

Transforming Mathematics Education

ALGEBRA II

An Integrated Approach

MODULE 9

Statistics

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

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9.1 What Is Normal?

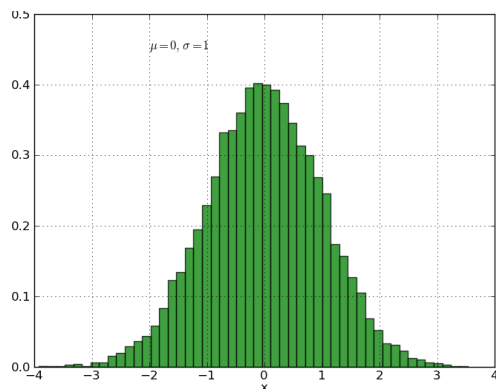
A Develop Understanding Task

One very important type of data distribution is called a “normal distribution.” In this case, the word “normal” has a special meaning for statistical distributions. In this task, you will be given pair of data distributions represented with histograms and distribution curves. In each pair, one distribution is normal and one is not. Your job is to compare each of the distributions given and come up with a list of features for normal distributions.

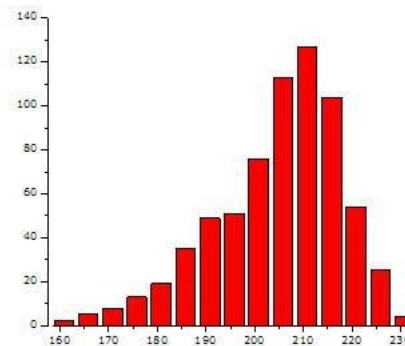


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<https://flic.kr/p/e65LUa>

1. This is approximately normal:

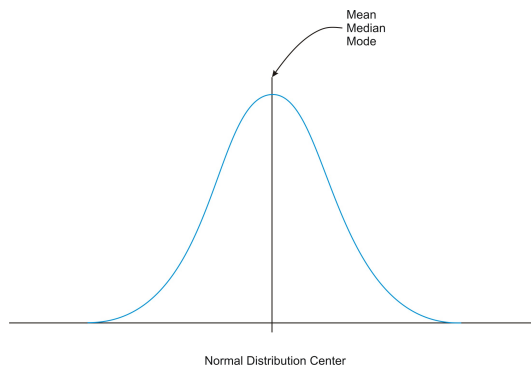


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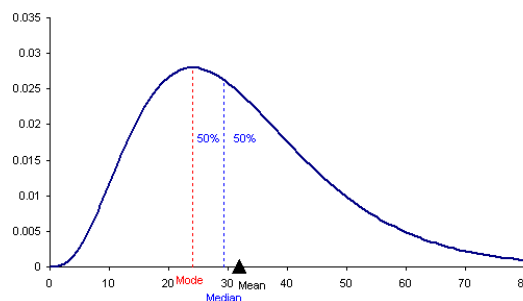


What differences do you see between these distributions?

2. This is normal:

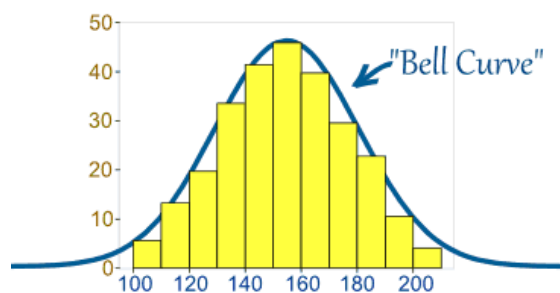


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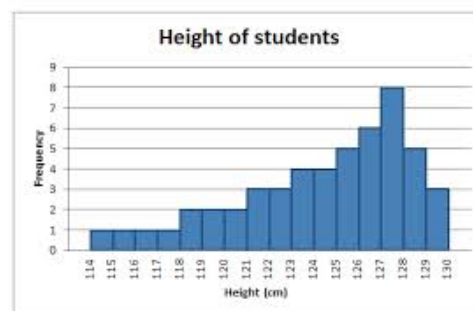


What differences do you see between these distributions?

3. This is approximately normal:

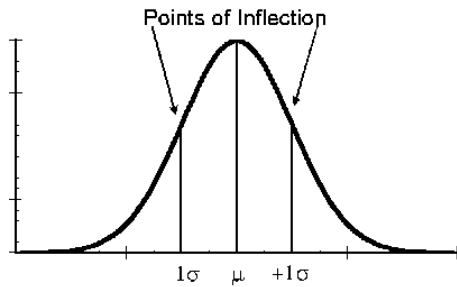


This is not:

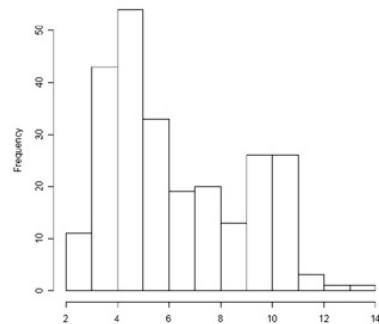


What differences do you see between these distributions?

4. This is normal:

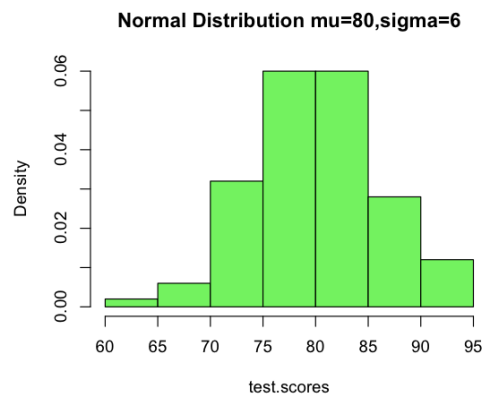


This is not:

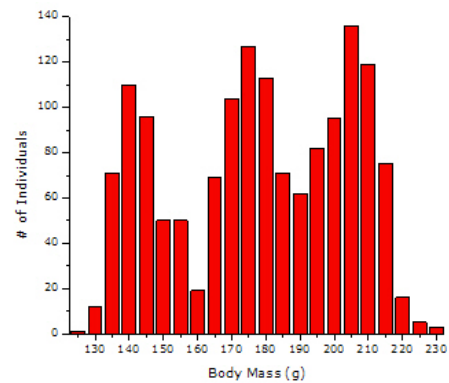


What differences do you see between these distributions?

5. This is approximately normal:

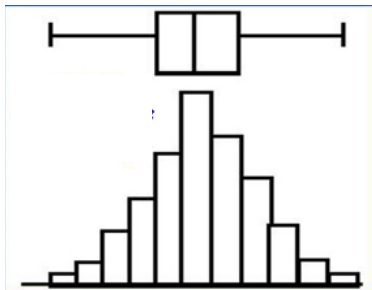


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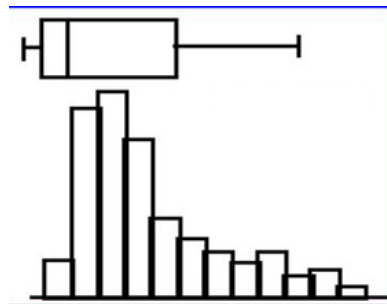


What differences do you see between these distributions?

6. This is approximately normal:

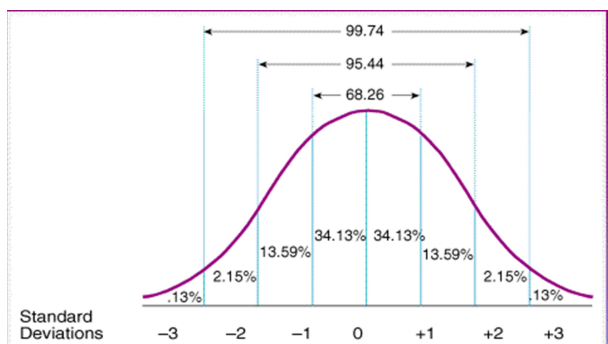


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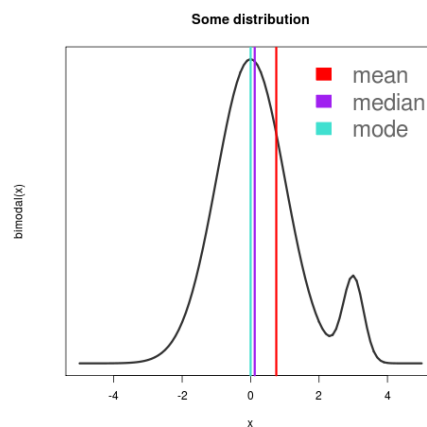
What differences do you see between these distributions?

