

Transforming Mathematics Education

# SECONDARY MATH ONE

An Integrated Approach

MODULE 3

# Features of Functions

MATHEMATICS VISION PROJECT. ORG

#### **The Mathematics Vision Project**

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# 3.1 Getting Ready for a Pool Party A Develop Understanding Task



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Sylvia has a small pool full of water that needs to be emptied and cleaned, then refilled for a pool party. During the process of getting the pool ready, Sylvia did all of the following activities, each during a different time interval.

|   | Filled the pool with a hose  |  |  |
|---|------------------------------|--|--|
| Removed water with a single bucket            | (same rate as emptying pool) |  |  |
| Drained water with a hose                     |                              |  |  |
| (same rate as filling pool)                   | Cleaned the empty pool       |  |  |
| Sylvia and her two friends removed water with |                              |  |  |
| her three buckets                             | Took a break                 |  |  |

- 1. Sketch a possible graph showing the height of the water level in the pool over time. Be sure to include all of activities Sylvia did to prepare the pool for the party. Remember that only one activity happened at a time. Think carefully about how each section of your graph will look, labeling where each activity occurs.
- 2. Create a story connecting Sylvia's process for emptying, cleaning, and then filling the pool to the graph you have created. Do your best to use appropriate math vocabulary.
- 3. Does your graph represent a function? Why or why not? Would all graphs created for this situation represent a function?



READY, SET, GO!

Name

Period

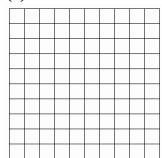
Date

#### **READY**

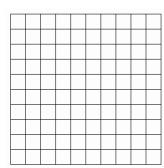
Topic: Graphing Linear and Exponential Functions

Graph each of the functions. Name 3 points that lie on each graph. Choose a scale for your graph that will make it possible to graph your 3 chosen points.

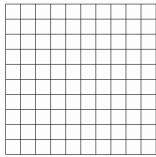
1. 
$$f(x) = -2x + 5$$



2. 
$$g(x) = 4 - 3x$$

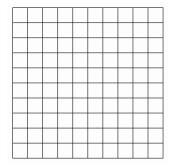


3. 
$$h(x) = 5(3)^x$$

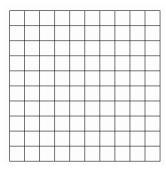


3 points:

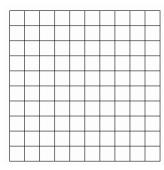
4. 
$$k(x) = 4(2)^x$$



5. 
$$v(t) = 2.5t - 4$$



6. 
$$f(x) = 8(3)^x$$



3 points:

3 points:

3 points:

SET

Topic: Describing attributes of a functions based on graphical representation

For each graph given match\_it to the contextual description that fits best. Then label the independent and dependent axis with the proper variables.

Graphs

7.



Contextual Descriptions

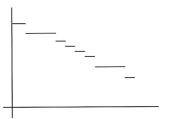
 a. The amount of money in a savings account where regular deposits and some withdrawals are made.

8.



b. The temperature of the oven on a day that mom bakes several batches of cookies.

9.



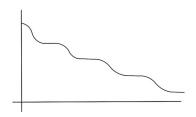
c. The amount of gasoline on hand at the gas station before a tanker truck delivers more.

10.



d. Watermelons are delivered to a farmer's market every Saturday morning. The number of watermelons available for sale on Thursday.

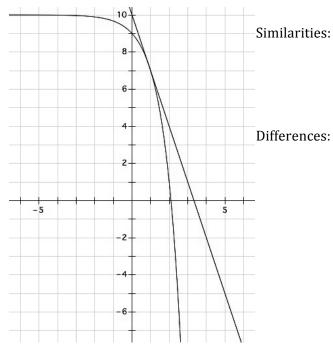
11.



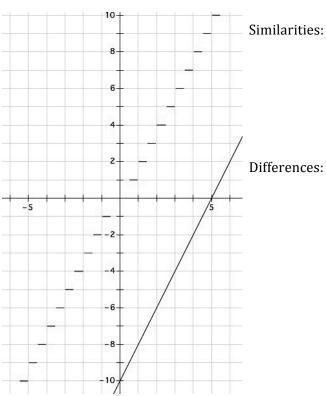
e. The amount of mileage recorded on the odometer of a delivery truck over a time period.

Given the pair of graphs on each coordinate grid, create a list of similarities the two graphs share and a list of differences. (Consider attributes like, continuous, discrete, increasing, decreasing, linear, exponential, restrictions on domain or range, etc.)

12.



13.



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3.1

GO

Topic: Solving equations

For each equation find the value of x that makes it true. (Hint for #20 and #22: when solving a linear equation, you need to get the term containing the variable alone on one side. When solving an exponential equation, you also need to get the term containing the variable alone on one side.)

14. 
$$10^x = 100,000$$

15. 
$$3x + 7 = 5x - 21$$

17. 
$$5x - 8 = 37$$

18. 
$$3^x = 81$$

$$19.3x - 12 = -4x + 23$$

20. 
$$10 = 2^x - 22$$

$$21. \ 243 = 8x + 3$$

22. 
$$5^x - 7 = 118$$

# 3.2 Floating Down the River A Solidify Understanding Task



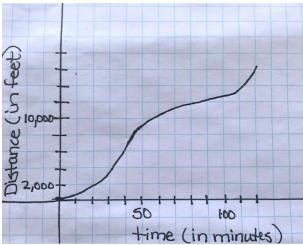
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Alonzo, Maria, and Sierra were floating in inner tubes down a river, enjoying their day. Alonzo noticed that sometimes the water level was higher in some places than in others. Maria noticed there were times they seemed to be moving faster than at other times. Sierra laughed and said "Math is everywhere!" To learn more about the river, Alonzo and Maria collected data throughout the trip.

| Time<br>(in minutes) | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
|----------------------|---|----|----|----|----|----|----|----|----|----|-----|-----|-----|
| Depth<br>(in feet)   | 4 | 6  | 8  | 10 | 6  | 5. | 4  | 5  | 7  | 12 | 9   | 6.5 | 5   |

1. Use the data collected by Alonzo to interpret the key features of this relationship.

Maria created a graph by collecting data on a GPS unit that told her the distance she had traveled over a period of time.



