mathematics vision project

Transforming Mathematics Education

SECONDARY MATH ONE

An Integrated Approach

MODULE 1

Sequences

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

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1.1 Checkerboard Borders

A Develop Understanding Task

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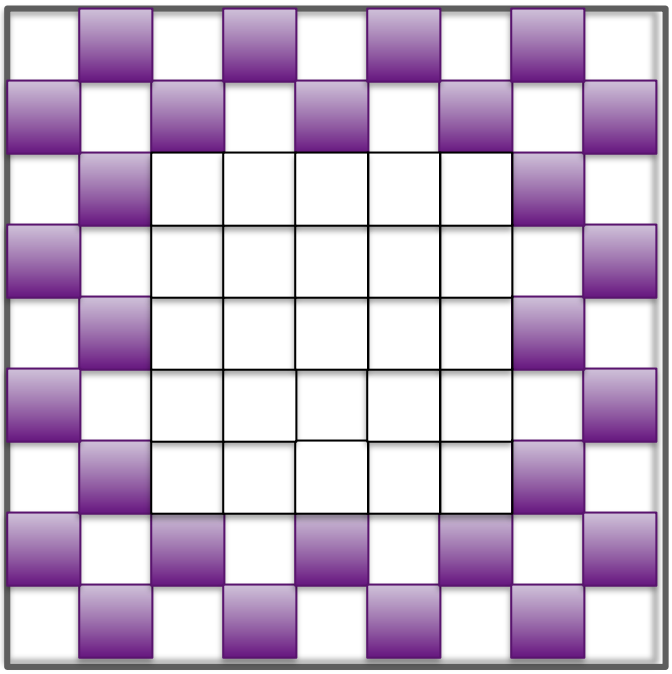
In preparation for back to school, the school administration plans to replace the tile in the cafeteria. They would like to have a checkerboard pattern of tiles two rows wide as a surround for the tables and serving carts.

Below is an example of the boarder that the administration is thinking of using to surround a square 5 x 5 set of tiles.

A. Find the number of colored tiles in the checkerboard border. Track your thinking and find a way of calculating the number of colored tiles in the border that is quick and efficient. Be prepared to share your strategy and justify your work.



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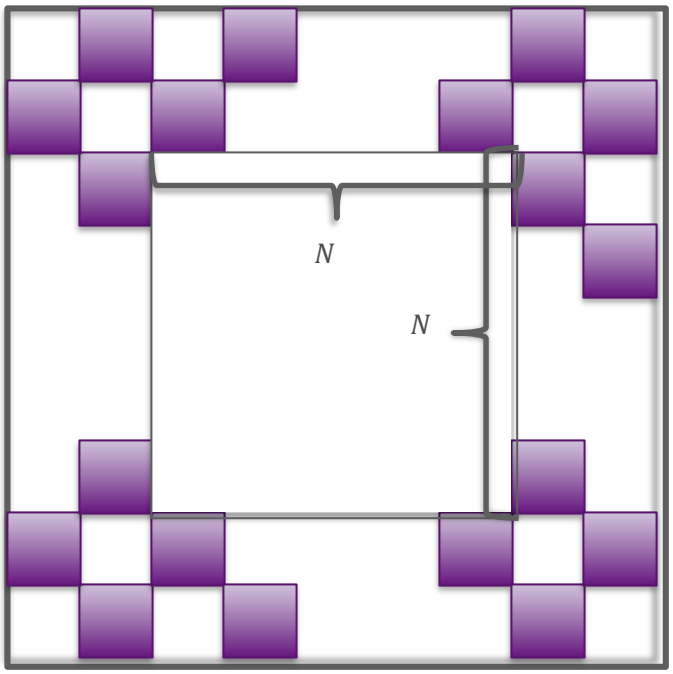
B. The contractor that was hired to lay the tile in the cafeteria is trying to generalize a way to calculate the number of colored tiles needed for a checkerboard border surrounding a square of tiles with any dimensions. To represent this general situation, the contractor started sketching the square below.

Find an expression for the number of colored border tiles needed for any N x N square center.



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1.1

READY, SET, GO! Name Period Date

READY

Topic: Recognizing Solutions to Equations

The solution to an equation is **the value of the variable** that makes the equation **true**. In the equation $9a + 17 = -21$, "a" is the variable. When $a = 2$, $9a + 17 \neq -19$, because $9(2) + 17 = 35$. Thus $a = 2$ is NOT a solution. However, when $a = -4$, the equation is true $9(-4) + 17 = -19$. Therefore, $a = -4$ must be the solution.

Identify which of the 3 possible numbers is the solution to the equation.

1. $3x + 7 = 13 \ (x = -2;\ x = 2;\ x = 5)$

2. $8 - 2b = -2 \ (b = -3; b = 0; b = 5)$

3. $5 + 4g + 8 = 1 \ (g = -3; g = -1; g = 2)$

4. $6t - 5 + 5t = 105 \ (t = 4; t = 7; t = 10)$

Some equations have two variables. You may recall seeing an equation written like the following: $y = 5x + 2$. We can let x equal a number and then work the problem with this x- value to determine the associated y- value. A solution to the equation must include both the x- value and the y- value. Often the answer is written as an **ordered pair**. The ***x*- value is always first**. Example: (x, y). The order matters!

Determine the y-value of each ordered pair based on the given *x*- value.

5. $y = 6x - 15;\ (8,\ \ ), (-1,\ \ ), (5,\ \ )$

6. $y = -4x + 9;\ (-5,\ \ ), (2,\ \ ), (4,\ \ )$

7. $y = 2x - 1;\ (-4,\ \ ), (0,\ \ ), (7,\ \ )$

8. $y = -x + 9;\ (-9,\ \ ), (1,\ \ ), (5,\ \ )$

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SET

Topic: Using a constant rate of change to complete a table of values

Fill in the table. Then write a sentence explaining how you figured out the values to put in each cell.

9. You run a business making birdhouses. You spend $600 to start your business, and it costs you $5.00 to make each birdhouse.

# of birdhouses	1	2	3	4	5	6	7
Total cost to build							

Explanation:

10. You make a $15 payment on your loan of $500 at the end of each month.

# of months	1	2	3	4	5	6	7
Amount of money owed							

Explanation:

11. You deposit $10 in a savings account at the end of each week.

# of weeks	1	2	3	4	5	6	7
Amount of money saved							

Explanation:

12. You are saving for a bike and can save $10 per week. You have $25 when you begin saving.

# of weeks	1	2	3	4	5	6	7
Amount of money saved							

Explanation:

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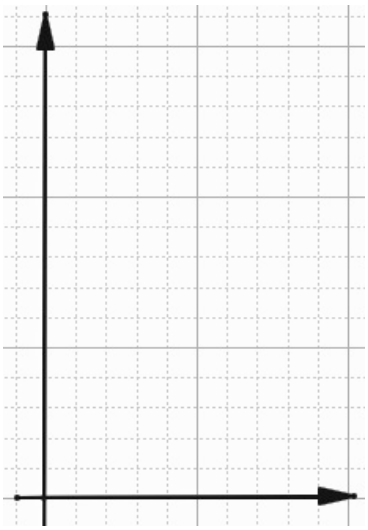
GO

Topic: Graph Linear Equations Given a Table of Values.

Graph the ordered pairs from the tables on the given graphs.

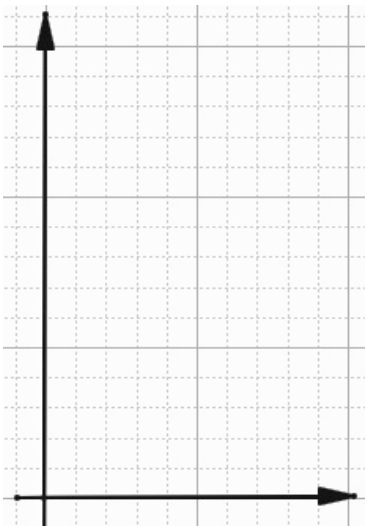
13.

x	y
0	3
2	7
3	9
5	13



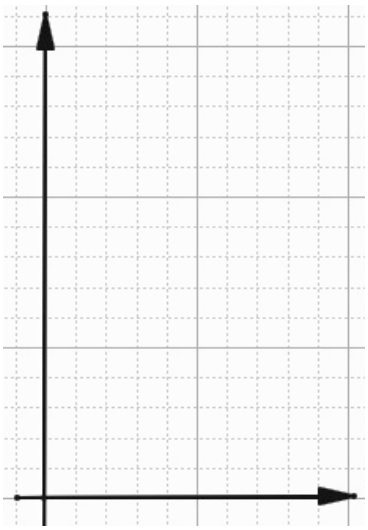
14.

