mathematics vision project

Transforming Mathematics Education

SECONDARY MATH ONE

An Integrated Approach

MODULE 4

Equations & Inequalities

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

Scott Hendrickson, Joleigh Honey, Barbara Kuehl, Travis Lemon, Janet Sutorius

MODULE 4 - TABLE OF CONTENTS

EQUATIONS & INEQUALITIES

4. 1 Cafeteria Actions and Reactions

A Develop Understanding Task

Elvira, the cafeteria manager, has just received a shipment of new trays with the school logo prominently displayed in the middle of the tray. After unloading 4 cartons of trays in the pizza line, she realizes that students are arriving for lunch and she will have to wait until lunch is over before unloading the remaining cartons. The new trays are very popular and in just a couple of minutes 24 students have passed through the pizza line and are showing off the school logo on the trays. At this time, Elvira decides to divide the remaining trays in the pizza line into 3 equal groups so she can also place some in the salad line and the sandwich line, hoping to attract students to the other lines. After doing so, she realizes that each of the three serving lines has only 12 of the new trays.

"That's not many trays for each line. I wonder how many trays there were in each of the cartons I unloaded?"

1. Help the cafeteria manager answer her question using the data in the story about each of the actions she took. Explain how you arrive at your solution.

Elvira is interested in collecting data about how many students use each of the tables during each lunch period. She has recorded some data on Post-It Notes to analyze later. Here are the notes she has recorded:

- Some students are sitting at the front table. (I got distracted by an incident in the back of the lunchroom, and forgot to record how many students.)
- Each of the students at the front table has been joined by a friend, doubling the number of students at the table.
- Four more students have just taken seats with the students at the front table.

- The students at the front table separated into three equal-sized groups and then two groups left, leaving only one-third of the students at the table.

- As the lunch period ends, there are still 12 students seated at the front table.

Elvira is wondering how many students were sitting at the front table when she wrote her first note. Unfortunately, she is not sure what order the middle three Post-It Notes were recorded in since they got stuck together in random order. She is wondering if it matters.

2. Does it matter which order the notes were recorded in? Determine how many students were originally sitting at the front table based on the sequence of notes that appears above. Then rearrange the middle three notes in a different order and determine what the new order implies about the number of students seated at the front table at the beginning.

3. Here are three different equations that could be written based on a particular sequence of notes. Examine each equation, and then list the order of the five notes that is represented by each equation. Find the solution for each equation.

   - $\frac{2(x+4)}{3}=12$

   - $2\left(\frac{x}{3}+4\right)=12$

   - $\frac{2x+4}{3}=12$

4.1

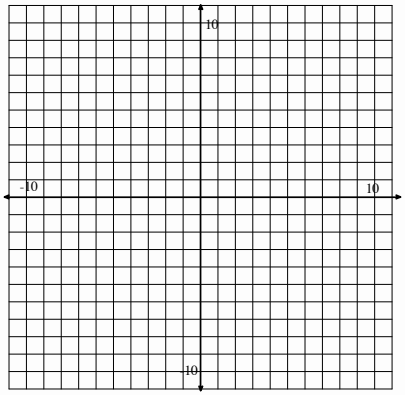
READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Solutions to an equation.

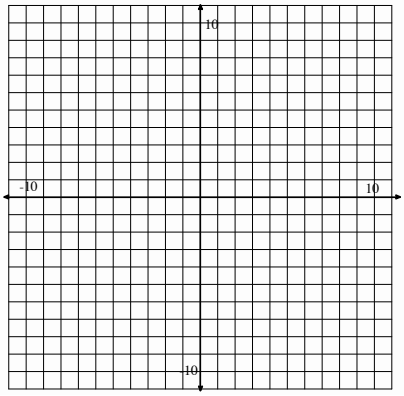
Graph the following equations on the coordinate grid. Determine if the given point is a solution to the equation?

1. $y = 5x - 2$



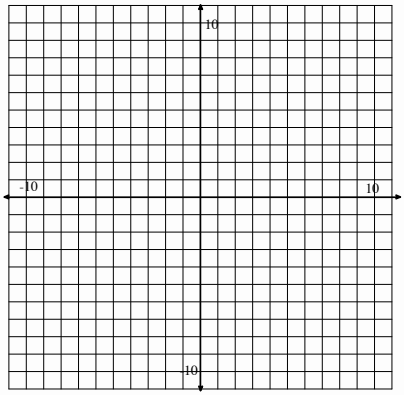
Point: (1 , 3) Yes? / No?

