Each curriculum in the Mathematics Vision Project materials is composed of two main components, the classroom experience, which is designed around the implementation of a specific type of task and the aligned "Ready, Set, Go!" homework assignment. Each task is accompanied by a set of teacher notes. The teacher notes identify the purpose of the lesson and describe the steps the teacher can take during the classroom experience to ensure that students engage in a rich learning event. Tasks are to be done in class and should not be assigned as homework. There is an aligned "Ready, Set, Go!" homework assignment for each task. It is the independent practice. Homework serves the student as a type of formative assessment. It is while doing the homework that the student can discern for himself if the mathematics done in class can be performed independently.

The MVP classroom experience begins by confronting students with an engaging task and then invites them to grapple with solving it. As students' ideas emerge, take form, and are shared, the teacher orchestrates the student discussions and explorations towards a focused mathematical goal. As conjectures are made and explored, they evolve into mathematical concepts that the community of learners begins to embrace as effective strategies for analyzing and solving problems. These strategies are eventually solidified into a body of practices and mathematical habits that belong to the students, because they were developed by the students, as an outcome of their own creative and logical thinking. This is how students learn mathematics. They learn by doing mathematics. They learn by needing mathematics. They learn by verbalizing the way they see the mathematical ideas connect and by listening to how their peers perceived the problem. Students then own the mathematics because it is a collective body of knowledge that they have developed over time through guided exploration.

This process describes the Learning Cycle, an instructional framework that allows students to build mathematical knowledge over time. This framework is flexible. Every progression does not follow the pattern of develop, solidify, practice. For instance, the first module on quadratics begins with a Develop Understanding Task. Many aspects of the definition of a quadratic surface in that task. Five solidify tasks follow the first task. Each of the Solidify tasks extends one of the key concepts that surfaced in the
beginning Develop Understanding Task. The module ends with a Practice Understanding Task that pulls all of the key concepts together into a complete definition of quadratic.

## The Learning Cycle

The diagram at the right illustrates the Comprehensive Mathematics Instructional Framework (CMI) around which the MVP curriculum has been developed. Every task in the curriculum is identified as one of the following:

- Develop Understanding Task
- Solidify Understanding Task
- Practice Understanding Task

A learning cycle begins with a single term, develop, which refers to bringing student thinking to the surface by activating prior knowledge, intuition, and insights to make sense of a problem.


> PRACTICE Understanding

DEVELOP Understanding

Develop Understanding Tasks are intended to generate ideas, strategies, and representations related to a new mathematical topic. Develop tasks contain multiple entry points for students, so that all students are able to use their intuition and logic to make sense of the problem and devise a strategy for organizing the information. In the second phase of the learning cycle, students will engage in Solidify Understanding tasks that will allow them to examine and extend the mathematical thinking that rose to the surface in the Develop Understanding task. The learning cycle will conclude with a Practice Understanding Task. It focuses students' attention on becoming fluent with the mathematics of the unit and refining the mathematics into formal definitions, properties, procedures, and models that are consistent with practices that exist outside the classroom.

In the CMI Framework the progression of the mathematics through the learning cycle is mapped out along a continuum of conceptual, procedural and representational understandings using the Continuum of Mathematical Understanding.

# Continuum of Mathematical Understanding 


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Mathematical understanding encompasses at least three connected but distinct domains as represented by the horizontal lines of the continuum: conceptualizing mathematics, doing mathematics, and representing mathematics. Mathematical understanding progresses continually along this continuum, but it is useful to note three sets of distinct landmarks of progression along the continuum that are associated with each of the three phases of the Learning Cycle. Emerging mental images are fragile as they are surfaced during students' initial experiences with tasks designed to elicit those images (Develop Understanding). In the continuum we refer to these fragile images as ideas, strategies, and representations. These ideas, strategies and representations need to be examined for accuracy and completeness, as well as extended and connected through multiple exposures and experiences until they become more tangible, solid and useful (Solidify Understanding). In the CMI Framework, ideas that have been examined for the understanding they reveal are called concepts; strategies that can be articulated and replicated are called algorithms; and useful representations are called tools. Once understanding has been developed and solidified, it needs further
refinement to become fluent and applicable to new situations and contexts (Practice Understanding). In the CMI Framework refined concepts become the definitions or properties of formal mathematics; algorithms that can be carried out flexibly and fluently are called procedures; and representations that embody essential mathematical understandings (either conceptual or procedural) are called models, such as "an area model for multiplication" or "the number line as a model of the set of real numbers." These definitions and properties, procedures, and models must be consistent with the broader mathematical "community of practice" that exists outside of the classroom.

The CMI Framework supports teachers in enacting the NCTM effective teaching practice: Build Procedural Fluency from Conceptual Understanding. However, the framework implies that the end-goal of mathematical instruction is not just procedural fluency; it also includes a deeper conceptual understanding of the properties and definitions on which procedures are based, and an ability to draw upon mathematical models more flexibly and fluently when representing one's mathematical understanding. The Learning Cycle component of the framework supports teachers in making curricular decisions that move students from individually-constructed ideas, strategies and representations towards a community of shared definitions, properties, procedures and models. The Continuum of Mathematical Understanding component of the framework emphasizes that there are multiple domains of mathematical understanding that need to be developed, solidified and practiced: the conceptual domain, which provides students with ways of thinking about mathematics; the procedural domain, which provides students with ways of doing mathematics; and the representational domain, which provides students with ways of making one's thinking visible. Together, both components of the CMI Framework promote student thinking to the forefront of mathematics instruction and highlight the decisionmaking role of the teacher in effectively selecting and sequencing tasks that build mathematical understanding and fluency over time.

Each module in the MVP educational program has been carefully designed and sequenced with rich mathematical tasks that have been formulated to generate the mathematical concepts within the core curriculum. Careful attention has been placed upon the way mathematical knowledge emerges, is
extended, and then becomes efficient, flexible, and accurate. Some tasks are developmental tasks while others are for solidifying or practicing the concepts. The sequencing of the tasks encourages students to notice relationships and make connections between the concepts. In this way, students perceive mathematics as a coherent whole.

