

*Transforming Mathematics Education*

# SECONDARY MATH ONE

*An Integrated Approach*

MODULE 7

## Congruence, Construction & Proof

MATHEMATICSVISIONPROJECT.ORG

### **The Mathematics Vision Project**

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**READY, SET, GO Homework: Congruence, Construction and Proof 7.6**



## 7.1 Under Construction

### *A Develop Understanding Task*



Anciently, one of the only tools builders and surveyors had for laying out a plot of land or the foundation of a building was a piece of rope.

There are two geometric figures you can create with a piece of rope: you can pull it tight to create a line segment, or you can fix one end, and—while extending the rope to its full length—trace out a circle with the other end. Geometric constructions have traditionally mimicked these two processes using an unmarked straightedge to create a line segment and a compass to trace out a circle (or sometimes a portion of a circle called an arc). Using only these two tools you can construct all kinds of geometric shapes.

Suppose you want to construct a rhombus using only a compass and straightedge. You might begin by drawing a line segment to define the length of a side, and drawing another ray from one of the endpoints of the line segment to define an angle, as in the following sketch.



Now the hard work begins. We can't just keep drawing line segments, because we have to be sure that all four sides of the rhombus are the same length. We have to stop drawing and start constructing.

### Constructing a rhombus

Knowing what you know about circles and line segments, how might you locate point  $C$  on the ray in the diagram above so the distance from  $B$  to  $C$  is the same as the distance from  $B$  to  $A$ ?

1. Describe how you will locate point  $C$  and how you know  $\overline{BC} \cong \overline{BA}$ , then construct point  $C$  on the diagram above.

Now that we have three of the four vertices of the rhombus, we need to locate point  $D$ , the fourth vertex.

2. Describe how you will locate point  $D$  and how you know  $\overline{CD} \cong \overline{DA} \cong \overline{AB}$ , then construct point  $D$  on the diagram above.

### Constructing a Square (A rhombus with right angles)

The only difference between constructing a rhombus and constructing a square is that a square contains right angles. Therefore, we need a way to construct perpendicular lines using only a compass and straightedge.

We will begin by inventing a way to construct a perpendicular bisector of a line segment.

3. Given  $\overline{RS}$  below, fold and crease the paper so that point  $R$  is reflected onto point  $S$ . Based on the definition of reflection, what do you know about this “crease line”?



You have “constructed” a perpendicular bisector of  $\overline{RS}$  by using a paper-folding strategy. Is there a way to construct this line using a compass and straightedge?

4. Experiment with the compass to see if you can develop a strategy to locate points on the “crease line”. When you have located at least two points on the “crease line” use the straightedge to finish your construction of the perpendicular bisector. Describe your strategy for locating points on the perpendicular bisector of  $\overline{RS}$ .

Now that you have created a line perpendicular to  $\overline{RS}$  we will use the right angle formed to construct a square.

5. Label the midpoint of  $\overline{RS}$  on the diagram above as point  $M$ . Using segment  $\overline{RM}$  as one side of the square, and the right angle formed by segment  $\overline{RM}$  and the perpendicular line drawn through point  $M$  as the beginning of a square. Finish constructing this square on the diagram above. (Hint: Remember that a square is also a rhombus, and you have already constructed a rhombus in the first part of this task.)

READY, SET, GO!

Name

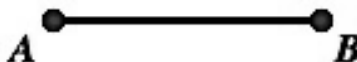
Period

Date

**READY**

Topic: Tools for construction and geometric work.

- Using your compass draw several concentric circles that have point A as a center and then draw those same sized concentric circles that have B as a center. What do you notice about where all the circles with center A intersect all the corresponding circles with center B?

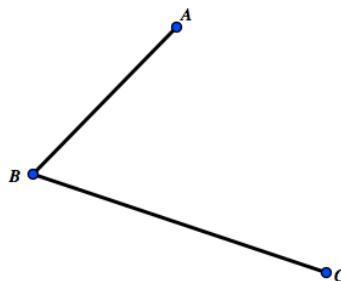
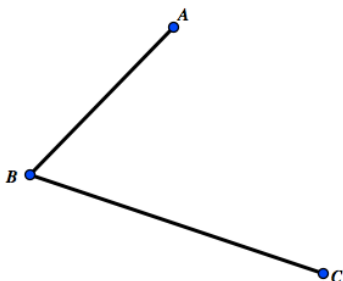


- In the problem above you have demonstrated one way to find the midpoint of a line segment. Explain another way that a line segment can be bisected without the use of circles.