2. $y = -\frac{1}{2}x + 8$



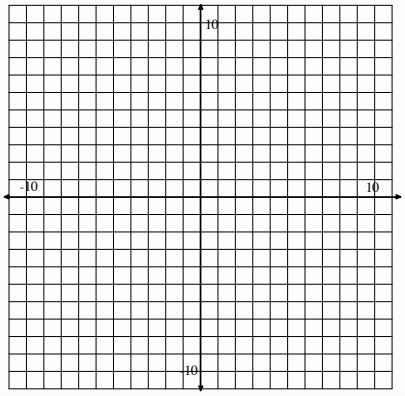
Point: (0 , 7) Yes? / No?

3. $y = -x + 4$



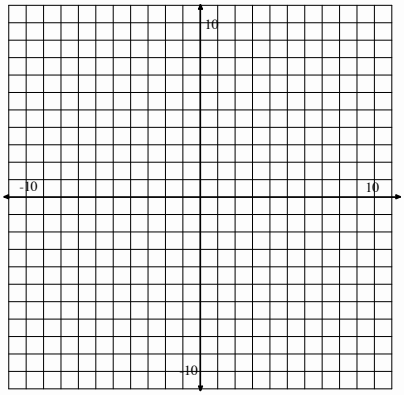
Point: (2 , 2) Yes? / No?

4. $y = x + 2$



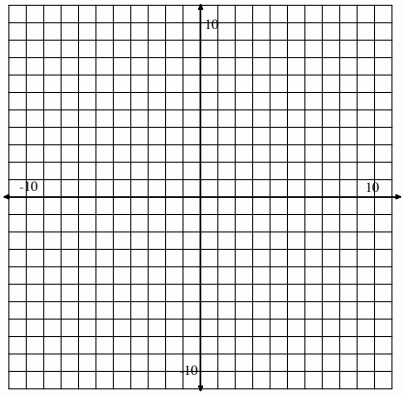
Point: (1 , 3) Yes? / No?

5. $y = \frac{5}{2}x - 7$



Point: (2 , -2) Yes? / No?

6. $y = -\frac{4}{3}x$



Point: (2 , -5) Yes? / No?

SET

Topic: Solve linear equations using parentheses.

Determine if the two expressions listed are equivalent. Explain your reasoning.

7. $14-(3a+2)$ $\quad$ $14-3a-2$	**8.** $4b-10$ $\quad$ $2(2b-5)$
9. $\frac{(x-7)}{4}$ $\quad$ $\frac{x}{4}-\frac{7}{4}$	**10.** $\frac{3(w-9)}{5}$ $\quad$ $\frac{3w}{5}-27$

11. Without solving, determine if the two equations below have the same solution. Explain why or why not?

$$3(x-5)=35 \text{ and } 3x-5=35.$$

12. Circle the expressions that are equivalent.

$\frac{4t-10}{2}$	$\frac{4t}{2}-10$	$2t-10$	$4t-5$

Solve for x.

13. $\frac{4(x-2)}{5}=20$ $\quad$ **14.** $4\left(\frac{x}{5}-2\right)=20$ $\quad$ **15.** $\frac{4x-2}{5}=20$

GO

Topic: Determine if a number is a solution to an equation.

Indicate whether the given value is a solution to the corresponding equation. Show your work.

16. $a=-3$; $\quad 4a+3=-9$ $\quad$ Yes?/No?

17. $x=\frac{4}{3}$; $\quad \frac{3}{4}x+\frac{1}{2}=\frac{3}{2}$ $\quad$ Yes?/No?

18. $y=2$; $\quad 2.5y-10=-0.5$ $\quad$ Yes?/No?

19. $z=-5$; $\quad 2(5-2z)=20-2(z-1)$ $\quad$ Yes?/No?

20. $w=\frac{1}{4}$; $\quad 4w=w+\frac{3}{4}$ $\quad$ Yes?/No?

21. $b=5$; $\quad 6x-2=4(x+2)$ $\quad$ Yes?/No?

4. 2 Elvira's Equations

A Solidify Understanding Task

Elvira, the cafeteria manager, likes to keep track of the things she can count or measure in the cafeteria. She hopes this will help her improve the efficiency of the cafeteria. To remind herself to keep track of important quantities, she has made a table of variables and descriptions of the things she wants to record. Here is a table of things she has decided to keep track of.

Symbol	**Meaning** (description of what the symbol means in context)	**Units** (what is counted or measured)
S	Number of students that buy lunch in the salad line	
W	Number of students that buy lunch in the sandwich line	
P	Number of students that buy lunch in the pizza line	
F	Number of food servers in the cafeteria	
M_T	Number of minutes it takes to serve lunch to all students	
C	Number of classes in the school	
P_L	Price per lunch	
A		
R		
T		
D_F		
M		

Elvira has written the following equation to describe a cafeteria relationship that seems meaningful to her. She has introduced a new variable A to describe this relationship.

$$A = \frac{S+W+P}{C}$$

1. What does A represent in terms of the school and the cafeteria? Record this information in the table above.

2. Using what you know about manipulating equations, solve this equation for *S*. Your solution will be of the form *S* = *an expression written in terms of the variables A, C, W* and *P.*

3. Does your expression for *S* make sense in terms of the meanings of the other variables? Explain why or why not.

Here is another one of Elvira's equations.

$$R = P_L(S + W + P)$$

4. What does *R* represent in terms of the school and the cafeteria? Record this information in the table above.

5. Using what you know about manipulating equations, solve this equation for P_L.

6. Does your expression for P_L make sense in terms of the meanings of the other variables? Explain why or why not.

7. Elvira notices that she uses the expression $S + W + P$ a lot in writing other expressions. She decides to represent this expression using the variable *T*, so that $T = S + W + P$. What does *T* represent in terms of the school and the cafeteria? Record this information in the table above.