x	y
0	14
4	10
7	7
9	5



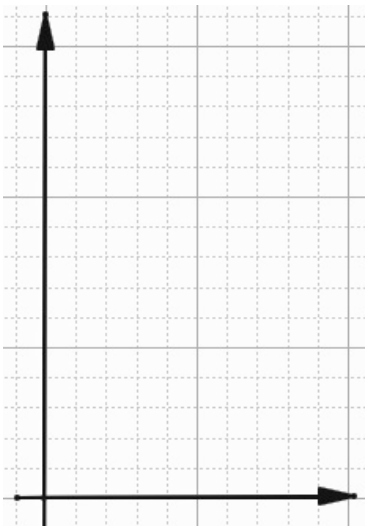
15.

x	y
2	11
4	10
6	9
8	8



16.

x	y
1	4
2	7
3	10
4	13



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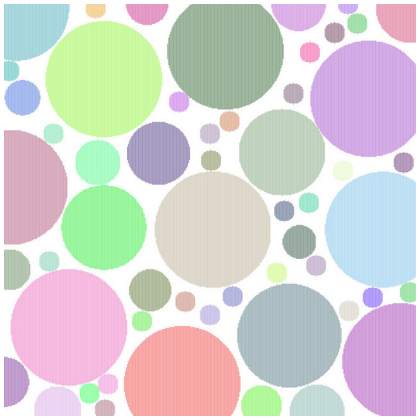
1.2 Growing Dots

A Develop Understanding Task



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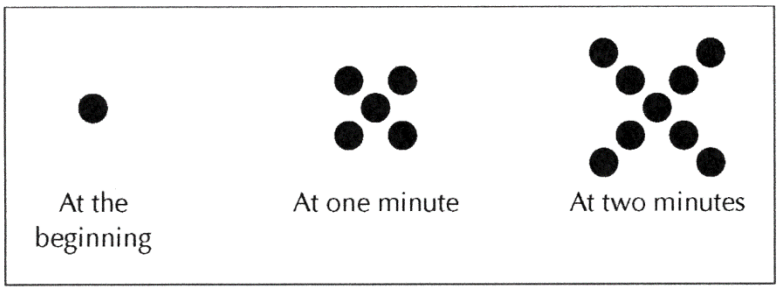

1. Describe the pattern that you see in the sequence of figures above.

2. Assuming the pattern continues in the same way, how many dots are there at 3 minutes?

3. How many dots are there at 100 minutes?

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4. How many dots are there at t minutes? Solve the problems by your preferred method. Your solution should indicate how many dots will be in the pattern at 3 minutes, 100 minutes, and t minutes. Be sure to show how your solution relates to the picture and how you arrived at your solution.

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1.2

READY, SET, GO! Name Period Date

READY

Topic: Using function notation

To **evaluate** an equation such as $y = 5x + 1$ when given a specific value for x, replace the variable x with the given value and work the problem to find the value of y.
Example: Find y when x = 2. Replace x with 2. $y = 5(2) + 1 = 10 + 1 = 11$.
Therefore, y = 11 when x = 2. The point $(2, 11)$ is one solution to the equation $y = 5x + 1$. Instead of using x *and* y in an equation, mathematicians often write $f(n) = 5n + 1$ because it can give more information. With this notation, the direction to find $f(2)$, means to replace the value of n with 2 and work the problem to find $f(n)$. The point $(n, f(n))$ is in the same location on the graph as (x, y), where n describes the location along the x–axis, and $f(n)$ is the height of the graph.

Given that $f(n) = 8n - 3$ and $g(n) = 3n - 10$, evaluate the following functions with the indicated values.

1. $f(5) =$
2. $g(5) =$
3. $f(-4) =$
4. $g(-4) =$
5. $f(0) =$
6. $g(0) =$
7. $f(1) =$
8. $g(1) =$

Topic: Looking for patterns of change
Complete each table by looking for the pattern.

9.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	2	4	8	16	32			

10.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	66	50	34	18				

11.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	160	80	40	20				

12.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	-9	-2	5	12				

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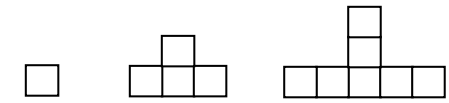
SET

Topic: Use variables to create equations that connect with visual patterns.

In the pictures below, each square represents one tile.



Step 1 Step 2 Step 3 Step 4 Step 5

13. Draw Step 4 and Step 5.

The students in a class were asked to find the number of tiles in a figure by describing how they saw the pattern of tiles changing at each step. Match each student's way of describing the pattern with the appropriate equation below. Note that *"s"* represents the step number and "*n*" represents the number of tiles.

(a) $\boldsymbol{n = (2s - 1) + (s - 1)}$ **(b)** $\boldsymbol{n = 3s - 2}$ **(c)** $\boldsymbol{n = s + 2(s - 1)}$

14. _____Dan explained that the middle "tower" is always the same as the step number. He also pointed out that the 2 arms on each side of the "tower" contain one less block than the step number.

15. _____ Sally counted the number of tiles at each step and made a table. She explained that the number of tiles in each figure was always 3 times the step number minus 2.

step number	1	2	3	4	5	6
number of tiles	1	4	7	10	13	16

16. _____ Nancy focused on the number of blocks in the base compared to the number of blocks above the base. She said the number of base blocks were the odd numbers starting at 1. And the number of tiles above the base followed the pattern 0, 1, 2, 3, 4. She organized her work in the table at the right.

Step number	# in base + #on top
1	1 + 0
2	3 + 1
3	5 + 2
4	7 + 3
5	9 + 4

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GO

Topic: The meaning of an exponent

Write each expression using an exponent.

17. $6\times6\times6\times6\times6$

18. $4\times4\times4$

19. $15\times15\times15\times15$

20. $\frac{1}{3}\times\frac{1}{3}$

A) Write each expression in expanded form. B) Then calculate the value of the expression.

21. 7^1

22. 3^2

23. 5^3

24. 10^4

25. $7(2)^3$

26. $10(8^2)$

27. $3(5)^4$

28. $16\left(\frac{1}{2}\right)^3$

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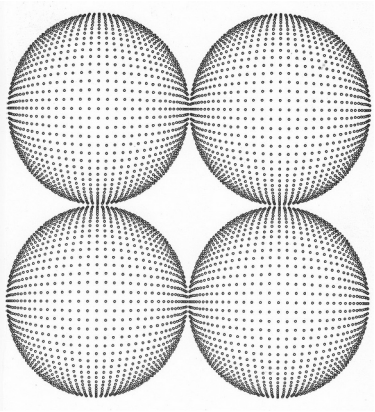
1.3 Growing, Growing Dots

A Develop Understanding Task



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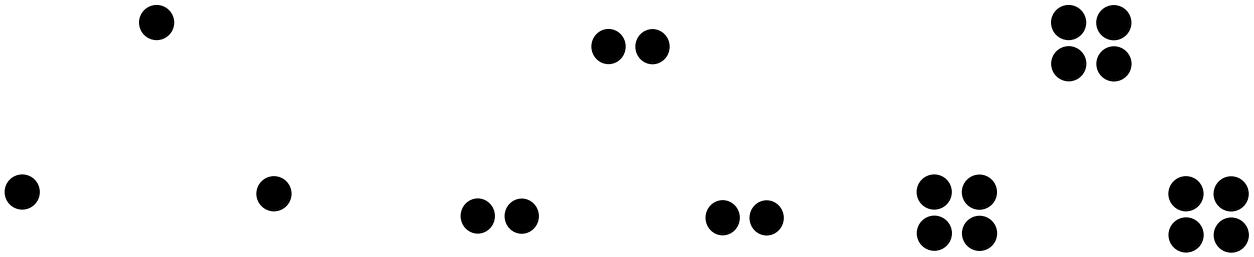
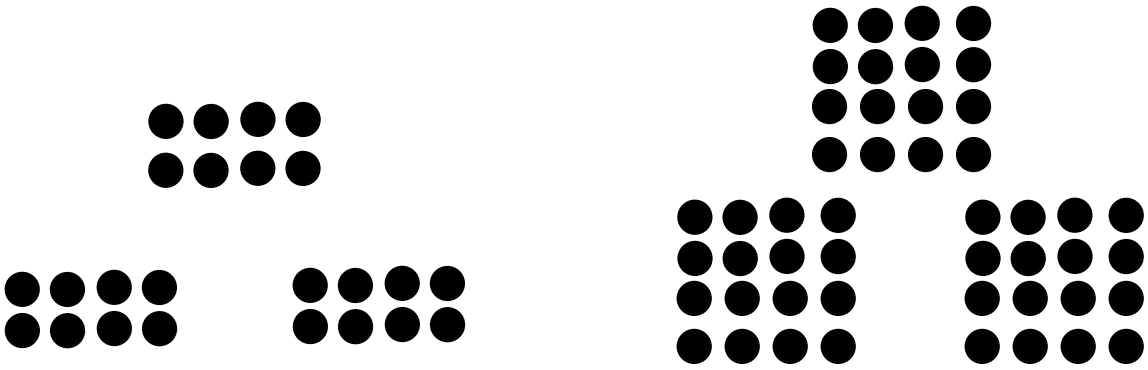


At the beginning

At one minute

At two minutes



At three minutes

At four minutes

1. Describe and label the pattern of change you see in the above sequence of figures.

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2. Assuming the sequence continues in the same way, how many dots are there at 5 minutes?