While the classroom experience is predominantly geared towards improving students' reasoning and sense-making skills, MVP regards mathematical understanding and procedural skill as being equally important. Hence, the "Ready, Set, Go!" homework assignments are focused on students practicing procedural skills and organizing principles to add structure to the ideas developed during the classroom experience. As in any discipline, practice is the refining element that brings fluency and agility to the skills of the participant. The Ready and the Go sections of the homework assignments have been designed to spiral a review of content, while the Set section focuses on consolidating the mathematics addressed in class that day. Each time a student engages in the homework assignment, it is expected that he or she will have the opportunity to reflect on the new learning from class and will practice the retrieval of ideas from the body of learning that has been growing over the school year, and even prior to the current school year. Recent research on learning has identified reflection and retrieval practice as being two key ingredients for durable learning. True learning should be long lasting and should grow out of previous understandings, extending over years of study. Hence, the "Go!" sections of the "Ready, Set, Go!" homework assignments will contain topics from previous lessons and prior years of mathematics instruction. Together the classroom experience and the "Ready, Set, Go!" homework assignments offer a powerful blend of new learning and maintained proficiency.

## The Teaching Cycle

The Learning Cycle depicts how students become proficient in the mathematics overtime. Each task represents at least one day of instruction. Therefore, a Learning Cycle may extend over several days or weeks of classroom instruction, however, each day the teacher is expected to frame the lesson around The Teaching Cycle. This cycle also has three components: Launch, Explore, and Discuss.

The Teaching Cycle may seem to be simple, but it
involves careful preparation and then deliberate implementation by the instructor.

## Discuss

Launch: How will you...

- hook and motivate students?
- provide schema for the task?
- describe the expectations for the finished task?

Explore: What will you . . .


- look for and listen for as you observe?
- accept as evidence of understanding?
- ask to stimulate, redirect, focus, and extend mathematical thinking?

Discuss: How will you ...

- select which students will present their solutions and strategies?
- determine what ideas to pursue?
- decide whether to contribute to the discourse or allow students to continue to struggle to make sense of a concept?

The diagram to the right depicts how the two instructional frameworks, the Teaching Cycle and the Learning

Cycle, fit together. The Teaching
Cycle occurs each day in the
classroom, while the Learning Cycle extends over days and possibly weeks as the unit develops.


The MVP curriculum and the CMI instructional framework together reflect current research on teaching and learning. Research in both cognitive science and mathematics education supports changes in the roles of the learner and the teacher. During instruction, students need to be developing specific reasoning habits that will serve them in other disciplines, real life, and their future careers. It is the teacher's role to provide opportunities for students to develop these skills. The CMI model provides a framework for both the teacher and the student to improve teaching and learning in the classroom.

|  | The Comprehensive Mathematics Instruction Model |  |
| :--- | :--- | :--- |
|  | Teacher's role | Student's role |
| Develop <br> Understanding | Focus learning on the goal of the task; provide <br> experiences using rich tasks; support <br> productive struggle; elicit and use evidence of <br> student thinking to orchestrate discussions <br> using the 5 practices* | Make sense of the context, organize <br> information, notice patterns, make <br> conjectures, invent strategies, create <br> arguments, engage in mathematical <br> discourse |
| Solidify <br> Understanding | Focus learning on the goal of the task; provide <br> experiences using rich tasks; support <br> productive struggle; elicit and use evidence of <br> student thinking to orchestrate discussions <br> using the 5 practices* | See structure; see regularities; attend <br> to precision; create and critique <br> arguments; adopt strategies, use <br> multiple representations, engage in <br> mathematical discourse |
| Understanding | Provide a vehicle for practice; provide <br> feedback; clarify misconceptions; confirm <br> mathematical and symbolic language; elicit <br> and use evidence of student thinking to <br> orchestrate discussions using the 5 practices* | Reason quantitatively; work <br> towards efficiency, flexibility, <br> accuracy; apply (model with <br> mathematics) |

*Five Practices for Orchestrating Productive Mathematical Discussion - $2^{\text {nd }}$ Edition, Margaret S. Smith and Mary K. Stein, NCTM, 2018

The eight effective teaching practices, as articulated in the NCTM publication Principles To Actions, Ensuring Mathematical Success for All (2014), describe a framework for improving instructional practice. The following figure shows how these eight practices can be incorporated into the Teaching Cycle. Note that seven of the practices fit naturally around the Teaching Cycle and can be implemented during each day of instruction, while building procedural fluency from conceptual understanding is a curriculum practice that describes the process of creating deep learning over time.

## A FRAMEWORK for a Lesson or TASK:

Moving from a conceptual foundation to procedural fluency
Comprehensive Mathematics Instruction Framework


Teaching Cycle

Pose Purposeful Questions

Elicit and Use Evidence of Student Thinking

Implement Tasks That Promote Reasoning and Problem Solving


- mathematics vision project

The Launch, Explore, Discuss sequence of the Teaching Cycle is the visible form of the daily, classroom experience. Yet, to make deep learning occur in the classroom, the teacher must carefully prepare for each aspect of the lesson. During the Launch the teacher must set the stage by informing students of the situation and the expectations of the task. During the Explore phase, as students are reasoning through the task, the teacher is busy moving from student to student, clarifying student questions and encouraging student work. As the teacher monitors student effort, he is also selecting and sequencing which work will move student
thinking towards the purpose of the lesson. During the Discuss phase, selected students share their mathematical thinking and strategies, while members of the class listen, question, and record strategies and key concepts. Throughout the lesson, it is the obligation of the teacher to connect the mathematics so that students leave class with the big ideas of the intended mathematical lesson. The following figure depicts how the framework of the five practices for orchestrating discourse fit within the Teaching Cycle. (Adapted from Five Practices for Orchestrating Productive Mathematical Discussion - Second Edition, Margaret S. Smith and Mary K. Stein, NCTM, 2018)

*Five Practices for Orchestrating Productive Mathematical Discussion - $2^{\text {nd }}$ Edition, Margaret S. Smith and Mary K. Stein, NCTM, 2018

## FEATURES

Each module begins with an annotated table of contents which identifies the key concepts that will be the focus of the module and the core standards that will be addressed. A set of teacher notes accompanies each task. The teacher notes outline each step of the lesson while following the framework of the Teaching Cycle. All of the teacher notes follow the same basic outline as described below:

## The Enhanced Teacher Notes include:

Purpose: Paying attention to the purpose of the task will help the teacher stay true to the progression of the module and refrain from trying to accomplish too much within the task.

Core Standards Focus: The MVP authors have taken a "multi-tasking approach" to the standards. While one task may focus on more than one standard, several tasks may hi-light a single standard. In this way a set of interrelated ideas or a sequence of strategies and skills can be fused into a meaningful whole. This "multi-tasking approach" to the standards also gives students multiple opportunities to master the standards.

