June 23, 2014

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Transforming Mathematics Education

Flexible & Engaging
Seamless Common Core Companion

The Mathematics Vision Project

- What MVP is
- How MVP fits the Core Standards
- Review the MVP materials

What is the Mathematics Vision Project?

- The Mathematics Vision Project is an educator-driven initiative to provide first-rate curriculum and professional development resources to secondary math teachers.

- Built from the ground-up by a team of award-winning, math-loving education experts, the material is flexible, engaging, and a seamless companion to the Common Core.

- Committed to life-long learning, the MVP team also offers educator training and resources to help teachers perform at their peak.

Who are we?
Meet the MVP Team

- **We believe math is engaging.** Our desire is to enable educators to teach their students through accessible, engaging, supportive curriculum.

- **We enjoy our work.** Our product has grown out of our passion for teaching mathematics and we love sharing it with our clients.

- **We know teachers need support.** We live the phrase "we're in this together." As educators face new challenges and requirements, we're there to support their next steps and, in turn, the success of their students.

What sets us apart from traditional publishers?

- Our materials are created from the ground-up to embody the focus, coherence, and rigor of the Common Core and immerse students in the Standards for Mathematical Practice.

- Our materials respect what is known about how students learn mathematics as well as the key ideas for how knowledge is organized and generated within the discipline of mathematics.

- Our curriculum is free and published under a Creative Commons license. Other support materials and professional development are available at mathematicsvisionproject.org

How are our materials designed?

- To meet the Core Standards
- Engage students in the Standards for Mathematical Practice

Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

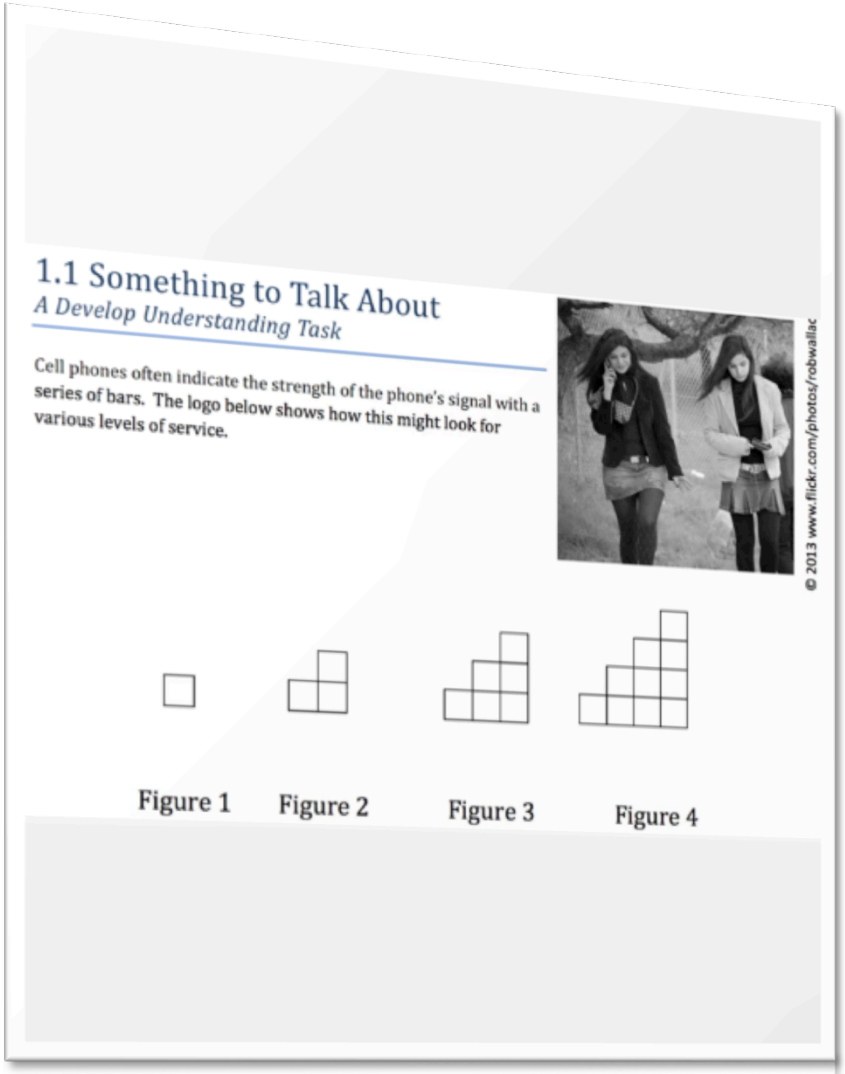
What needs to be in place to make this happen in the classroom?

