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Agenda:

- Questions we can ask to get students started
- Questions about student work
- Sense-making through questioning
- Questioning to extend student thinking
- Students questioning students

8 years ago –

- Before the Common Core
- Before the 8 Student Math Practices
- Before NCTM's 8 Effective Teaching Practices

Homework

Multiply.

1. $\frac{2}{7} \times \frac{3}{4}$ 2. $\frac{5}{9} \times \frac{6}{14}$ 3. $\frac{13}{19} \times \frac{5}{26}$

Divide.

4. $\frac{2}{3} \div \frac{5}{6}$ 5. $\frac{9}{16} \div \frac{3}{4}$ 6. $\frac{21}{25} \div \frac{15}{14}$

Critical Thinking:

7. How many bows that are 5/12 yards long can be made from 3 yards of ribbon?

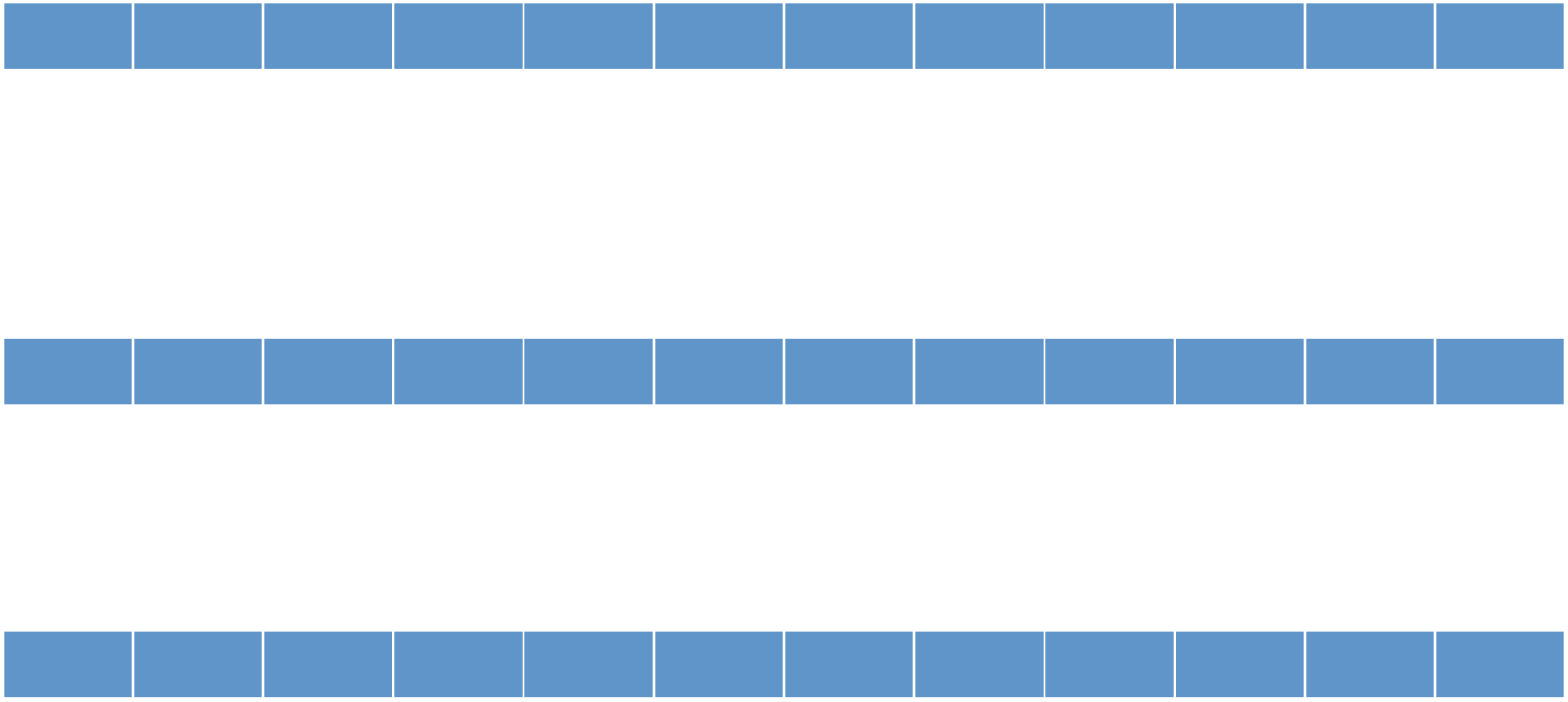
Critical Thinking: How many bows that are 5/12 yards long can be made from 3 yards of ribbon?

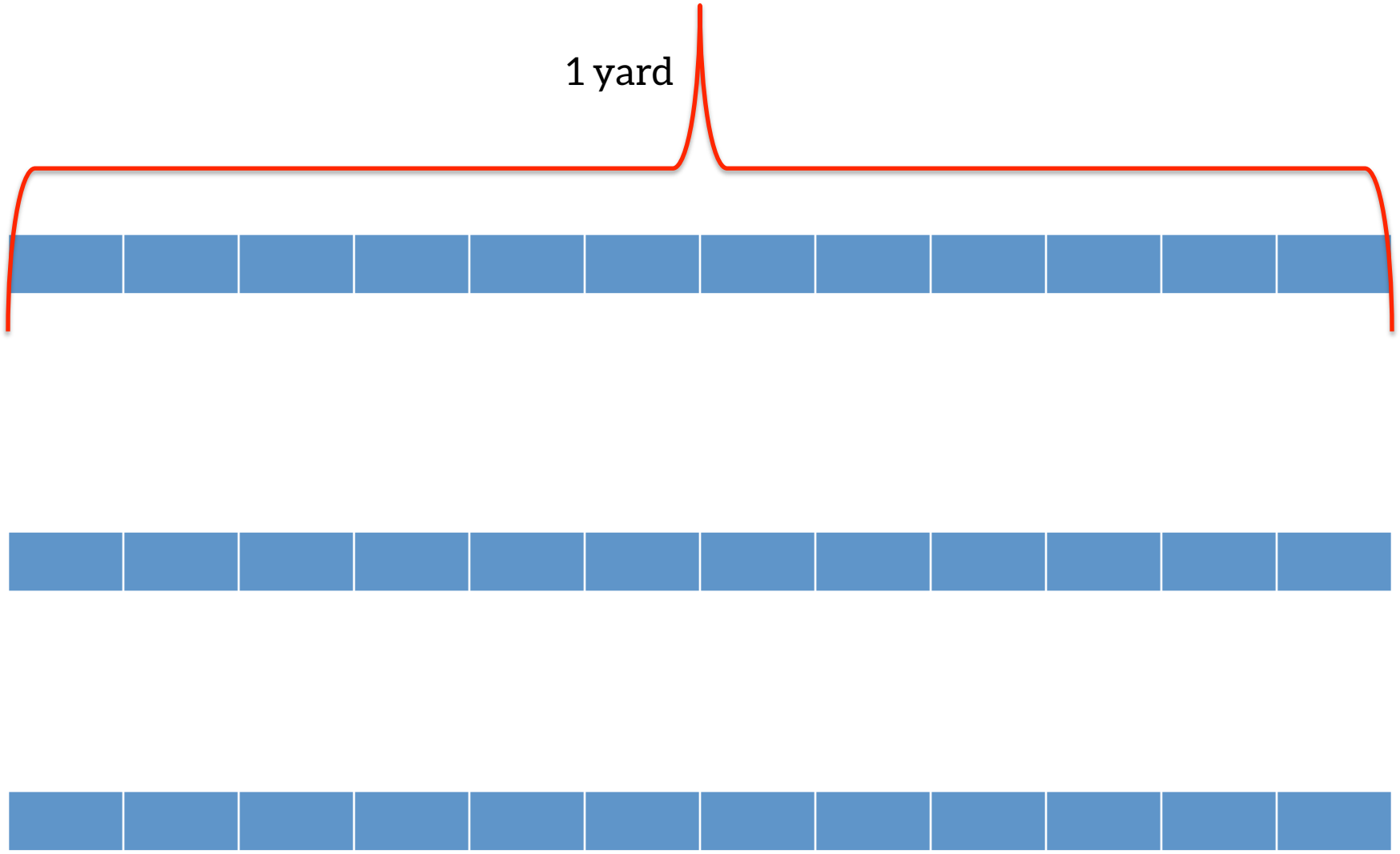
$7\frac{1}{5}$ $7\frac{1}{12}$

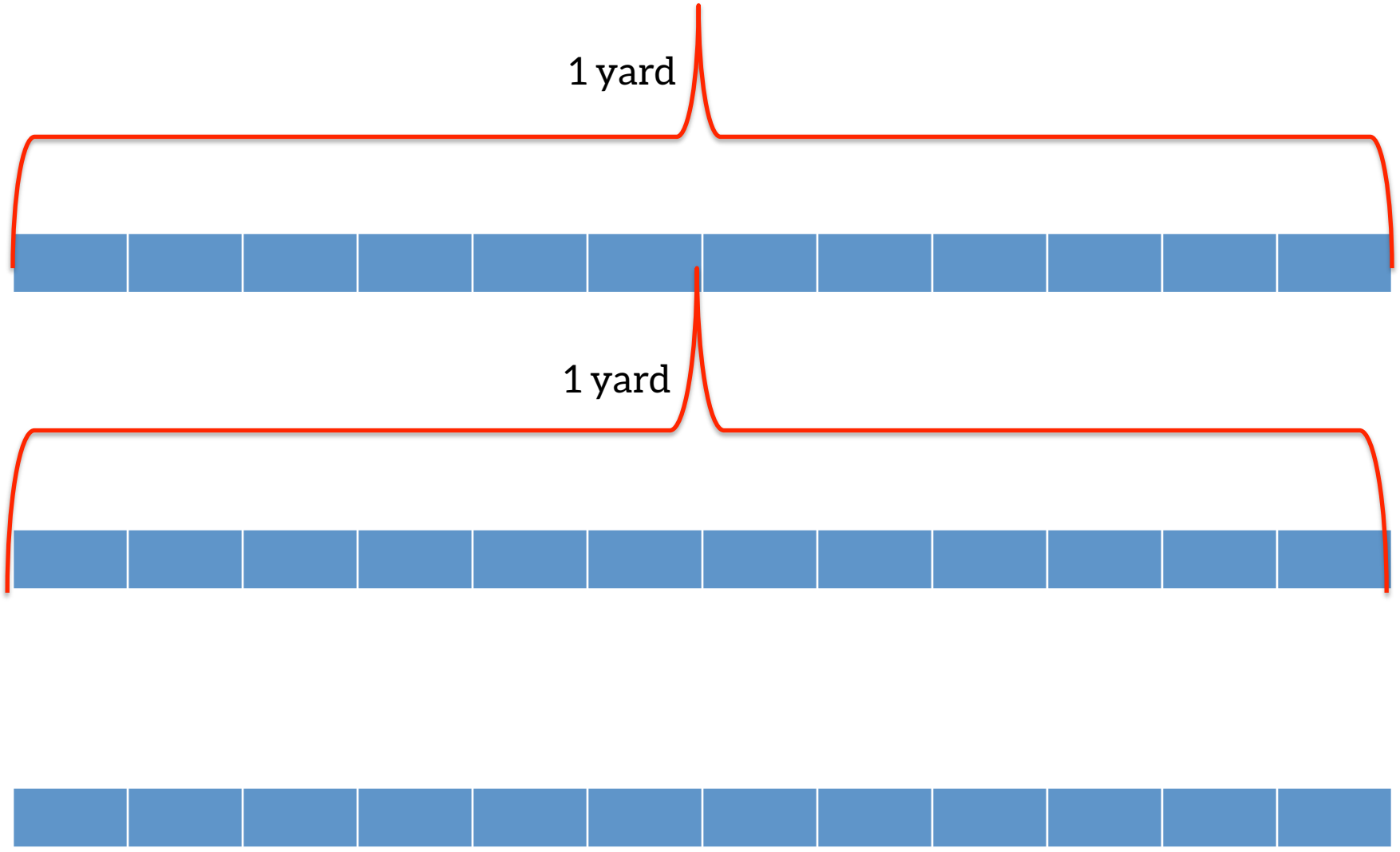
I knew that we were all doing something right because we all got 7 for the number of bows.

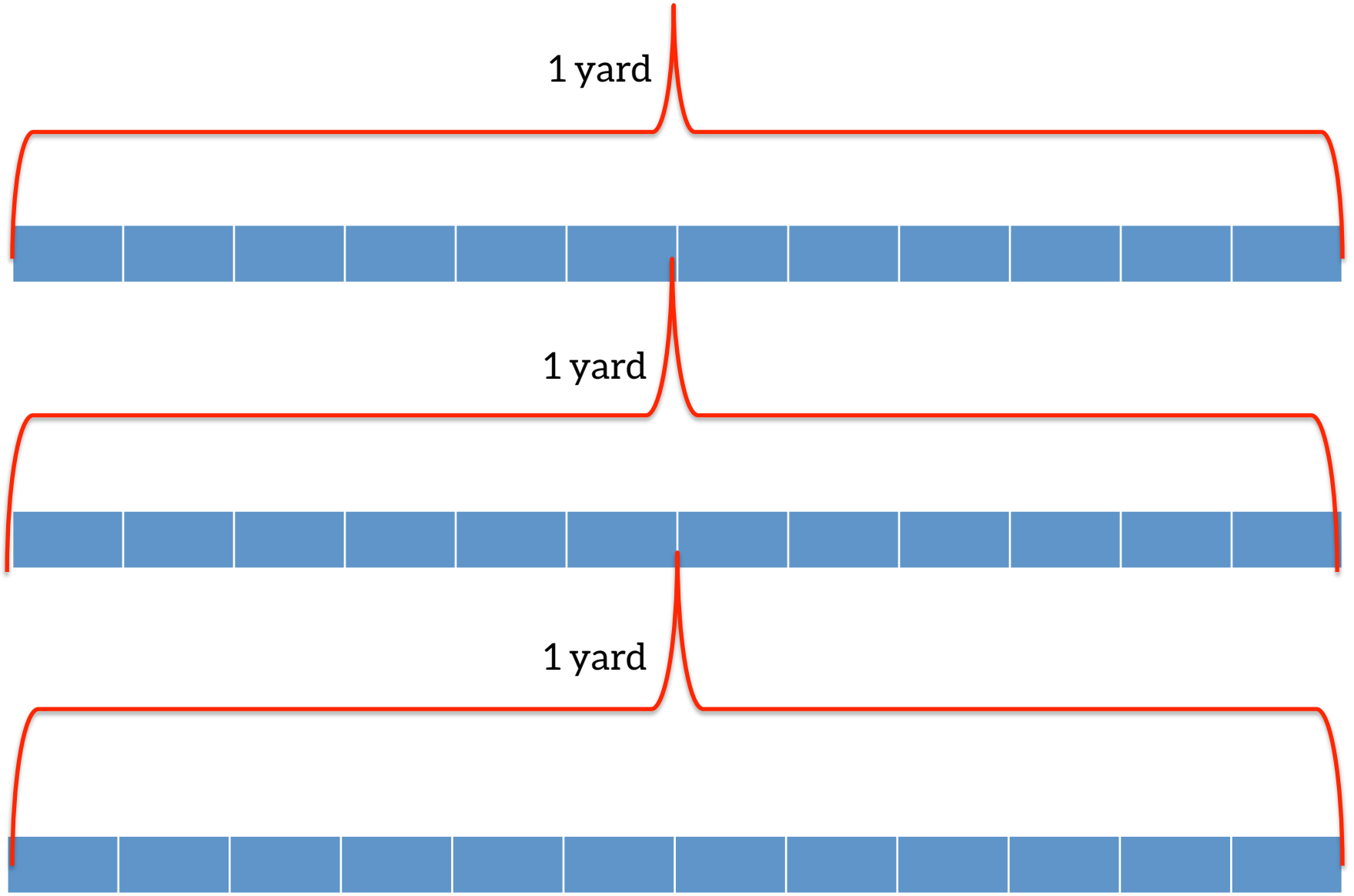
But what was going on with the fraction?

