June 19, 2014

mathematics
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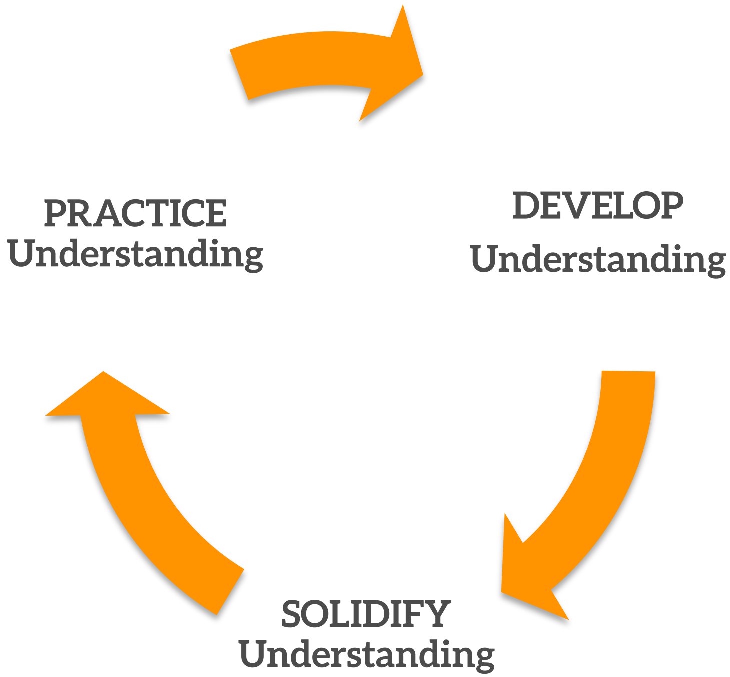
Transforming Mathematics Education

Flexible & Engaging
Seamless Common Core Companion

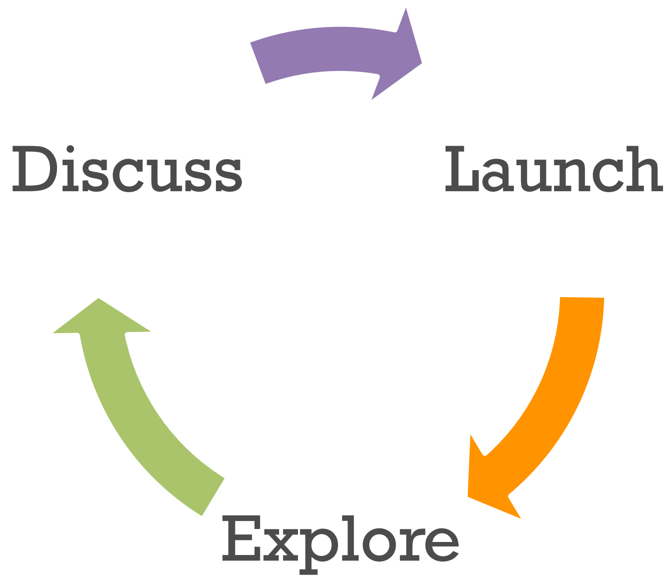
Task Sequencing

Comprehensive Mathematics Instruction Framework



- *Develop Understanding* tasks surface student thinking
- *Solidify Understanding* tasks examine and extend
- *Practice Understanding* tasks build fluency

The *Teaching Cycle*



The *Teaching Cycle*: Launch

How will you . . .

- hook and motivate students;
- provide schema (the problem setting, the mathematical context, and the challenge) for the mathematical task;
- provide tools, information, vocabulary, conventions and notations, as necessary; and
- describe what the expectations are for the finished task without giving away too much of the problem and leaving the potential of the task intact?

The *Teaching Cycle*: Explore

- How will you organize and encourage students to explore, investigate, experiment, look for patterns, make conjectures, collect and record data, participate in group discussions, and revisit and revise their thinking relative to the mathematical ideas intended to be elicited by the task?

