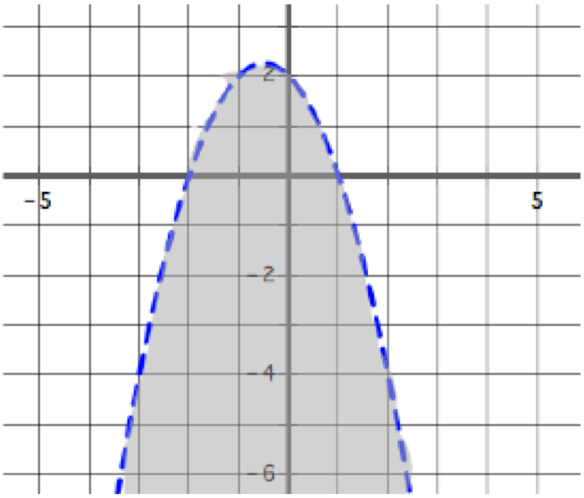

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<tr>
<td>20. $y > x^2 - x - 6$</td>
<td>21. $y < x^2 - 4$</td>
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<tr>
<td>22. $y < (x+2)(1-x)$</td>
<td>23. $y > 5 - 4x - x^2$</td>
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</table>

GO

Topic: Vertex form of quadratic equations

Write each quadratic function below in vertex form.

24. $f(x) = x^2 + 6x + 5$

25. $f(x) = (x + 3)(x - 5)$

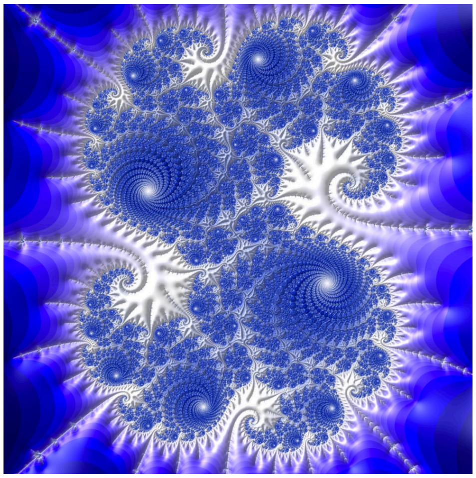
26. $f(x) = (x - 2)(x + 6)$

27. $f(x) = x^2 - 12x + 20$

28. $f(x) = 2x^2 + 16x + 8$

29. $f(x) = x^2 - 2x - 8$

3.8H Complex Computations



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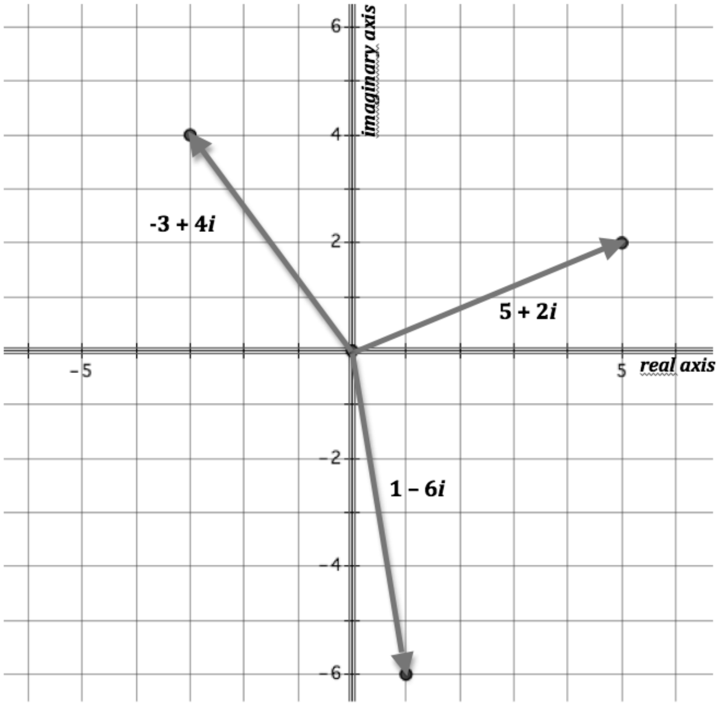
A Solidify Understanding Task

It is helpful to illustrate the arithmetic of complex numbers using a visual representation. To do so, we will introduce the complex plane.