1. Did anyone draw a diagram to help them think about the problem?




1 yard


1 yard
1 yard


1 yard
1 yard
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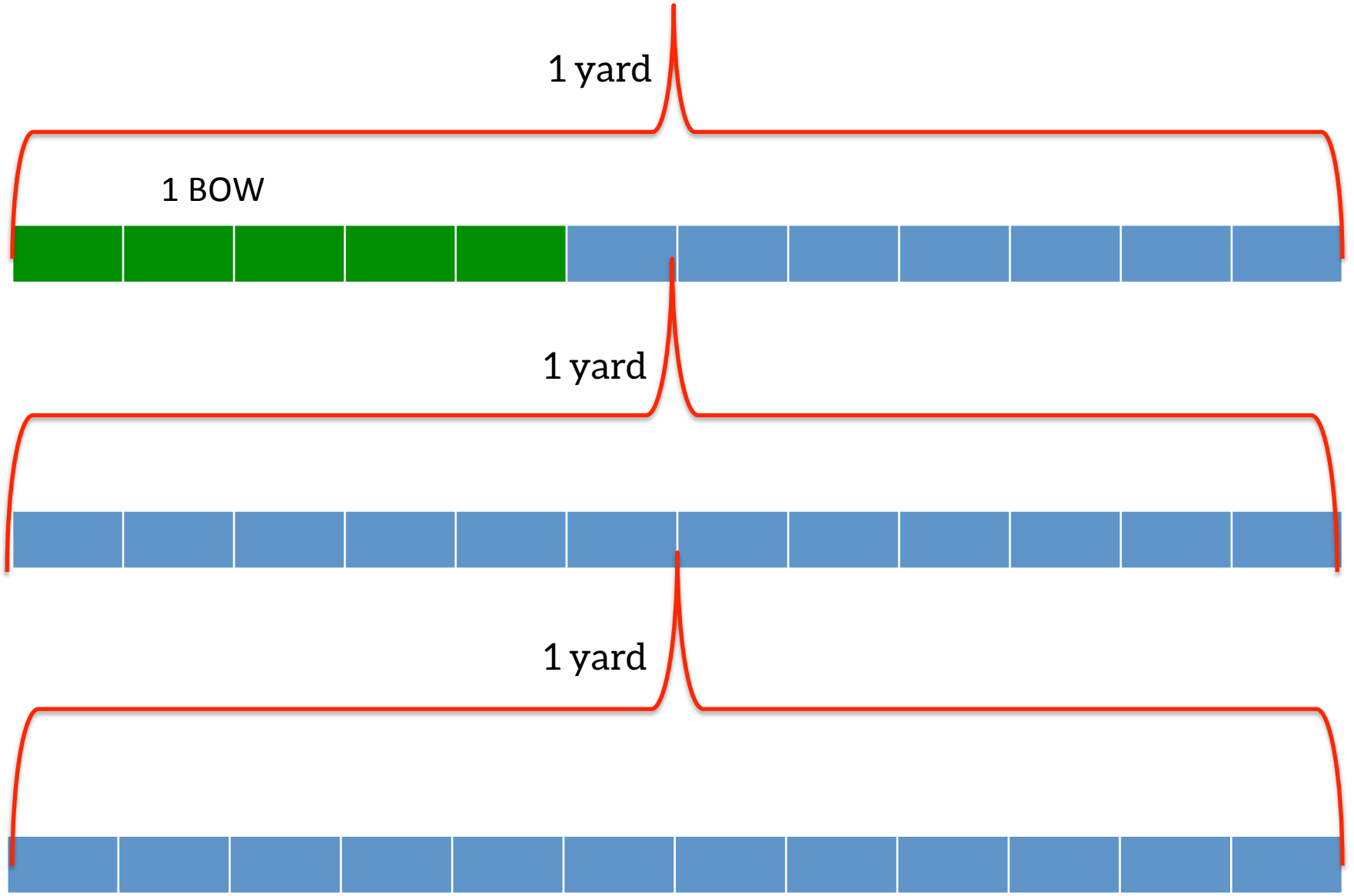
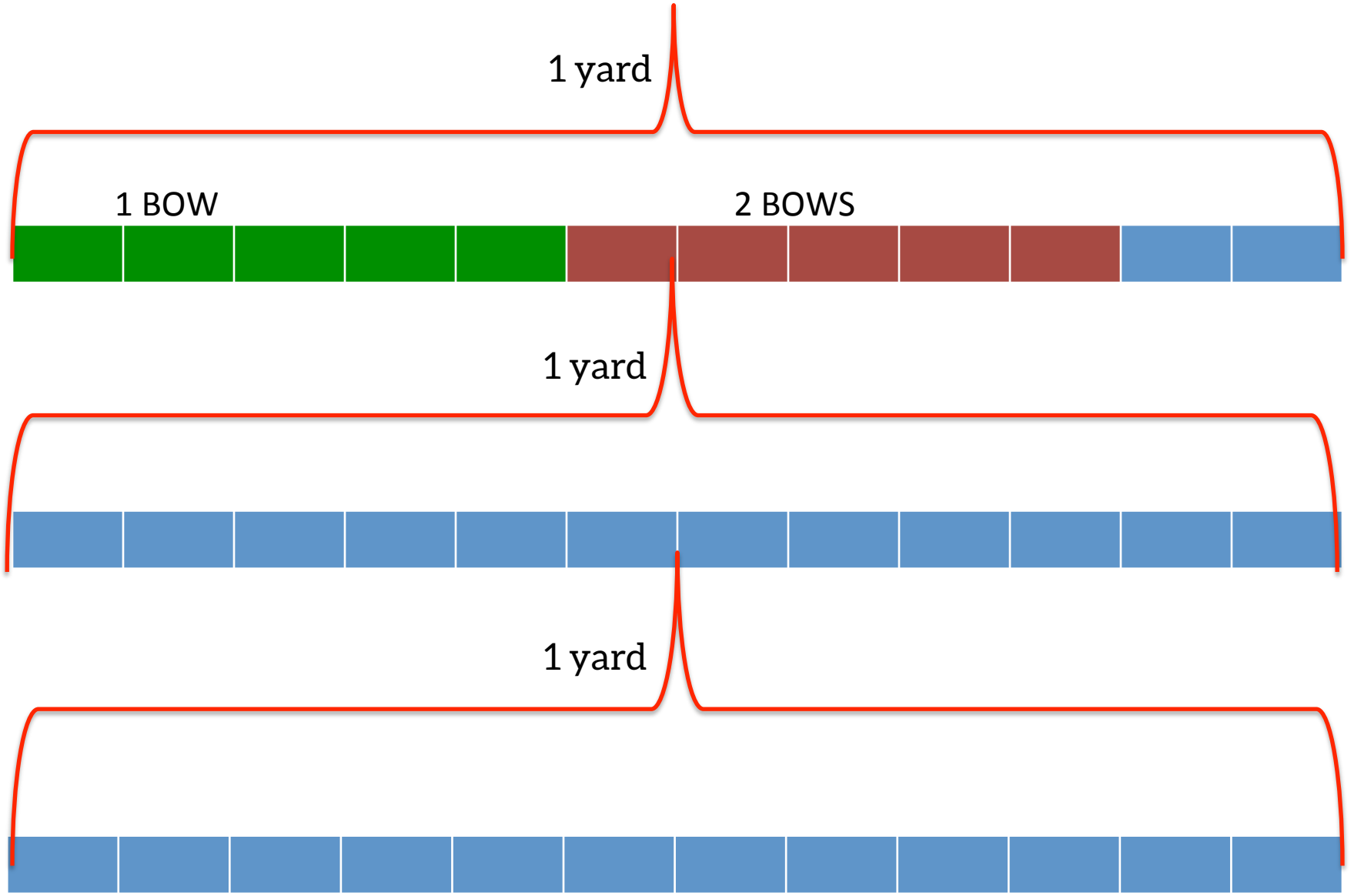
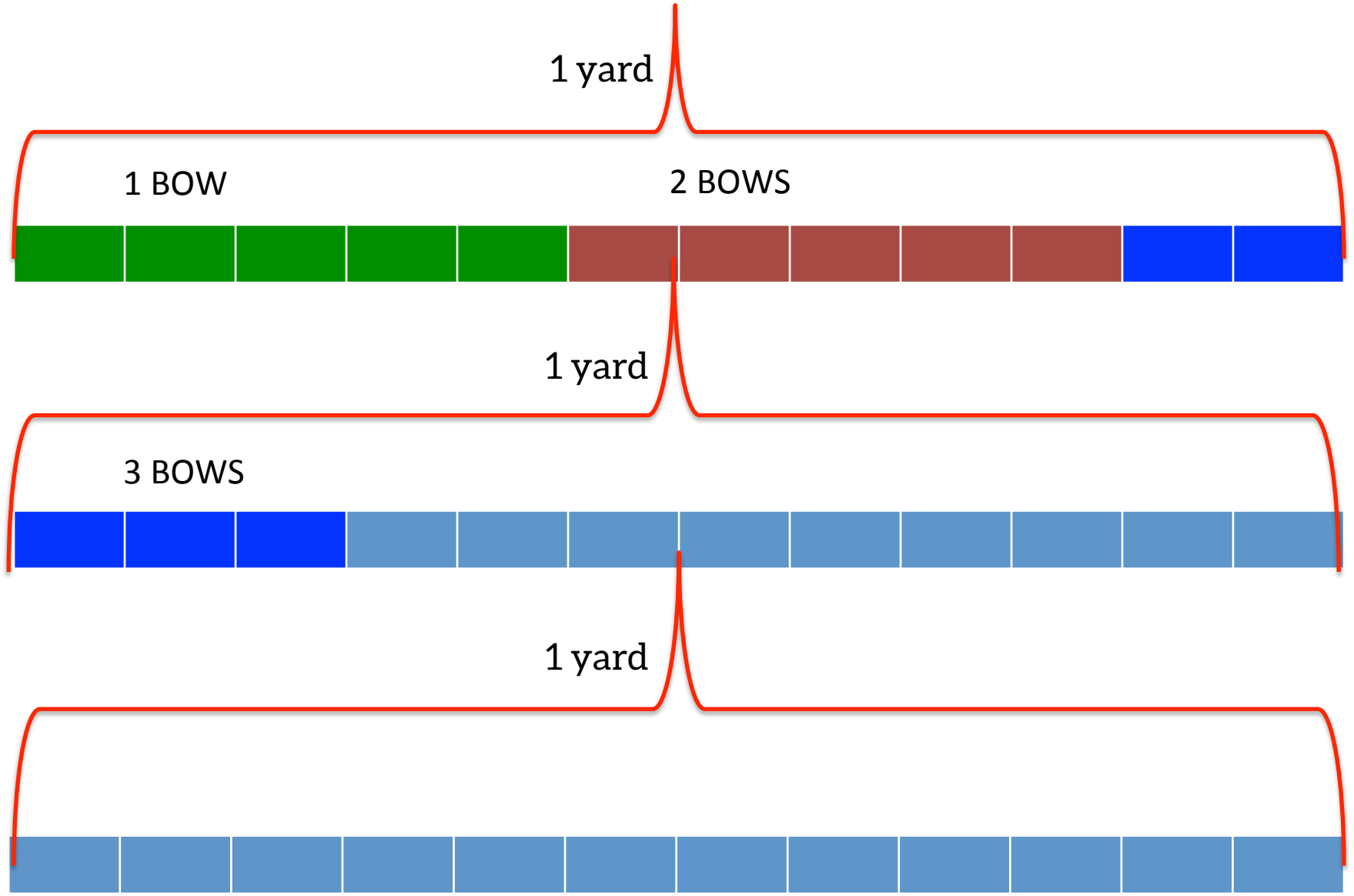
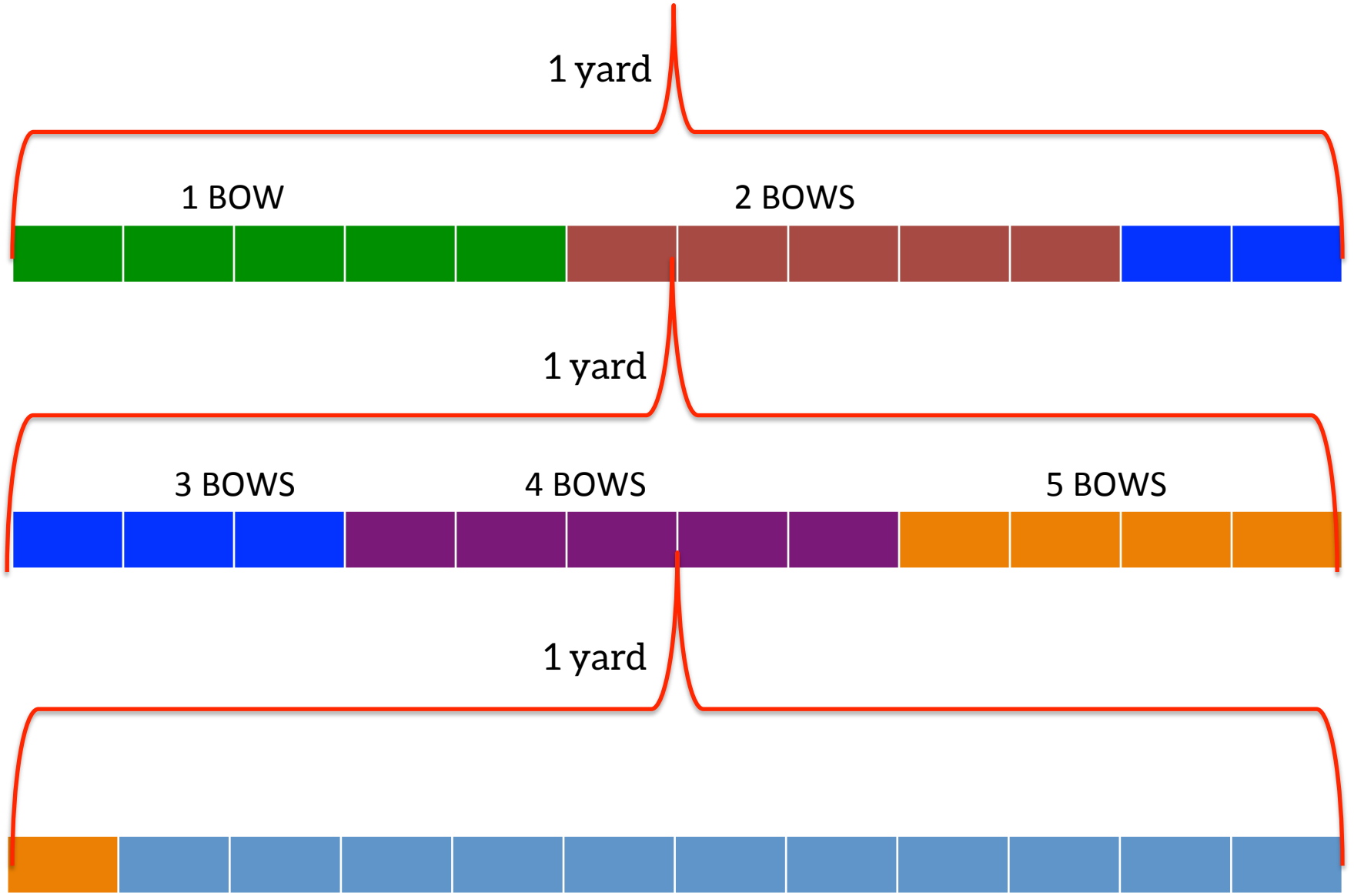

1 yard
1 BOW
1 yard
1 yard

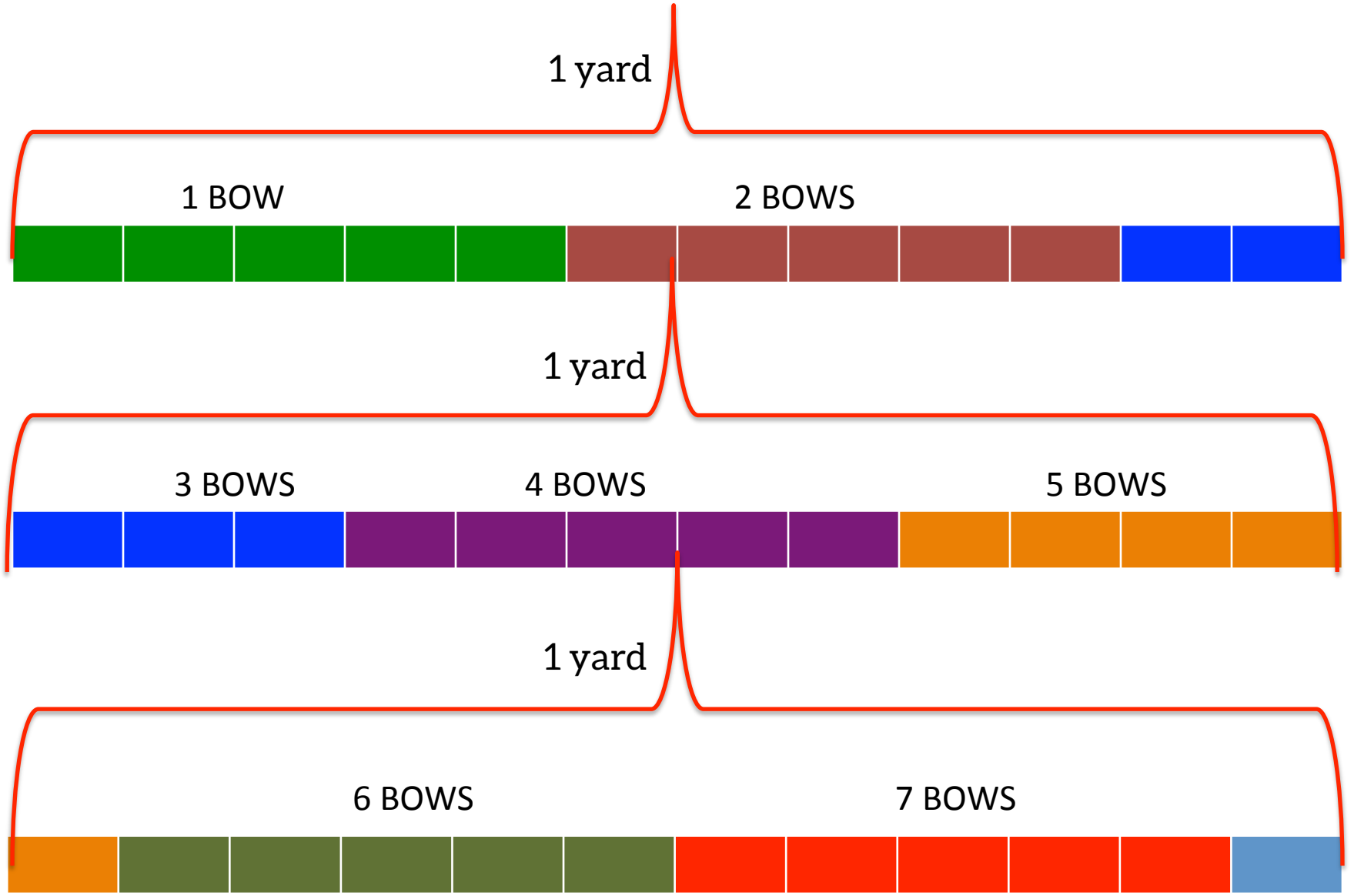

1 yard
1 BOW
2 BOWS
1 yard
1 yard

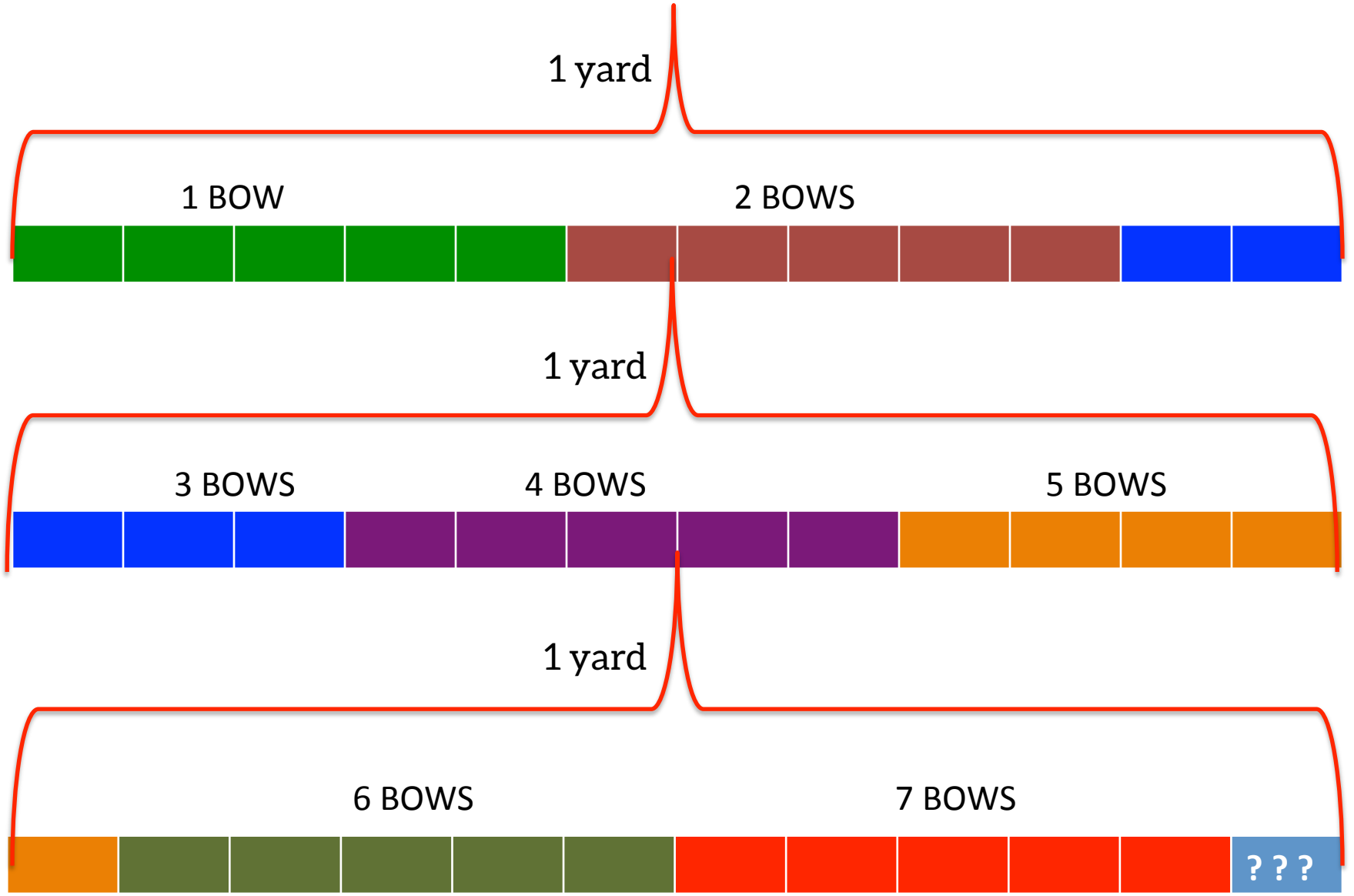

1 yard
1 BOW
2 BOWS
1 yard
3 BOWS
1 yard


1 yard
1 BOW
2 BOWS
1 yard
3 BOWS
4 BOWS
1 yard


1 yard
1 BOW
2 BOWS
1 yard
3 BOWS
4 BOWS
5 BOWS
1 yard


1 yard
1 BOW
2 BOWS
1 yard
3 BOWS
4 BOWS
5 BOWS
1 yard
6 BOWS


1 yard
1 BOW
2 BOWS
1 yard
3 BOWS
4 BOWS
5 BOWS
1 yard
6 BOWS
7 BOWS


1 yard
1 BOW
2 BOWS
1 yard
3 BOWS
4 BOWS
5 BOWS
1 yard
6 BOWS
7 BOWS
? ? ?

