April 14, 2015
11:15 – 12:15
Room 105

mathematics vision project

Sense-Making, the Ultimate Intervention

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More than 200 FREE tasks for high school students written to address every standard in the CCSS-M

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The curriculum reflects the latest research on how students learn mathematics.

The quiz is on the handout.

Problems 1 and 3 are the easiest, so you may want to do them first.

Problem #1. Fill in the blanks using the numbers in the box below the paragraph.

Thomas Jefferson was born on _____, April, _____. He was the author of the Declaration of Independence. He was also an architect, a scientist, and a farmer. In _____, at the age of _____, Thomas Jefferson became the _____d president of the United States.

1801 3 1743 58 13

#1. Put the numbers below in the correct order.

3, 13, 58, 1743, 1801

This is the correct order for #1.

13, 1743, 1801, 58, 3

What made it possible for you to put the numbers in the correct order?

Sense-making Depends on Context

"Number sense builds on students' natural insights and convinces them that mathematics makes sense, that it is not just a collection of rules."

"Teaching Number Sense" by Hilde Howden
Arithmetifc Teacher February 1989, pg.7

Problem #2

Barry scored _____, _____, _____, and _____ on his four mathematics tests. His average score was _____. Tanya scored _____, _____, and _____ on her first 3 mathematics tests. To have an average score _____ points greater than Barry's average score, Tanya must get _____ on her 4^{th} test. (Each of Barry's and Tanya's test scores were different.)

Problem #2 is a task with space inside it to think, and question, and learn

"For students to see mathematics as a subject of **learning**, not performing, they need tasks and questions in math class that have space to learn built in."

Jo Boaler, Stanford University

Problem #3

The printer left the decimal out of the answer. Use what you know to place the decimal in the correct location.

$$\begin{array}{r} 3.5 \\ \times\ 4.5 \\ \hline 1575 \end{array} \qquad \begin{array}{r} 5.5 \\ \times\ 3.2 \\ \hline 176 \end{array}$$

Problem #3

Most students missed the second problem even though they got the first one right.

$$\begin{array}{r} 3.5 \\ \times 4.5 \\ \hline 15.75 \end{array} \qquad \begin{array}{r} 5.5 \\ \times 3.2 \\ \hline 17.6 \end{array}$$

Sense-making Relies on Context

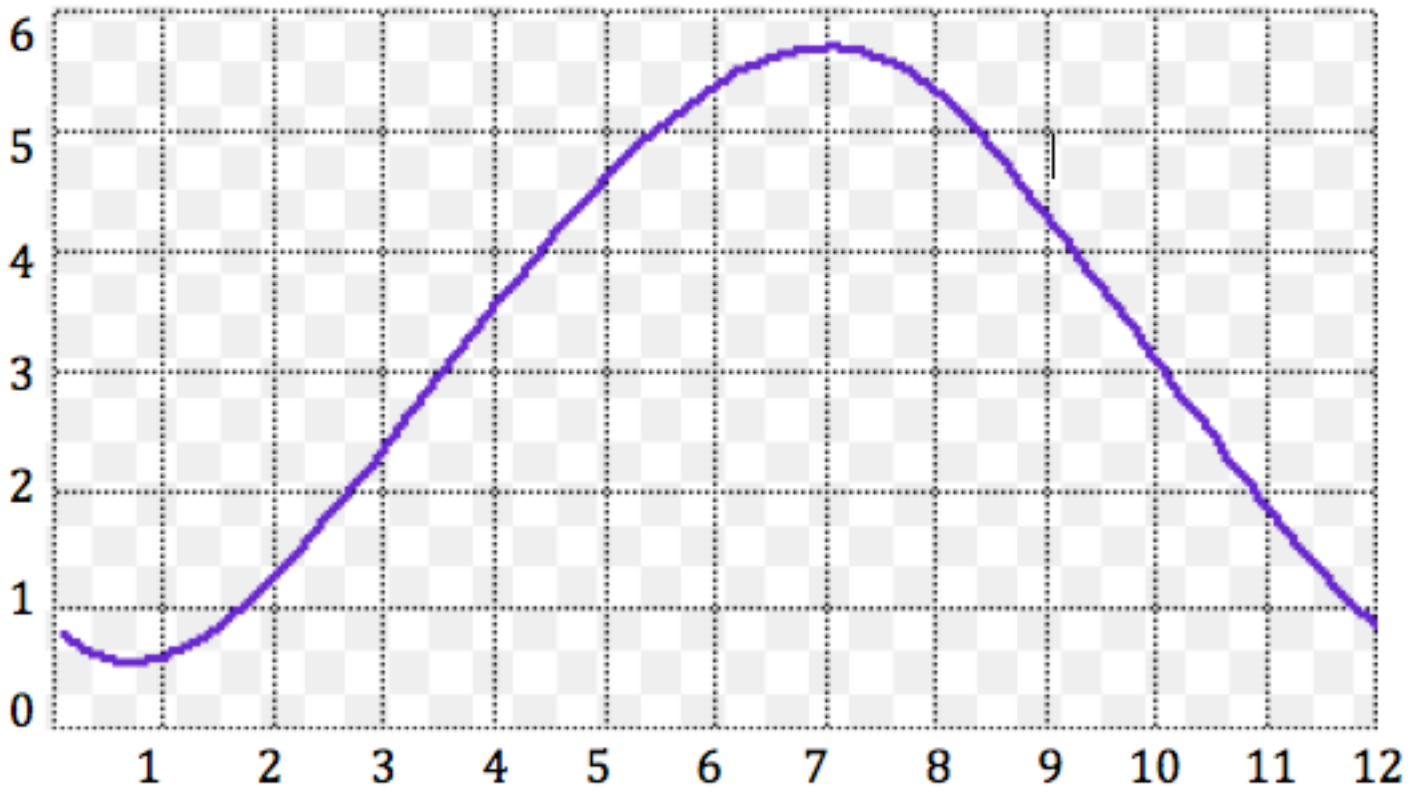
One research study revealed that students either do not have a reasonably developed number sense or, if they do have it, they

DO NOT APPLY IT TO TASKS IN A MATHEMATICAL CONTEXT.

In other words, math class is about following the rules instead of thinking.

