STRUCTURE OF THE CURRICULUM

Each curriculum in the Mathematics Vision Project materials is composed of two main components, the **classroom experience**, which is designed around the implementation of a specific type of task and the aligned **"Ready, Set, Go!" homework assignment**. Each task is accompanied by a set of teacher notes. The teacher notes identify the purpose of the lesson and describe the steps the teacher can take during the classroom experience to ensure that students engage in a rich learning event. Tasks are to be done in class and should not be assigned as homework. There is an aligned "Ready, Set, Go!" homework assignment for each task. It is the independent practice. Homework serves the student as a type of formative assessment. It is while doing the homework that the student can discern for himself if the mathematics done in class can be performed independently.

The MVP **classroom experience** begins by confronting students with an engaging task and then invites them to grapple with solving it. As students' ideas emerge, take form, and are shared, the teacher orchestrates the student discussions and explorations towards a focused mathematical goal. As conjectures are made and explored, they evolve into mathematical concepts that the community of learners begins to embrace as effective strategies for analyzing and solving problems. These strategies are eventually solidified into a body of practices and mathematical habits that belong to the students, because they were developed by the students, as an outcome of their own creative and logical thinking. This is how students learn mathematics. They learn by doing mathematics. They learn by needing mathematics. They learn by verbalizing the way they see the mathematical ideas connect and by listening to how their peers perceived the problem. Students then own the mathematics because it is a collective body of knowledge that they have developed over time through guided exploration.

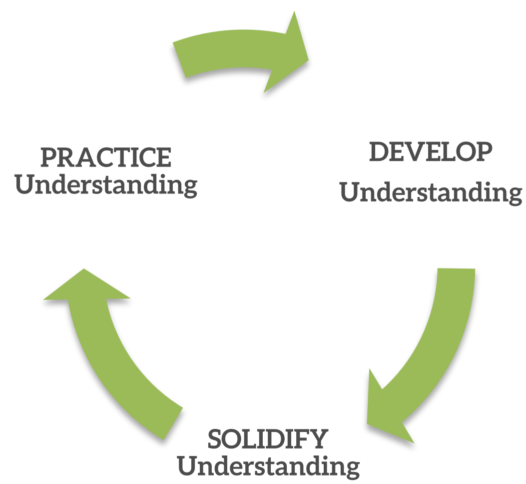
This process describes the **Learning Cycle**, an instructional framework that allows students to build mathematical knowledge over time. This framework is flexible. Every progression does not follow the pattern of develop, solidify, practice. For instance, the first module on quadratics begins with a Develop Understanding Task. Many aspects of the definition of a quadratic surface in that task. Five solidify tasks follow the first task. Each of the Solidify tasks extends one of the key concepts that surfaced in the beginning Develop Understanding Task. The module ends with a Practice Understanding Task that pulls all of the key concepts together into a complete definition of quadratic.

The Learning Cycle

The diagram at the right illustrates the Comprehensive Mathematics Instructional Framework (CMI) around which the MVP curriculum has been developed. Every task in the curriculum is identified as one of the following:

- Develop Understanding Task
- Solidify Understanding Task
- Practice Understanding Task

A learning cycle begins with a single term, *develop*, which refers to bringing student thinking to the surface by activating prior knowledge, intuition, and insights to make sense of a problem.

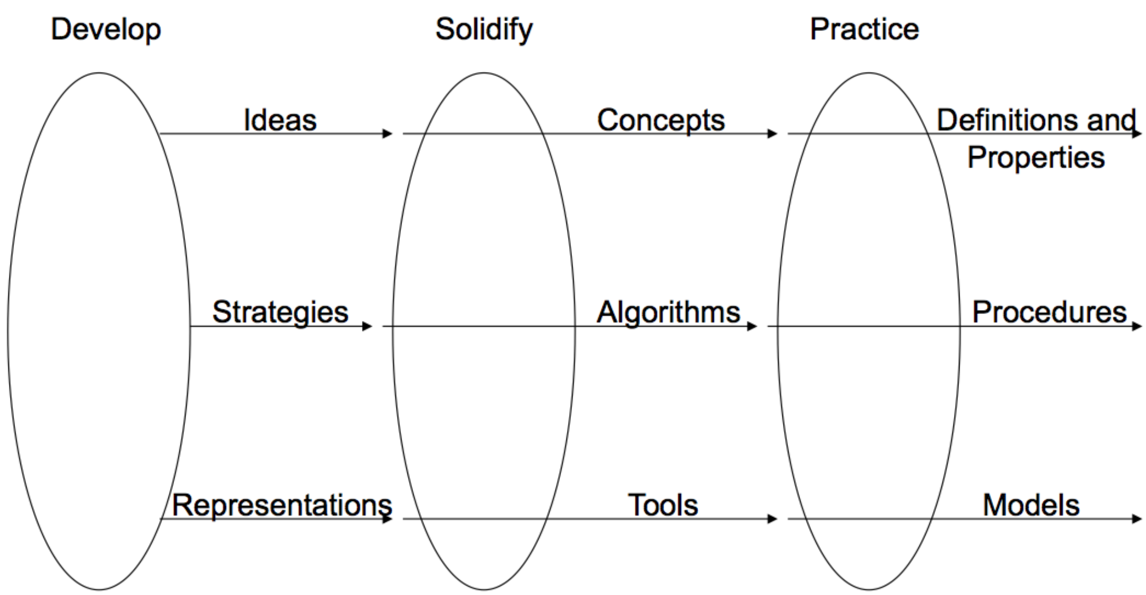


The Learning Cycle

Develop Understanding Tasks are intended to generate ideas, strategies, and representations related to a new mathematical topic. Develop tasks contain multiple entry points for students, so that all students are able to use their intuition and logic to make sense of the problem and devise a strategy for organizing the information. In the second phase of the learning cycle, students will engage in Solidify Understanding tasks that will allow them to examine and extend the mathematical thinking that rose to the surface in the Develop Understanding task. The learning cycle will conclude with a Practice Understanding Task. It focuses students' attention on becoming fluent with the mathematics of the unit and refining the mathematics into formal definitions, properties, procedures, and models that are consistent with practices that exist outside the classroom.

In the *CMI Framework* the progression of the mathematics through the *learning cycle* is mapped out along a continuum of conceptual, procedural and representational understandings using the *Continuum of Mathematical Understanding.*

Continuum of Mathematical Understanding



©BYU Public School Partnership, 2006

Mathematical understanding encompasses at least three connected but distinct domains as represented by the horizontal lines of the continuum: conceptualizing mathematics, doing mathematics, and representing mathematics. Mathematical understanding progresses continually along this continuum, but it is useful to note three sets of distinct landmarks of progression along the continuum that are associated with each of the three phases of the *Learning Cycle*. Emerging mental images are fragile as they are surfaced during students' initial experiences with tasks designed to elicit those images (*Develop Understanding*). In the continuum we refer to these fragile images as ideas, strategies, and representations. These ideas, strategies and representations need to be examined for accuracy and completeness, as well as extended and connected through multiple exposures and experiences until they become more tangible, solid and useful (*Solidify Understanding*). In the *CMI Framework*, ideas that have been examined for the understanding they reveal are called concepts; strategies that can be articulated and replicated are called algorithms; and useful representations are called tools. Once understanding has been developed and solidified, it needs further refinement to become fluent and applicable to new situations and contexts (*Practice Understanding*). In the *CMI Framework* refined concepts become the definitions or properties of formal mathematics; algorithms that can be carried out flexibly and fluently are called procedures; and representations that embody essential mathematical understandings (either conceptual or procedural) are called models, such as “an area model for multiplication” or “the number line as a model of the set of real numbers.” These definitions and properties, procedures, and models must be consistent with the broader mathematical “community of practice” that exists outside of the classroom.