I thought we
were good to
move on,
but I asked if
anyone still had
questions or had
thought about
the problem
differently and …

I know each yard contains 36 inches.
I have 3 yards so that's 108 inches total.
1 bow is 5/12 of a yard. 5/12 times 36 = 15 inches

Number of bows	Inches remaining
0	108 (subtract 15)
1	93 (subtract 15)
2	78 (subtract 15)
3	63 (subtract 15)
4	48 (subtract 15)
5	33 (subtract 15)
6	18 (subtract 15)
7	3 inches leftover

What is the meaning of the 3?
How does the 3 relate to the 1/12 answer?
How does the 3 relate to the 1/5 answer?

What is the meaning of the 3?
How does the 3 relate to the 1/12 answer?
How does the 3 relate to the 1/5 answer?

I've got 1/5 of a bow.

That's 1/12 of a yard.

We had 3 inches left.

What can we learn from this story?

Honor student thinking by believing that they have logical ideas and that they are trying to make sense of the mathematics. Then **ASK** about their thinking.

Recognize that students don't all think the same way and that's ok.

Students can think differently from their neighbor and the teacher and still be correct.

The students and teacher can learn from each other.

What kind of questions made this happen?

1. Agree or disagree?
2. Talk to me about this. Tell me more.
3. Did anyone draw a diagram?
4. Did anyone think about the problem differently or approach it in a different way?
5. What do the numbers mean?
6. How can the units help us think about the problem?

Moving 8 years forward to the present:

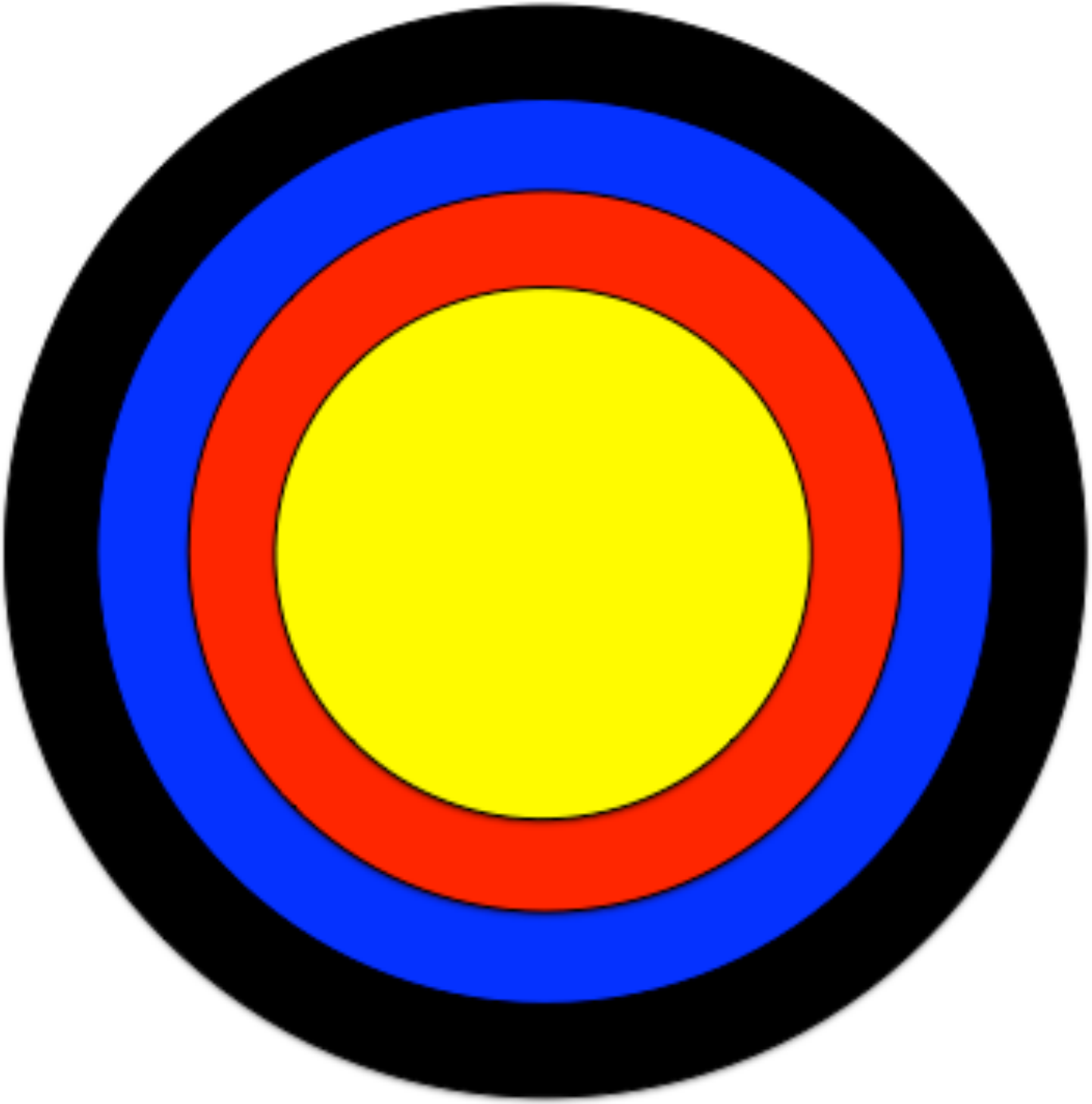
A lot has happened to redefine mathematics teaching and learning.

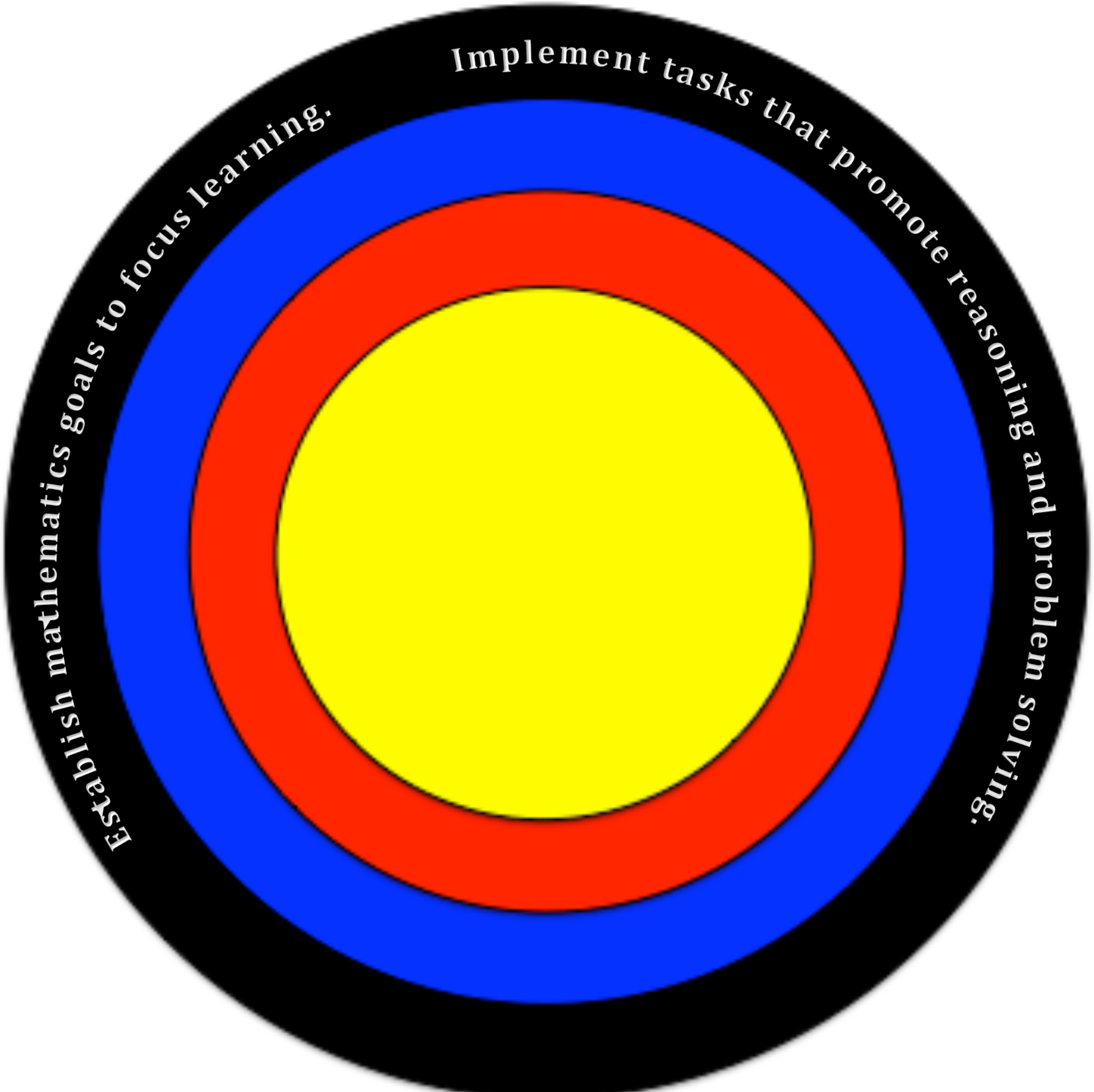
Mathematically proficient students:

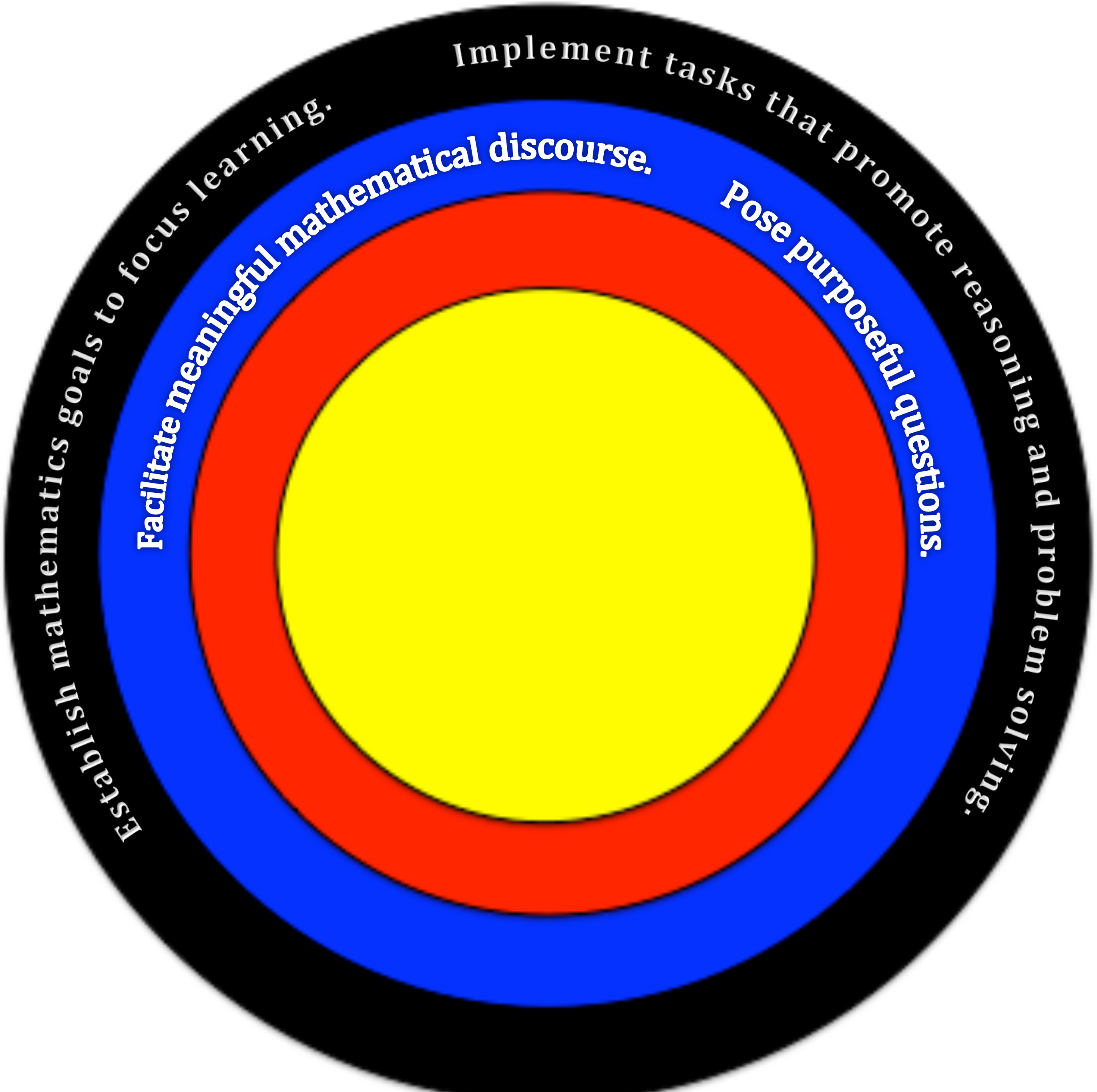
- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

The 8 Effective Teaching Practices (NCTM 2014)

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.




Establish mathematics goals to focus learning.
Implement tasks that promote reasoning and problem solving.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Facilitate meaningful mathematical discourse.
Pose purposeful questions.

Teacher's Role in Supporting Student Understanding

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- **Facilitate meaningful mathematical discourse.**

  Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

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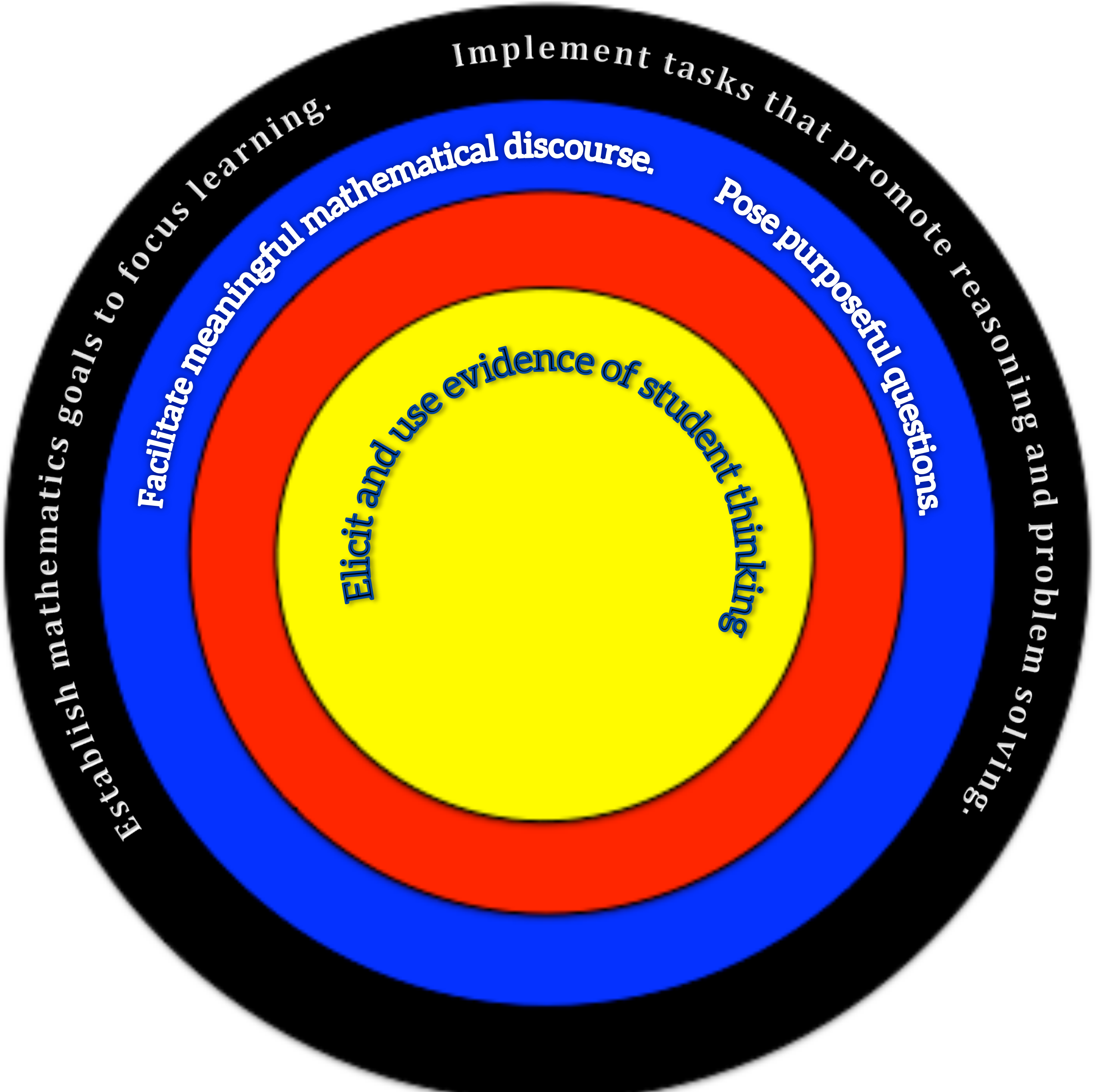

Implement tasks that promote reasoning and problem solving.
Establish mathematics goals to focus learning.
Facilitate meaningful mathematical discourse.
Pose purposeful questions.
Elicit and use evidence of student thinking

Why is sharing student work so important?

Elicit and use evidence of student thinking.

- Student work is student thinking made visible.
- Student work gives students something to talk about.
- Discourse should provide opportunities for students to think "out loud."
- Discourse should be conversations for learning.

3.1 Getting Ready for a Pool Party

A Develop Understanding Task

Sylvia has a small pool full of water that needs to be emptied and cleaned, then refilled for a pool party. During the process of getting the pool ready, Sylvia did all of the following activities, each during a different time interval.

3.1 Getting Ready for a Pool Party

A Develop Understanding Task

1. Sketch a possible graph showing the height of the water level in the pool over time. Be sure to include all of activities Sylvia did to prepare the pool for the party. Remember that only one activity happened at a time. Think carefully about how each section of your graph will look, labeling where each activity occurs.

