We’re All Language Learners: Advancing Academic Language Through the Learning Cycle

Barbara Kuehl
Mathematics Vision Project
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Transforming Mathematics Education
Supporting English Learners: The Right Pedagogical Approach

For all students, a large part of learning mathematics is learning the language of mathematics and using mathematics terminology meaningfully within academic conversations and written work. Understandably, students who are simultaneously learning the English language and mathematics need extra support. **Hands-on and authentic, problem-based instruction works best for English learners, and mathematics pedagogy fits well with this approach.**

_Making Mathematics Accessible to English Learners, A Guidebook for Teachers, 2009 WestEd, Carr, Carrol, Cremer, Gale, Lagunoff, Sexton_
Problems with Traditional Instruction

When instruction focuses on having students simply manipulate mathematical expressions and practice algorithms, it avoids the important cognitive challenges of understanding word problems and discussing mathematical ideas. This type of approach is generally not effective for any learner, but it is especially problematic for English learners because it does not involve them in the mathematical thinking and talking that support both language development and mathematics learning. The ultimate consequence is that English learners become marginalized in mathematics education and do not have the opportunity to become mathematically literate or choose a math-oriented career. (pg 5)

Making Mathematics Accessible to English Learners, A Guidebook for Teachers, 2009 WestEd, Carr, Carrol, Cremer, Gale, Lagunoff, Sexton
How do we learn language?

Chair

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How do we learn language in school?

We copy a definition into a notebook like this:

Erode: to eat into or away; destroy by slow consumption or disintegration

And then use it in sentences like this:

“My family eroded the turkey on Thanksgiving.”
How do we learn language in mathematics?

- **Contextual Language**: Language that is specific to a particular context
- **Bridging Language**: Language that may describe a specific representation
- **Official Math Language**: Language that is part of the mathematical register and would be recognized by anyone in the mathematical community
Task Sequencing
Comprehensive Mathematics Instruction Framework

- **Develop Understanding** tasks surface student thinking
- **Solidify Understanding** tasks examine and extend
- **Practice Understanding** tasks refine understanding and build fluency

Transforming Mathematics Education
3.1 Getting Ready for a Pool Party

A Develop Understanding Task

Sylvia has a small pool full of water that needs to be emptied and cleaned, then refilled for a pool party. During the process of getting the pool ready, Sylvia did all of the following activities, each during a different time interval.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Removed water with a single bucket</td>
<td>Filled the pool with a hose</td>
</tr>
<tr>
<td></td>
<td>(same rate as emptying pool)</td>
</tr>
<tr>
<td>Drained water with a hose</td>
<td>Cleaned the empty pool</td>
</tr>
<tr>
<td>(same rate as filling pool)</td>
<td></td>
</tr>
<tr>
<td>Sylvia and her two friends removed water with</td>
<td>Took a break</td>
</tr>
<tr>
<td>her three buckets</td>
<td></td>
</tr>
</tbody>
</table>

1. Sketch a possible graph showing the height of the water level in the pool over time. Be sure to include all of activities Sylvia did to prepare the pool for the party. Remember that only one activity happened at a time. Think carefully about how each section of your graph will look, labeling where each activity occurs.

2. Create a story connecting Sylvia’s process for emptying, cleaning, and then filling the pool to the graph you have created. Do your best to use appropriate math vocabulary.

3. Does your graph represent a function? Why or why not? Would all graphs created for this situation represent a function?
### 3.1 Getting Ready for a Pool Party

**A Develop Understanding Task**

Sylvia has a small pool full of water that needs to be emptied and cleaned, then refilled for a pool party. During the process of getting the pool ready, Sylvia did all of the following activities, each during a different time interval.

<table>
<thead>
<tr>
<th>Task</th>
<th>Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Removed water with a single bucket</td>
<td>1</td>
</tr>
<tr>
<td>Drained water with a hose (same rate as filling pool)</td>
<td>3</td>
</tr>
<tr>
<td>Sylvia and her two friends removed water with her three buckets</td>
<td>5</td>
</tr>
<tr>
<td>Filled the pool with a hose (same rate as emptying pool)</td>
<td>6</td>
</tr>
<tr>
<td>Cleaned the empty pool</td>
<td>5</td>
</tr>
<tr>
<td>Took a break</td>
<td>2</td>
</tr>
</tbody>
</table>
What other vocabulary is included in this task?

- y-intercept
- Rate of change
- Constant interval – 0 slope / minimum/ x-intercept
- Increasing
- Maximum
- Domain, Range

Transforming Mathematics Education
How do we learn language in mathematics?