7. This is normal:



What differences do you see between these distributions?

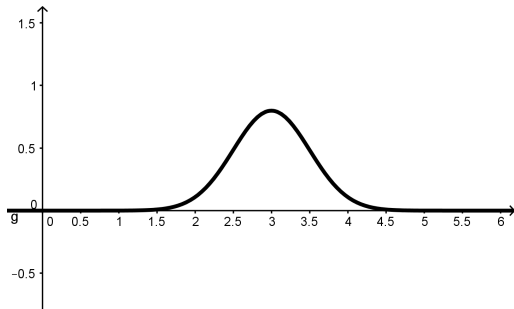
This is not:



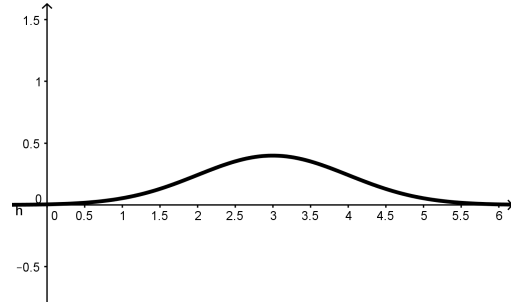
8. Based upon the examples you have seen in #1-7, what are the features of a normal distribution?

9. a. What does the standard deviation tell us about a distribution?
- b. Each of the distributions shown below are normal distributions with the same mean but a different standard deviation.

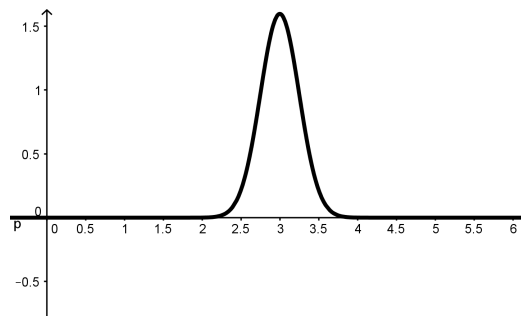
Mean = 3, Standard Deviation = 0.5



Mean = 3, Standard Deviation = 1



Mean = 3, Standard Deviation = 0.25

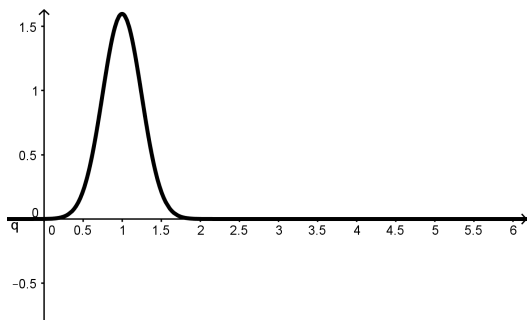


How does changing the standard deviation affect a normal curve? Why does it have this effect?

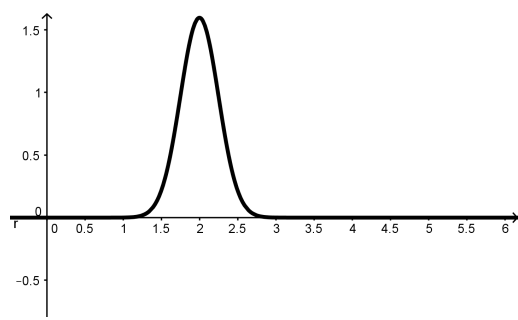
10. a. What does the mean tell us about a distribution?

b. Each of the distributions shown below are normal distributions with the same standard deviation but a different mean.

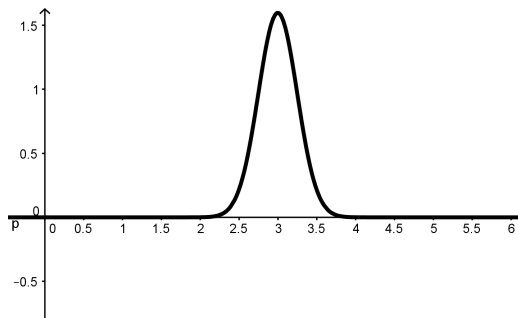
Mean = 1, Standard Deviation = 0.25



Mean = 2, Standard Deviation = 0.25



Mean = 3, Standard Deviation = 0.25



How does changing the mean affect a normal curve? Why does it have this effect?

11. Now that you have figured out some of the features of a normal distribution, determine if the following statements are true or false. In each case, explain your answer.

a. A normal distribution depends on the mean and the standard deviation.

True/False Why?

b. The mean, median, and mode are equal in a normal distribution.

True/False Why?

c. A normal distribution is bimodal.

True/False Why?

d. In a normal distribution, exactly 50% of the population is within one standard deviation of the mean.

True/False Why?

READY, SET, GO!

Name

Period

Date

READY

Topic: Working with standard deviation and percentiles

1. Jordan scores a 53 on his math test. The class average is 57 with a standard deviation of 2 points. How many standard deviations below the mean did Jordan score?
2. In Jordan's science class, he scored a 114. The class average was a 126 with a standard deviation of 6 points. How many standard deviations below the mean did Jordan score? In comparison to his peers, which test did Jordan perform better on?
3. Rank the data sets below in order of greatest standard deviation to smallest:

$$A = \{1, 2, 3, 4\} \quad B = \{2, 2, 2, 2\} \quad C = \{2, 4, 6, 8\} \quad D = \{4, 5, 6, 7\} \quad E = \{1, 1.5, 2, 2.5\}$$

4. Robin made it to the swimming finals for her state championship meet. The times in the finals were as follows:

$$\{2:10.3, \quad 2:12.5, \quad 2:12.7, \quad 2:12.38, \quad 2:20.45, \quad 2:21.43\}$$

If Robin's time was a 2:12.7, what percent of her competitors did she beat?

5. Remember that in statistics, μ is the symbol for mean and σ is the symbol for standard deviation. Using technology, identify the mean and standard deviation for the data set below:

$$\{1.23, \quad 1.3, \quad 1.1, \quad 1.48, \quad 1, \quad 1.14, \quad 5.21, \quad 5.1, \quad 4.63\}$$

$$\mu = \quad \sigma =$$

6. For the data in number 5, what time would fall one standard deviation above the mean?

Three standard deviations below the mean?

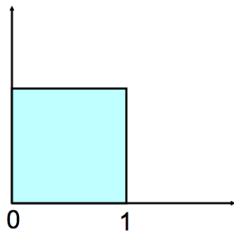
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SET

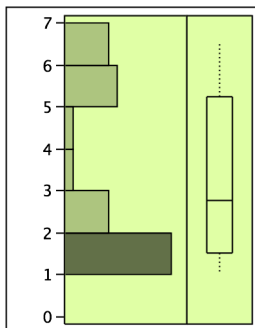
Topic: Identifying properties of the normal curve

For each distribution, identify the properties that match with a Normal Distribution, and then decide if the Normal curve could be used as a model for the distribution and explain why.

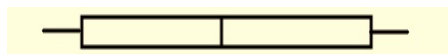
7. Normal Properties: Model with a Normal Curve? Yes or No



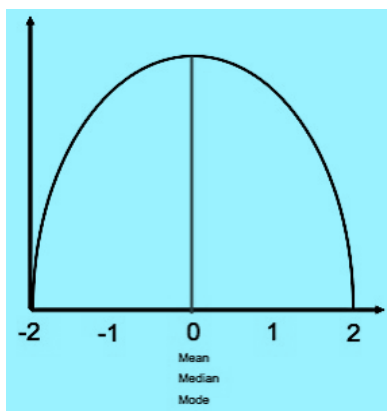
8. Normal Properties: Model with a Normal Curve? Yes or No



9. Normal Properties: Model with a Normal Curve? Yes or No

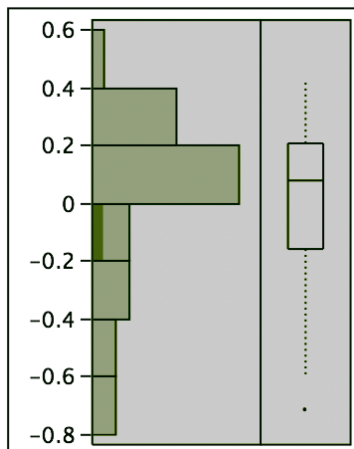


10. Normal Properties: Model with a Normal Curve? Yes or No



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11.