2. Using the graph created by Maria, describe the key features (increasing, decreasing, domain, range, maximum, minimum, intercepts) of this relationship.



#### Part II: Interpreting data

- 3. Sierra looked at the data collected by her two friends and made several of her own observations. Explain why you either agree or disagree with each observation made.
- a) The depth of the water increases and decreases throughout the 120 minutes of floating down the river.
- b) The distance traveled is always increasing.
- c) The distance traveled is a function of time.
- d) The distance traveled is greatest during the last ten minutes of the trip than during any other ten minute interval of time.
- e) The domain of the distance/time graph is all real numbers.
- f) The y-intercept of the depth of water over time function is (0,0).
- g) The distance traveled increases and decreases over time.
- h) The depth of the water is never 11 feet.
- i) The range of the distance/time graph is from [0, 15000].
- j) The domain of the depth of water with respect to time is from [0,120]
- k) The range of the depth of water over time is from [4,5].
- 1) The distance/ time graph has no maximum value.
- m) The depth of water reached a maximum at 30 minutes.

SECONDARY MATH I // MODULE 3

FEATURES OF FUNCTIONS - 3.2

READY, SET, GO!

Name

Period

Date

#### **READY**

Topic: Solve Linear Systems by Graphing

Graph each set of linear equations on the same set of axes. Name the coordinates of the point where the two lines intersect.

1. 
$$\begin{cases} f(x) = 2x - 7 \\ g(x) = -4x + 5 \end{cases}$$

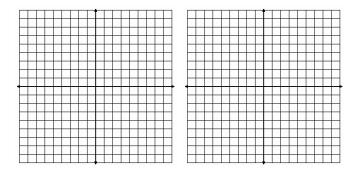
2. 
$$\begin{cases} f(x) = -5x - 2\\ g(x) = -2x + 1 \end{cases}$$
 3. 
$$\begin{cases} f(x) = -x - 2\\ g(x) = 2x + 10 \end{cases}$$

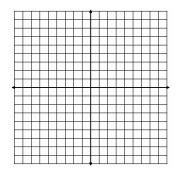
3. 
$$\begin{cases} f(x) = -x - 2 \\ g(x) = 2x + 10 \end{cases}$$

Point of intersection:

Point of intersection:

Point of intersection:





4. 
$$\begin{cases} f(x) = x - 5 \\ g(x) = -x + 1 \end{cases}$$

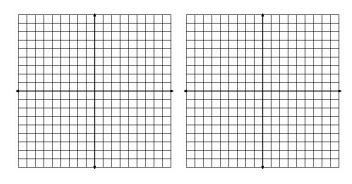
5. 
$$\begin{cases} f(x) = \frac{2}{3}x + 4 \\ g(x) = -\frac{1}{3}x + 1 \end{cases}$$
 6. 
$$\begin{cases} f(x) = x \\ g(x) = -x - 2 \end{cases}$$

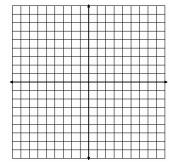
6. 
$$\begin{cases} f(x) = x \\ g(x) = -x - 2 \end{cases}$$

Point of intersection:

Point of intersection:

Point of intersection:



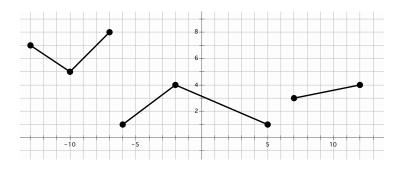


#### **SET**

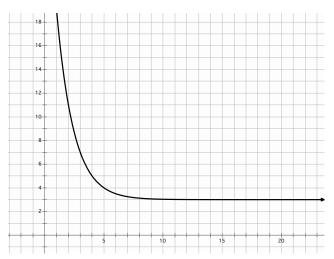
Topic: Describing attributes of a functions based on graphical representation

For each graph state 1)the interval(s) where it is increasing, decreasing, or constant 2)if it has a minimum or maximum, and 3)identify the domain and range. Use interval notation.

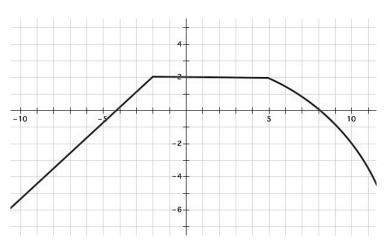
7. Description of function



8. Description of function



9. Description of function



GO

Topic: Creating both explicit and recursive equations

Write equations for the given tables in both recursive and explicit form.

