mathematics vision project

Transforming Mathematics Education

GEOMETRY

A Learning Cycle Approach

MODULE 6

Connecting Algebra and Geometry

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

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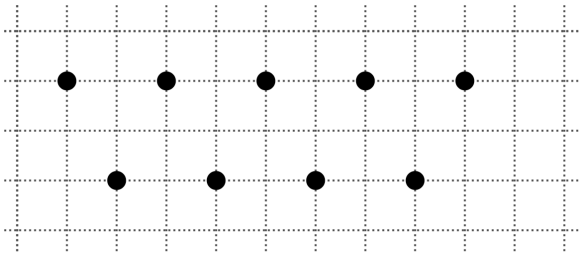
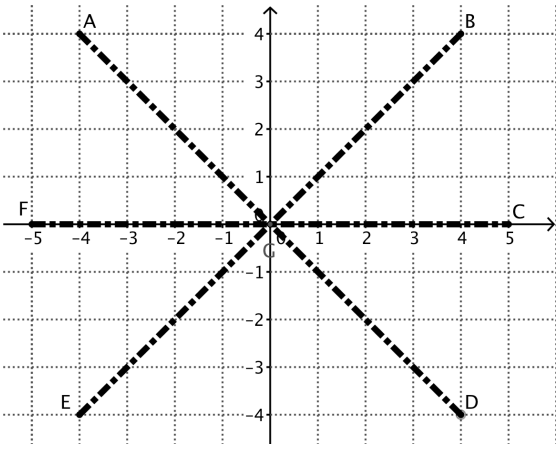
6.1 Go the Distance

A Develop Understanding Task

The performances of the Podunk High School drill team are very popular during half-time at the school's football and basketball games. When the Podunk High School drill team choreographs the dance moves that they will do on the football field, they lay out their positions on a grid like the one below:



In one of their dances, they plan to make patterns holding long, wide ribbons that will span from one dancer in the middle to six other dancers. On the grid, their pattern looks like this:



The question the dancers have is how long to make the ribbons. Gabriela (G) is standing in the center and some dancers think that the ribbon from Gabriela (G) to Courtney (C) will be shorter than the one from Gabriela (G) to Brittney (B).

1. How long does each ribbon need to be?

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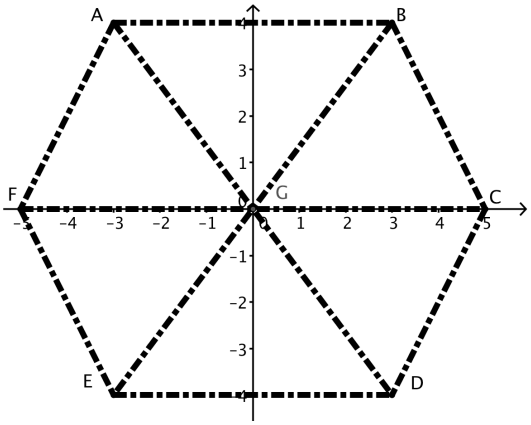
2. Explain how you found the length of each ribbon.

When they have finished with the ribbons in this position, they are considering using them to form a new pattern like this:



3. Will the ribbons they used in the previous pattern be long enough to go between Britney (B) and Courtney (C) in the new pattern? Explain your answer.

Gabriela notices that the calculations she is making for the length of the ribbons reminds her of math class. She says to the group, "Hey, I wonder if there is a process that we could use like what we have been doing to find the distance between any two points on the grid." She decides to think about it like this:

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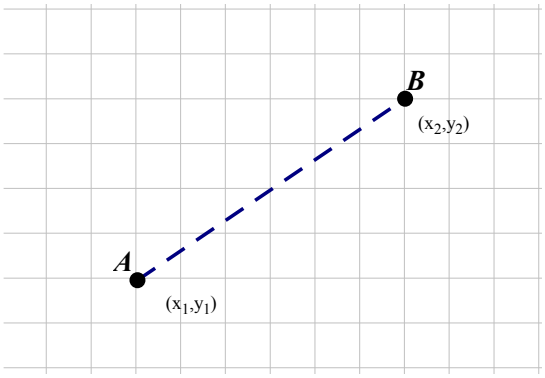
"I'm going to start with two points and draw the line between them that represents the distance that I'm looking for. Since these two points could be anywhere, I named them A (x_1,y_1) and B (x_2,y_2). Hmmmmm. . . . when I figured the length of the ribbons, what did I do next?"



4. Think back on the process you used to find the length of the ribbon and write down your steps here, in terms of (x_1, y_1) and (x_2, y_2).

5. Use the process you came up with in #4 to find the distance between two points located far enough away from each other that using your formula from #4 is more efficient than graphing and counting. For example find the distance between (-11, 25) and (23, -16)

6. Use your process to find the perimeter of the hexagon pattern shown in #3.

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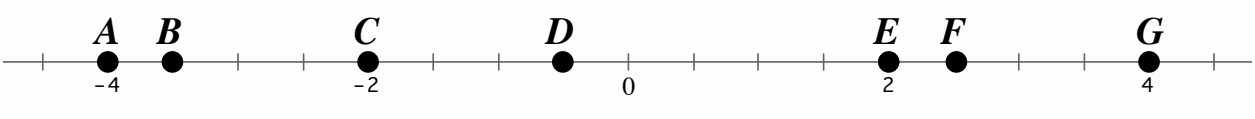
READY, SET, GO! **Name** **Period** **Date**

READY

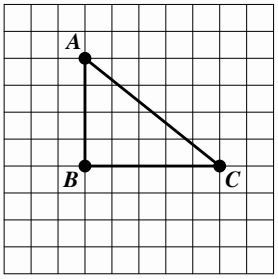
Topic: Finding the distance between two points

Use the number line to find the distance between the given points. (The notation AB means the distance between the points A and B.)

1. AE 2. CF 3. GB 4. CA 5. BF 6. EG



7. Describe a way to find the distance between two points on a number line without counting the spaces.

8. 

a. Find AB.

b. Find BC.

c. Find AC.

9. Why is it easier to find the distance between point A and point B and point B and point C than it is to find the distance between point A and point C?

10. Explain how to find the distance between point A and point C.

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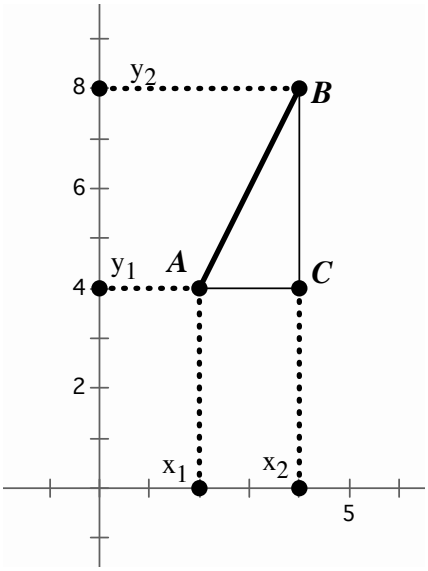
6.1

SET

Topic: Slope triangles and the distance formula

Triangle ABC is a slope triangle for the line segment AB where BC is the rise and AC is the run. Notice that the length of segment BC has a corresponding length on the y-axis and the length of AC has a corresponding length on the x-axis. The slope formula is written as $m = \frac{y_2 - y_1}{x_2 - x_1}$ *where m is the slope.*

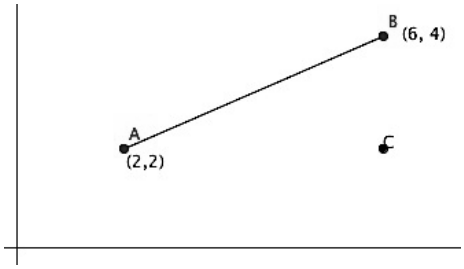


11. a. What does the value $(y_2 - y_1)$ tell you?

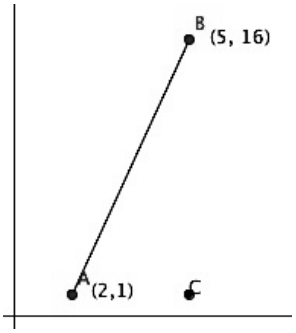
b. What does the value $(x_2 - x_1)$ tell you?

In the previous unit you found the length of a slanted line segment by drawing the slope triangle and then using the Pythagorean theorem on the two sides of the triangle. In this exercise, try to develop a more efficient method of calculating the length of a line segment by using the meaning of $(y_2 - y_1)$ *and* $(x_2 - x_1)$ *combined with the Pythagorean theorem.*

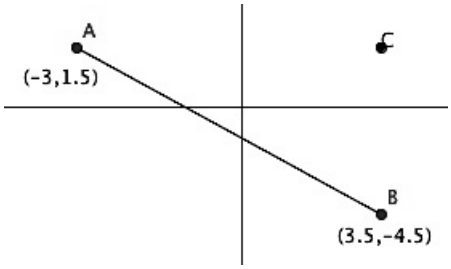
12. Find AB.



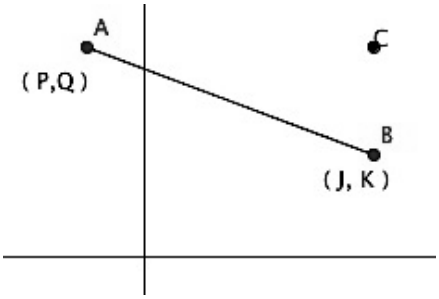
13. Find AB.



14. Find AB.



15. Find AB.



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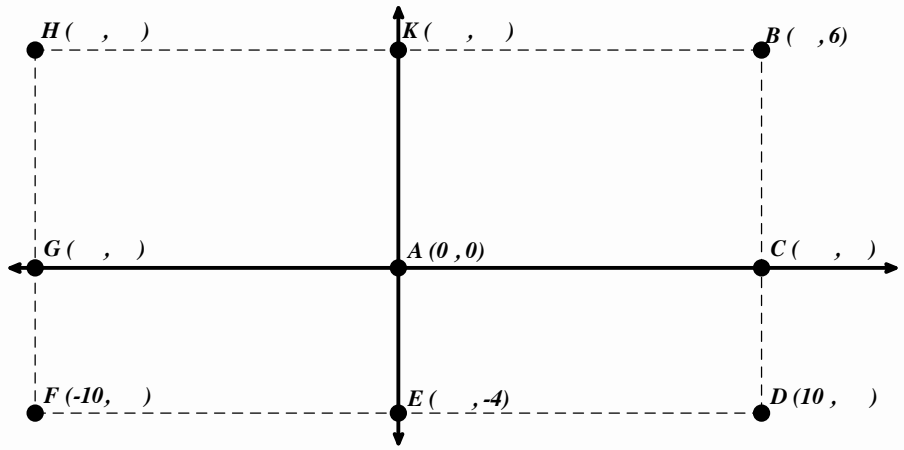
GO

Topic: Rectangular coordinates

Use the given information to fill in the missing coordinates. Then find the length of the indicated line segment.

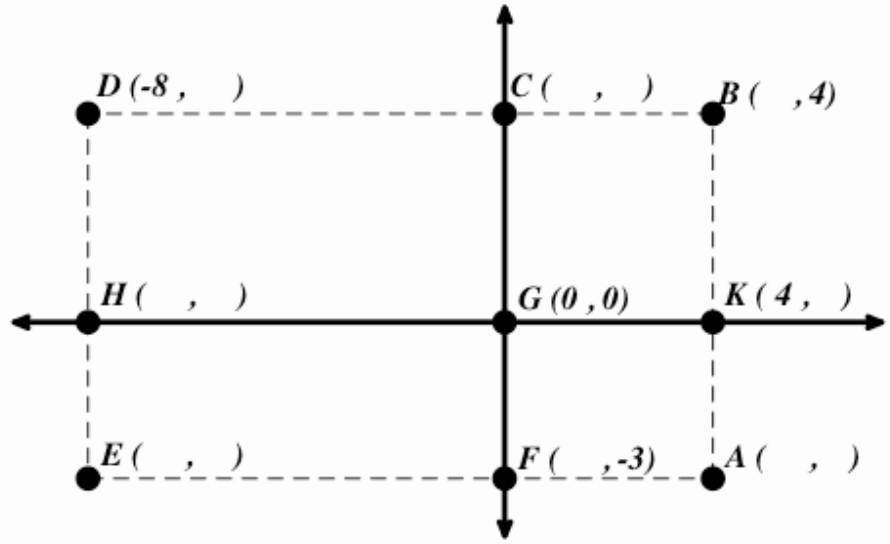
16. a) Find HB.

b) Find BD.