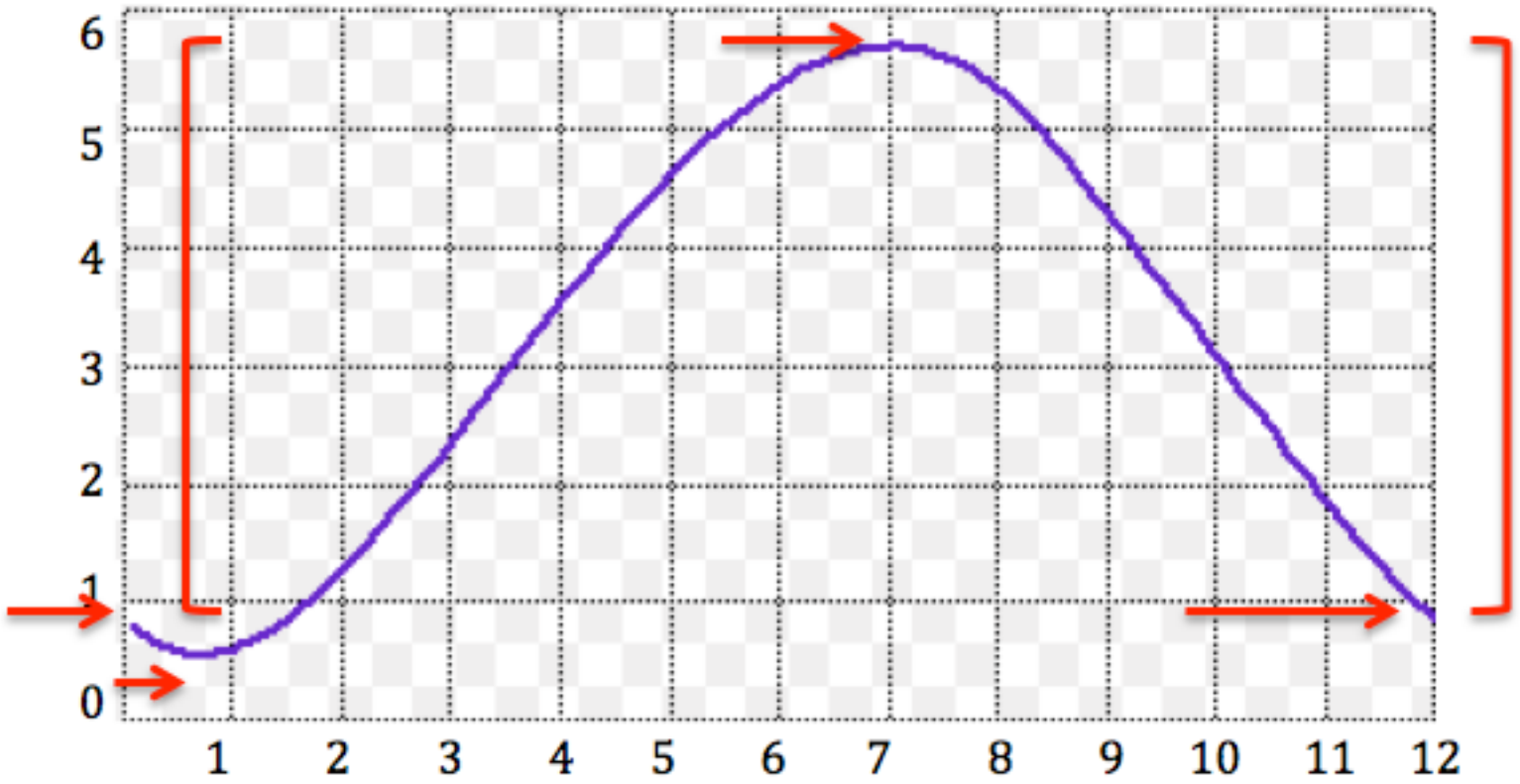
Practice Understanding: Building procedural fluency

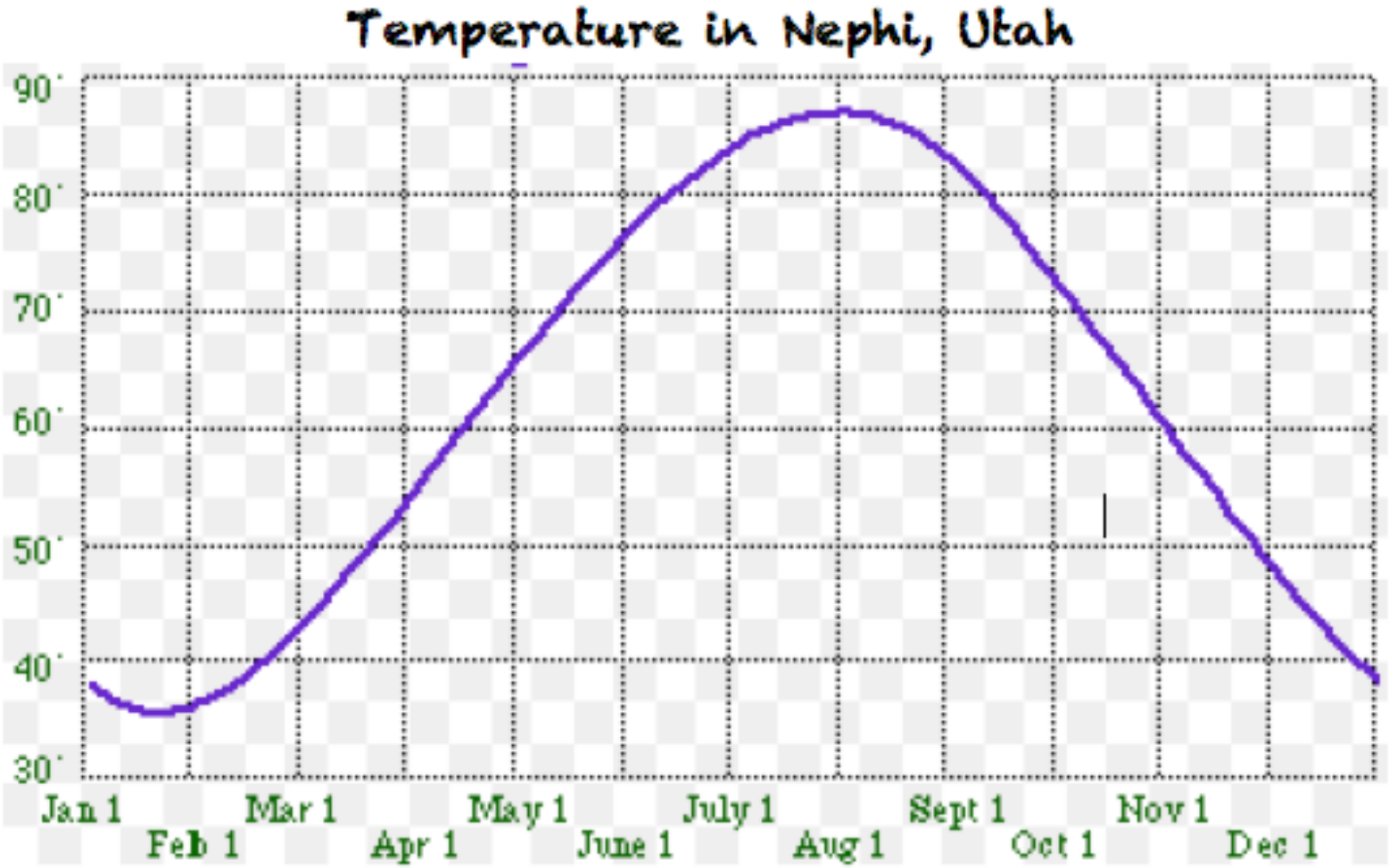
Write the interval(s) where the graph is increasing and where it is decreasing.



Most common student answers.

Decreasing (0.8, 0.6) and [5.75, 0.9] Increasing [0.6, 5.75)




Temperature in Nephi, Utah
90˚
80˚
70˚
60˚
50˚
40˚
30˚
Jan 1
Feb 1
Mar 1
Apr 1
May 1
June 1
July 1
Aug 1
Sept 1
Oct 1
Nov 1
Dec 1

Mathematics Teaching Practices

- **Establish mathematics goals to focus learning.**
- **Implement tasks that promote reasoning and problem solving.**
- **Use and connect mathematical representations.**
- **Facilitate meaningful mathematical discourse.**
- **Pose purposeful questions.**
- **Build procedural fluency from conceptual understanding.** Effective teaching of mathematics builds fluency with procedures on a **foundation of conceptual under**standing so that students, ***over time***, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
- **Support productive struggle in learning mathematics.**
- **Elicit and use evidence of student thinking.**

This is Isaac.
He was a student in my 9th grade class.

Isaac struggles in math. Sometimes he has the right answer but no work. Maybe he cheats. He doesn't like to write anything down and he seldom does his homework. He's lazy, mentally and physically. Good luck!