3. Write a recursive formula to describe how many dots there will be after t minutes.

4. Write an explicit formula to describe how many dots there will be after t minutes.

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READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Interpreting function notation

A) Use the given table to identify the indicated value for *n*. B) Then using the value for *n* that you determined in A, use the table to find the indicated value for B.

n	1	2	3	4	5	6	7	8	9	10
$f(n)$	-8	-3	2	7	12	17	22	27	32	37

1. $A)$ When $f(n) = 12$, what is the value of n?

   $B)$ What is the value of $f(n-1)$?

2. $A)$ When $f(n) = 17$, what is the value of n?

   $B)$ What is the value of $f(n-1)$?

3. $A)$ When $f(n) = 32$, what is the value of n?

   $B)$ What is the value of $f(n+1)$?

4. $A)$ When $f(n) = 2$, what is the value of n?

   $B)$ What is the value of $f(n+3)$?

5. $A)$ When $f(n) = 27$, what is the value of n?

   $B)$ What is the value of $f(n-6)$?

6. $A)$ When $f(n) = -8$, what is the value of n?

   $B)$ What is the value of $f(n+9)$?

SET

Topic: Comparing explicit and recursive equations

Use the given information to decide which equation will be the easiest to use to find the indicated value. Find the value and explain your choice.

7. Explicit equation: y = 3x + 7
   Recursive: $now = previous\ term + 3$

term #	1	2	3	4
value	10	13	16	

Find the value of the 4th term. _____
Explanation:

8. Explicit equation: y = 3x + 7
   Recursive: $now = previous\ term + 3$

term #	1	2	...	50
value	10	13	...	

Find the value of the 50th term._____
Explanation:

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9. The value of the 8th term is 78. The sequence is increasing by 10 at each step. Explicit equation: y = 10x – 2 Recursive: $now = previous\ term + 10$ Find the 20th term. _____ Explanation:	10. The value of the 8th term is 78. The sequence is increasing by 10 at each step. Explicit equation: y = 10x – 2 Recursive: $now = previous\ term + 10$ Find the 9th term. _____ Explanation:
11. The value of the 4th term is 80. The sequence is being doubled at each step. Explicit equation: $y = 5(2^x)$ Recursive: $now = previous\ term * 2$ Find the value of the 5th term. _____ Explanation:	12. The value of the 4th term is 80. The sequence is being doubled at each step. Explicit equation: $y = 5(2^x)$ Recursive: : $now = previous\ term * 2$ Find the value of the 7th term. _____ Explanation:

GO

Topic: Evaluating Exponential Equations

Evaluate the following equations when x = { 1, 2, 3, 4, 5 }. Organize your inputs and outputs into a table of values for each equation. Let x be the input and y be the output.

13. $y = 4^x$

x input	*y output*
1	
2	
3	
4	
5	

14 $y = (-3)^x$

x input	*y output*
1	
2	
3	
4	
5	

15. $y = -3^x$

x input	*y output*
1	
2	
3	
4	
5	

16. $y = 10^x$

x input	*y output*
1	
2	
3	
4	
5	

17. If $f(n) = 5^n$, *what is the value of* $f(4)$?

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1.4 Scott's Workout

A Solidify Understanding Task

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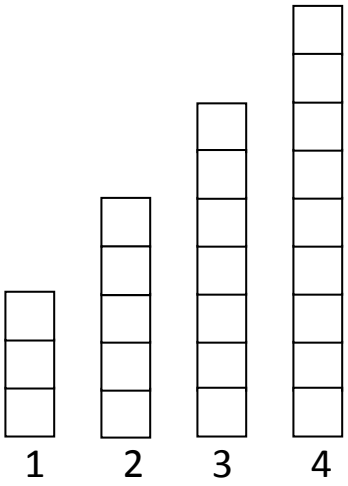
Scott has decided to add push-ups to his daily exercise routine. He is keeping track of the number of push-ups he completes each day in the bar graph below, with day one showing he completed three push-ups. After four days, Scott is certain he can continue this pattern of increasing the number of push-ups he completes each day.



1. How many push-ups will Scott do on day 10?

2. How many push-ups will Scott do on day n?

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3. Model the number of push-ups Scott will complete on any given day. Include both explicit and recursive equations.

4. Aly is also including push-ups in her workout and says she does more push-ups than Scott because she does fifteen push-ups every day. Is she correct? Explain.

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READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Use function notation to evaluate equations.

Evaluate the given equation for the indicated function values.

1. $f(n) = 5n + 8$
   $f(4) =$
   $f(-2) =$

2. $f(n) = -2n + 1$
   $f(10) =$
   $f(-1) =$

3. $f(n) = 6n - 3$
   $f(-5) =$
   $f(0) =$

4. $f(n) = -n$
   $f(9) =$
   $f(-11) =$

5. $f(n) = 5^n$
   $f(2) =$
   $f(3) =$

6. $f(n) = 3^n$
   $f(4) =$
   $f(1) =$

7. $f(n) = 10^n$
   $f(6) =$
   $f(0) =$

8. $f(n) = 2^n$
   $f(0) =$
   $f(5) =$

SET

Topic: Finding terms for a given sequence

Find the next 3 terms in each sequence. Identify the constant difference. Write a recursive function and an explicit function for each sequence. Circle where you see the common difference in both functions. (The first number is the 1st term, not the 0th term).

9. A) 3 , 8 , 13 , 18 , 23 , _____ , _____ , _____ , …
   B) Common Difference: _____
   C) Recursive Function: _____
   D) Explicit Function:_____

10. A) 11 , 9 , 7 , 5 , 3 , _____ , _____ , _____ , …
    B) Common Difference: _____
    C) Recursive Function: _____
    D) Explicit Function:_____

11. A) 3 , 1.5 , 0 , -1.5 , -3 , _____ , _____ , _____ , …
    B) Common Difference: _____
    C) Recursive Function: _____
    D) Explicit Function:_____

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GO

Topic: Reading a graph

Olaf is a mountain climber. The graph shows Olaf's location on the mountain beginning at noon. Use the information in the graph to answer the following questions.

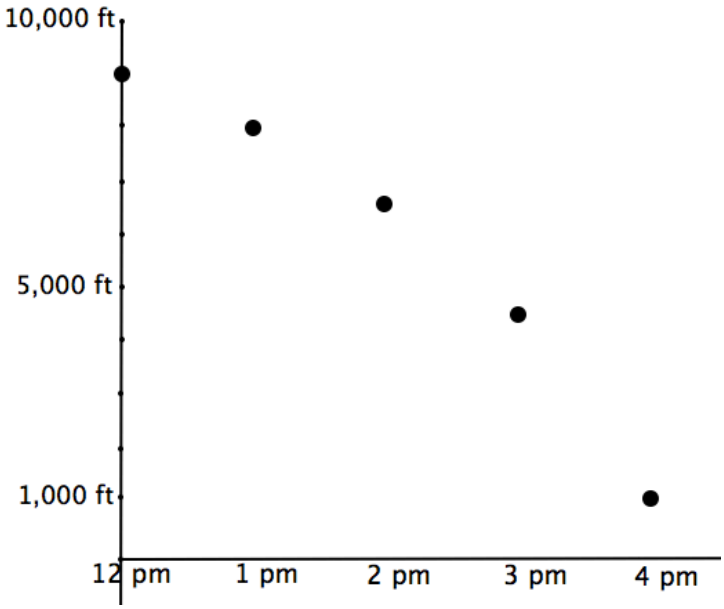
12. What was Olaf's elevation at noon?

