Learning is a consequence of thinking. Ron Ritchhart; (Making Thinking Visible)

Because of the abstract nature of mathematics, people have access to mathematical ideas only through the representations of those ideas. National Research Council (2001) p. 94

Using representations becomes the way in which students go about the process of learning.

Multiple representations are the vehicles through which students explore content.

Through the construction and sharing of representations, students mentally engage with the content through offering their ideas, explanations, justifications, interpretations, reasons, evidence, perspectives, alternatives, and questions.

Problem #1:
How many bows that are $\frac{5}{12}$ yards long can be made from 3 yards of ribbon?

Problem #2: Task 4.1 Cafeteria Actions and Reactions
Elvira, the cafeteria manager, has just received a shipment of new trays with the school logo prominently displayed in the middle of the tray. After unloading 4 cartons of trays in the pizza line, she realizes that students are arriving for lunch and she will have to wait until lunch is over before unloading the remaining cartons. The new trays are very popular and in just a couple of minutes 24 students have passed through the pizza line and are showing off the school logo on the trays. At this time, Elvira decides to divide the remaining trays in the pizza line into 3 equal groups so she can also place some in the salad line and the sandwich line, hoping to attract students to the other lines. After doing so, she realizes that each of the three serving lines has only 12 of the new trays.

• “That’s not many trays for each line; I wonder how many trays there were in each of the cartons I unloaded.”

Problem #3: If $x > y$, then which is greater $x + a$ or $y + a$?

Problem #4: I know that $5 > 4$. Which is greater $5x$ or $4x$?
Problem #5:
5. 1 Pet Sitters  A Develop Understanding Task

The Martinez twins, Carlos and Clarita, are trying to find a way to make money during summer vacation. When they overhear their aunt complaining about how difficult it is to find someone to care for her pets while she will be away on a trip, Carlos and Clarita know they have found the perfect solution. Not only do they have a large, unused storage shed on their property where they can house animals, they also have a spacious fenced backyard where the pets can play.

Carlos and Clarita are making a list of some of the issues they need to consider as part of their business plan to care for cats and dogs while their owners are on vacation.

- **Space:** Cat pens will require 6 ft$^2$ of space, while dog runs require 24 ft$^2$. Carlos and Clarita have up to 360 ft$^2$ available in the storage shed for pens and runs, while still leaving enough room to move around the cages.

- **Start-up Costs:** Carlos and Clarita plan to invest much of the $1280 they earned from their last business venture to purchase cat pens and dog runs. It will cost $32 for each cat pen and $80 for each dog run.

Of course, Carlos and Clarita want to make as much money as possible from their business, so they are trying to determine how many of each type of pet they should plan to accommodate. They plan to charge $8 per day for boarding each cat and $20 per day for each dog.

After surveying the community regarding the pet boarding needs, Carlos and Clarita are confident that they can keep all of their boarding spaces filled for the summer.

So the question is, how many of each type of pet should they prepare for? Their dad has suggested the same number of each, perhaps 12 cats and 12 dogs. Carlos thinks they should plan for more dogs, since they can charge more. Clarita thinks they should plan for more cats since they take less space and time, and therefore they can board more.

**What do you think?** What recommendations would you give to Carlos and Clarita, and what argument would you use to convince them that your recommendation is reasonable?