2.7 Shopping for Cats and Dogs

A Develop Understanding Task

Clarita is upset with Carlos because he has been buying cat and dog food without recording the price of each type of food in their accounting records. Instead, Carlos has just recorded the total price of each purchase, even though the total cost includes more that one type of food. Carlos is now trying to figure out the price of each type of food by reviewing some recent purchases. See if you can help him figure out the cost of particular items for each purchase, and be prepared to explain your reasoning to Carlos.

1. One week Carlos bought 3 bags of *Tabitha Tidbits* and 4 bags of *Figaro Flakes* for $43.00. The next week he bought 3 bags of *Tabitha Tidbits* and 6 bags of *Figaro Flakes* for $54.00. Based on this information, figure out the price of one bag of each type of cat food. Explain your reasoning.

2. One week Carlos bought 2 bags of *Brutus Bites* and 3 bags of *Lucky Licks* for $42.50. The next week he bought 5 bags of *Brutus Bites* and 6 bags of *Lucky Licks* for $94.25. Based on this information, figure out the price of one bag of each type of dog food. Explain your reasoning.

Once I have launched the task, I always ask my students to spend at least 5 minutes on their own working the task. During that time, I walk around looking for student strategies.

This is what Isaac had written on his worksheet.

1. One week Carlos bought 3 bags of *Tabitha Tidbits* and 4 bags of *Figaro Flakes* for $43.00. The next week he bought 3 bags of *Tabitha Tidbits* and 6 bags of *Figaro Flakes* for $54.00. Based on this information, figure out the price of one bag of each type of cat food. Explain your reasoning.

5,50

This is what he said.

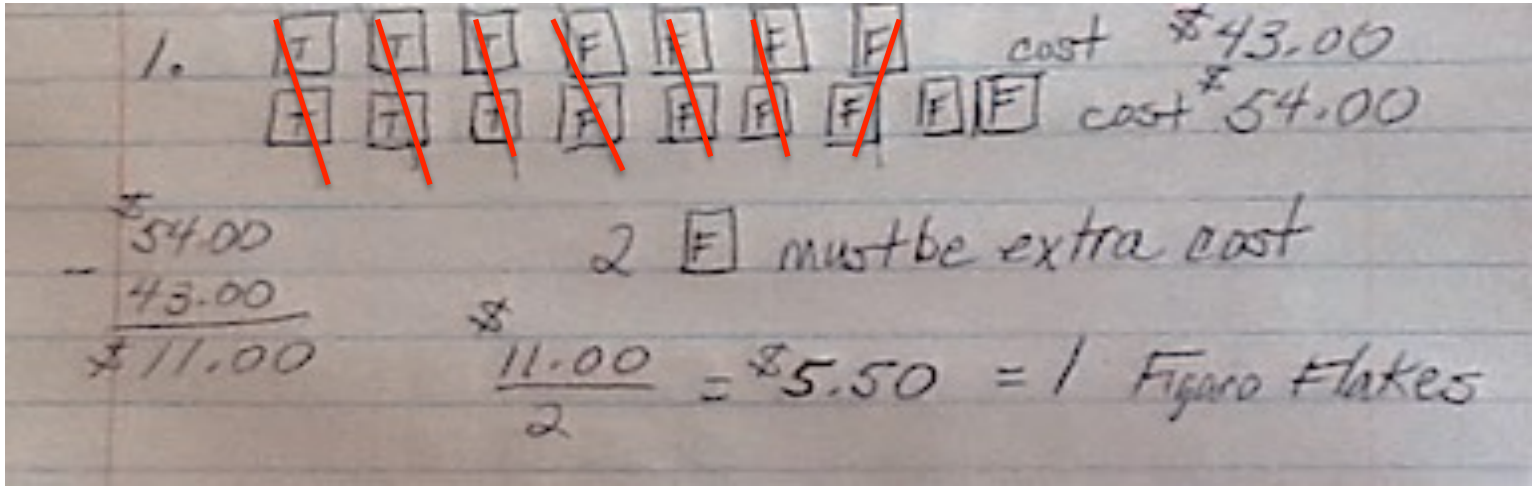
$5.50 is the cost of the Figaro Flakes.

It's obvious! Everything in the problem is the same except for the 2 extra Figaro Flakes so the extra cost must be them. I just subtracted the money in my head and got $11 so I divided by 2. Each bag costs $5.50. I don't need to write anything. I'm done.

The next step in my class:

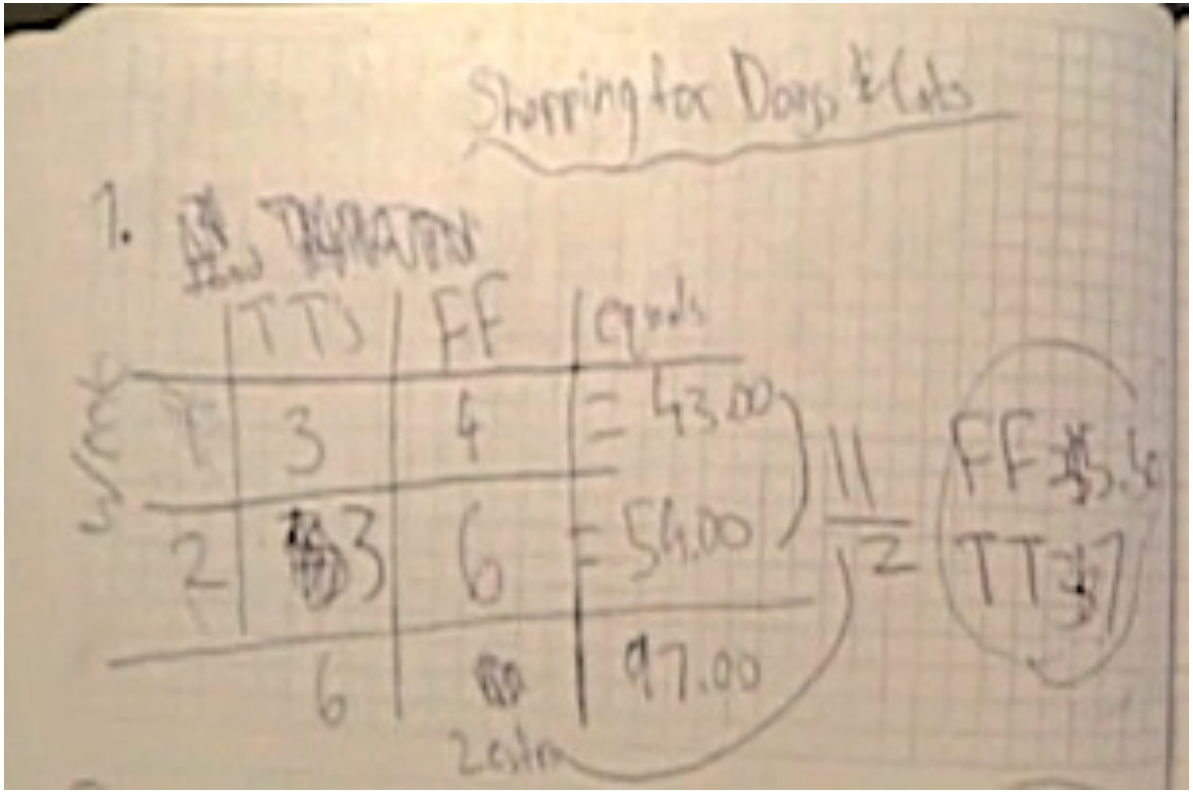
Talk to your partner or with your group and share your thinking and strategies.

(Isaac's group came up with this representation of the problem.)