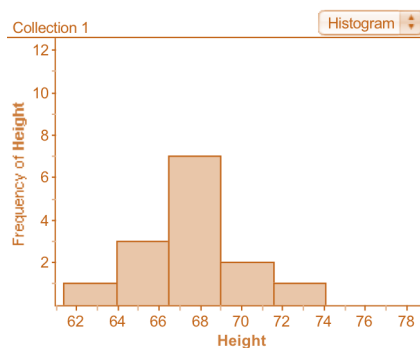


Mean = 0 Median = 0.1 Mode = 0.1

Normal Properties:
or No

Model with a Normal Curve? Yes

12.



Normal Properties:

Model with a Normal Curve? Yes or No

Mean: 68 Median: 68 Mode: 68

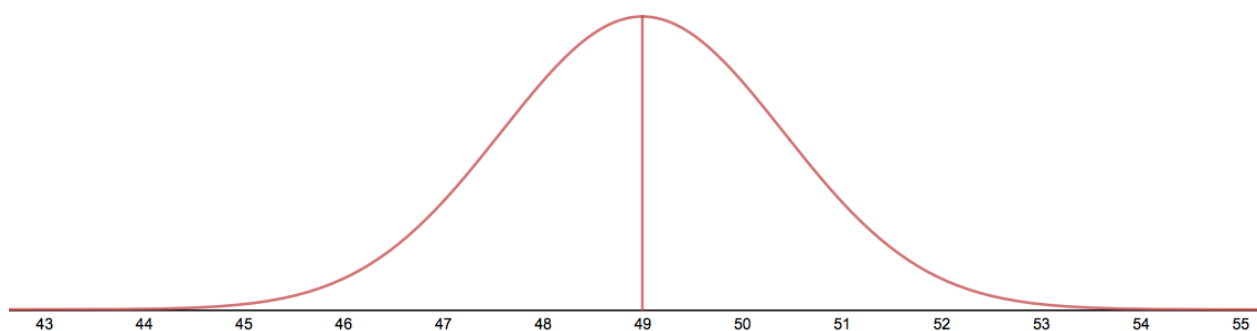
13. If two Normal distributions have the same standard deviation of 4.9 but different means of 3 and 6, how will the two Normal curves look in relation to each other?

Draw a sketch of each Normal curve below.

14. If two Normal distributions have the same mean of 3 but standard deviations of 1 and 4, how will they look in relation to each other? Draw a sketch of each Normal curve below.

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15. The Normal Curve given below has been labeled out three standard deviations. Estimate what one standard deviation is for this curve.



GO

Topic: Recalling inverses

Write the inverse of the given function in the same format as the given function:

16. $f(x) = 3x^2 + 2$

17. $g(x) = \frac{2x-7}{4}$

18. $h(x) = 3 + \sqrt{2x - 1}$

19.

x	y
12	24
14	38
-7	4
13	6
7	0

Determine if the following functions are inverses by finding $f(g(x))$ and $g(f(x))$.

20. $f(x) = 2x + 3$ and $g(x) = \frac{1}{2}x - \frac{3}{2}$

21. $f(x) = 2x^2 - 3$ and $g(x) = \sqrt{\frac{x^2}{2}} + 3$

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9.2 Just ACT Normal

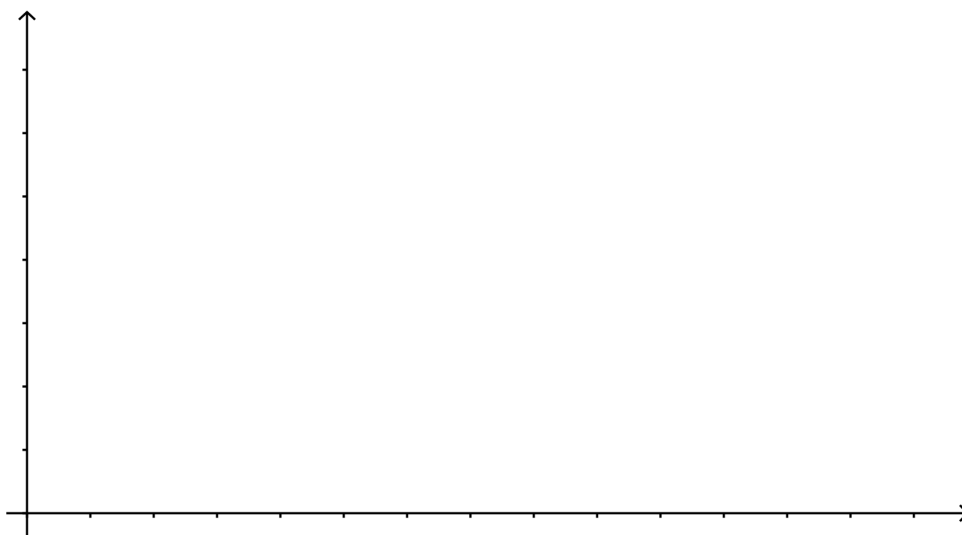
A Solidify Understanding Task

One of the most common examples of a normal distribution is the distribution of scores on standardized tests like the ACT. In 2010, the mean score was 21 and the standard deviation was 5.2 (Source: National Center for Education Statistics).



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1. Use this information to sketch a normal distribution curve for this test.



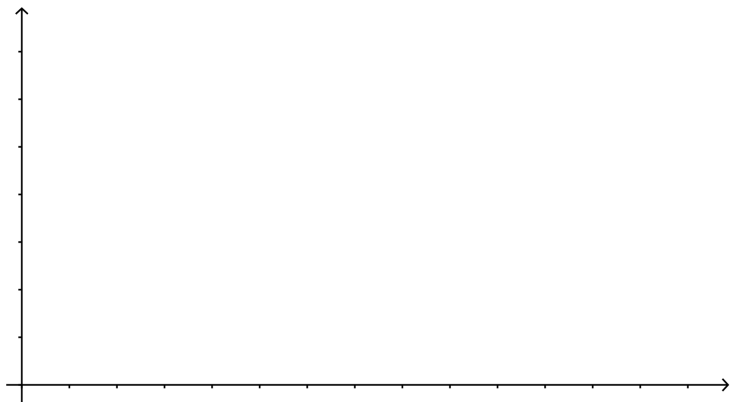
2. Use technology to check your graph. Did you get the points of inflection in the right places? (Make adjustments, if necessary.)
3. In “What Is Normal”, you learned that the 68 – 95 – 99.7 rule. Use the rule to answer the following questions:
 - a. What percentage of students scored below 21?
 - b. About what percentage of students scored above 16?
 - c. About what percentage of students scored between 11 and 26?

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4. Your friend, Calvin, would like to go to a very selective college that only admits the top 1% of all student applicants. Calvin has good grades and scored 33 on the test. Do you think that Calvin's ACT score gives him a good chance of being admitted? Explain your answer.

5. Many students like to eat microwave popcorn as they study for the ACT. Microwave popcorn producers assume that the time it takes for a kernel to pop is distributed normally with a mean of 120 seconds and a standard deviation of 13 for a standard microwave oven. If you're a devoted popcorn studier, you don't want a lot of un-popped kernels, but you know that if you leave the bag in long enough to be sure that all the kernels are popped, some of the popcorn will burn. How much time would you recommend for microwaving the popcorn? Use a normal distribution curve and the features of a normal distribution to explain your answer.



READY, SET, GO!

Name

Period

Date

READY

Topic: Thinking about the Law of Large Numbers

1. You and your friend are rolling one die over and over again. After 6 rolls, your friend has rolled four fives. Are you surprised by these results? Explain
2. After rolling the die 50 times, you know notice that you rolled a total of 20 fives. Are you surprised now? Explain.
3. You survey 100 people in your school and ask them if they feel your school has adequate parking. Only 30% of the sample feels the school has enough parking. If you have 728 students total in your school, how many would you expect out of all the student body that felt there was enough parking?