13. What was his elevation at 2 pm?

14. How many feet had Olaf descended from noon until 2 pm?

15. Olaf reached the base camp at 4 pm. What is the elevation of the base camp?

16. During which hour was Olaf descending the mountain the fastest? Explain how you know.



17. Is the value of $f(n)$ the time or the elevation?

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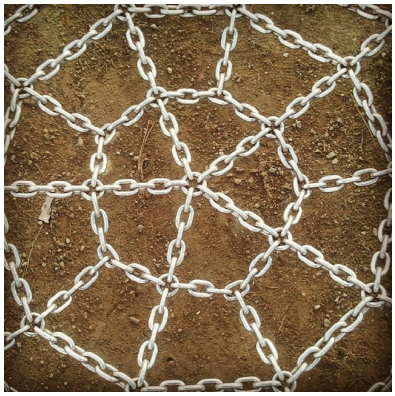
1.5 Don't Break the Chain

A Solidify Understanding Task



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Maybe you've received an email like this before:

> Hi! My name is Bill Weights, founder of Super Scooper Ice Cream. I am offering you a gift certificate for our signature "Super Bowl" (a $4.95 value) if you forward this letter to 10 people.
>
> When you have finished sending this letter to 10 people, a screen will come up. It will be your Super Bowl gift certificate. Print that screen out and bring it to your local Super Scooper Ice Cream store. The server will bring you the most wonderful ice cream creation in the world—a Super Bowl with three yummy ice cream flavors and three toppings!
>
> This is a sales promotion to get our name out to young people around the country. We believe this project can be a success, but only with your help. Thank you for your support.
>
> Sincerely,
>
> Bill Weights
> Founder of Super Scooper Ice Cream

These chain emails rely on each person that receives the email to forward it on. Have you ever wondered how many people might receive the email if the chain remains unbroken? To figure this out, assume that it takes a day for the email to be opened, forwarded, and then received by the next person. On day 1, Bill Weights starts by sending the email out to his 8 closest friends. They each forward it to 10 people so that on day 2 it is received by 80 people. The chain continues unbroken.

1. How many people will receive the email on day 7?

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2. How many people with receive the email on day n? Explain your answer with as many representations as possible.

3. If Bill gives away a Super Bowl that costs $4.95 to every person that receives the email during the first week, how much will he have spent?

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READY, SET, GO! **Name** **Period** **Date**

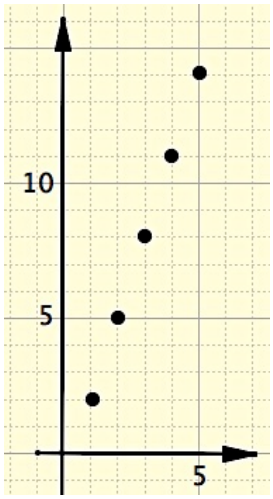
READY

Topic: Rates of change in a table and a graph

The same sequence is shown in both a table and a graph. Indicate on the table where you see the rate of change of the sequence. Then draw on the graph where you see the rate of change.

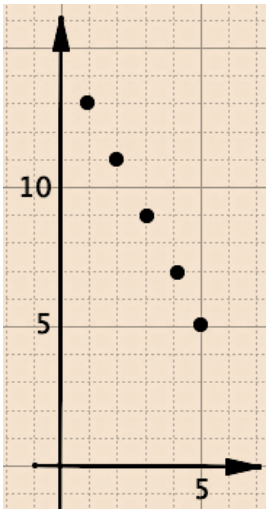
1.

n	$f(n)$
1	2
2	5
3	8
4	11
5	14



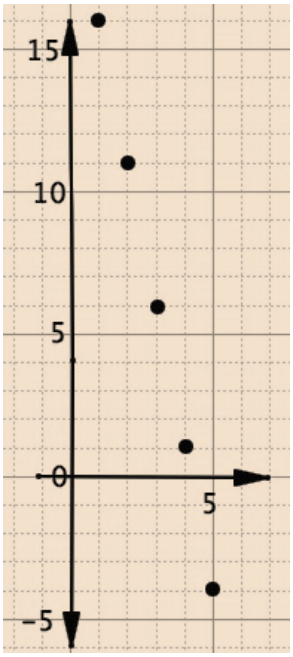
2.

n	$f(n)$
1	13
2	11
3	9
4	7
5	5



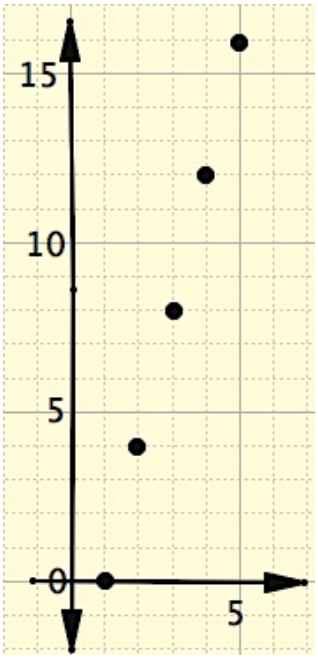
3.

n	$f(n)$
1	16
2	11
3	6
4	1
5	-4



4.

n	$f(n)$
1	0
2	4
3	8
4	12
5	16



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SET

Topic: Recursive and explicit functions of **geometric** sequences

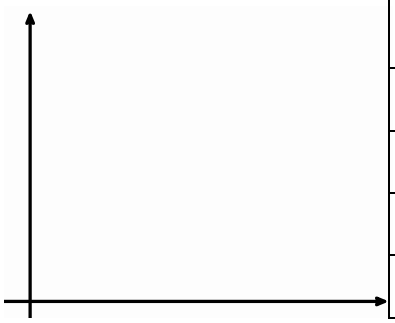
Below you are given various types of information. Write the recursive and explicit functions for each geometric sequence. Finally, graph each sequence, making sure you clearly label your axes.

5. 2 , 4 , 8 , 16 , …

Recursive:_____

Explicit:_____

6.

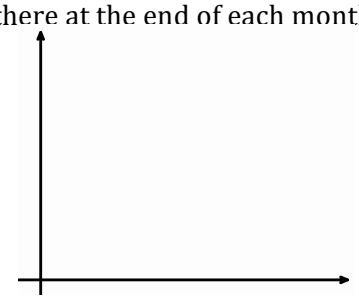


Time (days)	Number of cells
1	3
2	6
3	12
4	24

Recursive:_____

Explicit:_____

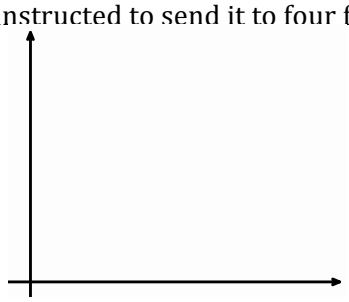
7. Claire has $300 in an account. She decides she is going to take out half of what's left in there at the end of each month.



Recursive:_____

Explicit:_____

8. Tania creates a chain letter and sends it to four friends. Each day each friend is then instructed to send it to four friends and so forth.



Recursive:_____

Explicit:_____

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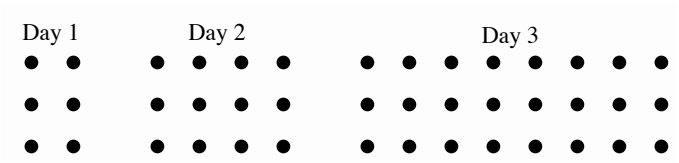
9.



Recursive:_____

Explicit:_____

GO

Topic: Recursive and explicit functions of **arithmetic** sequences

Below you are given various types of information. Write the recursive and explicit functions for each arithmetic sequence. Finally, graph each sequence, making sure you clearly label your axes.

10. 2 , 4 , 6 , 8 , …

Recursive:_____

Explicit:_____

11.

Time (days)	Number of cells
1	3
2	6
3	9
4	12

Recursive:_____

Explicit:_____

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12. Claire has $300 in an account. She decides she is going to take out $25 each month.

Recursive:_____

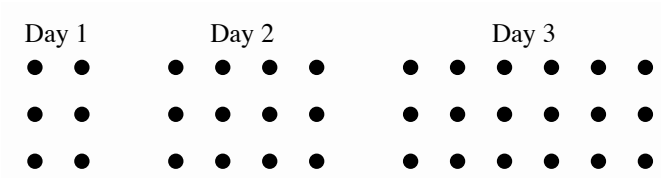
Explicit:_____

13. Each day Tania decides to do something nice for 2 strangers. What is the relationship between the number people helped and days?

Recursive:_____

Explicit:_____

14.