The *CMI Framework* supports teachers in enacting the NCTM effective teaching practice: *Build Procedural Fluency from Conceptual Understanding.* However, the framework implies that the end-goal of mathematical instruction is not just procedural fluency; it also includes a deeper conceptual understanding of the properties and definitions on which procedures are based, and an ability to draw upon mathematical models more flexibly and fluently when representing one’s mathematical understanding. The *Learning Cycle* component of the framework supports teachers in making curricular decisions that move students from individually-constructed ideas, strategies and representations towards a community of shared definitions, properties, procedures and models. The *Continuum of Mathematical Understanding* component of the framework emphasizes that there are multiple domains of mathematical understanding that need to be developed, solidified and practiced: the conceptual domain, which provides students with *ways of thinking about mathematics*; the procedural domain, which provides students with *ways of doing mathematics*; and the representational domain, which provides students with *ways of making one’s thinking visible*. Together, both components of the *CMI Framework* promote student thinking to the forefront of mathematics instruction and highlight the decision-making role of the teacher in effectively selecting and sequencing tasks that build mathematical understanding and fluency over time.

Each module in the **MVP** educational program has been carefully designed and sequenced with rich mathematical tasks that have been formulated to generate the mathematical concepts within the core curriculum. Careful attention has been placed upon the way mathematical knowledge emerges, is extended, and then becomes efficient, flexible, and accurate. Some tasks are developmental tasks while others are for solidifying or practicing the concepts. The sequencing of the tasks encourages students to notice relationships and make connections between the concepts. In this way, students perceive mathematics as a coherent whole.

While the classroom experience is predominantly geared towards improving students' reasoning and sense-making skills, MVP regards mathematical understanding and procedural skill as being equally important. Hence, the "**Ready, Set, Go**!" **homework assignments** are focused on students practicing procedural skills and organizing principles to add structure to the ideas developed during the classroom experience. As in any discipline, practice is the refining element that brings fluency and agility to the skills of the participant. The **Ready** and the **Go** sections of the homework assignments have been designed to spiral a review of content, while the **Set** section focuses on consolidating the mathematics addressed in class that day. Each time a student engages in the homework assignment, it is expected that he or she will have the opportunity to reflect on the new learning from class and will practice the retrieval of ideas from the body of learning that has been growing over the school year, and even prior to the current school year. Recent research on learning has identified reflection and retrieval practice as being two key ingredients for durable learning. True learning should be long lasting and should grow out of previous understandings, extending over years of study. Hence, the "**Go!**" sections of the **"Ready, Set, Go!" homework assignments** will contain topics from previous lessons and prior years of mathematics instruction. Together the **classroom experience** and the **"Ready, Set, Go!" homework assignments** offer a powerful blend of new learning and maintained proficiency.

The Teaching Cycle

The Learning Cycle depicts how students become proficient in the mathematics overtime. Each task represents at least one day of instruction. Therefore, a Learning Cycle may extend over several days or weeks of classroom instruction, however, each day the teacher is expected to frame the lesson around **The Teaching Cycle.** This cycle also has three components: **Launch, Explore**, and **Discuss.**

The Teaching Cycle may seem to be simple, but it involves careful preparation and then deliberate implementation by the instructor.

Launch: How will you . . .

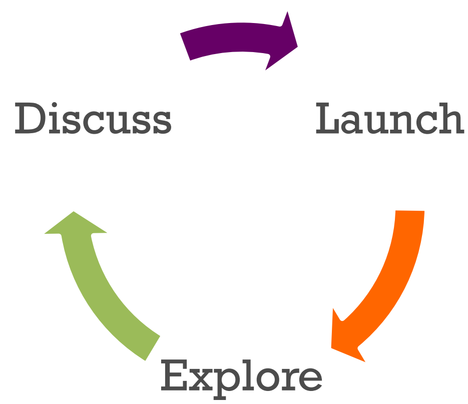
- hook and motivate students?
- provide schema for the task?
- describe the expectations for the finished task?

Explore: What will you . . .

- look for and listen for as you observe?
- accept as evidence of understanding?
- ask to stimulate, redirect, focus, and extend mathematical thinking?

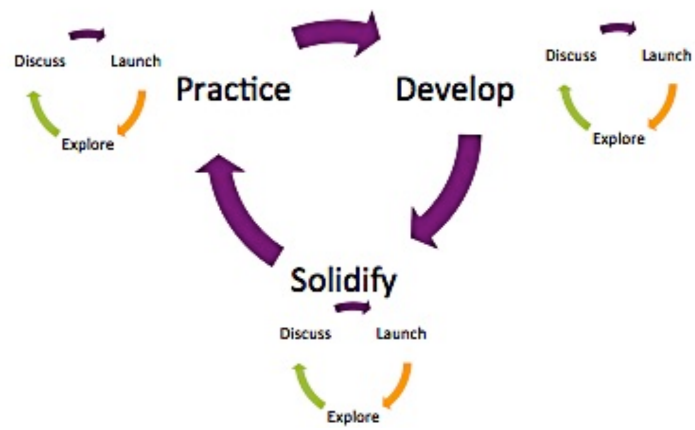
Discuss: How will you . . .

- select which students will present their solutions and strategies?
- determine what ideas to pursue?
- decide whether to contribute to the discourse or allow students to continue to struggle to make sense of a concept?



The Teaching Cycle

The diagram to the right depicts how the two instructional frameworks, the **Teaching Cycle** and the **Learning Cycle**, fit together. The **Teaching Cycle** occurs each day in the classroom, while the **Learning Cycle** extends over days and possibly weeks as the unit develops.

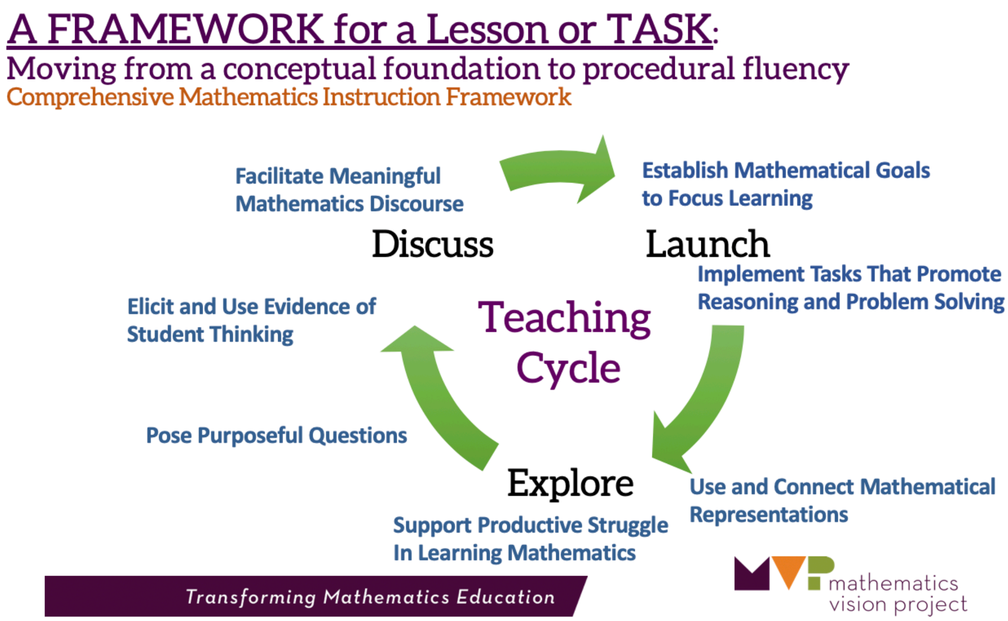


The MVP curriculum and the CMI instructional framework together reflect current research on teaching and learning. Research in both cognitive science and mathematics education supports changes in the roles of the learner and the teacher. During instruction, students need to be developing specific reasoning habits that will serve them in other disciplines, real life, and their future careers. It is the teacher's role to provide opportunities for students to develop these skills. The CMI model provides a framework for both the teacher and the student to improve teaching and learning in the classroom.