SET

Topic: Applying properties of normal distribution

4. The population of NBA players is *Normally distributed* with a mean of 6'7" and a standard deviation of 3.9 inches. (Wikipedia) Greg is considered unusually tall for his high school at 6' 3".
 - a. What percent of NBA players are taller than Greg?
 - b. What percent are shorter?
 - c. How tall would Greg have to be in order to be in the top 2.5% of NBA player heights?

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5. The average height of boys at Greg's school is 5'9" with a standard deviation of 2 ".
If we assume the population is normally distributed,
- d. What percent of students in the school are shorter than Greg?
 - e. What percent of students are between 5'5" and 5' 11"?
6. Jordan is drinking a cup of hot chocolate. From previous research, he knows that it takes an average time of 10 minutes for the hot chocolate to reach a temperature where his tongue will not burn. The time it takes the chocolate to cool varies Normally with a standard deviation of 2 minutes.
- a. How long should he wait to drink his hot chocolate if he wants to be 84% sure that he won't burn himself?
 - b. If he waits 8 minutes, what percent of the time will he burn his tongue?

GO

Topic: Applying the properties of logarithms

Use the properties of logarithms to expand the expression as a sum or difference and/or constant multiple of logarithms. (Assume all variables are positive.)

7. $\log_2 3x$

8. $\log_x \frac{5}{7}$

9. $\ln \sqrt[3]{x}$

10. $\log \frac{2x^2y^4}{3z^2}$

11. $\log_3 \frac{16x^2-36}{x^2}$

12. $\log \frac{x^2+12x+20}{5}$

13. $\log_3 27x^7$

14. $\log 10^5 \sqrt{y}$

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9.3 Y B Normal?

A Solidify Understanding Task

As a college admissions officer, you get to evaluate hundreds of applications from students that want to attend your school. Many of them have good grades, have participated in school activities, have done service within their communities, and all kinds of other attributes that would make them great candidates for attending the college you represent. One part of the application that is considered carefully is the applicants score on the college entrance examination. At the college you work for, some students have taken the ACT and some students have taken the SAT.

You have to make a final decision on two applicants. They are both wonderful students with the very same G.P.A. and class rankings. It all comes down to their test scores. Student A took the ACT and received a score of 29 in mathematics. Student B took the SAT and received a score of 680 in mathematics. Since you are an expert in college entrance exams, you know that both tests are designed to be normally distributed. A perfect ACT score is 36. The ACT mathematics section has a mean of 21 and standard deviation of 5.3. (Source: National Center for Education Statistics 2010) A perfect score on the SAT math section is 800. The SAT mathematics section has a mean of 516 and a standard deviation of 116. (Source: www.collegeboard.com 2010 Profile).

1. Based only on their test scores, which student would you choose and why?

This analysis is starting to make you hungry, so you call your friend in the Statistics Department at the university and ask her to go to lunch with you. During lunch, you tell her of your dilemma. The conversation goes something like this:

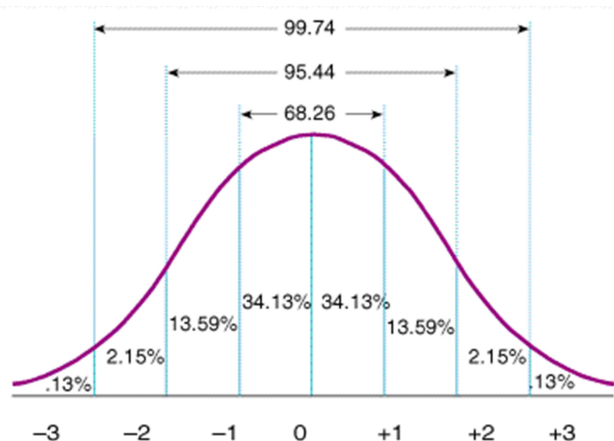


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You: I'm not sure that I'm making the right decision about which of two students to admit to the university. Their entrance exam scores seem like they're in about the same part of the distribution, but I don't know which one is better. It's like trying to figure out which bag of fruit weighs more when one is measured in kilograms and one is measured in pounds. They might look like about the same amount, but you can't tell the exact difference unless you put them on the same scale or convert them to the same units.

Statistician: Actually, there is a way to make comparisons on two different normal distributions that is like converting the scores to the same unit. The scale is called the "standard normal distribution". Since it was invented to make it easy to think about a normal distribution, they set it up so that the mean is 0 and the standard deviation is 1.

Here's what your statistician friend drew on her napkin to show you the standard normal distribution:



You: Well, that looks just like the way I always think of normal distributions.

Statistician: Yes, it's pretty simple. When we use this scale, we give things a z-score. A z-score of 1 means that it's 1 standard deviation above the mean. A z-score of -1.3 means that it is between 1 and 2 standard deviations below the mean. Easy-peasy.

What's even better is that when we have a z-score there are tables that will show the area under the curve to the left of that score. For a test score like the ACT or SAT, it shows the percentage of the population (or sample) that is below that score. I've got a z-score table right here in my purse. See, the z-score is -1.3, then 9.68% of the population scored less. You can also say that 90.32% of the population scored better, so -1.3 wouldn't be a very good score on a test.

Try it: Let's say you had two imaginary test takers, Jack and Jill. Jack's z-score was 1.49 and Jill's z-score was 0.89.

2. What percent of the test takers scored below Jack? What percent scored above Jack?
3. What percent of the test takers scored below Jill? What percent scored above Jill?
4. What percent of the test takers scored between Jack and Jill?
5. Jack and Jill's friend, Jason, scored -1.49. Find the number of test takers that scored above him without using a table or technology. Explain your strategy.

You: That's very cool, but the two scores I'm working with are not given as z-scores. Is there some way that I can transform values from some normal distribution like the scores on the ACT or SAT to z-scores?

Statistician: Sure. The scale wouldn't be so amazing if you couldn't use it for any normal distribution. There's a little formula for transforming a data point from any normal distribution to a standard normal distribution:

$$z\text{-score} = \frac{\text{data point} - \text{mean}}{\text{standard deviation}}$$

6. So, if you have an ACT score of 23. The mean score on the ACT is 21 and the standard deviation is 5.2. What would you estimate the z-score to be?
7. Let's use the formula to figure it out: $z\text{-score} = \frac{23-21}{5.2}$. How was your estimate? Explain why this value is reasonable.

You: That's great. I'm going back to the office to decide which student is admitted.

8. Compare the scores of Student A and Student B. Explain which student has the highest mathematics test score and why.

READY, SET, GO!

Name _____

Period _____

Date _____

READY

Topic: Using probability to predict from a sample

At South Beach High School, there are 2500 students attending. Mariana surveys 40 of her friends where they prefer to eat lunch. She created the following two-way table showing her results:

	9 th Grade	10 th Grade	11 th Grade	12 th Grade	Totals
School Cafeteria	18	6	2	1	28
Off Campus	2	4	3	4	12
Totals	20	10	5	5	40

Mariana plans to use her data to answer the following questions:

- I. Do students prefer to eat on campus or off campus overall?
- II. Is there a difference between grade levels for where students prefer to eat lunch?

1. In Mariana's sample, what percent of students prefer school lunch?

What percent prefer to eat off campus?

2. For each grade level in her sample, determine the percent of students that prefer school lunch and the percent that prefer off campus lunch. Do you notice anything unusual?

3. Based on her sample, Mariana concludes that students at South Beach High school overall like school lunch. Do you agree or disagree? Why?

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SET

Topic: Exploring z-scores

A company makes a mean monthly income of \$20,300 with a standard deviation of \$3,200. In one given month the company makes \$29,500.

4. Find the z-score.
5. Assuming the company's monthly income is *Normally distributed*, what percent of the time does the company make more than this amount? Less than?
6. What percent of the time does the company make between \$15,000 and \$25,000?
7. If the company needs to make \$16,400 in order to break even, how likely in a given month is the company to make a profit?

On the Wechsler Adult Intelligence Scale, an average IQ is 100 with a standard deviation of 15 units. (Source: http://en.wikipedia.org/wiki/Intelligence_quotient)

8. IQ scores between 90 and 109 are considered average. Assuming IQ scores follow a *Normal distribution*, what percent of people are considered average?
9. One measure of Genius is an IQ score of above 135. What percent of people are considered genius?
10. Einstein had an IQ score of 160. What is his z-score?
11. What is the probability of an individual having a higher IQ than Einstein?