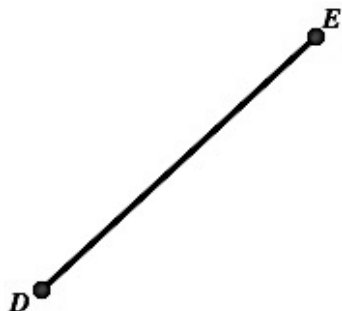
**SET**

Topic: Constructions with compass and straight edge.

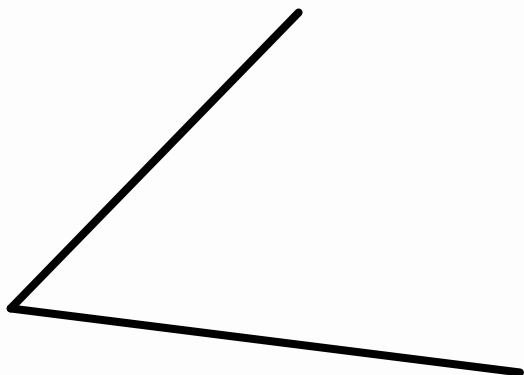
- Bisect the angle below do it with compass and straight edge as well as with paper folding.



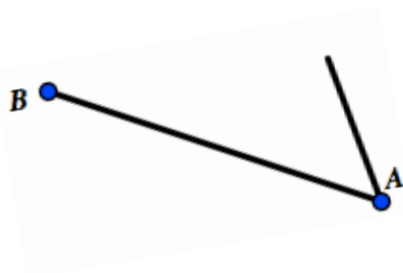
4. Copy the segment below using construction tools of compass and straight edge, label the image  $D'E'$ .



5. Copy the angle below using construction tool of compass and straight edge.



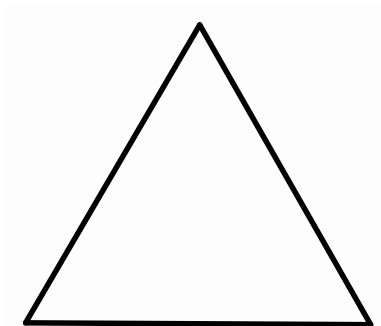
6. Construct a rhombus on the segment  $AB$  that is given below and that has point  $A$  as a vertex. Be sure to check that your final figure is a rhombus.



7. Construct a square on the segment CD that is given below. Be sure to check that your final figure is a square.



8. Given the equilateral triangle below, find the center of rotation of the triangle using compass and straight edge.



### GO

Topic: Solving systems of equations

Solve each system of equations. Utilize substitution, elimination, graphing or matrices.

9.  $\begin{cases} x = 11 + y \\ 2x + y = 19 \end{cases}$

10.  $\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases}$

11.  $\begin{cases} x + 2y = 11 \\ x - 4y = 2 \end{cases}$

12.  $\begin{cases} y = -x + 1 \\ y = 2x + 1 \end{cases}$

13.  $\begin{cases} y = -2x + 7 \\ -3x + y = -8 \end{cases}$

14.  $\begin{cases} 4x - y = 7 \\ -6x + 2y = 8 \end{cases}$

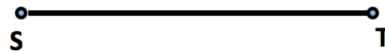
## 7.2 More Things Under Construction

### *A Develop Understanding Task*



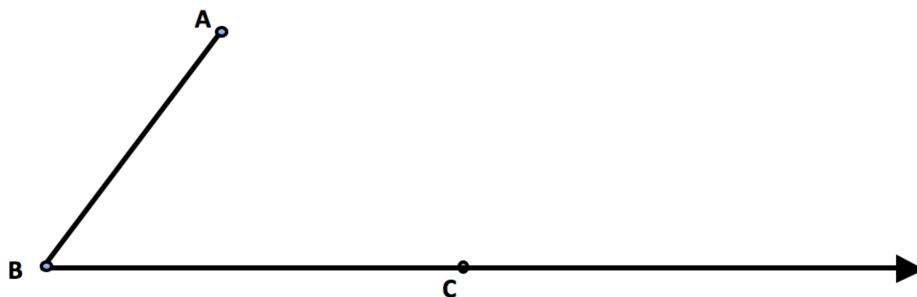
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<https://flic.kr/n/7eHdDP>