Elvira is having a meeting with the staff members who work in the lunchroom. She has created a couple of new equations for the food servers.

$$D_F = \frac{T \cdot P_L}{F} \qquad\qquad M = \frac{M_T}{T}$$

8. a. What does D_F represent in terms of the school and the cafeteria? Record this information in the table above.

   b. Solve this equation for P_L. Describe why your solution makes sense in terms of the other variables.

9. a. What does M represent in terms of the school and the cafeteria? Record this information in the table above.

   b. Solve this equation for T. Describe why your solution makes sense in terms of the other variables.

10. One of the staff members suggests that they need to write expressions for each of the following. Using the variables in the table, what would these expressions look like?

    a. The average number of students served each minute

    b. The average number of minutes students wait in the pizza line

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Isolate a variable with inverse operations.

Isolate the indicated variable and then fill in the blank for the statement that follows.

1. Solve for x; $ax = 7$ I can find $1x$ or x by _____ on both sides of the equation.
2. Solve for p; $8 + p = w$ I can find $1p$ or p by _____ on both sides of the equation.
3. Solve for m; $e = mc^2$ I can find $1m$ or m by _____ on both sides of the equation.
4. Solve for t; $d = rt$ I can find $1t$ or t by _____ on both sides of the equation.
5. Solve for r; $d = rt$ I can find r by _____ on both sides of the equation.
6. Solve for h; $7 - h = 0$ I can find h by _____ on both sides of the equation.
7. Solve for b; $b - 11 = 3$ I can find b by _____ on both sides of the equation.
8. Solve for y; $\frac{1}{2}y = k$ I can find y by _____ on both sides of the equation.
9. Solve for h; $A = \frac{bh}{2}$ I can find h by _____ on both sides of the equation.
10. Solve for x; $y = mx + b$ I can find x by _____ on both sides of the equation.

SET

Topic: Defining and interpreting variables and units of measure.

Jaxon likes to be organized, so he made the following chart. He has decided to keep track of the miles he runs and the time he spends running. He attends P.E. class on Monday, Wednesday, and Friday, but he goes to school everyday. *Fill in the Units column on the chart.*

Symbol	Meaning (Description of what the symbol means in context)	Units (What is counted or measured)
M	Number of miles ran in PE class on Mondays	
W	Number of miles ran PE class on Wednesdays	
F	Number of miles ran PE class on Fridays	
S	Number of miles from Jaxon's house to the school.	
H	Time (in hours) to travel to school	
t_M	Time (in minutes) spent running in PE on Monday	
t_W	Time (in minutes) spent running in PE on Wednesday	
t_F	Time (in minutes) spent running in PE on Friday	

Make meaning of the expressions below, write what they each mean!
If an expression does not make sense, say why.

11. $M + W + F$
12. $4(M + W + F)$
13. $2S$
14. $t_M + t_W + t_F$
15. $\frac{t_M + t_W + t_F}{3}$
16. $5(2H)$
17. $M + H$

GO

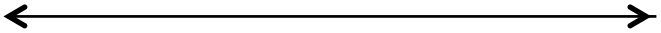
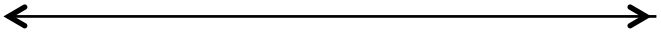
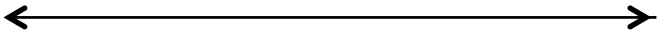
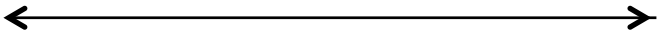
Topic: Set notation to interval notation. Inequalities on a number line.

Below you will find the domains of several different functions. The domains are described in either set notation or interval notation. Fill in the missing notation.

Set Notation	Interval Notation
18. $\{x \mid x \in \mathbb{R},\ -2 < x < 6\}$	
19.	[-4, 7]
20. $\{x \mid x \in \mathbb{R},\ x \geq -9\}$	
21.	(0, 13]
22. $\{x \mid x \in \mathbb{R},\ -15 \leq x \leq -8\}$	
23.	[-32, -15)
24.	$(-\infty, \infty)$

25. Which notation, interval or set, would most appropriate when working with a domain of whole numbers?

For each of the inequalities provided graph the values being described on the numbers line.