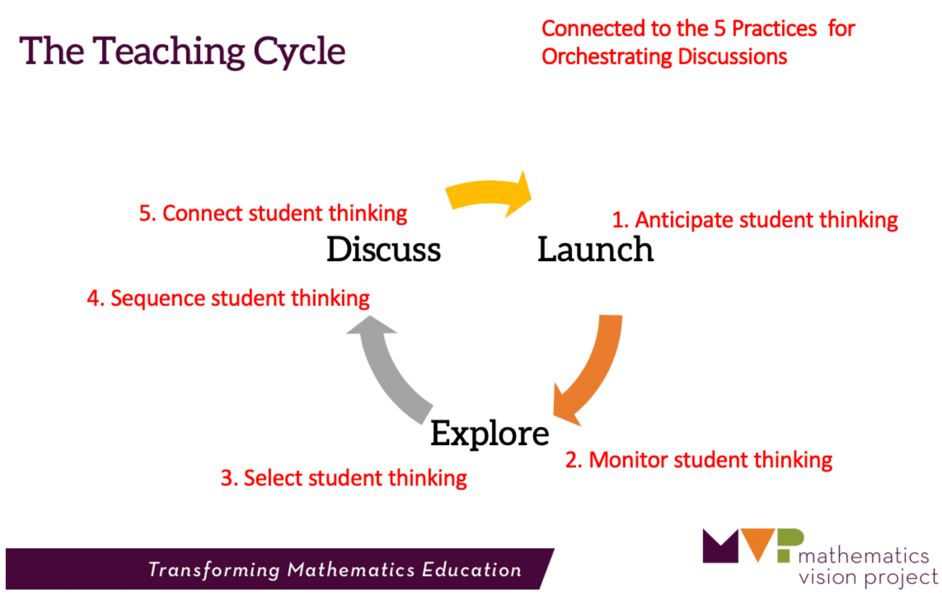
The Comprehensive Mathematics Instruction Model		
	Teacher's role	**Student's role**
Develop Understanding	Focus learning on the goal of the task; provide experiences using rich tasks; support productive struggle; elicit and use evidence of student thinking to orchestrate discussions using the 5 practices*	Make sense of the context, organize information, notice patterns, make conjectures, invent strategies, create arguments, engage in mathematical discourse
Solidify Understanding	Focus learning on the goal of the task; provide experiences using rich tasks; support productive struggle; elicit and use evidence of student thinking to orchestrate discussions using the 5 practices*	See structure; see regularities; attend to precision; create and critique arguments; adopt strategies, use multiple representations, engage in mathematical discourse
Practice Understanding	Provide a vehicle for practice; provide feedback; clarify misconceptions; confirm mathematical and symbolic language; elicit and use evidence of student thinking to orchestrate discussions using the 5 practices*	Reason quantitatively; work towards efficiency, flexibility, accuracy; apply (model with mathematics)

**Five Practices for Orchestrating Productive Mathematical Discussion* – 2nd Edition, Margaret S. Smith and Mary K. Stein, NCTM, 2018

The eight effective teaching practices, as articulated in the NCTM publication *Principles To Actions, Ensuring Mathematical Success for All* (2014), describe a framework for improving instructional practice. The following figure shows how these eight practices can be incorporated into the Teaching Cycle. Note that seven of the practices fit naturally around the Teaching Cycle and can be implemented during each day of instruction, while building procedural fluency from conceptual understanding is a curriculum practice that describes the process of creating deep learning over time.



The *Launch, Explore, Discuss* sequence of the Teaching Cycle is the visible form of the daily, classroom experience. Yet, to make deep learning occur in the classroom, the teacher must carefully prepare for each aspect of the lesson. During the *Launch* the teacher must set the stage by informing students of the situation and the expectations of the task. During the *Explore* phase, as students are reasoning through the task, the teacher is busy moving from student to student, clarifying student questions and encouraging student work. As the teacher monitors student effort, he is also selecting and sequencing which work will move student thinking towards the purpose of the lesson. During the *Discuss* phase, selected students share their mathematical thinking and strategies, while members of the class listen, question, and record strategies and key concepts. Throughout the lesson, it is the obligation of the teacher to connect the mathematics so that students leave class with the big ideas of the intended mathematical lesson. The following figure depicts how the framework of the five practices for orchestrating discourse fit within the Teaching Cycle. (Adapted from *Five Practices for Orchestrating Productive Mathematical Discussion* – Second Edition, Margaret S. Smith and Mary K. Stein, NCTM, 2018)



**Five Practices for Orchestrating Productive Mathematical Discussion* – 2nd Edition, Margaret S. Smith and Mary K. Stein, NCTM, 2018

FEATURES

Each module begins with an **annotated table of contents** which identifies the key concepts that will be the focus of the module and the core standards that will be addressed. A set of teacher notes accompanies each task. The teacher notes outline each step of the lesson while following the framework of the **Teaching Cycle**. All of the teacher notes follow the same basic outline as described below:

The Enhanced Teacher Notes include:

Purpose: Paying attention to the purpose of the task will help the teacher stay true to the progression of the module and refrain from trying to accomplish too much within the task.

Core Standards Focus: The MVP authors have taken a "multi-tasking approach" to the standards. While one task may focus on more than one standard, several tasks may hi-light a single standard. In this way a set of interrelated ideas or a sequence of strategies and skills can be fused into a meaningful whole. This "multi-tasking approach" to the standards also gives students multiple opportunities to master the standards.

Related Standards: The focus of a lesson may be on a specific standard, yet doing the mathematics may require students to draw on related standards.

Standards for Mathematical Practice: It is possible and even likely that students will implement all of the Standards for Mathematical Practice within a given lesson, however, different types of tasks naturally elicit certain practices. Those that seem to be the most likely to be drawn upon in the lesson have been identified in the teacher notes.

Essential question for students: Since all of the tasks are inquiry based, the essential question has been formulated to direct students' attention towards the purpose of the lesson without explicitly revealing the key ideas and strategies they should be producing.

The Teaching Cycle:

Launch (whole class): Suggestions for introducing the lesson to the students. Sometimes this is relating a story, while other times it's working the first problem together. The prompts for the tasks often involve a lot of reading. It is the teacher's obligation to make sure that students understand what they are expected to do or produce during the Explore stage of the learning.

Explore (small groups): While students are exploring, the teacher will be monitoring the individual students and groups, looking for student strategies that will promote the discussion about the mathematics of the task. This is also a time during which the teacher can assess what previously learned skills the students are bringing to the task. The teacher notes will make suggestions of what the teacher should be looking for during the Explore session.

Discuss: Here the teacher will find suggestions for orchestrating the discussion in order to achieve the purpose of the lesson. This is the time when key connections need to be made.