17. a) Find DB

b) Find CF



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6.2 Slippery Slopes

A Solidify Understanding Task

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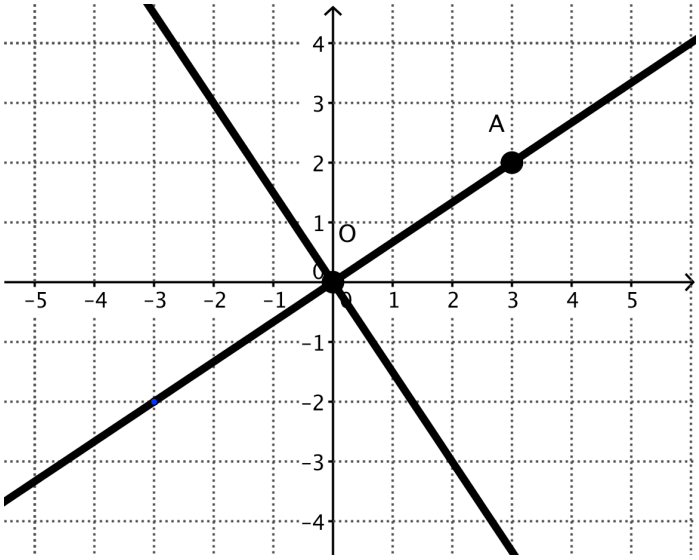
While working on "Is It Right?" in a previous module you looked at several examples that lead to the conclusion that the slopes of perpendicular lines are negative reciprocals. Your work here is to formalize this work into a proof. Let's start by thinking about two perpendicular lines that intersect at the origin, like these:



1. Start by drawing a right triangle with the segment $\overline{OA}$ as the hypotenuse. These are often called slope triangles. Based on the slope triangle that you have drawn, what is the slope of $\overleftrightarrow{OA}$?

2. Now, rotate the slope triangle 90° about the origin. What are the coordinates of the image of point A?

3. Using this new point, A', draw a slope triangle with hypotenuse $\overline{OA'}$. Based on the slope triangle, what is the slope of the line $\overleftrightarrow{OA'}$?

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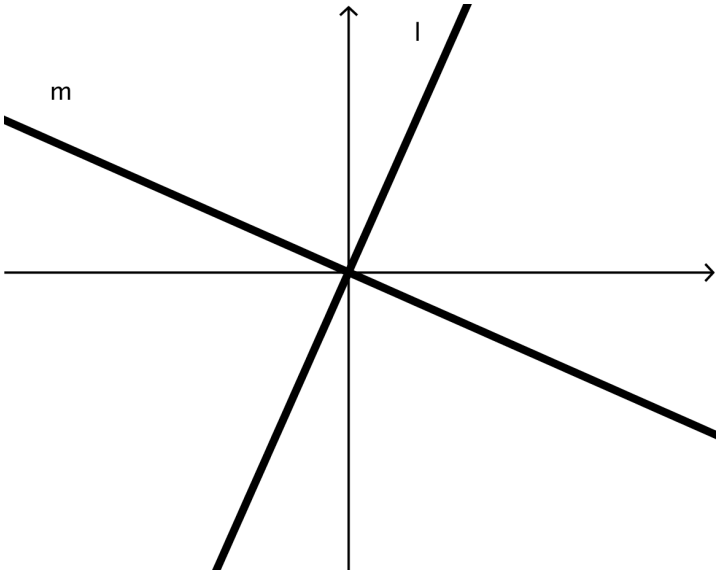
4. What is the relationship between these two slopes? How do you know?

5. Is the relationship changed if the two lines are translated so that the intersection is at (-5, 7)? How do you know?

To prove a theorem, we need to demonstrate that the property holds for any pair of perpendicular lines, not just a few specific examples. It is often done by drawing a very similar picture to the examples we have tried, but using variables instead of numbers. Using variables represents the idea that it doesn't matter which numbers we use, the relationship stays the same. Let's try that strategy with the theorem about perpendicular lines having slopes that are negative reciprocals.



- Lines *l* and *m* are constructed to be perpendicular.
- Start by labeling a point P on the line *l.*
- Label the coordinates of P.
- Draw the slope triangle from point P.
- Label the lengths of the sides of the slope triangle using variables like *a* and *b* for the run and the rise.

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6. What is the slope of line *l*?

   Rotate point P 90° about the origin, label it P' and mark it on line m. What are the coordinates of P'?

7. Draw the slope triangle from point P'. What are the lengths of the sides of the slope triangle? How do you know?

8. What is the slope of line *m*?

9. What is the relationship between the slopes of line *l* and line *m*? How do you know?

10. Is the relationship between the slopes changed if the intersection between line *l* and line *m* is translated to another location? How do you know?

11. Is the relationship between the slopes changed if lines *l* and *m* are rotated?

12. How do these steps demonstrate that the slopes of perpendicular lines are negative reciprocals for any pair of perpendicular lines?

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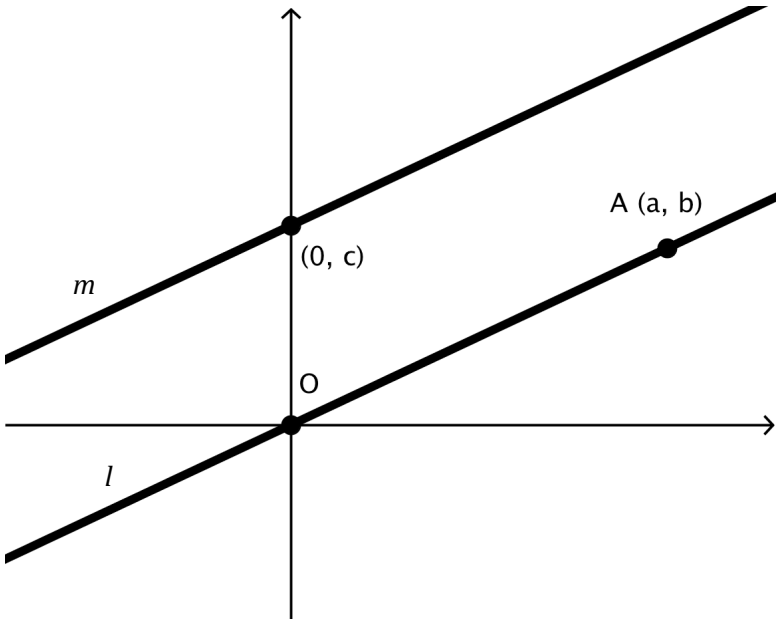
Think now about parallel lines like the ones below.



13. Draw the slope triangle from point A to the origin. What is the slope of $\overleftrightarrow{OA}$?

14. What transformation(s) maps the slope triangle with hypotenuse $\overline{OA}$ onto the other line m?

15. What must be true about the slope of line l? Why?

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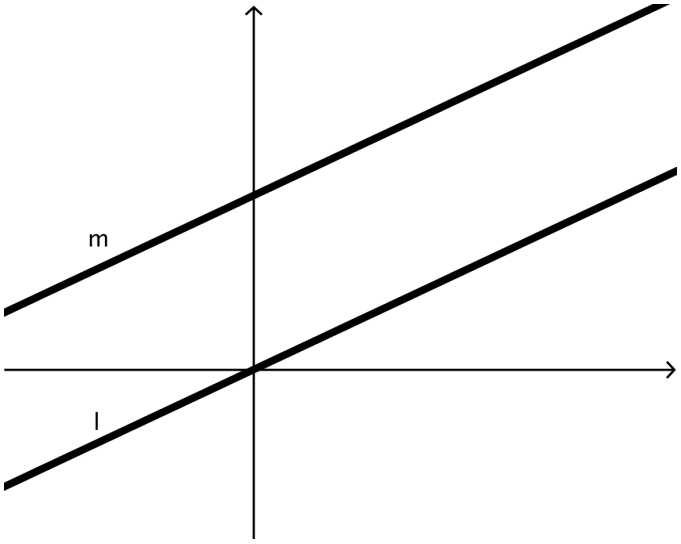
Now you're going to try to use this example to develop a proof, like you did with the perpendicular lines. Here are two lines that have been constructed to be parallel.



16. Show how you know that these two parallel lines have the same slope and explain why this proves that all parallel lines have the same slope.

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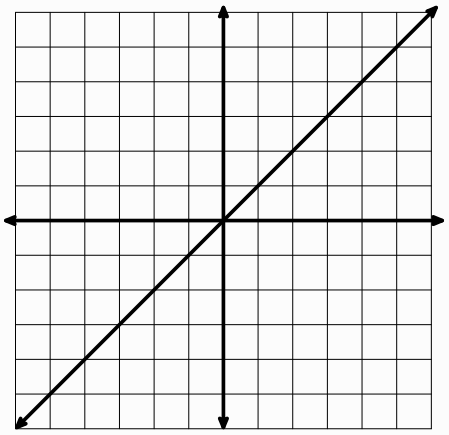
6.2

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Using translations to graph lines



The equation of the line in the graph is $y = x$.

1. a) On the same grid graph a parallel line that is 3 units above it .

   b) Write the equation for the new line in slope-intercept form.

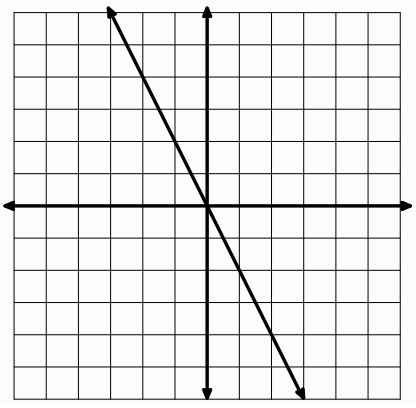
   c) Write the y-intercept of the new line as an ordered pair.

   d) Write the x-intercept of the new line as an ordered pair.

   e) Write the equation of the new line in point-slope form using the *y-intercept.*

   f) Write the equation of the new line in point-slope form using the *x-intercept.*

   g) Explain in what way the equations are the same and in what way they are different.



The graph at the right shows the line $y = -2x$.

2. a) On the same grid, graph a parallel line that is 4 units below it.

   b) Write the equation of the new line in slope-intercept form.

   c) Write the y-intercept of the new line as an ordered pair.

   d) Write the x-intercept of the new line as an ordered pair.

   e) Write the equation of the new line in point-slope form using the *y-intercept.*

   f) Write the equation of the new line in point-slope form using the *x-intercept.*

   g) Explain in what way the equations are the same and in what way they are different.

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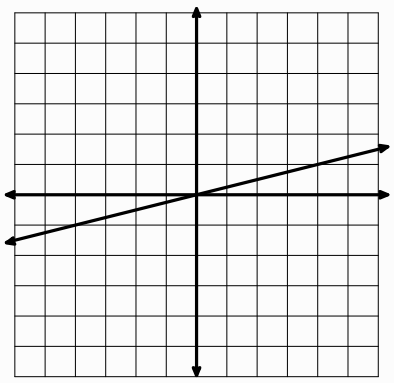
The graph at the right shows the line $y = \frac{1}{4}x$.