Related Standards: The focus of a lesson may be on a specific standard, yet doing the mathematics may require students to draw on related standards.
Standards for Mathematical Practice: It is possible and even likely that students will implement all of the Standards for Mathematical Practice within a given lesson, however, different types of tasks naturally elicit certain practices. Those that seem to be the most likely to be drawn upon in the lesson have been identified in the teacher notes.

Essential question for students: Since all of the tasks are inquiry based, the essential question has been formulated to direct students' attention towards the purpose of the lesson without explicitly revealing the key ideas and strategies they should be producing.

## The Teaching Cycle:

Launch (whole class): Suggestions for introducing the lesson to the students. Sometimes this is relating a story, while other times it's working the first problem together. The prompts for the tasks often involve a lot of reading. It is the teacher's obligation to make sure that students understand what they are expected to do or produce during the Explore stage of the learning.

Explore (small groups): While students are exploring, the teacher will be monitoring the individual students and groups, looking for student strategies that will promote the discussion about the
mathematics of the task. This is also a time during which the teacher can assess what previously learned skills the students are bringing to the task. The teacher notes will make suggestions of what the teacher should be looking for during the Explore session.

Discuss: Here the teacher will find suggestions for orchestrating the discussion in order to achieve the purpose of the lesson. This is the time when key connections need to be made.

Exit ticket for students: An exit slip can aid the teacher in checking for understanding. The items in the exit ticket could also be used as a warm-up in the subsequent lesson.

## Instructional Supports

ELL and equity suggestions: Equitable mathematics teaching maintains high standards of learning for all students. Instruction should affirm students' mathematical identities by honoring the multiple resources of mathematical learning present in the classroom. By following the plan of instruction included in the teacher notes, students' different mathematical strengths are used as a resource for learning. Additional strategies for providing equal opportunities for learning are offered where appropriate.

Interventions: These suggestions may lower the threshold for the task to accommodate students who don't know how to begin thinking about the task.

Challenge activity: The challenge activity is to provide a "high ceiling" for students who have finished early or need to be encouraged to think more deeply about the mathematics. Sometimes the last question in the task provides that extension, and it is not essential that it be completed by all students.

Additional Resources for Teachers: This could be a variety of things depending on the lesson. For instance, an app using GeoGebra has been developed for the rubber-band activity in the first task of Module 2 in the geometry course.

Sentence frame cards are available as an aid for students. The cards are intended to assist students in becoming self-directed thinkers by guiding their thinking and prompting the language needed for discourse about their mathematical work. The cards are structured around the Eight Student Practices for Mathematical Thinking. The cards are intended to support all learners, but they are particularly useful in supporting learners who struggle with language.

## Answer Key for each task:

The suggested mathematical approach for some of these tasks may require teachers
to look at the mathematics from a different perspective than they have ever done before. The best way to prepare to teach a task is to work the problem from the standpoint of the student. The answer key is provided as reassurance for the teacher.

Answer Key for each Ready, Set, Go! Homework assignment

## Additional Resources for Teachers and Students

The website www.rsgsupport.org contains a support video to match each Ready, Set, Go Homework assignment.

The Helps, Hints, and Explanations book provides an explanation for each type of homework problem and usually a worked example or two with annotation. Each "Ready, Set, Go!" homework assignment has an accompanying explanation in the Helps, Hints, and Explanations book.

## Assessments and Tools for the PLC

The assessment resources provide a more complete assessment package including: quick quizzes, self-assessments, performance tasks, and a bank of items that cover the topics of the module. When these tools are combined with the exit tickets and the formative assessment available from listening to students as they work on the tasks, teachers can really know what their students understand and can do.

The assessment book includes per module:

- Quick quizzes (mid-unit checks for understanding)
- Student self-assessments (identifies what the student should know and be able to do as he progresses through the module)
- A Module test
- A Performance task with teacher notes and a scoring rubric

The quick quizzes are short, multiple-choice and short answer assessments that give a snapshot of what students have learned in the module. They are designed to be given after a learning cycle is completed (in most cases), so the number of quick quizzes in a module varies. The quick
quizzes should be just that: quick. They can be given at the beginning or end of a period, still allowing time for other work to occur.

The self-assessments are a tool designed to help students know what they should be learning and to reflect on their progress. Like the quizzes, the self-assessments usually occur at the end of each learning cycle. They identify the mathematics that students should have learned and ask students to provide evidence (from their homework, their work on tasks, or problems from other assessments) that shows how well they have learned it, and to write what they will do to increase their understanding. The idea is to help students develop a growth mindset and gain ownership of their learning.

The Module Tests, provide a bank of items that can be used to design a summative assessment that reflects the work of the class. Some teachers like to create tests that have both the performance task and some more traditional items to ensure that all the standards of the module are assessed. There are about as many ways to assess as there are teachers, and most of the methods have merit. The key is to use all assessments as checkpoints to make instructional adjustments that will increase student learning.

The final tool is the performance task. There is one performance task provided for each module. This task incorporates the most important ideas of the module and asks students to use them flexibly. These tasks also provide an opportunity for students to communicate mathematically, using proper vocabulary and notation. An answer key and grading rubric are provided for each task, along with instructions for launching the task so that the task is accessible for each and every student. Some teachers like to give students the opportunity to work these tasks in pairs, mirroring the classroom experience. Others prefer to ask students to work individually to ensure that the results give a clear picture of what each student can do on their own.

The PLC tools include The Essentials Tracker and The Power of the Module.
The Essentials Tracker is a grid connecting the standards and the tasks. When a standard is addressed in a task, it is indicated with one of three letters, $D$ for developing, $S$ for solidifying, or $P$ for practicing. This helps teachers to see that the standards are addressed in more than one task. It also helps teachers to set an appropriate level of
expectation for students relative to the standard. $D$, for developing, indicates students' first exposure to the ideas and/or procedures of the standard, so teachers can expect new ideas to surface, although students may lack the notation or vocabulary that will be developed later. At the $S$ level, for solidifying, students will be sorting through ideas that have previously been surfaced, with support for examining and extending their understanding and clarifying their procedures. If the standard is addressed at the $P$ level, for practice, then students should be working on becoming efficient, accurate, and flexible as they demonstrate mastery of the standard.

