mathematics vision project

Transforming Mathematics Education

GEOMETRY

A Learning Cycle Approach

User's Guide

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

Scott Hendrickson, Joleigh Honey, Barbara Kuehl, Travis Lemon, Janet Sutorius

STRUCTURE OF THE MATERIALS

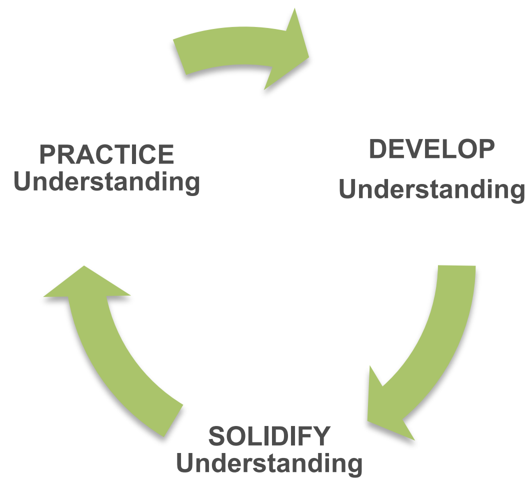
Each curriculum in the Mathematics Vision Project materials is composed of two main parts, the classroom experience, which is designed around the implementation of a specific type of task and the aligned "Ready, Set, Go!" homework assignment. Each task is accompanied by a set of teacher notes. The teacher notes identify the purpose of the lesson and then describe the steps the teacher can take during the classroom experience to ensure that students engage in a rich learning event. Tasks are to be done in class and should not be assigned as homework. There is an aligned "Ready, Set, Go!" homework assignment for each task. It is the independent practice. Homework serves the student as a type of formative assessment. It is while doing the homework that the student can discern for himself if the mathematics done in class can be performed independently.

The MVP classroom experience begins by confronting students with an engaging task and then invites them to grapple with solving it. As students' ideas emerge, take form, and are shared, the teacher orchestrates the student discussions and explorations towards a focused mathematical goal. As conjectures are made and explored, they evolve into mathematical concepts that the community of learners begins to embrace as effective strategies for analyzing and solving problems. These strategies are eventually solidified into a body of practices and mathematical habits that belong to the students, because, they were developed by the students, as an outcome of their own creative and logical thinking. This is how students learn mathematics. They learn by doing mathematics. They learn by needing mathematics. They learn by verbalizing the way they see the mathematical ideas connect and by listening to how their peers perceived the problem. Students then own the mathematics because it is a collective body of knowledge that they have developed over time through guided exploration. This process describes the Learning Cycle, an instructional framework that allows students to build mathematical knowledge over time. This framework is flexible. Every lesson does not follow the pattern of develop, solidify, practice. For instance, the first module on quadratics begins with a Develop Understanding Task. Many aspects of the definition of a quadratic, surface in that task. Five solidify tasks follow the first task. Each of the Solidify tasks extends one of the key concepts that surfaced in the beginning Develop Task. The module ends with a Practice task that pulls all of the key concepts together into a complete definition of quadratic.

The diagram at the right illustrates the Comprehensive Mathematics Instructional Framework (CMI) around which the MVP curriculum has been developed. Every task in the curriculum is identified as one of the following:

- *Develop Understanding Task*
- *Solidify Understanding Task*
- *Practice Understanding Task*

Each type of task serves a different purpose in moving a student's mathematical knowledge from a foundation of conceptual understanding to that of procedural fluency. Paying attention to the type of task will help teachers know if they have accomplished the goal of the lesson.



The Learning Cycle

*Each module in the **MVP** educational program has been carefully designed and sequenced with rich mathematical tasks that have been formulated to generate the mathematical concepts within the core curriculum. Careful attention has been placed upon the way mathematical knowledge emerges, is extended, and then becomes efficient, flexible, and accurate. Some tasks are developmental tasks while others are for solidifying or practicing the concepts. The sequencing of the tasks encourages students to notice relationships and make connections between the concepts. In this way, students perceive mathematics as a coherent whole.*