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Basic Structure of the Materials

- The underlying design of the materials distinguishes between *problems* and *exercises*
- Each problem or exercise has a purpose
  - *teach new knowledge*
  - *bring misconceptions to the surface*
  - *build skill or fluency*
  - *engage students in mathematical practice*
- Assignments are purposefully designed
  - *"Lessons have a few well-designed problems that progressively build and extend understanding."*

(K-8 Publishers' Criteria for the Common Core Standards in Mathematics)



Curriculum

In-Class Tasks
and
Ready, Set, Go!
Assignments

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In-Class Tasks and Ready-Set-Go Assignments

- In-class tasks are designed to be facilitated by a teacher, with students working together to build a balance of conceptual understanding and procedural skill.

- Ready-Set-Go assignments are independent practice that reinforce the work done in class and prepare students for upcoming work in class.

For Example:

Module 4 – Linear and Exponential Functions

4.1 Classroom Task: Connecting the Dots: Piggies and Pools – A Develop Understanding Task
Introducing continuous linear and exponential functions **(F.IF.3)**
Ready, Set, Go Homework: Linear and Exponential Functions 4.1

4.2 Classroom Task: Sorting Out the Change – A Solidify Understanding Task
Defining linear and exponential functions based upon the pattern of change **(F.LE.1, F.LE.2)**
Ready, Set, Go Homework: Linear and Exponential Functions 4.2

Problem

Exercise

4.3 Classroom Task: Where's My Change – A Practice Understanding Task
Identifying rates of change in linear and exponential functions **(F.LE.1, F.LE.2)**
Ready, Set, Go Homework: Linear and Exponential Functions 4.3

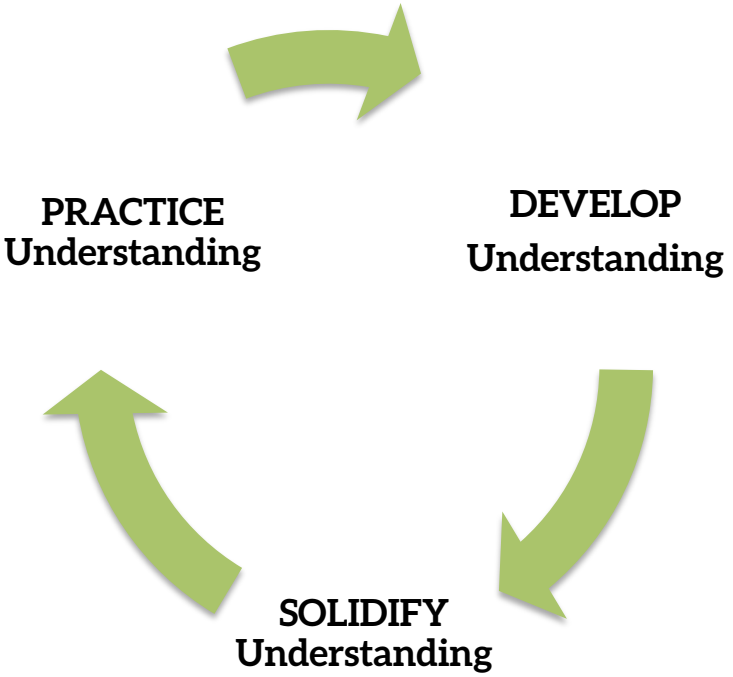
4.4 Classroom Task: Linear, Exponential or Neither – A Practice Understanding Task
Distinguishing between linear and exponential functions using various representations **(F.LE.3, F.LE.5)**
Ready, Set, Go Homework: Linear and Exponential Functions 4.4

4.5 Classroom Task: Getting Down to Business – A Solidify Understanding Task
Comparing the growth of linear and exponential functions **(F.LE.2, F.LE.3, F.LE.5, F.IF.7)**
Ready, Set, Go Homework: Linear and Exponential Functions 4.5