3. a) On the same grid, graph a parallel line that is 2 units below it.

   b) Write the equation of the new line in slope-intercept form.

   c) Write the y-intercept of the new line as an ordered pair.

   d) Write the x-intercept of the new line as an ordered pair.

   e) Write the equation of the new line in point-slope form using the *y-intercept.*

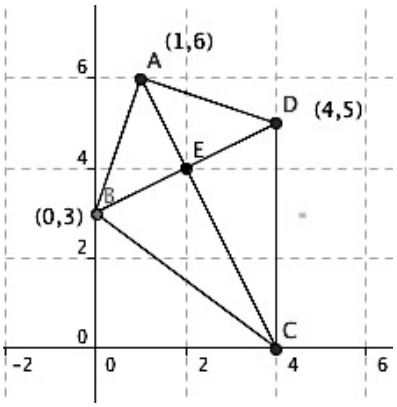
   f) Write the equation of the new line in point-slope form using the *x-intercept.*

   g) Explain in what way the equations are the same and in what way they are different.

SET

Topic: Verifying and proving geometric relationships

The quadrilateral at the right is called a **kite.**
Complete the mathematical statements about the kite using the given symbols. Prove each statement algebraically. (A symbol may be used more than once.)



$\cong$ $\perp$ $\parallel$ $<$ $>$ $=$

Proof

4. $\overline{BC}$_____$\overline{DC}$ _____

5. $\overline{BD}$_____$\overline{AC}$ _____

6. $\overline{AB}$_____$\overline{BC}$ _____

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7. ΔABC_____ ΔADC

8. $\overline{BE}$_____$\overline{ED}$

9. $\overline{AE}$_____$\overline{ED}$

10. $\overline{AC}$_____$\overline{BD}$

GO

Topic: Writing equations of lines

Use the given information to write the equation of the line in standard form. $(Ax + By = C)$

11. $Slope: -\frac{1}{4}$ $point\ (12, 5)$

12. $P(11, -3),\ Q(6, 2)$

13. $x - intercept: -2;\ y - intercept: -3$

14. $All\ x\ values\ are\ (-7).\ Y\ is\ any\ number.$

15. $Slope: \frac{1}{2};\ x - intercept: 5$

16. $E\ (-10, 17),\ G\ (13, 17)$

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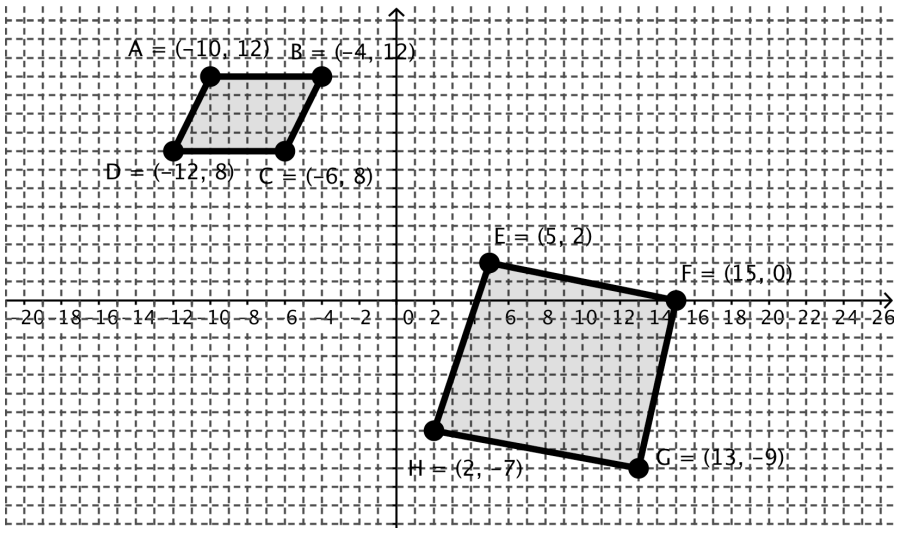
6.3 Prove It!

A Practice Understanding Task

In this task you need to use all the things you know about quadrilaterals, distance, and slope to prove that the shapes are parallelograms, rectangles, rhombi, or squares. Be systematic and be sure that you give all the evidence necessary to verify your claim.

1.



a. Is ABCD a parallelogram? Explain how you know.

b. Is EFGH a parallelogram? Explain how you know.

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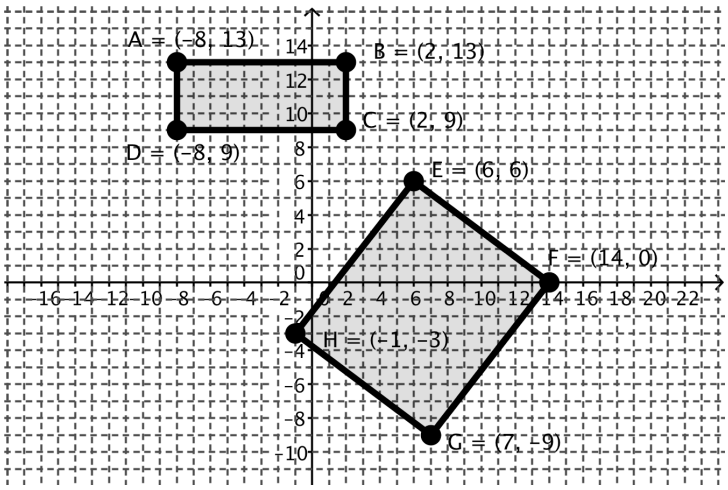
2.



a. Is ABCD a rectangle? Explain how you know.

b. Is EFGH a rectangle? Explain how you know.

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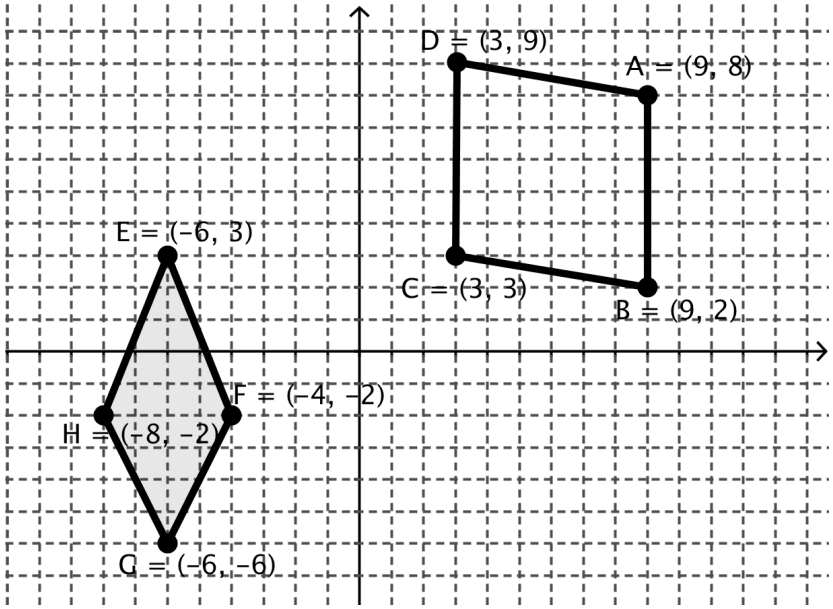
3.



a. Is ABCD a rhombus? Explain how you know.

b. Is EFGH a rhombus? Explain how you know.

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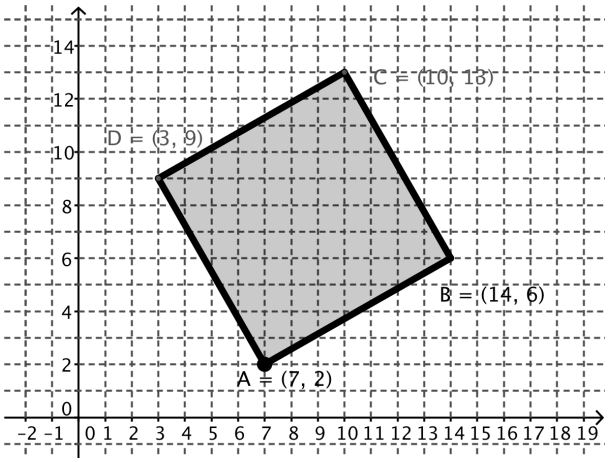
4.



a. Is ABCD a square? Explain how you know.

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6.3

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Interpreting tables of value as ordered pairs.

Find the value of $f(x)$ for the given domain. Write x and $f(x)$ as an ordered pair.

1. $f(x) = 3x - 2$

x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

2. $f(x) = x^2$

x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

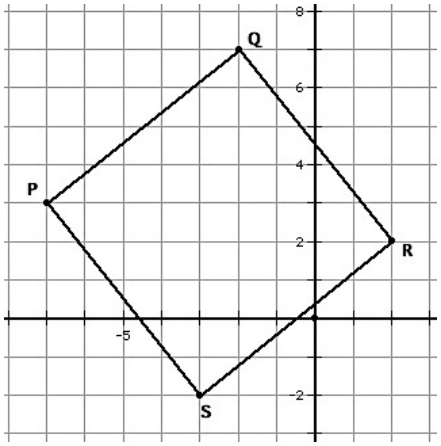
3. $f(x) = 5^x$

x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

SET

Topic: Identifying specific quadrilaterals

4. a) Is the figure at the right a rectangle? Justify your answer.



b) Is the figure at the right a rhombus? Justify your answer.

c) Is the figure at the right a square? Justify your answer.

GO

Topic: Calculating perimeters of geometric shapes

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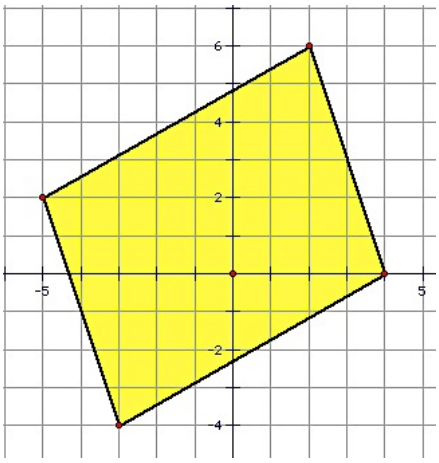
Find the perimeter of each figure below. Round answers to the nearest hundredth.

5.