The Power of the Module shows the focus or target for each task in the module and the topics of each section of the homework: Ready (to prepare for upcoming tasks), Set (to solidify the work done in the task), and Go (to reinforce previously-learned skills). This tool can help teachers see the opportunities for recall and rehearsal built into the program, along with the progression of mathematical ideas in the tasks. It also provides a "quick glance" or overview of the module, which will help teachers anticipate upcoming mathematical content. By working the tasks, then creating and discussing the Power of a Module outlines as a team, teachers will come to trust the materials and understand the progressions of mathematics that students will have the opportunity learn.

## TECHNOLOGY

Technology is an important tool to be used as part of the MVP curriculum. In their description of the CCSS Standard for Mathematical Practice 5, "Use appropriate tools strategically," the core authors specifically name graphing calculators, computer algebra systems, statistical packages, and dynamic geometry software. They suggest that these tools could be used by students to explore and deepen their understanding of concepts, analyze graphs of functions, visualize mathematical models, and test various assumptions and compare predictions with data. Tasks in MVP provide opportunities for using technological tools in each of the ways described. The use of calculators may also help students to quickly make calculations so that their attention remains focused on the analytical work of the task. The curriculum is designed so that students may use technology that is widely available including graphing calculators or free computer apps such as Desmos or Geogebra. Making technology an integral tool for mathematical thinking enriches the work and provides students with opportunities to engage with SMP 5.

The Secondary Mathematics III course is written to align with the third of three courses in the integrated pathway of the Common Core State Standards, as described in Appendix A. Like all courses in the integrated pathway, it contains standards from each of the conceptual categories in the standards, including:

- Number and quantity;
- Algebra;
- Functions;
- Geometry; and
- Statistics and probability.

The major purpose of Secondary Math III is for students to pull together and apply the accumulation of learning that they have from their previous courses. Students add to their catalog of functions types to include polynomial, rational, logarithmic, and trigonometric. They expand their understanding of right triangle trigonometry to include circular trigonometry and general triangles. They apply methods from probability and statistics to draw inferences and conclusions from data. And, finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. The Mathematical Practice Standards apply throughout each course and, together with the content standards, create mathematical learning experiences based upon reasoning and sensemaking, building perseverance and problem-solving skills, and rich in mathematical discourse.

The standards indicated in the CCSS with a ( + ) sign are addressed with additional tasks in Secondary Math III Honors. The Honors version of the course includes all the same tasks as Secondary Math III, with the additional tasks embedded into the modules where they fit conceptually.

Standards specified in the Widely Accepted Prerequisites (WAP’s) included in the High School Publishers Criteria for the Common Core State Standards for Mathematics constitute the bulk of the curriculum in Secondary Math III. The F-IF standards for interpreting functions are extensively addressed in Modules 1-4 with inverse functions, logarithms, polynomial and rational functions. The functions modules also contain a number of opportunities for students to understand structure of expressions and use various algebraic forms to model a situation or to highlight a given feature, the work defined in the A-SSE standards.

In the narrative that follows, the specific approach and details of the mathematics in the curriculum is described by conceptual category in roughly the same order as the categories are addressed in the curriculum. The additional work of the Honors course is clearly identified.

## Conceptual Category: Functions

The MVP curriculum takes a coherent approach to functions across grade levels. The following big ideas of functions are introduced in Secondary Math I, reinforced in Secondary Math II and fully realized with several different function types in Secondary Math III:

- Functions are categorized by their rates of change.
- The key features of functions are tools for analysis:
- Domain and range
- Intervals of increase and decrease
- Maxima and minima
- x and y intercepts
- Continuity
- Functions can be transformed in the same, predictable way.
- Different algebraic forms of functions have purpose in different situations.
- Functions can be combined together (using basic operations or composed) to make new functions, usually retaining some of the features of both functions.

In the previous two courses, students did extensive work with linear, exponential, and quadratic functions. They also learned about piecewise and absolute value functions and were introduced to inverse functions. Secondary Math III, Module 1, Functions and Their Inverses, reviews the features of linear, exponential, and quadratic functions and general inverse relationships. The idea that the inputs and outputs are reversed in inverse functions is reinforced in the module using tables, graphs, equations, and story context. Students consider situations when the inverse is not a function and learn about invertibility. Students write equations of inverse functions, recognizing that inverse functions have inverse operations in the reverse order. As students use a story context to reason about the inverse of an exponential function, the concept of a logarithm is introduced.

Module 2, Logarithmic Functions, picks up where Module 1 leaves off. Students begin to understand logarithms by drawing upon their experiences with inverses and exponential functions to evaluate, approximate, and order logarithmic expressions such $\log _{2} 8$ and $\log _{2} 20$. Through this experience, students recognize some basic properties of logarithms like $\log _{b} b=1, \log _{b} 1=$ 0 , and $\log _{b} b^{n}=n$. They use known log values to graph logarithmic parent functions such as $y=$ $\log _{2} x$ and then use the parent functions, recognizing the vertical asymptote and the anchor point $(1,0)$ to graph transformations such as $y=1+\log _{2}(x-3)$. The addition, subtraction, and multiplication properties for logarithms are derived from recognizing transformations of graphs of equivalent functions. Students use the log properties to write equivalent expressions and evaluate unknown log quantities using known log values. Students also solve simple exponential and log equations algebraically and using tables and graphs.

The Honors tasks in Module 2 introduce exponential functions with base $e$ and natural logarithm functions. Students model continuous growth situations and solve equations using natural logs. More complicated exponential and log equations are introduced with support for solving them, along with analysis of common misconceptions.

Module 3, Polynomial Functions, begins with a task that links linear, quadratic, and cubic functions together by highlighting the rates of change of each function type and using a story context to show that a linear function is the sum of a constant, a quadratic function is the accumulation or sum of a linear function, and a cubic function is the sum of a quadratic function. Students generalize the pattern they see that linear functions have a constant first difference, quadratic functions have a constant second difference, and cubic functions have a constant third difference, to predict that the pattern continues for quartic polynomials and the rest of the polynomial family. As the module proceeds, students graph $y=x^{3}$, identify its features, and transform the graph. They use the Fundamental Theorem of Algebra and their previous experience with quadratics to identify the number of possible roots, both real and complex, for a given polynomial. Students employ polynomial operations such as division to find roots and write equations using given roots.

Students compare end behavior of polynomials and learn to predict the end behavior. In the context of predicting end behavior for even and odd-powered polynomials, they are introduced to the concept of even and odd functions. The module ends with students synthesizing their understanding of end behavior and roots to write equations and graph polynomials given various information.