4.6 Classroom Task: Growing, Growing, Gone – A Solidify Understanding Task
Comparing linear and exponential models of population **(F.BF.1, F.BF.2, F.LE.1, F.LE.2, F.LE.3)**
Ready, Set, Go Homework: Linear and Exponential Functions 4.6

Task Sequencing

Comprehensive Mathematics Instruction Framework



- *Develop Understanding* tasks surface student thinking
- *Solidify Understanding* tasks examine and extend
- *Practice Understanding* tasks build fluency

For Example:

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Exercises

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Shopping for Cats and Dogs

2.7 Shopping for Cats and Dogs

A Develop Understanding Task

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Clarita is upset with Carlos because he has been buying cat and dog food without recording the price of each type of food in their accounting records. Instead, Carlos has just recorded the total price of each purchase, even though the total cost includes more that one type of food. Carlos is now trying to figure out the price of each type of food by reviewing some recent purchases. See if you can help him figure out the cost of particular items for each purchase, and be prepared to explain your reasoning to Carlos.

1. One week Carlos bought 3 bags of *Tabitha Tidbits* and 4 bags of *Figaro Flakes* for $43.00. The next week he bought 3 bags of *Tabitha Tidbits* and 6 bags of *Figaro Flakes* for $54.00. Based on this information, figure out the price of one bag of each type of cat food. Explain your reasoning.

2. One week Carlos bought 2 bags of *Brutus Bites* and 3 bags of *Lucky Licks* for $42.50. The next week he bought 5 bags of *Brutus Bites* and 6 bags of *Lucky Licks* for $94.25. Based on this information, figure out the price of one bag of each type of dog food. Explain your reasoning.

3. Carlos purchased 6 dog leashes and 6 cat brushes for $45.00 for Clarita to use while pampering the pets. Later in the summer he purchased 3 additional dog leashes and 2 cat brushes for $19.00. Based on this information, figure out the price of each item. Explain your reasoning.

4. One week Carlos bought 2 packages of dog bones and 4 packages of cat treats for $18.50. Because the finicky cats didn't like the cat treats, the next week Carlos returned 3 unopened packages of cat treats and bought 2 more packages of dog bones. After being refunded for the cat treats, Carlos only had to pay $1.00 for his purchase. Based on this information, figure out the price of each item. Explain your reasoning.

5. Carlos has noticed that because each of his purchases have been somewhat similar, it has been easy to figure out the cost of each item. However, his last set of receipts has him puzzled. One week he tried out cheaper brands of cat and dog food. On Monday he purchased 3 small bags of cat food and 5 small bags of dog food for $22.75. Because he went through the small bags quite quickly, he had to return to the store on Thursday to buy 2 more small bags of cat food and 3 more small bags of dog food, which cost him $14.25. Based on this information, figure out the price of each bag of the cheaper cat and dog food. Explain your reasoning.

Summarize the strategies you have used to reason about the price of individual items in the problems given above. What are some key ideas that seem helpful?

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Can You Get to the Point Too?

2.8 Can You Get to the Point, Too?

A Solidify Understanding Task

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Part 1
In "Shopping for Cats and Dogs," Carlos found a way to find the cost of individual items when given the purchase price of two different combinations of those items. He would like to make his strategy more efficient by writing it out using symbols and algebra. Help him formalize his strategy by doing the following:

- For each scenario in "Shopping for Cats and Dogs" write a **system of equations** to represent the two purchases.
- Show how your strategies for finding the cost of individual items could be represented by manipulating the equations in the system. Write out intermediate steps symbolically, so that someone else could follow your work.
- Once you find the price of one of the items in the combination, show how you would find the price of the other item.

Part 2
Writing out each system of equations reminded Carlos of his work with solving systems of equations graphically. Show how each scenario in "Shopping for Cats and Dogs" can be represented graphically, and how the cost of each item shows up in the graphs.

Part 3
Carlos also realized that the algebraic strategy he created in part 1 could be used to find the points of intersection for the "Pet Sitters" constraints. Use the **elimination of variables** method developed in part 1 to find the point of intersection for each of the following pairs of "Pet Sitter" constraints.

- *Start-up costs* and *space* constraints
- *Pampering time* and *feeding time* constraints
- Any other pair of "Pet Sitter" constraints of your choice

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Can You Get to the Point, Too?