- What will you look for and listen for as you observe students?

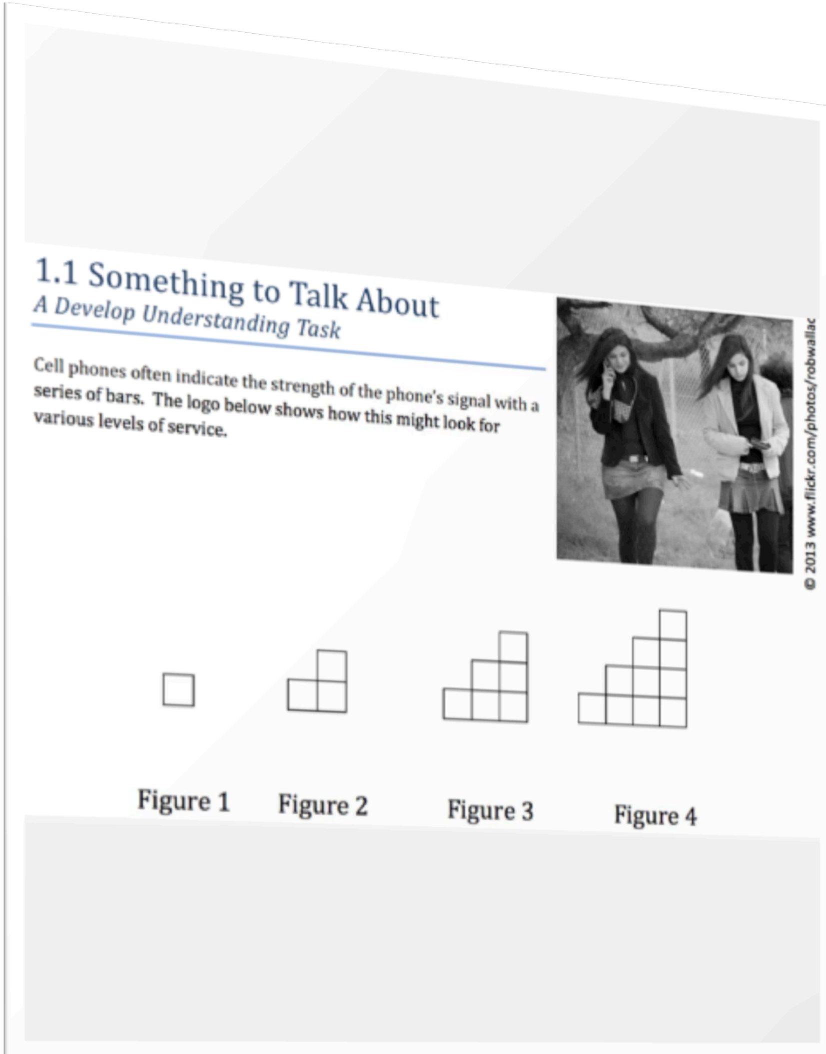
- What will you accept as evidence of student understanding?

- What questions will you ask to stimulate, redirect, focus, and extend the students' mathematical thinking?

The *Teaching Cycle*: Discuss

- How will you select which students will present and discuss their solutions and strategies?
- How will you determine what ideas to pursue in depth and what to defer for another time?
- How will you decide whether to contribute to the discourse by providing additional information (e.g., vocabulary, conventions, notation), suggesting other models, demonstrating alternative strategies, clarifying difficult issues; or to allow students to continue to struggle to make sense of an idea or concept?

Something to Talk About



1.1 Something to Talk About
A Develop Understanding Task

Cell phones often indicate the strength of the phone's signal with a series of bars. The logo below shows how this might look for various levels of service.

