mathematics vision project

Transforming Mathematics Education

# GEOMETRY

*A Learning Cycle Approach*

Teacher's Notes

MODULE 8

# Probability

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**The Mathematics Vision Project**

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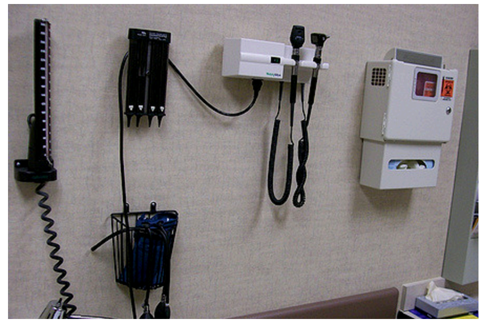
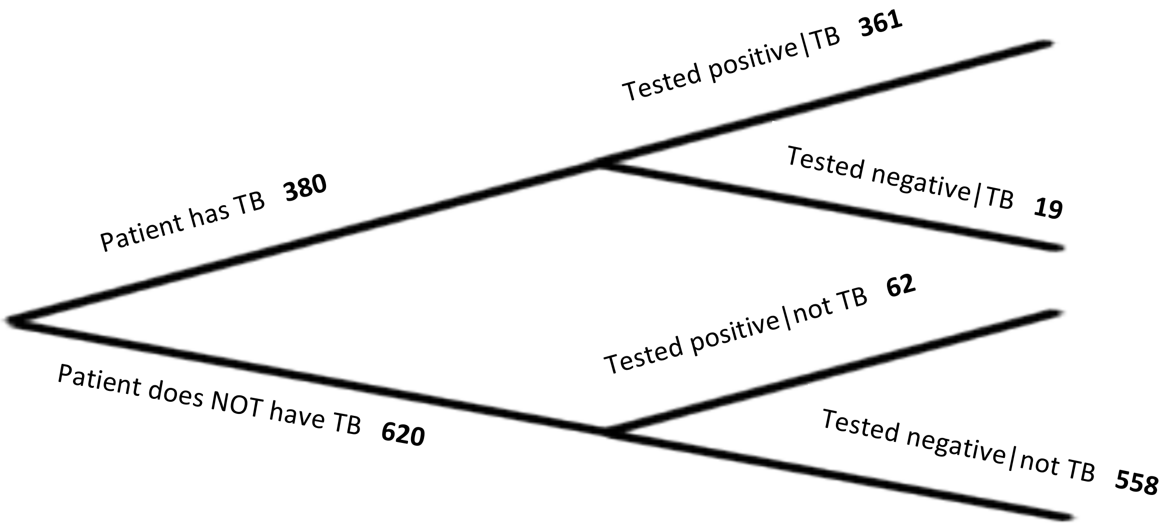




Tuberculosis (TB) can be tested in a variety of ways, including a skin test. If a person has turberculosis antibodies, then they are considered to have TB. Below is a tree diagram representing data based on 1,000 people who have been given a skin test for turberculosis.




1. What observations do you notice about TB tests based on the tree diagram?

2. You may have noticed that 380 patients have TB, yet not all 380 patients with TB tested positive. In statistics, the notation: “Tested negative | TB” means ‘*the number of patients who tested negative, given that they have TB’.* Determine the probability that a person who has TB could receive a negative result compared to others who have TB. What does this mean?

This is an example of **conditional probability**, which is the measure of an event, given that another event has occurred.

3. Write several other probability and conditional probability statements based on the tree diagram.

Part of understanding the world around us is being able to analyze data and explain it to others.

4. Based on the probability statements from the tree diagram, what would you say to a friend regarding the validity of their results if they are testing for TB using a skin test and the result came back positive?

5. In this situation, explain the consequences of errors (having a test with incorrect results).

6. If a health test is not 100% certain, why might it be beneficial to have the results lean more toward a false positive?

7. Is a sample space of 200 enough to indicate whether or not this is true for an entire population?

# 8.1 TB or Not TB? – Teacher Notes

## *A Develop Understanding Task*

**Purpose**: The purpose of this task is for students to analyze and make sense of data. Students will connect their prior understandings of tree diagrams and frequency tables (from earlier grades) to analyze data from a tree diagram and explain the results to others. The focus of this task is to highlight the information revealed as a result of the conditional probability statements. Questions such as 'How does the subgroup information tell us a more complete story?' should be addressed in this task.

**Note to teacher:** Throughout the module, students will be analyzing data but will have other areas of focus (such as representations, the addition rule, and notation). Therefore, it is important that the focus of this task is for students to make sense of the context given the representation and to write/communicate general probability statements and conditional probability statements.

**Core Standards Focus**:

**S.CP.6:** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

**S.MD.7 (+):** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

**Related Standards: S.CP.3, S.CP. 4, S.CP.5**

**Standards for Mathematical Practice:**

**SMP 1 – Make sense of problems and persevere in solving them**
**SMP 2 – Reason abstractly and quantitatively**
**SMP 6 – Attend to precision**
**SMP 7 – Look and make use of structure**

**The Teaching Cycle:**

**Launch (Whole Class):**

Start this task by sharing how important it is to know how to read and make sense of data. In the health care field, there are lots of tests used to determine whether or not a patient has an infection or a disease. It is also important to note that not all tests are accurate 100% of the time, which is why there is sometimes more than one test to determine a diagnosis. Read the directions from the task and have students make observations about the tree diagram (question 1). If students are not familiar with the symbol used for conditional probability that is given on the tree diagram, explain this to them (Tested positive|TB means "the number of patients who tested positive, given that they have TB").

**Explore (Small Group):**

Give students time to make sense of the diagram independently and then have them work in pairs to create statements. If a group seems stumped, you can ask questions like:

- What information does this diagram tell you?
- Is this TB test always 100% accurate? How do you know?
- Do most people test positive or negative based on the skin test results?
- What other information can you determine?

It is best to stay away from specific questions at this time as part of this task is for students to make sense and interpret the data themselves. The low threshold of this task is that most students should be able to create statements such as "36.1% of the population tested have TB antibodies and tested positive". As you monitor, if you notice students are not writing any conditional statements, probe groups that are finishing quickly to analyze the results of a specific branch. For example, 'What do you know about patients who test positive, given that they have TB?' or to explain the difference, in context, between these probabilities: 620/1000, 62/620 and 62/1000. Be sure to have students write out their statements in their journal and not just say them. This assists students in communicating conditional probabilities. If they need prompting, you may wish to give an example or two using phrases such as "out of" or "given that" or "If… then".

As you prepare for the whole group discussion, choose a group to chart some of their statements related to the total number of people who took the skin test. Also select students to share their conditional statements who have articulated understanding.

**Discuss (Whole Class):**

For the whole group discussion, the goal is for students to be able to answer "How accurate is the tuberculosis (TB) test?" using the data provided to write probability statements. There are several ways to do this, with the following as one possibility:

First, have group who charted statements related to the total share and explain their statements. Highlights of this conversation include results of certain aspects of the sample (percent who have antibodies, percent who test positive, etc.) and that the test is accurate 'most' of the time. While this is good information, taking a deeper look can reveal more information. Next, sequence the students you selected earlier to share their conditional statements and ask them what their statement reveals. Be sure each statement is accurate (not just computation, but also how it is written/spoken), and make the appropriate adjustments for those who need it. During this part of the discussion is also an appropriate time to introduce probability notation as students have likely not seen this in the past.