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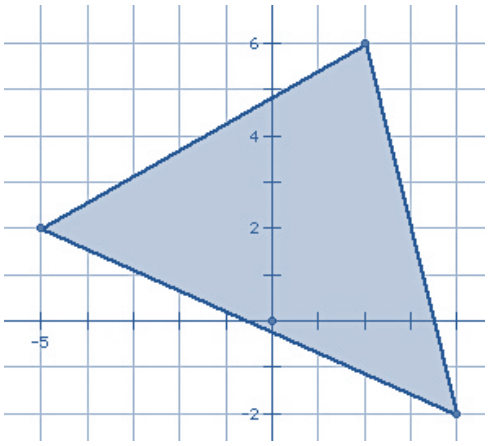

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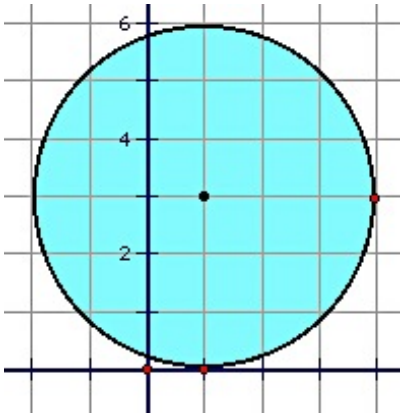

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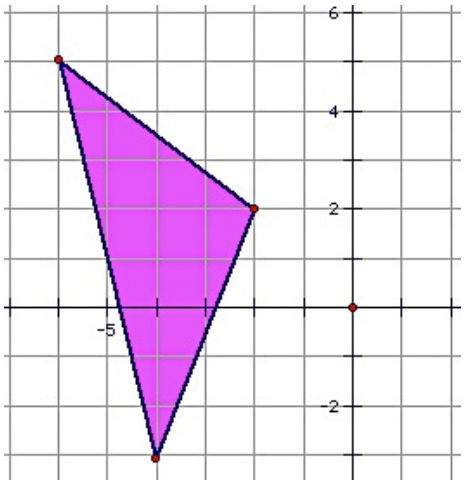

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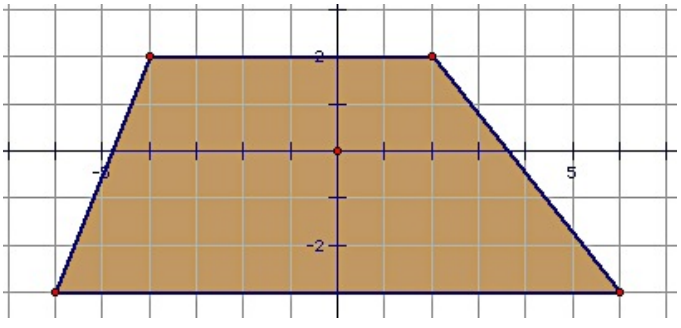

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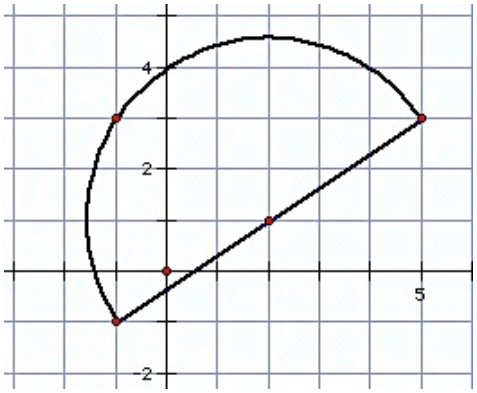

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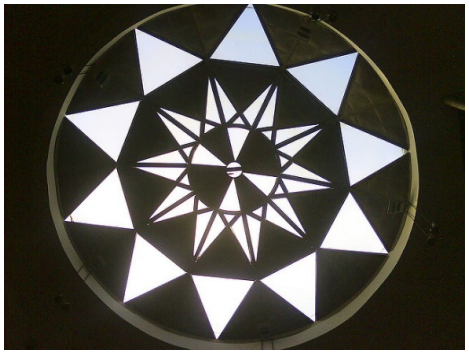

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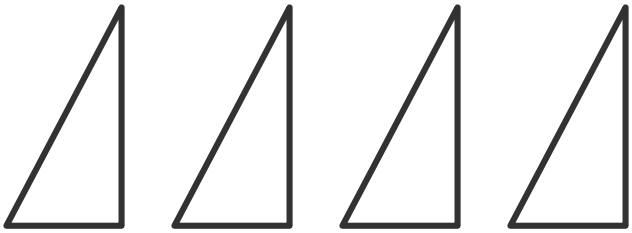
6.4 Circling Triangles (Or Triangulating Circles)

A Develop Understanding Task

Using the corner of a piece of colored paper and a ruler, cut a right triangle with a 6" hypotenuse, like so:

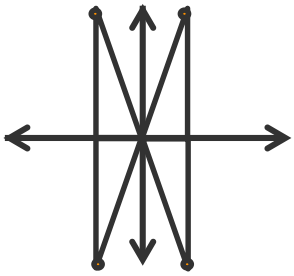
Use this triangle as a pattern to cut three more just like it, so that you have a total of four congruent triangles.



1. Choose one of the legs of the first triangle and label it *x* and label the other leg *y*. What is the relationship between the three sides of the triangle?

2. When you are told to do so, take your triangles up to the board and place each of them on the coordinate axis like this:

   Mark the point at the end of each hypotenuse with a pin.



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3. What shape is formed by the pins after the class has posted all of their triangles? Why would this construction create this shape?

4. What are the coordinates of the pin that you placed in:
   a. the first quadrant?
   b. the second quadrant?
   c. the third quadrant?
   d. the fourth quadrant?

5. Now that the triangles have been placed on the coordinate plane, some of your triangles have sides that are of length $-x$ or $-y$. Is the relationship $x^2 + y^2 = 6^2$ still true for these triangles? Why or why not?

6. What would be the equation of the graph that is the set on all points that are 6" away from the origin?

7. Is the point (0, -6) on the graph? How about the point (3, 5.193)? How can you tell?

8. If the graph is translated 3 units to the right and 2 units up, what would be the equation of the new graph? Explain how you found the equation.

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6.4

READY, SET, GO! Name Period Date

READY

Topic: Factoring special products

Factor the following as the difference of 2 squares or as a perfect square trinomial. Do not factor if they are neither.

$b^2 - 49$

$b^2 - 2b + 1$

$b^2 + 10b + 25$

$x^2 - y^2$

$x^2 - 2xy + y^2$

$25x^2 - 49y^2$

$36x^2 + 60xy + 25y^2$

$81a^2 - 16d^2$

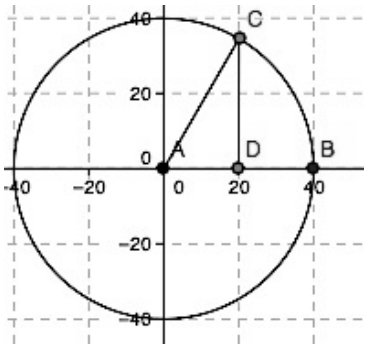
$144x^2 - 312xy + 169y^2$

SET

Topic: Writing the equations of circles

Write the equation of each circle centered at the origin.

10.



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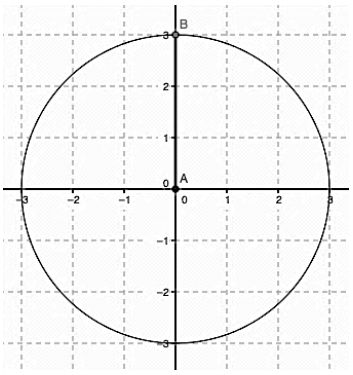

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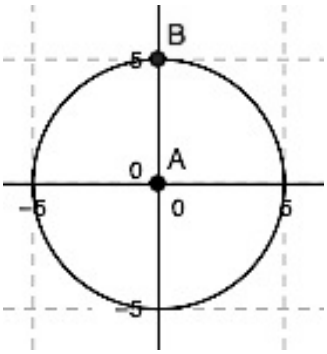

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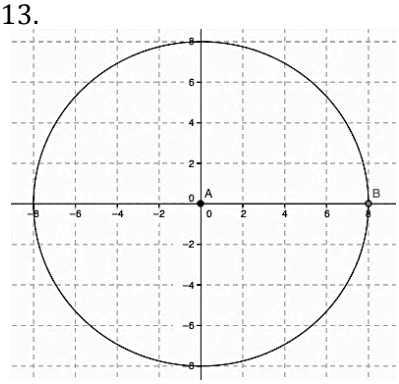

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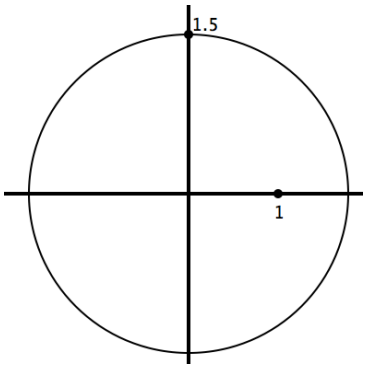

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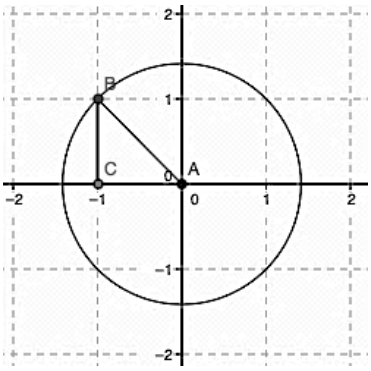

GO

Topic: Verifying Pythagorean triples

Identify which sets of numbers could be the sides of a right triangle. Show your work.

16. $\{9, 12, 15\}$

17. $\{9, 10, \sqrt{19}\}$

18. $\{1, \sqrt{3}, 2\}$

19. $\{2, 4, 6\}$

20. $\{\sqrt{3}, 4, 5\}$

21. $\{10, 24, 26\}$

22. $\{\sqrt{2}, \sqrt{7}, 3\}$

23. $\{2\sqrt{2}, 5\sqrt{3}, 9\}$

24. $\{4ab^3\sqrt{10}, 6ab^3, 14ab^3\}$

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6.5 Getting Centered

A Solidify Understanding Task

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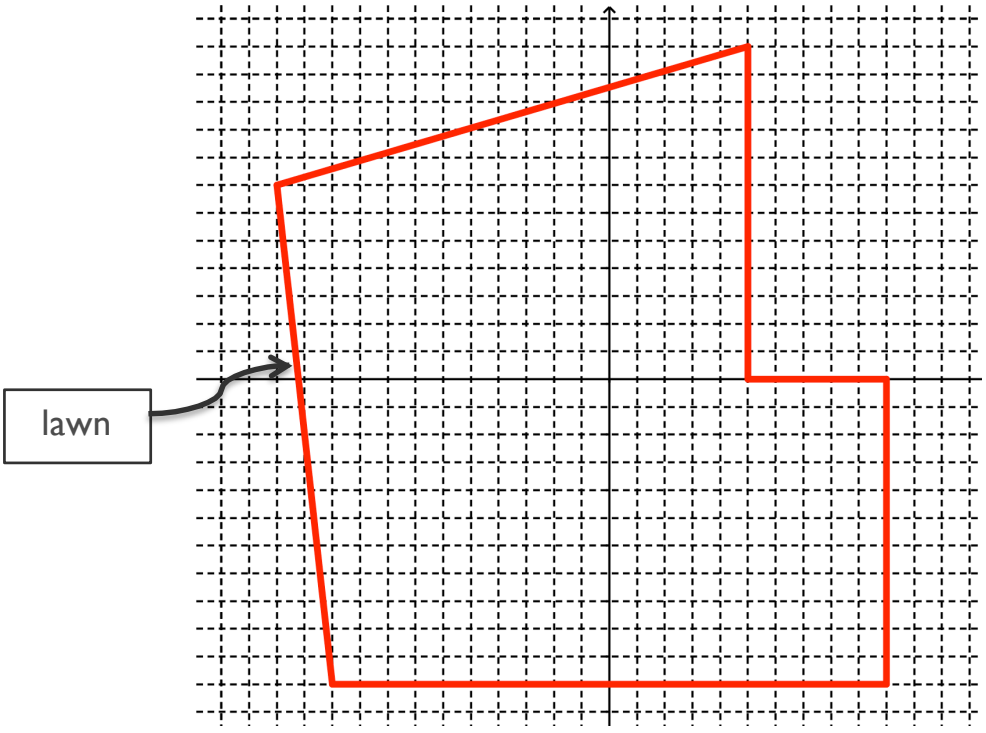
Malik's family has decided to put in a new sprinkling system in their yard. Malik has volunteered to lay the system out. Sprinklers are available at the hardware store in the following sizes:

- Full circle, maximum 15' radius
- Half circle, maximum 15' radius
- Quarter circle, maximum 15' radius

All of the sprinklers can be adjusted so that they spray a smaller radius. Malik needs to be sure that the entire yard gets watered, which he knows will require that some of the circular water patterns will overlap. He gets out a piece of graph paper and begins with a scale diagram of the yard. In this diagram, the length of the side of each square represents 5 feet.



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1. As he begins to think about locating sprinklers on the lawn, his parents tell him to try to cover the whole lawn with the fewest number of sprinklers possible so that they can save some money. The equation of the first circle that Malik draws to represent the area watered by the sprinkler is:

   $$(x + 25)^2 + (y + 20)^2 = 225$$

   Draw this circle on the diagram using a compass.