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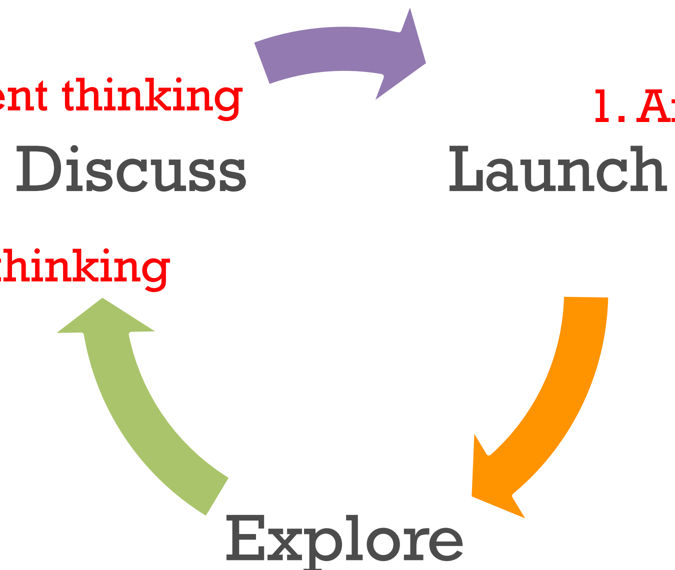
Figure 1 Figure 2 Figure 3 Figure 4

The *Teaching Cycle*

Connected to the 5 practices of Orchestrating Discussions

5. Connect student thinking

1. Anticipate student thinking



4. Sequence student thinking

2. Monitor student thinking

3. Select student thinking

Five Practices for Facilitating Mathematical Discourse

1. **Anticipate** student thinking. Work the task yourself.

2. **Monitor** students as they work. Circulate around the room and ask students how they are thinking about what they have written.

3. **Select** students for the classroom discussion. Have a method for keeping track of who you have selected

4. **Sequence** student work.

5. **Connect.** Help students to make connections among the ideas presented.