While the classroom experience is predominantly geared towards improving students' reasoning and sense-making skills, MVP regards mathematical understanding and procedural skill as being equally important. Hence, the "Ready, Set, Go!" homework assignments are focused on students practicing procedural skills and organizing principles to add structure to the ideas developed during the classroom experience. As in any discipline, practice is the refining element that brings fluency and agility to the skills of the participant. The Ready and the Go sections of the homework *assignments have been designed to continue to spiral a review of content, while the Set section focuses on consolidating the mathematics addressed in class that day. Each day when a student engages in the homework assignment, it is expected that he or she will have the opportunity to reflect on the new learning from class and will practice the retrieval of ideas from the body of learning that has been growing over the school year, and even prior to the current school year. Recent research on learning has identified reflection and retrieval practice as being two key ingredients for durable learning. True learning should be long lasting and should grow out of previous understandings, extending over years of study. Hence, the "Go!" sections of the "Ready, Set, Go!" homework assignments will contain topics from previous lessons and prior years of mathematics instruction. Together the classroom experience and the "Ready, Set, Go!" homework assignments offer a powerful blend of new learning and maintained proficiency.*

The Learning Cycle depicts how students become proficient in the mathematics overtime. Each task represents at least one day of instruction. Therefore, a Learning Cycle may extend over several days or weeks of classroom instruction, however, each day the teacher is expected to frame the lesson around The Teaching Cycle. This cycle also has three components: Launch, Explore, and Discuss.

The Teaching Cycle may seem to be simple, but it involves careful preparation and then deliberate implementation by the instructor.

Launch: How will you . . .

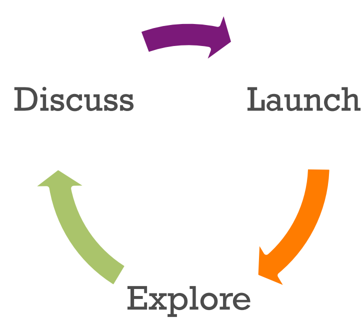
- *hook and motivate students?*
- *provide schema for the task?*
- *describe the expectations for the finished task?*

Explore: What will you . . .

- *look for and listen for as you observe?*
- *accept as evidence of understanding?*
- *ask to stimulate, redirect, focus, and extend mathematical thinking?*

Discuss: How will you . . .

- *select which students will present their*

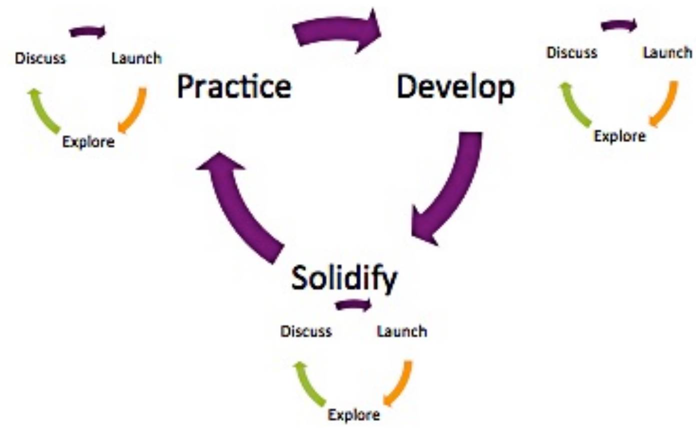


The Teaching Cycle

The Teacher Notes are orgainzed to reflect the three components of the Teaching Cycle. Look for the headings Launch, Explore, and Discuss.

solutions and strategies?
- *determine what ideas to pursue?*
- *decide whether to contribute to the discourse or allow students to continue to struggle to make sense of a concept?*

The diagram to the right depicts how the two instructional frameworks, the Teaching Cycle and the Learning Cycle, fit together. The Teaching Cycle occurs each day in the classroom, while the Learning Cycle extends over days and possibly weeks as the unit develops.



CONTENT

Transformational Geometry: Progressions and Practices

The big ideas addressed in the Geometry course include:

1. ***The definitions of the rigid motion and dilation transformations form an intuitive, yet precise, foundation for geometric proof.***
2. ***Defining congruence and similarity in terms of a sequence of transformations that carries one geometric figure onto another can be universally applied to both polygon and non-polygon shapes (e.g., circles, parabolas), whereas defining congruence and similarity in terms of corresponding proportional sides and corresponding congruent angles cannot.***
3. ***Making one's thinking visible through a diagram or visual representation is an important mathematical tool, and an essential part of geometric thinking.***
4. ***Adding auxiliary figures to a diagram, such as adding a circle to reveal congruent line segments, is an important strategy for seeing geometric structure.***
5. ***Working with geometric figures on a coordinate grid reveals the relationships between algebraic and geometric ideas.***
6. ***Experimenting with transformations and symmetry surfaces conjectures and provides insight into proving statements about properties of geometric figures.***
7. ***Generating compass and straightedge constructions, as well as constructions made with other tools, provide pathways for thinking through the logical sequence of ideas required for more formal justification and proof.***
8. ***Proofs, regardless of the format they are written in, consist of a logical sequence of ideas that a community of practice has already accepted as true (e.g., definitions, assumptions, previously proved statements), in order to establish the validity of another new idea or conjecture.***
9. ***Triangle congruence criteria (ASA, SAS and SSS) and the relationships of angles formed by two parallel lines cut by a transversal can be justified using rigid motion transformations. Once established, these ideas become useful tools in geometric proofs, although transformational thinking remains as a viable option for justifying geometric conjectures.***
10. ***Observations about conditions under which corresponding image and pre-image lines remain parallel after a transformation are an essential part of transformational proofs.***
11. ***The structure of a proof (e.g., flow diagram, two column, narrative, annotated diagram, etc.) is less important than the validity of the argument, which should be carefully analyzed and critiqued in order to make it better.***
12. ***Right triangle trigonometry emerges naturally from work with dilations and similarity.***