Discussion items that you may wish to make sure come out:

1. Meaning of error (false positive/false negative) and why this particular data leans more toward a false positive.
2. The law of large numbers. The sample size of this data is 1,000. What does this mean for an individual who has a skin test for TB?
3. How does exploring conditional probabilities allow for investigation of the accuracy of medical tests?

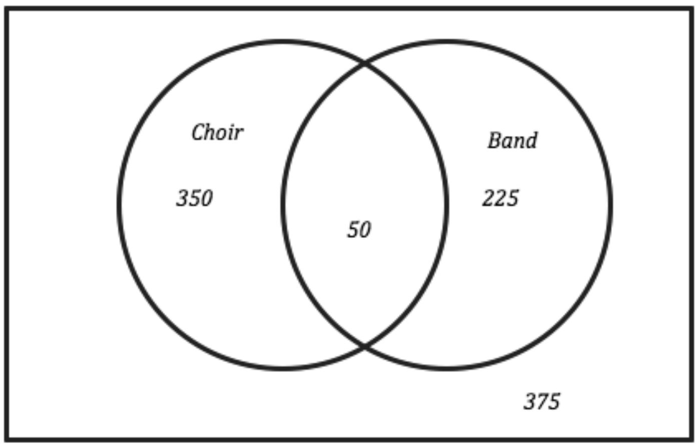
**Aligned Ready, Set, Go: Probability 8.1**

READY, SET, GO! Name Period Date

## READY

Topic: Venn Diagrams, how to create and read.

**For each Venn Diagram provided answer the questions.**



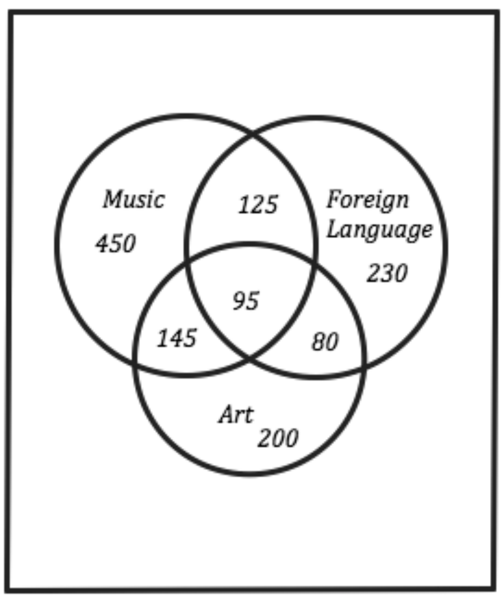

1. How many students were surveyed?

2. What were the students asked?

3. How many students are in both choir and band?

4. How many students are not in either choir or band?

5. What is the probability that a randomly selected student would be in band?




This Venn Diagram represents enrollment in some of the elective courses.

6. What does the 95 in the center tell you?

7. What does the 145 tell you?

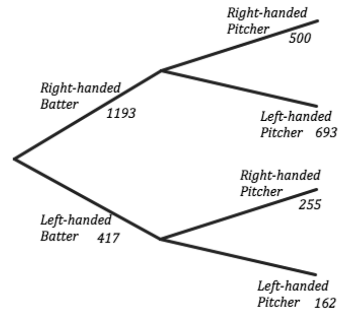
8. How many total students are represented in the diagram?

9. Which elective class has the least number of students enrolled?

## *SET*

Topic: Interpreting a tree diagram to determine probability

**Given the tree diagram below answer the questions and determine the probabilities. The diagram represents the number of plate appearances during the first month of a minor league baseball season.**




10. How many times did a batter come to the plate during this time period?

11. Based on this data, if you are a left-handed batter what is the probability that you will face a right-handed pitcher?

12. Based on this data, if you are a right-handed batter what is the probability that you will face a left-handed pitcher?

13. What is the probability that a left-handed pitcher will be throwing for any given plate appearance?

14. What is the probability that a left-handed batter would be at the plate for any given plate appearance?

    What observations do you make about the data? Is there any amount that seems to be overly abundant? What might account for this?

## *GO*

Topic: Basic Probability

**Find the probability of achieving success with each of the events below.**

15. Rolling an even number on standard six-sided die.

16. Drawing a black card from a standard deck of cards.

17. Flipping a coin and getting Heads three times in a row.

18. Rolling a die and getting a four.

19. Drawing an ace from a deck of cards.

20. Rolling a die twice in a row and getting two threes.

21. From a bag containing 3 blue, 2 red, and 5 white marbles. Pulling out a red marble.

CC BY Steve Snodgrass
https://flic.kr/p/dAmJrc

# 8.2 Chocolate versus Vanilla

## *A Solidify Understanding Task*

Danielle loves chocolate ice cream much more than vanilla and was explaining to her best friend Raquel that so does most of the world. Raquel disagreed and thought vanilla is much better. To settle the argument, they created a survey asking people to choose their favorite ice cream flavor between chocolate and vanilla. After completing the survey, the following results came back:

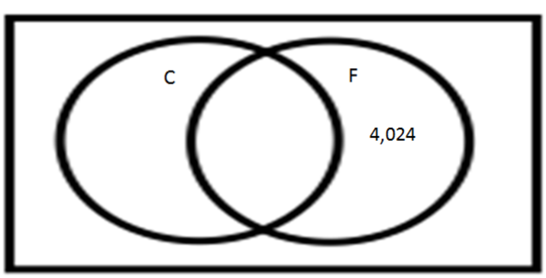
- There were 8,756 females and 6,010 males who responded.
- Out of all the males, 59.7% chose vanilla over chocolate.
- 4,732 females chose chocolate.

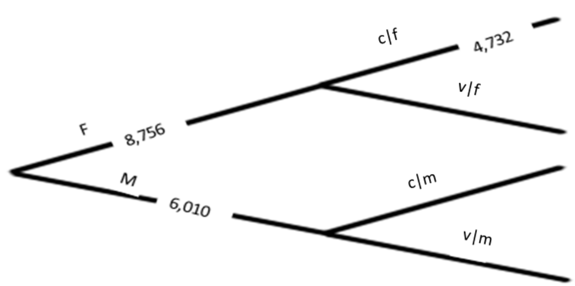
1. Upon first observations, which flavor do you think "won"? ________________. Write a sentence describing what you see at 'first glance' that makes you think this.

2. Raquel started to organize the data in the following two-way table. See if you can help complete this (using counts and not percentages):

| | Chocolate | Vanilla | Total |
|---|---|---|---|
| Female | | | 8,756 |
| Male | | | 6,010 |
| Total | | | |

3. Organize the same data in a Venn diagram and a tree diagram.



4. Using your organized data representations, write probabilities that help support your claim regarding the preferred flavor of ice cream. For each probability, write a complete statement as well as the corresponding probability notation.

5. Looking over the three representations (tree diagram, two-way table, and Venn diagram), what probabilities seem to be easier to see in each? What probabilities are hidden or hard to see?

| **Highlighted (easier to see)** | **Hidden** |
|---|---|
| Tree diagram | Tree diagram |
| Two-way table | Two-way table |
| Venn diagram | Venn diagram |

6. Getting back to ice cream. Do you think this is enough information to proclaim the statement that one ice cream is favored over another? Explain.

# 8.2 Chocolate versus Vanilla – Teacher Notes

## *A Solidify Understanding Task*

**Purpose**: The purpose of this task is for students to interpret information provided that allows them to make sense of and organize data in a tree diagram, a two-way table, and a Venn diagram. Students will solidify their understanding of conditional probability by writing statements supported by data collected to justify the flavor of ice cream preferred by most. In this task, students will:

- Organize data into a tree diagram, two-way table, and a Venn diagram
- Calculate probabilities and conditional probabilities of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model
- Highlight the different representations and become more familiar with what each representation highlights and conceals.
- Continue to become more familiar with probability notation.
- Make decisions about meaning of data.