Removed water with a single bucket	**Filled the pool with a hose (same rate as emptying pool)**
Drained water with a hose (same rate as filling pool)	**Cleaned the empty pool**
Sylvia and her two friends removed water with three buckets	**Took a break**

When looking at student work, ask yourself:

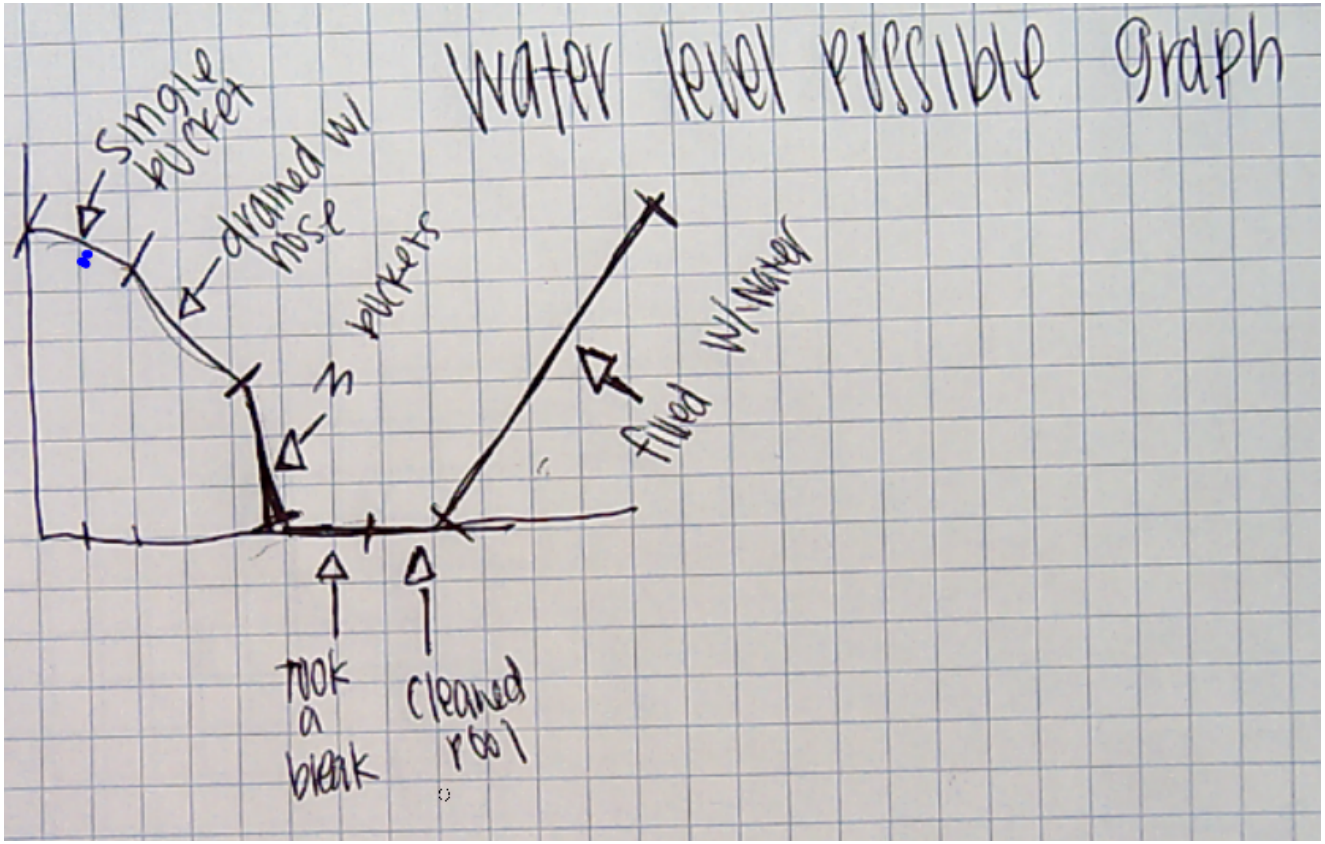
What is right about this work?

What does the student know based on this work?

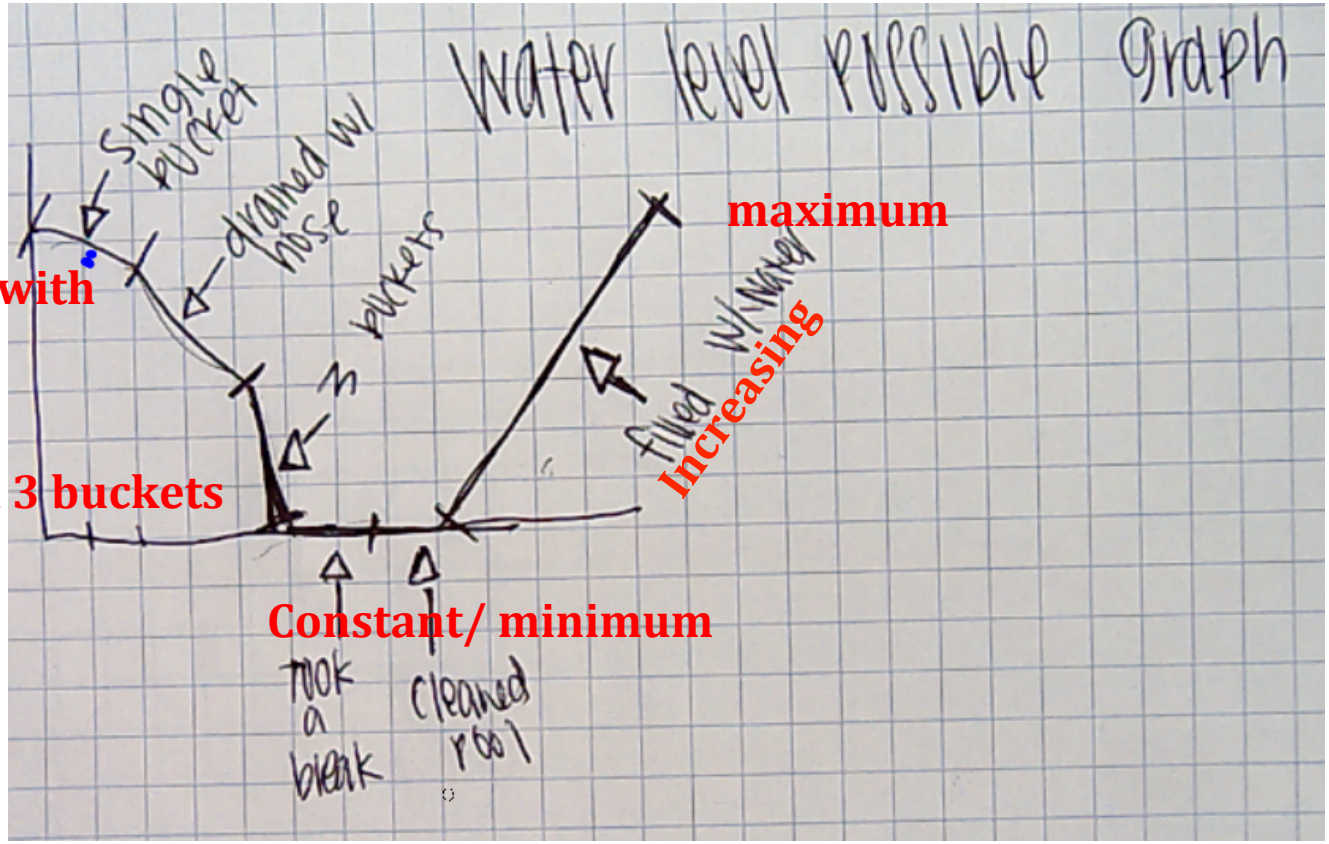
What would I like the student to tell me about his/her work?

What about this work gives us opportunities to discuss the mathematics of the purpose of the lesson?

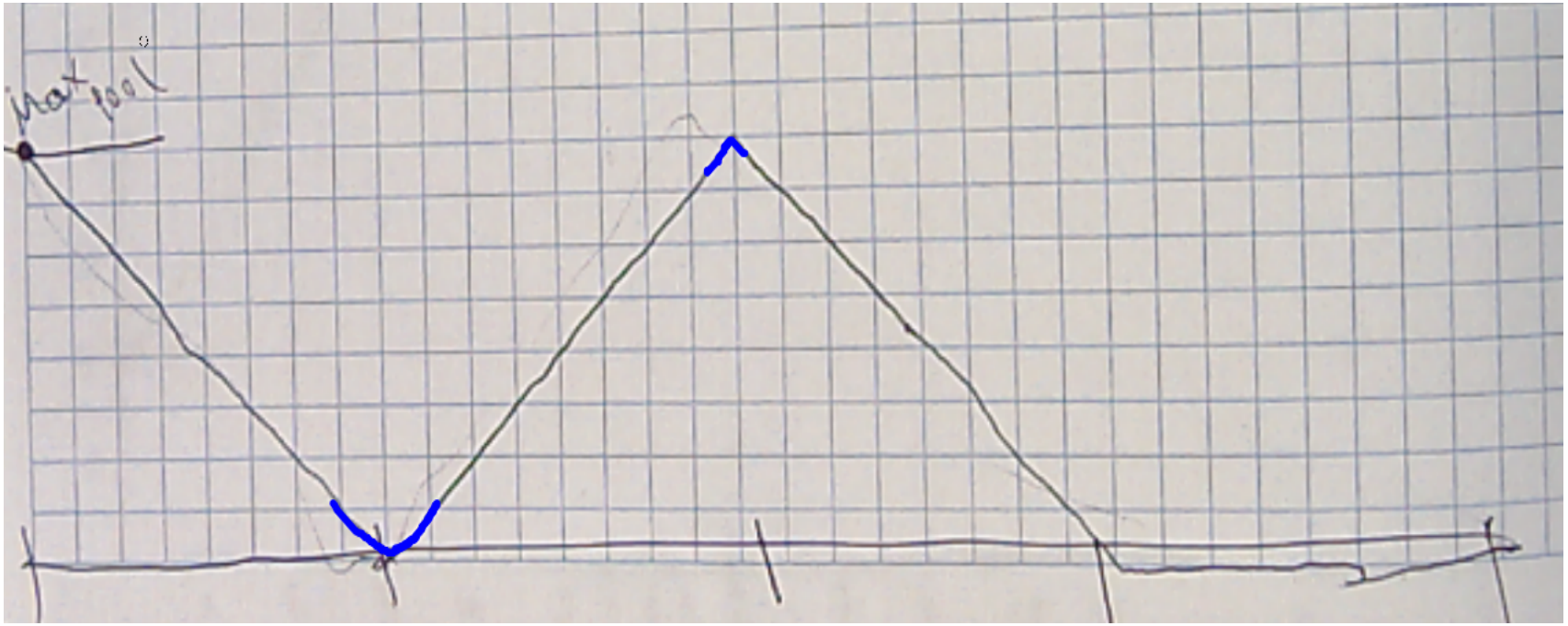
The goal of this task is to surface the vocabulary that connects to the graph of a function.



What would you want to talk about in this graph?



What is right about this student work?
What would you like to ask this student?



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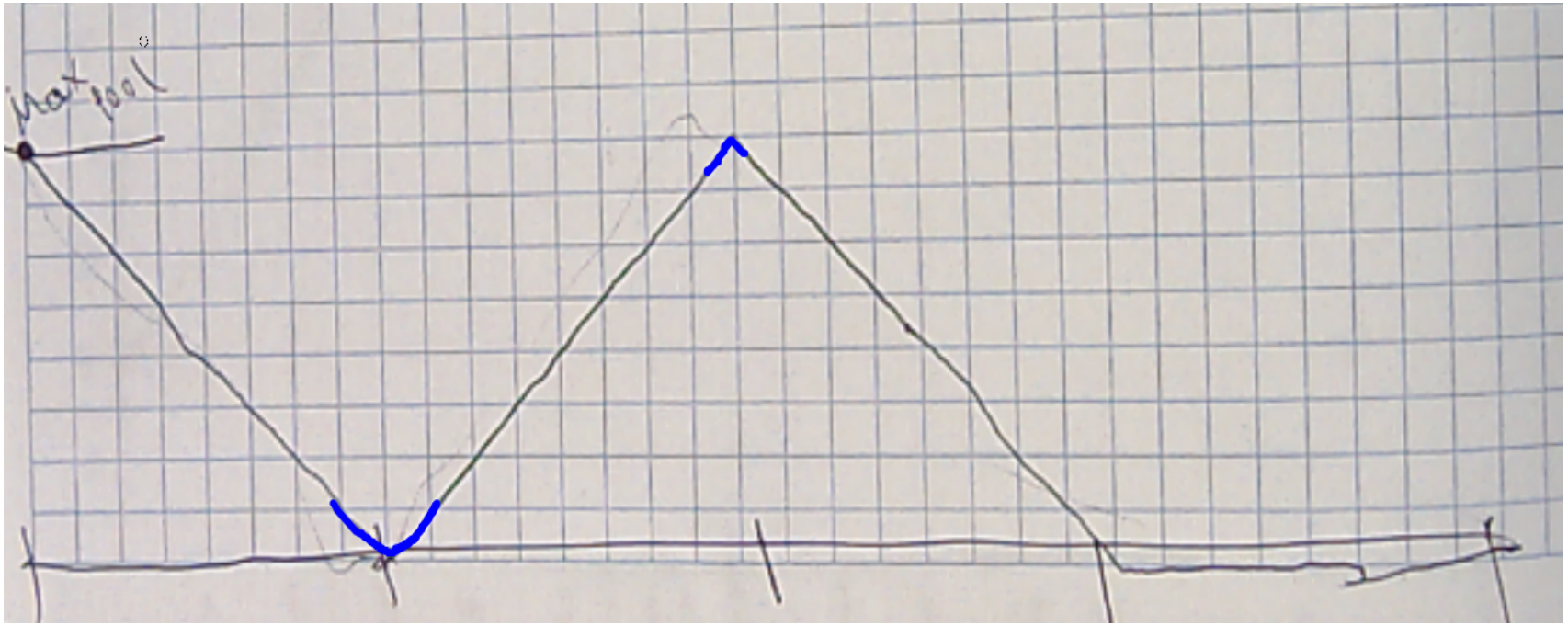
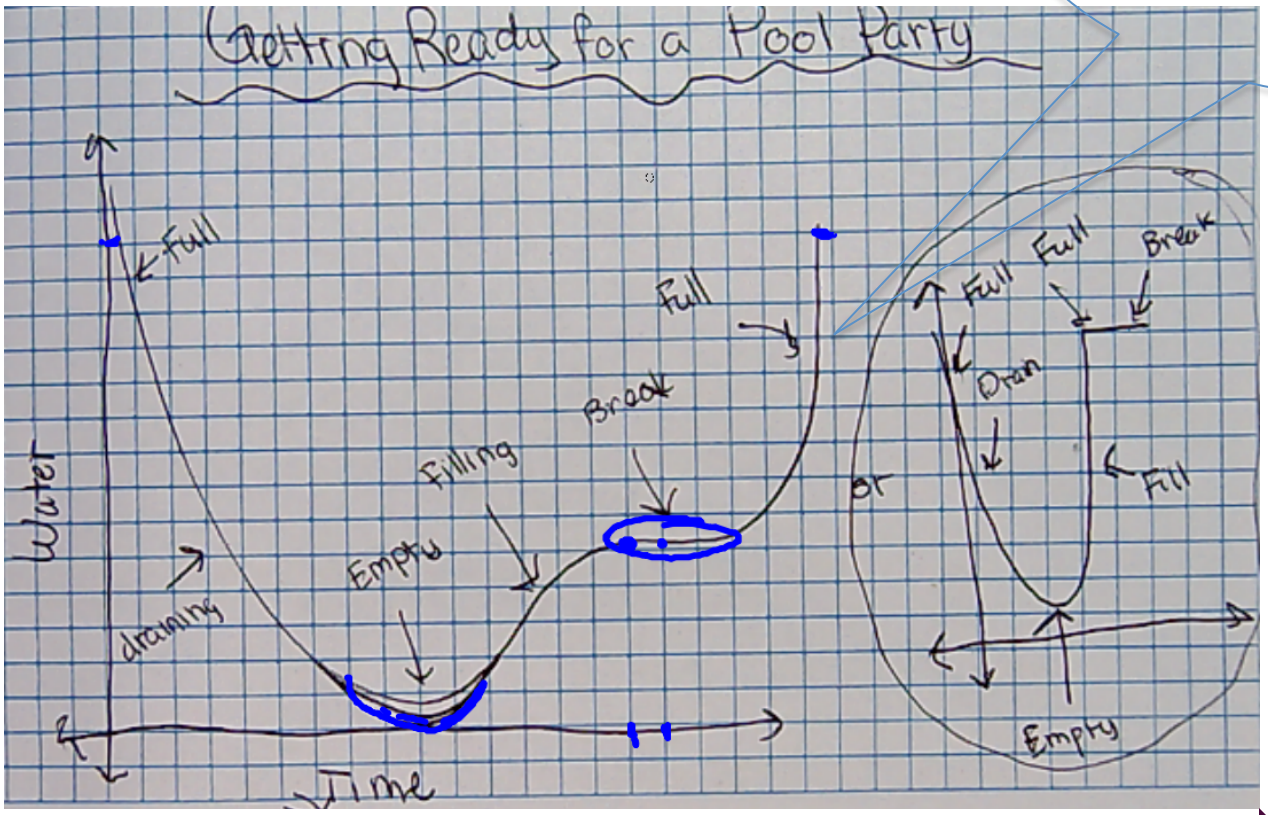


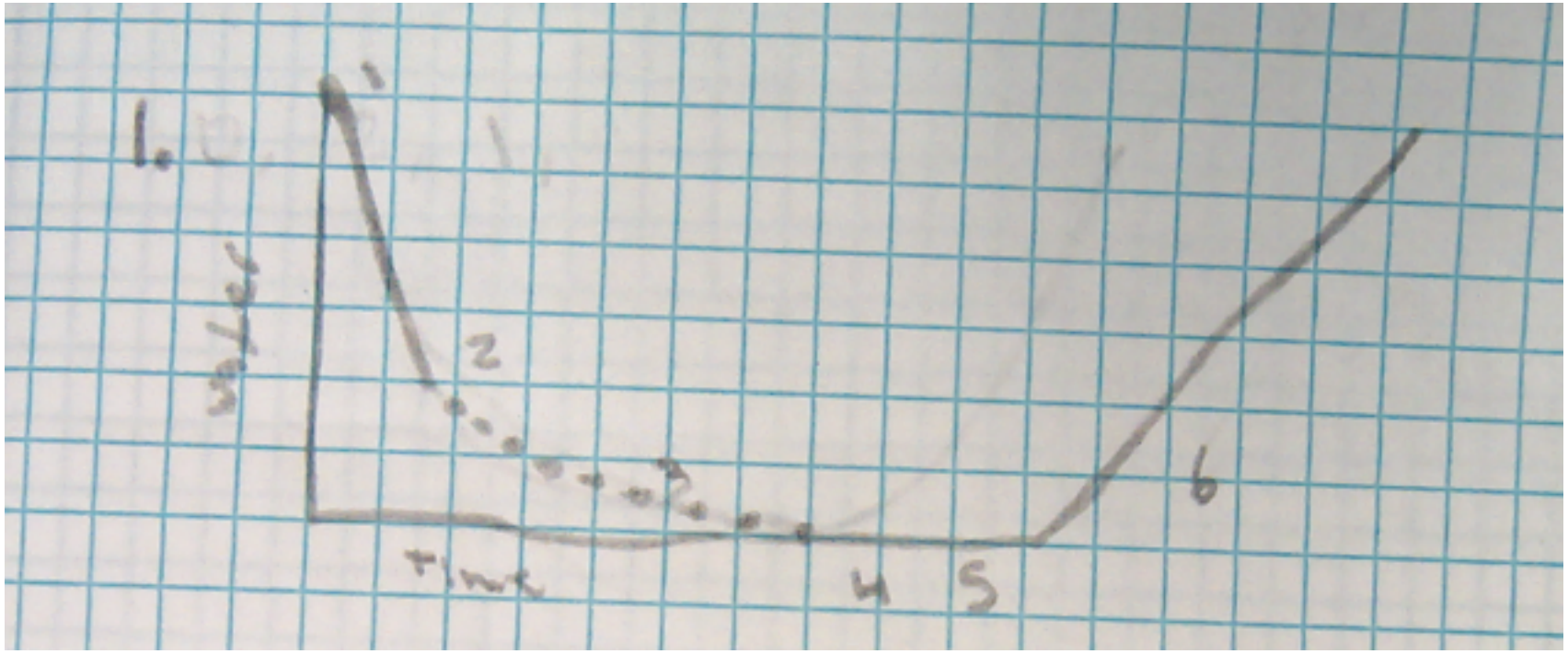
A slope of 0 does not mean the graph is on the y = 0 line.
Is there a story that could go with this graph?