Mr. Lemon's Classroom Problem #1

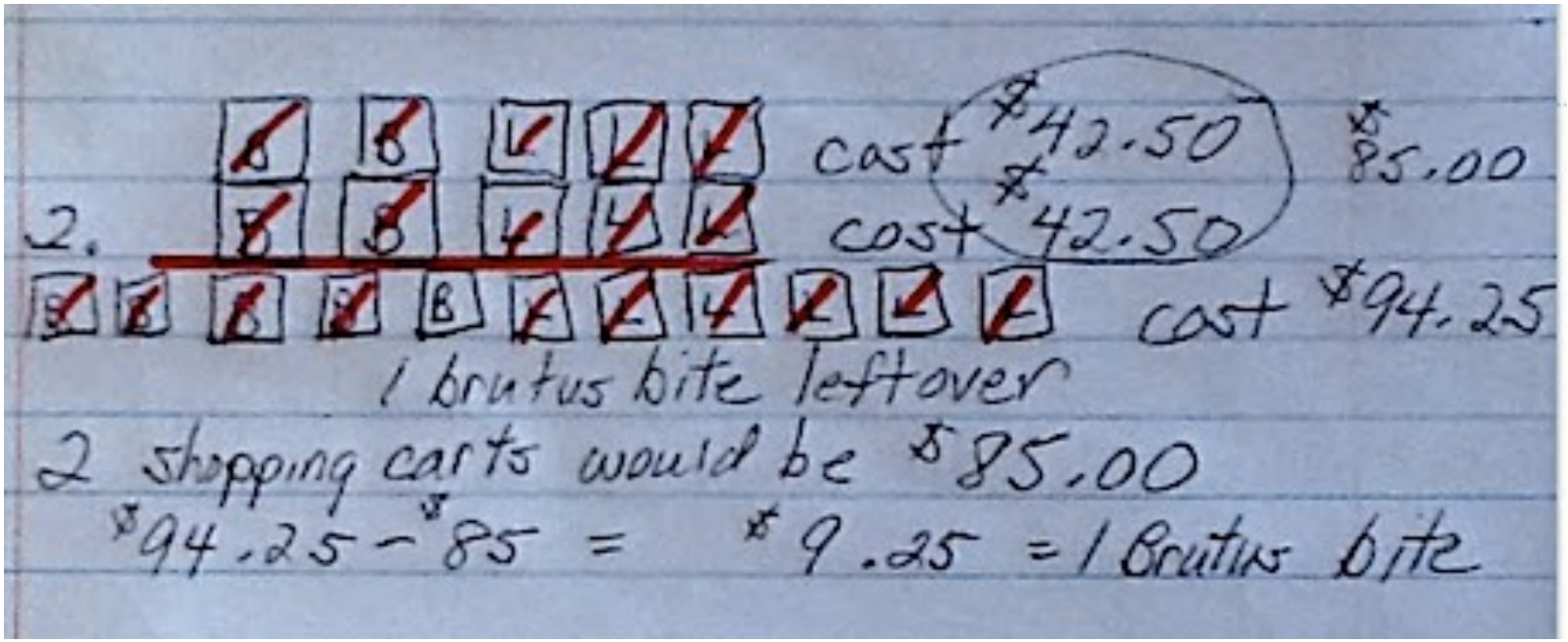
Student work for 2.7



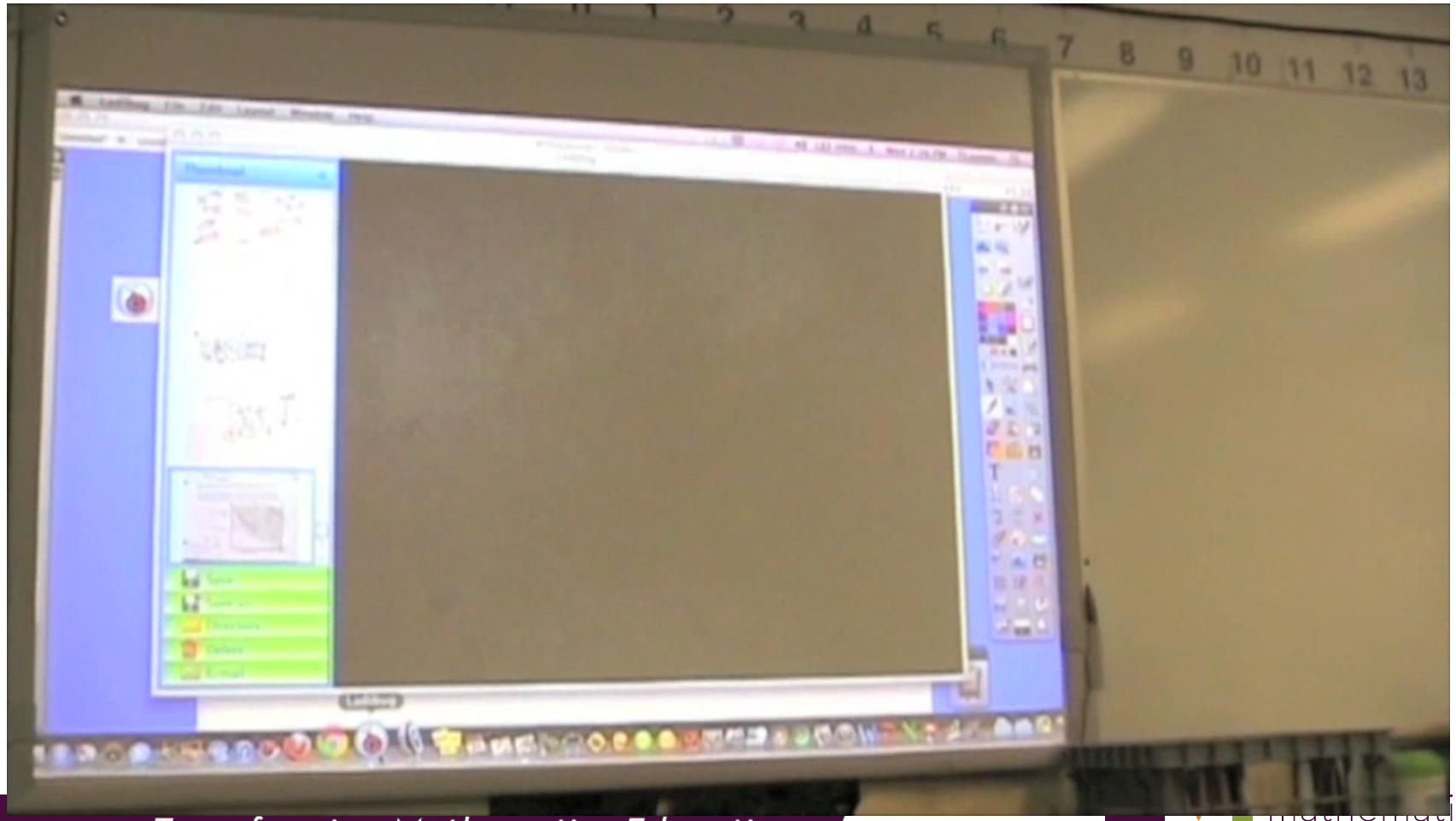
In my class I had asked the students to do problems 1 & 2. Once they had a strategy for thinking about the problem in #1, I suggested they use that strategy to think about #2.

- Isaac's strategy of just doing it in his head became more difficult. It was harder to keep all of the numbers straight. So having a way to organize the numbers started becoming important to him.