Recursive:_____

Explicit:_____

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1.6 Something to Chew On

A Solidify Understanding Task

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The Food-Mart grocery store has a candy machine like the one pictured here. Each time a child inserts a quarter, 7 candies come out of the machine. The machine holds 15 pounds of candy. Each pound of candy contains about 180 individual candies.

1. Represent the number of candies in the machine for any given number of customers. About how many customers will there be before the machine is empty?

2. Represent the amount of money in the machine for any given number of customers.

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3. To avoid theft, the store owners don't want to let too much money collect in the machine, so they take all the money out when they think the machine has about $25 in it. The tricky part is that the store owners can't tell how much money is actually in the machine without opening it up, so they choose when to remove the money by judging how many candies are left in the machine. About how full should the machine look when they take the money out? How do you know?

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READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Finding the common difference

Find the missing terms for each arithmetic sequence and state the common difference.

1. 5, 11, _____ , 23, 29, _____...

   Common Difference = _____

2. 7, 3, -1, _____ , _____ , -13...

   Common Difference = _____

3. 8, _____ , _____ , 47, 60...

   Common Difference = _____

4. 0, _____ , _____ , 2, $\frac{8}{3}$ ...

   Common Difference = _____

5. 5, _____ , _____ , _____ , 25...

   Common Difference = _____

6. 3, _____ , _____ , _____ , -13 ...

   Common Difference = _____

SET

Topic: Writing the recursive function

Two consecutive terms in an arithmetic sequence are given. Find the recursive function.

7. If $f(3) = 5$ *and* $f(4) = 8$ ...

   $f(5)$ = _____. $f(6)$ = _____. Recursive Function: _____

8. If $f(2) = 20$ *and* $f(3) = 12$ ...

   $f(4)$ = _____. $f(5)$ = _____. Recursive Function: _____

9. If $f(5) = 3.7$ *and* $f(6) = 8.7$ ...

   $f(7)$ = _____. $f(8)$ = _____. Recursive Function: _____

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Two consecutive terms in a <u>geometric</u> sequence are given. Find the recursive function.

10. If $f(3) = 5$ *and* $f(4) = 10$ …

$f(5)$ = _____. $f(6)$ = _____. Recursive Function: _____

11. If $f(2) = 20$ *and* $f(3) = 10$ …

$f(4)$ = _____. $f(5)$ = _____. Recursive Function: _____

12. If $f(5) = 20.58$ *and* $f(6) = 2.94$ …

$f(7)$ = _____. $f(8)$ = _____. Recursive Function: _____

GO

Topic: Evaluating using function notation

Find the indicated values of $f(n)$.

13. $f(n) = 2^n$ Find $f(5)$ *and* $f(0)$.

14. $f(n) = 5^n$ Find $f(4)$ *and* $f(1)$.

15. $f(n) = (-2)^n$ Find $f(3)$ *and* $f(0)$.

16. $f(n) = -2^n$ Find $f(3)$ *and* $f(0)$.

17. In what way are the problems in #15 and #16 different?

18. $f(n) = 3 + 4(n - 1)$ Find $f(5)$ and $f(0)$.

19. $f(n) = 2(n - 1) + 6$ Find $f(1)$ and $f(6)$.

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1.7 Chew On This

A Solidify Understanding Task

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Mr. and Mrs. Gloop want their son, Augustus, to do his homework every day. Augustus loves to eat candy, so his parents have decided to motivate him to do his homework by giving him candies for each day that the homework is complete. Mr. Gloop says that on the first day that Augustus turns in his homework, he will give him 10 candies. On the second day he promises to give 20 candies, on the third day he will give 30 candies, and so on.

1. Write both a recursive and an explicit formula that shows the number of candies that Augustus earns on any given day with his father's plan.

2. Use a formula to find how many candies Augustus will get on day 30 in this plan.

Augustus looks in the mirror and decides that he is gaining weight. He is afraid that all that candy will just make it worse, so he tells his parents that it would be ok if they just give him 1 candy on the first day, 2 on the second day, continuing to double the amount each day as he completes his homework. Mr. and Mrs. Gloop like Augustus' plan and agree to it.

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3. Model the amount of candy that Augustus would get each day he reaches his goals with the new plan.

4. Use your model to predict the number of candies that Augustus would earn on the 30th day with this plan.

5. Write both a recursive and an explicit formula that shows the number of candies that Augustus earns on any given day with this plan.

Augustus is generally selfish and somewhat unpopular at school. He decides that he could improve his image by sharing his candy with everyone at school. When he has a pile of 100,000 candies, he generously plans to give away 60% of the candies that are in the pile each day. Although Augustus

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may be earning more candies for doing his homework, he is only giving away candies from the pile that started with 100,000. (He's not that generous.)

6. How many pieces of candy will be left on day 4? On day 8?

7. Model the amount of candy that would be left in the pile each day.

8. How many days will it take for the candy to be gone?

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READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Distinguishing between arithmetic and geometric sequences

Find the missing values for each arithmetic or geometric sequence. Underline whether it has a constant difference or a constant ratio. State the value of the constant difference or ratio. Indicate if the sequence is arithmetic or geometric by circling the correct answer.

1. 5, 10, 15, ___ , 25, 30, ___...

Common difference or ratio?

Common Difference/ratio = _____

Arithmetic or geometric?

2. 20, 10, ___ , 2.5, ___,...

Common difference or ratio?

Common Difference/ratio = _____

Arithmetic or geometric?

3. 2, 5, 8, ___ , 14, ___, ...

Common difference or ratio?

Common Difference/ratio = _____

Arithmetic or geometric?

4. 30, 24 , ___ , 12, 6, ...

Common difference or ratio?

Common Difference/ratio = _____

Arithmetic or geometric?

SET

Topic: Recursive and explicit equations

Determine whether the given information represents an arithmetic or geometric sequence. Then write the recursive and the explicit equation for each.

5. 2, 4, 6, 8, ...

Arithmetic or geometric?

Recursive:
_____

Explicit:
_____

6. 2, 4, 8, 16, ...

Arithmetic or geometric?

Recursive:
_____

Explicit:
_____

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7.

Time (in days)	Number of dots
1	3
2	7
3	11
4	15

Arithmetic or geometric?

Recursive:

_____

Explicit:

_____

8.

Time (in days)	Number of cells
1	5
2	8
3	12.8
4	20.48

Arithmetic or geometric?

Recursive:

_____

Explicit:

_____

9. Michelle likes chocolate but it causes acne. She chooses to limit herself to three chocolate bars every 5 days. (So, she eats part of a bar each day.)

Arithmetic or geometric?

Recursive:

_____

Explicit:

_____

10. Scott decides to add running to his exercise routine and runs a total of one mile his first week. He plans to double the number of miles he runs each week.

Arithmetic or geometric?

Recursive:

_____

Explicit:

_____

11. Vanessa has $60 to spend on rides at the state fair. Each ride costs $4.

Arithmetic or geometric?

Recursive:

_____

Explicit:

_____

12. Cami invested $6,000 into an account that earns 10% interest each year. (Hint: Make a table of values to help yourself.)
Arithmetic or geometric?

Recursive:

_____

Explicit:

_____

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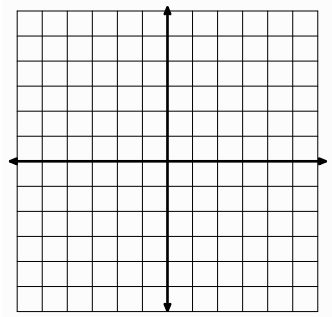
GO

Topic: Graphing and counting slope between two points.

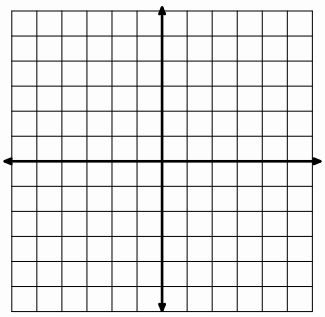
For the following problems two points and a slope are given. Plot and label the 2 points on the graph. Draw the line segment between them. Then sketch on the graph how you count the slope of the line by moving up or down and then sideways from one point to the other.