26. $x < 6$	
27. $x > 5$	
28. $x \geq -9$	
29. $-7 \leq x < 0$	
30. $3 \leq x \leq 25$	
31. $-15 < x \leq 8$	

4. 3 Solving Equations Literally

A Practice Understanding Task

Solve each of the following equations for *x*:

1. $\frac{3x+2}{5}=7$

2. $\frac{3x+2y}{5}=7$

3. $\frac{4x}{3}-5=11$

4. $\frac{4x}{3}-5y=11$

5. $\frac{2}{5}(x+3)=6$

6. $\frac{2}{5}(x+y)=6$

7. $2(3x+4)=4x+12$

8. $2(3x+4y)=4x+12y$

Write a verbal description for each step of the equation solving process used to solve the following equations for *x*. Your description should include statements about how you know what to do next. For example, you might write, "*First I* _____, *because* _____…"

9. $\frac{ax+b}{c}-d=e$

10. $r\cdot\sqrt{\frac{mx}{n}+s}=t$

4.3

READY, SET, GO! Name Period Date

READY

Topic: Solving Inequalities.

Use the inequality $-9 < 2$ to complete each row in the table.

Apply each operation to the original inequality $-9 < 2$	Result	Is the resulting inequality true or false?
Example: Add 3 to both sides	-9+3 < 2+3 → -6 < 5	True
1. Subtract 7 from both sides.		
2. Add 15 to both sides.		
3. Add -10 to both sides.		
4. Multiply both sides by 10.		
5. Divide both sides by 5.		
6. Multiply both sides by -6.		
7. Divide both sides by -3.		

8. What operations when performed on an inequality, reverse the inequality? (Be very specific!)

SET

Topic: Solve literal equations that require more than one step.
Solve for the indicated variable. Show your work!!!

9. Solve for h. $Q = 25\pi h$

10. Solve for h. $Q = \pi r^2 h$

11. Solve for m. $y = 7m + 6$

12. Solve for m. $y = mx + b$

13. Solve for z. $A = (z + 7)3$

14. Solve for z. $A = (z + 7)w$

15. Solve for x. $\frac{x+2}{7} = 4$

16. Solve for x. $\frac{x+2y}{7} = 4$

17. Solve for x. $\frac{2x}{5} - 9 = 6$

18. Solve for x. $\frac{2x}{5} - 9y = 6$

19. Solve for x. $\frac{3}{4}(x - 2) = 12$

20. Solve for x. $\frac{3}{4}(x - 2y) = 12$

GO

Topic: Identifying x-intercepts and y-intercepts

Locate the x-intercept and y-intercept in the table. Write each as an ordered pair.

21.

x	y
-4	12
-3	10
-2	8
-1	6
0	4
1	2
2	0

x – intercept:

y – intercept:

22.

x	y
0	-6
3	-5
6	-4
9	-3
12	-2
15	-1
18	0

x – intercept:

y – intercept:

23.

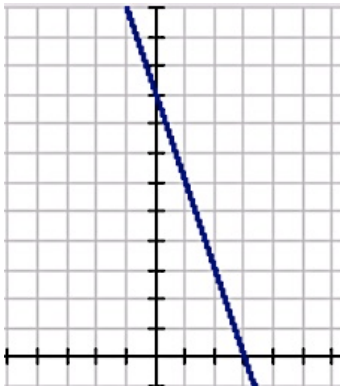
x	y
-3	10
-2	8
-1	6
0	4
1	2
2	0
3	-2

x – intercept:

y – intercept:

Locate the x-intercept and the y-intercept in the graph. Write each as an ordered pair.

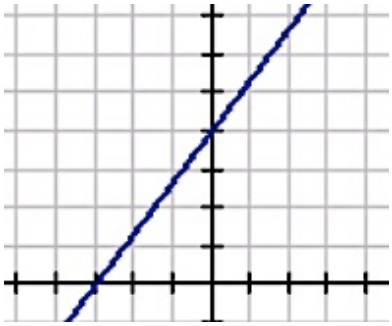
24.



x – intercept:

y – intercept:

25.



x – intercept:

y – intercept:

Solve each equation for x. Provide the justifications for each step. See the first example as a reminder for the types of justifications that might be used.

Example:

$3x - 6 = 15$	Justification
$+6 \quad +6$	Addition Property of equality
$\frac{3x}{3} = \frac{21}{3}$	Division Property of equality
$x = 7$	

26.

$4x - 10 = 2$	Justification

27.

$-16 = 3x + 11$	Justification

28.

$6 - 2x = 10$	Justification

29.

$6x + 3 = x + 18$	Justification

30.

$3x - 10 = 2x + 12$	Justification

31.

$12x + 3y = 15$	Justification

32.