As shown in the figure below, the complex plane consists of a horizontal axis representing the set of real numbers and a vertical axis representing the set of imaginary numbers. Since a complex number $a + bi$ has both a real component and an imaginary component, it can be represented as a point in the plane with coordinates (a, b). It can also be represented by a position vector with its tail located at the point (0, 0) and its head located at the point (a, b), as shown in the diagram. It will be useful to be able to move back and forth between both geometric representations of a complex number in the complex plane—sometimes representing the complex number as a single point, and sometimes as a vector.

You may want to review the Secondary Math 1 task, *The Arithmetic of Vectors*, so you can draw upon those ideas in the following work.



The Modulus of a Complex Number

It is often useful to be able to compare the magnitude of two different numbers. For example, collecting $25 in revenue will not pay off a $45 debt, since $|25| < |-45|$. Note that in this example we used the absolute value of signed numbers to compare the magnitude of the revenue and the debt. Since -45 lies farther from 0 along a real number line than 25, the debt is greater than the revenue. In a similar way, we can compare the relative magnitudes of complex numbers by determining how far they lie away from the origin, the point (0, 0), in the complex plane. We refer to the magnitude of a complex number as its **modulus**, and symbolize this length with the notation $|a + bi|$.

1. Find the modulus of each of the complex numbers shown in the figure above.

2. State a rule, either in words or using algebraic notation, for finding the modulus of any complex number $a + bi$.

Adding and subtracting complex numbers

3. Experiment with the vector representation of complex numbers to develop and justify an algebraic rule for adding two complex numbers: $(a + bi) + (c + di)$. How do your representations of addition of vectors on the complex plane help to explain your algebraic rule for adding complex numbers?

4. How would you represent the additive inverse of a complex number on the complex plane? How would you represent the additive inverse algebraically?

5. If we think of subtraction as adding the additive inverse of a number, use the vector representation of complex numbers to develop and justify an algebraic rule for subtracting two complex numbers: $(a + bi) - (c + di)$. How do your representations of the additive inverse of a complex number and the addition of vectors on the complex plane help to explain your algebraic rule for subtracting complex numbers?

Multiplying complex numbers

One way to think about multiplication on the complex plane is to treat the first factor in the multiplication as a scale factor.

6. Provide a few examples of multiplying a complex number by a real number scale factor: $a(c + di)$. For example, what happens to the vector representation of a complex number when the scale factor a is 4? ½? -2?

7. Provide a few examples of multiplying a complex number by an imaginary scale factor: $bi(c + di)$. For example, what happens to the vector representation when the scale factor bi is i? $2i$? $-3i$?

8. Experiment with the vector representation of complex numbers to justify the following rule for multiplying complex numbers:

$$(a + bi)(c + di) = a(c + di) + bi(c + di) = ac - bd + (ad + bc)i.$$

9. How do the geometric observations you made in question 6, question 7 and question 3 show up in this work?

The conjugate of a complex number

The conjugate of a complex number $a + bi$ is the complex number $a - bi$. The conjugate of a complex number is represented with the notation $\overline{a + bi}$.

10. Illustrate an example of a complex number and its conjugate in the complex plane using vector representations.

11. Illustrate finding the sum of a complex number and its conjugate in the complex plane using vector representations.

12. Illustrate finding the product of a complex number and its conjugate in the complex plane using vector representations. (Use the geometric observations you made in questions 6-8 to guide your work.)

13. If z is a complex number and $\bar{z}$ is its conjugate, how are the moduli $|z|$ and $|\bar{z}|$ related?

14. Use either a geometric or algebraic argument to complete and justify the following statements for any complex number $a + bi$:

    - The sum of a complex number and its conjugate is always the real number _________ .

    - The product of a complex number and its conjugate is always the real number ________ .

The division of complex numbers

Dividing a complex number by a real number is the same as multiplying the complex number by the multiplicative inverse of the divisor. That is, $\frac{a+bi}{c} = \frac{1}{c}(a+bi) = \frac{a}{c} + \frac{b}{c}i$. Therefore, division of a complex number by a real number can be thought of in terms of multiplying the complex number by a real-valued scale factor, an idea we explored in question 6.

We have also observed that multiplying a complex number by its conjugate always gives us a real number result. We make use of this fact to change a problem involving division by a complex number into an equivalent problem in which the divisor is a real number.