13. A(2 , -1) and B(4 , 2)



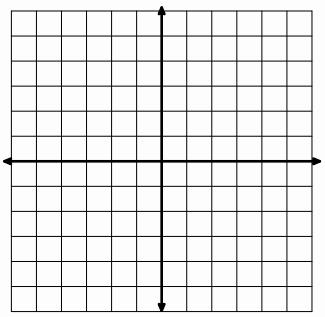
Slope: $m = \frac{3}{2}$

14. H(-2 , 1) and K(2 , 5)



Slope: $m = 1 \; or \; \frac{1}{1}$

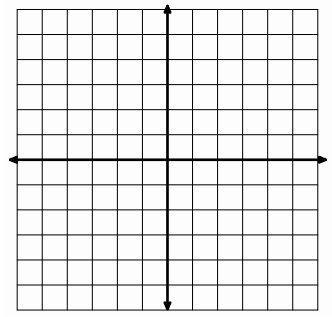
15. P(0 , 0) and Q(3 , 6)



Slope: $m = 2 \; or \; \frac{2}{1}$

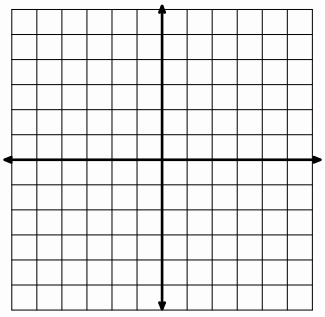
For the following problems, two points are given. Plot and label these points on the graph. Then count the slope.

16. C(-3 , 0) and D(0 , 5)



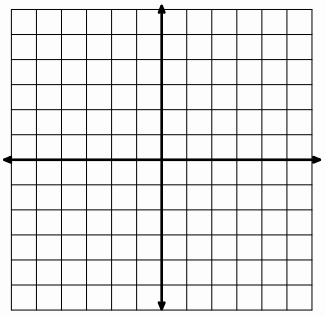
Slope: $m =$

17. E(-2 , -1) and N(-4 , 4)



Slope: $m =$

18. S(0 , 3) and W(1 , 6)



Slope: $m =$

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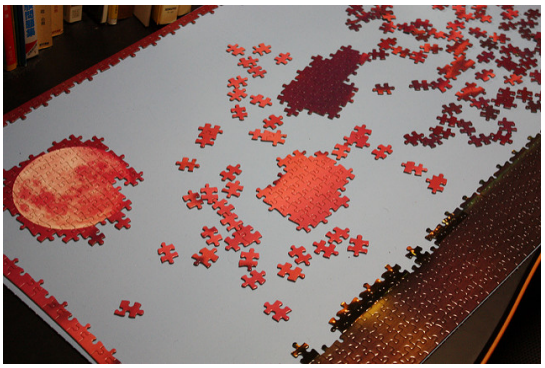
1.8 What Comes Next? What Comes Later?



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A Practice Understanding Task

For each of the following tables,

- describe how to find the next term in the sequence,
- write a recursive rule for the function,
- describe how the features identified in the recursive rule can be used to write an explicit rule for the function, and
- write an explicit rule for the function.
- identify if the function is arithmetic, geometric or neither

Example:

x	y
0	5
1	8
2	11
3	14
4	?
…	…
n	?

- To find the next term: add 3 to the previous term
- Recursive rule: $f(0) = 5, f(n) = f(n-1) + 3$
- To find the n^{th} term: start with 5 and add 3 n times
- Explicit rule: $f(n) = 5 + 3n$
- Arithmetic , geometric, or neither? Arithmetic

Function A

x	y
1	5
2	10
3	20
4	40
5	?
…	…
n	?

1. How to find the next term: _____
2. Recursive rule: _____
3. To find the n^{th} term: _____
4. Explicit rule: _____
5. Arithmetic, geometric, or neither? _____

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Function B

6. How to find the next term: _____

7. Recursive rule: _____

8. To find the n^{th} term: _____

9. Explicit rule: _____

10. Arithmetic, geometric, or neither? _____

x	y
1	-8
2	-17
3	-26
4	-35
5	-44
6	-53
…	…
n	

Function C

11. To find the next term: _____

12. Recursive rule: _____

13. To find the n^{th} term: _____

14. Explicit rule: _____

15. Arithmetic, geometric, or neither? _____

x	y
1	2
2	6
3	18
4	54
5	162
6	486
…	…
n	

Function D

16. To find the next term: _____

17. Recursive rule: _____

18. To find the n^{th} term: _____

19. Explicit rule: _____

20. Arithmetic, geometric, or neither? _____

x	***y***
1	3
2	15
3	27
4	39
5	51
6	?
…	…
n	?

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Function E

21. To find the next term: _____

22. Recursive rule: _____

23. To find the n^{th} term: _____

24. Explicit rule: _____

25. Arithmetic, geometric, or neither? _____

x	y
0	1
1	$1\frac{3}{5}$
2	$2\frac{1}{5}$
3	$2\frac{4}{5}$
4	$3\frac{2}{5}$
5	4
…	…
n	

Function F

26. To find the next term: _____

27. Recursive rule: _____

28. To find the n^{th} term: _____

29. Explicit rule: _____

30. Arithmetic, geometric, or neither? _____

x	y
0	3
1	4
2	7
3	12
4	19
5	?
…	…
n	?

Function G

31. To find the next term: _____

32. Recursive rule: _____

33. To find the n^{th} term: _____

34. Explicit rule: _____

35. Arithmetic, geometric, or neither? _____

x	y
1	10
2	2
3	$\frac{2}{5}$
4	$\frac{2}{25}$
5	$\frac{2}{125}$
6	$\frac{2}{625}$
…	…
n	

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Function H

36. To find the next term: _____

37. Recursive rule: _____

38. To find the n^{th} term: _____

39. Explicit rule: _____

40. Arithmetic, geometric, or neither? _____

x	y
1	-1
2	0.2
3	-0.04
4	0.008
5	-0.0016
6	0.00032
...	...
n	

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READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Common Ratios

Find the common ratio for each geometric sequence.

1. 2, 4, 8, 16…

2. $\frac{1}{2}$, 1, 2, 4, 8…

3. -5, 10, -20, 40…

4. 10, 5, 2.5, 1.25…

SET

Topic: Recursive and explicit equations

Fill in the blanks for each table; then write the recursive and explicit equation for each sequence.

5. Table 1

x	1	2	3	4	5
y	5	7	9		

Recursive: _____ Explicit: _____

6. Table 2

x	y
1	-2
2	-4
3	-6
4	
5	

Recursive:

Explicit:

7. Table 3

x	y
1	3
2	9
3	27
4	
5	

Recursive:

Explicit

8. Table 4

x	y
1	27
2	9
3	3
4	
5	

Recursive:

Explicit:

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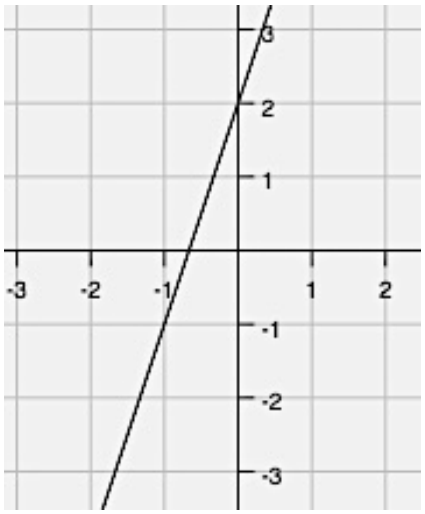
GO

Topic: Writing equations of lines given a graph.

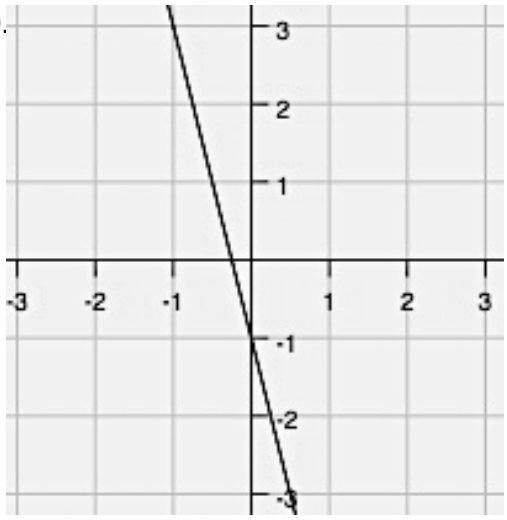
Write each equation of the line in $y = mx + b$ form. Name the value of m and b. Recall that m is the slope or rate of change and b is the y-intercept.

9.



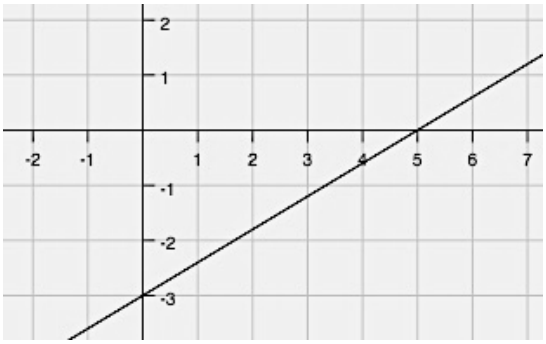
$m =$ $b =$ Equation:

10.