In Module 3, students saw that dividing two polynomials sometimes resulted in a remainder, which could be written as a fraction. In Module 4, Rational Expressions and Functions, students work with the fractions that are ratios of polynomials, rational expressions and functions. The module begins with a story context to provide conceptual understanding of the simple rational function, $y=\frac{1}{x^{\prime}}$ for both large values of $x$ and for $x$ approaching zero. The context provides meaning for both the horizontal and vertical asymptotes and supports students in thinking about dividing by fractions. The module continues with students transforming the graph of $y=\frac{1}{x}$ and then introduces more complicated rational functions. The rational functions are categorized by comparing the degree of the numerator and the denominator so that students learn to predict horizontal, slant, and vertical asymptotes, and find x-intercepts. Students develop a strategy for determining and keeping track of the behavior of the function near the vertical asymptotes so that they can easily graph any rational function. Technology is used throughout the module to support students in making conjectures and reasoning about rational functions.

The next functions module is Module 6, Modeling Periodic Behavior. In this module students use a Ferris wheel as a context for constructing conceptual understanding of circular trigonometry. They begin by calculating heights on the Ferris wheel, progress to calculating the heights at a given time on the Ferris wheel, and then, graphing the heights to show a sine function. This progression of concepts takes students from static right triangle trigonometry that they learned in Secondary Math II to defining a more dynamic sine function in terms of angles of rotation. At the end of the learning cycle, students also consider the shadows cast at a giving time, which produces a cosine function. The second learning cycle of the module re-establishes and extends students' understanding of radians, which were introduced in Secondary Math II. They find arc lengths on concentric circles and compare the ratio of arc length to radius on the circles, noticing that they are same for a given angle of rotation. This learning cycle culminates in defining sine and cosine on the unit circle, in terms of angles given in radians.

When graphing the heights on the Ferris wheel in Module 6, students considered the effect of changing the height of the center of the wheel, resulting in a vertical shift of the graph. In Module 7, Trigonometric Functions, Equations, and Identities, students work with more trigonometric graphs, beginning with the familiar Ferris wheel context. In this case, the Ferris wheel is used to introduce a horizontal shift of the graph. The first learning cycle continues with more graphical modeling using sine and cosine, with students learning to work fluently and flexibly with all the transformations of the graphs of the function. An additional task in the Honors course extends the tangent function for angles of rotations, and introduces secant, cosecant, and cotangent functions.

In Secondary Math II, students discovered the Pythagorean Identity for sine and cosine. In the second learning cycle of Module 7, students use diagrams to find more identities including the cofunction identities and the identities that are related to odd and even functions. After making
arguments to establish the basic identities, students use the identities to rewrite trigonometric expressions. In the Honors course, students also develop and use the addition and double angle identities to solve equations. Module 7 continues in the Honors course to introduce inverse trig functions with restricted domains.

Module 8, Modeling with Functions, focuses on a big idea of functions: Functions can be combined together (using basic operations or composed) to make new functions, usually retaining some of the features of both functions. The module begins with students taking a closer look at transformations using tables. They compare function notation with geometric notation. As the learning cycle proceeds, students combine functions using basic operations, noticing and predicting the graphs. They model complex situations by combining functions with arithmetic operations. In the second learning cycle, composition of functions is introduced and students do more modeling combining a variety of function types using both composition and arithmetic operations. In the Honors course, students learn about parametric functions, both graphing and writing parametric equations.

## Conceptual Category: Number and Quantity

Most of the standards in Number and Quantity are addressed in Secondary Math I and II. In Secondary Math III, extend the work done with complex numbers and quadratic functions in Secondary Math II to higher-powered polynomials. In Module 3, Polynomial Functions, students use the Fundamental Theorem of Algebra to predict the number of roots of a polynomial. They find real and complex roots. They use the relationship between roots and factors to write equations of polynomials in factored and standard form, given a known root.

In Module 7, Trigonometric Functions, Equations, and Identities, in the Honors course, students learn about polar coordinates. They write complex numbers in polar form and use them to multiply, divide and find complex roots.

## Conceptual Category: Algebra

In Module 3, Polynomial Functions, students learn to identify and classify polynomials. They compare polynomials to integers and learn that their structures are analogous. Students perform the basic operations of addition, subtraction, multiplication and division with conceptual connections made to how the operations work with integers. Students extend their work with area models for two binomials from Secondary Math II to higher powered expressions. They also learn to expand binomials using patterns in Pascal's Triangle. Besides performing the basic operations with polynomials, students are introduced to the idea of closure. They construct arguments about statements regarding closure of the set of polynomials under given operations and learn that polynomials are closed under the same operations as integers.

In Module 4, students compare rational expressions to rational numbers and perform operations on rational expressions using the same properties. Students learn to simplify rational expressions and to rewrite rational expressions given in the form $a(x) / b(x)$ into the form $q(x)+r(x) / b(x)$ to facilitate graphing the function using transformations or other strategies, depending on the relationship between $a(x)$ and $b(x)$. Students model real situations using rational functions and
solve rational equations. They learn to recognize extraneous solutions and to interpret solutions based upon the context.

## Conceptual Category: Geometry

The geometry of Secondary Math III is primarily focused on using the principles of geometry, including transformation, to model real situations. The first learning cycle in Module 5, Geometric Modeling, begins with students visualizing two-dimensional cross sections of three-dimensional objects and solids of rotation. They learn to approximate the volume of an irregular solid by decomposing it into cylinders, frustrums, and cones with volumes that can be easily calculated.

In the second learning cycle of Module 5, students extend their understanding of right triangle trigonometry from Secondary Math II to general triangles. They begin with a study of special right triangles and proceed to finding the sides and angles of some triangle by decomposing them into right triangles so that the Pythagorean Theorem or right triangle trigonometry can be used. This strategy supports students in deriving the Law of Cosines and the Law of Sines, which they then apply to finding sides and angles for triangles. In the final task of the module, students explore the ambiguous case of Law of Sines and develop formulas for the area of triangles using the Law of Sines and the Law of Cosines.

## Conceptual Category: Statistics and Probability

In Secondary Math III, students combine all their experience with data and probability from previous courses to make inferences and draw conclusions from data. Module 9, Statistics, begins with a learning cycle where students construct the concept of normal distributions, understanding the effect of modifying either the mean or standard deviation. Students learn to compare distributions using z -scores, and to determine whether a particular point in a normal distribution is typical or unusual. The module continues by introducing methods of sampling and comparing the validity of each method for selecting a sample that is representative of the population. Students learn about different study methods and select an appropriate study type and sampling method for a given parameter of interest. In the last task, students draw conclusions about the likelihood of a given event, based on a simulation.