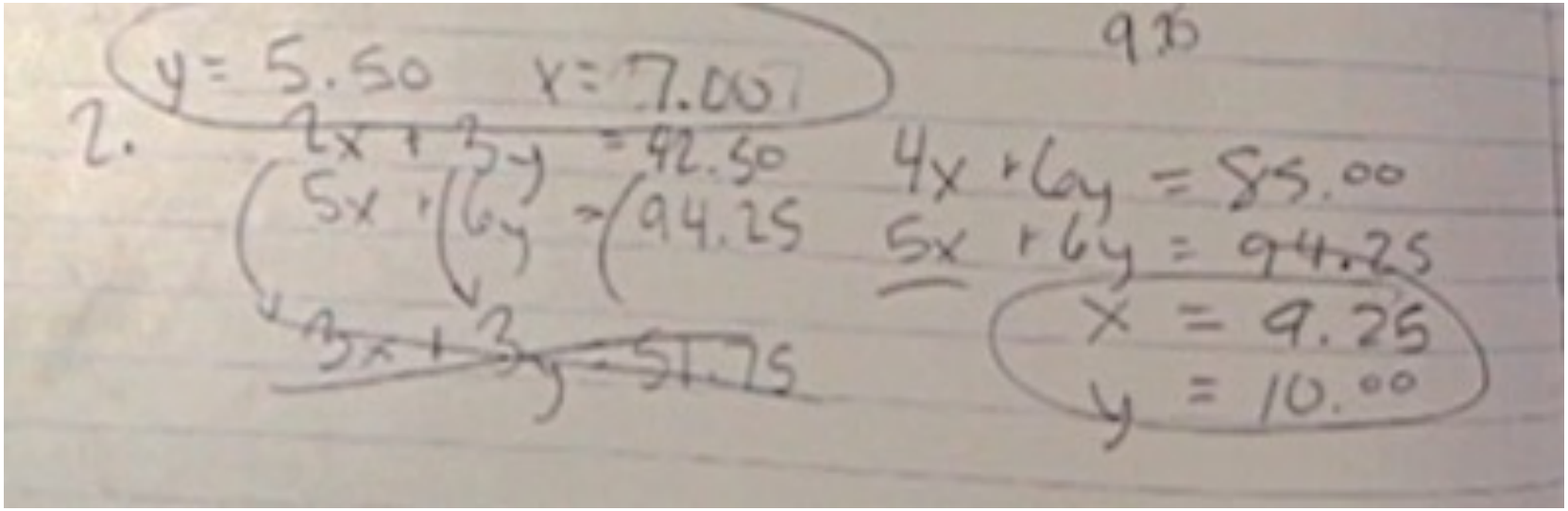
Isaac's group work for 2.7 Problem #2



Mr. Lemon's class for 2.7 Problem #2



Building Procedural Fluency



Standards for Mathematical Practice

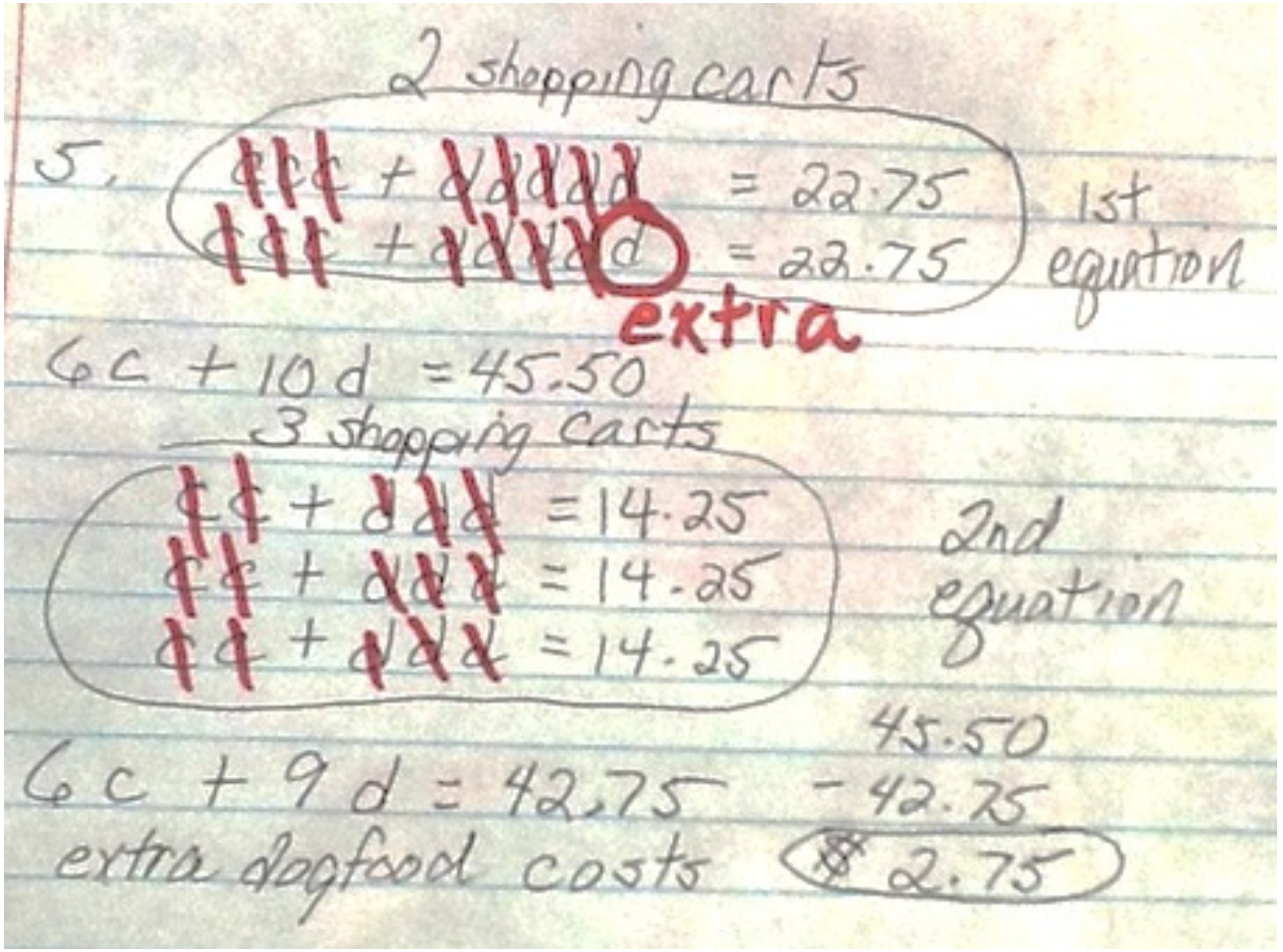
- **Make sense of problems** and **persevere** in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

How do you see the conceptual foundation being developed from problem 1 to problem 5?