2. Lay out a possible configuration for the sprinkling system that includes the first sprinkler pattern that you drew in #1.

3. Find the equation of each of the full circles that you have drawn.

Malik wrote the equation of one of the circles and just because he likes messing with the algebra, he did this:

Original equation: $(x - 3)^2 + (y + 2)^2 = 225$

$x^2 - 6x + 9 + y^2 + 4y + 4 = 225$

$x^2 + y^2 - 6x + 4y - 212 = 0$

Malik thought, "That's pretty cool. It's like a different form of the equation. I guess that there could be different forms of the equation of a circle like there are different forms of the equation of a parabola or the equation of a line." He showed his equation to his sister, Sapana,

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and she thought he was nuts. Sapana, said, "That's a crazy equation. I can't even tell where the center is or the length of the radius anymore." Malik said, "Now it's like a puzzle for you. I'll give you an equation in the new form. I'll bet you can't figure out where the center is."

Sapana said, "Of course, I can. I'll just do the same thing you did, but work backwards."

4. Malik gave Sapana this equation of a circle:

$$x^2 + y^2 - 4x + 10y + 20 = 0$$

Help Sapana find the center and the length of the radius of the circle.

5. Sapana said, "Ok. I made one for you. What's the center and length of the radius for this circle?"

$$x^2 + y^2 + 6x - 14y - 42 = 0$$

6. Sapana said, "I still don't know why this form of the equation might be useful. When we had different forms for other equations like lines and parabolas, each of the various forms highlighted different features of the relationship." Why might this form of the equation of a circle be useful?

$$x^2 + y^2 + Ax + By + C = 0$$

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6.5

READY, SET, GO! Name Period Date

READY

Topic: Making perfect square trinomials

Fill in the number that completes the square. Then write the trinomial in factored form.

1. x^2+6x+_____

2. x^2-14x+_____

3. x^2-50x+_____

4. x^2-28x+_____

On the next set, leave the number that completes the square as a fraction. Then write the trinomial in factored form.

5. x^2-11x+_____

6. x^2+7x+_____

7. x^2+15x+_____

8. $x^2+\frac{2}{3}x+$_____

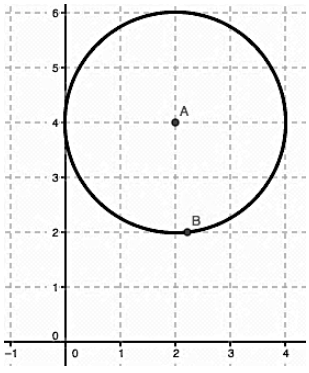
9. $x^2-\frac{1}{5}x+$_____

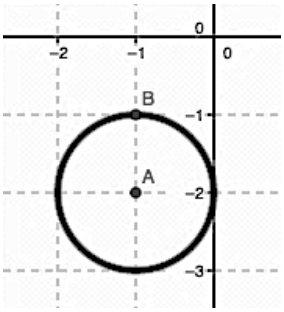
10. $x^2-\frac{3}{4}x+$_____

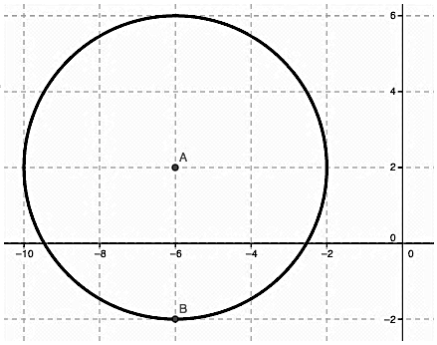
SET

Topic: Writing equations of circles with center (h, k) and radius *r*.

Write the equation of each circle.

11. 

12. 

13. 

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Write the equation of the circle with the given center and radius. Then write it in expanded form.

14. Center: (5, 2) Radius: 13

15. Center: (-6, -10) Radius: 9

16. Center: (0, 8) Radius: 15

17. Center: (19, -13) Radius: 1

18. Center: (-1, 2) Radius: 10

19. Center: (-3, -4) Radius: 8

Go

Topic: Verifying if a point is a solution

Identify which point is a solution to the given equation. Show your work.

20. $y = \frac{4}{5}x - 2$ a. (-15, -14) b. (10, 10)

21. $y = 3|x|$ a. (-4, -12) b. $(-\sqrt{5}, 3\sqrt{5})$

22. $y = x^2 + 8$ a. $(\sqrt{7}, 15)$ b. $(\sqrt{7}, -1)$

23. $y = -4x^2 + 120$ a. $(5\sqrt{3}, -180)$ b. $(5\sqrt{3}, 40)$

24. $x^2 + y^2 = 9$ a. $(8, -1)$ b. $(-2, \sqrt{5})$

25. $4x^2 - y^2 = 16$ a. $(-3, \sqrt{10})$ b. $(-2\sqrt{2}, 4)$

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6.6 Circle Challenges

A Practice Understanding Task

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Once Malik and Sapana started challenging each other with circle equations, they got a little more creative with their ideas. See if you can work out the challenges that they gave each other to solve. Be sure to justify all of your answers.

1. Malik's challenge:
   What is the equation of the circle with center (-13,-16) and containing the point (-10,-16) on the circle?

2. Sapana's challenge:
   The points (0, 5) and (0,-5) are the endpoints of the diameter of a circle. The point $(3, y)$ is on the circle. What is a value for y?

3. Malik's challenge:
   Find the equation of a circle with center in the first quadrant and is tangent to the lines $x = 8$, $y = 3$, and $x = 14$.

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4. Sapana's challenge:
   The points (4,-1) and (-6,7) are the endpoints of the diameter of a circle. What is the equation of the circle?

5. Malik's challenge:
   Is the point (5,1) inside, outside, or on the circle $x^2 - 6x + y^2 + 8y = 24$? How do you know?

6. Sapana's challenge:
   The circle defined by $(x-1)^2 + (y+4)^2 = 16$ is translated 5 units to the left and 2 units down. Write the equation of the resulting circle.

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7. Malik's challenge:

   There are two circles, the first with center (3,3) and radius r_1, and the second with center (3, 1) and radius r_2.

   a. Find values r_1 and r_2 of so that the first circle is completely enclosed by the second circle.

   b. Find one value of r_1 and one value of r_2 so that the two circles intersect at two points.

   c. Find one value of r_1 and one value of r_2 so that the two circles intersect at exactly one point.

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6.6

READY, SET, GO! Name Period Date

READY

Topic: Finding the distance between two points

Simplify. Use the distance formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ to find the distance between the given points. Leave your answer in simplest radical form.

1. $A(18,-12)\ B(10,4)$
2. $G(-11,-9)\ H(-3,7)$
3. $J(14,-20)\ K(5,5)$
4. $M(1,3)\ P(-2,7)$
5. $Q(8,2)\ R(3,7)$
6. $S(-11,2\sqrt{2})\ T(-5,-4\sqrt{2})$
7. $W(-12,-2\sqrt{2})\ Z(-7,-3\sqrt{2})$

SET

Topic: Writing equations of circles

Use the information provided to write the equation of the circle in standard form,

$$(x-h)^2+(y-h)^2=r^2$$

8. Center $(-16,-5)$ and the circumference is 22π
9. Center $(13,-27)$ and the area is 196π
10. Diameter measures 15 units and the center is at the intersection of y = x + 7 and y = 2x – 5
11. Lies in quadrant 2 Tangent to $x=-12$ *and* $x=-4$

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12. Center (-14 , 9) Point on circle (1 , 11)

13. Center lies on the y axis Tangent to y = -2 and y = -17

14. Three points on the circle are $(-8,5),(3,-6),(14,5)$

15. I know three points on the circle are (-7,6), (9,6), and (-4,13). I think that the equation of the circle is $(x-1)^2+(y-6)^2=64$. Is this the correct equation for the circle? Justify your answer.

GO

Topic: Finding the value of ***B*** in a quadratic in the form of Ax^2+Bx+C in order to create a perfect square trinomial.

Find the value of *B* that will make a perfect square trinomial. Then write the trinomial in factored form.

16. $x^2+_____x+36$

17. $x^2+_____x+100$

18. $x^2+_____x+225$

19. $9x^2+_____x+225$

20. $16x^2+_____x+169$

21. $x^2+_____x+5$

22. $x^2+_____x+\frac{25}{4}$

23. $x^2+_____x+\frac{9}{4}$

24. $x^2+_____x+\frac{49}{4}$

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6.7 Directing Our Focus

A Develop Understanding Task

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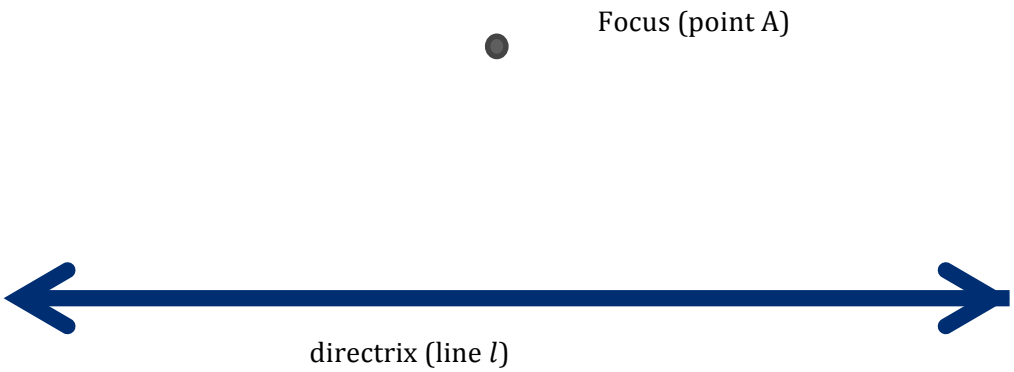
On a board in your classroom, your teacher has set up a point and a line like this:



We're going to call the line a directrix and the point a focus. They've been labeled on the drawing.

Similar to the circles task, the class is going to construct a geometric figure using the focus (point A) and directrix (line l).

1. Cut two pieces of string with the same length.

2. Mark the midpoint of each piece of string with a marker.

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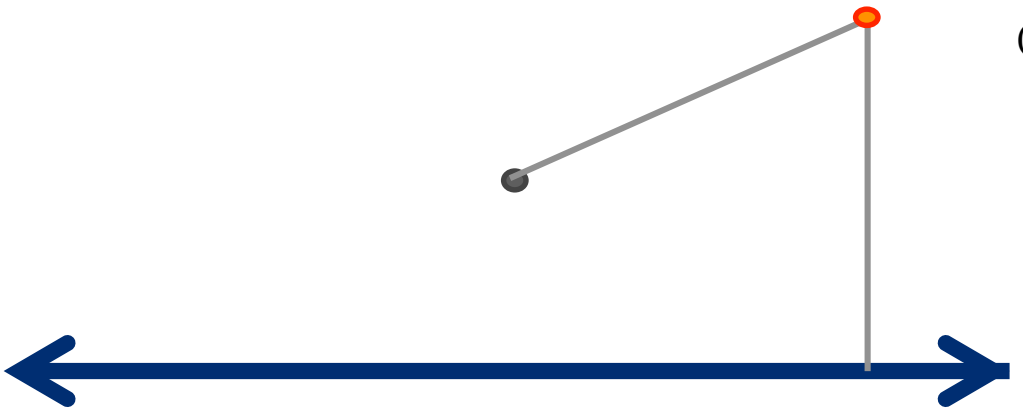
3. Position the string on the board so that the midpoint is equidistant from the focus (point A) and the directrix (line l), which means that it must be perpendicular to the directrix. While holding the string in this position, put a pin through the midpoint. Depending on the size of your string, it will look something like this:



4. Using your second string, use the same procedure to post a pin on the other side of the focus.

5. As your classmates post their strings, what geometric figure do you predict will be made by the tacks (the collection of all points like (x, y) show in the figure above)? Why?