**Core Standards Focus**:

**S.CP.4:** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

**S.CP.6:** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

**Related Standards: S.CP.3, S.CP.5 , S.MD.6**

**Standards for Mathematical Practice:**

**SMP 3 – Construct viable arguments and critique the reasoning of others**

**SMP 4 – Model with mathematics**

**SMP 7 – Look and make use of structure**

**The Teaching Cycle:**

**Launch (Whole Class):**

Begin by reading the context of the problem and the data provided by the survey. Have students individually make a decision to question 1. This is their 'estimation' using their first impression reasoning. Give students wait time to write their first impression. To launch the rest of the task, explain to students that they will be organizing the data into the three representations (tree diagram, Venn diagram, and two way table). They will use this information to further make claims regarding the 'favorite ice cream flavor'.

**Explore (Small Group):**

As you monitor, listen for students to make sense of the data provided so that they can complete the tree diagram. Allow for time so that students can determine what information is still needed and how to go about finding this. If after a couple of minutes you notice a group that seems to be stuck as to how to begin, ask probing questions such as "What part of the tree diagram needs to still be completed?" followed by "How could you find the solution to that part of the diagram (such as the number of females who prefer vanilla ice cream) or (the number of males who prefer vanilla ice cream)?" Also look for the common error a group could make by calculating 59.7% of the total to determine the number of males who like Vanilla ice cream. To clarify, have the group explain the

diagram and their calculation. Likely, they will notice their mistake and recalculate to find 59.7% of the males instead of the total. If not, ask them to explain the meaning of the bullet point in the given data that reads "Out of all the males, 59.7% chose vanilla over chocolate." As students move from completing one representation to the others (tree, Venn, table), listen for explanations they have for determining values in each representation and any connections they make between representations (as you monitor, select students to share based on these explanations). Below is the completed two way table to assist in making sure student calculations are accurate as well as a couple of probability statements students may choose to use.

| | Chocolate | Vanilla | Total |
|---|---|---|---|
| Female | 4,732 | 4,024 | 8,756 |
| Male | 2,420 | 3,590 | 6,010 |
| Total | 7,152 | 7,614 | 14,766 |

For those who chose vanilla:

P(vanilla) = 7614/14766
P(vanilla|male) = 3590/6010

Begin the whole group discussion after most students have completed part I.

**Discuss (Whole Class):**

Sequence students to share their strategies for organizing data. Include connections they notice between representations. Use misconceptions students may have had during small groups to highlight how to determine values. For each situation, encourage students to use appropriate academic language.

**Explore Part II (Small Groups):**

Have students complete questions 5 and 6. Students should be able to articulate how each representation organizes the data to highlight certain information. The goal of this portion of the task is for students to become more comfortable with each representation so they can choose which representation to use in the future when organizing data. If students seem stuck, ask them to write a probability that is 'easier' to see in the tree diagram than in the other two representations. Likewise, write a probability that is 'easier' to see in the two way table or that is 'easier' to see in the Venn Diagram.

**Discuss Part II (Whole Class):**

Complete the chart as a whole group after students have had time to think about this themselves. To conclude, answer question six by discussing the law of large numbers and randomness.

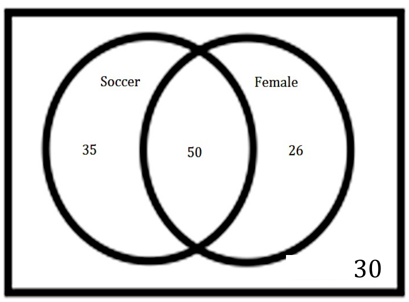
**Aligned Ready, Set, Go: Probability 8.2**

READY, SET, GO! Name Period Date

## *READY*

Topic: Analyzing data given in a Venn Diagram.
**Use the Venn Diagrams below to answer the following questions. (Hint: you may use the same data provided in the two-way table from question 3 on the next page to help make sense of the Venn Diagram)**

*The following Venn Diagram represents the relationship between favorite sport (Soccer or Baseball) and gender (Female or Male).*




1. How many people said soccer is their favorite sport?

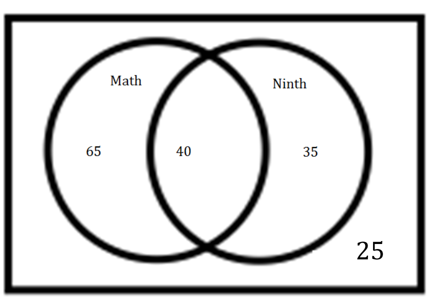
2. How many females are in the data?

3. How many males chose baseball?

4. What is the probability that a person would say soccer is their favorite sport? P(soccer) =

5. What is the probability that a female would say soccer is their favorite sport? ("Out of all females, ____% say soccer is their favorite sport") P(soccer | female) =

*The following Venn Diagram represents the relationship between favorite subject (Math or Science) and grade level (Ninth or Tenth). Using this data, answer the following questions.*




6. How many people said math is their favorite subject?

7. How many tenth graders are in the data?

8. How many ninth graders chose science?

9. What is the probability that a person would say science is their favorite subject? P(s) =

10. What is the probability that a tenth grader would say science is their favorite subject? ("If you are a tenth grader, then the probability of science being your favorite subject is _____ %") P(science | tenth)=

## *SET*

Topic: Writing conditional statements from two-way tables

11. Complete the table and write three conditional statements.

| | Soccer | Baseball | Total |
|---|---|---|---|
| Male | | 30 | |
| Female | 50 | | 76 |
| Total | 85 | | |

12. Complete the table about preferred genre of reading and write three conditional statements.

| | Fiction | Non-Fiction | Total |
|---|---|---|---|
| Male | | 10 | |
| Female | 50 | | 60 |
| Total | 85 | | |

13. Complete the table about favorite color of M&M's and write three conditional statements.

| | Blue | Green | Red | Other | Total |
|---|---|---|---|---|---|
| Male | 15 | 20 | 15 | | 60 |
| Female | 30 | 20 | | 10 | |
| Total | 45 | | | | 130 |

14. Use the information provided to make a tree diagram, a two-way table and a Venn Diagram.

- Data was collected at the movie theater last fall. Not about movies but clothes.
- 6,525 people were observed.
- 3,123 had on shorts and the rest had on pants
- 45% of those wearing shorts were denim.
- Of those wearing pants 88% were denim.

## *GO*

Topic: Basic Probability
**Find the desired values.**

15. What is half of one-third?

16. What is one-third of two-fifths?

17. What is one-fourth of four-sevenths?

18. What percent is $\frac{5}{8}$?

19. What is 35% of 50?

20. Seventy is 60% of what number?

21. Write $\frac{7}{12}$ as a percent.

22. Write $\frac{1}{6}$ as a percent.

23. What is 52% of 1,200?

24. What percent is 32 of 160?

25. Sixty is what percent of 250?

26. What percent of 350 is 50?

# 8.3 Fried Freddy's

## *A Solidify Understanding Task*



Danielle was surprised by the results of the survey to determine the 'favorite ice cream' between chocolate and vanilla (See task 9.2 *Chocolate vs. Vanilla*). The reason, she explains, is that she had asked several of her friends and the results were as follows:

| | Chocolate | Vanilla | Total |
|---|---|---|---|
| Female | 23 | 10 | 33 |
| Male | 6 | 8 | 14 |
| Total | 29 | 18 | 47 |

1. In this situation, chocolate is most preferred. How would you explain to her that this data may be less 'valid' compared to the data from the previous survey?