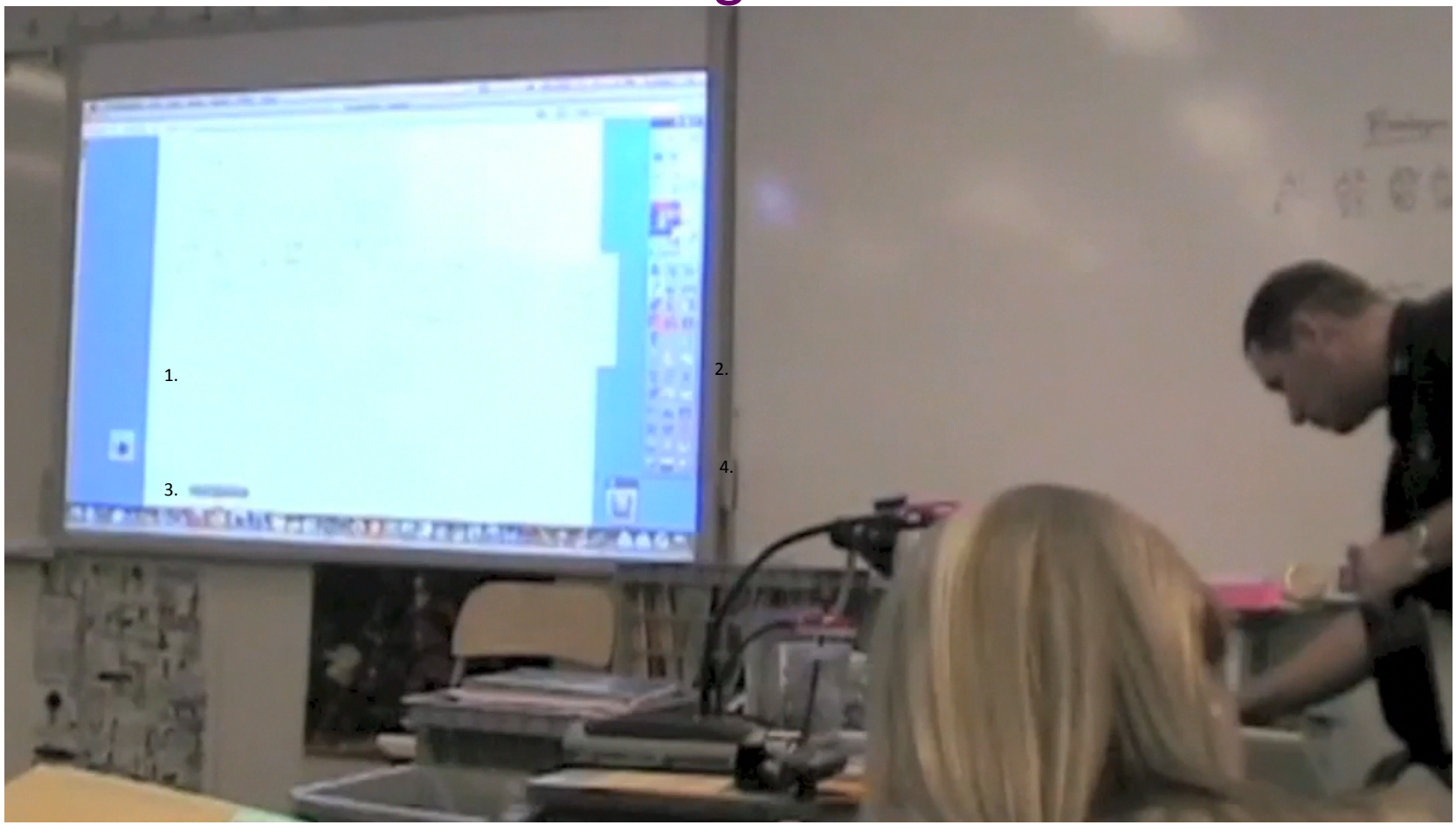
4. One week Carlos bought 2 packages of dog bones and 4 packages of cat treats for $18.50. Because the finicky cats didn't like the cat treats, the next week Carlos returned 3 unopened packages of cat treats and bought 2 more packages of dog bones. After being refunded for the cat treats, Carlos only had to pay $1.00 for his purchase. Based on this information, figure out the price of each item. Explain your reasoning.

5. Carlos has noticed that because each of his purchases have been somewhat similar, it has been easy to figure out the cost of each item. However, his last set of receipts has him puzzled. One week he tried out cheaper brands of cat and dog food. On Monday he purchased 3 small bags of cat food and 5 small bags of dog food for $22.75. Because he went through the small bags quite quickly, he had to return to the store on Thursday to buy 2 more small bags of cat food and 3 more small bags of dog food, which cost him $14.25. Based on this information, figure out the price of each bag of the cheaper cat and dog food. Explain your reasoning.

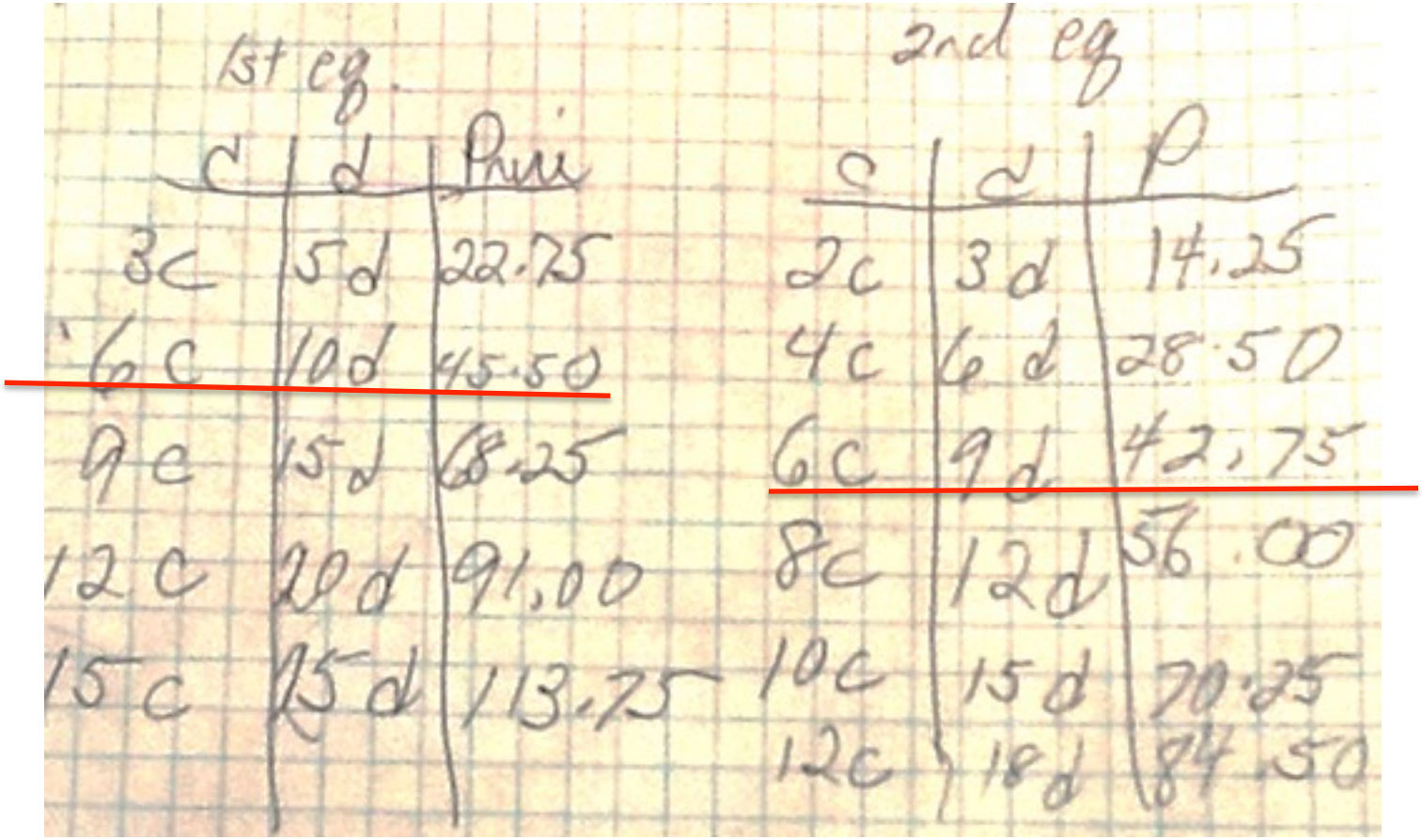
Could Isaac's group's strategy still work on problem 5?