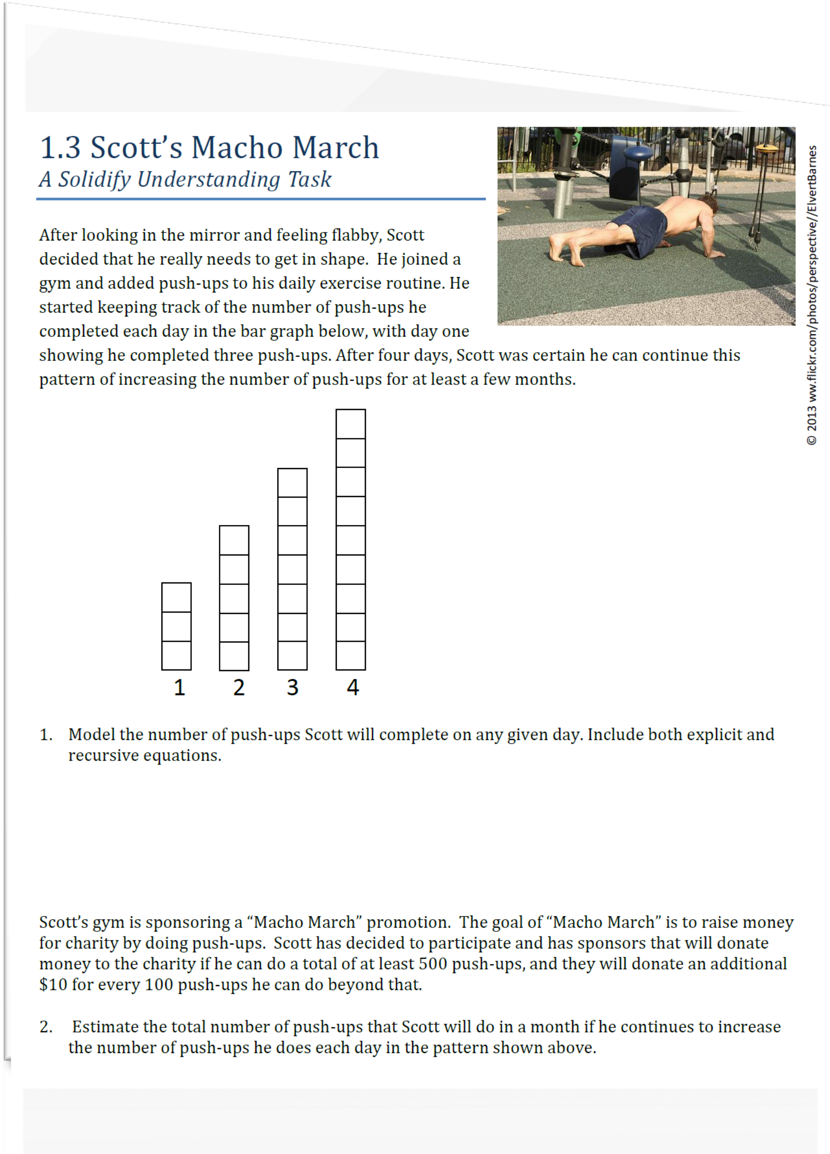
The 0 Practice

Have clear mathematical goals and a task that supports the goals.

Something to Talk About

- How is the rate of growth for this context different from the rate of growth in *Growing Dots* and *Growing, Growing Dots*?
- How does the rate of growth show up in different representations? (e.g., in a table, in a graph, in a recursive rule)
- How can we represent this sequence with an explicit equation?
- How does the structure of the diagram support this work?

Scott's Macho March



1.3 Scott's Macho March

A Solidify Understanding Task

After looking in the mirror and feeling flabby, Scott decided that he really needs to get in shape. He joined a gym and added push-ups to his daily exercise routine. He started keeping track of the number of push-ups he completed each day in the bar graph below, with day one showing he completed three push-ups. After four days, Scott was certain he can continue this pattern of increasing the number of push-ups for at least a few months.

1. Model the number of push-ups Scott will complete on any given day. Include both explicit and recursive equations.

Scott's gym is sponsoring a "Macho March" promotion. The goal of "Macho March" is to raise money for charity by doing push-ups. Scott has decided to participate and has sponsors that will donate money to the charity if he can do a total of at least 500 push-ups, and they will donate an additional $10 for every 100 push-ups he can do beyond that.

2. Estimate the total number of push-ups that Scott will do in a month if he continues to increase the number of push-ups he does each day in the pattern shown above.

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Scott's Macho March

- How did you extend your thinking that emerged in *Something to Talk About* as you worked on this task?

- What ideas, strategies or representations got solidified for you?

- What connections can be made between the work of the first part of the task (describing push ups per day) and the last part of the task (describing the accumulated push ups for the month)? Why do these relationships exist?

The Tortoise and the Hare

1.6 The Tortoise and the Hare

A Solidify Understanding Task

In the children's story of the tortoise and the hare, the hare mocks the tortoise for being slow. The tortoise replies, "Slow and steady wins the race." The hare says, "We'll just see about that," and challenges the tortoise to a race. The distance from the starting line of the hare is given by the function:

$d = t^2$ (d in meters and t in seconds)

Because the hare is so confident that he can beat the tortoise, he gives the tortoise a 1 meter head start. The distance from the starting line of the tortoise including the head start is given by the function:

$d = 2^t$ (d in meters and t in seconds)

1. At what time does the hare catch up to the tortoise?

2. If the race course is very long, who wins: the tortoise or the hare? Why?

3. At what time(s) are they tied?

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Tortoise and the Hare

- How are the contexts of the three tasks worked on today similar, and how are they different?
- How are the quadratic representations we have seen today similar and how are they different?
- How does quadratic and exponential behavior compare?

What Are the Features of Quadratic Functions?