15. Explain why $\frac{a+bi}{c+di}$ is equivalent to $\frac{(a+bi)}{(c+di)} \cdot \frac{(c-di)}{(c-di)}$.

16. Use this idea to find the quotient $\frac{3+5i}{4+2i}$.

We have been using a vector representation of complex numbers in the complex plane in the previous problems. In the following problems we will represent complex numbers simply as points in the complex plane.

Finding the distance between two complex numbers

To find the distance between two points on a real number line, we find the absolute value of the difference between their coordinates. (Illustrate this idea with a couple of examples.)

In a similar way, we define the distance between two complex numbers in the complex plane as the modulus of the difference between them.

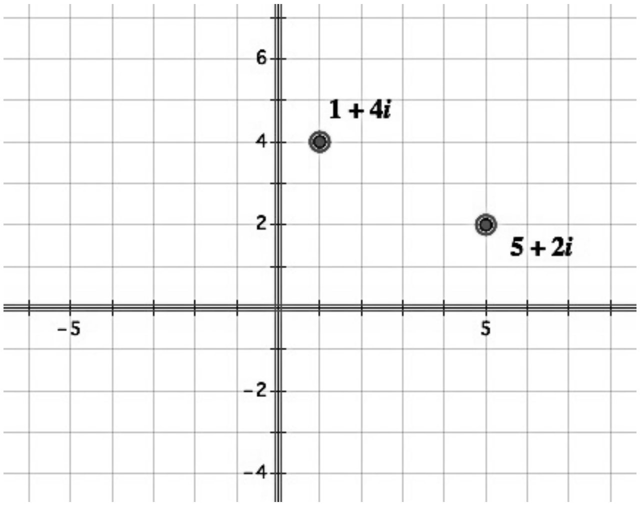
17. Find the distance between the two complex numbers plotted on the complex plane below.

Finding the average of two complex numbers

The average of two real numbers $\frac{x_1 + x_2}{2}$ is located at the midpoint of the segment connecting the two real numbers on the real number line. (Illustrate this idea with a couple of examples.)

In a similar way, we define the average of two complex numbers to be the midpoint of the segment connecting the two complex numbers in the complex plane.

18. Find the average of the two complex numbers plotted on the complex plane below.



3.8H Complex Computations – Teacher Notes

A Solidify Understanding Task

Purpose The purpose of this task is to examine the arithmetic of complex numbers, using the complex plane to help make sense of the procedures for adding, subtracting, multiplying and dividing complex numbers. Students will also define and calculate the distance between two complex numbers, including the distance from the origin to a complex number in the complex plane, and will examine the meaning of the average of two complex numbers.
Note: You may want to review the Secondary Math 1 task, *The Arithmetic of Vectors*, with students so they can draw upon those ideas in the following work.

Core Standards Focus:

N.CN.3 Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

N.CN.4 Represent complex numbers on the complex plane in rectangular form (including real and imaginary numbers).

N.CN.5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.

N.CN.6 Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Standards for Mathematical Practice:

SMP 5 – Use appropriate tools strategically

SMP 8 – Look for and make use of structure

The Teaching Cycle:

Launch (Whole Class):

Point out to students that the arithmetic of complex numbers can be understood from both an arithmetic and geometric perspective by introducing the complex plane. Help them recognize how a complex number $a + bi$ can be visualized on the complex plane as either a single point whose coordinates are (a, b) or as a vector with a horizontal component of a and a vertical component of b. Once students understand how to represent complex numbers on the complex plane, set them to work on the first section of the task: the modulus of a complex number. Give students a couple of minutes to read the definition of modulus and to determine a rule for finding the modulus (questions 1 and 2). This should be fairly easy and quick for students, since the rule is just another application of the Pythagorean theorem. Once this has been discussed, students can begin work on the remainder of the task.