Tell me about your graph.

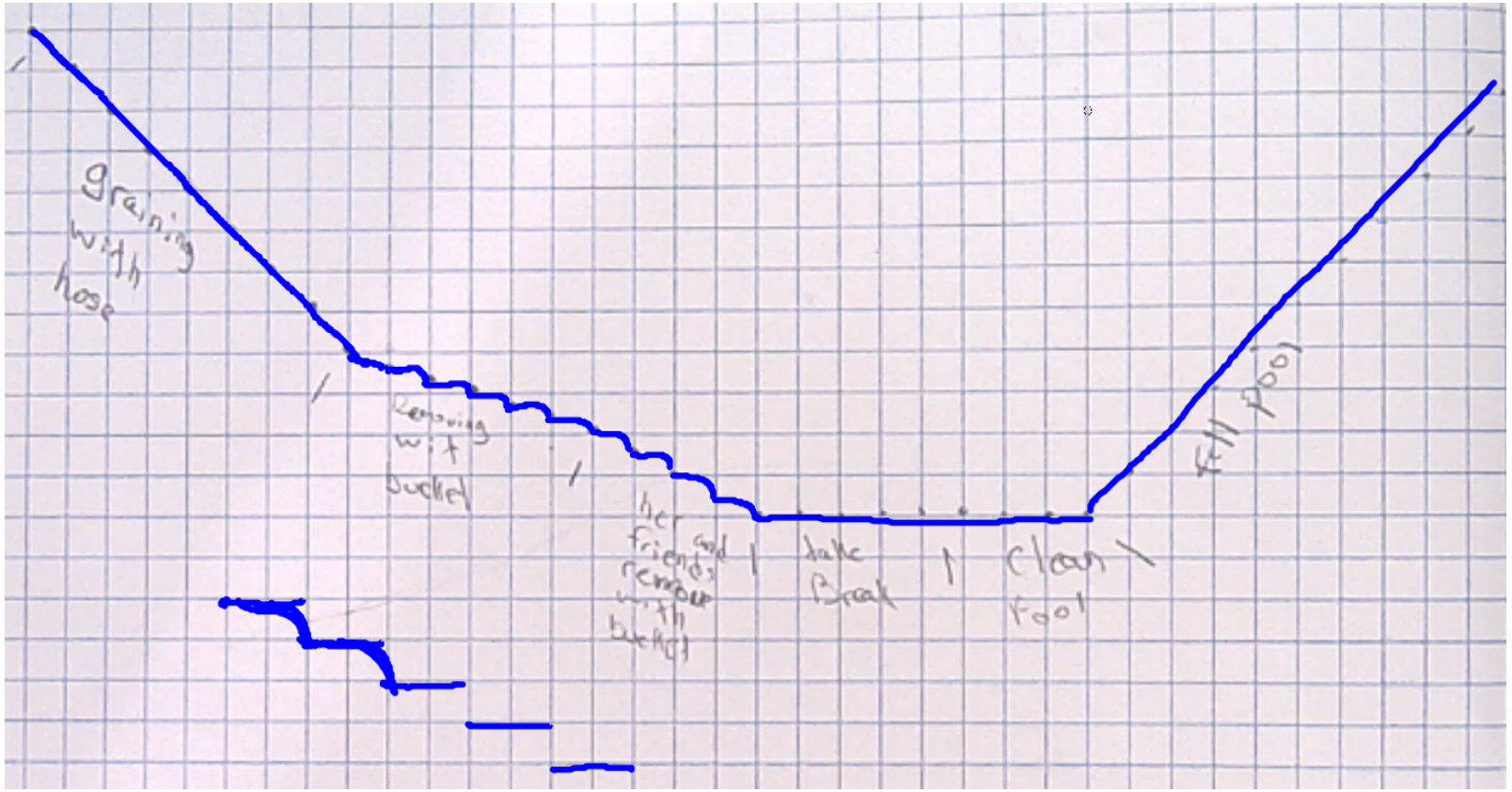
My graph shows the shallow end and the deep end of the pool.

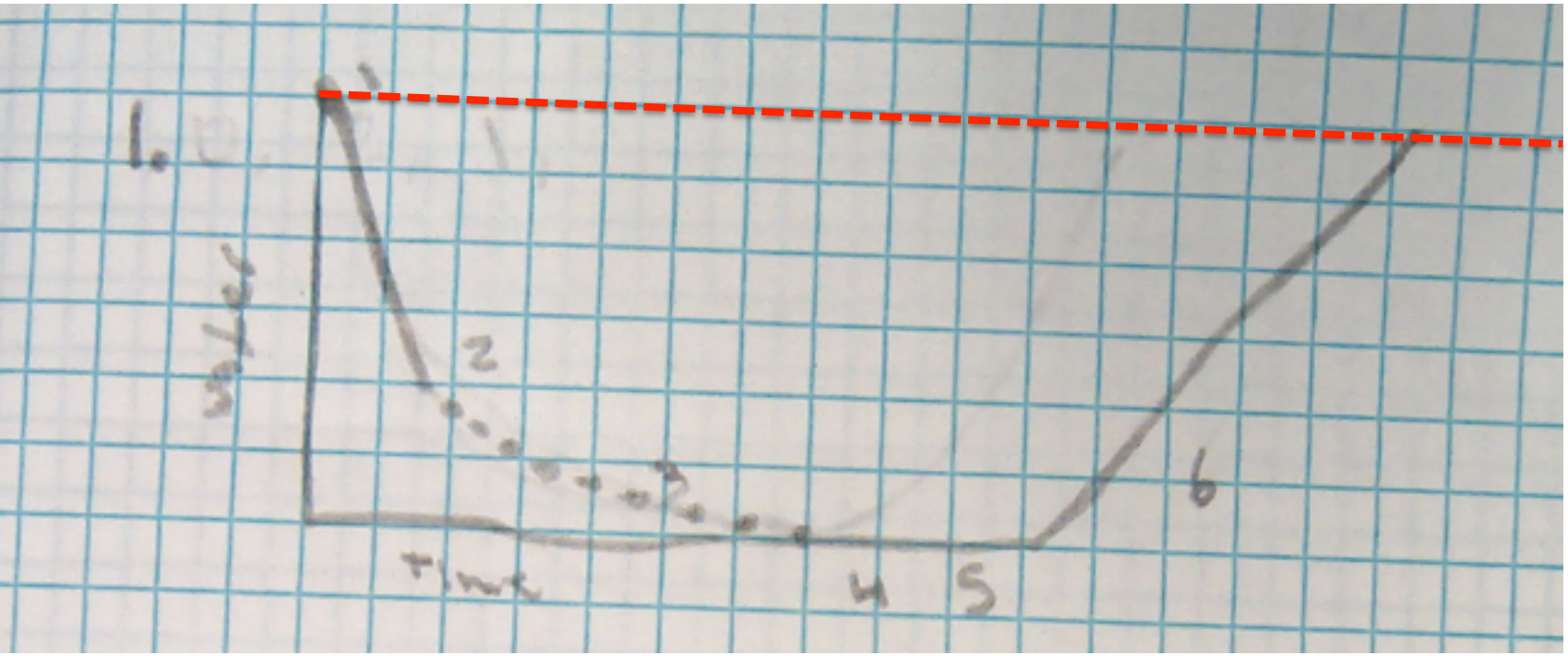


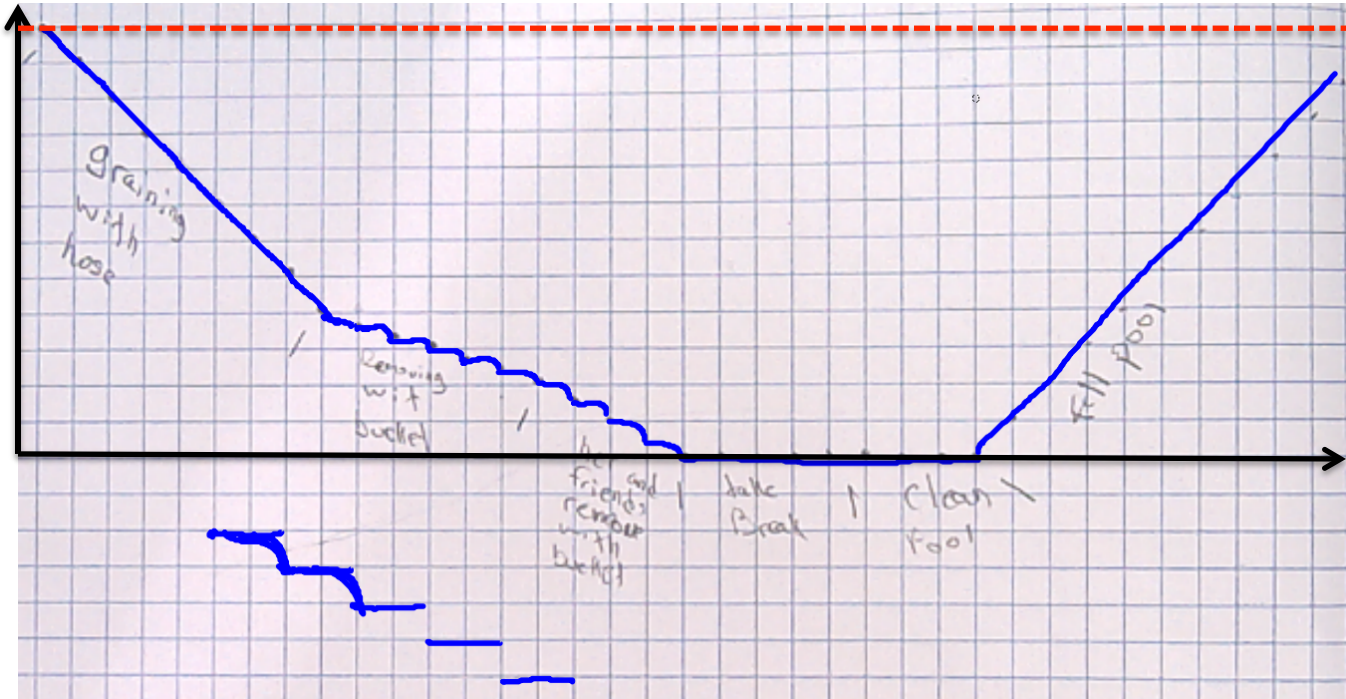
What do you think the student is thinking about in this graph?



What would help this graph communicate its story better?

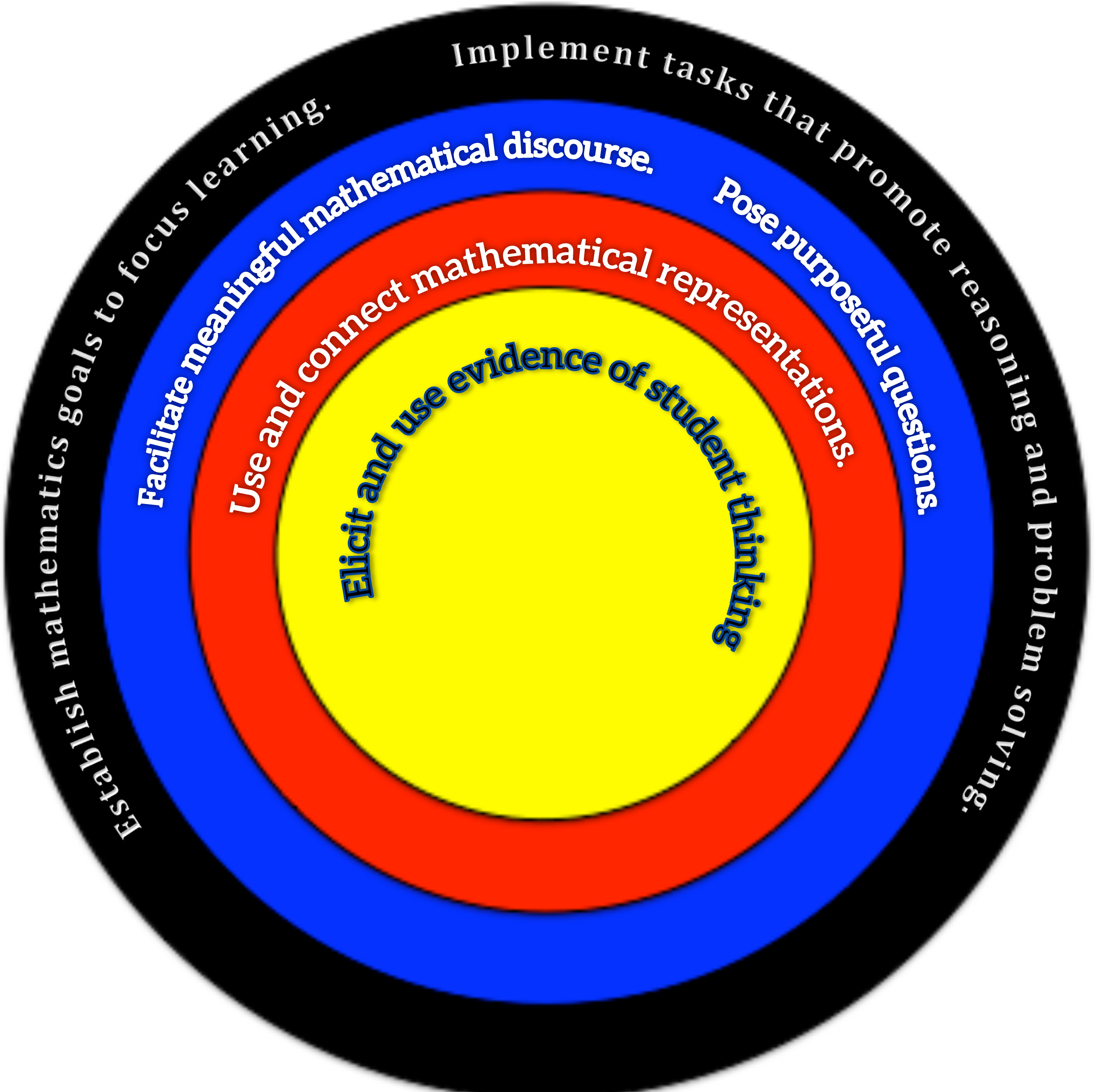



water
time
1
2
3
4
5
6


graining with hose
Cleaning with bucket
Fill pool
clean pool

Learning from each other's work.

- What can I learn from my classmate's representation?
- Do I need to rethink my interpretation?
- Do I need to add something to my graph?
- Are there math vocabulary words that I need to add to my notes?
- Do I understand everything that we talked about in class?



Implement tasks that promote reasoning and problem solving.
Establish mathematics goals to focus learning.
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Pose purposeful questions.
Use and connect mathematical representations.
Elicit and use evidence of student thinking

What is the power of multiple representations?

- **Use and connect mathematical representations.** Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

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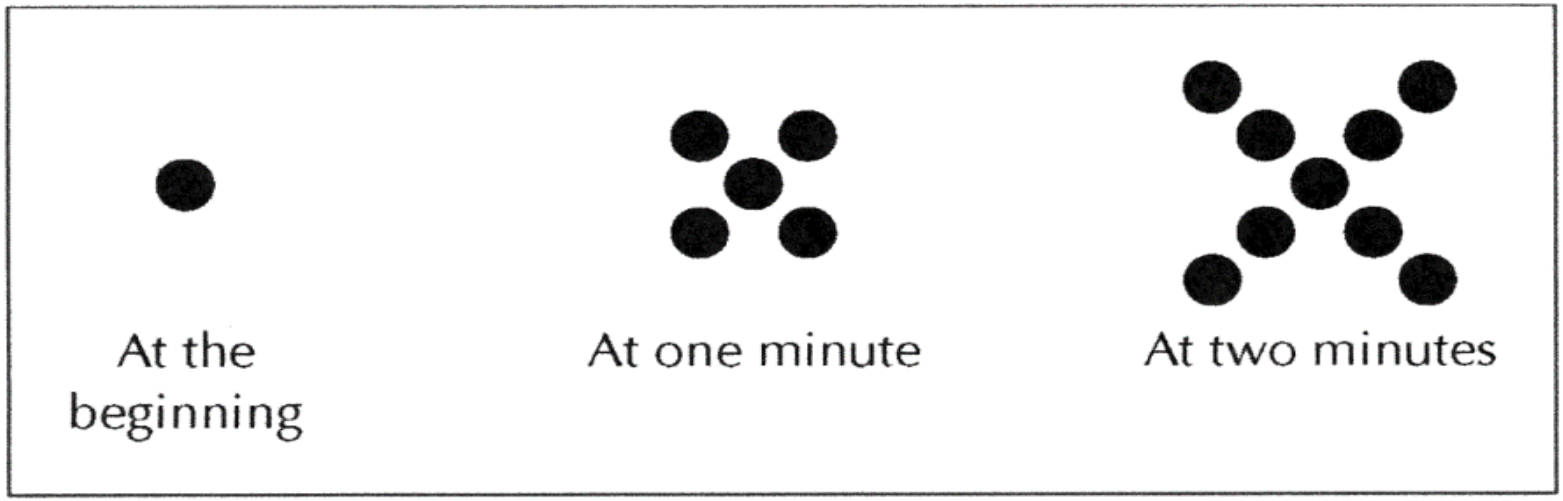
What is the power of multiple representations?

- **Use and connect mathematical representations**. Effective teaching of mathematics engages students in making connections among mathematical representations **to deepen understanding of mathematics concepts and procedures** and **as tools for problem solving – and it gives students a direction to take to promote productive struggle.**

What kind of representations would we want students to create?

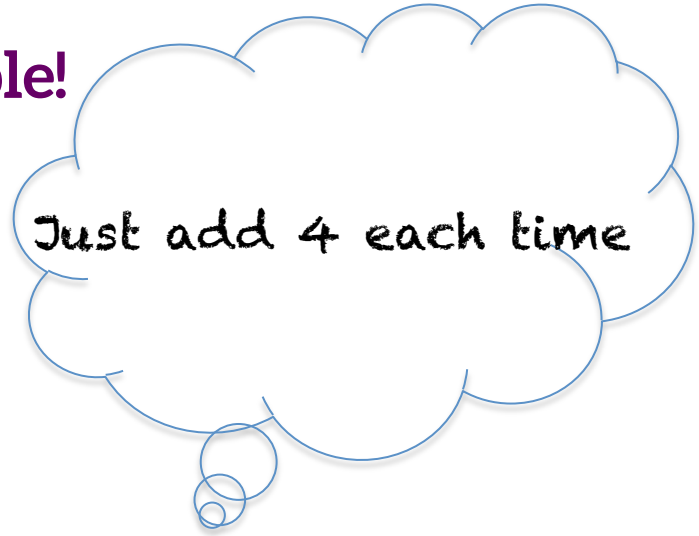
- Use words to describe the situation
- Table
- Graph
- Equations – both explicit and recursive
- Notation
- Diagram
- Picture

1.2 Growing Dots



1. Describe the pattern that you see in the sequence of figures above.
2. Assuming the pattern continues in the same way, how many dots are there at 3 minutes?
3. How many dots are there at 100 minutes?
4. How many dots are there at t minutes?