Part 1

- In "Shopping for Cats and Dogs," Carlos found a way to find the cost of individual items when given the purchase price of two different combinations of those items. He would like to make his strategy more efficient by writing it out using symbols and algebra. Help him formalize his strategy by doing the following:
- For each scenario in "Shopping for Cats and Dogs" write a **system of equations** to represent the two purchases.
- Show how your strategies for finding the cost of individual items could be represented by manipulating the equations in the system. Write out intermediate steps symbolically, so that someone else could follow your work.
- Once you find the price of one of the items in the combination, show how you would find the price of the other item.
- (Focus on question #3 only for your graphical representation.)

Can You Get to the Point, Too

Part 2

- Writing out each system of equations reminded Carlos of his work with solving systems of equations graphically. Show how each scenario in "Shopping for Cats and Dogs" can be represented graphically, and how the cost of each item shows up in the graphs.

(Focus on question #3 only for your graphical representation.)

Taken Out of Context

Write a shopping scenario similar to those in "Shopping for Cats and Dogs" to fit each of the following systems of equations. Then use the elimination of variables method you invented in "Can You Get to the Point, Too" to solve the system. Some of the systems may have interesting or unusual solutions. See if you can explain them in terms of the shopping scenarios you wrote.

1. $\begin{cases} 3x + 4y = 23 \\ 5x + 3y = 31 \end{cases}$

2. $\begin{cases} 2x + 3y = 14 \\ 4x + 6y = 28 \end{cases}$

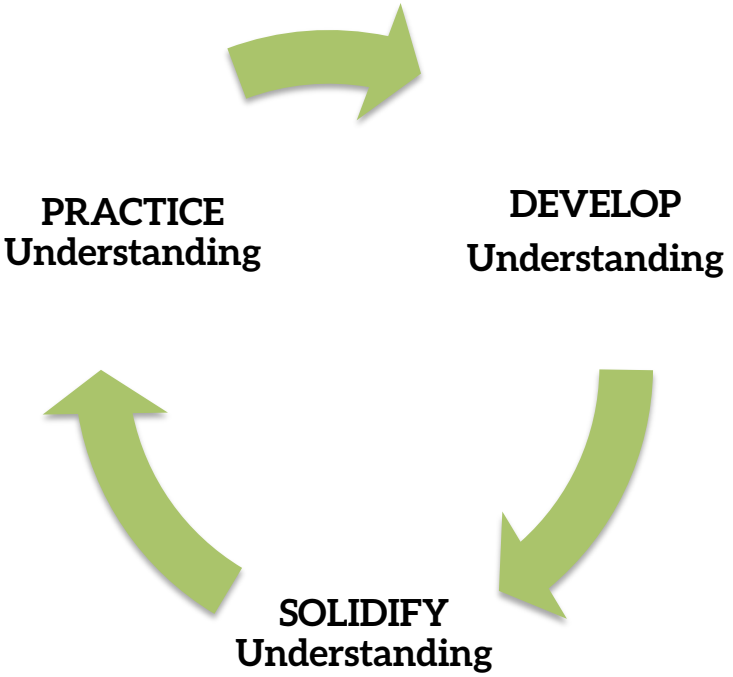
3. $\begin{cases} 3x + 2y = 20 \\ 9x + 6y = 35 \end{cases}$

4. $\begin{cases} 4x + 2y = 8 \\ 5x + 3y = 9 \end{cases}$

Three of Carlos' and Clarita's friends are purchasing school supplies at the bookstore. Stan buys a notebook, three packages of pencils and two markers for $7.50. Jan buys two notebooks, six packages of pencils and five markers for $15.50. Fran buys a notebook, two packages of pencils and two markers for $6.25. How much do each of these three items cost?
Explain in words or with symbols how you can use your intuitive reasoning about these purchases to find the price of each item.