Exit ticket for students: An exit slip can aid the teacher in checking for understanding. The items in the exit ticket could also be used as a warm-up in the subsequent lesson.

Instructional Supports

ELL and equity suggestions: Equitable mathematics teaching maintains high standards of learning for all students. Instruction should affirm students' mathematical identities by honoring the multiple resources of mathematical learning present in the classroom. By following the plan of instruction included in the teacher notes, students' different mathematical strengths are used as a resource for learning. Additional strategies for providing equal opportunities for learning are offered where appropriate.

Interventions: These suggestions may lower the threshold for the task to accommodate students who don't know how to begin thinking about the task.

Challenge activity: The challenge activity is to provide a "high ceiling" for students who have finished early or need to be encouraged to think more deeply about the mathematics. Sometimes the last question in the task provides that extension, and it is not essential that it be completed by all students.

Additional Resources for Teachers: This could be a variety of things depending on the lesson. For instance, an app using GeoGebra has been developed for the rubber-band activity in the first task of Module 2 in the geometry course.

Sentence frame cards are available as an aid for students. The cards are intended to assist students in becoming self-directed thinkers by guiding their thinking and prompting the language needed for discourse about their mathematical work. The cards are structured around the Eight Student Practices for Mathematical Thinking. The cards are intended to support all learners, but they are particularly useful in supporting learners who struggle with language.

Answer Key for each task:

The suggested mathematical approach for some of these tasks may require teachers to look at the mathematics from a different perspective than they have ever done before. The best way to prepare to teach a task is to work the problem from the standpoint of the student. The answer key is provided as reassurance for the teacher.

Answer Key for each Ready, Set, Go! Homework assignment

Additional Resources for Teachers and Students

The website www.rsgsupport.org contains a support video to match each Ready, Set, Go Homework assignment.

The Helps, Hints, and Explanations book provides an explanation for each type of homework problem and usually a worked example or two with annotation. Each "Ready, Set, Go!" homework assignment has an accompanying explanation in the Helps, Hints, and Explanations book.

Assessments and Tools for the PLC

The assessment resources provide a more complete assessment package including: quick quizzes, self-assessments, performance tasks, and a bank of items that cover the topics of the module. When these tools are combined with the exit tickets and the formative assessment available from listening to students as they work on the tasks, teachers can really know what their students understand and can do.

The assessment book includes per module:

- Quick quizzes (mid-unit checks for understanding)
- Student self-assessments (identifies what the student should know and be able to do as he progresses through the module)
- A Module test
- A Performance task with teacher notes and a scoring rubric

The **quick quizzes** are short, multiple-choice and short answer assessments that give a snapshot of what students have learned in the module. They are designed to be given after a learning cycle is completed (in most cases), so the number of quick quizzes in a module varies. The quick quizzes should be just that: quick. They can be given at the beginning or end of a period, still allowing time for other work to occur.

The **self-assessments** are a tool designed to help students know what they should be learning and to reflect on their progress. Like the quizzes, the self-assessments usually occur at the end of each learning cycle. They identify the mathematics that students should have learned and ask students to provide evidence (from their homework, their work on tasks, or problems from other assessments) that shows how well they have learned it, and to write what they will do to increase their understanding. The idea is to help students develop a growth mindset and gain ownership of their learning.

The **Module Tests**, provide a bank of items that can be used to design a summative assessment that reflects the work of the class. Some teachers like to create tests that have both the performance task and some more traditional items to ensure that all the standards of the module are assessed. There are about as many ways to assess as there are teachers, and most of the methods have merit. The key is to use all assessments as checkpoints to make instructional adjustments that will increase student learning.

The final tool is the **performance task**. There is one performance task provided for each module. This task incorporates the most important ideas of the module and asks students to use them flexibly. These tasks also provide an opportunity for students to communicate mathematically, using proper vocabulary and notation. An answer key and grading rubric are provided for each task, along with instructions for launching the task so that the task is accessible for each and every student. Some teachers like to give students the opportunity to work these tasks in pairs, mirroring the classroom experience. Others prefer to ask students to work individually to ensure that the results give a clear picture of what each student can do on their own.

The PLC tools include **The Essentials Tracker** and **The Power of the Module**.

> **The Essentials Tracker** is a grid connecting the standards and the tasks. When a standard is addressed in a task, it is indicated with one of three letters, *D* for developing, *S* for solidifying, or *P* for practicing. This helps teachers to see that the standards are addressed in more than one task. It also helps teachers to set an appropriate level of

expectation for students relative to the standard. *D,* for developing, indicates students' first exposure to the ideas and/or procedures of the standard, so teachers can expect new ideas to surface, although students may lack the notation or vocabulary that will be developed later. At the *S* level, for solidifying, students will be sorting through ideas that have previously been surfaced, with support for examining and extending their understanding and clarifying their procedures. If the standard is addressed at the *P* level, for practice, then students should be working on becoming efficient, accurate, and flexible as they demonstrate mastery of the standard.

The Power of the Module shows the focus or target for each task in the module and the topics of each section of the homework: *Ready* (to prepare for upcoming tasks), *Set* (to solidify the work done in the task), and *Go* (to reinforce previously-learned skills). This tool can help teachers see the opportunities for recall and rehearsal built into the program, along with the progression of mathematical ideas in the tasks. It also provides a "quick glance" or overview of the module, which will help teachers anticipate upcoming mathematical content. By working the tasks, then creating and discussing the **Power of a Module** outlines as a team, teachers will come to trust the materials and understand the progressions of mathematics that students will have the opportunity learn.

TECHNOLOGY

Technology is an important tool to be used as part of the MVP curriculum. In their description of the CCSS Standard for Mathematical Practice 5, "Use appropriate tools strategically," the core authors specifically name graphing calculators, computer algebra systems, statistical packages, and dynamic geometry software. They suggest that these tools could be used by students to explore and deepen their understanding of concepts, analyze graphs of functions, visualize mathematical models, and test various assumptions and compare predictions with data. Tasks in MVP provide opportunities for using technological tools in each of the ways described. The use of calculators may also help students to quickly make calculations so that their attention remains focused on the analytical work of the task. The curriculum is designed so that students may use technology that is widely available including graphing calculators or free computer apps such as Desmos or Geogebra. Making technology an integral tool for mathematical thinking enriches the work and provides students with opportunities to engage with SMP 5.

The Secondary Mathematics II course is written to align with the second of three courses in the integrated pathway of the Common Core State Standards, as described in Appendix A. Like all courses in the integrated pathway, it contains standards from each of the conceptual categories in the standards, including:

- Number and quantity;
- Algebra;
- Functions;
- Geometry; and
- Statistics and probability.

The focus of Secondary Math II is on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships from Secondary Mathematics I. The need for extending the set of rational numbers arises and real and complex numbers are introduced so that all quadratic equations can be solved. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. Circles, with their quadratic algebraic representations, round out the course. The Mathematical Practice Standards apply throughout each course and, together with the content standards, create mathematical learning experiences based upon reasoning and sensemaking, building perseverance and problem solving skills, and rich in mathematical discourse.

The standards indicated in the CCSS with a (+) sign are addressed with additional tasks in Secondary Math II Honors. The Honors version of the course includes all the same tasks as Secondary Math II, with the additional tasks embedded into the modules where they fit conceptually.