Students make their thinking visible!



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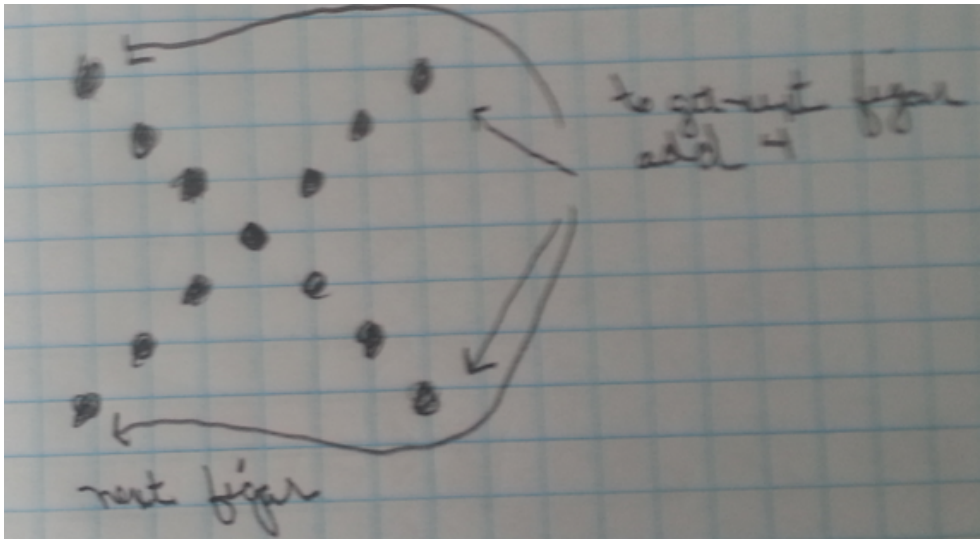
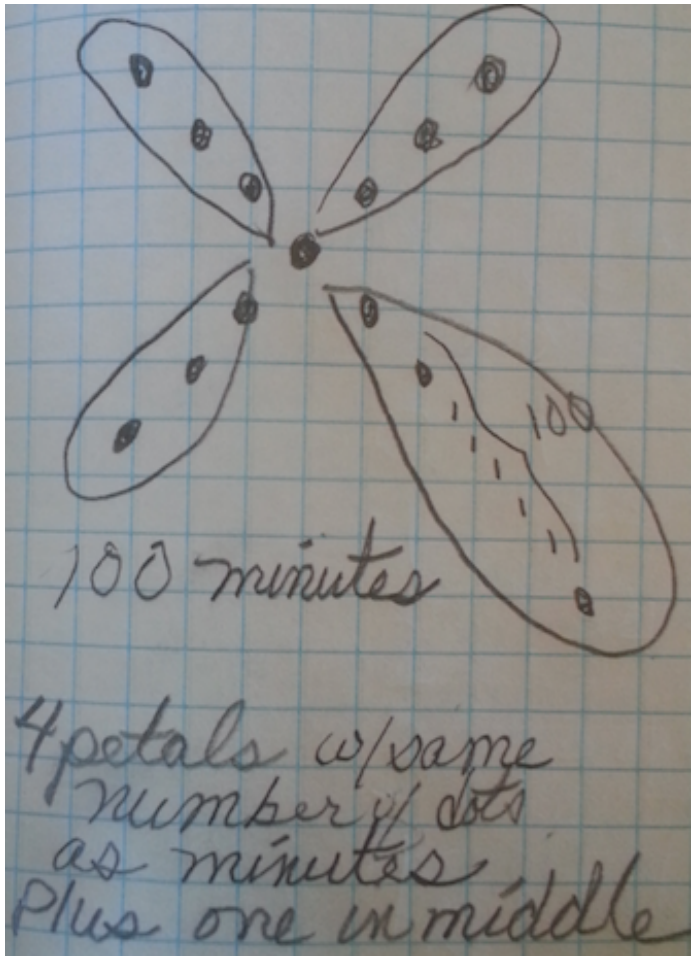
2. Assuming the pattern continues in the same way, how many dots are there at 3 minutes?



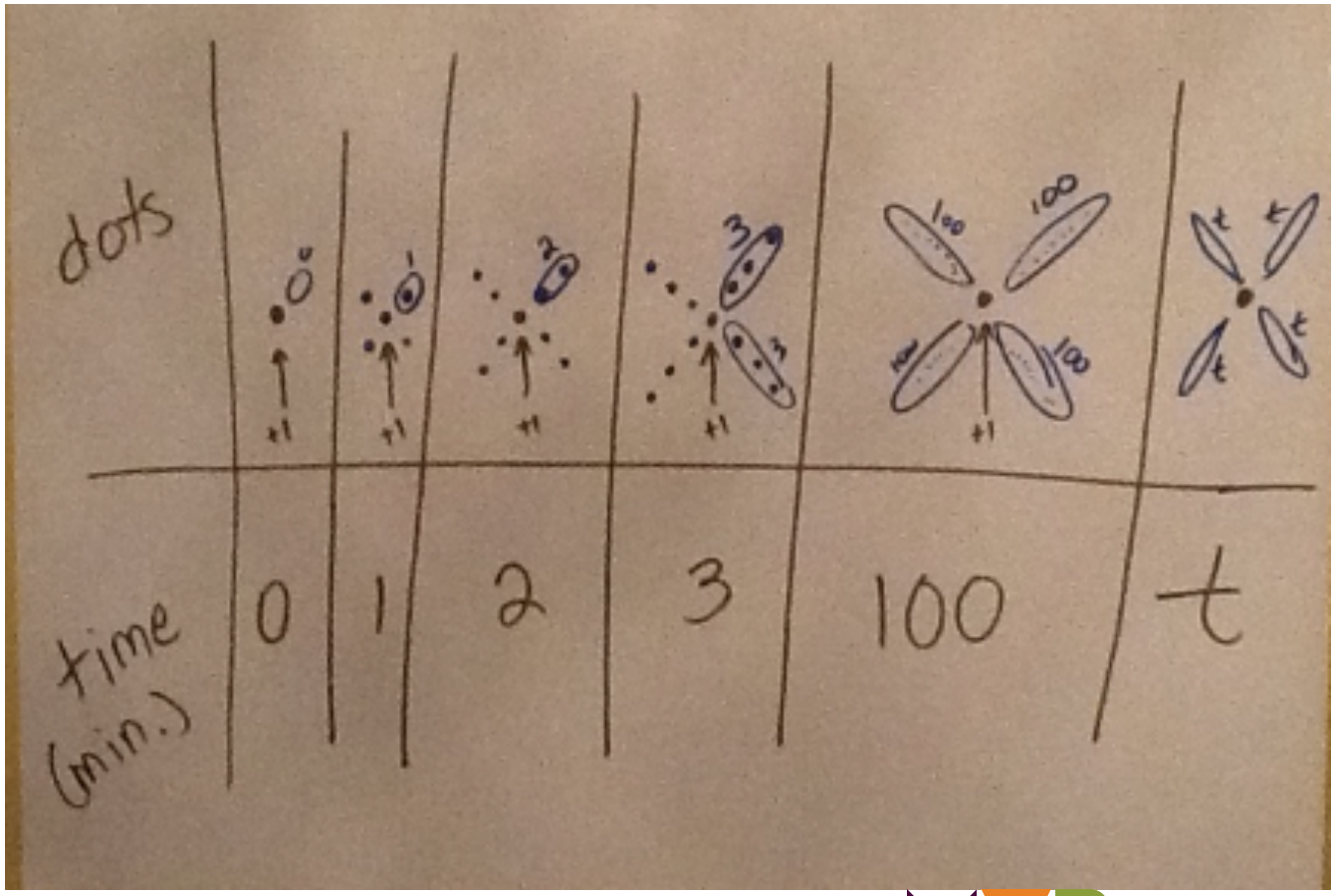
What did you notice and what did this make you think?



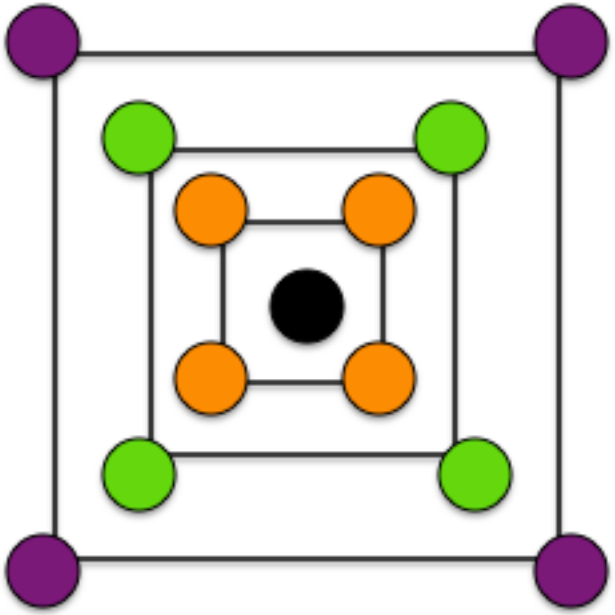
- I saw the pattern as a flower.
- There are always 4 petals.
- The number of dots on the petals is changing.
- The number of dots matches the time.
- Plus, there is always 1 dot in the center.
- At 100 min. I have 401 dots.

Deriving my equation from the table.

$D = 4(t) + 1$



I didn't see a flower. I saw boxes growing out from the center.



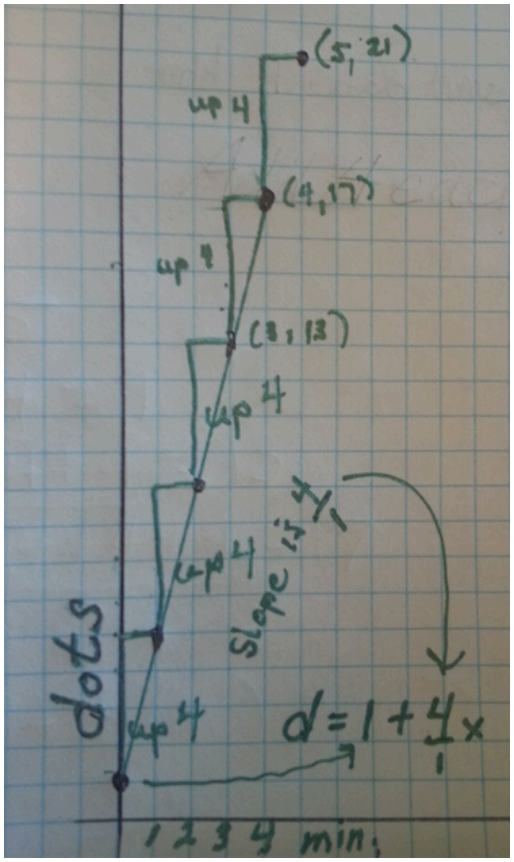
- I started with 1 dot in the center.
- Each minute I got a new box around the previous figure.
- There are always 4 dots, one in each corner of the box.
- The number of boxes is changing.
- The number of boxes matches the time.
- **Next = previous figure + 4**
- **D = 1 + *b*(4)**

What can be learned from a table or a graph?

time	# dots
0	1
1	5
2	9
3	13
4	17

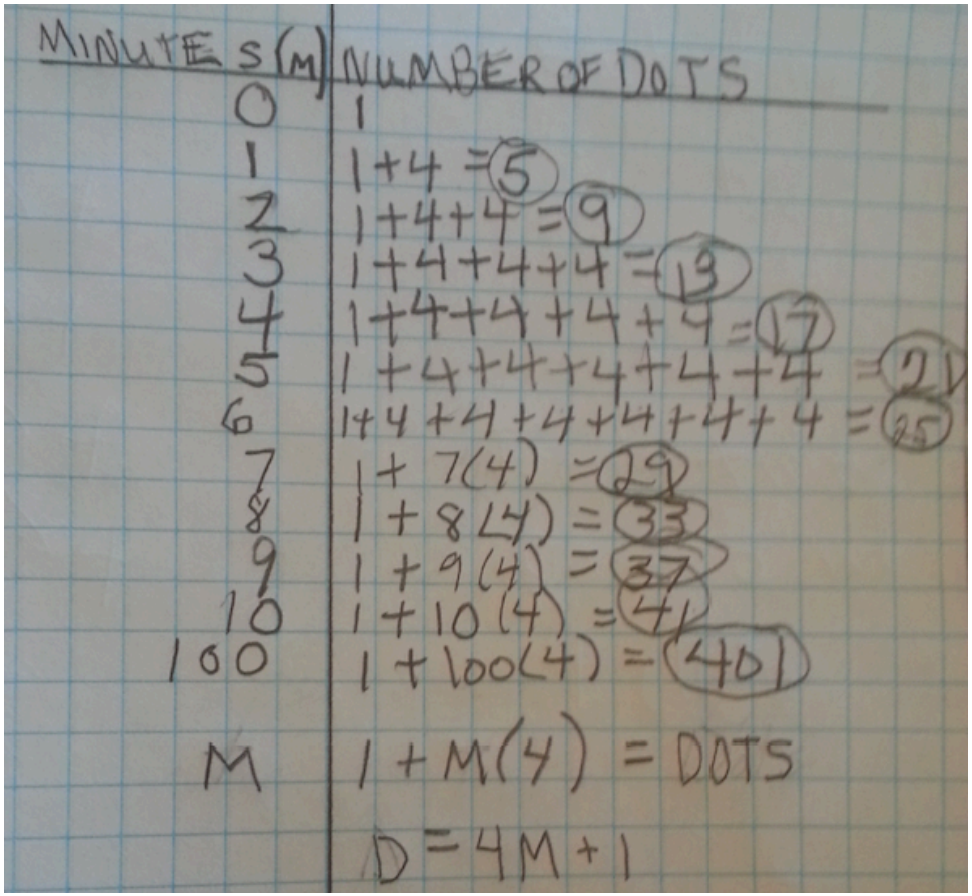
+4 +4 +4 +4

The table shows that I add 4. But I don't see why that's multiplication in the equation.



I can count that the slope is 4/1. I just know that you multiply slope in the equation $y=mx+b$.

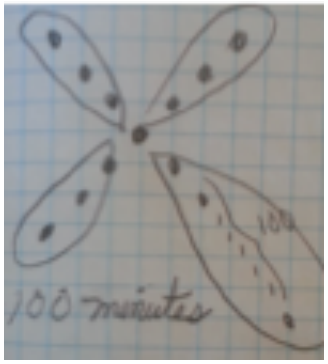
Could I view the numbers in the table in a different way?

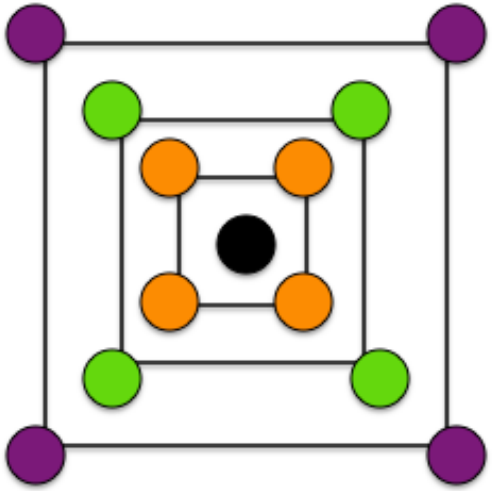


I started with 1. Then I added 4 each time. The number of 4's I add at each minute matches the number of minutes, M in my equation.

<u>Repeated addition is multiplication.</u>

Connecting the equation to the equation.





$D = 4(t) + 1$

$D = 1 + t(4)$

Does *t* represent the same thing in each of the equations?

Connecting the type of equation to the table.

Time (minutes)	Number of dots
0 →	1
1 →	5
2 →	9
3 →	13
4 →	17
5 →	21
6 →	25

Time (minutes)	Number of dots	
0	1	
1	5	Add 4
2	9	
3	13	Add 4
4	17	Add 4
5	21	
6	25	

D = 4*t* + 1

Next = previous + 4

The student work uses the number 4 in many ways.

- Just add 4 each time.
- It's the number of petals in the flower.
- It's the number of dots at the corner of the boxes.
- It's the slope in the graph.
- It's +4 looking down the table.
- It's multiply by 4 in the explicit equation.
- It's plus 4 in the recursive equation.

Questions you can ask to get students started.

- What do you notice? What does that make you think?
- Can you express your observations in words?
- What is the same? What is different?
- Did you try making a table, a graph, or diagram to help you think about the problem?

Questions you can ask when students are sharing.

- Tell us about your work.
- How did you see the pattern changing?
- What did you see in the diagram?

Sentence frames lead students to making connections and help them voice their thinking.

- I used _____ to represent _____ in the problem because _____

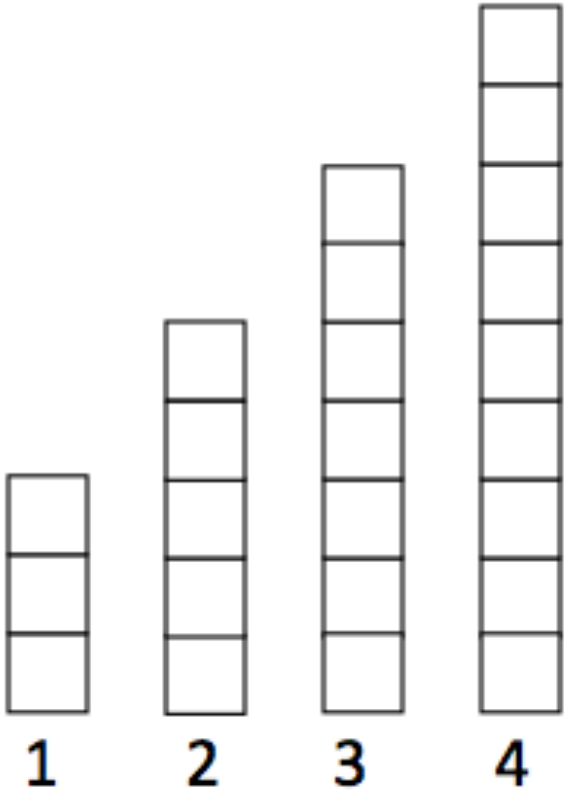
- The connection I see between the _____(representation) and the _____(representation) is _____

- In my equation, _____ means _____
- In my graph, _____ means _____
- In my picture _____ means _____
- In the story _____ means _____
- In my table _____ means _____

- There is a relationship between _____ and _____ because _____

1.4 Scott's Workout

Scott has decided to add push-ups to his daily exercise routine. He is keeping track of the number of push-ups he completes each day in the bar graph, with day one showing he completed three push-ups. After four days, Scott is certain he can continue this pattern of increasing the number of push-ups he completes each day.