Task Sequencing

Comprehensive Mathematics Instruction Framework

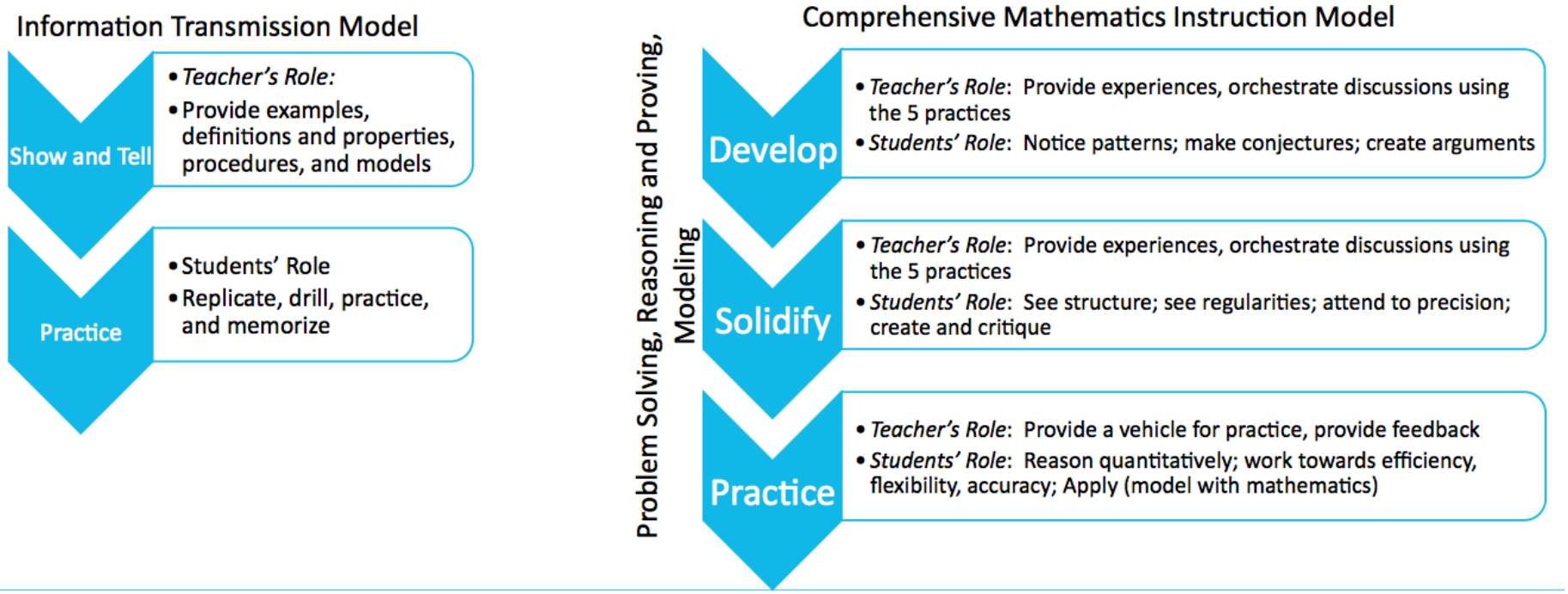


- *Develop Understanding* tasks surface student thinking
- *Solidify Understanding* tasks examine and extend
- *Practice Understanding* tasks build fluency

Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

How does the experience of the learner change throughout the learning cycle?



Develop Understanding Tasks

- Low threshold, high ceiling (easy entry, but extendable for all learners)
- Contextualized (problematic story context, diagrams, symbols)
- Multiple pathways to solutions or multiple solutions
- Surface student thinking (misconceptions and correct thinking)
- Purposeful selection of the vocabulary, numbers, etc. to reveal rather than obscure the mathematics
- Introduction of a number of representations

Solidify Understanding Tasks

Features of the task (context, scaffolding questions, constraints) focuses students' attention on:

- looking for patterns and making use of structure
- looking for repeated reasoning and expressing regularities as generalized methods
- attending to precision in language and use of symbols
- constructing viable arguments and critiquing the reasoning of others
- using representations and tools strategically for the purpose of developing deeper levels of understanding of mathematical ideas, strategies, and/or representations

Practice Understanding Tasks

Practice tasks focused on acquiring fluency:

- Task involves either reproducing previously learned facts, definitions, rules, formulas or models; OR drawing upon previously learned facts, definitions, rules, formulas or models; OR committing facts, definitions, rules, formulas or models to memory

- An appropriate vehicle of practice is selected (e.g., routines, games, worksheets, etc.) which allows for reproducing, drawing upon, or committing to memory previously examined mathematics

- Task focuses on a broad definition of fluency: accuracy, efficiency, flexibility, automaticity

Practice Understanding Tasks

Practice tasks focused on refining understanding:

- Task allows student to use reasoning habits to contextualize (symbolic to real-world) and decontextualize (real-world to symbolic) problems and situations.
- Tasks involve sufficient complexity to refine mathematical thinking beyond rote memorization
- The task requires a high level of cognitive demand because students are required to draw upon multiple concepts and procedures, make use of structure and recognize complex relationships among facts, definitions, rules, formulas and/or models

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Ready, Set, Go Homework: Linear and Exponential Functions 4.6

Develop

Solidify

Practice

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A Multi-tasking Approach to Learning

- Each standard is addressed in more than one task.
- Each task addresses more than one standard.
- Because the materials are built from the ground-up, every standard is fully addressed.