Standards specified in the Widely Accepted Prerequisites (WAP's) included in the High School Publishers Criteria for the Common Core State Standards for Mathematics constitute the bulk of the curriculum in Secondary Math II. The F-IF standards for interpreting quadratic functions are extensively addressed in Modules 1-2. Module 5 contains the emphasized Geometry standards, G-CO.1 G-CO.9 G-CO.10, which address proving statements about lines, angles, and triangles. Module 6 contains G-SRT.B G-SRT.C addressing the topics of similarity and right triangle trigonometry. Students develop a rich understanding of these terms as they use them to reason about transformations, construction, and features of triangles and quadrilaterals. All of the domains in the Algebra Conceptual Category are included in the WAP's. These domains constitute all of the work in Modules 3 and much of Module 2.

In the narrative that follows, the specific approach and details of the mathematics in the curriculum is described by conceptual category in roughly the same order as the categories are addressed in the curriculum. The additional work of the Honors course is clearly identified.

Conceptual Category: Functions

In Math I, students did extensive work with linear and exponential functions. They learned to work flexibly and fluently among representations for these two function types including tables, graphs, equations (both recursive and explicit), diagrams, and story contexts. Students used rates of change to distinguish between linear and exponential functions and to compare their behavior for large values of x. They learned that an arithmetic sequence is a discrete linear function and that a geometric sequence is a discrete exponential function.

The definition of function as a relationship in which each input has a unique output was formalized in Secondary Math I. Besides their work with linear and exponential functions, students learned that functions can be models for many situations and that in each situation it is useful to identify features such as:

- x and y intercepts;
- Domain and range;
- Continuity;
- Intervals of increase and decrease; and
- Maxima and minima.

They were also introduced to the idea that functions can be transformed, although they used only vertical transformations.

Module 1, Quadratic Functions, picks up where students left off with linear and exponential functions, using the same types of diagrams and representations to introduce quadratic functions. The entire module focuses on the features of quadratic functions, comparing quadratics to other functions, and representing quadratic functions. The first tasks in Module 1 use diagrams to help students write equations, both explicit and recursive, for quadratic functions. They use tables to identify the rate of change, noticing that the second difference is constant, making the first difference linear in a quadratic function. This idea is extended to the entire class of polynomials in Secondary Math III. In the first two tasks, students are also introduced to a related idea: that a quadratic function can be the sum of an arithmetic sequence or linear function. As the module advances, students use quadratic functions to model continuous contexts with a quadratic curve that has a maximum. There are lessons in the module that compare linear and quadratic functions and exponential and quadratic functions. The final task of the module asks students to distinguish between linear, exponential, and quadratic, given a single representation for the function, to create other representations for the function, and to identify the rate of change exhibited by the function.

Module 2, Structures of Expressions, is both a functions module and an algebra module. It is designed to extend students' knowledge of quadratic functions and to reinforce two big ideas of functions:

- Functions can be transformed in the same, predictable way.
- Different algebraic forms have purpose in different situations.

The module begins with students using technology to explore transformations of the graph of $f(x) = x^2$. They learn about horizontal and vertical translations, reflections over the x-axis, and vertical stretching or shrinking. These transformations are combined and students learn to identify the vertex, line of symmetry, reflection, and vertical stretch factor from equations in the form: $f(x) = a(x-h)^2 + k$. The curriculum encourages students to use a quick-graph method to be able to fluently identify the features and produce an accurate graph of any equation in vertex form.

Students soon experience equations that are not in vertex form, which may require them to change forms. Completing the square is introduced as a method for this purpose, using area models so that students have a visual model to rely on for the procedure. Students are also introduced to factoring using area models and come to understand that factored form can also be useful for graphing quadratics using the x-intercepts and the symmetry of the parabola to find the vertex. The only Honors task in the module addresses factoring trinomials in which the lead term has a coefficient that is not equal to 1. The module ends with students learning to be efficient in identifying the form that will be easiest to use in a given situation and moving flexibly between forms of quadratic equations. The idea that different quadratic forms are useful in graphing is extended in Module 3, Solving Quadratic and Other Equations, when students use different forms of quadratic equations to find the roots or solve quadratic equations of a single variable.

After all the work with quadratics in Modules 1-3, Module 4, More Functions, More Features, shifts focus to consider piecewise functions, absolute value functions, and inverse functions. Understanding of piecewise functions is built from students' understanding of graphs and the stories that they can tell. Students learn to write functions for contexts in which rates change, making a piecewise function. Point-slope form of the equation of a line is frequently used and connected to students' previous experience with transformation of functions. Students write and linear absolute value functions as piecewise functions and learn to graph linear absolute value functions with transformations. They also graph non-linear absolute value functions to more deeply understand the meaning of absolute and the effect it has when composed with other functions.

The last learning cycle in Module 4 introduces students to inverse functions. It begins with a context in which two people each keep track of their own bike ride in different units, one in minutes per mile, one in miles per minute. As students model the two different methods for thinking about the bike rides, they notice that the inputs and outputs are reversed, making the graphs reflections over the $y = x$ line. These initial ideas about inverses are generalized in subsequent tasks in the module so that students learn to graph and write the inverse for simple functions. In Secondary Math III, students delve more deeply into inverses, writing inverses for more complicated functions, understanding invertibility, and extending the general ideas of inverses to find the inverse of an exponential function, which a logarithmic function.

Conceptual Category: Number and Quantity

In eighth grade, students learned about the properties of exponents and were introduced to integer exponents. Students used integer exponents in Secondary Math I as they created tables and wrote equations for geometric sequences and exponential functions in Modules 1 and 2. In Secondary Math II, Module 3, Solving Quadratic and Other Equations, exponential functions provide a context for thinking about the outputs that lie between integer exponents, and find that they can be named with rational exponents. In the first learning cycle of Module 3, rational exponents are connected to roots, which students learned about in eighth grade and used with the distance formula in Secondary Math I. As they work with rational exponents and roots, students find that the rules are directly analogous. In the final task of the learning cycle, students use both rational exponents and roots to work with the same expression, with the goal being to find the form that is most efficient or useful for a given expression and operation.

As students are solving quadratic equations Module 3, they encounter equations that give solutions like: $x = 3 \pm \sqrt{-4}$. Students know from previous experience that square roots are undefined for negative values, so these solutions present a problem. This problem is resolved by introducing imaginary numbers and using them to write solutions to quadratic equations.

Conceptual Category: Algebra

The overall approach in the MVP curriculum to algebra is to give algebraic work meaning and purpose by embedding it in story context, modeling, and functions. This approach is evident in Module 2, Structures of Expressions, which is described in the Functions section, and is also illustrated by Module 3, Solving Quadratic and Other Equations. Module 3 uses students' experience with graphing quadratics to derive a method for finding roots and x-intercepts that becomes the quadratic formula. Students learn to use the quadratic formula, along with other methods such as factoring or taking the square root of both sides to solve equations and inequalities accurately and efficiently.

In the Honors tasks in Module 3 students solve quadratic inequalities. The tasks also extend the algebraic work with imaginary numbers to the complex plane, representing both the complex numbers and their operations. If students have been in Secondary Math I Honors, they have learned to use matrices. In Module 3 of Secondary Math II Honors, they use inverse matrices to solve systems of equations.