FEATURES

Each module begins with an overview of the module and a table of contents. The overview introduces the teacher to the key concepts that will be the focus of the module. It also relates the new learning to the store of prior knowledge that the student should be bringing with him. The Learning Cycles in the module are identified with a brief description of their content and an explanation of how the learning should progress through the module. Finally, the overview offers some instructional supports, including suggestions for the launch, language support, and suggestions for discourse. A set of "sentence frame cards" has been included as an aid for students. The cards are intended to assist students in becoming self-directed thinkers by guiding their thinking and prompting the language needed for discourse about their mathematical work. The table of contents identifies the name and type of task. It also identifies the standards from the core curriculum that will be developed in the task.

A set of teacher notes accompanies each task. The teacher notes outline each step of the lesson while following the framework of the Teaching Cycle. All of the teacher notes follow the same basic outline as described below:

Teacher Notes:

Purpose: Paying attention to the purpose of the task will help the teacher stay true to the progression of the module and keep the teacher from trying to accomplish too much within the task.

Core Standards Focus: The MVP authors have taken a "multi-tasking approach" to the standards. While one task may focus on more than one standard, several tasks may hi-light a single standard. In this way a set of interrelated ideas or a sequence of strategies and skills can be fused into a meaningful whole.

Related Standards: The focus of a lesson may be on a specific standard, yet doing the mathematics may require students to draw on related standards.

Standards for Mathematical Practice: It is possible and even likely that students will draw on all of the Standards for Mathematical Practice, however, different types of tasks naturally elicit certain practices. Those that seem to be the most likely to be drawn upon in the lesson have been identified.

Additional Resources for Teachers: This could be a variety of things depending on the lesson. For instance, an app using geogebra has been developed for the rubber-band activity in the first task of Module 2 in the geometry course.

The Teaching Cycle:

Essential question for students: Since all of the tasks are inquiry based, the essential question has been formulated to direct students' attention towards the purpose of the lesson without directly revealing the key ideas and strategies they should be producing.

Launch (whole class): Suggestions for introducing the lesson to the students. Sometimes this is relating a story, while other times it's working the first problem together. The prompts for the tasks often involve a lot of reading. It is the teacher's obligation to make sure that students understand what they are expected to do or produce during the Explore stage of the learning.

Explore (small groups): While students are exploring, the teacher will be monitoring the individual students and groups, looking for student strategies that will promote the discussion about the mathematics of the task. This is also a time during which the teacher can assess what previously learned skills the students are bringing to the task. The teacher notes will make suggestions of what the teacher should be looking for during the Explore session.

Discuss: Here the teacher will find suggestions for orchestrating the discussion in order to achieve the purpose of the lesson. This is the time when key connections need to be made.

Exit ticket for students: An exit slip can aid the teacher in checking for understanding. The items in the exit ticket could also be used as a warm-up in the subsequent lesson.

Instructional Supports

Adaptations

Interventions: These suggestions may lower the threshold for the task to accommodate students who don't know how to begin thinking about the task.

Challenge activity: The challenge activity is to provide a "high ceiling" for students who have finished early or need to be encouraged to think more deeply about the mathematics. Sometimes the last question in the task provides that extension and is not necessary to be completed by all students.

Answer Key for each task:

The suggested mathematical approach for some of these tasks may require teachers to look at the mathematics from a different perspective than they have ever done before. The best way to prepare to teach a task is to work the problem from the standpoint of the student. The answer key is provided as reassurance for the teacher.

Each "Ready, Set, Go!" homework assignments includes a section called Helps, Hints, and Explanations.

The Helps, Hints, and Explanations section provides an explanation for each type of homework problem and usually a worked example or two with annotation.