Materials cannot match the contours of the standards by approaching each individual content standard as a separate event. (K-8 Publishers' Criteria for the Common Core Standards in Mathematics)

For Example:

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The same standards appear in multiple tasks

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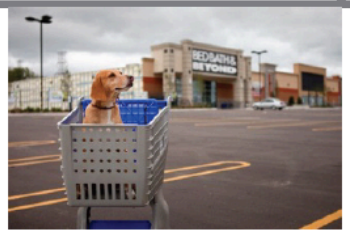
Ready-Set-Go

Name: Systems | 2.7

Ready, Set, Go!



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Ready

Topic: Exponents

Write the following in exponential notation.

1. $4 \times 4 \times 4 \times 4 \times 4$
2. $3x \cdot 3x \cdot 3x \cdot 3x$

Find each value.

3. 2^3
4. 3^3
5. 2^5
6. $(-2)^3$
7. 4^3

Ready-Set-Go

Set

Topic: Solving systems

8. Nadia and Peter visit the candy store. Nadia buys three candy bars and four fruit roll-ups for $2.84. Peter also buys three candy bars, but can only afford one additional fruit roll-up. His purchase costs $1.79. What is the cost of a candy bar and a fruit roll-up individually?

9. A farmer noticed that his chickens were loose and were running around with the cows in the cow pen. He quickly counted 100 heads and 270 legs. How many chickens did he have and how many cows?

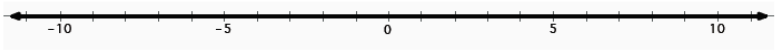
Ready-Set-Go

Go

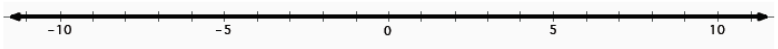
Topic: Solve one variable inequalities.

Solve the following inequalities. Write the solution set in *interval notation* and graph the solution set on a number line.

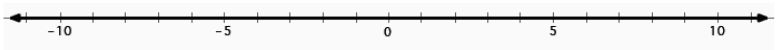
10. $4x + 10 < 2x + 14$



11. $2x + 6 > 55 - 5x$



12. $2(\frac{x}{4} + 3) > 6(x - 1)$



13. $9x + 4 \leq -2(x + \frac{1}{2})$

Solve each inequality. Give the solution in *inequality notation* and *set notation*.

14. $-\frac{x}{3} > -\frac{10}{9}$

15. $5x > 8x + 27$

16. $\frac{x}{4} > \frac{5}{4}$

17. $3x - 7 \geq 3(x - 7)$

18. $2x < 7x - 36$

19. $5 - x < 9 + x$

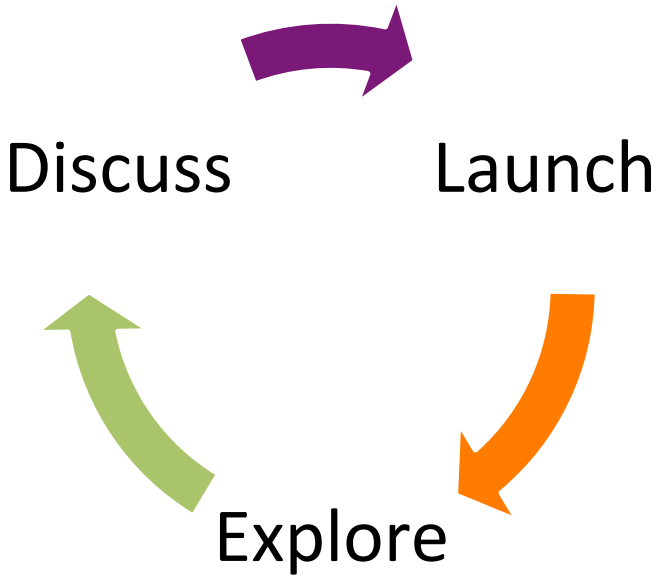
Check out the Ready – Set - Go

Ready: Get ready for upcoming lessons

Set: Reinforce what was learned in today's lesson

Go: Practice previously-learned skills

The *Teaching Cycle*



Shopping for Cats and Dogs Video

The *Teaching Cycle*: Launch

How will you . . .