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GO

Topic: Sketching Polynomials

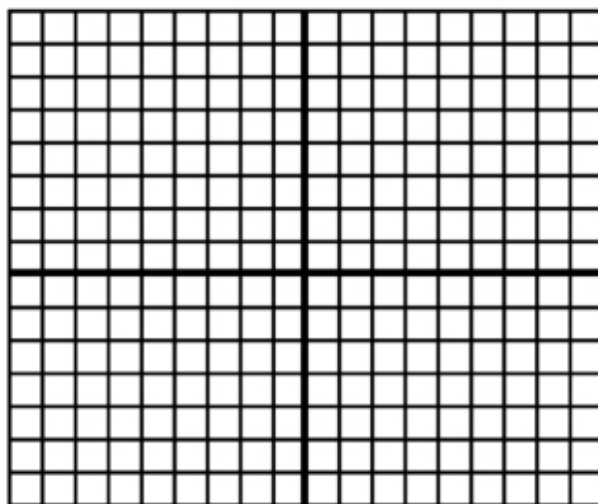
Without using technology, sketch the graph of the polynomial function with the given characteristics. If the equation is not given, write it in standard form.

12. Degree 4

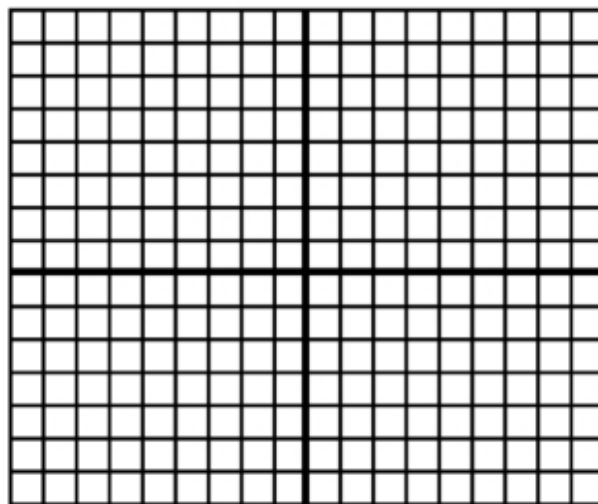
Roots: -1 multiplicity 2, 5, -2

y - intercept: 20

Equation:

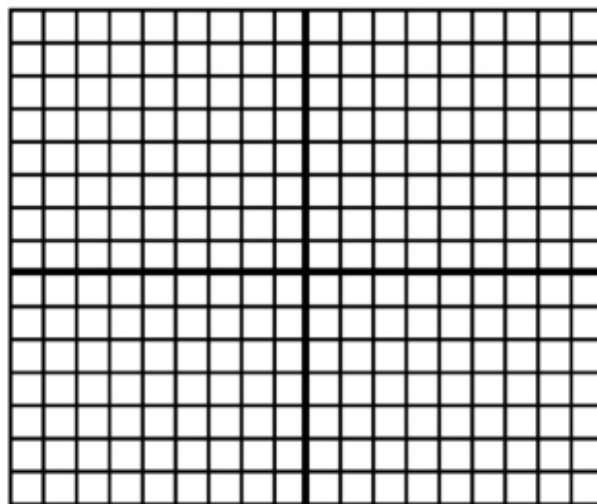


13. $f(x) = (x + 2)(x - 3)^3$



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14. $g(x) = -x(x - 3)^2(x + 5)(x - 5)$



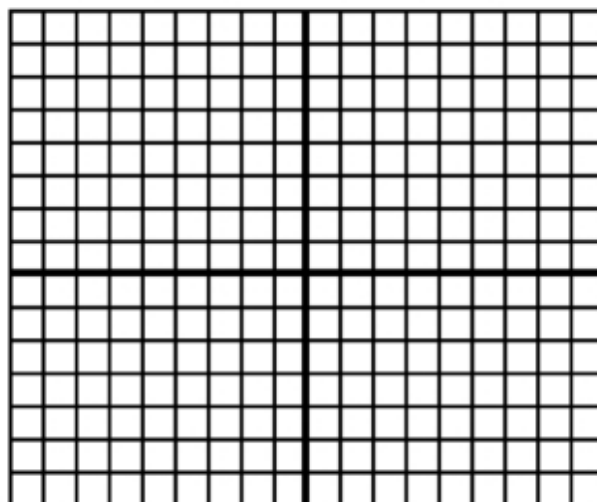
15. Degree 3

$$f(-1) = 10$$

$$f(2i) = 0$$

$$f(3) = 0$$

Equation:



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9.4 Wow, That's Weird!

A Practice Understanding Task

Each of the stories below are based upon normal distributions. Rank order these stories from most unusual to most average. (1 is the most unusual, 8 is the most average.) In each case, explain your ranking.

A. The number of red loops in a box of Tutti-Frutti-O's is normally distributed with mean of 800 loops and standard deviation 120. Tony bought a new box, opened it, and counted 1243 red loops. (It didn't really matter because all the colors are the same flavor anyway.)

Rank _____ Explanation: _____

B. The weight of house cats is normally distributed with a mean of 10 pounds and standard deviation 2.1 pounds. My cat, Big Boy, weighs 6 pounds.

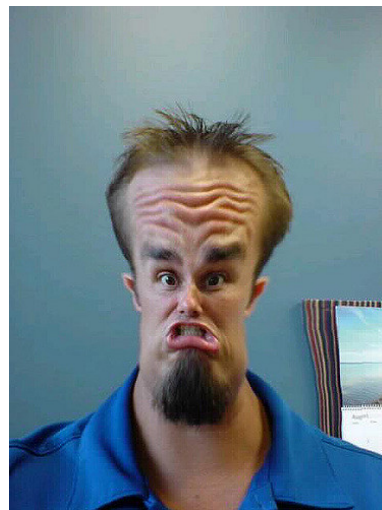
Rank _____ Explanation: _____

C. The lifetime of a battery is normally distributed with a mean life of 40 hours and a standard deviation of 1.2 hours. I just bought a battery and it died after just 20 hours

Rank _____ Explanation: _____

D. The amount that a human fingernail grows in a year is normally distributed with a mean length of 3.5 cm and a standard deviation of 0.63 cm. My neighbor's thumbnail grew all year without breaking and it is 4.6 cm long with stars and stripes painted on it.

Rank _____ Explanation: _____



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E. My little brother was digging in the garden and found a giant earthworm that was 35 cm long. The length of earthworms is normally distributed with a mean length of 14 cm and a standard deviation of 5.3 cm.

Rank _____ Explanation: _____

F. The mean length of a human pregnancy is 268 days with a standard deviation of 16 days. My aunt just had a premature baby delivered after only 245 days.

Rank _____ Explanation: _____

G. IQ scores for young adults on a famous IQ test are distributed normally with a mean of 110 and a standard deviation of 25. I'm pretty smart and my IQ is 135.

Rank _____ Explanation: _____

H. The army measured head sizes among male soldiers and found that the distribution is pretty close to normal with a mean of 22.8 inches and standard deviation of 1.1 inches. Little Joe was almost too small to get into the army because his head size was only 20.6 inches.

Rank _____ Explanation: _____

READY, SET, GO!

Name

Period

Date

READY

Topic: Filling in two-way tables

The data below is the data from Mrs. Hender's class. Students needed to score 60% or better to pass the test.

1 st hour:	2 nd hour:	3 rd hour:
72, 83, 56, 63, 89, 92, 92, 67, 88, 84, 67, 97, 96, 100, 84, 82	80, 83, 81, 81, 67, 90, 70, 71, 72, 77, 81, 85, 86, 77, 74, 51	51, 45, 67, 83, 99, 100, 94, 52, 48, 46, 100, 59, 65, 56, 72, 63

1. Make a two-way frequency table showing how many students passed the test and how many failed each class.

	1 st	2 nd	3 rd	Total
Passed				
Failed				

2. What percent of students passed Mrs. Hender's test in each class?
What is the total percent of all classes that passed?
3. Combine the data from all three classes to create a histogram using technology. Sketch your histogram below. What features of the Normal curve does your histogram have?
4. If Mrs. Hender's were going to predict her total pass rate using only 2nd hour, would she make an accurate prediction? Explain why or why not.