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| Module 1 Functions and Their Inverses | 2 weeks of instruction |
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| 1.1 Brutus Bites Back - A Develop Understanding Task Develops the concept of inverse functions in a linear modeling context using tables, graphs, and equations. (F.BF.1, F.BF.4, F.BF.4a) | 1-80 minute period <br> 2-45 to 50 minute periods |
| 1.2 Flipping Ferraris - A Solidify Understanding Task Extends the concepts of inverse functions in a quadratic modeling context with a focus on domain and range and whether a function is invertible in a given domain. (F.BF.1, F.BF.4, F.BF.4c, F.BF.4d) | 1-80 minute period <br> 2-45 to 50 minute periods |
| 1.3 Tracking the Tortoise - A Solidify Understanding Task Solidifies the concepts of inverse function in an exponential modeling context and surfaces ideas about logarithms. (F.BF.1, F.BF.4, F.BF.4c, F.BF.4d) | 1-80 minute period <br> 2-45 to 50 minute periods |
| Quick Quiz 1 \& Self-Assessment (formative) | 20 minutes |
| 1.4 Pulling a Rabbit Out of a Hat - A Solidify Understanding Task <br> Uses function machines to model functions and their inverses. Focus on finding inverse functions and verifying that two functions are inverses. (F.BF.4, F.BF.4a, F.BF.4b) | 1-80 minute period <br> 2-45 to 50 minute periods |
| 1.5 Inverse Universe - A Practice Understanding Task Uses tables, graphs, equations, and written descriptions of functions to match functions and their inverses together and to verify the inverse relationship between two functions. (F.BF.4a, F.BF.4b, F.BF.4c, F.BF.4d) | 1-80 minute period <br> 2-45 to 50 minute periods |
| Quick Quiz 2 \& Self-Assessment (formative) | 20 minutes |
| Module 1 Assessment and Performance Assessment | 1-45 to 50 minute period e |


| Module 2 Logarithmic Functions | 4 weeks of instruction |
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| 2.1 Log Logic - A Develop Understanding Task <br> Evaluate and compare logarithmic expressions. (F.BF.5, F.LE.4) | 1-80 minute period <br> $2-45$ to 50 minute periods |
| 2.2 Falling Off a Log-A Solidify Understanding Task | $1-80$ minute period <br> $2-45$ to 50 minute periods |


| Graph logarithmic functions with transformations (F.BF.5, <br> F.IF.7a) |  |
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| Quick Quiz 1 \& Self-Assessment (formative) | $\mathbf{2 0}$ minutes |
| 2.3 Chopping Logs - A Solidify Understanding Task <br> Explore properties of logarithms. (F.IF.8, F.LE.4) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| 2.4 Log-Arithm-etic- A Solidify Understanding Task <br> Use log properties to evaluate expressions. (F.IF.8, F.LE.4) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| 2.5 Powerful Tens - A Practice Understanding Task <br> Solve exponential and logarithmic functions in base 10 using <br> technology. (F.LE.4) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| Quick Quiz 2 \& Self-Assessment (formative) | $\mathbf{2 0 ~ m i n u t e s ~}$ |
| 2.6H Compounding the Problem- A Develop Understanding <br> Task <br> Develop the number $e$. (F.LE.4) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| 2.7 Logs Go Viral - A Solidify Understanding Task <br> Use base e exponential functions and natural logarithms to <br> model continuous growth and decay (F.LE.4) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| 2.8H Chose This, Not That - A Solidify Understanding Task <br> Solve exponential and logarithmic equations. | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| 2.9H Don't Forget Your Login - A Practice Understanding Task <br> Solve and categorize exponential and logarithmic equations. | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| Module 2 Test \& Performance Assessment | $\mathbf{1 - 4 5}$ to 50 minute period each |


| Module 3 Polynomial Functions | $\mathbf{3}$ weeks of instruction |
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| 3.1 Scott's March Madness - A Develop Understanding Task <br> Introduce polynomial functions and their rates of change <br> (F.BF.1, F.LE.3, A.CED.2) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| 3.2 You-mix Cubes - A Solidify Understanding Task <br> Graph $y=x^{3}$ with transformations and compare to $y=x^{2 .}$ <br> (F.BF.3, F.IF.4, F.IF.5, F.IF.7) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| Quick Quiz 1 \& Self-Assessment (formative) | $\mathbf{2 0}$ minutes |
| 3.3 Building Strong Roots - A Solidify Understanding Task | $\mathbf{1 - 8 0}$ minute period |


| Understand the Fundamental Theorem of Algebra and apply it to <br> cubic functions to find roots. (A.SSE.1, A.APR.3, N.CN.9) | $\mathbf{2 - 4 5}$ to $\mathbf{5 0}$ minute periods |
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| 3.4 Getting to the Root of the Problem - A Solidify <br> Understanding Task <br> Find the roots of polynomials and write polynomial equations in <br> factored form. (A.APR.3, N.CN.8, N.CN.9) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to $\mathbf{5 0}$ minute periods |
| 3.5 Is This the End? - A Solidify Understanding Task <br> Examine the end behavior of polynomials and determine <br> whether they are even or odd. (F.LE.3, A.SSE.1, F.IF.4, F.BF.3) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to $\mathbf{5 0}$ minute periods |
| 3.6 Puzzling Over Polynomials - A Practice Understanding Task <br> Analyze polynomials, determine roots, end behavior, and write <br> equations (A-APR.3, N-CN.8, N-CN.9, A-CED.2) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to $\mathbf{5 0}$ minute periods |
| Quick Quiz 2 \& Self-Assessment (formative) | $\mathbf{2 0}$ minutes |
| Module 3 Test \& Performance Assessment | $\mathbf{1 - 4 5}$ to $\mathbf{5 0}$ minute period each |