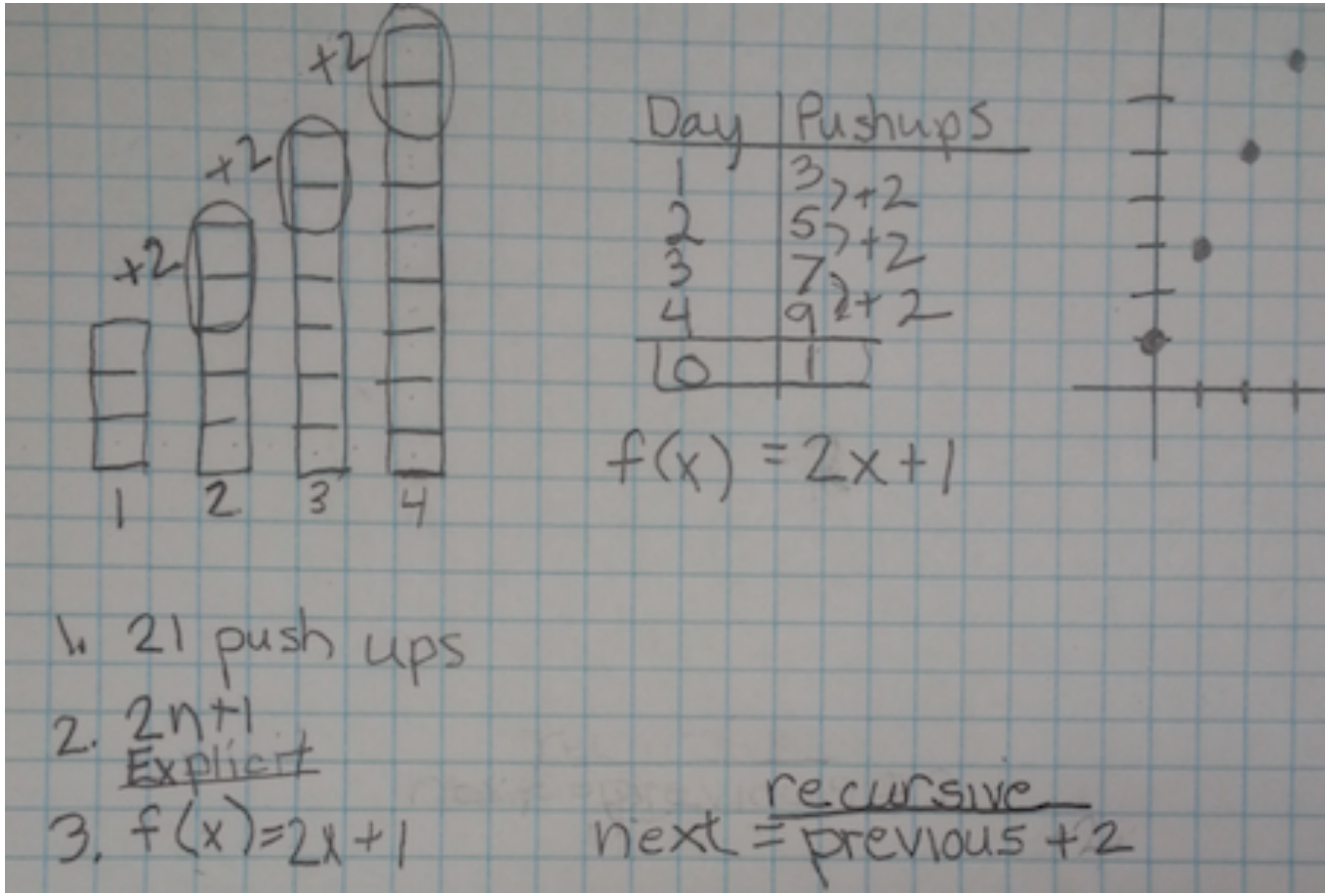
1.4 Scott's Workout

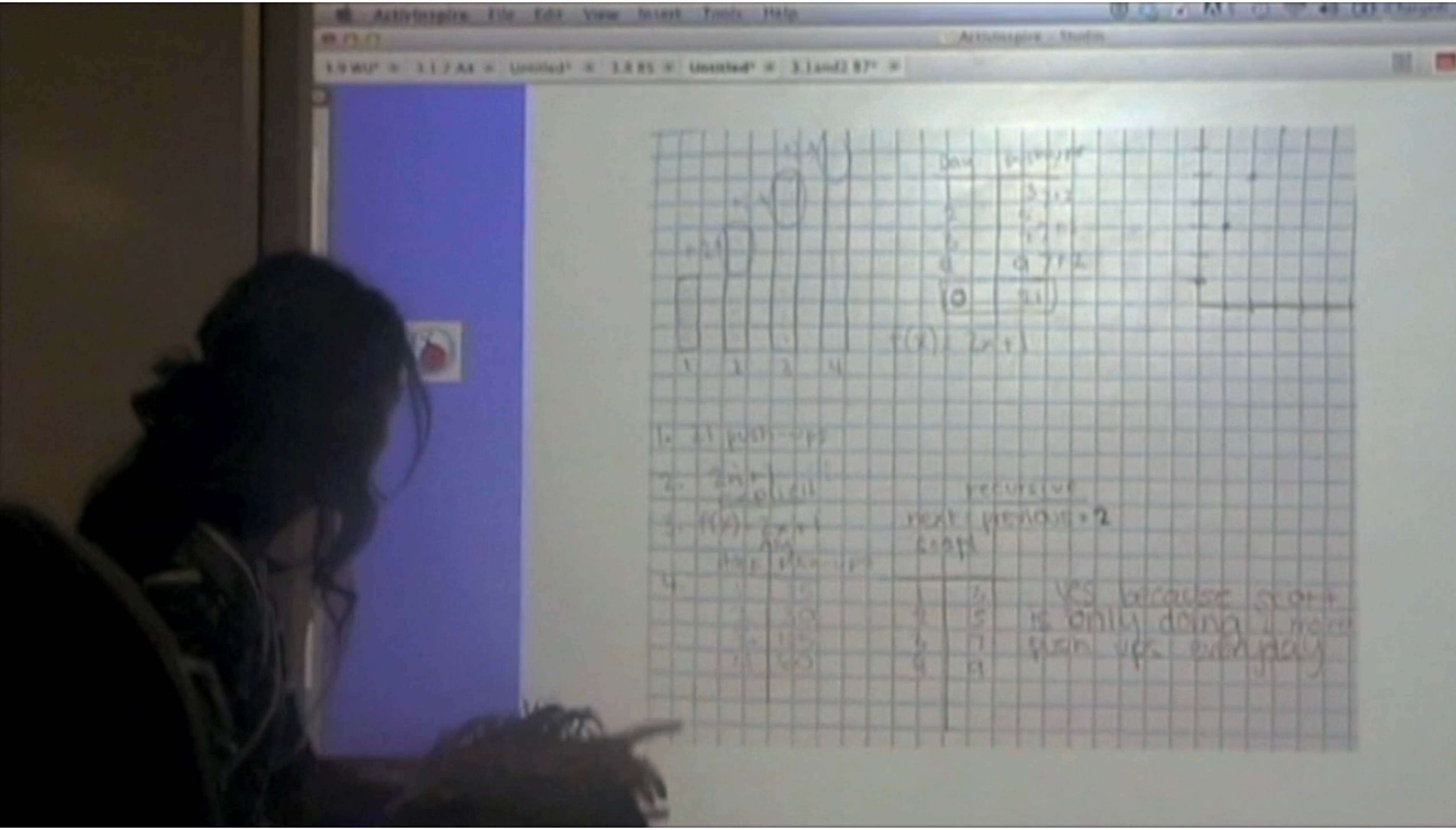
1. How many push-ups will Scott do on day 10?

2. How many push-ups will Scott do on day n?

3. Model the number of push-ups Scott will complete on any given day.
   Include both explicit and recursive equations.

1.4 Scott's Workout Student Work



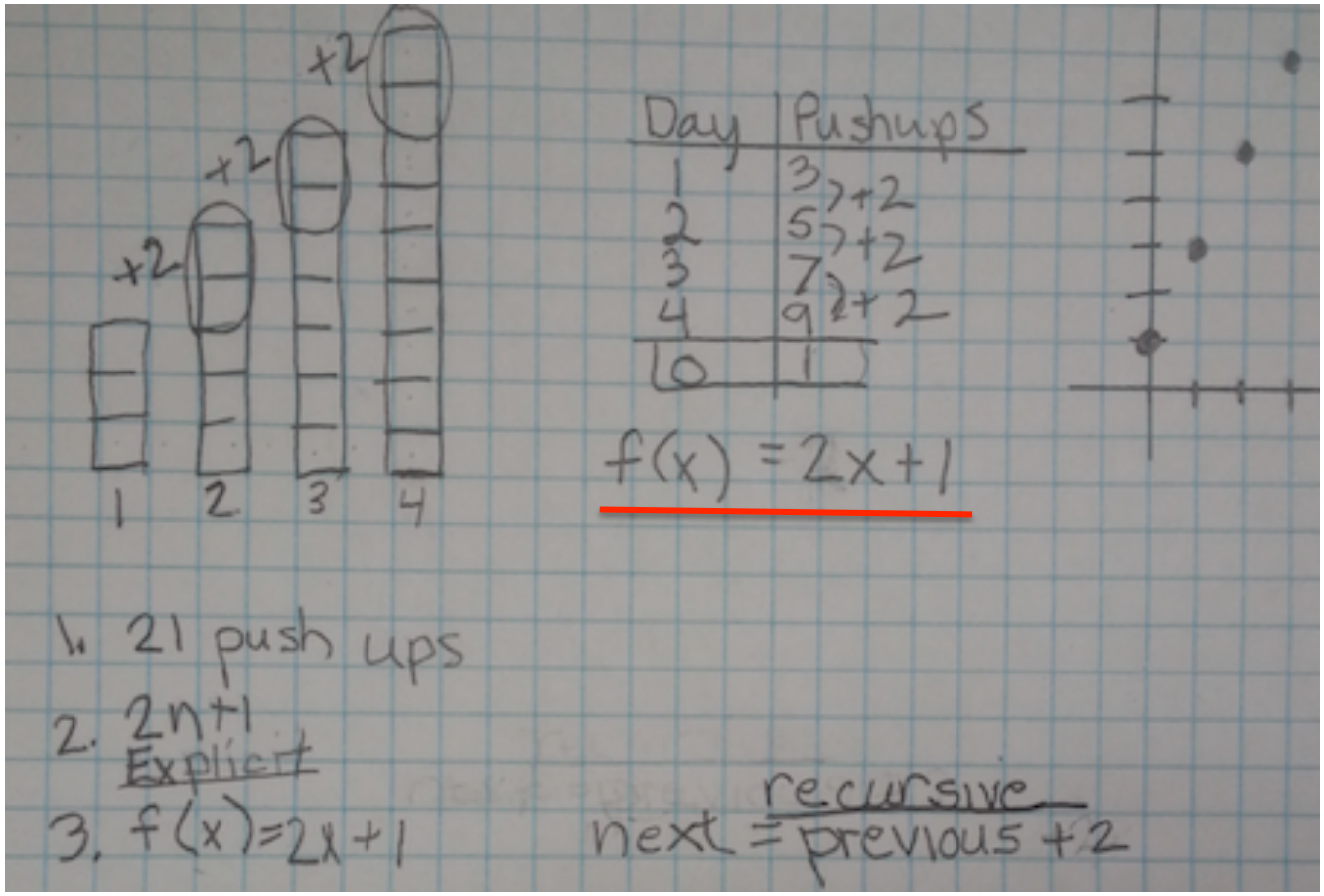

recursive
next = previous + 2
Yes because Scott
is only doing 2 more
push ups each day

Students making connections as part of the sense-making process.

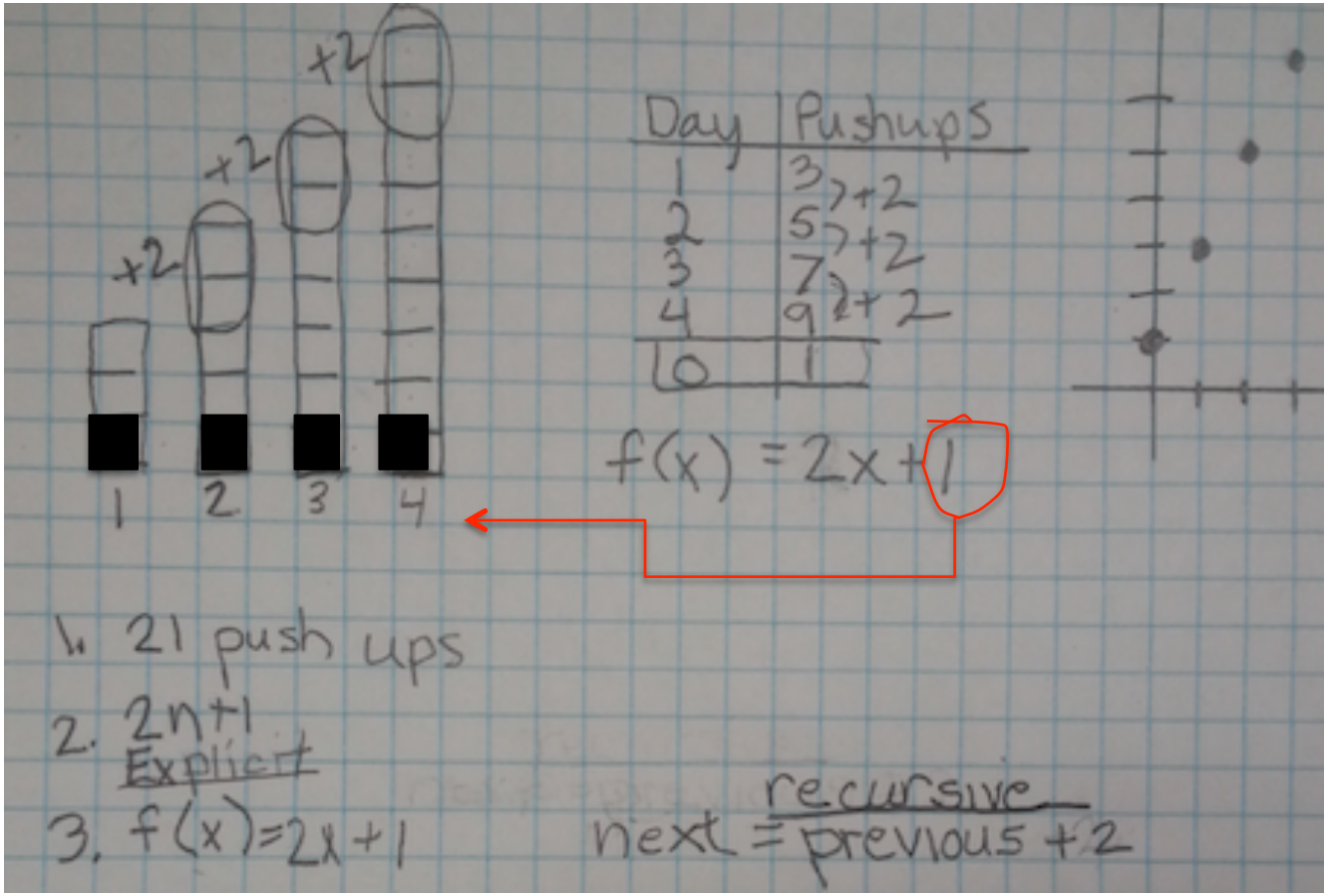
What was important about Lainie asking her question?

What kind of connection are the students trying to make?

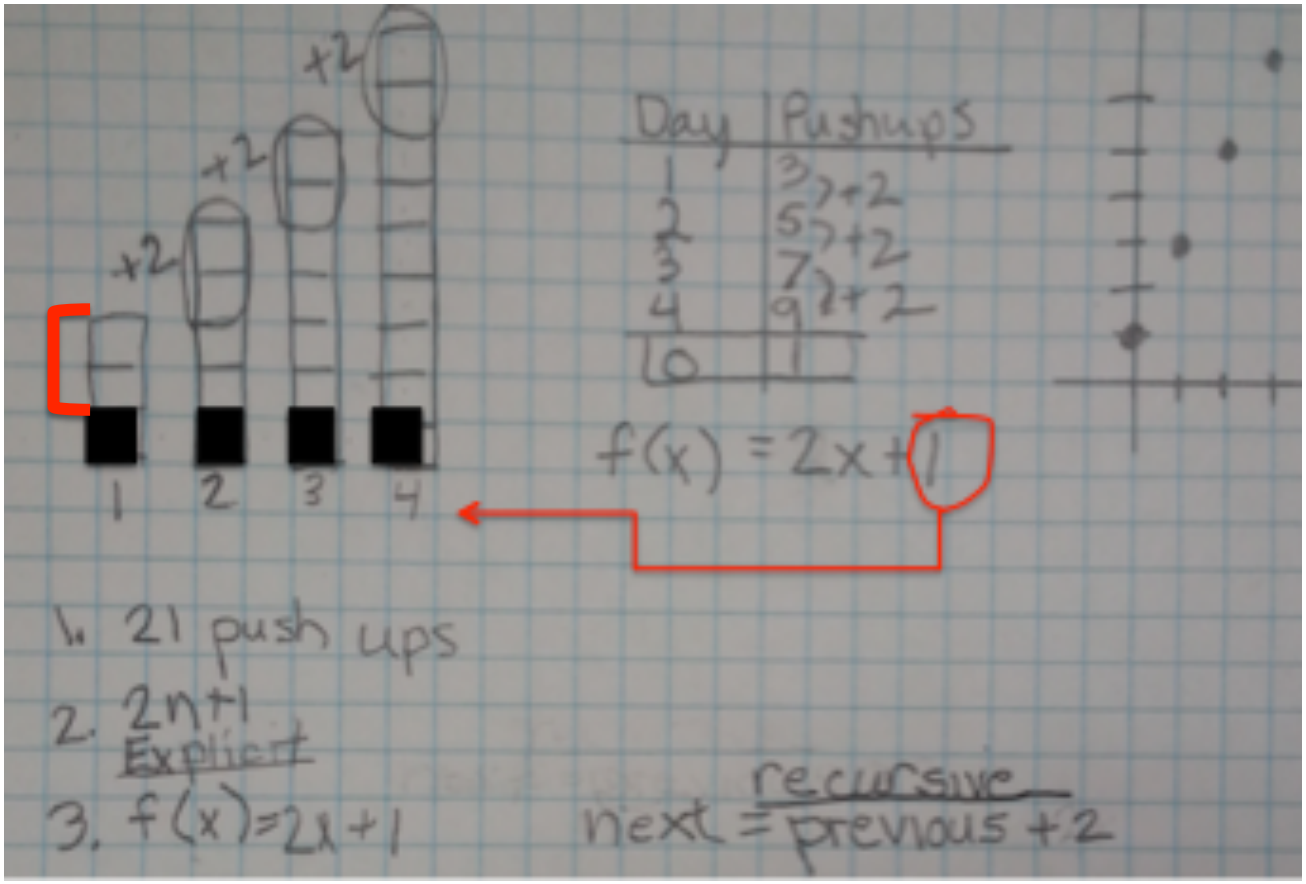
How does the equation connect to the diagram?