$X(B + 7) = 9$	Justification

4.4 Greater Than?

A Develop Understanding Task

For each situation you are given a mathematical statement and two expressions beneath it.

1. Decide which of the two expressions is greater, if the expressions are equal, or if the relationship cannot be determined from the statement.
2. Write an equation or inequality that shows your answer.
3. Explain why your answer is correct.

Watch out—this gets tricky!

Example:
Statement: $x = 8$
Which is greater? $x + 5$ or $3x + 2$
Answer: $3x + 2 > x + 5$ because if $x = 8, 3x + 2 = 26, x + 5 = 13$ and $26 > 13$.

Try it yourself:

1. Statement: $y < x$
   Which is greater? $x - y$ or $y - x$

2. Statement: $2x - 3 > 7$
   Which is greater? 5 or x

3. Statement: $10 - 2x < 6$
   Which is greater? x or 2

4. Statement: $4x \leq 0$
   Which is greater? 1 or x

5. Statement: n is an integer
   Which is greater? n or $-n$

6. Statement $x > y$
   Which is greater? $x + a$ or $y + a$

7. Statement: $x > y$
   Which is greater? $x - a$ or $y - a$

8. Statement: $5 > 4$
   Which is greater? $5x$ or $4x$

9. Statement: $5 > 4$
   Which is greater? $\frac{5}{x}$ or $\frac{4}{x}$

10. Statement: $0 < x < 10$ and $0 < y < 12$
    Which is greater? x or y

11. Statement: $3^{n+2} \geq 27$
    Which is greater? n or 1

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Write an equation from a context. Interpret notation for inequalities.

Write an equation that describes the story. Then answer the question asked by the story.

1. Virginia's Painting Service charges $10 per job and $0.20 per square foot. If Virginia earned $50 for painting one job, how many square feet did she paint at the job?

2. Renting the ice-skating rink for a party costs $200 plus $4 per person. If the final charge for Dane's birthday party was $324, how many people attended his birthday party?

Indicate if the following statements are <u>true</u> or <u>false</u>. Explain your thinking.

3. The notation $12 < x$ means the same thing as $x < 12$. It works just like $12 = x \text{ and } x = 12$.

4. The inequality $-2(x + 10) \geq 75$ says the same thing as $-2x - 20 \geq 75$. I can multiply by -2 on the left side without reversing the inequality symbol.

5. When solving the inequality $10x + 22 < 2$, the second step should say $10x > -20$ because I added -22 to both sides and I got a negative number on the right.

6. When solving the inequality $-5x \geq 45$, the answer is $x \leq -9$ because I divided both sides of the inequality by a negative number.

7. The words that describe the inequality $x \geq 100$ are "*x is greater than or equal to 100.*"

SET

Topic: Solve inequalities. Verify that given numbers are elements of the solution set.

Solve for x. (Show your work.) Indicate if the given value of x is an element of the solution set.

8. $2x - 9 < 3$

Is this value part of the solution set? $x = 6$; *yes*? *no*?

9. $4x + 25 > 13$

Is this value part of the solution set? $x = -5$; *yes*? *no*?

10. $6x - 4 \leq -28$

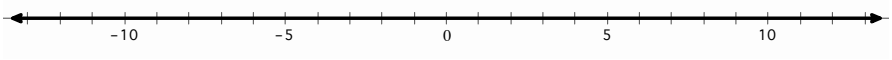
Is this value part of the solution set? $x = -10$; *yes*? *no*?

11. $3x - 5 \geq -5$

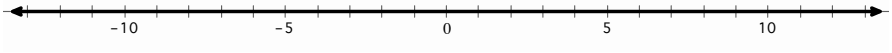
Is this value part of the solution set? $x = 1$; *yes*? *no*?

Solve each inequality and graph the solution on the number line.

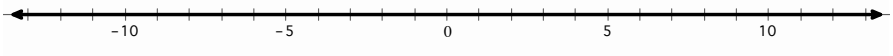
12. $x + 9 \leq 7$



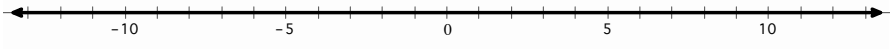
13. $-3x - 4 > 2$



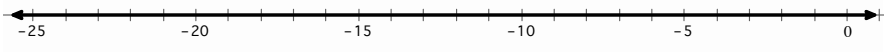
14. $3x < -6$



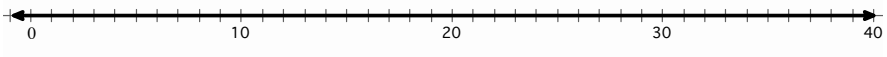
15. $\frac{x}{5} > -\frac{3}{10}$



16. $-10x > 150$



17. $\frac{x}{-7} \geq -5$



Solve each multi-step inequality.