Explore (Small Group):

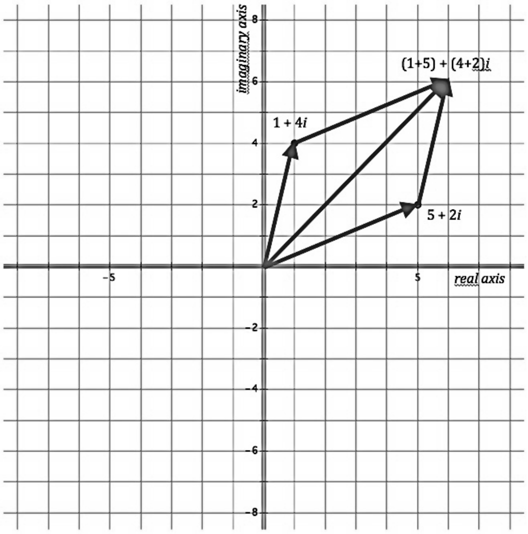
Note: The task contains several sections. In each section the students explore a different idea about the arithmetic of complex numbers. Break up the individual and whole group explorations with whole class discussions about each section of the task. If students are making adequate progress you might want to have the first whole group discussion after the sections on addition, subtraction and multiplication of complex numbers. You can then introduce the idea of the conjugate before having students work on the sections on complex conjugates and division of complex numbers. Following a whole group discussion on division you can introduce the last section by pointing out that the last two ideas are based on representing complex numbers as points in the complex plane, rather than vectors. Then have students work on the remainder of the task.

Recall from our work in Secondary Math 1 that a vector has magnitude and direction, but no specific location. Therefore, when we use a vector representation for a complex number we can translate the vector to different locations in the complex plane, and it will still represent the same complex number. This will be helpful in explaining the addition of vectors in terms of adding the real parts (i.e., the horizontal components of the vectors) and the imaginary parts (the vertical components of the vectors). From Secondary Math 1 students are also familiar with the idea of the dilation of a vector—stretching or shrinking its length—when the vector is multiplied by a scale factor. This should support students' work on the multiplication of complex numbers.

As you monitor students make sure they are experimenting with a variety of complex numbers—with both positive and negative values for the parameters *a* and *b*. Students should verify that any conjectures they make when adding or multiplying complex numbers whose vectors lie in the first quadrant still work when one or more of the complex vectors lie in quadrants II, III or IV.

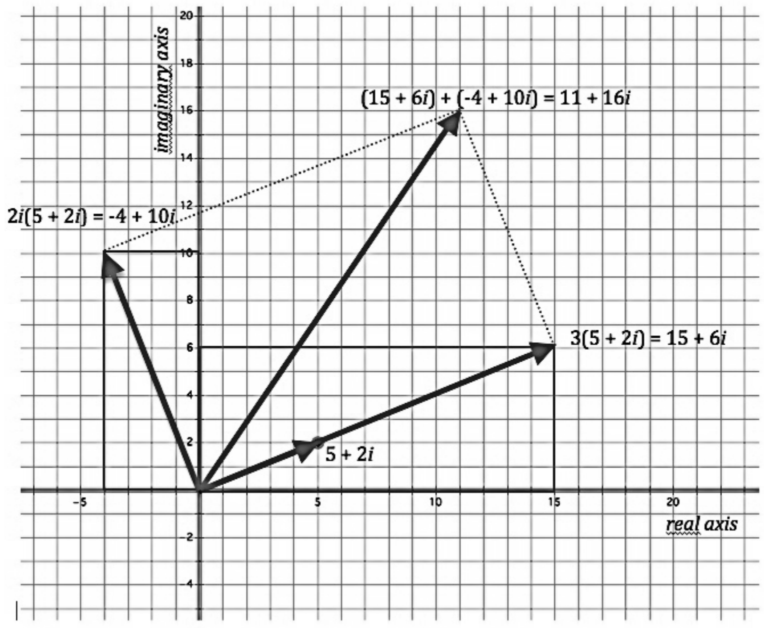
You should watch for the following ideas to emerge during the exploration:

1. We add complex numbers by adding the real terms and the imaginary terms. This can be justified on a diagram by showing that when two complex vectors are placed tail to head, the horizontal components of the vectors add together and the vertical components of the vectors add together to produce the resultant vector. This is equivalent to the parallelogram rule for adding vectors.



2. We subtract complex numbers by adding the additive inverse of the complex number that is being subtracted. On a vector diagram the additive inverse of a complex number has the same modulus and is rotated 180° about the origin from the original complex number.

3. We multiply complex numbers algebraically as follows:
   $(a+bi)(c+di) = a(c+di) + bi(c+di) = ac + adi + cbi + bdi^2 = (ac-bd) + (ad+cb)i$.
   This can be justified on a diagram by treating the first complex number as a scale factor. Multiplying $c + di$ by a stretches or shrinks the complex vector $c + di$ by a factor of a. Multiplying $c + di$ by bi rotates the complex vector $c + di$ 90° counterclockwise and stretches the vector by a factor of b. These two "scaled" vectors are then added by the parallelogram rule to get the final product.