Video of Hannah's Thinking

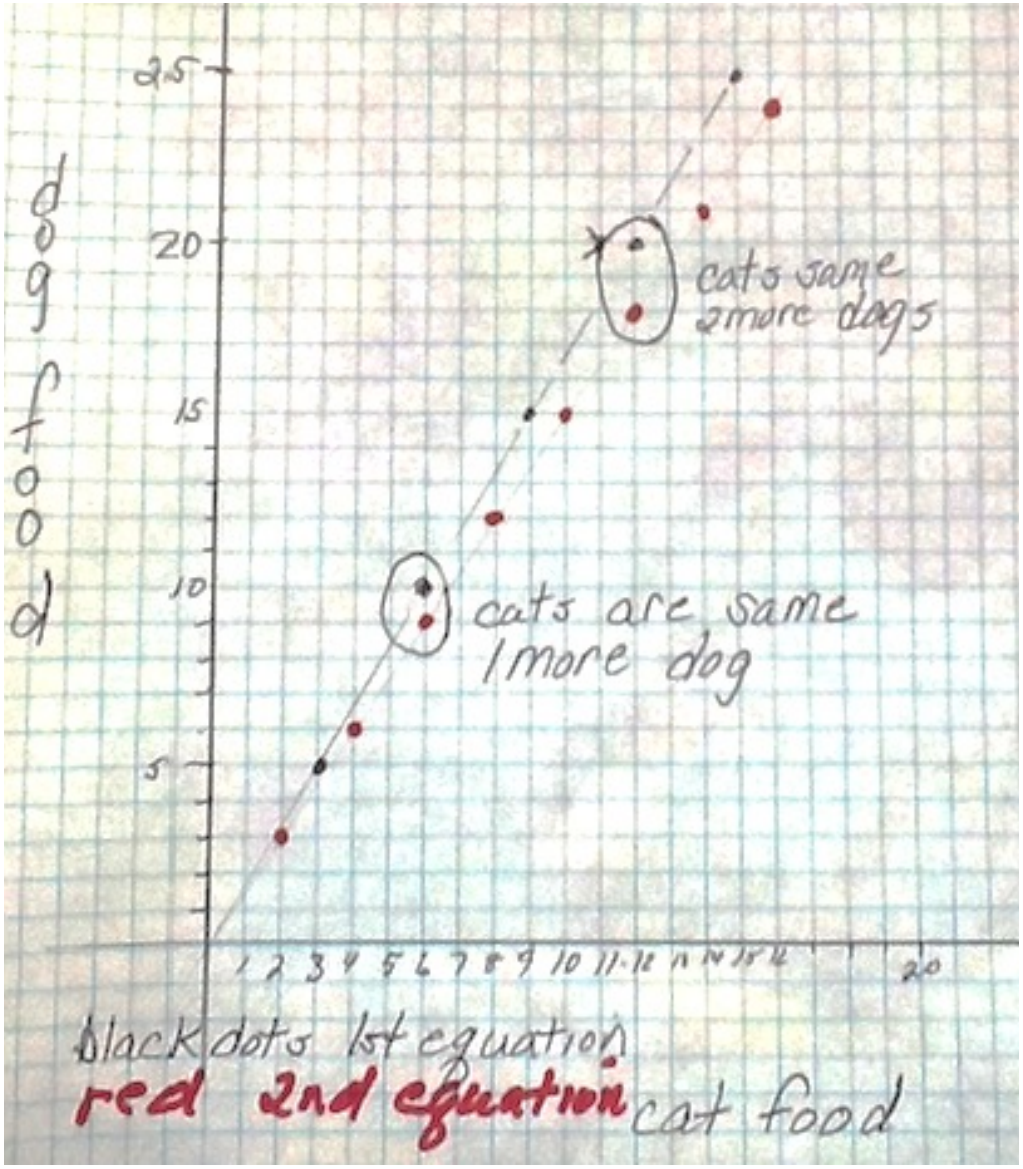


Here's the rest of the story!



They weren't finished yet.

They graphed the numbers from their table and discovered that when the cats were the same, they could see how many more dogs there were. And maybe they could use the other point with the same # of cats instead. They also wondered where the price was expressed in the graph.



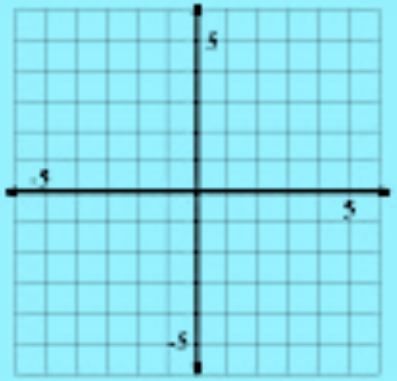
Building Procedural Fluency

Set

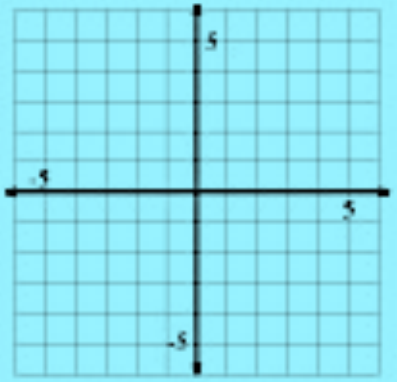
Topic: Solve systems of equations

Solve the following systems of equations using *elimination* of variables, then justify graphically.

7. $\begin{cases} 2x + 0.5y = 3 \\ x + 2y = 8.5 \end{cases}$



8. $\begin{cases} 3x + 5y = -1 \\ x + 2y = -1 \end{cases}$



Building Procedural Fluency

2. 8 Can You Get to the Point, Too?

A Solidify Understanding Task

Part 1

In "Shopping for Cats and Dogs," Carlos found a way to find the cost of individual items when given the purchase price of two different combinations of those items. He would like to make his strategy more efficient by writing it out using symbols and algebra. Help him formalize his strategy by doing the following:

- For each scenario in "Shopping for Cats and Dogs" write a **system of equations** to represent the two purchases.
- Show how your strategies for finding the cost of individual items could be represented by manipulating the equations in the system. Write out intermediate steps symbolically, so that someone else could follow your work.
- Once you find the price of one of the items in the combination, show how you would find the price of the other item.

Building Procedural Fluency

2.10 Taken Out of Context

A Practice Understanding Task

Write a shopping scenario similar to those in "Shopping for Cats and Dogs" to fit each of the following systems of equations. Then use the elimination of variables method you invented in "Can You Get to the Point, Too" to solve the system. Some of the systems may have interesting or unusual solutions. See if you can explain them in terms of the shopping scenarios you wrote.

1. $\begin{cases} 3x+4y=23 \\ 5x+3y=31 \end{cases}$

2. $\begin{cases} 2x+3y=14 \\ 4x+6y=28 \end{cases}$

3. $\begin{cases} 3x+2y=20 \\ 9x+6y=35 \end{cases}$

4. $\begin{cases} 4x+2y=8 \\ 5x+3y=9 \end{cases}$

Most teachers answer this way and are done.

2.10 Taken Out of Context

A Practice Understanding Task

Write a shopping scenario similar to those in "Shopping for Cats and Dogs" to fit each of the following systems of equations. Then use the elimination of variables method you invented in "Can You Get to the Point, Too" to solve the system. Some of the systems may have interesting or unusual solutions. See if you can explain them in terms of the shopping scenarios you wrote.

1. $\begin{cases} 3x+4y=23 \\ 5x+3y=31 \end{cases}$

2. $\begin{cases} 2x+3y=14 \\ 4x+6y=28 \end{cases}$

Same line, Many solutions

3. $\begin{cases} 3x+2y=20 \\ 9x+6y=35 \end{cases}$

Parallel, no solutions

4. $\begin{cases} 4x+2y=8 \\ 5x+3y=9 \end{cases}$

Students are still sense-making.

2.10 Taken Out of Context

A Practice Understanding Task

Write a shopping scenario similar to those in "Shopping for Cats and Dogs" to fit each of the following systems of equations. Then use the elimination of variables method you invented in "Can You Get to the Point, Too" to solve the system. Some of the systems may have interesting or unusual solutions. See if you can explain them in terms of the shopping scenarios you wrote.