10

| n | f(n) |
|---|------|
| 1 | 5    |
| 2 | 2    |
| 3 | -1   |

11.

| n | f(n) |
|---|------|
| 1 | 6    |
| 2 | 12   |
| 3 | 24   |

12.

| n | f(n) |
|---|------|
| 0 | -13  |
| 2 | -5   |
| 3 | -1   |

Explicit:

Explicit:

**Explicit:** 

Recursive:

Recursive:

Recursive:

13.

| n | f(n) |
|---|------|
| 1 | 5    |
| 4 | 11   |
| 5 | 13   |

14.

| n | f(n)    |
|---|---------|
| 2 | 5       |
| 7 | 15,625  |
| 9 | 390,625 |

15.

| n | f(n) |
|---|------|
| 0 | -4   |
| 1 | -16  |
| 2 | -64  |

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Explicit:

Explicit:

Explicit:

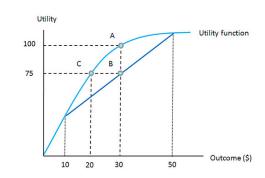
Recursive:

Recursive:

Recursive:

# 3.3 Features of Functions

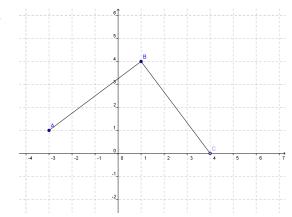
### A Practice Understanding Task



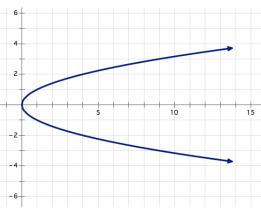
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For each graph, determine if the relationship represents a function, and if so, state the key features of the function (*key features include intercepts, intervals where the function is increasing or decreasing, relative maximums and minimums, symmetries, domain and range, and end behavior*).

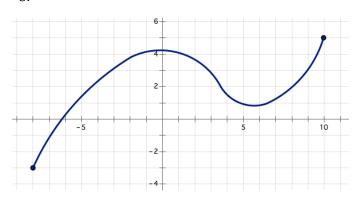
1.



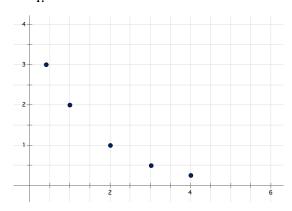
2.



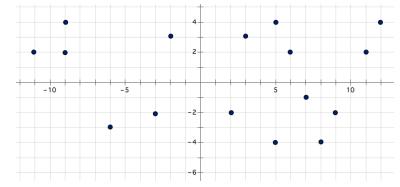
3.



4.



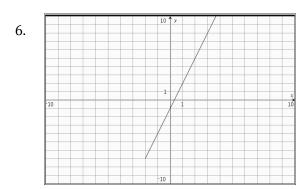
5.

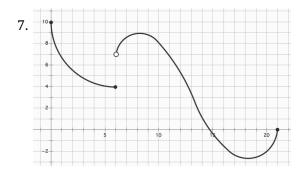


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- 8. The table on the right represents a continuous function defined on the interval from [0, 6].
  - a) Determine the domain, range, x and y intercepts.
  - b) Based on the table, identify the minimum value and where it is located.

| х | f(x) |
|---|------|
| 0 | 2    |
| 1 | -3   |
| 2 | 0    |
| 3 | 2    |
| 4 | 6    |
| 5 | 12   |
| 6 | 20   |

- 9. The table represents a discrete function defined on the interval from [1,5].
  - a) Determine the domain, range, x and y intercepts.
  - b) Based on the table, identify the minimum value and where it is located.

| x | f(x) |
|---|------|
| 1 | 4    |
| 2 | 10   |
| 3 | 5    |
| 4 | 8    |
| 5 | 3    |
|   |      |

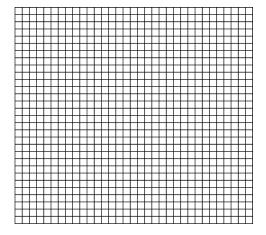
Describe the key features for each situation.

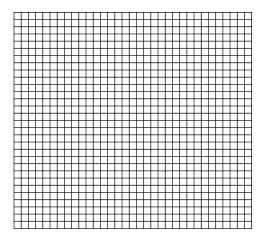
- 10. The amount of daylight (in hours) dependent on the month of the year.
- 11. The first term in a sequence is 36. Each consecutive term is exactly 1/2 of the previous term.
- 12. Marcus bought a \$900 couch on a six months, interest free payment plan. He makes \$50 payments to the loan each week.
- 13. The first term in a sequence is 36. Each consecutive term is 1/2 less than the previous term.
- 14. An empty 15 gallon tank is being filled with gasoline at a rate of 2 gallons per minute.

For each equation, sketch a graph and describe the key features of the graph.

15. 
$$f(x) = -2x + 4$$
, when  $x \ge 0$ 

16. 
$$g(x) = 3^x$$





READY, SET, GO!

Name

Period

Date

#### READY

Topic: Find the point of intersection for two lines by looking at the table.

Fill in the table of values for each of the linear functions. Then circle the point of intersection of the two lines in each table.

$$1. f(x) = 3x - 5$$

$$g(x) = x + 1$$

$$g(x) = x + 1$$
 2.  $f(x) = x + 2$ 

$$g(x) = 2x$$

| х | f(x) |
|---|------|
| 0 |      |
| 1 |      |
| 2 |      |
| 3 |      |
| 4 |      |

| x | g(x) |
|---|------|
| 0 |      |
| 1 |      |
| 2 |      |
| 3 |      |
| 4 |      |

| х | f(x) |
|---|------|
|   |      |
|   |      |
|   |      |
|   |      |
|   |      |

| х | g(x) |
|---|------|
|   |      |
|   |      |
|   |      |
|   |      |
|   |      |

$$3. f(x) = 3x - 4$$

$$g(x) = 3x - 4 \qquad g(x) = -2x + 6$$

4. 
$$f(x) = 4x - 9$$

$$g(x) = 2x + 1$$

| x | f(x) |
|---|------|
| 1 |      |
| 2 |      |
| 3 |      |
| 4 |      |
| 5 |      |

$$\begin{array}{c|cccc}
x & g(x) \\
\hline
1 & & \\
2 & & \\
\hline
3 & & \\
4 & & \\
\hline
5 & & \\
\end{array}$$

| x | g(x) |
|---|------|
|   |      |
|   |      |
|   |      |
|   |      |
|   |      |

#### SET

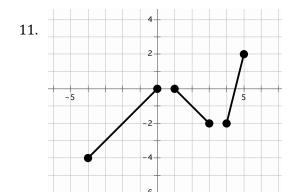
Topic: Attributes of linear and exponential functions.