- hook and motivate students;
- provide schema (the problem setting, the mathematical context, and the challenge) for the mathematical task;
- provide tools, information, vocabulary, conventions and notations, as necessary; and
- describe what the expectations are for the finished task without giving away too much of the problem and leaving the potential of the task intact?

The *Teaching Cycle*: Explore

- How will you organize and encourage students to explore, investigate, experiment, look for patterns, make conjectures, collect and record data, participate in group discussions, and revisit and revise their thinking relative to the mathematical ideas intended to be elicited by the task?

- What will you look for and listen for as you observe students?

- What will you accept as evidence of student understanding?

- What questions will you ask to stimulate, redirect, focus, and extend the students' mathematical thinking?

The *Teaching Cycle*: Discuss

- How will you select which students will present and discuss their solutions and strategies?
- How will you determine what ideas to pursue in depth and what to defer for another time?
- How will you decide whether to contribute to the discourse by providing additional information (e.g., vocabulary, conventions, notation), suggesting other models, demonstrating alternative strategies, clarifying difficult issues; or to allow students to continue to struggle to make sense of an idea or concept?

Five Practices for Facilitating Mathematical Discourse

1. Anticipate student thinking – work the task yourself
2. Monitor students as they work – circulate around the room and ask students how they are thinking about what they have written
3. Select students for the classroom discussion – have a method for keeping track of who you have selected
4. Sequence student work
5. Connect – help students to make connections among the ideas presented

Checkerboard Borders

1.1 Checkerboard Borders

A Develop Understanding Task

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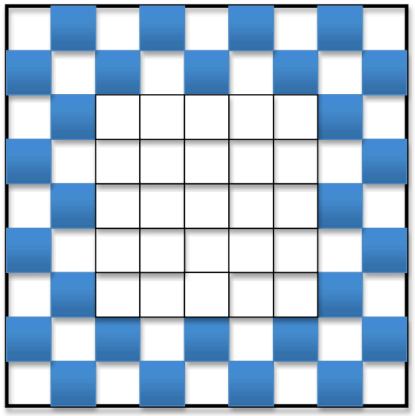
In preparation for back to school, the school administration has planned to replace the tile in the cafeteria. They would like to have a checkerboard pattern of tiles two rows wide as a surround for the tables and serving carts.

Below is an example of the boarder that the administration is thinking of using to surround a square 5 x 5 set of tiles.

A. Find the number of colored tiles in the checkerboard border. Track your thinking and find a way of calculating the number of colored tiles in the border that is quick and efficient. Be prepared to share your strategy and justify your work.



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The 0 Practice

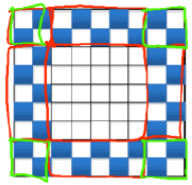
- Have clear mathematical goals and a task that supports the goals.

Checkerboard Borders

Purpose: The focus of this task is on the generation of multiple expressions that connect with the visuals provided for the checkerboard borders. These expressions will also provide opportunity to discuss equivalent expressions and review the skills students have previously learned about simplifying expressions and using variables.