Using a sufficiently large number of trials helps us estimate the probability of an event happening. If the sample is large enough, we can say that we have an estimated probability outcome for the probability of an event happening. If the sample is not randomly selected (only asking your friends) or not large enough (collecting four data points is not enough information to estimate long run probabilities), then one should not estimate large scale probabilities. Sometimes, our sample increases in size over time. Below is an example of data that is collected over time, so the estimated probability outcome becomes more precise as the sample increases over time.

Freddy loves fried food. His passion for the perfect fried food recipes led to him opening the restaurant, "Fried Freddies." His two main dishes are focused around fish or chicken. Knowing he also had to open up his menu to people who prefer to have their food grilled instead of fried, he created the following menu board:

**Fried Freddy's** $7.95

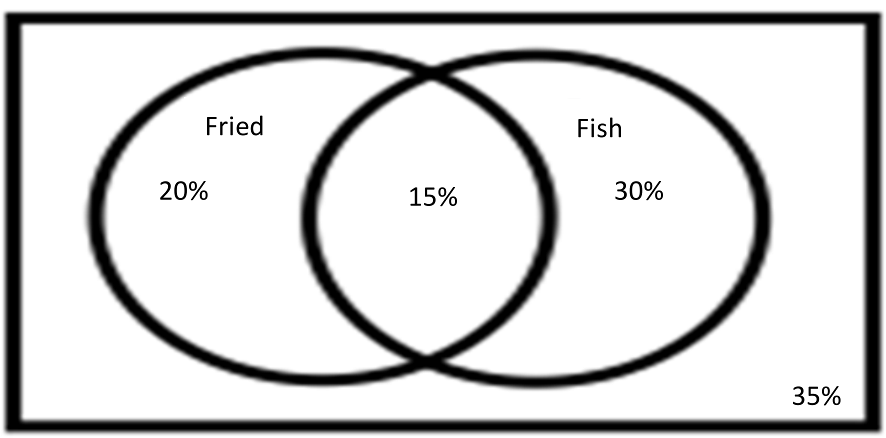
*Choose dish: Chicken or Fish*

*Choose cooking preference: Grilled or Fried*

After being open for six months, Freddy realized he was having more food waste than he should because he was not predicting how much of each he should prepare in advance. His business friend, Tyrell, said he could help.

2. What information do you think Tyrell would need?

Luckily, Freddy uses a computer to take orders each day so Tyrell had lots of data to pull from. After determining the average number of customers Freddy serves each day, Tyrell created the following Venn diagram to show Freddy the food preference of his customers:

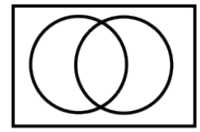


To make sense of the diagram, Freddy computed the following probability statements:

3. What is the probability that a randomly selected customer would order fish?

   P(fish) =

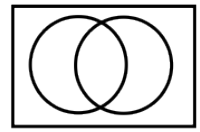
   Shade the part of the diagram that models this solution.



4. What is the probability that a randomly selected customer would order fried fish?

   P(fried ∩ fish) = P(fried and fish) =

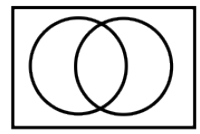
   Shade the part of the diagram that models this solution.



5. What is the probability that a person prefers fried chicken?

   P(fried∩chicken) = P(fried and chicken) =

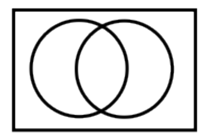
   Shade the part of the diagram that models this solution.



6. What is the estimated probability that a randomly selected customer would want their fish grilled?

   P(grilled and fish) = P(____________________) =

   Shade the part of the diagram that models this solution.

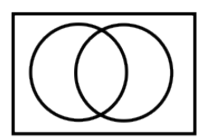


7. If Freddy serves 100 meals at lunch on a particular day, how many orders of fish should he prepare with his famous fried recipe?

8. What is the probability that a randomly selected person would choose fish or fried?

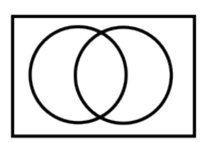
   P(fried∪fish) = P(fried or fish) =

   Shade the part of the diagram that models this solution.



9. What is the probability that a randomly selected person would NOT choose fish or fried?

   Shade the part of the diagram that models this solution.



# 8.3 Fried Freddy's – Teacher Notes

## *A Solidify Understanding Task*

**Purpose**: A purpose of this task is for students to gain a stronger understanding the law of large numbers and how this helps to estimate probable outcomes. Another purpose is for students to solidify their understanding around the following ideas:

- Whether or not there is enough data to estimate outcomes.
- Distinguish between a general probability, a conditional probability, and the addition rule.
- Use a Venn diagram to analyze data and to write various probability statements (unions, intersections, complements).
- Apply the Addition Rule and interpret the answer in terms of the model.
- Use estimated outcomes to make recommendations and decisions.

**Core Standards Focus**:

**S.CP.1:** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or", "and", "not").

**S.CP.4:** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

**S.CP.6:** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

**S.CP.7:** Apply the Addition Rule and interpret the answer in terms of the model.

**Related Standards: S.CP.3, S.CP.5**

**Standards for Mathematical Practice**

**SMP 1 – Make sense of problems and persevere in solving them**

**SMP 4 – Model with mathematics**

**SMP 7 – Look and make use of structure**

**The Teaching Cycle**
**Launch (Whole Class):**
Have students answer the first question about the validity of data individually, then discuss as a group how important sample is when estimating outcomes. Read the context of Fried Freddy, then have students answer the question about what information Tyrell would need to pull together data. Depending on your class, you may also wish to have students label each section of the Venn diagram and make sure everyone understands this prior to having students answer the questions related to the diagram.

**Explore (Small Group):**

As you monitor, listen for students to make sense of the probability statements they are answering and look for the solution area to be shaded on their models. Help students who are struggling by suggesting they create a different representation (such as a two-way table- see below).

| | Fish | Chicken | total |
|---|---|---|---|
| Fried | 15% | 20% | 35% |
| Grilled | 30% | 35% | 65% |
| Total | 45% | 55% | 100% |

Since the purpose of this task is for students to recognize and use probability statements to interpret data, plan to have the whole group discussion focus on questions that relate to standards S.CP.1 and S.CP. 7.

**Discuss (Whole Class):**

Choose students to answer questions relating to the data from the Venn diagram. Spend most time on questions relating to unions, intersections, complements, and the Addition Rule. Be sure to use this vocabulary and have student's record unfamiliar vocabulary in their journal. The most time may be spent on questions six through nine. At the end of the lesson, review probability notation and discuss the similarities and differences between the rules of probability (unions, intersections, complements, Addition Rule, conditional probability) and other vocabulary (mutually exclusive, joint, disjoint).

**Aligned Ready, Set, Go: Probability 8.3**

READY, SET, GO! Name Period Date

## *READY*

Topic: Independent and Dependent Events

**In some of the situations described below the first event effects the subsequent event (dependent events). In others each of the events is completely independent of the others (independent events). Determine which situations are dependent and which are independent.**

1. A coin is flipped twice. The first event is the first flip and the second event is the next flip.

2. A bag of marbles contains 3 blue marbles, 6 red marbles and 2 yellow marbles. Two of the marbles are drawn out of the bag. The first event is the first marble taken out the second event is the second marble taken out.

3. An attempt to find the probability of there being a right-handed or a left-handed batter at the plate in a baseball game. The first event is the 1$^{st}$ batter to come to the plate. The second event is the second player to come up to the plate.