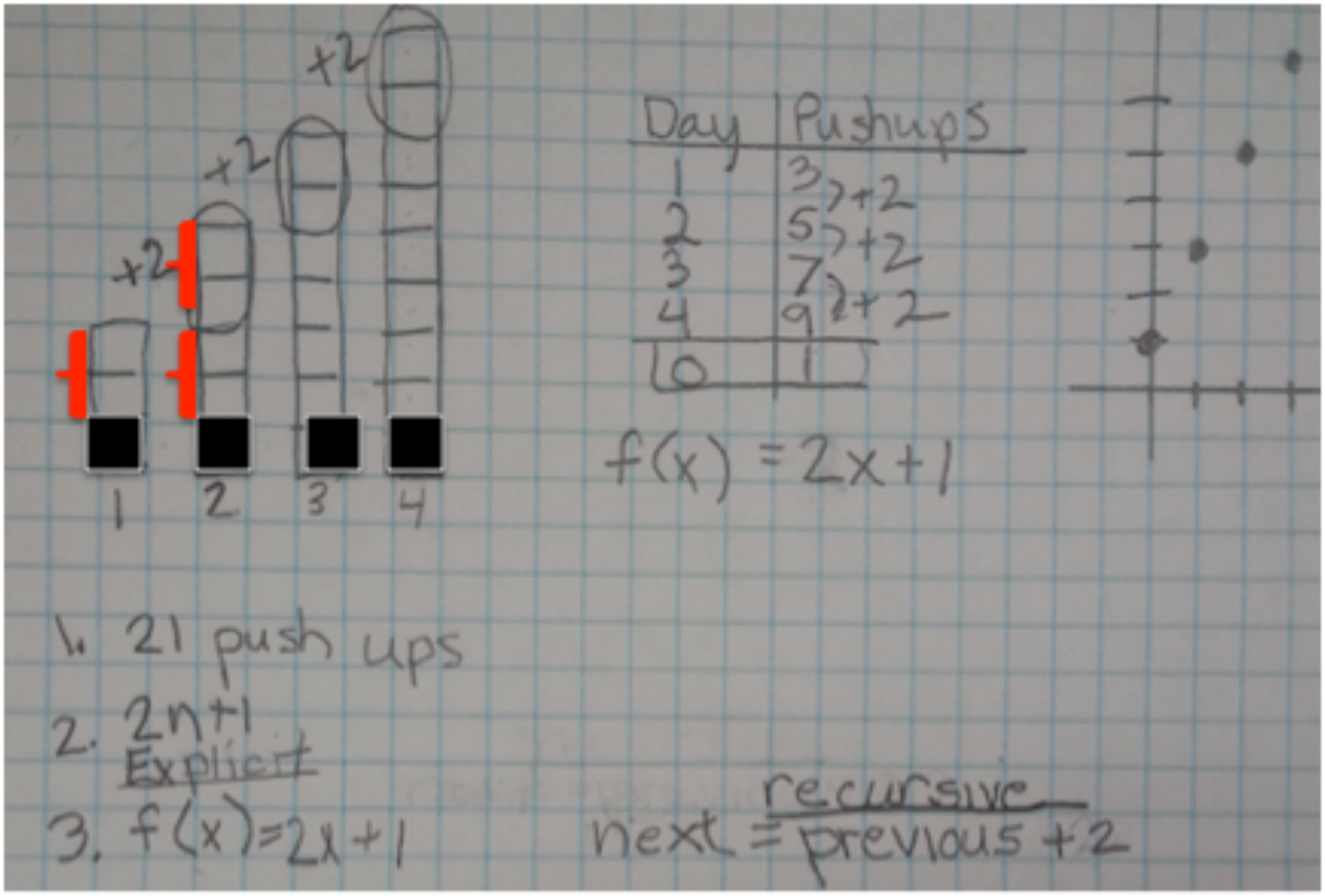
How does the equation connect to the diagram?



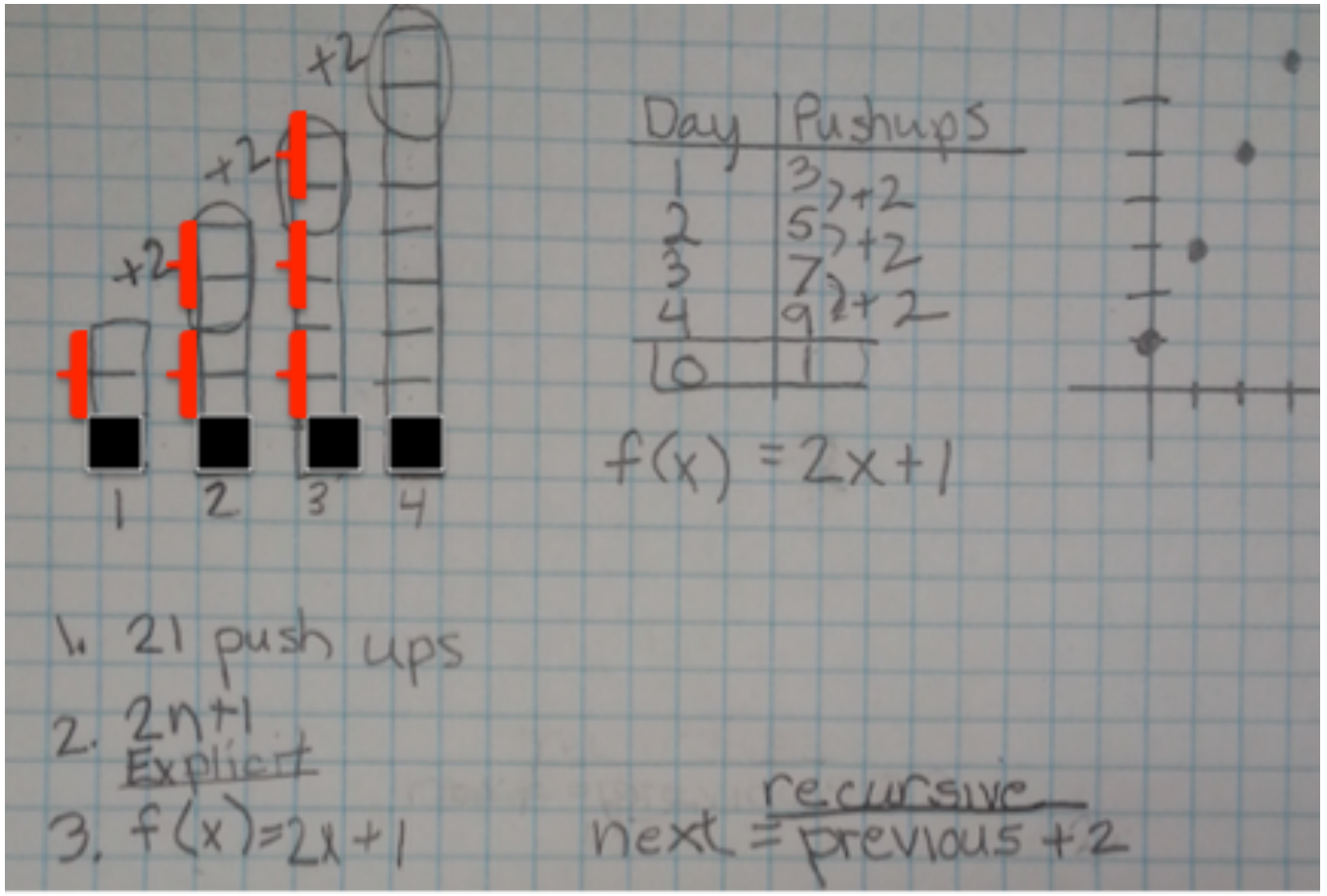
One group of 2.



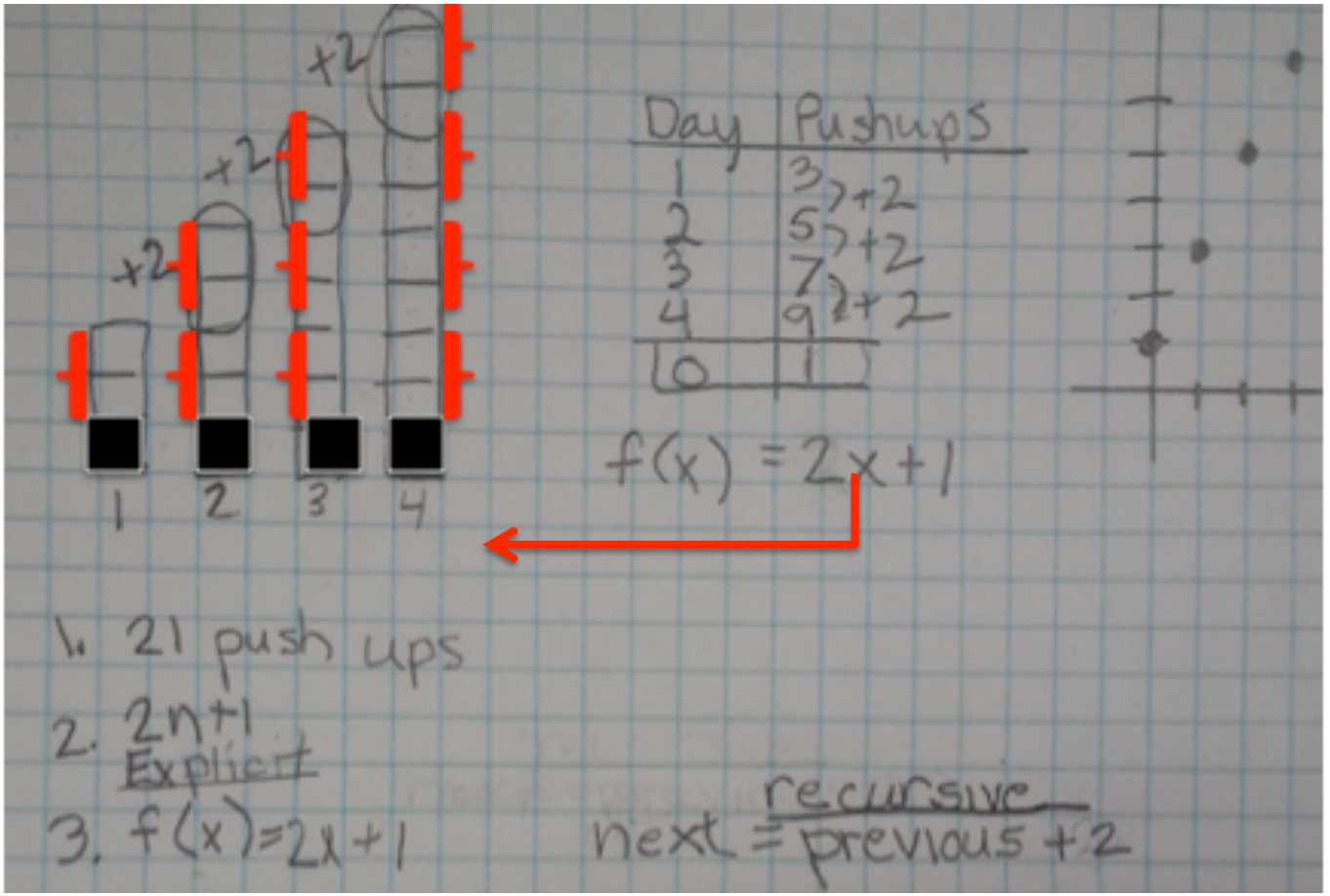
2 groups of 2


+2
+2
+2
+2
1
2
3
4
Day | Pushups
1 | 3
2 | 5 +2
3 | 7 +2
4 | 9 +2
10 | 1
f(x) = 2x + 1
1. 21 push ups
2. 2n+1
Explicit
3. f(x)=2x+1
next = previous + 2
recursive

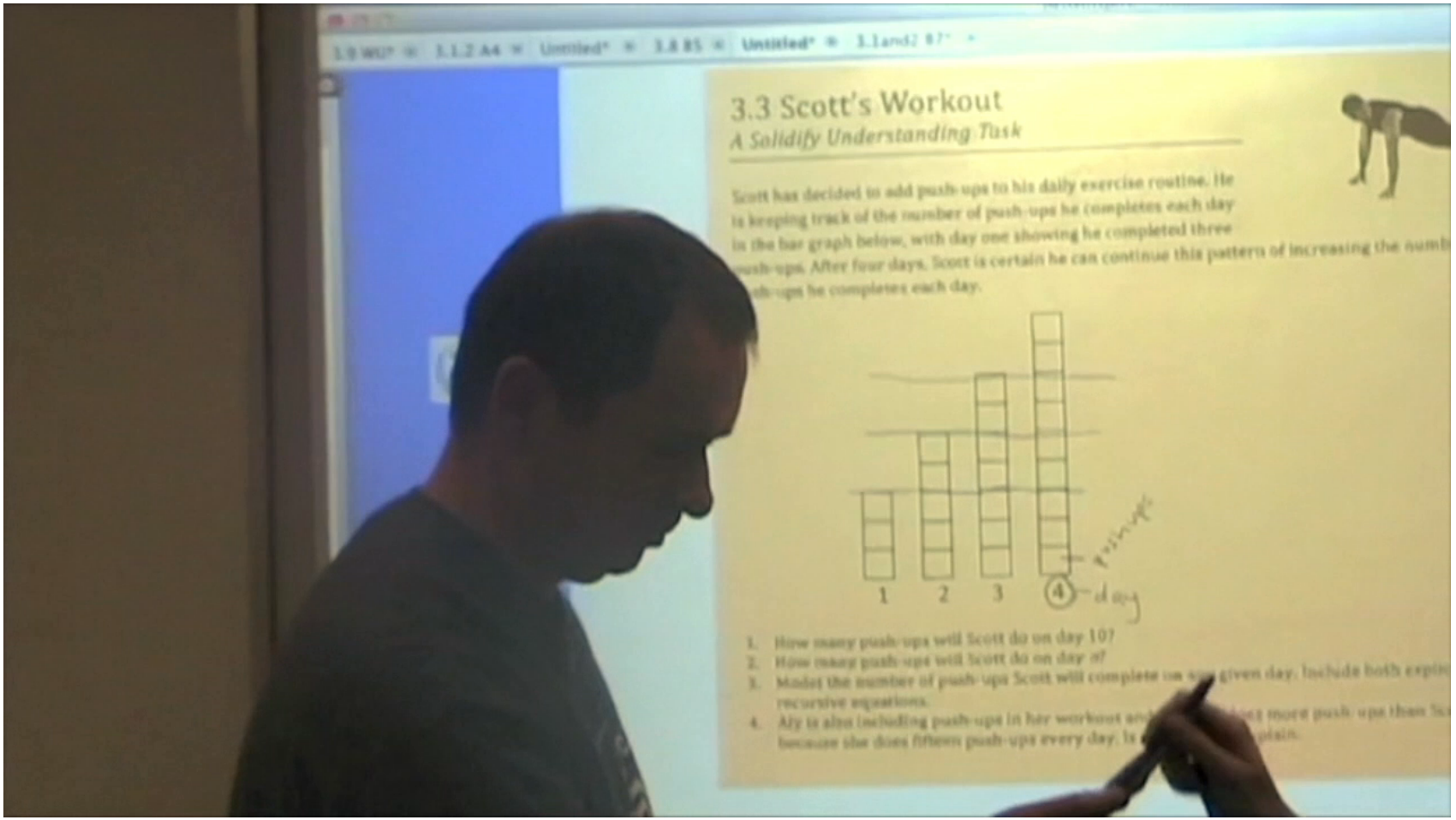
3 groups of 2


x2
x2
x2
1
2
3
4
Day | Pushups
1 | 3
2 | 5
3 | 7
4 | 9
10 | 1
+2
+2
+2
f(x) = 2x + 1
1. 21 push ups
2. 2n+1
Explicit
3. f(x)=2x+1
next = previous + 2
recursive

4 groups of 2. The number of groups matches the day which is x.



Equations and number expressions tell a story.



Students making connections as part of the sense-making process.

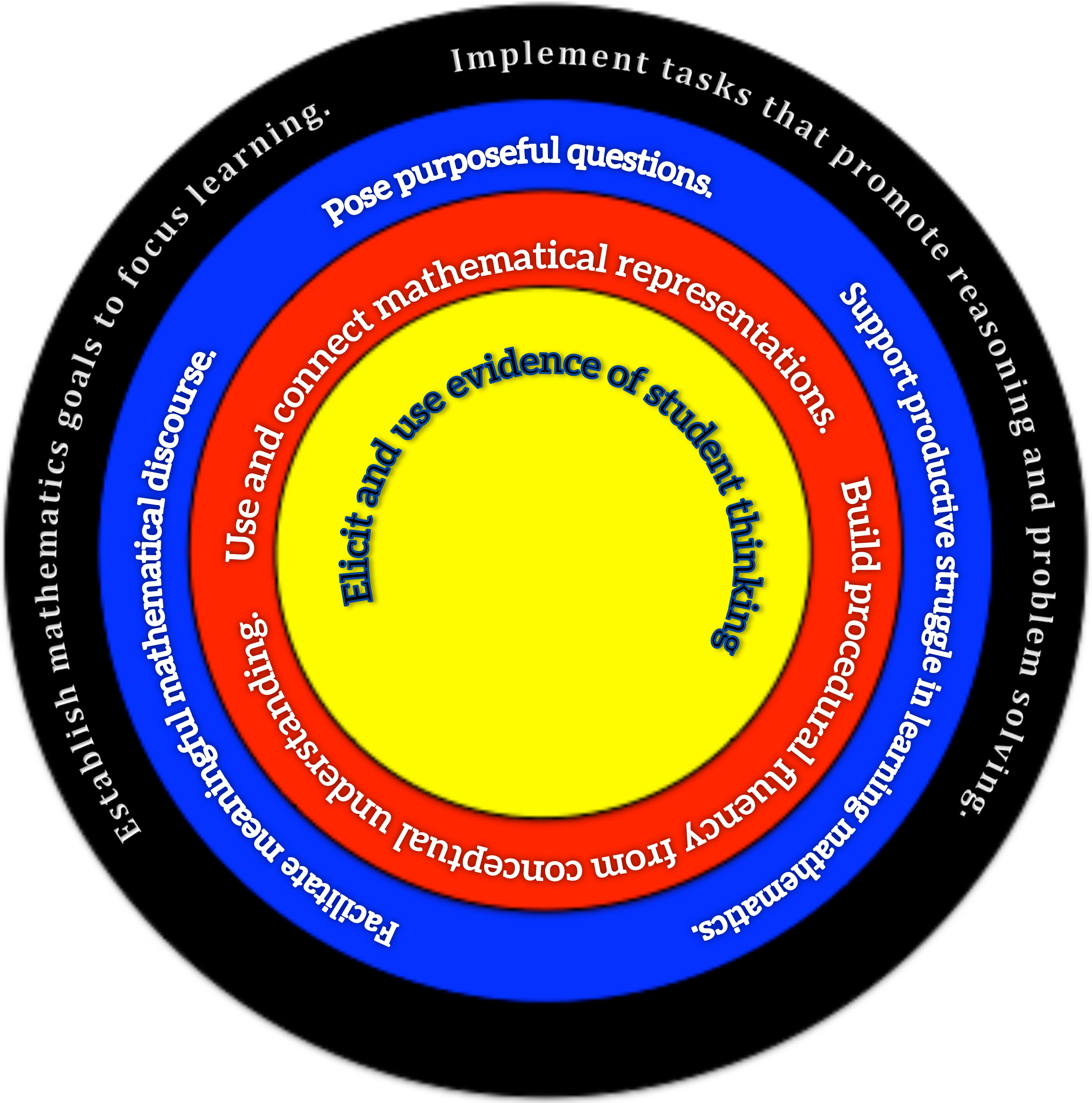
1. How can I connect the numbers to the story?
2. How does my equation connect to the way I see the model?
3. What story matches the numbers?
4. What did my classmate see that I didn't?

There is something inherently important about this classroom discourse.

Mathematics is supposed to make sense.

The mathematical model develops out of a situation.

Numbers connect to and describe what is happening.


Implement tasks that promote reasoning and problem solving.
Establish mathematics goals to focus learning.
Pose purposeful questions.
Support productive struggle in learning mathematics.
Facilitate meaningful mathematical discourse.
Use and connect mathematical representations.
Build procedural fluency from conceptual understanding.
Elicit and use evidence of student thinking.

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