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Core Subject Area: Secondary III Mathematics

## Mathematics, Secondary III

| Standard | Designated Section |
| :---: | :---: |
| Domain: Number and Quantity |  |
| Use Complex numbers in polynomial identities and equations. |  |
| N.CN. 8 Extend polynomial identities to the complex numbers. For example, rewrite $x^{\mathbf{2}}+$ 4 as $(x+2 i)(x-2 i)$. | Module 3 Task 8 Getting to the Root of the Problem <br> Module 3 Task 10 Puzzling Over Polynomials <br> ${ }^{* *}$ N.CN. 8 is throughout Module 3 and 4, both in the tasks and in the RSG's. |
| N.CN. 9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. | Module 3 Task 7 Building Stronger Roots <br> Module 3 Task 8 Getting to the Root of the Problem <br> Module 3 Task 10 Puzzling Over Polynomials |
| Domain: Algebra |  |
| Interpret the structure of expressions. |  |
| A.SSE. 1 Interpret expressions that represent a quantity in terms of its context. $\star$ | Module 1 Task 1 Checkerboard Borders |


| a. Interpret parts of an expression, such as terms, factors, and coefficients. <br> b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^{n}$ as the product of $P$ and $a$ factor not depending on $P$. | Module 2 Task 5 Making My Point <br> Module 3 Task 7 Building Stronger Roots <br> Module 3 Task 9 Is This the End? |
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| A.SSE. 2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{\mathbf{4}}$ as $\left(x^{\mathbf{2}}\right)^{\mathbf{2}}-\left(y^{\mathbf{2}}\right)^{2}$, thus recognizing it as a difference of squares that can be factored as $\left(x^{2}-y^{2}\right)\left(x^{2}+y^{2}\right)$. |  |
| Write expressions in equivalent forms to solve problems. |  |
| A.SSE. 4 Understand the formula for the sum of a series and use the formula to solve problems. <br> a. Derive the formula for the sum of an arithmetic series. <br> b. Derive the formula for the sum of a geometric series, and use the formula to solve problems. Extend to infinite geometric series. For example, calculate mortgage payments. |  |
| Perform arithmetic operations on polynomials. |  |
| A.APR. 1 Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | Module 3 Task 3 It All Adds Up <br> Module 3 Task 4 Pascal's Pride <br> Module 3 Task 5 Divide and Conquer <br> Module 3 Task 6 Sorry, We're Closed |
| Understand the relationship between zeros and factors of polynomials. |  |
| A.APR. 2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x-a$ is a factor of $p(x)$. | Module 3 Task 5 Divide and Conquer |
| A.APR. 3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. | Module 3 Task 7 Building Stronger Roots <br> Module 3 Task 8 Getting to the Root of the Problem <br> Module 3 Task 10 Puzzling Over Polynomials |


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| Use polynomial identities to solve problems. |  |
| A.APR. 4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $\left(x^{2}+y^{2}\right)^{2}=\left(x^{2}-y^{2}\right)^{2}+(2 x y)^{\mathbf{2}}$ can be used to generate Pythagorean triples. |  |
| A.APR. 5 Know and apply the Binomial Theorem for the expansion of $(x+y)^{\boldsymbol{n}}$ in powers of $x$ and $y$ for a positive integer $n$, where $x$ and $y$ are any given numbers, with coefficients determined by example by Pascal's Triangle. | Module 3 Task 4 Pascal's Pride |
| Rewrite rational expressions. |  |
| A.APR. 6 Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x)+\frac{r(x)}{b(x)}$ where $a(x), b(x), q(x)$ and $r(x)$ are polynomials with degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or for the more complicated examples, a computer algebra system. |  |
| A.APR. 7 Understand the rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply and divide rational expressions. |  |
| Create equations that describe numbers or relationships. |  |
| A.CED. 1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. |  |
| A.CED. 2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | Module 3 Task 1 Scott's March Madness Module 3 Task 10 Puzzling Over Polynomials |
| A.CED. 3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. |  |

A.CED. 4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V =IR to highlight resistance $R$.
Understand solving equations as a process of reasoning and explain the reasoning.
A.REI. 2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
Represent and solve equations and inequalities graphically.
A.REI. 11 Explain why the $x$-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

## Domain: Function

Interpret functions that arise in applications in terms of a context.
F.IF. 4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. $\star$
F.IF. 5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function. $\star$
F.IF. 6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. $\star$
Analyze functions using different representations.
F.IF. 7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. $\star$

Module 3 Task 2 You-mix Cubes
Module 3 Task 9 Is This the End?
Module 6 Task 4 More Ferris Wheels

Module 3 Task 2 You-mix Cubes

| b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. <br> e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | Module 3 Task 3 It All Adds Up |
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| F.IF. 8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. | Module 2 Task 3 Chopping Logs <br> Module 2 Task 4 Log-Arithm-etic |
| F.IF. 9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. |  |
| Build a function that models a relationship between two quantities. |  |
| F.BF. 1 Write a function that describes a relationship between two quantities.* <br> b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. | Module 1 Task 1 Brutus Bites Back <br> Module 1 Task 2 Flipping Ferraris <br> Module 1 Task 3 Tracking the Tortoise <br> Module 3 Task 1 Scott's March Madness <br> Module 3 Task 3 It All Adds Up <br> Module 3 Task 6 Sorry, We're Closed <br> Module 8 Task 2 Imagineering <br> Module 8 Task 3 The Bungee Jump Simulator <br> Module 8 Task 4 Composing and Decomposing <br> Module 8 Task 5 Translating My Composition <br> Module 8 Task 6 Different Combinations |
| Build new functions that exist from existing functions. |  |
| F.BF. 3 Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+$ $k$ ) for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. | Module 2 Task 2 Falling Off a Log <br> Module 3 Task 2 You-mix Cubes <br> Module 3 Task 9 Is This the End? <br> Module 6 Task 4 More Ferris Wheels <br> Module 7 Task 1 High Noon and Sunset Shadows |