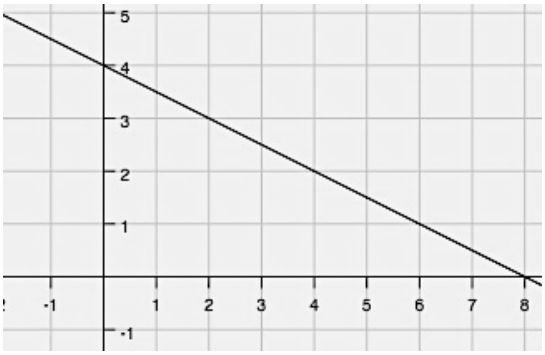
$m =$ $b =$ Equation:

11.



$m =$ $b =$ Equation:

12.



$m =$ $b =$ Equation:

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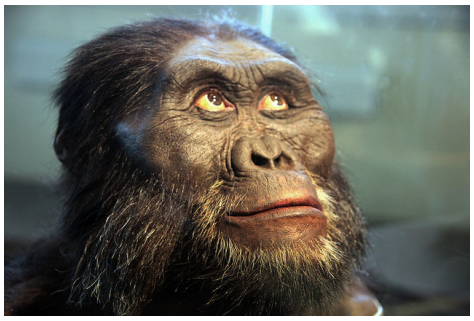
1.9 What Does It *Mean*?

A Solidify Understanding Task



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Each of the tables below represents an arithmetic sequence. Find the missing terms in the sequence, showing your method.

1.

x	1	2	3
y	5		11

2.

x	1	2	3	4	5
y	18				-10

3.

x	1	2	3	4	5	6	7
y	12						-6

4. Describe your method for finding the missing terms. Will the method always work? How do you know?

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Here are a few more arithmetic sequences with missing terms. Complete each table, either using the method you developed previously or by finding a new method.

5.

x	1	2	3	4
y	50			86

6.

x	1	2	3	4	5	6
y	40					10

7.

x	1	2	3	4	5	6	7	8
y	-23							5

8. The missing terms in an arithmetic sequence are called "arithmetic means". For example, in the problem above, you might say, "Find the 6 arithmetic means between -23 and 5". Describe a method that will work to find arithmetic means and explain why this method works.

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READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Comparing arithmetic and geometric sequences

1. How are arithmetic and geometric sequences similar?

2. How are they different?

SET

Topic: Finding missing terms in an Arithmetic sequence

Each of the tables below represents an arithmetic sequence. Find the missing terms in the sequence, showing your method.

3. Table 1

x	1	2	3
y	3		12

4. Table 2

x	y
1	2
2	
3	
4	26

5. Table 3

x	y
1	24
2	
3	6
4	

6. Table 4

x	y
1	16
2	
3	
4	4
5	

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GO

Topic: Sequences

Determine the recursive and explicit equations for each. (if the sequence is not arithmetic or geometric, identify it as neither and don't write the equations).

7. 5, 9, 13, 17,... This sequence is: Arithmetic , Geometric , Neither

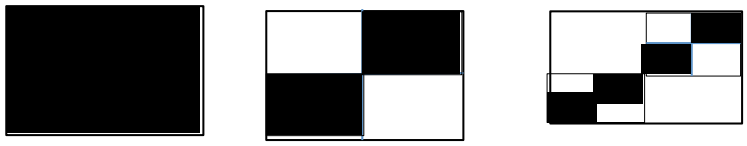
Recursive Equation: _____ Explicit Equation: _____

8. 60, 30, 0, -30 ,... This sequence is: Arithmetic , Geometric , Neither

Recursive Equation: _____ Explicit Equation: _____

9. 60, 30, 15, $\frac{15}{2}$, ,... This sequence is: Arithmetic , Geometric , Neither

Recursive Equation: _____ Explicit Equation: _____

10. 

(The number of black tiles above) This sequence is: Arithmetic , Geometric , Neither

Recursive Equation: _____ Explicit Equation: _____

11.. 4, 7, 12, 19, ,... This sequence is: Arithmetic , Geometric , Neither

Recursive Equation: _____ Explicit Equation: _____

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1.10 Geometric Meanies

A Practice Understanding Task

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Each of the tables below represents a geometric sequence. Find the missing terms in the sequence, showing your method.

Table 1

x	1	2	3
y	3		12

Is the missing term that you identified the only answer? Why or why not?

Table 2

x	1	2	3	4
y	7			875

Are the missing terms that you identified the only answers? Why or why not?

Table 3

x	1	2	3	4	5
y	6				96

Are the missing terms that you identified the only answers? Why or why not?

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Table 4

x	1	2	3	4	5	6
y	4					972

Are the missing terms that you identified the only answers? Why or why not?

A. Describe your method for finding the geometric means.

B. How can you tell if there will be more than one solution for the geometric means?

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READY

Topic: Arithmetic and geometric sequences

For each set of sequences, find the first five terms. Then compare the growth of the arithmetic sequence and the geometric sequence. Which grows faster? When?

1. Arithmetic sequence: $f(1) = 2$, common difference, $d = 3$
   Geometric sequence: $g(1) = 2$, common ratio, $r = 3$

Arithmetic	Geometric
$f(1) =$	$g(1) =$
$f(2) =$	$g(2) =$
$f(3) =$	$g(3) =$
$f(4) =$	$g(4) =$
$f(5) =$	$g(5) =$

a) Which value do you think will be more, $f(100)$ or $g(100)$? b) Why?

2. Arithmetic sequence: $f(1) = 2$, common difference, $d = 10$
   Geometric sequence: $g(1) = 128$, common ratio, $r = \frac{1}{2}$

Arithmetic	Geometric
$f(1) =$	$g(1) =$
$f(2) =$	$g(2) =$
$f(3) =$	$g(3) =$
$f(4) =$	$g(4) =$
$f(5) =$	$g(5) =$

a) Which value do you think will be more, $f(100)$ or $g(100)$? b) Why?

3. Arithmetic sequence: $f(1) = 20, d = 10$
   Geometric sequence: $g(1) = 2, r = 2$

Arithmetic	Geometric
$f(1) =$	$g(1) =$
$f(2) =$	$g(2) =$
$f(3) =$	$g(3) =$
$f(4) =$	$g(4) =$
$f(5) =$	$g(5) =$

a) Which value do you think will be more, $f(100)$ or $g(100)$? b) Why?

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4. Arithmetic sequence: $f(1) = 50$, common difference, $d = -10$
Geometric sequence: $g(1) = 1$, common ratio, $r = 2$

Arithmetic	Geometric
$f(1) =$	$g(1) =$
$f(2) =$	$g(2) =$
$f(3) =$	$g(3) =$
$f(4) =$	$g(4) =$
$f(5) =$	$g(5) =$

a) Which value do you think will be more, $f(100)$ or $g(100)$? b) Why?

5. Arithmetic sequence: $f(1) = 64$, common difference, $d = -2$
Geometric sequence: $g(1) = 64$, common ratio, $r = \frac{1}{2}$

Arithmetic	Geometric
$f(1) =$	$g(1) =$
$f(2) =$	$g(2) =$
$f(3) =$	$g(3) =$
$f(4) =$	$g(4) =$
$f(5) =$	$g(5) =$

a) Which value do you think will be more, $f(100)$ or $g(100)$? b) Why?

6. Considering arithmetic and geometric sequences, would there ever be a time that a geometric sequence does not out grow an arithmetic sequence in the long run as the number of terms of the sequences becomes really large? Explain.

SET

Topic: Finding missing terms in a geometric sequence

Each of the tables below represents a *geometric* sequence. Find the missing terms in the sequence. Show your method.

7. Table 1

x	1	2	3
y	3		12

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8. Table 2

x	y
1	2
2	
3	
4	54

9. Table 3

x	y
1	5
2	
3	20
4	

10. Table 4

x	y
1	4
2	
3	
4	
5	324

GO

Topic: Writing the explicit equations of a geometric sequence

Given the following information, determine the explicit equation for each geometric sequence.

11. $f(1) = 8, common\ ratio\ r = 2$

12. $f(1) = 4, f(n) = 3f(n-1)$

13. $f(n) = 4f(n-1);\ f(1) = \frac{5}{3}$

14. Which geometric sequence above has the greatest value at $f(100)$?

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1.11 I Know … What Do You Know?