18. $x - 5 > 2x + 3$

19. $\frac{3(x-4)}{12} \leq \frac{2x}{3}$

20. $2(x - 3) \leq 3x - 2$

GO

Topic: Use substitution to solve linear systems

Solve each system of equations by using substitution.

Example: $\begin{cases} y = 12 \\ 2x - y = 14 \end{cases}$

The first equation states that $y = 12$. That information can be used in the second equation to find the value of x by replacing y with 12. The second equation now says $\mathbf{2}\boldsymbol{x} - (\mathbf{12}) = \mathbf{14}$. Solve this new equation by adding 12 to both sides and then dividing by 2. The result is x = 13.

21. $\begin{cases} y = 5 \\ -x + y = 1 \end{cases}$

22. $\begin{cases} x = 8 \\ 5x + 2y = 0 \end{cases}$

23. $\begin{cases} 2y = 10 \\ 4x - 2y = 50 \end{cases}$

24. $\begin{cases} 3x = 12 \\ 4x - y = 5 \end{cases}$

25. $\begin{cases} y = 2x - 5 \\ y = x + 8 \end{cases}$

26. $\begin{cases} 3x = 9 \\ 5x + y = -5 \end{cases}$

4.5 May I Have More, Please?

A Solidify Understanding Task

Elvira, the cafeteria manager, has to be careful with her spending and manages the cafeteria so that they can serve the best food at the lowest cost. To do this, Elvira keeps good records and analyzes all of her budgets.

1. Elvira's cafeteria has those cute little cartons of milk that are typical of school lunch. The milk supplier charges $0.35 per carton of milk, in addition to a delivery charge of $75. What is the maximum number of milk cartons that Elvira can buy if she has budgeted $500 for milk?
    a. Write and solve an inequality that models this situation.

    b. Describe in words the quantities that would work in this situation.

    c. Write your answer in both interval and set notation.

2. Students love to put ranch dressing on everything, so Elvira needs to keep plenty in stock. The students eat about 2.25 gallons of ranch each day! Elvira started the school year with 130 gallon of ranch dressing. She needs to have at least 20 gallons left when she reorders to have enough in stock until the new order comes. For how many days will her ranch dressing supply last before she needs to reorder?
    a. Write and solve an inequality that models this situation.

b. Describe in words the quantities that would work in this situation.

c. Write your answer in both interval and set notation.

3. The prices on many of the cafeteria foods change during the year. Elvira finds that she has ordered veggie burgers four times and paid $78, $72, $87, and $90 on the orders. To stay within her budget, Elvira needs to be sure that the average order of veggie burgers is not more than $82. How much can she spend on the fifth order to keep the average order within her budget?
   a. Write and solve an inequality that models this situation.

   b. Describe in words the quantities that would work in this situation.

   c. Write your answer in both interval and set notation.

4. Elvira can purchase ready-made pizzas for $14.50 each. If she makes them in the cafeteria, she can spend $44.20 on ingredients and $6.25 per pizza on labor. For how many pizzas is it cheaper for the cafeteria to make the pizzas themselves rather than buy them ready-made?
   a. Write and solve an inequality that models this situation.

   b. Describe in words the quantities that would work in this situation.

   c. Write your answer in both interval and set notation.

5. Elvira is comparing prices between two different suppliers of fresh lettuce. Val's Veggies charges $250 for delivery plus $1.50 per bag of lettuce. Sally's Salads charges $100 for delivery plus $4.00 per bag of lettuce. How many bags of lettuce must be purchased for Val's Veggies to be the cheaper option?
    a. Write and solve an inequality that models this situation.

    b. Describe in words the quantities that would work in this situation.

    c. Write your answer in both interval and set notation.

6. Each student that buys school lunch pays $2.75. The cafeteria typically brings in between $1168.75 and $1438.25. How many students does the cafeteria usually serve?
    a. Model this situation using an inequality.

    b. Describe in words the quantities that would work in this situation.

    c. Write your answer in both interval and set notation.

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Interpret phrases that imply an inequality.
Rewrite the given "word sentence" as a "math sentence." Each math sentence will use one of the following symbols: $>$, $<$, $\leq$, $\geq$. **Use "x" in place of the number.**

	Word Sentence	Math Sentence
Example:	I am thinking of a number that is greater than 13.	$x > 13$
1.	I am thinking of a number that is at least 13.	
2.	I am thinking of a number that is no fewer than 13.	
3.	I am thinking of a number that does not exceed 13.	
4.	I am thinking of a number that is at most 13.	
5.	I am thinking of a number that is no more than 13.	
6.	I am thinking of a number that is fewer than 13.	
7.	I am thinking of a number that is not above 13.	
8.	I am thinking of a number that is less than 13.	
9.	I am thinking of a number that is not under 13.	
10.	I am thinking of a number that is not greater than 13.	

SET

Topic: Write and solve inequalities from a context.