1. $\begin{cases} 3x+4y=23 \\ 5x+3y=31 \end{cases}$

They must have gone to different stores. Or they had a coupon because the cost is inconsistent.

2. $\begin{cases} 2x+3y=14 \\ 4x+6y=28 \end{cases}$

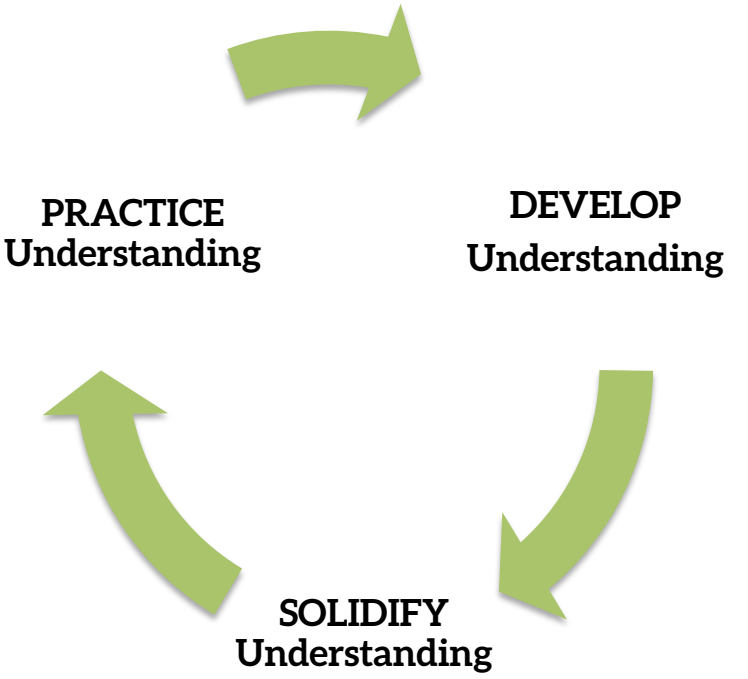
The 2nd time they went to the store they bought twice as much. It's consistent.

3. $\begin{cases} 3x+2y=20 \\ 9x+6y=35 \end{cases}$

4. $\begin{cases} 4x+2y=8 \\ 5x+3y=9 \end{cases}$

Task Sequencing: Moving from a conceptual foundation to procedural fluency

Comprehensive Mathematics Instruction Framework



- *Develop Understanding* tasks surface student thinking
- *Solidify Understanding* tasks examine and extend
- *Practice Understanding* tasks build fluency

The Power of the Learning Cycle

- "[t]he process of trying to solve a problem without the benefit of having been taught how is called ***generative learning***, meaning that the learner is generating the answer rather than recalling it." p.94

Research supports beginning new learning with *Develop Understanding Tasks*

Even if you're unsure, thinking about alternatives before you hit on (or are given) the correct answer will help you. Wrestling with the question, you rack your brain for something that might give you an idea. You may get curious, even stumped or frustrated and acutely aware of the hole in your knowledge that needs filling. When you're then shown the solution, a light goes on.

- Unsuccessful attempts to solve a problem encourage deep processing of the answer when it is later supplied, creating fertile ground for its encoding, in a way that simply reading the answer cannot. It's better to solve a problem than to memorize a solution and supply the incorrect answer than not to make the attempt.

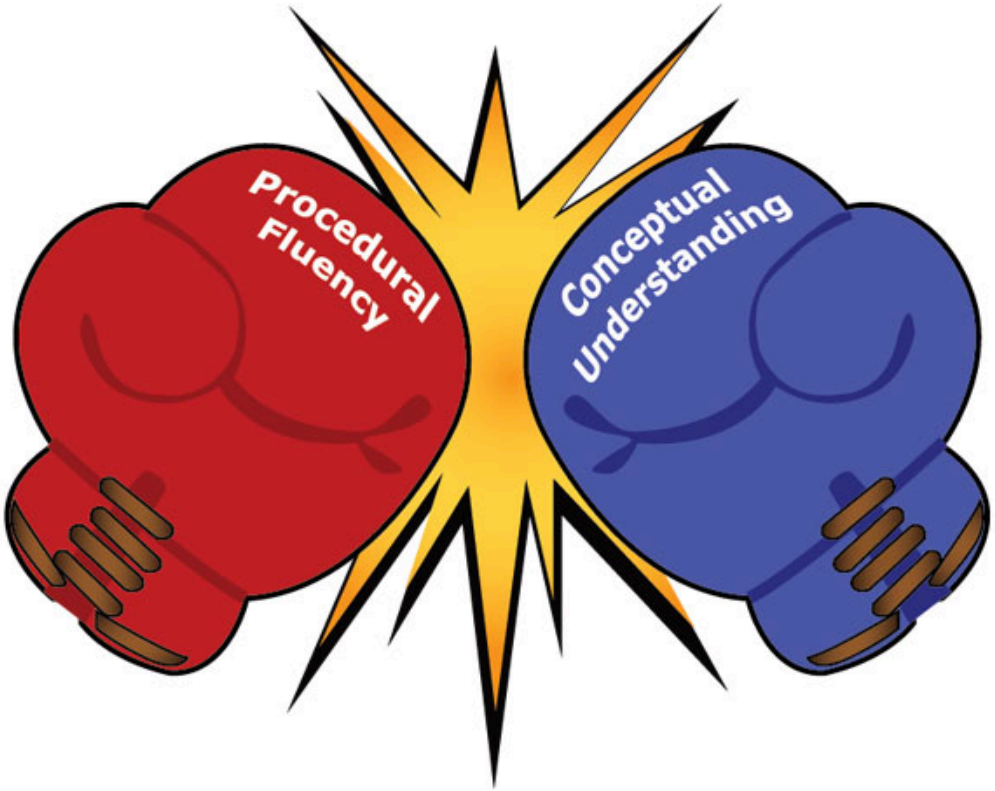
Research in neuroscience and cognitive psychology has revealed much new information about how we learn.

- How readily you can recall knowledge from your internal archives is determined by **context**, by recent use, and by the number and **vividness of cues that you have linked to the knowledge** and can call on to help bring it forth. From *Make it Stick*

We have a lot of misconceptions about learning math in America.

- Students must be shown how to do procedural math. It can't be easily figured out.
- Students must learn "the math" before they can apply it in a story problem.
- **Procedures are easier to learn than application**

Build Procedural Fluency from Conceptual Understanding



Build Procedural Fluency from Conceptual Understanding

A public misconception about mathematics is that contextual and procedural approaches to learning mathematics are in competition with each other. The demand to get **back to basics** is a demand to focus only on procedures. Yet the research tell us that "Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding."

(from Principles to Action)

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Additional Presentations from Mathematics Vision Project

8 by 8: Connecting Teaching Practices and Student Mathematical Practices

Today: 2:15 – 3:15 p.m. Room 153 C

Connecting Progressions and Practices with High Quality Tasks

Wednesday: 2:15 – 3:15 p.m. Room 152

Thomas Edison on education

"The most necessary task of civilization is to teach people how to think. It should be the primary purpose of our public schools. . . The trouble with our way of educating is that it does not give elasticity to the mind. It casts the brain into a mold. It insists that the child must accept. It does not encourage original thought or reasoning, and it lays more stress on memory than observation."

Mathematics Teaching Practice #6

Build procedural fluency from conceptual understanding.

Effective teaching of mathematics builds fluency with procedures on a **foundation** of conceptual understanding so that students, **over time**, become skillful in using procedures flexibly as they solve contextual and mathematical problems.