Checkerboard Borders
Student Work

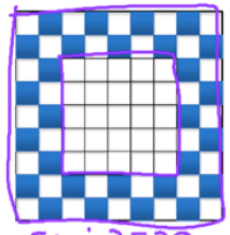
Student A



4 sides
4 corners

+(5×4
2×4
20+8=28

Student B

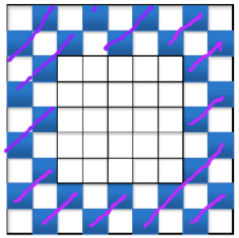


9×9-5×5
=area of checkers

56÷2=28

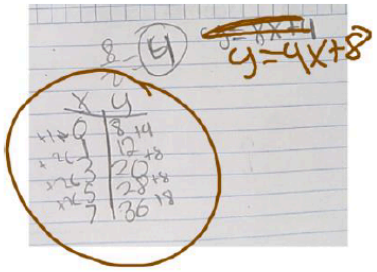
$C=\frac{(b+4)^2-b^2}{2}$

Student C

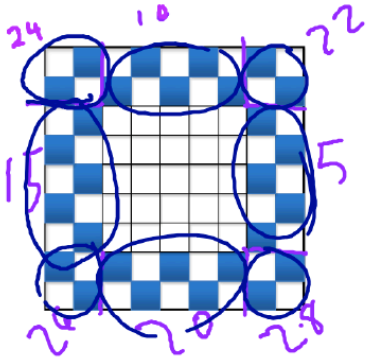


8 rows
7×4=28
4(x+2)=C

Student D

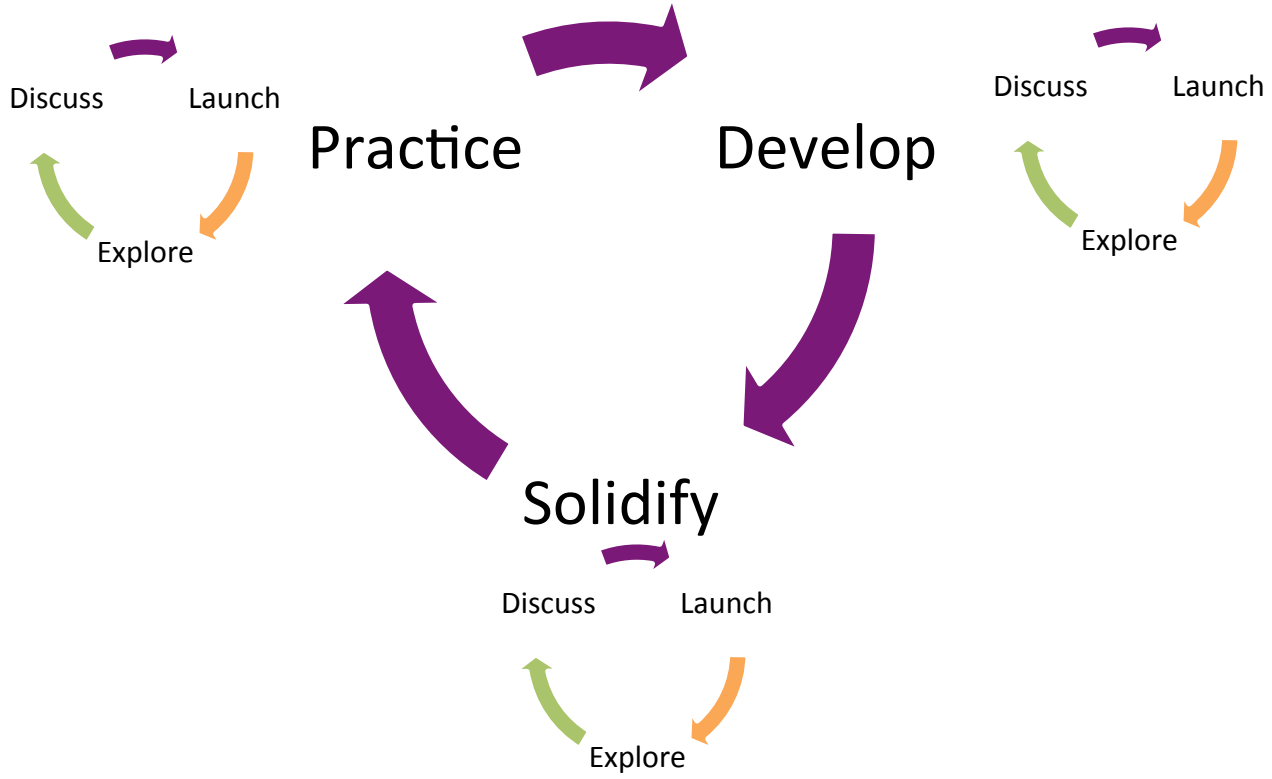


Student E



Student F

Looking at Student Work

- Examine the student work using the observation notes guide.
- Select student work that you would want to have shared in the whole class discussion.
- Sequence the work for the whole class discussion.
- Annotate your poster to highlight the connections you would want to make across the samples of student work.

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