11. To take sweepstakes for the largest pumpkin crop at the Riverside County Fair, the average weight of Ethan's two pumpkins must be greater than 875 lbs. One of his pumpkins weighs 903 lbs. What is the least amount of pounds the second pumpkin could weigh in order for Ethan to win the prize?

    a) Write an inequality that models this situation. Be sure to define your variables.

    b) Describe in words the quantities that would work in this situation.

    c) Write your answer in both interval and set notation.

12. The average of Aaron's three test scores must be at least 93 to earn an A in the class. Aaron scored 89 on the first test and 94 on the second test. What scores can Aaron get on his third test to guarantee an A in the class? (The highest possible score is 100.)

    a) Write and solve an inequality that models this situation. Be sure to define your variables.

    b) Describe in words the quantities that would work in this situation.

    c) Write your answer in both interval and set notation.

13. A cell phone company offers a plan that costs $35.99 and includes unlimited texting. Another company offers a plan that costs $19.99 and charges $0.25 per text. For what number of texts does the second company's plan cost more than the first company's plan?

    a) Write and solve an inequality that models this situation. Be sure to define your variables.

    b) Describe in words the quantities that would work in this situation.

    c) Write your answer in both interval and set notation.

GO

Topic: Use substitution to solve linear systems

Solve each system of equations by using substitution.

Example: $\begin{cases} y = x + 3 \\ 2x - y = 14 \end{cases}$

The first equation states that $y = x + 3$. *That information can be used in the second equation to find the value of x by replacing y with* $x + 3$. *The second equation now says* $\mathbf{2x - (x + 3) = 14}$. *Solve this new equation by first distributing the negative over* $(x + 3)$. *The new equation will be* $\mathbf{2x - x - 3 = 14}$. *Combine like terms. You will get the equivalent equation* $\mathbf{x - 3 = 14}$. *Add 3 to both sides. You should get* $\mathbf{x = 17}$. *But you still don't know the value of y. Now that you know the value of x, you can use either equation to figure out the value of* y. *Since the first equation is simpler, you may want to substitute the known value of x (recall that* $x = 17$*) into it. It should be easy to see what y equals.* $\mathbf{y = (17) + 3 = 20}$.

21. $\begin{cases} y = x + 5 \\ 2x + y = -1 \end{cases}$

22. $\begin{cases} x = y - 1 \\ 5x + 2y = 9 \end{cases}$

23. $\begin{cases} y = 10 - x \\ 4x - 2y = 40 \end{cases}$

24. $\begin{cases} x = 1 + y \\ 4x - y = 7 \end{cases}$

4.6 Taking Sides

A Practice Understanding Task

Joaquin and Serena work together productively in their math class. They both contribute their thinking and when they disagree, they both give their reasons and decide together who is right. In their math class right now, they are working on inequalities. Recently they had a discussion that went something like this:

> Joaquin: The problem says that "6 less than a number is greater than 4." I think that we should just follow the words and write: $6 - n > 4$.
>
> Serena: I don't think that works because if n is 20 and you do 6 less than that you get 20 – 6 = 14. I think we should write $n - 6 > 4$
>
> Joaquin: Oh, you're right. Then it makes sense that the solution will be $n > 10$, which means we can choose any number greater than 10.

The situations below are a few more of the disagreements and questions that Joaquin and Serena have. Your job is to decide how to answer their questions, decide who is right, and give a mathematical explanation of your reasoning.

1. Joaquin and Serena are assigned to graph the inequality $x \geq -7$.
   Joaquin thinks the graph should have an open dot -7.
   Serena thinks the graph should have a closed dot at -7.
   Explain who is correct and why.

2. Joaquin and Serena are looking at the problem $3x + 1 > 0$.
   Serena says that the inequality is always true because multiplying a number by three and then adding one to it makes the number greater than zero.
   Is she right? Explain why or why not.

3. The word problem that Joaquin and Serena are working on says, "4 greater than x".
Joaquin says that they should write: $4 > x$.
Serena says they should write: $4 + x$
Explain who is correct and why.

4. Joaquin is thinking hard about equations and inequalities and comes up with this idea:
If $45 + 47 = t$, then $t = 45 + 47$.
So, if $45 + 47 < t$, then $t < 45 + 47$.
Is he right? Explain why or why not.

5. Joaquin's question in #4 made Serena think about other similarities and differences in equations and inequalities. Serena wonders about the equation $-\frac{x}{3} = 4$ and the inequality $-\frac{x}{3} > 4$. Explain to Serena ways that solving these two problems are alike and ways that they are different. How are the solutions to the problems alike and different?

6. Joaquin solved $-15q \leq 135$ by adding 15 to each side of the inequality. Serena said that he was wrong. Who do you think is right and why?

Joaquin's solution was $q \leq 150$. He checked his work by substituting 150 for q in the original inequality. Does this prove that Joaquin is right? Explain why or why not.