6. Where is the vertex of the figure located? How do you know?

7. Where is the line of symmetry located? How do you know?

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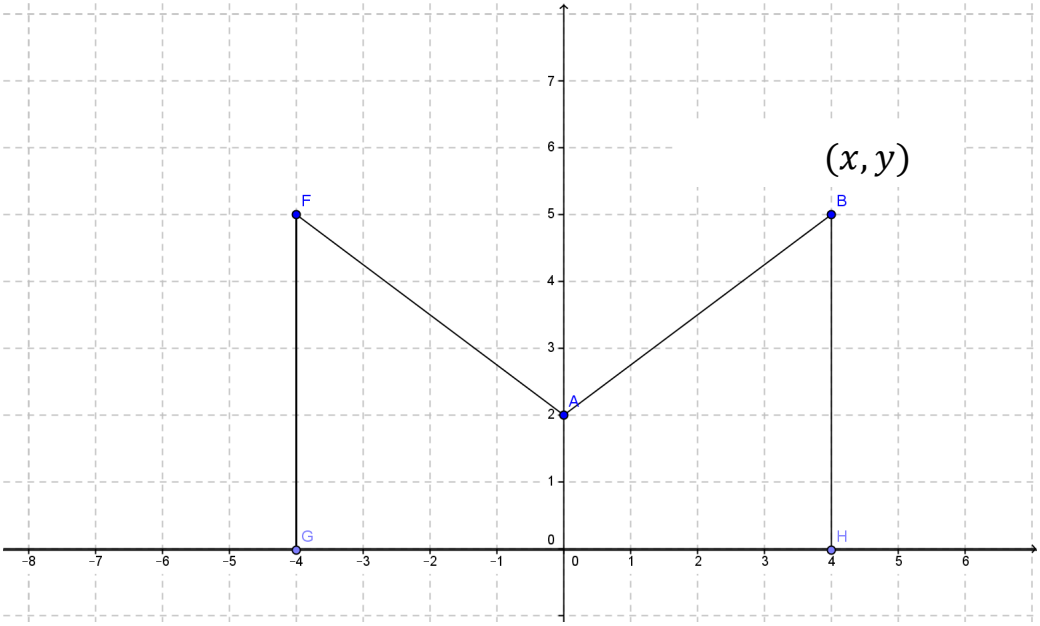
8. Consider the following construction with focus point A and the x-axis as the directrix. Use a ruler to complete the construction of the parabola in the same way that the class constructed the parabola with string.



9. You have just constructed a parabola based upon the definition: A parabola is the set of all points (x, y) equidistant from a line l (the directrix) and a point not on the line (the focus). Use this definition to write the equation of the parabola above, using the point (x, y) to represent any point on the parabola.

10. How would the parabola change if the focus was moved up, away from the directrix?

11. How would the parabola change if the focus were to be moved down, toward the directrix?

12. How would the parabola change if the focus were to be moved down, below the directrix?

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6.7

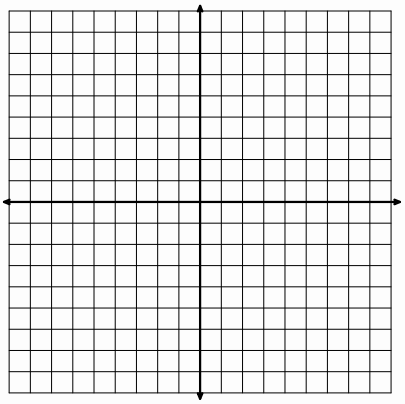
READY, SET, GO! Name Period Date

READY

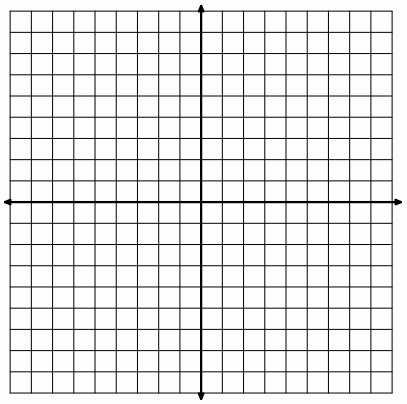
Topic: Graphing Quadratics

Graph each set of functions on the same coordinate axes. Describe in what way the graphs are the same and in what way they are different.

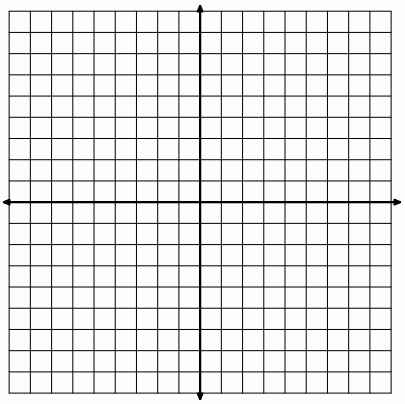
1. $y = x^2, y = 2x^2, y = 4x^2$



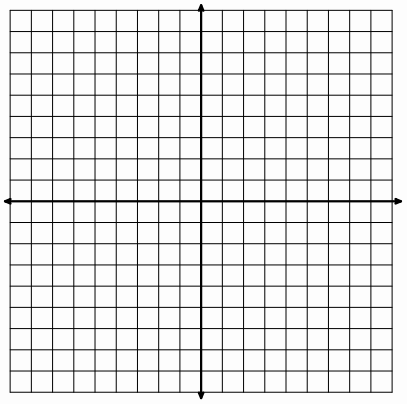
2. $y = \frac{1}{4}x^2, y = -x^2, y = -4x^2$



3. $y = \frac{1}{4}x^2, y = x^2 - 2, y = \frac{1}{4}x^2 - 2, y = 4x^2 - 2$



4. $y = x^2, y = -x^2, y = x^2 + 2, y = -x^2 + 2$



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SET

Topic: Sketching a parabola using the conic definition.

Use the conic definition of a parabola to sketch a parabola defined by the given focus *F* and the equation of the directrix.

Begin by graphing the focus, the directrix, and point P_1. Use the distance formula to find FP_1 and find the vertical distance between P_1 and the directrix by identifying point H on the directrix and counting the distance. Locate the point P_2, (the point on the parabola that is a reflection of P_1 across the axis of symmetry.) Locate the vertex V. Since the vertex is a point on the parabola, it must lie equidistant between the focus and the directrix. Sketch the parabola. Hint: the parabola always "hugs" the focus.

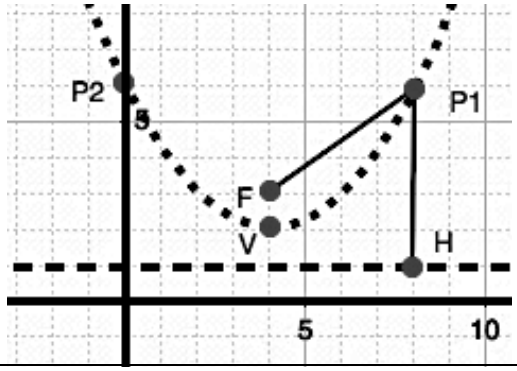
Example: ***F*** (4,3), ***P_1*** (8, 6), y = 1

FP_1 = $\sqrt{(4-8)^2+(3-6)^2} = \sqrt{16+9} = \sqrt{25} = 5$

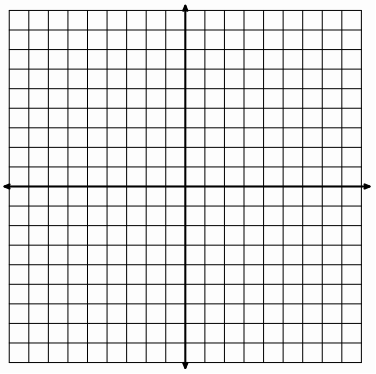
P_1H = 5

P_2 is located at (0, 6)

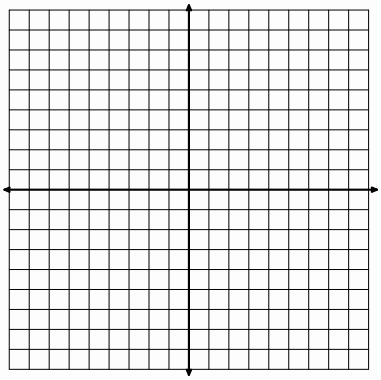
V is located at (4, 2)



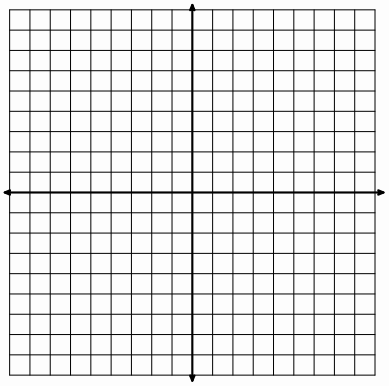
5. ***F*** (1,-1), ***P_1*** (3, -1) y = -3



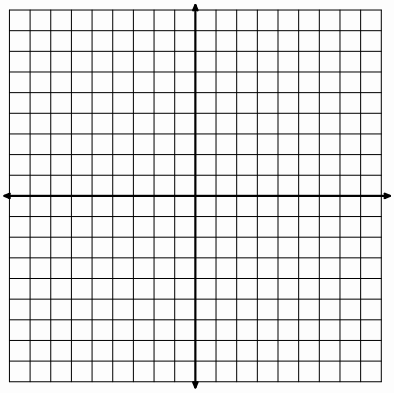
6. ***F*** (-5,3), ***P_1*** (-1, 3) y = 7



7. ***F*** (2,1), ***P_1*** (-2, 1) y =-3



8. ***F*** (1, -1), ***P_1*** (-9, -1) y = 9



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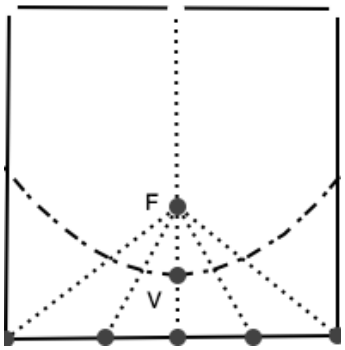
9. Find a square piece of paper (a post-it note will work). Fold the square in half vertically and put a dot anywhere on the fold. Let the edge of the paper be the directrix and the dot be the focus. Fold the edge of the paper (the directrix) up to the dot repeatedly from different points along the edge. The fold lines between the focus and the edge should make a parabola.



Experiment with a new paper and move the focus.
Use your experiments to answer the following questions.

10. How would the parabola change if the focus were moved up, away from the directrix?

11. How would the parabola change if the focus were moved down, toward the directrix?

12. How would the parabola change if the focus were moved down, below the directrix?

GO

Topic: Finding the center and radius of a circle.

Write each equation so that it shows the center (*h, k*) and radius *r* of the circle. This called the standard form of a circle. $(x-h)^2+(y-k)^2=r^2$

13. $x^2+y^2+4y-12=0$

14. $x^2+y^2-6x-3=0$

15. $x^2+y^2+8x+4y-5=0$

16. $x^2+y^2-6x-10y-2=0$

17. $x^2+y^2-6y-7=0$

18. $x^2+y^2-4x+8y+6=0$

19. $x^2+y^2-4x+6y-72=0$

20. $x^2+y^2+12x+6y-59=0$

21. $x^2+y^2-2x+10y+21=0$

22. $4x^2+4y^2+4x-4y-1=0$

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6.8 Functioning With Parabolas

A Solidify Understanding Task

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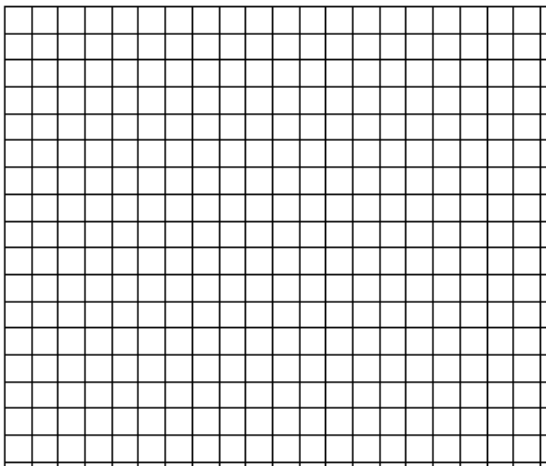
Sketch the graph (accurately), find the vertex and use the geometric definition of a parabola to find the equation of these parabolas.