Determine if the statement is true or false. If it is false, explain why.

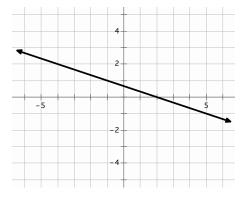
- 5. All linear functions are increasing.
- 6. Arithmetic sequences are an example of linear functions.
- 7. Exponential functions have a domain that includes all real numbers.
- 8. Geometric sequences have a domain that includes all integers.
- 9. The range for an exponential function includes all real numbers.
- 10. All linear relationships are functions with a domain and range containing all real numbers.

GO

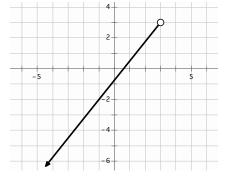
Topic: Determine the domain of a function from a graphical representation. For each graph state the domain of the function. Use interval notation.



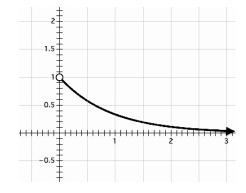
12.



13.



14.



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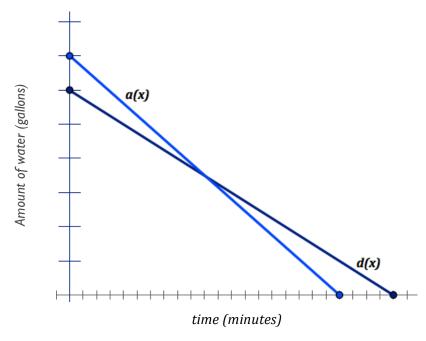
### 3.4 The Water Park

### A Solidify Understanding Task



https://flic.kr/p/sRGBDX

Aly and Dayne work at a water park and have to drain the water at the end of each month for the ride they supervise. Each uses a pump to remove the water from the small pool at the bottom of their ride. The graph below represents the amount of water in Aly's pool, a(x), and Dayne's pool, d(x), over time.



#### Part I

1. Make as many observations as possible with the information given in the graph above.



#### Part II

Dayne figured out that the pump he uses drains water at a rate of 1000 gallons per minute and takes 24 minutes to drain.

- 2. Write the equation to represent the draining of Dayne's pool, d(x). What does each part of the equation mean?
- 3. Based on this new information, correctly label the graph above.
- 4. For what values of *x* make sense in this situation? (Use interval notation to write the domain of the amount of water in Dayne's pool).
- 5. Determine the range, or output values, that make sense in this situation. (Use interval notation to write the range of the amount of water in Dayne's pool).
- 6. Write the equation used to represent the draining of Aly's pool, a(x). Using interval notation, state the domain and range for the function, a(x) as well as the domain and range of the situation. Compare the two domains by describing the constraints made by the situation.

#### Part III

Based on the graph and corresponding equations for each pool, answer the following questions.

- 7. When is a(x) = d(x)? What does this mean?
- 8. Find a(5). What does this mean?
- 9. If d(x)=2000, then x=\_\_\_. What does this mean?
- 10. When is a(x) > d(x)? What does this mean?



3.4

READY, SET, GO!

Name

Period

Date

#### **READY**

Topic: Attributes of linear and exponential functions.

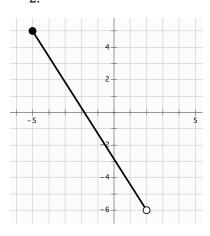
1. Comparing and contrasting linear and exponential functions. Provide a comparison between linear and exponential functions, be sure to include as many characteristics of each function as possible and be clear about the similarities and differences between these functions.

#### SET

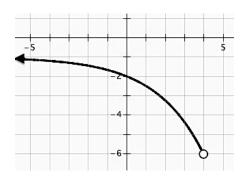
Topic: Identifying attributes of functions from their graphs.

For each graph, identify the domain, range and whether or not the function is increasing or decreasing. Use interval notation when you state the domain and range.

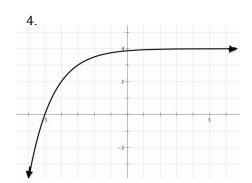


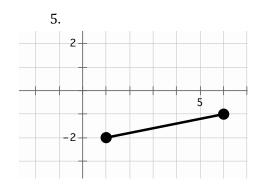


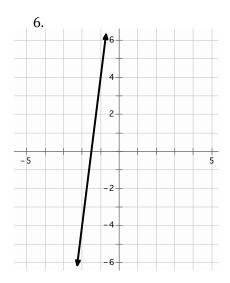
#### 3.

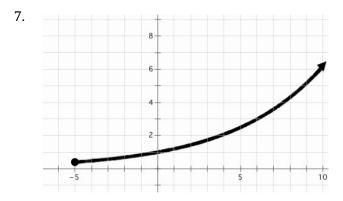


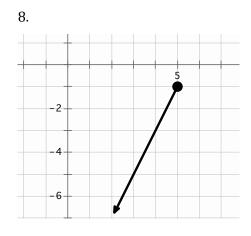
3.4

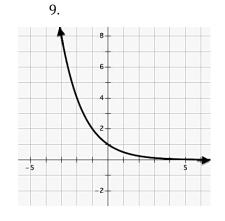












GO

Topic: Finding equations for functions.