| Module 4 Rational Expressions and Functions | $\mathbf{3}$ weeks of instructions |
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| 4.1 Winner, Winner - A Develop Understanding Task <br> Introducing rational functions and asymptotic behavior (F.IF.7d <br> A.CED.2, F.IF.5) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| 4.2 Shift and Stretch - A Solidify Understanding Task <br> Applying transformations to the graph of $f(x)=\frac{1}{x}$. (F.BF.3, <br> F.IF.7d, A.CED.2) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| Quick Quiz 1 \& Self-Assessment (formative) | $\mathbf{2 0}$ minutes |
| 4.3 Rational Thinking - A Solidify Understanding Task <br> Discovering the relationship between the degree of the <br> numerator and denominator and the horizontal asymptotes. <br> (F.IF.7d A.CED.2, F.IF.5) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| 4.4 Are You Rational? - A Solidify Understanding Task <br> Reducing rational functions and identifying improper rational <br> functions and writing them in an equivalent form. (A.APR.6, <br> A.APR.7, A.SSE.3) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| Quick Quiz 1 \& Self-Assessment (formative) | $\mathbf{2 0}$ minutes |
| 4.5 Just Act Rational - A Solidify Understanding Task | $\mathbf{1 - 8 0 \text { minute period }}$ |


| Adding, subtracting, multiplying, and dividing rational expressions. (A.APR.7, A.SSE.3) | 2-45 to 50 minute periods |
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| 4.6 Sign on the Dotted Line - A Practice Understanding Task Developing a strategy for determining the behavior near the asymptotes and graphing rational functions. (F.IF.4, F.IF.7d) | 1-80 minute period <br> 2-45 to 50 minute periods |
| Quick Quiz 2 \& Self-Assessment (formative) | 20 minutes |
| 4.7 We All Scream - A Practice Understanding Task Modeling with rational functions, and solving equations that contain rational expressions. (A.REI.A.2, A.SSE.3) | 1-80 minute period 2-45 to 50 minute periods |
| Quick Quiz 3 \& Self-Assessment (formative) | 20 minutes |
| Module 4 Test \& Performance Assessment | 1-45 to 50 minute period each |


| Module 5 Modeling with Geometry | 3 weeks of instruction |
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| 5.1 Any Way You Slice It - A Develop Understanding Task <br> Visualizing two-dimensional cross sections of three dimensional <br> objects (G.GMD.4) | $1-80$ minute period <br> $2-45$ to 50 minute periods |
| 5.2 Any Way You Spin It - Develop Understanding Task <br> Visualizing solids of revolution (G.GMD.4) | $1-80$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| 5.3 Take Another Spin - A Solidify Understanding Task <br> Approximating volumes of solids of revolution with cylinders and <br> frustums (G.MG.1, G.GMD.4) | $1-80$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| 5.4 You Nailed It! - A Practice Understanding Task <br> Solving problems using geometric modeling (G.MG.1, G.MG.2, <br> G.MG.3) | $1-80$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| Quick Quiz 1 \& Self-Assessment (formative) | 20 minutes |
| 5.5 Special Rights-A Solidify Understanding Task <br> Examining the relationship of sides in special right triangles <br> (G.SRT.11) | $1-80$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| 5.6 More Than Right - A Develop Understanding Task | $1-80$ minute period |


| Developing strategies for solving non-right triangles (G.SRT.10, <br> G.SRT.11) | $\mathbf{2 - 4 5}$ to $\mathbf{5 0}$ minute periods |
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| 5.7 Justify the Laws - A Solidify Understanding Task <br> Examining the Law of Cosines and the Law of Sines (G.SRT.10, <br> G.SRT.11) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to $\mathbf{5 0}$ minute periods |
| 5.8 Triangle Areas by Trig - A Practice Understanding Task <br> Finding the missing sides, angles and areas of general triangles <br> (G.SRT.9, G.SRT.10, G.SRT.11) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| Quick Quiz 2 \& Self-Assessment (formative) | $\mathbf{2 0}$ minutes |
| Module 5 Test \& Performance Assessment | $\mathbf{1 - 4 5}$ to $\mathbf{5 0}$ minute period each |


| Module 6 Modeling Periodic Behavior | 4 Weeks of Instruction |
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| 6.1 George W. Ferris' Day Off - A Develop Understanding Task Using reference triangles, right triangle trigonometry and the symmetry of a circle to find the $y$-coordinates of points on a circular path (F.TF.5) | 1-80 minute period 2-45 to 50 minute periods |
| 6.2 "Sine" Language - A Solidify Understanding Task <br> Using reference triangles, right triangle trigonometry, angular speed and the symmetry of a circle to find the $y$-coordinates of points on a circular path at given instances in time-an introduction to the circular trigonometric functions (F.TF.5) | 1-80 minute period 2-45 to 50 minute periods |
| 6.3 More "Sine" Language - A Solidify Understanding Task Extending the definition of sine from a right triangle trigonometric ratio to a function of an angle of rotation (F.TF.2) | 1-80 minute period <br> 2-45 to 50 minute periods |
| 6.4 More Ferris Wheels- A Solidify Understanding Task <br> Graphing a sine function to model circular motion and relating features of the graph to the parameters of the function (F.TF.5, F.IF.4, F.BF.3) | 1-80 minute period <br> 2-45 to 50 minute periods |
| 6.5 Moving Shadows- A Practice Understanding Task <br> Extending the definition of the cosine from a right triangle trigonometric ratio to a function of an angle of rotation (F.TF.2, F.TF.5) | 1-80 minute period <br> 2-45 to 50 minute periods |
| Quick Quiz 1 \& Self-Assessment (formative) | 20 minutes |


| 6.6 Diggin' It- A Develop Understanding Task <br> Introducing radians as a unit for measuring angles on concentric <br> circles (F.TF.1, F.TF.2) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
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| 6.7 Staking It- A Solidify Understanding Task <br> Introducing radians as a unit for measuring angles on concentric <br> circles (F.TF.1, F.TF.2) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| 6.8 "Sine"ing and "Cosine"ing It- A Solidify Understanding Task <br> Introducing radians as a unit for measuring angles on concentric <br> circles (F.TF.1, F.TF.2) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| 6.9 Water Wheels and the Unit Circle- A Practice <br> Understanding Task <br> Defining sine and cosine on the unit circle in terms of angles of <br> rotation measured in radians <br> (F.TF.1, F.TF.2) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| Quick Quiz 2 \& Self-Assessment (formative) | $\mathbf{2 0 ~ m i n u t e s ~}$ |
| Module 6 Test \& Performance Assessment | $\mathbf{1 - 4 5 \text { to } 5 0 \text { minute period each }}$ |