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SET

Topic: Using features of the normal curve to rank data

5. Five track athletes are in the running for the Athletic Performance of the Year award. A panel of coaches is trying to decide which athlete is the most deserving to win the award. Rank each athlete below by the given information. Assume all distributions follow a Normal Curve.
- Javier threw the Javelin 215 ft. The average Javelin throw is 152.08 ft. with a standard deviation of 15.85 ft.
 - Chance ran a 400m time of 46.99 seconds. The average 400m time was 52.6, with a standard deviation of 1.01 seconds.
 - Derick ran a 36.26 in the 300m Hurdles. The average time was 41.77 with a standard deviation of 1.49 seconds.
 - Chad ran a 100m time of 10.59 seconds. The average time was 11.603 seconds with a standard deviation of .29 seconds.
 - Kayden threw the discus 180 ft. The average throw was 122.4 ft. with a standard deviation of 14.38 ft.

GO

Topic: Solving logarithmic equations

Solve each equation below for x by applying properties for exponents and logarithms.

6. $2^{x-5} = 128$

7. $\left(\frac{1}{243}\right)^x = 27$

8. $3^{x+2} = 27^{x-3}$

9. $\log(2x + 4) - \log(3x) = 0$

10. $\log_2(2x^2 + 4x - 2) - \log_2 10 = 0$

11. $\frac{\ln(x+7)}{\ln(2x-3)} = 1$

12. $\frac{\log(4x+2)}{\log 15} = 1$

13. $\frac{\log_3(3x+6)}{\log_3 81} = 1$

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9.5 Would You Like to Try a Sample?

A Develop Understanding Task



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In the task *Wow! That's Weird!*, you saw a number of statistics for things like the average weight of a house cat. You know it would be impossible to measure all the house cats to find their average weights, but scientists still claim to know it.

You've probably heard it many times before: "Survey results show that 54% of Americans believe that. . ." You're sure that you didn't participate in the survey and neither did anyone you know, and yet, the researchers claim that the survey represents the beliefs of all Americans.

How can this be possible? In the next few tasks, we'll explore how statistics allow us to draw conclusions about an entire group without actually working with the entire group. Sometimes the results make sense and other times you might think that they just can't be right. We will learn how to make judgments about statistical studies, based on the methods that have been used.

First, we need to get our terms straight. When we talk about the entire group that we are interested in, that is called the **population**. When some members of the group are selected to represent the entire group, that is called a **sample**. The thing we are interested in knowing about the population is the **parameter of interest**.

For each of the scenarios below, identify the population, the sample and the population parameter of interest.

1. A grocery store wants to know the average number of items that shoppers purchase in each visit to the store. They decide to count the items in the cart of every twentieth person through the check stand.

Population _____

Sample _____

Parameter of interest _____

2. A team of biologists wants to know the average weight of fish in a lake. They decide to drop a net and measure all the fish caught in three different locations in the lake.

Population _____

Sample _____

Parameter of interest _____

3. There are lots of different ways that a sample can be chosen from a population. Group the following examples of ways to select a sample into six categories.

A. You are in charge of school activities. You want to know what activities students would prefer to participate in during the school year. You decide to put the name of each student in the school into a big bowl. You draw 100 names and ask those students to respond to a survey about the activities they prefer.

B. You are in charge of school activities. You want to know what activities students would prefer to participate in during the school year. You assign each student in the school a number. You randomly select a starting number among the first 10 numbers and then select every tenth student in the list from that point forward.

C. You are in charge of school activities. You want to know what activities students would prefer to participate in during the school year. You use the rolls from each homeroom class. You go through each homeroom class, drawing 2 names from each class. You ask those students to respond to a survey about the activities they prefer.

D. You are in charge of school activities. You want to know what activities students would prefer to participate in during the school year. You get the list of all the homeroom classes and randomly select 5 classes. You go to each of the classes selected and survey all the students in that class.

E. You are in charge of school activities. You want to know what activities students would prefer to participate in during the school year. You stand in the cafeteria during your lunch break and ask students in they would be willing to participate in your survey as they walk by.

F. You are in charge of school activities. You want to know what activities students would prefer to participate in during the school year. You make a lot of copies of the survey about the

G. You are interested in finding out the percent of residents in the city that have experienced a robbery in the past year. Using the city property records, you assign each residence a number. You use a random number generator to give you a list of numbers. You look up the police reports for each residence selected.

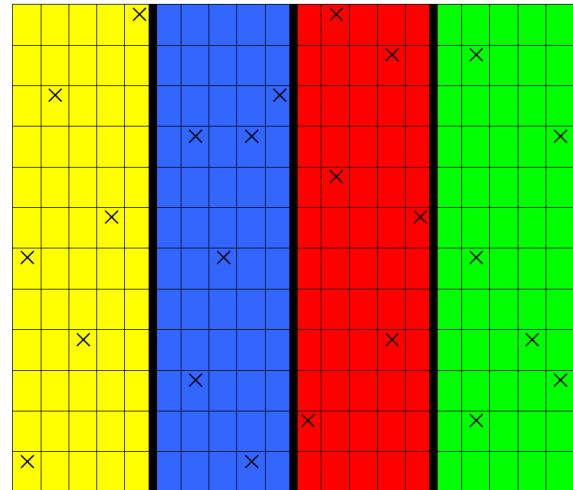
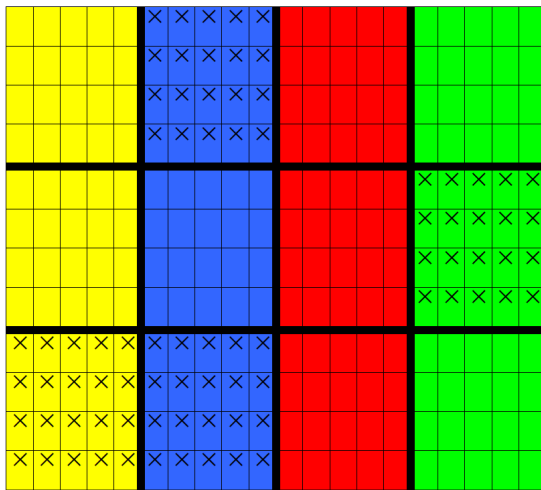
I. An auto analyst is conducting a satisfaction survey, sampling from a list of 10,000 new car buyers. The list includes 2,500 Ford buyers, 2,500 GM buyers, 2,500 Honda buyers, and 2,500 Toyota buyers. The analyst selects a sample of 400 car buyers, by randomly sampling 100 buyers of each brand.

K. A restaurant owner wants to find out the average number of dishes ordered at each table served on Friday evenings, their busiest time. She decides to collect and analyze every fifth receipt of the night, starting at 6:00 p.m.

A 10x10 grid with 10 'X' marks placed at the following intersections (row, column) starting from the top-left corner (0,0):

- (0, 1)
- (1, 3)
- (1, 8)
- (2, 9)
- (3, 5)
- (4, 2)
- (5, 9)
- (6, 1)
- (6, 6)
- (8, 2)
- (9, 6)

0.



READY, SET, GO!

Name

Period

Date

READY

Topic: Contrasting association and causation

When collecting data, statisticians are often interested in making predictions. Sometimes they simply want to know if one variable is related or is **associated** with another variable. (Can you predict one variable given information on the other one). Other times, they want to determine if one variable actually **causes** a change in another variable. For each example below, decide whether the variables simply explain each other, or if you think one variable would cause the other to change.

1. As the amount of food Ollie the elephant eats increases her weight also increases.
(Associated/Causes)
2. As Popsicle sales go up in the summer, the number of people drowning also increases.
(Associated/Causes)
3. As Erika's feet grow longer, she grows taller. (Associated/Causes)
4. As Tabatha gets older, her reading score improves in school. (Associated/Causes)

SET

Topic: Identifying population, sample, and parameter

For each scenario below, identify the *population*, *sample* and *parameter* of interest.

5. The local school board wants to get parents to evaluate teachers. They select 100 parents and find that 89% approve of their child's teacher.

Population:

Sample:

Parameter:

6. Jarret wants to know the average height of the students in his school. There are 753 students in his high school; he finds the heights of 52 of them.

Population:

Sample:

Parameter:

7. A government official is interested in the percent of people at JFK airport that are searched by security. He watches 300 people go through security and observes 42 that are searched.