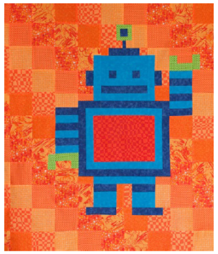
- Based on your work in these three tasks, what are some of the defining features of quadratic functions?
- More generally what ideas about function can we add to the work form Secondary Math I ?
- To this point, what similarities and differences do you notice in the nature of the work from Math I and Math II?

from *Infinity + 1*

“I believe in numbers. The ones you can see and the ones you can't. The real and the imaginary, the rational and the irrational, and every point on lines that go on forever. Numbers have never let me down. They don't waffle. They don't lie. They don't pretend to be what they're not. They're timeless.”

2.1 Transformers: Shifty y's

A Develop Understanding Task



Optima is designing a robot quilt for her new grandson. She plans for the robot to have a square face. The amount of fabric that she needs for the face will depend on the area of the face, so Optima decides to model the area of the robot's face mathematically. She knows that the area A of a square with side length x is modeled by the function, $A(x) = x^2$.

1. What is the domain of the function $A(x)$ in this context?

2. Match each statement about the area to the function that models it:

Matching Equation (A,B, C, or D)	Statement	Function Equation
	The length of any given side is increased by 5 units.	A) $A = 5x^2$
	The length of any given side is multiplied by 5 units.	B) $A = (x+5)^2$
	The area of a square is increased by 5 square units.	C) $A = (5x)^2$
	The area of a square is multiplied by 5.	D) $A = x^2 + 5$

Optima started thinking about the graph of $y = x^2$ (in the domain of all real numbers) and wondering about how changes to the equation of the function like adding 5 or multiplying by 5 affect the graph. She decided to make predictions about the effects and then check them out.

3. Predict how the graphs of each of the following equations will be the same or different from the graph of $y = x^2$.

	Similarities to the graph of $y = x^2$	**Differences from the graph of $y = x^2$**
$y = 5x^2$		
$y = (x+5)^2$		
$y = (5x)^2$		
$y = x^2 + 5$		

Transformers: Shifty y's

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Transformers: Shifty y's

- What is the goal of this task?
- What ideas are available for students after doing this task?

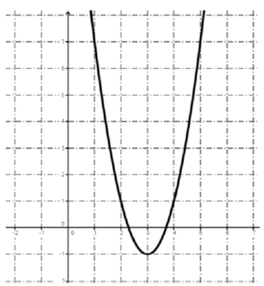
2.2 Transformers: More Than Meets the y's

A Solidify Understanding Task

Write the equation for each problem below. Use a second representation to check your equation.

1. The area of a square with side length *x*, where the side length is decreased by 3, the area is multiplied by 2 and then 4 square units are added to the area.

2.



3.

x	*f(x)*
-4	7
-3	2
-2	-1
-1	-2
0	-1
1	2
2	7
3	14
4	23

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Transformers: More Than Meets the y's

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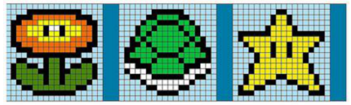
Building the Perfect Square

2.3 Building the Perfect Square

A Solidify Understanding Task

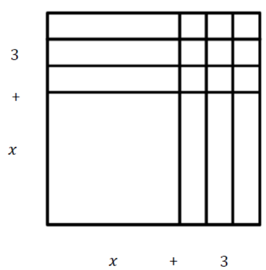


Part 1: Quadratic Quilts

Optima has a quilt shop where she sells many colorful quilt blocks for people who want to make their own quilts. She has quilt designs that are made so that the can be sized to fit any bed. She bases her designs on quilt squares that can vary in size, so she calls the length of the side for the basic square x, and the area of the basic square is the function $A(x) = x^2$. In this way, she can customize the designs by making bigger squares or smaller squares.

1. If Optima adds 3 inches to the side of the square, what is the area of the square?

When Optima draws a pattern for the square in problem #1, it looks like this:



2. Use both the diagram and the equation, $A(x) = (x+3)^2$ to explain why the area of the quilt block square, $A(x)$, is also equal to the $x^2 + 6x + 9$.