| Module $\mathbf{7}$ Trigonometric Functions, Equations \& Identities | $\mathbf{5}$ weeks of instruction |
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| 7.1 High Noon and Sunset Shadows - A Develop Understanding <br> Task <br> Introducing the horizontal shift of a trigonometric function in a <br> modeling context (F.TF.5, F.BF.3) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to $\mathbf{5 0}$ minute periods |
| 7.2 High Tide - A Solidify Understanding Task <br> Using trigonometric graphs and inverse trigonometric functions <br> to model periodic behavior (F.TF.5, F.BF.4) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| 7.3 Getting on the Right Wavelength - A Practice <br> Understanding Task <br> Practice using trigonometric graphs and inverse trigonometric <br> functions to model periodic behavior (F.TF.5, F.BF.3, F.BF.4) | $\mathbf{1 - 8 0}$ minute period |


| Quick Quiz 1 \& Self-Assessment (formative) | 20 minutes |
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| 7.4H off on a Tangent - A Develop and Solidify Understanding Task <br> Extending the definition of tangent from a right triangle trigonometric ratio to a function of an angle of rotation measured in degrees or radians; introducing the reciprocal trig functions: secant, cosecant and cotangent (F.TF.2, F.TF.3, F.TF.4, F.IF.5) | 1-80 minute period <br> 2-45 to 50 minute periods |
| 7.5 Maintaining Your Identity- A Develop Understanding Task Using diagrams to introduce fundamental trig identities, including identities related to odd and even functions (F.TF.3, F.TF.4, F.TF.8) | 1-80 minute period <br> 2-45 to 50 minute periods |
| 7.6 Hidden identities - A Practice Understanding Task Using fundamental trig identities to change the form of trig expressions, and as an aid to solving trig equations (F.TF.7+) | 1-80 minute period <br> 2-45 to 50 minute periods |
| Quick Quiz 2 \& Self-Assessment (formative) | 20 minutes |
| 7.7H Double Identity - A Solidify Understanding Task Extending trig identities to include the addition, subtract and double identities for sine, cosine and tangent (F.TF.9+) | 1-80 minute period <br> 2-45 to 50 minute periods |
| 7.8H The Amazing Inverse Trig Function Race - A Solidify Understanding Task <br> Extending students' thinking about inverse trig functions and examining the graph of the inverse sine, cosine and tangent functions (F.TF.6+, F.TF.7+) | 1-80 minute period <br> 2-45 to 50 minute periods |
| 7.9H More Hidden Identities - A Practice Understanding Task Using trig identities to change the form of trig expressions, and as an aid to solving trig equations (F.TF.7+) | 1-80 minute period <br> 2-45 to 50 minute periods |
| Quick Quiz 3 \& Self-Assessment (formative) | 20 minutes |
| 7.10H Polar Planes - A Develop Understanding TaskIntroducing polar coordinates and polar grids (N.CN.4+, N.CN.5+) | 1-80 minute period <br> 2-45 to 50 minute periods |
| 7.11H Complex Polar Forms - A Solidify Understanding Task Introducing and using the polar form of complex numbers to multiply, divide and find roots of complex numbers (N.CN.4+, N.CN.5+) | 1-80 minute period <br> 2-45 to 50 minute periods |


| Quick Quiz 4 \& Self-Assessment (formative) | 20 minutes |
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| Module 7 Test \& Performance Assessment | $\mathbf{1 - 4 5}$ to 50 minute period each |


| Module 8 Modeling With Functions | 3 weeks of instruction |
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| 8.1 Function Family Reunion - A Solidify Understanding Task <br> Examining transformations of a variety of familiar functions using <br> tables (F.BF.3, G.CO.2) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| 8.2 Imagineering - A Develop Understanding Task <br> Predicting the shape of a graph that is the sum or product of <br> familiar functions (F.BF.1b) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| 8.3 The Bungee Jump Simulator - A Solidify Understanding Task <br> Combining a variety of functions using arithmetic operations to <br> model complex behavior (F.BF.1b) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| Quick Quiz 1 \& Self-Assessment (formative) | $\mathbf{2 0 ~ m i n u t e s ~}$ |
| 8.4 Composing and Decomposing- A Develop Understanding <br> Task <br> Combining a variety of functions using function composition to <br> model complex behavior (F.BF.1c) | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| 8.5 Translating My Composition - A Solidify Understanding <br> Task | $\mathbf{1 - 8 0}$ minute period <br> Extending function transformations by composing and <br> decomposing functions (F.BF.1c, F.BF.3) |
| 8.6 Different Combinations - A Practice Understanding Task <br> Combining functions defined by tables, graphs or equations <br> using function composition and/or arithmetic operations | $\mathbf{1 - 8 0}$ minute period <br> $\mathbf{2 - 4 5}$ to 50 minute periods |
| (F.BF.1b. F.BF.1c) |  |$\quad$| Quick Quiz 2 \& Self-Assessment (formative) |
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| Quick Quiz 3 \& Self-Assessment (formative) | 20 minutes |
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| Module 8 Test \& Performance Assessment | $1-45$ to 50 minute period each |


| Module 9 Statistics | 4 Weeks of Instruction |
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| 9.1 What Is Normal? - A Develop Understanding Task Understanding normal distributions and identify their features (S.ID.4) | 1-80 minute period <br> 2-45 to 50 minute periods |
| 9.2 Just ACT Normal - A Solidify Understanding Task Using the features of a normal distribution to make decisions (S.ID.4) | 1-80 minute period <br> 2-45 to 50 minute periods |
| 9.3 Y B Normal? - A Solidify Understanding Task Introducing z scores to compare normal distributions (S.ID.4) | 1-80 minute period <br> 2-45 to 50 minute periods |
| 9.4 Wow, That's Weird - A Practice Understanding Task Comparing normal distributions using z scores and understanding of mean and standard deviation (S.ID.4) | 1-80 minute period 2-45 to 50 minute periods |
| Quick Quiz 1 \& Self-Assessment (formative) | 20 minutes |
| 9.5 Would You Like to Try a Sample - A Develop Understanding Task Understanding and identifying different methods of sampling (S.IC.1) | 1-80 minute period 2-45 to 50 minute periods |
| 9.6 Let's Investigate - A Solidify Understanding Task Identifying the difference between survey, observational studies, and experiments (S.IC.1, S.IC.2) | 1-80 minute period <br> 2-45 to 50 minute periods |
| 9.7 Slacker's Simulation - A Solidify Understanding Task Using simulation to estimate the likelihood of an event (S.IC.2, S.IC.3) | 1-80 minute period <br> 2-45 to 50 minute periods |
| Quick Quiz 2 \& Self-Assessment (formative) | 20 minutes |
| Module 9 Test \& Performance Assessment | 1-45 to 50 minute period each |