4. A standard die is rolled twice. The first event is the first roll and the second event is the second roll.

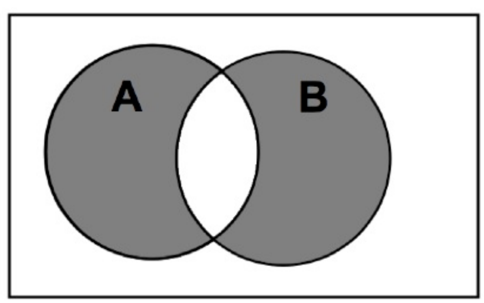
5. Two cards are drawn from a standard deck of cards. The first event is the first card that is drawn the second event is the second card that is drawn.

## *SET*

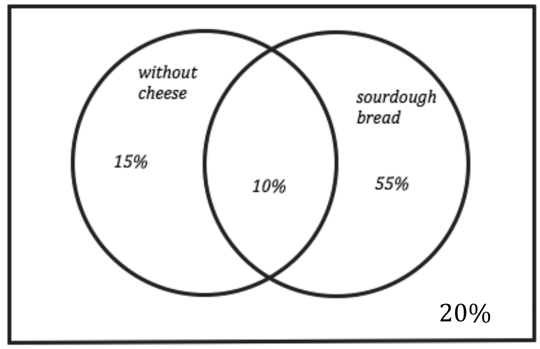
Topic: Addition Rule, Interpreting a Venn Diagram

6. Sally was assigned to create a Venn diagram to represent $P(A \text{ or } B)$. Sally first writes $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, what does this mean? Explain each part.

7. Sally then creates the following diagram.
Sally's Venn diagram is incorrect. Why?



The Venn diagram to the right shows the data collected at a sandwich shop for the last six months with respect to the type of bread people ordered (sourdough or wheat) and whether or not they got cheese on their sandwich. Use this data to create a two-way frequency table and answer the questions.




8. Two-way frequency table

| | | | |
|---|---|---|---|
| | | | |
| | | | |
| | | | |

9. What is the probability that a randomly selected customer would order sourdough bread?
   P(sourdough bread) =

10. What is the probability that a randomly selected customer would order sourdough bread without cheese?
    P(sourdough ∩ no cheese) = P(sourdough and no cheese) =

11. What is the probability that a person prefers wheat bread without cheese?
    P(wheat ∩ no cheese) = P(wheat and no cheese) =

12. What is the estimated probability that a randomly selected customer would want their sandwich with cheese?
    P(sourdough cheese and wheat cheese) = P(__________________) =

13. If they serve 100 sandwiches at lunch on a particular day, how many orders with sourdough should be prepared without cheese?

14. What is the probability that a randomly selected person would choose sourdough or without cheese?
    P(sourdough ∪ no cheese) = P(sourdough or no cheese) =

15. What is the probability that a randomly selected person would NOT choose sourdourgh or no cheese?

## *GO*

Topic: Equivalent Ratios and Proportions

**Use the given ratio to set up a proportion and find the desired value.**

16. If 3 out of 5 students eat school lunch then how many students would be expected to eat school lunch at a school with 750 students?

17. In a well developed and carried out survey it was found that 4 out of 10 students have a pair of sunglasses. How many students would you expect to have a pair of sunglasses out of a group of 45 students?

18. Data collected at a local mall indicted that 7 out of 20 men observed were wearing a hat. How many would you expect to have been wearing hats if 7500 men were to be at the mall on a similar day?

# 8.4 Visualizing with Venn

## *A Solidify Understanding Task*




One of the attributes of Venn diagram's is that it can be easy to see the relationships within the data. In this task, we will create multiple Venn diagrams using data and determine the events that create diagrams to either have an intersection or for them to be mutually exclusive.

1. The following data represents the number of men and women passengers aboard the Titanic and whether or not they survived. Fill in the blanks for this table:

| | Survived | Did not survive | Total |
|---|---|---|---|
| Men | | 659 | 805 |
| Women | 296 | | |
| Total | 442 | 765 | 1207 |

2. Using the data above, create a Venn diagram for each of the following:
   a. Men vs Women
   b. Women vs Survived
   c. You choose the conditions

3. Create two probability statements using each of your Venn diagrams from question 2.

4. Create and label three different Venn diagrams using the following data. Create at least one that is mutually exclusive and at least one that has an intersection.

   Sample size: 100

   P(girl) = $\frac{42}{100}$

   P(girl or art) = $\left(\frac{42}{100} + \frac{30}{100}\right) - \frac{12}{100}$

   P(art) = $\frac{30}{100}$

   P(not art) =

   P(boy) =

5. Describe the conditions that create mutually exclusive Venn diagrams and those that create intersections.

6. What conjecture can you make regarding the best way to create a Venn diagram from data to highlight probabilities?

# 8.4 Visualizing with Venn- Teacher Notes

## *A Solidify Understanding Task*

**Purpose**: The purpose of this task is to have students create and analyze attributes of Venn diagrams, then make sense of the data. Students will add academic vocabulary (mutually exclusive, joint, disjoint) and will distinguish between conditional probability and using the addition rule. At the end of the task, students will be able to:

- Create Venn Diagrams that highlight specific data.
- Understand mutually exclusive, joint (intersection), and the Addition Rule using Venn diagrams.
- Distinguish between conditional probability and the Addition Rule.
- Get out vocabulary such as joint, disjoint, mutually exclusive, Addition Rule, and conditional probability.
- Analyzing data from a two way table and from probability notation to create various Venn Diagrams

### Core Standards Focus:

**S.CP.6:** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

**S.CP.7:** Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

**Related Standards: S.CP.3, S.CP. 4, S.CP.5**

**Standards for Mathematical Practice:**

**SMP 1 – Make sense of problems and persevere in solving them**

**SMP 3 – Construct viable arguments and critique the reasoning of others**

**SMP 6 – Attend to precision**

**The Teaching Cycle:**

**Launch (Whole Class):**
Have students share what they know about Venn diagrams to access background knowledge, then have them begin working on the task in small groups.

**Explore (Small Group):**
As you monitor, check for student understanding of how to write a Venn diagram when given the constraints (questions two and three). These questions scaffold students by telling them which categories to use when creating the diagram, then having them interpret what kind of information they can see from each. The intention of these models is for students to recognize that when they are choosing categories for a Venn diagram, that the data is more meaningful when they select from different category types. Listen for students to make the following conjectures regarding creating Venn diagrams: If two events are related and have overlapping data, the intersection is easy to see and several probability statements can be made. Likewise, if the two events being compared do not have overlap, then those two events are mutually exclusive. Also listen for students to become clear as to the probabilities that are easier to see in a Venn diagram than in other models.

**Discuss (Whole Class):**
**Vocabulary to solidify during discussion using student work:** mutually exclusive, Addition Rule, intersections, complements, disjoint, and joint.