Find both the explicit and recursive equations for the tables below.

10.

| n | f(n) |
|---|------|
| 1 | 3    |
| 2 | 5    |
| 3 | 7    |
| 4 | 9    |

11.

| n | f(n) |
|---|------|
| 2 | 4    |
| 3 | 8    |
| 4 | 16   |
| 5 | 32   |

12.

| n | f(n) |
|---|------|
| 6 | 23   |
| 7 | 19   |
| 8 | 15   |
| 9 | 11   |

Explicit:

Recursive:

13.

| n | f(n) |
|---|------|
| 1 | 1    |
| 2 | 3    |
| 3 | 9    |

Explicit:

Recursive:

14.

| n | f(n) |
|---|------|
| 3 | 8    |
| 4 | 4    |
| 5 | 2    |

Explicit:

Recursive:

15.

| n  | f(n) |
|----|------|
| 6  | 7    |
| 9  | 13   |
| 12 | 19   |

Explicit:

Recursive:

16.

| n | f(n) |
|---|------|
| 2 | 40   |
| 4 | 32   |
| 8 | 16   |

Explicit:

Recursive:

17.

| n | f(n) |
|---|------|
| 2 | 16   |
| 3 | 4    |
| 4 | 1    |

Explicit:

Recursive:

18.

| n  | f(n) |
|----|------|
| 17 | 5    |
| 20 | 10   |
| 26 | 20   |

Explicit: Recursive: Explicit:

Recursive:

Explicit:

Recursive:

### 3.5 Pooling it Together

### A Solidify Understanding Task

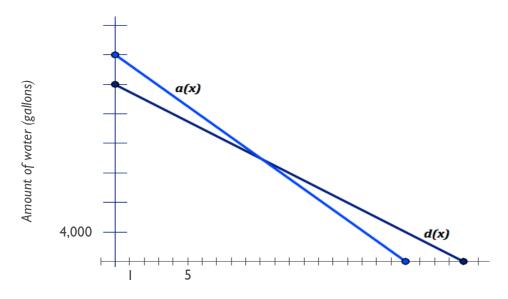
Aly and Dayne work at a water park and have to drain the water at the end of each month for the ride they supervise. Each uses a



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pump to remove the water from the small pool at the bottom of their ride. The graph below represents the amount of water in Aly's pool, a(x), and Dayne's pool, d(x), over time. In this scenario, they decided to work together to drain their pools and created the equation:

$$g(x) = a(x) + d(x).$$



Answer the following questions about g(x).

- 1. What does g(x) represent?
- 2. Create the graph of g(x) on a new set of axes using the graphs of a(x) and d(x). Identify g(x) and label (scale, axes).

| 3. | Write the equation for the function $g(x)$ using the graph you created. Compare this equation    |
|----|--|
|    | to the algebraic representation of finding the sum of the equations for $a(x)$ and $d(x)$ . (The |
|    | equations were created in the last task, "The Water Park" task).                                 |

- 4. Should the algebraic equation of g(x) be the same as the algebraic function created from the graph? Why or why not?
- 5. Use both the graphical as well as the algebraic representation to describe characteristics of g(x) and explain what each characteristic means (each intercept, domain and range for this situation and for the equation, maxima and minima, whether or not g(x) is a function, etc.)
- 6. Explain why adding the two values of the y-intercepts together in a(x) and d(x) can be used to find the y-intercept in g(x).
- 7. Can a similar method be used to find the x-intercepts? Explain.



3.5

READY, SET, GO!

Name

Period

Date

#### READY

Topic: Interpreting function notation to find the output or input based on what is given

For each function, find the indicated values.

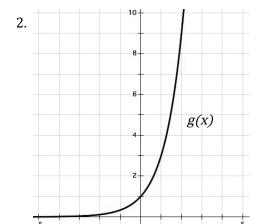
1. Given: 
$$h(t) = 2t - 5$$

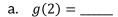
$$a. h(-4) =$$

b. 
$$h(t) = 23$$
,  $t = _____$ 

c. 
$$h(13) =$$
\_\_\_\_

a. 
$$h(-4) =$$
 b.  $h(t) = 23$ ,  $t =$  c.  $h(13) =$  d.  $h(t) = -33$ ,  $t =$ 

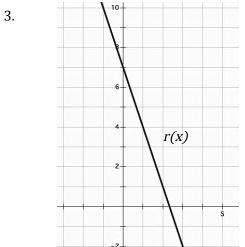




b. 
$$g(x) = 3$$
,  $x = _____$ 

c. 
$$g(0) = _____$$

d. Write the explicit rule for g(x).



a. 
$$r(-1) =$$
\_\_\_\_\_

b. 
$$r(x) = 4$$
,  $x = _____$ 

c. 
$$r(2) = _{---}$$

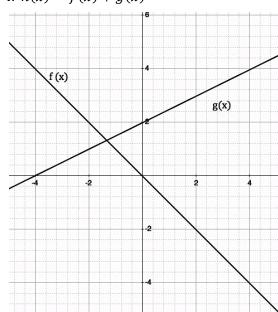
d. Write the explicit rule for r(x).

**SET** 

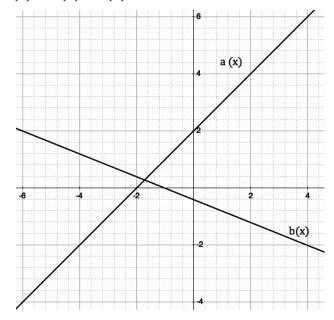
Topic: Adding functions

Two functions are graphed. Graph a new function on the same grid by adding the two given functions.