(3 + 2*i*)(5 + 2*i*) = 3(5 + 2*i*) + 2*i*(5 + 2*i*) = (15 + 6*i*) + (-4 + 10*i*) = 11 + 16*i*

4. Multiplying two complex conjugates together, $(a+bi)(a-bi)$, always produces the real number $a^2 + b^2$. This can be illustrated on a vector diagram using the multiplication strategy described in #3.

5. Division of complex numbers can be treated from an algebraic perspective, as described in the task.

6. Finding the distance between two complex numbers is analogous to finding the distance between two points in the real plane using the distance formula.

7. Finding the midpoint between two complex numbers is analogous to finding the midpoint of a segment in the real plane using the midpoint formula.

Discuss (Whole Class):

Select students to present their arithmetic rules and supporting illustrations on the complex plane for each of 1-7 listed above. While students will probably present specific examples of each procedure, it is important to focus the discussion on why each procedure works for all complex numbers. This can be accomplished by using generic illustrations, rather than specific values for *a, b, c,* and *d* in $a + bi$ and $c + di$

.

You may need to help students see why multiplying a complex number by *i* rotates the vector 90° counterclockwise. This is a consequence of $i^2 = -1$, so the real part of $a + bi$ becomes the imaginary part when multiplied by *i*, and the imaginary part becomes the real part, but with the opposite sign.

Aligned Ready, Set, Go: ***Numbers and Operations 3.8H***

READY, SET, GO! Name Period Date

READY

Topic: Solving systems of linear equations

Solve each system of equations using substitution.

1.
$$\begin{cases} y = 3x \\ y = -2x - 15 \end{cases}$$

2.
$$\begin{cases} 3x + y = 21 \\ y = -2x - 15 \end{cases}$$

3.
$$\begin{cases} 3x + 2y = 7 \\ x - 2y = -3 \end{cases}$$

Solve each system of equations using elimination.

4.
$$\begin{cases} 5x - y = 13 \\ -2x + y = -1 \end{cases}$$

5.
$$\begin{cases} 3x + y = 21 \\ -3x + 5y = -3 \end{cases}$$

6.
$$\begin{cases} 3x + 2y = 7 \\ x + y = 2 \end{cases}$$

Create an augmented matrix for each system of equations and then use row reductions to solve the system.

7.
$$\begin{cases} 2x + y = 7 \\ -2x + y = -1 \end{cases}$$

8.
$$\begin{cases} 3x - 4y = 11 \\ -3x + 5y = -3 \end{cases}$$

9.
$$\begin{cases} 5x - y = 13 \\ -2x + y = -1 \end{cases}$$

SET

Topic: Operations with imaginary numbers

Perform the indicated operations on the complex numbers.

10. $(3+4i)+(2-5i)$

11. $(6-4i)-(7+2i)$

12. $3(5+2i)$

13. $(9-2i)(1+3i)$

14. $4(3-2i)-(5+3i)$

15. $(2-5i)(2+5i)$

Use the conjugate of each denominator to rationalize the denominators and write an equivalent fraction.

16. $\dfrac{3-5i}{2+5i}$

17. $\dfrac{6+7i}{4-3i}$

18. $\dfrac{2-3i}{1-6i}$

Find the modulus for each complex number.

19. $3-5i$

20. $4-3i$

21. $-4+3i$

22. If the graphical representation of the operations between two complex numbers results in a value along the y-axis or imaginary axis, what must be true about the two complex numbers?

23. If the graphical representation of the operations between two complex numbers results in a value along the x-axis or real number axis, what must be true about the two complex numbers?

GO

Topic: Solving Quadratics

24. List the strategies that can be used to solve quadratic equations. Explain when each of the strategies would be most efficient. Give an example of a quadratic that would be most efficiently solved for each.

Solve the quadratics below using an appropriate method.

25. $x^2 + 9x + 18 = 0$

26. $x^2 - 2x - 3 = 0$

27. $2x^2 - 5x + 3 = 0$

28. $(x-2)(x+3) = 0$

29. $10x^2 - x + 9 = 0$

30. $(x-2)^2 = 20$