For the whole group discussion, have students share what they have learned about creating Venn diagrams. Chart this information, and then press students to explain how they would use data in the future to create a Venn diagram that would produce the data they were seeking. Ask students to share some probability statements they created and have them explain how the Venn diagram was helpful in seeing the probability. Focus on conditional probability statements, the Addition Rule, and data that highlights intersections and complements.
**Aligned Ready, Set, Go: Probability 8.4**

READY, SET, GO! Name Period Date

## READY

Topic: Products of probabilities, multiplying and dividing fractions
**Find the products or quotients below.**

1. $\frac{1}{2} \cdot \frac{2}{3}$

2. $\frac{3}{5} \cdot \frac{1}{3}$

3. $\frac{7}{10} \cdot \frac{2}{5}$

4. $\frac{8}{7} \cdot \frac{3}{4}$

5. $\frac{\frac{1}{3}}{\frac{1}{2}}$

6. $\frac{2}{5} \div \frac{2}{3}$

7. $P(A) = \frac{3}{4}$ $P(B) = \frac{1}{2}$

   $P(A) * P(B) =$

8. $P(A) = \frac{1}{6}$ $P(B) = \frac{1}{3}$

   $P(A) * P(B) =$

## SET

Topic: Connecting representations of events for probability
**For each situation, one of the representations (two-way table, Venn diagram, tree diagram, context or probability notation) is provided. Use the provided information to complete the remaining representations.**

9. Are you Blue?

**Notation**

Key:
Male = M Female = F
Blue = B Not Blue = N

Sample size = 200

$P(B) = 84/200$ $P(M) = 64/200$

$P(F|B) = 48/84$ $P(B|F) =$

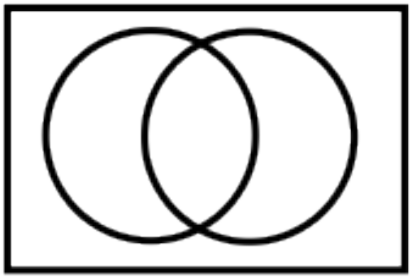
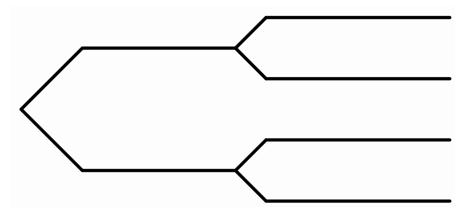
$P(M \cap B) =$ $P(M \cup B) =$

**2-way Table**

| | Blue | Not Blue | Total |
|---|---|---|---|
| Male | | | |
| Female | | | |
| Total | | | |

(Continued on the next page)

(Continued from the last page)

| Venn Diagram | Tree Diagram |
|---|---|
|  |  |
| Write three observations you can make about this data. | |

10. Right and left handedness of a group.

**Notation**

Key:
Male = M      Female = F
Lefty = L      Righty = R

Sample size = 100 people

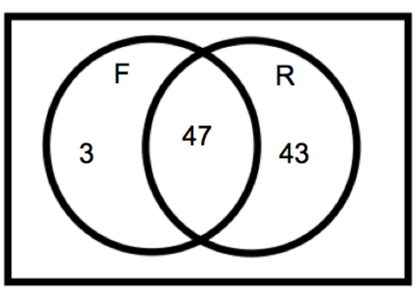
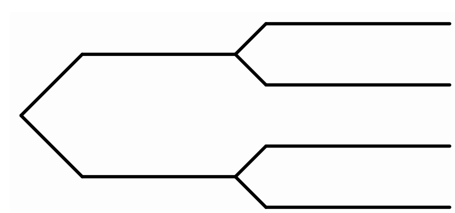
P(L) =      P(M) =

P(F) =      P(L|F) =

P(L|M) =

**2-way Table**

| | Lefty | Righty | Total |
|---|---|---|---|
| Male | | | |
| Female | | | |
| Total | | | |

| Venn Diagram | Tree Diagram |
|---|---|
|  |  |
| Write three conditional statements regarding this data. | |

11. The most important meal of the day.

**Notation**

Key:
Male = M    Female = F
Eats Breakfast = E    Doesn't Eat Breakfast = D

Sample size =
P(E) =    P(E|M) =

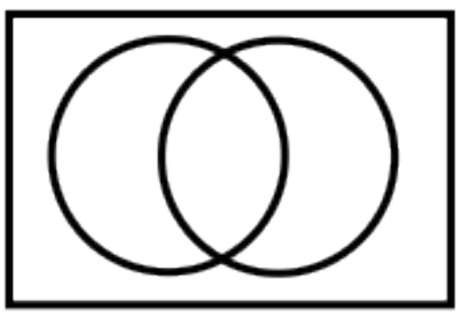
P(E∩M)=    P(E|F) =

P(E∩F) =

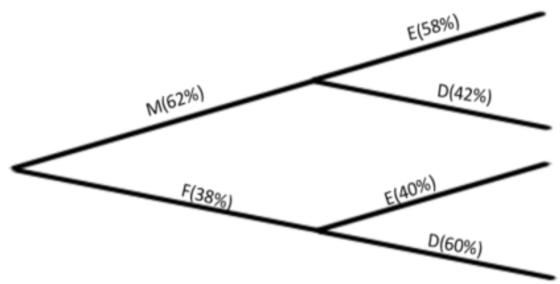
**2-way Table**

| | Eats | Doesn't | Total |
|---|---|---|---|
| Male | | | |
| Female | | | |
| Total | | | 685 |

**Venn Diagram**



**Tree Diagram**




Does this data surprise you? Why or why not.

## GO

Topic: Writing conditional statements from two-way tables

12. Complete the table and write three conditional statements.

| | Biking | Swimming | Total |
|---|---|---|---|
| Male | | 50 | |
| Female | 35 | | 76 |
| Total | 85 | | |

13. Complete the table about preferred genre of reading and write three conditional statements.

| | Ice Cream | Cake | Total |
|---|---|---|---|
| Male | | 20 | |
| Female | 10 | | 60 |
| Total | 85 | | |

14. Complete the table about eye color and write three conditional statements.

| | Blue | Green | Brown | Other | Total |
|---|---|---|---|---|---|
| Male | 55 | 20 | 15 | | 100 |
| Female | | 20 | | 10 | |
| Total | | | 75 | | 230 |

# 8.5 Freddy Revisited

## *A Solidify Understanding Task*



In task 9.3 *Fried Freddy's,* Tyrell helped Freddy in determining the amount and type of food Freddy should prepare each day for his restaurant. As a result, Freddy's food waste decreased dramatically. As time went by, Freddy noticed that another factor he needed to consider was the day of the week. He noticed that he was overpreparing during the week and sometimes underpreparing on the weekend. Tyrell and Freddy worked together and started collecting data to find the average number of orders he received of chicken and fish on a weekday and compared it to the average number of orders he received of each on the weekend. After two months, they had enough information to create the two way table below:

| | Fish | Chicken | Total |
|---|---|---|---|
| Weekday | 65 | 79 | 144 |
| Weekend | 88 | 107 | 195 |
| Total | 153 | 186 | 339 |

1. What observations can be made from the table (include probability statements)?

2. What do you notice about the probability statements?

3. Based on the data, if Freddy had a sales promotion and anticipated 500 orders in a given week, how many of each (chicken and fish) should he order?

# 8.5 Freddy Revisited – Teacher Notes

## *A Solidify Understanding Task*

**Purpose**: The purpose of this task is for students to determine if two events are independent. In this task, students are asked to interpret the amount of fish and chicken Freddy should prepare on any given day. The goal is for students to recognize that Freddy sells more food on a weekend day than he does on a weekday, however, the percentage of each food type stays the same. In other words, the likelihood that a randomly selected customer would order chicken is independent as to whether or not it is a weekday or a weekend.

**Core Standards Focus**:

**S.CP.2:** Understand that two events $A$ and $B$ are independent if the probability of $A$ and $B$ occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

**S.CP.3:** Understand the conditional probability of $A$ given $B$ as $P(A \text{ and } B)/P(B)$, and interpret independence of $A$ and $B$ as saying that the conditional probability of $A$ given $B$ is the same as the probability of $A$, and the conditional probability of $B$ given $A$ is the same as the probability of $B$.