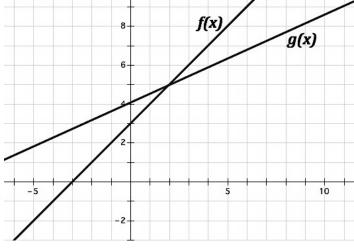
4. h(x) = f(x) + g(x)



5. s(x) = a(x) + b(x)

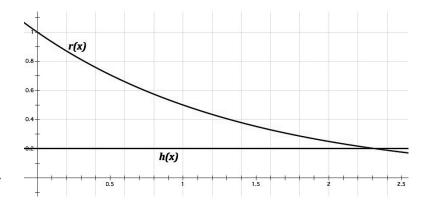


- 5. Use the graph to answer the following questions.
- a. Where does f(x) = g(x)?
- b. What is f(4) + g(4)?
- c. What is g(-2) f(-2)?
- d. State the interval where g(x) > f(x).



6. Use the graph to answer the following questions.

- a. Where is r(x) > h(x)?
- b. What is r(1) h(1)?
- c. What is r(0) + h(0)?
- d. Write an explicit rule for r(x) and for h(x).
- e. Sketch r(x) h(x) on the graph.



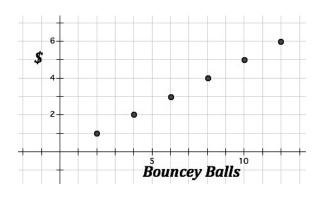
GO

Topic: Distinguishing between discrete and continuous functions

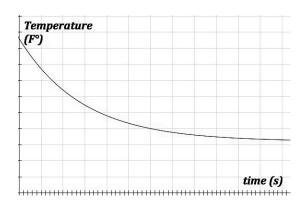
For each context or representation determine whether it is discrete or continuous or could be modeled best in a discrete or continuous way. Justify your answer.

8. Susan puts exactly \$5 a week in her piggy bank.

9.



10.



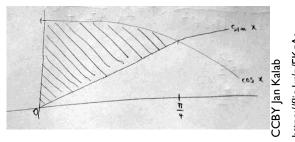
- 11. Marshal tracks the number of hits he gets each baseball game and is recording his total number of hits for the season in a table.
- 12. The distance you have traveled since the day began.

13.

| Number of gumballs | Cost |
|--------------------|------|
| 5                  | 10¢  |
| 10                 | 20¢  |
| 15                 | 30¢  |
| 20                 | 40¢  |

14. Stephen deposited \$1,000 in a savings account at the bank when he turned 21. He deposits \$100 each month. He plans to never withdraw any money until the balance is \$150,000.

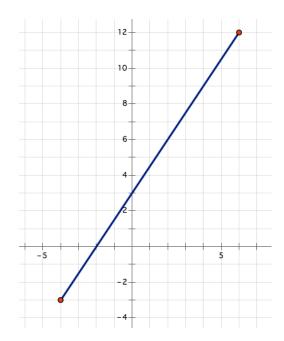
### 3.6 Interpreting Functions A Practice Understanding Task



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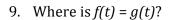
Given the graph of f(x), answer the following questions. Unless otherwise specified, restrict the domain of the function to what you see in the graph below. Approximations are appropriate answers.

- 1. What is f(2)?
- 2. For what values, if any, does f(x) = 3?
- 3. What is the x-intercept?
- 4. What is the domain of f(x)?
- 5. On what intervals is f(x) > 0?
- 6. On what intervals is f(x) increasing?
- 7. On what intervals is f(x) decreasing?
- 8. For what values, if any, is f(x) > 3?





Consider the linear graph of f(t) and the nonlinear graph of g(t) to answer questions 9-14. Approximations are appropriate answers.



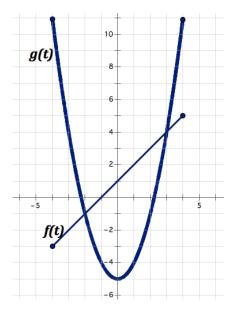
10. Where is 
$$f(t) > g(t)$$
?

11. What is 
$$f(0) + g(0)$$
?

12. What is 
$$f(-1) + g(-1)$$
?

13. Which is greater: 
$$f(0)$$
 or  $g(-3)$ ?

14. Graph: 
$$f(t) + g(t)$$
 from [-1, 3]



The following table of values represents two continuous functions, f(x) and g(x). Use the table to answer the following questions:

| X  | f(x) | g(x) |
|----|------|------|
| -5 | 44   | -13  |
| -4 | 30   | -9   |
| -3 | 20   | -5   |
| -2 | 12   | -1   |
| -1 | 6    | 3    |
| 0  | 2    | 7    |
| 1  | 0    | 11   |
| 2  | 0    | 15   |
| 3  | 2    | 19   |
| 4  | 6    | 23   |
| 5  | 12   | 27   |
| 6  | 20   | 31   |

- 15. What is g(-3)?
- 16. For what value(s) is f(x) = 0?
- 17. For what values does f(x) seem to be increasing?
- 18. On what interval is g(x) > f(x)
- 19. Which function is changing faster in the interval [-5, -1]? Why?

Use the following relationships to answer the questions below.

$$h(x) = 2^x$$

$$h(x) = 2^x$$
  $f(x) = 3x - 2$   $g(x) = 8$   $x = 4$   $y = 5x + 1$ 

$$g(x) = 8$$

$$x = 4$$

$$y = 5x + 1$$

- 20. Which of the above relations are functions? Explain.
- 21. Find f(2), g(2), and h(2).
- 22. Write the equation for g(x) + h(x).
- 23. Where is g(x) < h(x)?
- 24. Where is f(x) increasing?
- 25. Which of the above functions has the fastest growth rate?