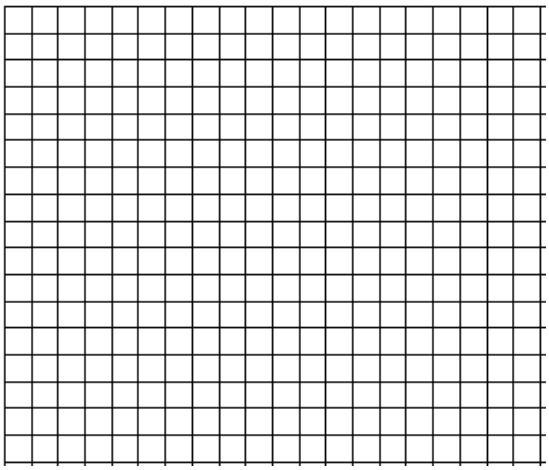
1. Directrix $y = -4$, Focus A(2, -2)



Vertex _____

Equation:

2. Directrix $y = 2$, Focus A(-1, 0)



Vertex _____

Equation:

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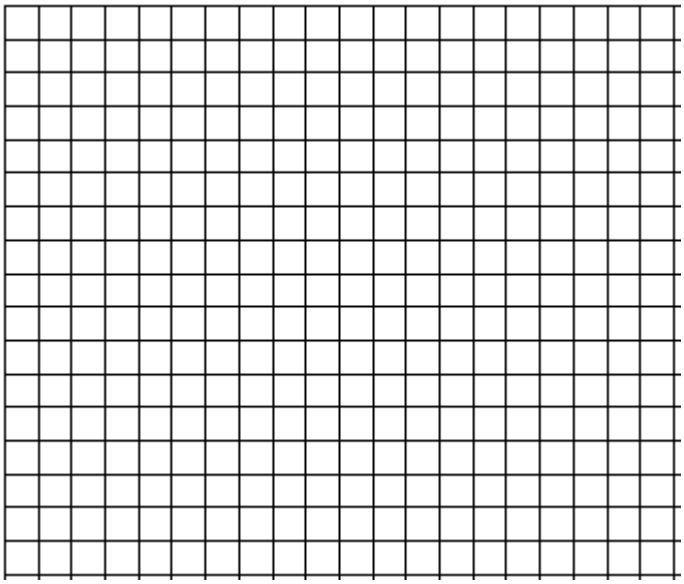
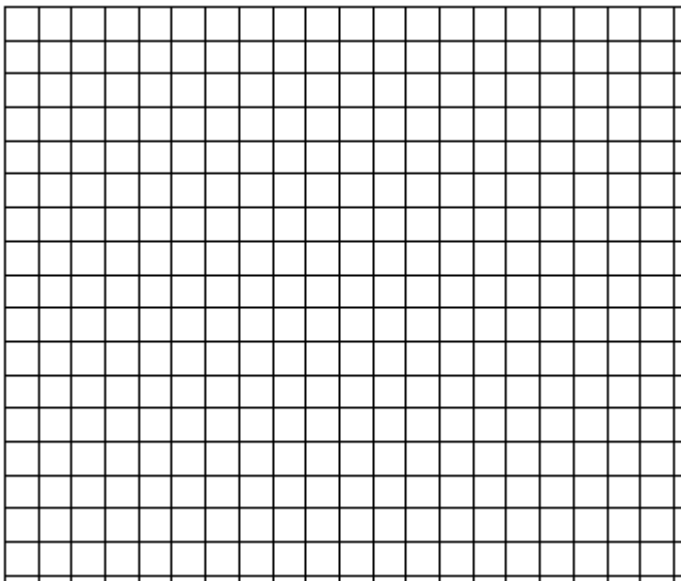
3. Directrix $y = 3$, Focus A(1, 7)



Vertex _____

Equation:

3. Directrix $y = 3$, Focus A(2, -1)



Vertex _____

Equation:

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4. Given the focus and directrix, how can you find the vertex of the parabola?

5. Given the focus and directrix, how can you tell if the parabola opens up or down?

6. How do you see the distance between the focus and the vertex (or the vertex and the directrix) showing up in the equations that you have written?

7. Describe a pattern for writing the equation of a parabola given the focus and directrix.

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8. Annika wonders why we are suddenly thinking about parabolas in a completely different way than when we did quadratic functions. She wonders how these different ways of thinking match up. For instance, when we talked about quadratic functions earlier we started with $y = x^2$. "Hmmmm. .... I wonder where the focus and directrix would be on this function," she thought. Help Annika find the focus and directrix for $y = x^2$.

9. Annika thinks, "Ok, I can see that you can find the focus and directrix for a quadratic function, but what about these new parabolas. Are they quadratic functions? When we work with families of functions, they are defined by their rates of change. For instance, we can tell a linear function because it has a constant rate of change." How would you answer Annika? Are these new parabolas quadratic functions? Justify your answer using several representations and the parabolas in problems 1-4 as examples.

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6.8

READY, SET, GO! Name Period Date

READY

Topic: Standard form of a quadratic.

Verify that the given point lies on the graph of the parabola described by the equation. (Show your work.)

1. $(6,0)$ $y = 2x^2 - 9x - 18$

2. $(-2,49)$ $y = 25x^2 + 30x + 9$

3. $(5,53)$ $y = 3x^2 - 4x - 2$

4. $(8,2)$ $y = \frac{1}{4}x^2 - x - 6$

SET

Topic: Equation of parabola based on the geometric definition

5. Verify that $(y-1) = \frac{1}{4}x^2$ is the equation of the parabola in *figure 1* by plugging in the 3 points V (0,1), C (4,5) and E (2,2). Show your work for each point!

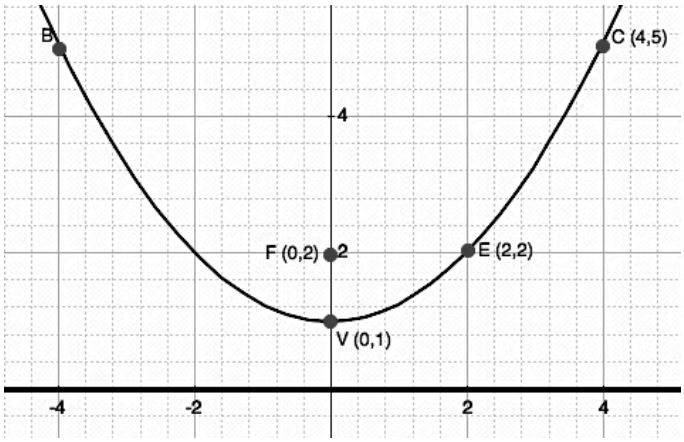

Figure 1

6. If you didn't know that (0,1) was the vertex of the parabola, could you have found it by just looking at the equation? Explain.

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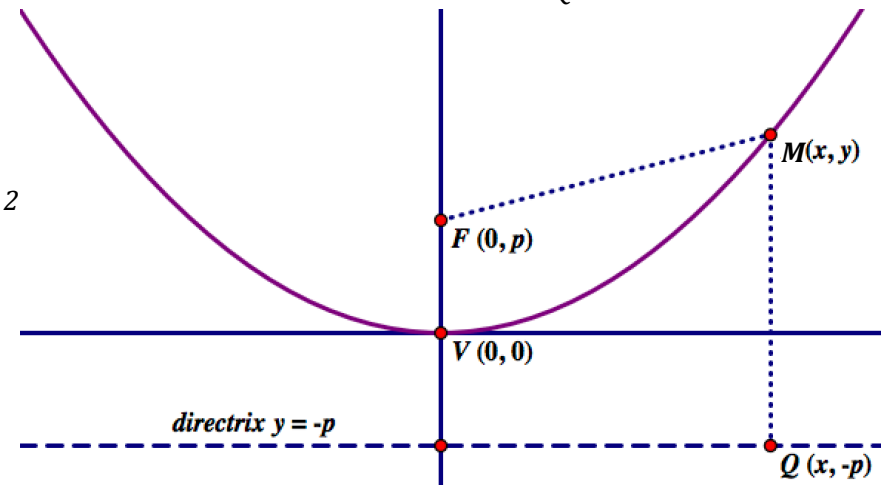
7. Use the diagram in *figure 2* to derive the general equation of a parabola based on the **geometric definition** of a parabola. Remember that the definition states that MF = MQ.

Figure 2

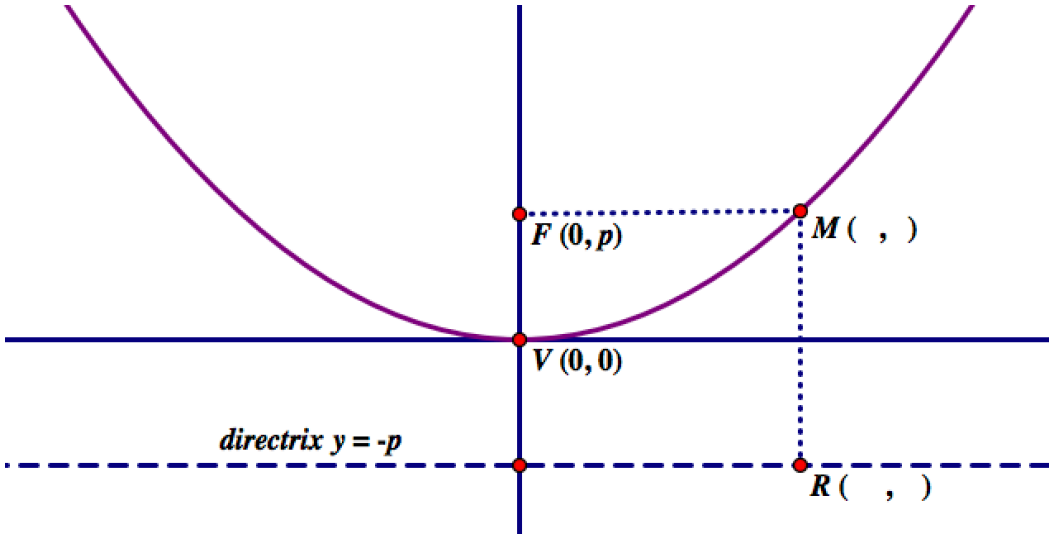


8. Recall the equation in #5, $(y-1)=\frac{1}{4}x^2$, what is the value of ***p***?

9. In general, what is the value of ***p*** for any parabola?

10. In *figure 3*, the point M is the same height as the focus and $\overline{FM} \cong \overline{MR}$. How do the coordinates of this point compare with the coordinates of the focus?
Fill in the missing coordinates for M and R in the diagram.

Figure 3



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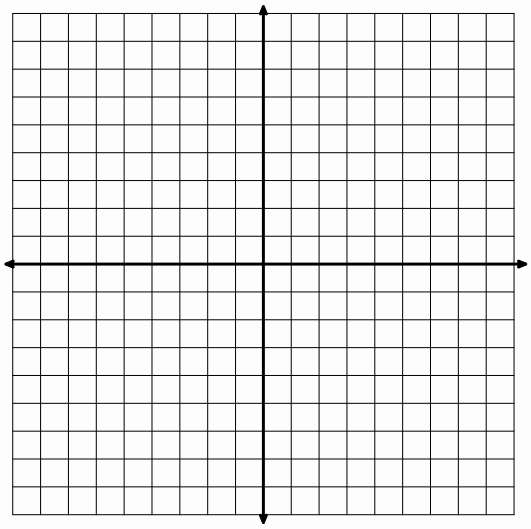
Sketch the graph by finding the vertex and the point M and R (the reflection of M) as defined in the diagram above. Use the geometric definition of a parabola to find the equation of these parabolas.