Conceptual Category: Geometry

The standards for geometry in the integrated pathway are carefully designed to allow students to experiment and construct general ideas about shapes and how they transform in eighth grade, moving towards formalizing definitions of rigid transformations and congruence in Secondary Math I through reasoning with diagrams, and then proving theorems and formalizing definitions of dilation and similarity in Secondary Math II. True to the vision of the standards, the MVP curriculum takes a transformational approach to the standards, developing transformations and construction as tools for reasoning and proof that are used in addition to the traditional axiomatic tools of geometry. The curriculum provides students many opportunities to use their intuitive understanding about geometry and experiment with compass, protractor, patty paper, rulers, graph paper, dynamic geometry software and other physical tools to make and justify conjectures.

In Secondary Math II, Geometry begins with Module 5, Geometric Figures. Formal proof is introduced in this module, beginning with students understanding the ways of knowing continuum:

1. Based on authority
2. Based on experience with a few examples
3. Based on reasoning from a diagram
4. Based on statements accepted as true by the community of practice, including postulates, definitions and theorems.

Students experience each of the ways of knowing in the module, learning to evaluate the strength of a mathematical argument. The fourth way of knowing, which is mathematical proof, has traditionally been taught using the two-column proof format. As suggested by the CCSS, the MVP curriculum also introduces other forms of proof including paragraph proofs and flow proofs. With the addition of transformation and construction as tools for creating geometric arguments and the availability of more open forms of proof, geometric proofs become accessible to all students. Once students have been introduced to different ways of reasoning and making arguments in Module 5, they use their understanding of congruence and the congruent triangle criteria from Secondary Math I to prove statements about other figures including equilateral triangles and various quadrilaterals. Many of the ideas about congruence, symmetry, and properties of quadrilaterals that were surfaced in Secondary Math I are proved in Module 5 of Secondary Math II.

Module 6, Similarity and Right Triangle Trigonometry, introduces the last of the transformations, dilation. A big idea of Module 6 is that two figures are similar if a sequence of rigid transformation and dilations exists that maps one figure onto the other. Students begin the module by learning about the features of a dilation, including the effects of changing the scale factor and/or point of dilation. They use the definition of a dilation to establish the AA similarity criterion and understand that corresponding sides of similar figures will be proportional. In the second learning cycle of the module, students prove theorems about the angles that occur when two parallel lines are cut by a transversal. They also develop a method for finding the midpoint or dividing a segment into other proportional pieces. In the final task of the learning cycle, students use similarity to prove the Pythagorean Theorem and find geometric means in right triangles.

The third learning cycle of Module 6 explores right triangle trigonometry. The definitions of sine, cosine, and tangent are introduced, and students use relationships between sine and cosine to construct the Pythagorean Identity for sine and cosine. They solve right triangles in both abstract and real-world situations.

Module 7, Circles: A Geometric Perspective, is composed of four learning cycles. In the first learning cycle, students use rotations and perpendicular bisectors to find the center of a circle. The task also introduces the terms associated with circles, including arcs, chords, secants, tangent, radius, diameter, etc. Students use these terms throughout the module as they explore features and develop conjectures about circles. The learning cycle proceeds with students showing that all circles are similar and making conjectures about central angles, inscribed angles and circumscribed angles.

The second learning cycle in Module 7 builds on the circle relationships that students have learned so far in the module to develop a formula for the perimeter and area of a regular polygon. Using intuitive ideas of limits, students extend these formulas to understand the formulas for the circumference and area of a circle.

The third learning cycle addresses relationships among central angles, radii, arcs, and sectors. Students calculate arc length and the area of a sector. Students learn that radians are another way to describe angles and to make conversions between degrees to radians. Radians are introduced in Secondary Math II as part of understanding proportional relationships in circles. Radians are not used in circular trigonometry until Secondary Math III.

The final learning cycle in Module 7 is an intuitive approach to volume of prisms, pyramids, and cylinders. Students informally consider dissection as a method for deriving volume formulas for solid figures and to understand Cavalieri's Principle for calculating the volume of oblique geometric solids.

Module 8, Circles and Other Conics, takes an algebraic approach to solving problems with circles, parabolas, and in the Honors course, ellipses and hyperbolas. The module includes several hands-on explorations to develop the equations for circles, parabolas, and ellipses. In the first learning cycle, students build a circle from right triangles and use the Pythagorean Theorem to derive the equation of a circle. They use the equation of a circle to determine if a given point is on a circle, to graphs circles, and to write equations given specific information about a circle.

The second learning cycle focuses on parabolas as a set of points defined by a focus and directrix. Students construct parabolas using this definition and discover relationships that help them to write equations. Students consider both parabolas with a horizontal directrix and those with a vertical directrix. They compare parabolas considered from a geometric perspective to their previous experience with parabolas from a functions perspective. The module also includes Honors tasks that involve students in deriving and using the equation of an ellipse and the equation of a hyperbola.

Conceptual Category: Statistics and Probability

Students do a great deal of work in probability in grade 7, which informs the work of Secondary Math II. From seventh grade, they have experience developing probability models and testing the models with experiments. They learned that probabilities are numbers between 0 and 1, with events becoming more likely as the probability approaches 1. They represented sample spaces for simple situations and used simulations to determine frequencies for compound events.

Module 9, Probability, extends students' work in representing and analyzing data to understand concepts in probability. In the module, students use representations such as tree diagrams, Venn diagrams, and two-way frequency tables to draw conclusions about the likelihood of an event. Students learn about conditional probability, writing statements, using both words and probability notation, about real situations. They use probability statements to complete Venn diagrams and use them to draw conclusions. They understand terms such as mutually exclusive, union, and intersection, in contexts that give them meaning and help them to visualize the terms with diagrams. Students learn about independence and how to determine if events are independent using a Venn diagram, a tree diagram, or a two-way frequency table.

Module 1 Quadratic Functions	**3 Weeks of Instruction**
1.1 Something to Talk About – A Develop Understanding Task An introduction to quadratic functions, designed to elicit representations and surface a new type of pattern and change (F.BF.1, A.SSE.1, A.CED.2)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
1.2 I Rule – A Solidify Understanding Task Solidification of quadratic functions begins as quadratic patterns are examined in multiple representations and contrasted with linear relationships (F.BF.1, A.SSE.1, A.CED.2)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
1.3 Scott's Macho March – A Solidify Understanding Task Focus specifically on the nature of change between values in a quadratic being linear (F-BF, F-LE)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 1 & Self-Assessment (formative)	**20 minutes**
1.4 Rabbit Run– A Solidify Understanding Task Focus on maximum/minimum point as well as domain and range for quadratics (F.BF.1, A.SSE.1, A.CED.2)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
1.5 The Tortoise and the Hare– A Solidify Understanding Task Comparing quadratic and exponential functions to clarify and distinguish between each type of growth as well as how that growth appears in each of their representations (F.BF.1, A.SSE.1, A.CED.2, F.LE.3)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 2 & Self-Assessment (formative)	**20 minutes**
1.6 How Does it Grow – A Practice Understanding Task Incorporating quadratics with the understandings of linear and exponential functions (F.LE.1, F.LE.2, F.LE.3)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Module 6 Test & Performance Assessment	**1 – 45 to 50 minute period each**