Population:

Sample:

Parameter:

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For each scenario, identify what type of sampling was used to obtain the sample. Explain whether or not you think the sample will be representative of the population it was sampled from:

- | | |
|--|---|
| 8. Elvira surveys the first 60 students in the lunch line to determine if students at the school are satisfied with school lunch. | Type of sample:

Representative? Explain. |
| 9. Elvira selects every 5 th student in the lunch line to determine if students at the school are satisfied with school lunch. | Type of sample:

Representative? Explain. |
| 10. Elvira randomly selects 7 different tables in the lunchroom and surveys every student on the table to determine if students at the school are satisfied with school lunch. | Type of sample:

Representative? Explain. |
| 11. Elvira assigns every student in the school a number and randomly selects 60 students to survey to determine if student at the school are satisfied with school lunch. | Type of sample:

Representative? Explain. |
| 12. Elvira wants to determine if students are satisfied with school lunch. She leaves surveys on a table for students to answer as the walk by. | Type of sample:

Representative? Explain. |
| 13. Elvira wants to determine if students are satisfied with school lunch. She wants to include input from each grade level at the high school. She randomly surveys 25 freshman, 25 sophomores, 25 juniors, and 25 seniors. | Type of sample:

Representative? Explain. |

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GO

Topic: Graphing trig functions

For each function identify the amplitude, period, horizontal shift, and the vertical shift.

14. $f(t) = 120 \cos\left(\frac{\pi}{4}(t - 3)\right) + 30$

Amplitude:

Period:

Horizontal Shift:

Vertical Shift:

15. $f(t) = 3.5 \sin\left(\frac{\pi}{6}\left(t + \frac{1}{3}\right)\right) + 7$

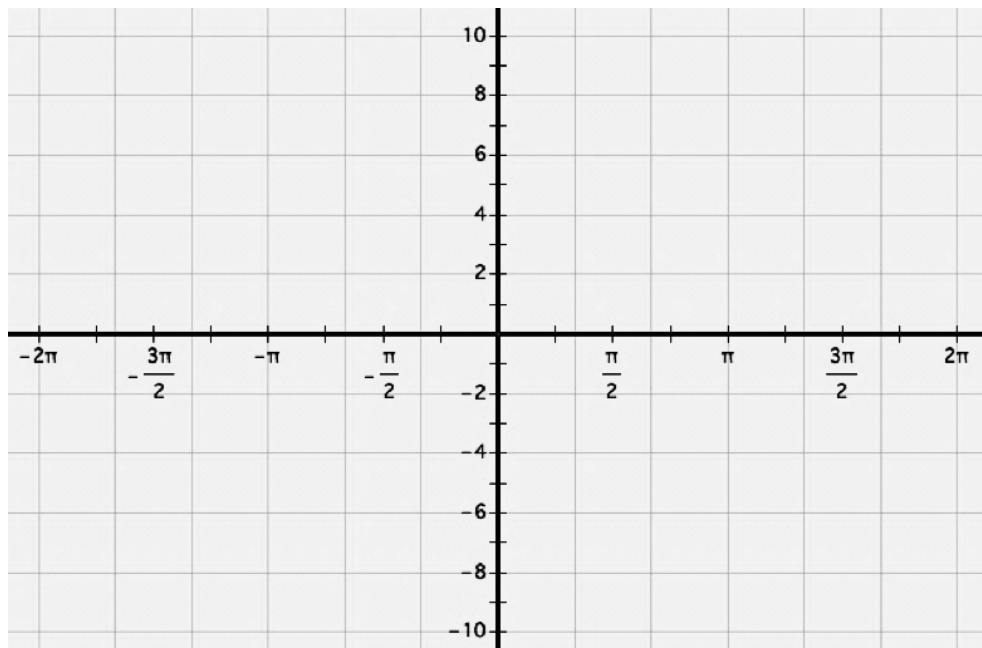
Amplitude:

Period:

Horizontal Shift:

Vertical Shift:

16. Graph one full period of $f(x) = 8 \sin(x - \pi) - 2$.



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9.6 Let's Investigate

A Solidify Understanding Task

When we want to draw conclusions about some population, there are at least two different statistical ideas to consider. We learned about sampling in *Would You Like to Try a Sample*, since it is usually more practical to sample the population rather than somehow measure everyone or everything in the population.



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The second thing to consider is how to measure the parameter of interest, the thing we want to know about the population. Sometimes it's obvious, like if you want to know the average weight of a population, you determine a sample and then put each of the subjects on a scale. Three other techniques are the following:

- **Surveys:** When they want to know how people feel, what their preferences are, what they own, how much they make, etc., researchers often construct a survey to ask the people in the sample about the parameter of interest.
 - **Observational Studies:** In this type of study, researchers observe the behavior of the participants/subjects without trying to influence it in any way so they can learn about the parameter of interest.
 - **Experiments:** In an experiment, researchers manipulate the variables to try to determine cause and effect.
1. Imagine that you want to know whether a new diet plan is effective in helping people lose weight. You might choose any of the three methods to determine this.
- If you used a survey, you could simply ask people that had tried the diet plan if they lost weight.
 - If you used an observational study, you might monitor volunteers that try the diet plan and measure how much weight they lost (or gained).
 - If you used an experiment, you might randomly assign participants to two groups. One group (the control group) eats as they normally would and the other group (the experimental group) eats according to the diet plan. At the end of two months, the two groups are compared to see the average weight gain or loss in each group.

Based on these three examples,

- a. What are some possible advantages and disadvantages of surveys?
- b. What are some possible advantages and disadvantages of observational studies?
- c. What are some possible advantages and disadvantage of experiments?

2. Identify which method is illustrated by each example:

- a. To determine whether drinking orange juice prevents colds, researchers randomly assigned participants to a group that drank no orange juice or a group that drank two glasses of orange juice a day. They measured the number of colds that each group had over the course of the year and compared the results of the two groups.
- b. To determine whether exercise reduces the number of headaches, researchers randomly selected a group of participants and recorded the number of hours each participant exercised and the number of headaches each participant experienced.
- c. To determine the effectiveness of a new advertising campaign, a restaurant asked every tenth customer if they had seen the advertisement, and if it had influenced their decision to visit the restaurant.
- d. To determine if a new drug is an effective treatment for the flu, researchers randomly selected two groups of people that had the flu. One group was given a placebo (a sugar pill that has no physical effect) and one group was given the new drug. Researchers measured the number of days that participants experienced flu symptoms and compared the two groups to see if they were different.
- e. To determine if higher speed limits cause more traffic fatalities, researchers compared the number of traffic deaths on randomly selected stretches of highway with 65 mph speed limits to the number of traffic deaths on an equal number of randomly selected stretches of highway with 75 mph speed limits.

3. Describe how you might select a sample and use a survey to investigate which soft drink people prefer: Fizzy Pop or Kooky Kola.
4. Describe how you might select a sample and use an observational study to investigate which soft drink people prefer: Fizzy Pop or Kooky Kola.
5. Describe how you might select a sample and use an experiment to investigate if consuming large quantities of Kooky Kola is associated with having headaches.
6. Describe the method you would use to determine if excessive texting is associated with bad grades. Explain why you chose that method and what conclusions could be drawn from the study.

READY, SET, GO!

Name

Period

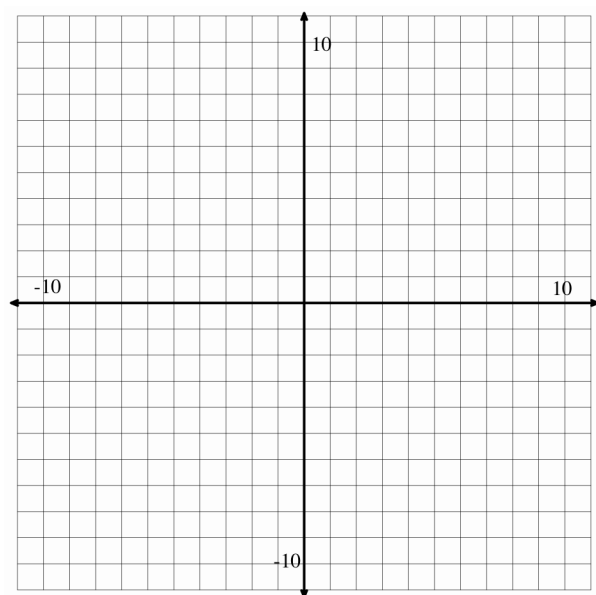
Date

READY

Topic: Solving systems of linear equations

1. Solve the system of equations
$$\begin{cases} 5x - 3y = 3 \\ 2x + y = 10 \end{cases}$$

a. By graphing



b. By substitution

c. By elimination

SET

Topic: Distinguishing between surveys, observational studies, and experiments

For the following scenarios, identify each situation as a *survey*, *observational study*, or an *experiment*.