|  | Module 7 Task 3 Getting on the Right Wavelength <br> Module 8 Task 1 Function Family Reunion <br> Module 8 Task 5 Translating My Composition |
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| F.BF. 4 Find inverse functions. <br> a. a. Solve an equation of the form $f(x)=c$ for a simple function $f$ that has an inverse and write an expression for the inverse. For example, $f(x)=2 \times 3$ or $f(x)$ $=(x+1) /(x-1)$ for $x \neq 1$. | Module 1 Task 1 Brutus Bites Back <br> Module 1 Task 2 Flipping Ferraris <br> Module 1 Task 3 Tracking the Tortoise <br> Module 1 Task 4 Pulling a Rabbit Out of a Hat <br> Module 1 Task 5 Inverse Universe <br> Module 7 Task 2 High Tide <br> Module 7 Task 3 Getting on the Right Wavelength |
| F.BF.5(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. | Module 2 Task 1 Log Logic <br> Module 2 Task 2 Falling Off a Log |
| Construct and compare linear, quadratic and exponential models and solve problems. |  |
| F.LE. 3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. | Module 3 Task 1 Scott's March Madness <br> Module 3 Task 9 Is This the End? |
| F.LE. 4 For exponential models, express as a logarithm the solution to a $b c t=d$ where $a$, $c$, and $d$ are numbers and the base $b$ is 2,10 , or $e$; evaluate the logarithm using technology. | Module 2 Task 3 Chopping Logs <br> Module 2 Task 4 Log-Arithm-etic <br> Module 2 Task 5 Powerful Tens |
| Extend the domain of trigonometric functions using the unit circle. |  |
| F.TF. 1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. | Module 6 Task 6 Diggin' It <br> Module 6 Task 7 Staking It <br> Module 6 Task 8 "Sine"ing and "Cosine"ing It <br> Module 6 Task 9 Water Wheels and Unit Circle |
| F.TF. 2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. | Module 6 Task 3 More "Sine" Language <br> Module 6 Task 5 Moving Shadows <br> Module 6 Task 6 Diggin' It <br> Module 6 Task 7 Staking It <br> Module 6 Task 8 "Sine"ing and "Cosine"ing It |


|  | Module 6 Task 9 Water Wheels and Unit Circle Module 7 Task 4 Off on a Tangent |
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| Model periodic phenomena with trigonometric functions. |  |
| F.TF. 5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. | Module 6 Task 1 George W. Ferris' Day Off <br> Module 6 Task 2 "Sine" Language <br> Module 6 Task 4 More Ferris Wheels <br> Module 6 Task 5 Moving Shadows <br> Module 7 Task 1 High Noon and Sunset Shadows <br> Module 7 Task 2 High Tide <br> Module 7 Task 3 Getting on the Right Wavelength <br> Module 7 Task 4 Off on a Tangent |
| F.TF. 7 Use inverse functions to solve trigonometric equations that arise in modeling context; evaluate the solutions using technology and interpret them in terms of context. Limit solutions to a given interval. | Module 7 Task 6 Hidden Identities |
| Domain: Geometry |  |
| Apply trigonometry to general triangles. |  |
| G.SRT. 9 Derive the formula $\mathrm{A}=1 / 2 \mathrm{ab} \sin (\mathrm{C})$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. | Module 5 Task 8 Triangles Areas by Trig |
| G.SRT. 10 Prove the Laws of Sines and Cosines and use them to solve problems. | Module 5 Task 6 More Than Right <br> Module 5 Task 7 Justifying the Laws <br> Module 5 Task 8 Triangles Areas by Trig |
| G.SRT. 11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). | Module 5 Task 5 Special Rights <br> Module 5 Task 6 More Than Right <br> Module 5 Task 7 Justifying the Laws <br> Module 5 Task 8 Triangles Areas by Trig |
| Visualize relationships between two dimensional and three-dimensional objects. |  |


| G.GMD. 4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of twodimensional objects. | Module 5 Task 1 Any Way You Slice It Module 5 Task 2 Any Way You Spin It Module 5 Task 3 Take Another Spin |
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| Apply geometric concepts in modeling situations. |  |
| G.MG. 1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). | Module 5 Task 3 Take Another Spin <br> Module 5 Task 4 You Nailed It |
| G.MG. 2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). | Module 5 Task 4 You Nailed It |
| G.MG. 3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios) | Module 5 Task 4 You Nailed It |
| Domain: Statistics |  |
| Summarize, represent and interpret data on a single count or measurement system. |  |
| S.ID. 4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. | Module 9 Task 1 What is Normal? <br> Module 9 Task 2 Just Act Normal <br> Module 9 Task 3 Y B Normal? <br> Module 9 Task 4 Wow! That's Weird! |
| Understand and evaluate random processes underlying statistical experiments. |  |
| S.IC. 1 Understand that statistics allows inferences to be made about population parameters based on a random sample from that population. | Module 9 Task 5 Would You Like to Try a Sample? <br> Module 9 Task 6 Let's Investigate <br> Module 9 Task 7 Slacker's Simulation |
| S.IC. 2 Decide if a specified model is consistent with results from a given datagenerating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5 . Would a result of five tails in a row cause you to question the model? | Module 9 Task 6 Let's Investigate <br> Module 9 Task 7 Slacker's Simulation |

Make inferences and justify conclusions from sample surveys, experiments and observational studies.
S.IC. 3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
S.IC. 4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. S.IC. 6 Evaluate reports based on data.