Module 2 Structures of Expressions	**5 weeks of instruction**

SECONDARY MATH 2
SCOPE & SEQUENCE

2.1 Transformers: Shifty y's – A Develop Understanding Task Connecting transformations to quadratic functions and parabolas (F.IF.7, F.BF.3)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
2.2 Transformers: More Than Meets the y's – A Solidify Understanding Task Working with vertex form of a quadratic, connecting the components to transformations (F.IF.7, F.BF.3)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 1 & Self-Assessment (formative)	**20 minutes**
2.3 Building the Perfect Square – A Develop Understanding Task Visual and algebraic approaches to completing the square (F.IF.8)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
2.4 A Square Deal– A Solidify Understanding Task Visual and algebraic approaches to completing the square (F.IF.8)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
2.5 Be There or Be Square– A Practice Understanding Task Visual and algebraic approaches to completing the square (F.IF.8)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 2 & Self-Assessment (formative)	**20 minutes**
2.6 Factor Fixin' – A Solidify Understanding Task Connecting the factored and expanded forms of a quadratic (F.IF.8, F.BF.1, A.SSE.3)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
2.7 The x Factor – A Solidify Understanding Task Connecting the factored and expanded or standard forms of a quadratic (F.IF.8, F.BF.1, A.SSE.3)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
2.8H The Wow Factor – A Solidify Understanding Task Connecting the factored and expanded forms of a quadratic when a-value is not equal to one. (F.IF.8, A.SSE.3)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
2.9 Lining Up Quadratics – A Solidify Understanding Task Focus on the vertex and intercepts for quadratics (F.IF.8, F.BF.1, A.SSE.3)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 3 & Self-Assessment (formative)	**20 minutes**
2.10 I've Got a Fill-in – A Practice Understanding Task Building fluency in rewriting and connecting different forms of a quadratic (F.IF.8, F.BF.1, A.SSE.3)	**1 - 80 minute period** **2 – 45 to 50 minute periods**

Mathematics Vision Project
Licensed under the Creative Commons Attribution CC BY 4.0
mathematicsvisionproject.org

2.11 Throwing an Interception – A Develop Understanding Task Developing the Quadratic Formula as a way for finding x-intercepts and roots of quadratic functions (A.REI.4, A.CED.4)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
2.12 Curbside Rivalry – A Solidify Understanding Task Examining how different forms of a quadratic expression can facilitate the solving of quadratic equations (A.REI.4, A.REI.7, A.CED.1, A.CED.4)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
2.13 Perfecting My Quads – A Solidify Understanding Task Building fluency with solving of quadratic equations (A.REI.4, A.REI.7, A.CED.1, A.CED.4)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Module 2 Test & Performance Assessment	**1 – 45 to 50 minute period each**

Module 3 Solving Quadratic Equations & Other Equations	**4 weeks of instruction**
3.1 How Do You Know That? – A Develop Understanding Task An introduction to proof illustrated by the triangle interior angle sum theorem (G.CO.10)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
3.2 Do You See What I See? – A Develop Understanding Task Reasoning from a diagram to develop proof-like arguments about lines and angles, triangles and parallelograms (G.CO.9, G.CO.10, G.CO.11)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
3.3 It's All in Your Head – A Solidify Understanding Task Organizing proofs about lines, angles and triangles using flow diagrams and two-column proof formats (G.CO.9, G.CO.10)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
3.4 Parallelism Preserved and Protected – A Solidify Understanding Task Examining parallelism from a transformational perspective (G.CO.9)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
3.5 Claims and Conjectures – A Solidify Understanding Task Generating conjectures from a diagram about lines, angles and triangles (G.CO.9, G.CO.10)	**1 - 80 minute period** **2 – 45 to 50 minute periods**

3.6 Justification and Proof – A Practice Understanding Task Writing formal proofs to prove conjectures about lines, angles and triangles (G.CO.9, G.CO.10)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 1 & Self-Assessment (formative)	**20 minutes**
3.7 Parallelogram Conjectures and Proof – A Solidify Understanding Task Proving conjectures about parallelograms (G.CO.11)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
3.8 Guess My Parallelogram – A Practice Understanding Task Identifying parallelograms from information about the diagonals (G.CO.11)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
3.9 Centers of a Triangle– A Practice Understanding Task Reading and writing proofs about the concurrency of medians, angle bisectors and perpendicular bisectors of the sides of a triangle (G.CO.10)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 2 & Self-Assessment (formative)	**20 minutes**
Module 3 Test & Performance Assessment	**1 – 45 to 50 minute period each**

Module 4 More Functions, More Features	**3 weeks of instruction**
4.1 Some of This, Some of That – A Develop Understanding Task Use prior knowledge of functions to develop understanding of piecewise functions (F.IF.7b)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
4.2 Bike Lovers – A Solidify Understanding Task Solidification of graphing and writing equations for piecewise functions (F.IF.5, F.IF.7b)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 1 & Self-Assessment (formative)	**20 minutes**
4.3 More Functions with Features – A Solidify Understanding Task Incorporating absolute value as piecewise-defined functions (F.IF.7b)	**1 - 80 minute period** **2 – 45 to 50 minute periods**

4.4 Reflections of a Bike Lover– A Practice Understanding Task Fluency with domain, range, absolute value and piecewise-defined functions (F.IF.5, F.IF.7B)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 2 & Self-Assessment (formative)	**20 minutes**
4.5 What's Your Pace? – A Develop Understanding Task Comparing input and output values to develop understanding of inverse functions (F.BF.4)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
4.6 Bernie's Bikes – A Solidify Understanding Task Solidifying inverse functions using multiple representations (F.BF.4)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 3 & Self-Assessment (formative)	**20 minutes**
4.7 More Features, More Functions – A Practice Understanding Task Using prior knowledge to identify features of a function as well as to create functions when given features (F.IF.4)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Module 4 Test & Performance Assessment	**1 – 45 to 50 minute period each**

Module 5 Geometric Figures	**4 weeks of instruction**
5.1 How Do You Know That? – A Develop Understanding Task An introduction to proof illustrated by the triangle interior angle sum theorem (G.CO.10)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
5.2 Do You See What I See? – A Develop Understanding Task Reasoning from a diagram to develop proof-like arguments about lines and angles, triangles and parallelograms (G.CO.9, G.CO.10, G.CO.11)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
5.3 It's All in Your Head – A Solidify Understanding Task Organizing proofs about lines, angles and triangles using flow diagrams and two-column proof formats (G.CO.9, G.CO.10)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
5.4 Parallelism Preserved and Protected – A Solidify Understanding Task Examining parallelism from a transformational perspective (G.CO.9)	**1 - 80 minute period** **2 – 45 to 50 minute periods**

SECONDARY MATH 2
SCOPE & SEQUENCE

5.5 Claims and Conjectures – A Solidify Understanding Task Generating conjectures from a diagram about lines, angles and triangles (G.CO.9, G.CO.10)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
5.6 Justification and Proof – A Practice Understanding Task Writing formal proofs to prove conjectures about lines, angles and triangles (G.CO.9, G.CO.10)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 1 & Self-Assessment (formative)	**20 minutes**
5.7 Parallelogram Conjectures and Proof – A Solidify Understanding Task Proving conjectures about parallelograms (G.CO.11)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
5.8 Guess My Parallelogram – A Practice Understanding Task Identifying parallelograms from information about the diagonals (G.CO.11)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
5.9 Centers of a Triangle– A Practice Understanding Task Reading and writing proofs about the concurrency of medians, angle bisectors and perpendicular bisectors of the sides of a triangle (G.CO.10)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 2 & Self-Assessment (formative)	**20 minutes**
Module 5 Test & Performance Assessment	**1 – 45 to 50 minute period each**

Module 6 Similarity and Right Triangle Trigonometry	**5 weeks of instruction**
6.1 Photocopy Faux Pas – A Develop Understanding Task Describing the essential features of a dilation (G.SRT.1)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
6.2 Triangle Dilations – A Solidify Understanding Task Examining proportionality relationships in triangles that are known to be similar to each other based on dilations (G.SRT.2, G.SRT.4)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
6.3 Similar Triangles and Other Figures – A Solidify Understanding Task	**1 - 80 minute period** **2 – 45 to 50 minute periods**