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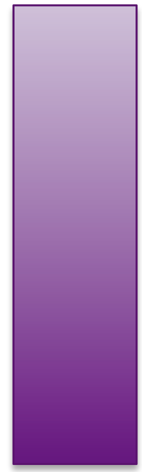
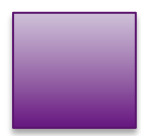
Putting Pieces Together



Rectangle
length = x units
width = 1 unit
Area = x square units

Square
side length = x units
Area = x^2 square units



Square
side length = 1 unit
Area = 1 square unit

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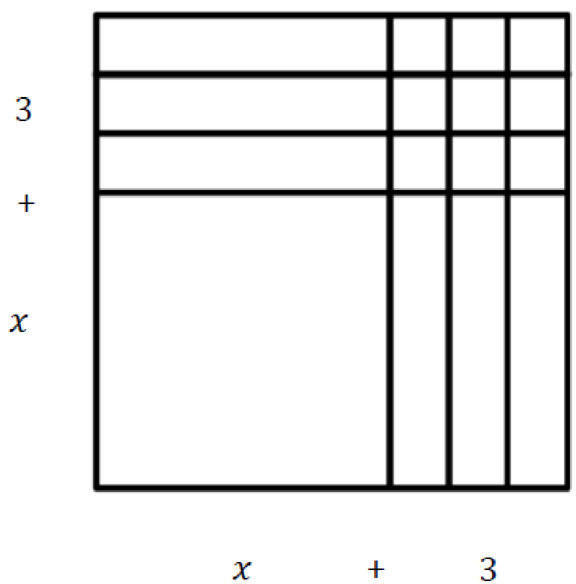


Building the Perfect Square

- What is the connection between completing the square and transformations?

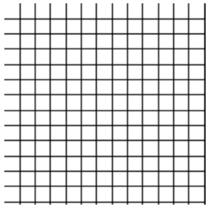
Throwing an Interception

3.5 Throwing an Interception

A Develop Understanding Task

The x-intercept(s) of the graph of a function $f(x)$ are often very important because they are the solution to the equation $f(x)=0$. In past tasks, we learned how to find the x-intercepts of the function by factoring, which works great for some functions, but not for others. In this task we are going to work on a process to find the x-intercepts of any quadratic function that has them. We'll start by thinking about what we already know about a few specific quadratic functions and then use what we know to generalize to all quadratic functions with x-intercepts.

1. What can you say about the graph of the function $f(x)=x^2-2x-3$?
   a. Graph the function

   b. What is the equation of the line of symmetry?
   c. What is the vertex of the function?

2. Now let's think specifically about the x-intercepts.
   a. What are the x-intercepts of $f(x)=x^2-2x-3$?
   b. How far are the x-intercepts from the line of symmetry?
   c. If you knew the line of symmetry was the line $x = h$, and you know how far the x-intercepts are from the line of symmetry, how would you find the actual x-intercepts?
   d. How far above the vertex are the x-intercepts?
   e. What is the value of $f(x)$ at the x-intercepts?

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Throwing an Interception

- What is the main focus of this task?
- How would you describe the approach to finding the x-intercepts?
- How is the need for the quadratic formula motivated?

Throwing an Interception

- What is the main focus of this task?
- How is the need for the quadratic formula motivated?

Building Procedural Fluency from Conceptual Understanding

"Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems."

Principles to Actions: Ensuring Mathematical Success for All, National Council Teachers of Mathematics 2014

Building Procedural Fluency from Conceptual Understanding

What are the foundational concepts for the procedures in these tasks?

- Graphing parabolas using transformations
- Completing the square
- Quadratic formula

Building Procedural Fluency from Conceptual Understanding

- How does the conceptual approach based on students' knowledge of functions provide access to the procedures?
- How does the conceptual approach based on students' knowledge of functions provide motivation for the procedures?