Joaquin is still skeptical and believes that he is right. Find a number that satisfies his solution but does not satisfy the original inequality.

7. Serena is checking her work with Joaquin and finds that they disagree on a problem. Here's what Serena wrote:

$$3x + 3 \leq -2x + 5$$
$$3x \leq -2x + 2$$
$$x \leq 2$$

Is she right? Explain why or why not?

8. Joaquin and Serena are having trouble solving $-4(3m - 1) \geq 2(m + 3)$
Explain how they should solve the inequality, showing all the necessary steps and identifying the properties you would use.

9. Joaquin and Serena know that some equations are true for any value of the variable and some equations are never true, no matter what value is chosen for the variable. They are wondering about inequalities. What could you tell them about the following inequalities? Do they have solutions? What are they? How would you graph their solutions on a number line?

a. $4s + 6 \geq 6 + 4s$

b. $3r + 5 > 3r - 2$

c. $4(n + 1) < 4n - 3$

10. The partners are given the literal inequality $ax + b > c$ to solve for x. Joaquin says that he will solve it just like an equation. Serena says that he needs to be careful because if a is a negative number, the solution will be different. What do you say? What are the solutions for the inequality?

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Solving equations and inequalities from a context.

Write the given situation as an equation or inequality and then solve it.

1. The local amusement park sells summer memberships for $50 each. Normal admission to the park costs $25; admission for members costs $15.

   a. If Darren wants to spend no more than $100 on trips to the amusement park this summer, how many visits can he make if he buys a membership with part of that money?

   b. How many visits can he make if he pay normal admission instead?

   c. If he increases his budget to $160, how many visits can he make as a member?

   d. How many can he make as a non-member with the increased budget of $160?

2. Jade just took a math test with 20 questions, each question is worth an equal number of points. The test is worth 100 points total.

   a. Write an equation that can be used to calculate Jade's score based on the number of questions she got right on the test.

   b. If a score of 70 points earns a grade of *C*-, how many questions would Jade need to get right to get at least a *C*- on the test?

   c. If a score of 83 points earns a grade of *B*, how many questions would Jade need to get right to get at least a *B* on the test?

   d. Suppose Jade got a score of 60% and then was allowed to retake the test. On the retake, she got all the questions right that she got right the first time, and also got half the questions right that she got wrong the first time. What percent of the questions did Jade get right, in total, on the retake?

SET

Topic: Solve and justify one variable inequalities

Solve each inequality, justifying each step you use.

3.

$-5x < 35$	Justification

4.

$x + 68 \geq 75$	Justification

5.

$2x - 4 \leq 10$	Justification

6.

$5 - 4x \leq 17$	Justification

7.

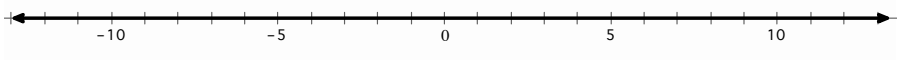
$\frac{x}{-3} > -\frac{10}{9}$	Justification

8.

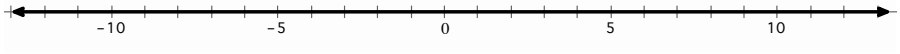
$2(x - 3) \leq 3x - 2$	Justification

Solve each inequality and graph the solution on the number line.

9. $x - 8 > -20$



10. $x + 11 > 13$



Solve each multi-step inequality.

11. $4x + 3 < -1$

12. $4 - 6x \leq 2(2x + 3)$

13. $5(4x + 3) \geq 9(x - 2) - x$

14. $\frac{2}{3}x - \frac{1}{2}(4x - 1) \geq x + 2(x - 3)$

Topic: Solve literal equations

15. Solve the following equation for C: $F = \frac{9}{5}C + 32$

16. Given $V = \frac{1}{3}\pi r^2 h$, rewrite the formula to isolate the variable r.

17. The area formula of a regular polygon is $A = \frac{1}{2}Pa$. The variable a represents the apothem and P represents the perimeter of the polygon. Solve the equation for the apothem, a.

18. The equation $y = mx + b$ is the equation of a line. Isolate the variable b.

19. The equation for the circumference c of a circle with radius r is $c = 2\pi r$. Solve the equation for the radius, r.

20. The equation for the area of a circle A based on diameter d is $A = \pi\frac{d^2}{4}$. Solve the equation to isolate the diameter, d.

GO

Topic: Solve systems of equations by graphing

Graph both lines on the same coordinate grid. Identify the point of intersection. Then test the x and y values of the point of intersection in the two equations.

21. $\begin{cases} y = 2x + 5 \\ -x + y = 1 \end{cases}$

22. $\begin{cases} 10 + y = 3x \\ 2x + y = 0 \end{cases}$

23. $\begin{cases} x + y = 9 \\ x - y = -7 \end{cases}$