Mathematics Vision Project
Licensed under the Creative Commons Attribution CC BY 4.0
mathematicsvisionproject.org

Comparing definitions of similarity based on dilations and relationships between corresponding sides and angles (G.SRT.2, G.SRT.3)	
6.4 Cut by a Transversal – A Solidify Understanding Task Examining proportionality relationships of segments when two transversals intersect sets of parallel lines (G.SRT.4)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
6.5 Measured Reasoning – A Practice Understanding Task Applying theorems about lines, angles and proportional relationships when parallel lines are crossed by multiple transversals (G.CO.9, G.CO.10, G.SRT.4, G.SRT.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 1 & Self-Assessment (formative)	**20 minutes**
6.6 Yard Work in Segments – A Solidify Understanding Task Applying understanding of similar and congruent triangles to find the midpoint or any point on a line segment that partitions the segment into a given ratio (G.GPE.6)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
6.7 Pythagoras by Proportions – A Practice Understanding Task Using similar triangles to prove the Pythagorean theorem and theorems about geometric means in right triangles (G.SRT.4, G.SRT.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 2 & Self-Assessment (formative)	**20 minutes**
6.8 Are Relationships Predictable? – A Develop Understanding Task Developing an understanding of right triangle trigonometric relationships based on similar triangles (G.SRT.6, G.SRT.8)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
6.9 Relationships with Meaning– A Solidify Understanding Task Finding relationships between the sine and cosine ratios for right triangles, including the Pythagorean identity (G.SRT.6, G.SRT.7, F.TF.8)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
6.10 Finding the Value of a Relationship– A Solidify Understanding Task Solving for unknowns in right triangles using trigonometric ratios (G.SRT.7, G.SRT.8)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
6.11 Solving Right Triangles Using Trigonometric Relationships– A Practice Understanding Task Setting up and solving right triangles to model real world contexts (G.SRT.6, G.SRT.7, F.TF.8)	**1 - 80 minute period** **2 – 45 to 50 minute periods**

Quick Quiz 3 & Self-Assessment (formative)	**20 minutes**
Module 6 Test & Performance Assessment	**1 – 45 to 50 minute period each**

Module 7 Circles: A Geometric Perspective	**5 weeks of instructions**
7.1 Centered – A Develop Understanding Task Searching for centers of rotation using perpendicular bisectors as a tool (G.C.2)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
7.2 Circle Dilations – A Solidify Understanding Task Proving circles similar (G.C.1)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
7.3 Cyclic Polygons – A Solidify Understanding Task Examining relationships between central angles, inscribed angles, circumscribed angles and their arcs (G.C.2, G.C.3, G.C.4)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
7.4 Planning the Gazebo – A Practice Understanding Task Developing formulas for perimeter and area of regular polygons (G.GMD.1)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
7.5 From Polygons to Circles – A Solidify Understanding Task **Justifying formulas for circumference and area of circles using intuitive limit arguments (G.GMD.1)**	**1 - 80 minute period** **2 – 45 to 50 minute periods**
7.6 Circular Reasoning – A Practice Understanding Task Applying and practicing circle relationships (G.C.2)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 1 & Self-Assessment (formative)	**20 minutes**
7.7 Pied! – A Develop Understanding Task Using proportional reasoning to calculate arc length and area of sectors (G.C.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
7.8 Madison's Round Garden – A Practice and Develop Understanding Task Using the ratio of arc length to radius to develop radians as a way of measuring angles (G.C.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
7.9 Rays and Radians – A Solidify and Practice Understanding Task Converting between degree measure and radian measure of an angle (G.C.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**

7.10 Sand Castles – A Solidify Understanding Task Examining the proportionality relationships of lengths, areas and volumes when geometric figures are scaled up (G.GMD.1, G.GMD.3)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
7.11 Footprints in the Sand– A Develop Understanding Task Examining informal, dissection arguments for the volume formulas of prisms, pyramids and cylinders (G.GMD.1, G.GMD.3)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
7.12 Cavelieri to the Rescue – A Solidify Understanding Task Examining informal, dissection arguments based on Cavelieri's principle for the volume formulas of oblique prisms, pyramids and cylinders (G.GMD.2)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 2 & Self-Assessment (formative)	**20 minutes**
Module 7 Test & Performance Assessment	**1 – 45 to 50 minute period each**

Module 8 Circles & Other Conics	**3 weeks of instruction**
8.1 Circling Triangles – A Develop Understanding Task Deriving the equation of a circle using the Pythagorean Theorem (G.GPE.1)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
8.2 Getting Centered – A Solidify Understanding Task Complete the square to find the center and radius of a circle given by an equation (G.GPE.1)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
8.3 Circle Challenges – A Practice Understanding Task Writing the equation of a circle given various information (G.GPE.1)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 1 & Self-Assessment (formative)	**20 minutes**
8.4 Directing Our Focus– A Develop Understanding Task Derive the equation of a parabola given a focus and directrix (G.GPE.2)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
8.5 Functioning With Parabolas – A Solidify Understanding Task Connecting the equations of parabolas to prior work with	**1 - 80 minute period** **2 – 45 to 50 minute periods**

SECONDARY MATH 2
SCOPE & SEQUENCE

quadratic functions (G.GPE.2)	
8.6 Turn It Around – A Solidify Understanding Task Writing the equation of a parabola with a vertical directrix, and constructing an argument that all parabolas are similar (G.GPE.2)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 2 & Self-Assessment (formative)	**20 minutes**
8.7H Operating on a Shoestring – A Solidify Understanding Task Exploring features of ellipses and writing the equation of an ellipse using the fact that the sum of the distances from the foci is constant. (G.GPE.3) (+)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
8.8H What Happens If?– A Solidify Understanding Task Exploring features of hyperbolas writing the equation of a hyperbola using the fact that the difference of the distances from the foci is constant. (G.GPE.3) (+)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Module 8 Test & Performance Assessment	**1 – 45 to 50 minute period each**

Module 9 Probability	**3 weeks of instruction**
9.1 TB or Not TB – A Develop Understanding Task Estimating conditional probabilities and interpreting the meaning of a set of data (S.CP.6, S.MD.7+)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
9.2 Chocolate Versus Vanilla – A Solidify Understanding Task Examining conditional probability using multiple representations (S.CP.6)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
9.3 Fried Freddy's – A Solidify Understanding Task Using sample to estimate probabilities (S.CP.2, S.CP.6)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
9.4 Visualizing with Venn– A Practice Understanding Task Creating Venn diagram's using data while examining the addition rule for probability (S.CP.6, S.CP.7)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Quick Quiz 1 & Self-Assessment (formative)	**20 minutes**
9.5 Freddy Revisited – A Solidify Understanding Task Examining independence of events using two-way tables	**1 - 80 minute period** **2 – 45 to 50 minute periods**

Mathematics Vision Project
Licensed under the Creative Commons Attribution CC BY 4.0
mathematicsvisionproject.org

mathematics vision project

(S.CP.2, S.CP.3, S.CP.4, S.CP.5)	
9.6 Striving for Independence – A Practice Understanding Task Using data in various representations to determine independence (S.CP.2, S.CP.3, S.CP.4, S.CP.5)	**1 - 80 minute period** **2 – 45 to 50 minute periods**
Module 9 Test & Performance Assessment	**1 – 45 to 50 minute period each**