**S.CP.4:** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

**S.CP.5:** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

**Related Standards: S.CP.6, S.MD.7+**

**Standards for Mathematical Practice:**

**SMP 6 – Attend to precision**
**SMP 8 – Look for and express regularity in repeated reasoning**

**The Teaching Cycle:**

**Launch (Whole Class):**
Begin with reminding students about the task *Fried Freddy's* and how Tyrell helped Freddy determine the amount of chicken and fish he should prepare each day. Launch into this task by sharing the scenario, and then have students answer the three questions.

**Explore (Small Group):**
In small groups, have students make as many observations as possible as to the amount of food Freddy should prepare. Some observations may be simple (such as 'Freddy serves more chicken than fish'), but other observations **may** include the following:

- Additive observation: Freddy serves more chicken than fish: there are 14 more orders of chicken than fish each weekday.
- Probability statement (multiplicative observation): Given that it is a weekday, 55% of Freddy's business orders are chicken.
- Probability statement (multiplicative observation): Given that it is a weekend, 55% of Freddy's business orders are chicken.
- Probability statement (multiplicative observation): 55% of Freddy's business orders are chicken (regardless of whether it is a weekday or weekend).

- Additive: There are 23 more orders of fish on the weekend than during the week.

- Additive: There are 28 more orders of chicken on the weekend than during the week.
- Additive: There are 51 more orders on the weekend than during the week.

- Multiplicative: There is a 35.4 % increase in the number of orders for fish on the weekend than during the week.
- Multiplicative: There is a 35.4 % increase in the number of orders for chicken on the weekend than during the week.
- Multiplicative: There is a 35.4 % increase in the number of orders for food on the weekend than during the week.

- Probability statement (multiplicative observation): 45% of Freddy's business orders fish, regardless of whether or not it is a weekday or a weekend.

There are many observations students can make at this point. The goal is to distinguish observations that are probability statements versus 'additive' observations.

**Discuss (Whole Class):**
Begin the discussion phase by having students share observations in general, then hone in on the probability statements. Select a student to share who has noticed that the conditional probability is the same as a general probability statement. At this point, introduce the vocabulary of independent events and explain that two events are considered independent when the conditional probability of $A$ given $B$ is the same as the probability of $A$. Next, show the conditional probability test for independence using notation (for example: P(fish|weekday) = f(fish)). Ask students to consider the example just shared and to determine if the example shows two independent events. Have students share with their partner and explain why the events are independent or not.

Have another student share a different observation using a probability statement, and then ask the class to find out if these are independent events. At the end of this task, all students should be able to recognize that fish is independent from the day of the week because the percent of people who prefer fish, given that it is a weekday (or weekend) is the same as the percent of people who prefer fish in general.

Use notation to show this relationship. Do the same with chicken given that it is a weekday. Show this data using a tree diagram and a Venn diagram. What do students notice regarding independen ce when data is presented in a particular representation? Again, use notation to show independence and then connect this to each representation you have drawn. Conclude the task by writing independence statements, drawing a representation model that matches and connecting to standards **S.CP.2, S.CP.3, S.CP.4,** and **S.CP.5.**

**Aligned Ready, Set, Go: Probability 8.5**

READY, SET, GO! Name Period Date

## READY

Topic: Quadratic functions
**Find the x-intercepts, y-intercept, line of symmetry and vertex for the quadratic functions.**

1. $f(x) = x^2 + 8x - 9$
2. $g(x) = x^2 - 3x - 5$
3. $h(x) = 2x^2 + 5x - 3$
4. $k(x) = x^2 + 6x - 9$
5. $p(x) = (x + 5)^2 - 2$
6. $q(x) = (x + 7)(x - 5)$

## SET

Topic: Independence
***Determining the independence of events can sometimes be done by becoming familiar with the context in which the events occur and the nature of the events. There are also some ways of determining independence of events based on equivalent probabilities.***

- *Two events, A and B, are independent if* $P(A \text{ and } B) = P(A) \cdot P(B)$
- *Additionally, two events, A and B, are independent if* $P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = P(A)$

**Use these two ways of determining independent events to determine independence in the problems below and answer the questions.**

7. P (A and B) = $\frac{3}{5}$
   P (A) = $\frac{1}{2}$
   P (B) = $\frac{3}{10}$

8. P (A) = $\frac{1}{5}$
   P (A and B) = $\frac{1}{6}$
   P (B) = $\frac{1}{3}$

9. P (A) = $\frac{1}{2}$
   P (A and B) = $\frac{1}{5}$
   P (B) = $\frac{2}{5}$

10. P (A and B) = $\frac{2}{5}$
    P (A) = $\frac{1}{4}$
    P (B) = $\frac{4}{5}$

## *GO*

Topic: Find Probabilities from a two-way table

**The following data represents the number of men and women passengers aboard the titanic and whether or not they survived.**

| | Survived | Did not survive | Total |
|---|---|---|---|
| Men | 146 | 659 | 805 |
| Women | 296 | 106 | 402 |
| Total | 442 | 765 | 1207 |

11. P(w) =
12. P(s) =
13. P(s|w) =
14. P(w or s) =
15. P(w or m) =
16. P(ns|w) =
17. P(m ∩ ns) =

# 8.6 Striving for Independence

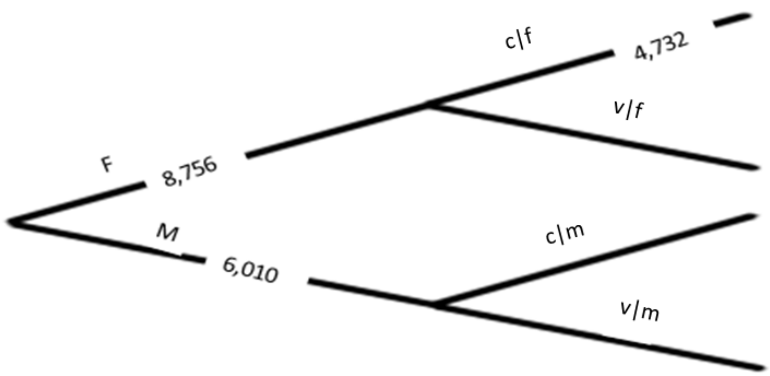
## *A Practice Understanding Task*



Answer the questions below using your knowledge of conditional probability (the probability of $A$ given $B$ as $P(A \text{ and } B)/P(B)$ ) as well as the definition of independence. Two events (A and B) are said to be independent if $P(A|B) = P(A)$ *and* $P(B|A) = P(B)$. Keep track of how you are determining independence for each type of representation.

1. Out of the 2000 students who attend a certain high school, 1400 students own cell phones, 1000 own a tablet, and 800 have both. Create a Venn diagram model for this situation. Use proper probability notation as you answer the questions below.
   a) What is the probability that a randomly selected student owns a cell phone?
   b) What is the probability that a randomly selected students owns both a cell phone and a tablet?
   c) If a randomly selected student owns a cell phone, what is the probability that this student also owns a tablet?
   d) How are questions b and c different?
   e) Are the outcomes, *owns a cell phone* and *owns a tablet,* independent? Explain.

2. Below is a partially completed tree diagram from the task *Chocolate vs Vanilla.*
   a) Circle the parts of the diagram that would be used to determine if choosing chocolate is independent of being a male or female.
   b) Complete the diagram so that choosing chocolate is independent of being male or female.