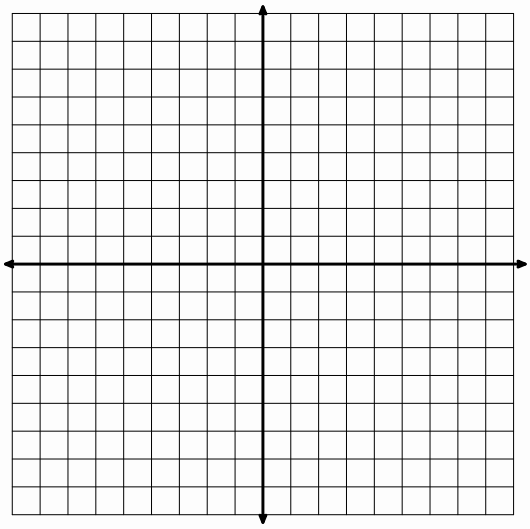
11. Directrix y = 9, Focus F(-3, 7)

Vertex _____
Equation _____



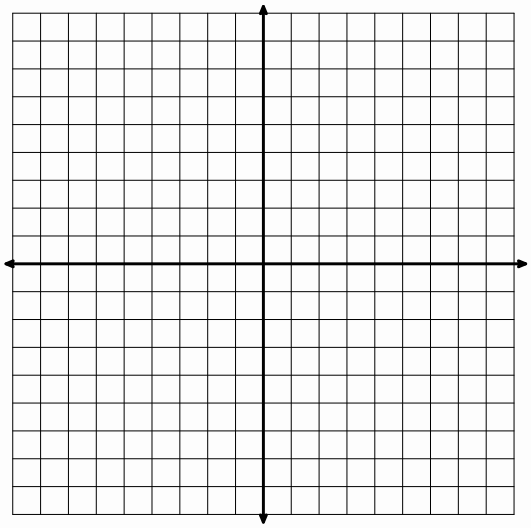
12. Directrix y = -6, Focus F(2, -2)

Vertex _____
Equation _____



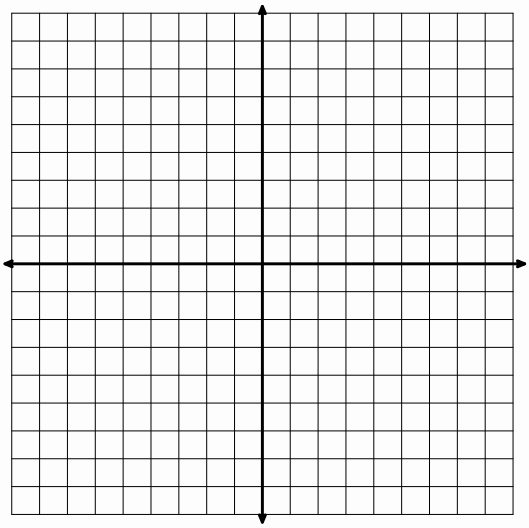
13. Directrix y = 5, Focus F(-4, -1)

Vertex _____
Equation _____



14. Directrix y = -1, Focus F(4, -3)

Vertex _____
Equation _____



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GO

Topic: Finding minimum and minimum values for quadratics

Find the maximum or minimum value of the quadratic. Indicate which it is.

15. $y = x^2 + 6x - 5$

16. $y = 3x^2 - 12x + 8$

17. $y = -\frac{1}{2}x^2 + 10x + 13$

18. $y = -5x^2 + 20x - 11$

19. $y = \frac{7}{2}x^2 - 21x - 3$

20. $y = -\frac{3}{2}x^2 + 9x + 25$

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6.9 Turn It Around

A Solidify Understanding Task

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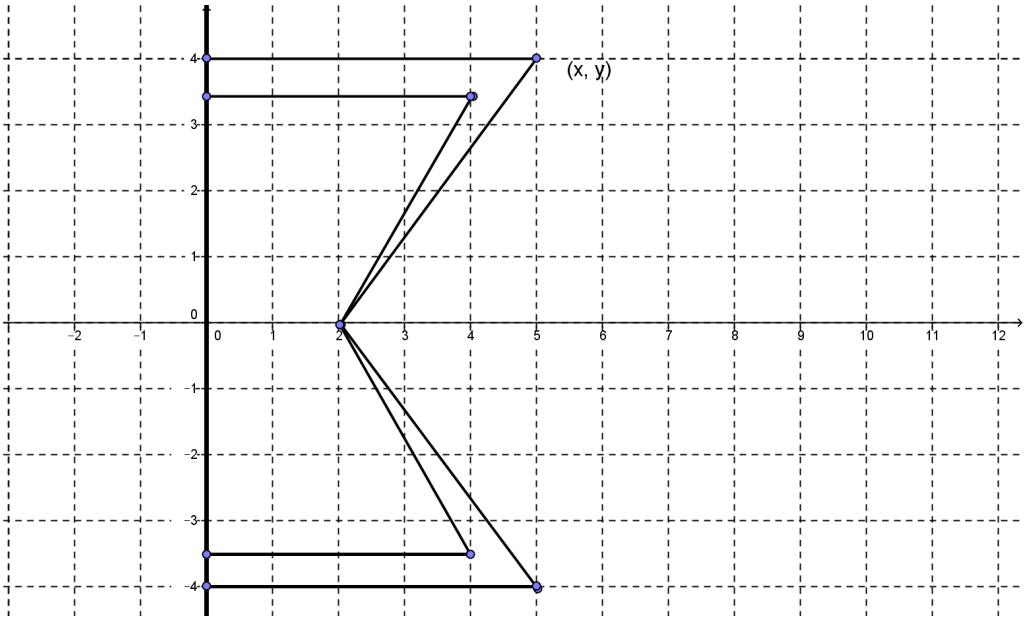
Annika is thinking more about the geometric view of parabolas that she has been working on in math class. She thinks, "Now I see how all the parabolas that come from graphing quadratic functions could also come from a given focus and directrix. I notice that all the parabolas have opened up or down when the directrix is horizontal. I wonder what would happen if I rotated the focus and directrix 90 degrees so that the directrix is vertical. How would that look? What would the equation be? Hmmm…." So Annika starts trying to construct a parabola with a vertical directrix. Here's the beginning of her drawing. Use a ruler to complete Annika's drawing.



1. Use the definition of a parabola to write the equation of Annika's parabola.

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2. What similarities do you see to the equations of parabolas that open up or down? What differences do you see?

3. Try another one: Write the equation of the parabola with directrix $x = 4$ and focus (0, 3).

4. One more for good measure: Write the equation of the parabola with directrix $x = -3$ and focus (-2, -5).

5. How can you predict if a parabola will open left, right, up, or down?

6. How can you tell how wide or narrow a parabola is?

7. Annika has two big questions left. Write and explain your answers to these questions.
   a. Are all parabolas functions?

   b. Are all parabolas similar?

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6.9

READY, SET, GO! Name Period Date

READY

Topic: Circles Review

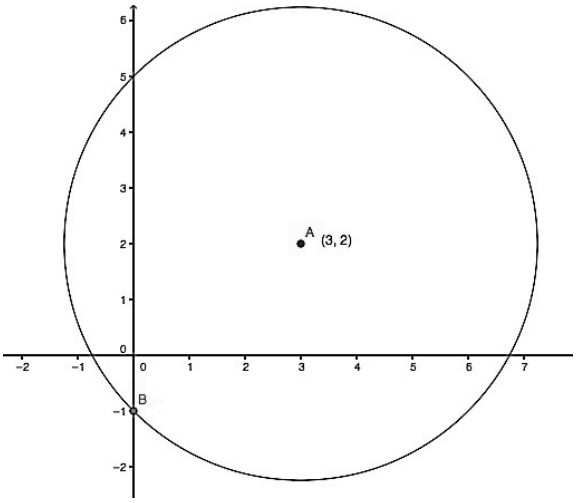
Use the given information to write the equation of the circle in standard form.

1. Center: (-5, -8) , Radius: 11

2. Endpoints of the diameter: (6, 0) and (2, -4)

3. Center (-5, 4) : Point on the circle (-9, 1)

4. Equation of the circle in the diagram to the right.

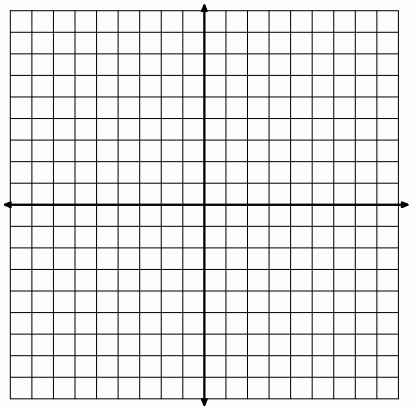


SET

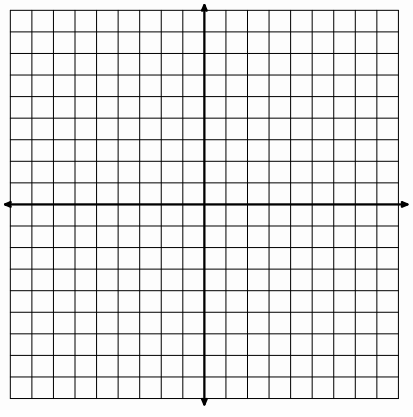
Topic: Writing equations of horizontal parabolas.

Use the focus F, point M, a point on the parabola, and the equation of the directrix to sketch the parabola (label your points) and write the equation. Put your equation in the form $x = \frac{1}{4p}(y-k)^2 + h$ where "p" is the distance from the focus to the vertex.

5. ***F*** (1,0), ***M*** (1,4) x = -3



6. ***F*** (3,1), ***M*** (3,-5) x = 9



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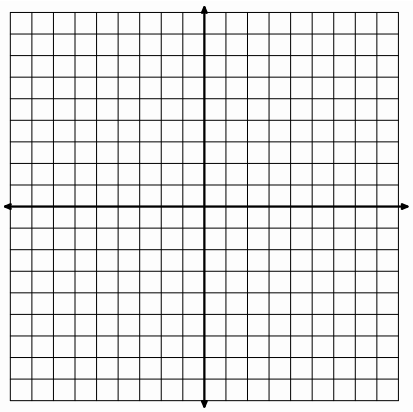
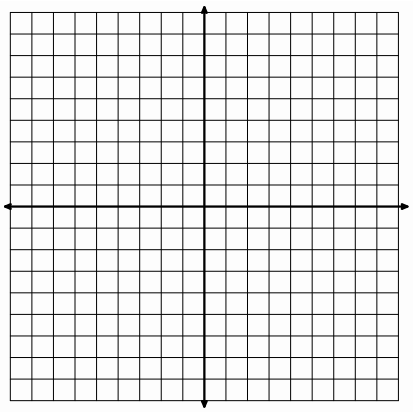
7. ***F*** (7,-5), ***M*** (4,-1) x = 9



8. ***F*** (-1,2), ***M*** (6,-9) x = -7



GO

Topic: Identifying key features of a quadratic written in vertex form

State (a) the coordinates of the vertex, (b) the equation of the axis of symmetry, (c) the domain, and (d) the range for each of the following functions.

9. $f(x)=(x-3)^2+5$

10. $f(x)=(x+1)^2-2$

11. $f(x)=-(x-3)^2-7$

12. $f(x)=-3\left(x-\frac{3}{4}\right)^2+\frac{4}{5}$

13. $f(x)=\frac{1}{2}(x-4)^2+1$

14. $f(x)=\frac{1}{4}(x+2)^2-4$

15. Compare the vertex form of a quadratic to the geometric definition of a parabola based on the focus and directrix. Describe how they are similar and how they are different.

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