Create a graph for each of the following functions, using the given conditions

- 26. This function has the following features: f(2) is positive; f(-2) = 0, f(x) is always Increasing and has a domain of All Real Numbers.
- 27. This function has the following features: f(3) > f(6); f(1) = 0; f(2) = 4; f(x) is increasing from [-5, 3); has a domain from [-5, 10]
- 28. This function has the following features: f(x) has a constant rate of change; f(5) = 0
- 29. Create your own conditions- have at least three and then create examples where the solution could be different graphs.



3.6

READY, SET, GO!

Name

Period

Date

#### **READY**

Topic: Solving Systems by Substitution

In prior work the meaning of f(x) = g(x) was discussed. This means to find the point where the two equations are <u>equal</u> and where the two graphs intersect. It is possible to find the point of intersection algebraically instead of graphing the two lines. Since f(x) = g(x), it's possible to set each equation equal to the other and solve for x.

**Example:** Find the point of intersection of function f(x) = 3x + 4 and function g(x) = 4x + 1.

Since, f(x) = g(x), let 3x + 4 = 4x + 1. Then solve for x.

3x + 4 = 4x + 1 Subtract 3x and 1 from both sides of the equation.

$$\frac{-3x - 1 = -3x - 1}{0x + 3 = 1x + 0}$$

$$3 = 1x$$

Now let x = 3 in each equation to find f(x) and g(x) when x = 3.

 $f(3) = 3(3) + 4 \rightarrow 9 + 4 = 13$  and  $g(3) = 4(3) + 1 \rightarrow 12 + 1 = 13$ 

When x = 3, f(3) and g(3) both equal 13. The point of intersection is (3, 13).

Find the point of intersection for f(x) and g(x) using the algebraic method in the example above.

1. 
$$f(x) = -5x + 12$$
 and  $g(x) = -2x - 3$ 

2. 
$$f(x) = \frac{1}{2}x + 2$$
 and  $g(x) = 2x - 7$ 

3. 
$$f(x) = -\frac{2}{3}x + 5$$
 and  $g(x) = -x + 7$ 

4. 
$$f(x) = x - 6$$
 and  $g(x) = -x - 6$ 



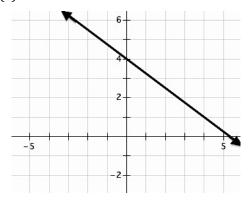
31

#### SET

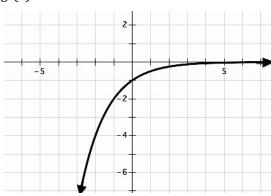
Topic: Describing attributes of a functions based on graphical representation

Use the graph of each function provided to find the indicated values.

5. f(x)



6. 
$$g(x)$$



a. 
$$f(4) =$$
\_\_\_\_\_

a. 
$$f(4) =$$
\_\_\_\_\_ b.  $f(-4) =$ \_\_\_\_\_

c. 
$$f(x) = 4$$
,  $x = _____ d. f(x) = 7$ ,  $x = ______$ 

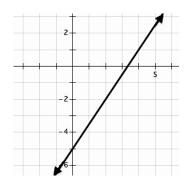
$$d f(y) = 7 y =$$

a. 
$$g(-1) =$$
\_\_\_\_\_ b.  $g(-3) =$ \_\_\_\_\_

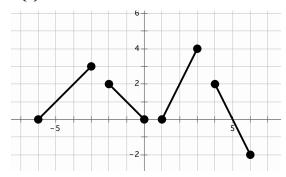
c. 
$$g(x) = -4$$
  $x = _____$ 

c. 
$$g(x) = -4$$
  $x = ____ d. g(x) = -1$ ,  $x = _____$ 

7. h(x)



8. 
$$d(x)$$



a. 
$$h(0) = _____$$

a. 
$$h(0) =$$
\_\_\_\_\_ b.  $h(3) =$ \_\_\_\_\_

c. 
$$h(x) = 1$$
,  $x =$ \_\_\_\_\_ d.  $h(x) = -2$ ,  $x =$ \_\_\_\_

$$d. h(x) = -2.x =$$

a. 
$$d(-5) =$$
\_\_\_\_\_ b.  $d(4) =$ \_\_\_\_\_

b. 
$$d(4) =$$
\_\_\_\_\_

c. 
$$d(x) = 4$$
,  $x = _____ d. d(x) = 0$ ,  $x = _____$ 

d. 
$$d(x) = 0$$
,  $x = ____$ 

3.6

#### For each situation either create a function or use the given function to find and interpret solutions.

- 9. Fran collected data on the number of feet she could walk each second and wrote the following rule to model her walking rate d(t) = 4t.
  - a. What is Fran looking for if she writes d(12) =\_\_\_\_?
  - b. In this situation what does d(t) = 100 tell you?
  - c. How can the function rule be used to indicate a time of 16 seconds was walked?
  - d. How can the function rule be used to indicate that a distance of 200 feet was walked?

10.Mr. Multbank has developed a population growth model for the rodents in the field by his house. He believes that starting each spring the population can be modeled based on the number of weeks with the function  $p(t) = 8(2^t)$ .

Find 
$$p(t) = 128$$
. Find  $p(4)$ .

Find 
$$p(4)$$
.

Find 
$$p(10)$$
.

- d. Find the number of weeks it will take for the population to be over 20,000.
- e. In a year with 16 weeks of summer, how many rodents would he expect by the end of the summer using Mr. Multbank's model?