3. Use the data from the Titantic below to answer the following questions.

| | Survived | Did not survive | Total |
|---|---|---|---|
| Men | 146 | 659 | 805 |
| Women | 296 | 106 | 402 |
| Total | 442 | 765 | 1207 |

a) Determine if survival is independent of being male for this data. Explain or show why or why not. If it is not independent determine how many men would need to survive in order to make it independent.

4. Determine whether the second scenario would be dependent or independent of the first scenario. Explain.

a) Rolling a six-sided die, then drawing a card from a deck of 52 cards.
b) Drawing a card from a deck of 52 cards, then drawing another card from the same deck.
c) Rolling a six-sided die, then rolling it again.
d) Pulling a marble out of a bag, replacing it, then pulling a marble out of the same bag.
e) Having 20 treats in five different flavors for a soccer team, with each player taking a treat.

# 8.6 Striving for Independence – Teacher Notes

## *A Practice Understanding Task*

**Purpose**: The purpose of this task is for students to practice determining whether one event is independent of another event. Students will use data from different representations, plus make sense of whether or not one scenario would be independent of another. In the end, students will explain how to quickly determine independence from a Venn diagram, a tree diagram, and a two-way table.

**Core Standards Focus**:

**S.CP.2:** Understand that two events $A$ and $B$ are independent if the probability of $A$ and $B$ occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

**S.CP.3** Understand the conditional probability of $A$ given $B$ as $P(A$ and $B)/P(B)$, and interpret independence of $A$ and $B$ as saying that the conditional probability of $A$ given $B$ is the same as the probability of $A$, and the conditional probability of $B$ given $A$ is the same as the probability of $B$.

**S.CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

**S.CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

**S.CP.6:** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

**Related Standards: S.CP.1, S.CP.7**

**Standards for Mathematical Practice:**

**SMP 1 – Make sense of problems and persevere in solving them**
**SMP 2 – Reason abstractly and quantitatively**
**SMP 4 – Model with mathematics**
**SMP 7 – Look and make use of structure**

**The Teaching Cycle:**

**Launch (Whole Class):**
Start this task by reviewing the definition of independence, then have students work in pairs to practice using probability and determining if two events are independent using data from different representations.

**Explore (Small Group):**
As you monitor, look for students to use their knowledge of conditional probability and the formula for independence to determine whether two events are independent for each situation. Select students to share how the representation and notation are connected, and explain whether the two events are independent.

**Discuss (Whole Class):**
The goal of the whole group discussion is for students to be able to fluently write conditional probability statements, to make sense of conditional probability using the different representations they have been using throughout this module, and to determine if two events are independent. Using the groups you selected during the explore phase, sequence the order of how they share their problem to reach the goals of the task. For each situation, connect the representation being used to the formula for independence.

**Aligned Ready, Set, Go: Probability 8.6**

READY, SET, GO! Name Period Date

## *READY*

Topic: Solving quadratics
**Solve each of the quadratics below using an appropriate method.**

1. $m^2 + 15m + 56 = 0$

2. $5x^2 - 3x + 7 = 0$

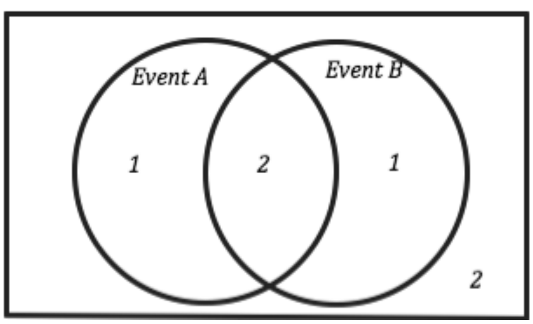
3. $x^2 - 10x + 21 = 0$

4. $6x^2 + 7x - 5 = 0$

## *SET*

Topic: Representing Independent Events in Venn Diagrams
**In each of the Venn Diagrams the number of outcomes for each event are given, use the provided information to determine the conditional probabilities or independence. The numbers in the Venn Diagram indicate the number of outcomes in that part of the sample space.**

5.




a. How many total outcomes are possible?

b. P(A) =

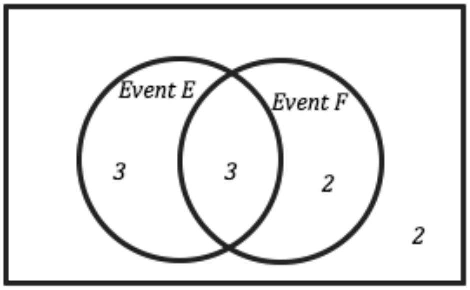
c. P(B) =

d. P(A∩B) =

e. P(A|B) =

f. Are events A and B independent events? Why or why not?

6.




a. How many total outcomes are possible?

b. P(E) =

c. P(F) =

d. P(E∩F) =

e. P(E|F) =

f. Are events E and F independent events? Why or why not?

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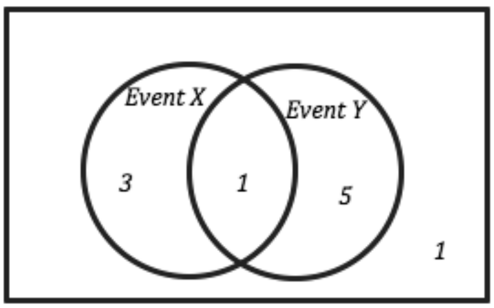
7.




a. How many total outcomes are possible?

b. P(X) =

c. P(Y) =

d. P(X∩Y) =

e. P(X|Y) =

f. Are events X and Y independent events? Why or why not?

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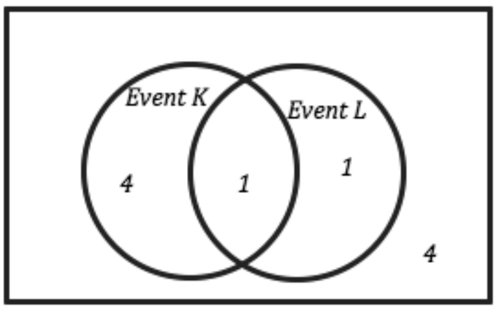
8.




a. How many total outcomes are possible?

b. P(K) =

c. P(L) =

d. P(K∩L) =

e. P(K|L) =

f. Are events K and L independent events? Why or why not?

## GO

Topic: Conditional Probability and Independence

***Data gathered on the shopping patterns during the months of April and May of high school students from Peanut Village revealed the following. 38% of students purchased a new pair of shorts (call this event H), 15% of students purchased a new pair of sunglasses (call this event G) and 6% of students purchased both a pair of short and a pair of sunglasses.***

9. Find the probability that a student purchased a pair of sunglasses given that you know they purchased a pair of shorts. P (G|H) =

10. Find the probability that a student purchased a pair of shorts or purchased a new pair of sunglasses. P (H ∪ G) =

11. Given the condition that you know a student has purchased at least one of the items. What is the probability that they purchased only one of the items?

12. Are the two events H and G independent of one another? Why or Why not?

**Given the data collected from 200 individuals concerning whether or not to extend the length of the school year in the table below answer the questions.**

| | **For** | **Against** | **No Opinion** | |
|---|---|---|---|---|
| Youth (5 to 19) | 7 | 35 | 12 | |
| Adults (20 to 55) | 30 | 27 | 20 | |
| Seniors (55 +) | 25 | 16 | 28 | |
| | | | | 200 |

13. Given that condition that a person is an adult what is the probability that they are in favor of extending the school year? P(For|Adult) =

14. Given the condition that a person is against extending the school year what is the probability they are a Senior? P (Senior|Against) =

15. What is the probability that a person has no opinion given that they are a youth?
P (no opinion| youth) =