A Practice Understanding Task

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In each of the problems below I share some of the information that I know about a sequence. Your job is to add all the things that you know about the sequence from the information that I have given. Depending on the sequence, some of the things you may be able to figure out for the sequence are:

- a table;
- a graph;
- an explicit equation;
- a recursive formula;
- the constant ratio or constant difference between consecutive terms;
- any terms that are missing;
- the type of sequence;
- a story context.

Try to find as many as you can for each sequence, but you must have at least 4 things for each.

1. I know that: the recursive formula for the sequence is $f(1) = -12,\ f(n) = f(n-1) + 4$
   What do you know?

2. I know that: the first 5 terms of the sequence are 0, -6, -12, -18, -24 . . .
   What do you know?

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3. I know that: the explicit formula for the sequence is $f(n) = -10(3)^n$

   What do you know?

4. I know that: The first 4 terms of the sequence are 2, 3, 4.5, 6.75 . . .

   What do you know?

5. I know that: the sequence is arithmetic and $f(3) = 10$ and $f(7) = 26$

   What do you know?

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6. I know that: the sequence is a model for the perimeter of the following figures:

Figure 1

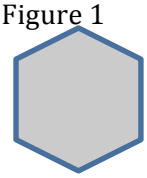


Figure 2

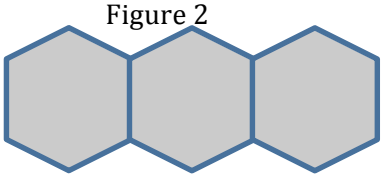
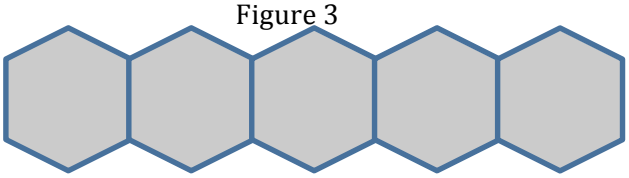


Figure 3



Length of each side = 1

What do you know?

7. I know that: it is a sequence where $f(1) = 5$ and the constant ratio between terms is -2.

What do you know?

8. I know that: the sequence models the value of a car that originally cost $26,500, but loses 10% of its value each year.

What do you know?

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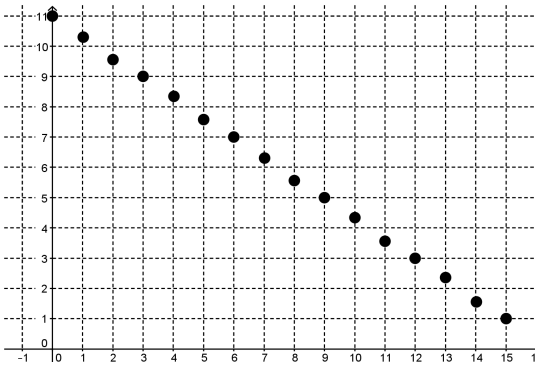
9. I know that: the first term of the sequence is -2, and the fifth term is $-\frac{1}{8}$.

   What do you know?

10. I know that: a graph of the sequence is:

    What do you know?



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READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Comparing linear equations and arithmetic sequences

1. Describe the similarities and differences between linear equations and arithmetic sequences.

Similarities	Differences

SET

Topic: Representations of arithmetic sequences

Use the given information to complete the other representations for each arithmetic sequence.

2. **Recursive Equation:**

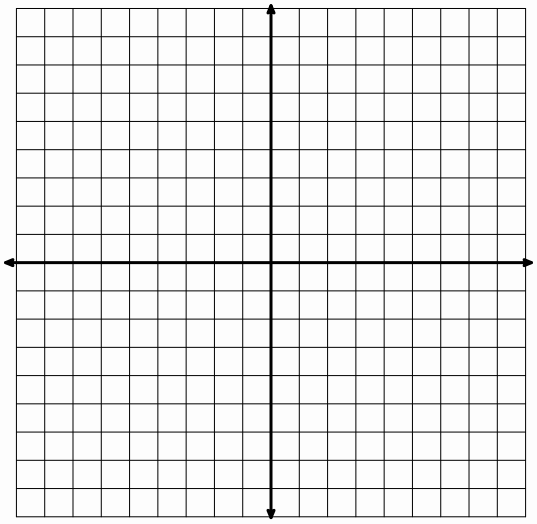
Explicit Equation:

Table

Days	Cost
1	8
2	16
3	24
4	32

Create a context

Graph



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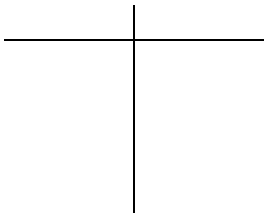
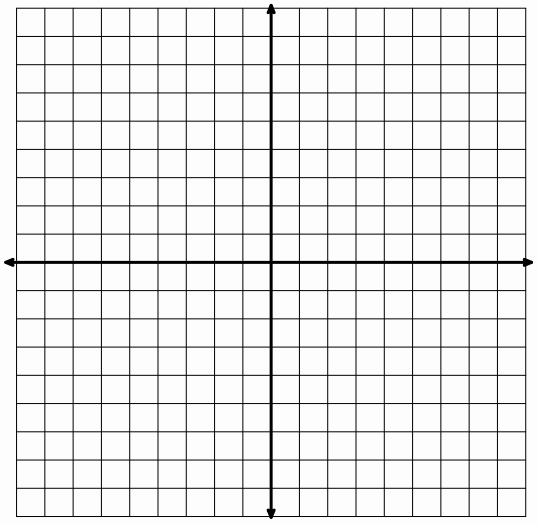
3. **Recursive Equation:** $f(1) = 4, \quad f(n) = f(n-1) + 3$

Explicit Equation:

Graph

Table





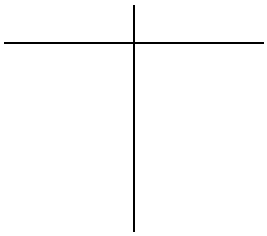
Create a context

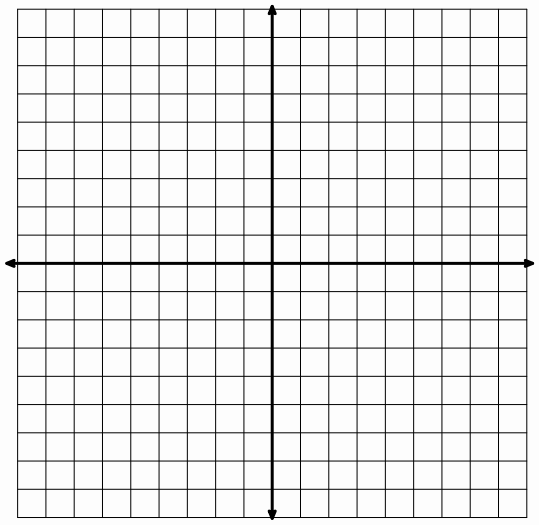
4. **Recursive Equation:**

Graph

Explicit Equation: $f(n) = 4 + 5(n-1)$

Table





Create a context

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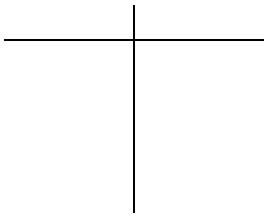
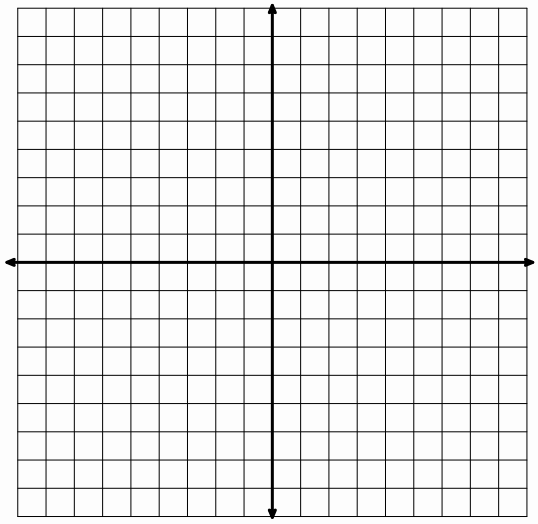
5. **Recursive Equation:** **Graph**

Explicit Equation:

Table





Create a context

Janet wants to know how many seats are in each row of the theater. Jamal lets her know that each row has 2 seats more than the row in front of it. The first row has 14 seats.

GO

Topic: Writing explicit equations

Given the recursive equation for each arithmetic sequence, write the explicit equation.

6. $f(n) = f(n-1) - 2; f(1) = 8$

7. $f(n) = 5 + f(n-1); f(1) = 0$

8. $f(n) = f(n-1) + 1;\ f(1) = \frac{5}{3}$

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