What are some factors that could change the actual result from your estimate?

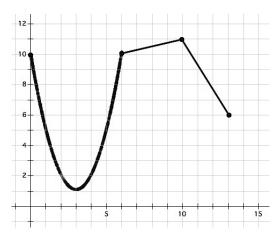
3.6

GO

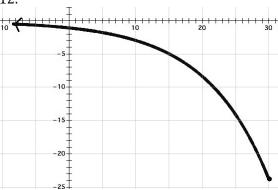
Topic: Describe features of functions from the graphical representation.

For each graph given provide a description of the function. Be sure to consider the following: decreasing/increasing, min/max, domain/range, etc.

11. Description of function:

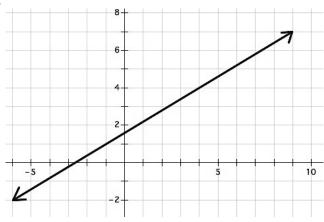


12.



Description of function:

13.



Description of function:

Mathematics Vision Project

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### 3.7 To Function or Not to Function

### A Practice Understanding Task



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Identify the two variables for each situation and determine which is independent and which is dependent. Then, determine if the relationship is a function and justify your reasoning.

| A person's name versus their social security number. | 2. A person's social security number versus their name.                   | 3. The cost of gas versus the amount of gas pumped.   |
|--|---|---|
| 4.<br>{ (3,6), (4, 10), (8,12) }                     | 5. The temperature in degrees Fahrenheit with respect to the time of day. | 6.  distance days 6 2 10 4 6 5 9 8  |
| 7. The area of a circle as it relates to the radius. | 8. 3 5 5 5 7 9 9  | 9. The volume of water in a given cylinder is dependent on the height of water in cylinder. |

| 10. The size of the radius of a circle dependent on the area.                                | 11. Students letter grade dependent on the percent earned.                                | 12. The length of fence needed with respect to the amount of rectangular area to be enclosed. |
|--|---|---|
| 13. The explicit formula for the recursive situation below: f(1) = 3 and f(n + 1) = f(n) + 4 | 14. If x is a rational number, then f(x) = 1  If x is an irrational number, then f(x) = 0 | 15. The national debt with respect to time.   |

3.7

READY, SET, GO!

Name

Period

Date

#### **READY**

Topic: Determine domain and range and whether the relation is a function or not. **Determine if each set of ordered pairs is a function or not and then state the domain and range.** 

Determine if each set of ordered pairs is a function, then state the domain and range.

1. { (-7, 2), (3, 5), (8, 4), (-6, 5), (-2, 3)}

Function: Yes / No

Domain:

Range:

2. { (9, 2), (0, 4), (4, 0), (5, 3), (2, 7) (0, -3), (3, -1)}

Function: Yes / No

Domain:

Range:

 $3.\{(1,2),(2,3),(3,4),(4,5),(5,6),(6,7),(7,8),(8,9)\}$ 

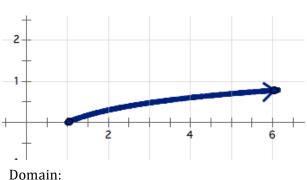
Function: Yes / No

Domain:

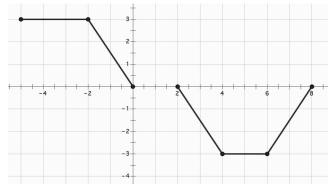
Range:

Determine the domain and range for each of the given functions.

4.



5.



Range:

Domain:

Range:

3.7

6. f(x) = -2x + 7

7.  $g(x) = 3(5)^x$ 

8. The elements in the table define the entire function.

Domain:

Range:

Domain:

Range:

Domain:

Range:

| X | h(x) |
|---|------|
| 1 | 9    |
| 2 | 98   |
| 3 | 987  |
| 4 | 9876 |

SET

Topic: Determine whether or not the relationship is a function.

#### Determine the domain and range then determine whether or not the relationship is a function.

- 9. The distance a person is from the ground related to time as they ride a Ferris Wheel.
- 10. The amount of daylight during a day throughout the calendar year.
- 11. The value of a Volkswagen Bug convertible from time of first purchase in 1978 to now.
- 12. A person's name and their phone number.
- 13. The stadium in which a football player is playing related to the outcome of the game.

GO

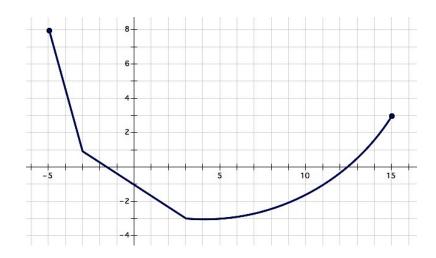
Topic: Determine the features of functions.

14. Describe the function in the graph.

Write the intervals where it is decreasing and/or increasing.

Identify the min and/or max.

State the domain and range.





3.7

15. For each situation use the given function to find and interpret solutions.

Hope has been tracking the progress of her family as they travel across the country, she knows they are driving 78 miles per hour, during their vacation and she has created a function, d(t)= 78t to model the progress they are making.

- a. What would Hope be attempting to find if she writes d(4) = 78(4)?
- b. What would the expression d(t) = 450 mean in this situation?
- c. What would the expression d(3.5) mean in this situation?
- d. How could Hope use the function to find the time it would take to travel 800 miles?

#### 16. Use the given representation of the functions to answer the questions.

- a. Where does f(x) = g(x)?
- b. What is g(0) + f(0)?
- c. On what interval(s) is g(x) > f(x)?
- d. What is g(-8) + f(-8)?