- To determine if a new pain medication is effective, researchers randomly assign two groups of people to use the pain medication in group 1 and a placebo in group 2. Both groups are asked to rate their pain and the results are compared.

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3. Officials want to determine if raising the speed limit from 75 mph to 80 mph will have an impact on safety. To determine this, they watch a stretch of the highway when the speed limit is 75 and see how many accidents there are. Then they observe the number of accidents over a period of time on the same stretch of highway for a speed limit of 80 mph. They then compare the difference.
4. To determine if a new sandwich on the menu is preferred more than the original, the manager of the restaurant takes a random sample of customers that have tried both sandwiches and asks them which sandwich they like best.
5. A newspaper wants to know what its customer satisfaction is. It randomly selects 500 customers and asks them.

Mrs. Goodmore wants to know if doing homework actually helps students do better on their unit exams.

6. Describe how Mrs. Goodmore could carry out a survey to determine if homework actually helps. Explain the role of randomization in your design.
7. Describe how Mrs. Goodmore could carry out an observational study to determine if homework helps test scores.
8. Describe how Mrs. Goodmore could carry out an experiment to determine if homework helps test scores. Explain how you will use randomization in your design and how you will use a control.
9. If Mrs. Goodmore wants to determine if homework causes test scores to rise, which method would be best? Why?

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GO

Topic: Recalling normal curves

The average resting heart rate of a young adult is approximately 70 beats per minute with a standard deviation of 10 beats per minute. Assuming resting heart rate follows a Normal Distribution, answer the following questions.

10. Draw and label the Normal curve that describes this distribution. Be sure to label the mean, and the measurements 1, 2, and 3 standard deviations out from the mean.

11. What percent of people have a heart rate between 55 and 80 beats per minute? Label these points on your Normal curve above and shade in the area that represents the percent of people with heartbeats between 55 and 80 beats per minute.

12. If a resting heart rate above 80 beats per minute is considered unhealthy, what percent of people have an unhealthy heart rate?

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9.7 Slacker's Simulation

A Solidify Understanding Task

I know a student who forgot about the upcoming history test and did not study at all. To protect his identity, I'll just call him Slacker. When I reminded Slacker that we had a test in the next class, he said that he wasn't worried because the test has 10 true/false questions. Slacker said that he would totally guess on every question, and since he's always lucky, he thinks he will get at least 8 out of 10. That's what he did on the last quiz and it worked great.



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I'm skeptical, but Slacker said, "Hey, sometimes you flip a coin and it seems like you just keep getting heads. You may only have a 50/50 chance of getting heads, but you still might get heads several times in a row. I think this is just about the same thing. I could get lucky."

1. What do you think of Slacker's claim? Is it possible for him to get 8 out of 10 questions right? Explain.

I thought about it for a minute and said, "Slacker, I think you're on to something. I'm not sure that you will get 80% on the test, but I agree that the situation is just like a coin flip. It's either one way or the other and they are both equally likely if you're just guessing." My idea is to use a coin flip to simulate the T/F test situation. We can try it many times and see how often we get 8 out of 10 questions right. I'm going to say that if the coin lands on heads, then you guessed the problem correctly. If it lands on tails, then you got it wrong.

2. Try it a few times yourself. To save a little time, just flip 10 coins at once and count up the number of heads for each test.

	# Correct (Heads)	# Incorrect (Tails)	% Correct
Test 1			

Test 2			
Test 3			
Test 4			
Test 5			

Did you get 8 out of 10 correct in any of your trials?

3. Based on your trials, do you think Slacker has a good chance of getting 80% correct?
4. Collect the data from the entire class and display it using technology. Now what do you think of Slacker's chances of getting 80% correct? Explain why.
5. What would you expect the graph to look like if you continued to collect samples? Why?
6. Based upon your understanding of this distribution, what would you estimate the likelihood of Slacker getting 80% on the test without studying?

READY, SET, GO!

Name

Period

Date

READY

Topic: Recalling row operations in systems of linear equations

For each combination of matrices, write out the justification that allows you to write the new matrix based on the three manipulations we can perform on the equations in a system.

- Replace an equation in the system with a constant multiple of that equation
- Replace an equation in the system with the sum or difference of the two equations
- Replace an equation with the sum of that equation and a multiple of the other

For example, the following matrix transformation can be justified by writing “I replaced the first row of the matrix by multiplying the first row by $\frac{1}{6}$.”

$$\begin{bmatrix} 6 & 6 & 45.00 \\ 3 & 2 & 19.00 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 7.50 \\ 3 & 2 & 19.00 \end{bmatrix}$$

1. $\begin{bmatrix} -5 & 2 & 9 \\ 10 & 4 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} -15 & 6 & 27 \\ 30 & 12 & 24 \end{bmatrix}$	2. $\begin{bmatrix} 8 & 12 & 5 \\ 1 & -6 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 10 & 0 & 9 \\ 1 & -6 & 2 \end{bmatrix}$
4. $\begin{bmatrix} 1 & 2 & 5 \\ 1 & 4 & 10 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 5 \\ 0 & 2 & 5 \end{bmatrix}$	5. $\begin{bmatrix} 16 & 22 & 19 \\ 10 & 44 & 38 \end{bmatrix} \Rightarrow \begin{bmatrix} 16 & 22 & 19 \\ 11 & 0 & 0 \end{bmatrix}$

SET

Topic: Using a simulation

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In 1963, NBC started to host a game called *Let's Make a Deal!* Contestants were given three doors to choose from. Behind one door was a prize. After selecting one door, the contestant was shown what was behind one of the doors they did not select. The contestant is then asked if they would like to stick with the door they first selected, or switch to the remaining one.

6. Which strategy do you think would result in the best chance of selecting the winning door?
Should the contestant switch doors, or stick with the first one they chose?

Go to the following website: http://nlvm.usu.edu/en/nav/category_g_3_t_2.html

Select the applet stick or switch.

7. Play the game 20 times using the stick method and 20 times using the switch method. Record your wins and losses in the table below:

	Stick	Switch	Total
Win			
Lose			
Total			

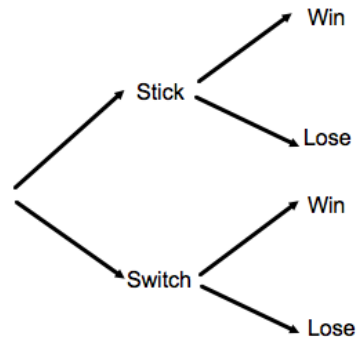
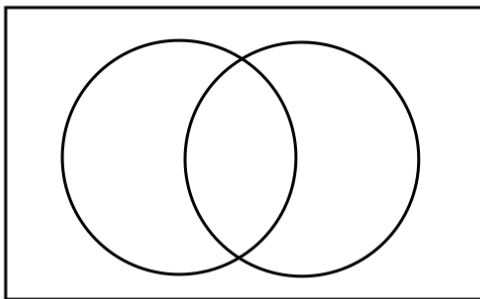
8. Based on the simulation, what is $P(\text{winning}|\text{stick}) =$
9. Based on the simulation, what is $P(\text{winning}|\text{switch}) =$
10. Click on the multiple games tab. Simulate 100 games for each strategy. What is the probability of winning using each method?

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GO

Topic: Reviewing probability

7. For your two-way table in problem 3, create a Venn diagram and a tree diagram below.



8. $P(\text{winning}) =$

14. $P(\text{winning} \cap \text{sticking}) =$

9. $P(\text{winning} \cup \text{sticking}) =$

16. $P(\text{loosing} | \text{sticking}) =$

10. $P(\text{winning or loosing}) =$

11. Are the events winning and sticking independent of each other? Justify your answer using probabilities.

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