**Contextual Language**
- Language that is specific to a particular context
- "Minimum is when the pool is empty, maximum is when the pool is full"

**Bridging Language**
- Language that may describe a specific representation

**Official Math Language**
- Language that is part of the mathematical register and would be recognized by anyone in the mathematical community

*Transforming Mathematics Education*
Task Sequencing
Comprehensive Mathematics Instruction Framework

PRACTICE Understanding

DEVELOP Understanding

SOLIDIFY Understanding

Transforming Mathematics Education
3.2 Floating Down the River

A Solidify Understanding Task

Alonzo, Maria, and Sierra were floating in inner tubes down a river, enjoying their day. Alonzo noticed that sometimes the water level was higher in some places than in others. Maria noticed there were times they seemed to be moving faster than at other times. Sierra laughed and said “Math is everywhere!” To learn more about the river, Alonzo and Maria collected data throughout the trip.

<table>
<thead>
<tr>
<th>Time (in minutes)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
<th>110</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (in feet)</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>8</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>12</td>
<td>9</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

1. Use the data collected by Alonzo to interpret the key features of this relationship.

Maria created a graph by collecting data on a GPS unit that told her the distance she had traveled over a period of time.

2. Using the graph created by Maria, describe the key features (increasing, decreasing, domain, range, maximum, minimum, intercepts) of this relationship.
Math Language Routine: Notice and Wonder

<table>
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<tr>
<th>Time (in minutes)</th>
<th>0</th>
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<th>40</th>
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<td>12</td>
<td>9</td>
<td>6.5</td>
<td>5</td>
</tr>
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</table>
2 Floating Down the River
A Solidify Understanding Task

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Use the data collected by Alonzo to interpret the key features of this relationship.

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<th>120</th>
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<td>10</td>
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<td>5</td>
<td>7</td>
<td>12</td>
<td>9</td>
<td>6.5</td>
<td>5</td>
</tr>
</tbody>
</table>

- **Increasing**: [0, 30], [60, 90]
- **Decreasing**: [30, 60], [90, 120]
- **Maximum**: 12
- **Minimum**: 4
- **y-intercept**: 4
- **Range**: [4, 12]
- **Increasing**: $y = 4 - y = 10$, $y = 4 - y = 7$
- **Decreasing**: $y = 9 - y = 4$, $y = 7 - y = 5$

$x \in \mathbb{R}, x \geq 0.5 \geq [0, 120]$, $x \leq 120$
Floating Down the River
A Solidify Understanding Task

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Floating Down the River – Language Supports

Complete the following sentences:

• A maximum is a point on a graph that ________________.
• To find a minimum on a table, we ____________________.
How do we learn language in mathematics?

- **Contextual Language**
  - Language that is specific to a particular context
  - “The minimum is when the pool is empty, the maximum is when the pool is full”

- **Bridging Language**
  - Language that may describe a specific representation
  - “The minimum is the lowest number in the table.”

- **Official Math Language**
  - Language that is part of the mathematical register and would be recognized by anyone in the mathematical community

Transforming Mathematics Education
Task Sequencing
Comprehensive Mathematics Instruction Framework

Practice Understanding

Develop Understanding

Solidify Understanding

Transforming Mathematics Education
3.3 Features of Functions

A Practice Understanding Task

For each graph, determine if the relationship represents a function, and if so, state the key features of the function (key features include intercepts, intervals where the function is increasing or decreasing, relative maximums and minimums, symmetries, domain and range, and end behavior).

1. 

2. 

3. 

4. 

5. 

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3.3 Features of Functions

A Practice Understanding Task

For each graph, determine if the relationship represents a function, and if so, state the key features of the function (key features include intercepts, intervals where the function is increasing or decreasing, relative maximums and minimums, symmetries, domain and range, and end behavior).

1. 

2. 

3. 

4. 

5. 

Mathematics Language Routine: Compare and Connect

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Co-Construct a Definition

Definition and features:

Identify it in a graph:

Maximum of a function

Identify it in an equation:

Identify it in a table:
How do we learn language in mathematics?

**Contextual Language**
- Language that is specific to a particular context
- “The minimum is when the pool is empty, the maximum is when the pool is full”

**Bridging Language**
- Language that may describe a specific representation
- “The minimum is the lowest number in the table.”

**Official Math Language**
- Language that is part of the mathematical register and would be recognized by anyone in the mathematical community
- “The minimum of a function is the smallest y-value in the domain.”
Task Sequencing
Comprehensive Mathematics Instruction Framework

PRACTICE Understanding

DEVELOP Understanding

SOLIDIFY Understanding

Transforming Mathematics Education
Best Instruction Practices for English Learners:

- Provide a rich, meaning centered context for students to use language, with many visual representations, hands-on activities, and language supports.
- Provide ample opportunities for high-quality interaction between English learners and native English speakers that encourage English learners to share their knowledge and experience, hear other students rephrase what the teacher said, and apply new language.
- Use high-frequency vocabulary that students know and gradually introduce more academic vocabulary as they progress in the lesson and their language skills.
- Integrate listening, speaking, reading, and writing skills across instruction, and assist English learners to make a bridge between oral and written language.
One more MVP Session at NCTM:

Learning Progressions with Punch: Learning Occurs Over Time

Travis Lemon, MVP Author
Session 646
Saturday, April 6
Hilton Bayfront, Indigo E
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