Like a rhombus, an equilateral triangle has three congruent sides. Show and describe how you might locate the third vertex point on an equilateral triangle, given  $\overline{ST}$  below as one side of the equilateral triangle.



### Constructing a Parallelogram

To construct a parallelogram we will need to be able to construct a line parallel to a given line through a given point. For example, suppose we want to construct a line parallel to segment  $\overline{AB}$  through point  $C$  on the diagram below. Since we have observed that parallel lines have the same slope, the line through point  $C$  will be parallel to  $\overline{AB}$  only if the angle formed by the line and  $\overrightarrow{BC}$  is congruent to  $\angle ABC$ . Can you describe and illustrate a strategy that will construct an angle with vertex at point  $C$  and a side parallel to  $\overline{AB}$ ?



### Constructing a Hexagon Inscribed in a Circle

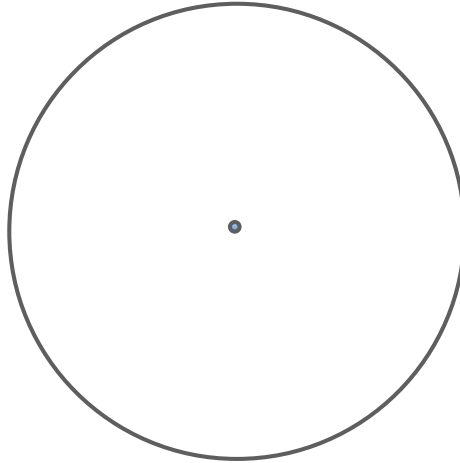
Because regular polygons have rotational symmetry, they can be *inscribed* in a circle. The *circumscribed* circle has its center at the center of rotation and passes through all of the vertices of the regular polygon.

We might begin constructing a hexagon by noticing that a hexagon can be decomposed into six congruent equilateral triangles, formed by three of its lines of symmetry.

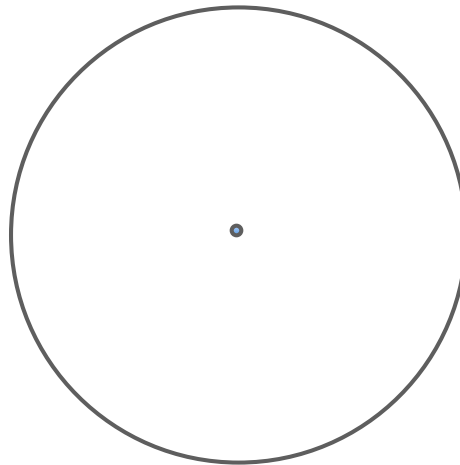
1. Sketch a diagram of such a decomposition.
2. Based on your sketch, where is the center of the circle that would circumscribe the hexagon?
3. The six vertices of the hexagon lie on the circle in which the regular hexagon is inscribed. The six sides of the hexagon are *chords* of the circle. How are the lengths of these chords related to the lengths of the radii from the center of the circle to the vertices of the hexagon? That is, how do you know that the six triangles formed by drawing the three lines of symmetry are equilateral triangles? (Hint: Considering angles of rotation, can you convince yourself that these six triangles are equiangular, and therefore equilateral?)



4. Based on this analysis of the regular hexagon and its circumscribed circle, illustrate and describe a process for constructing a hexagon inscribed in the circle given below.



5. Modify your work with the hexagon to construct an equilateral triangle inscribed in the circle given below.



6. Describe how you might construct a square inscribed in a circle.

READY, SET, GO!

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Period \_\_\_\_\_

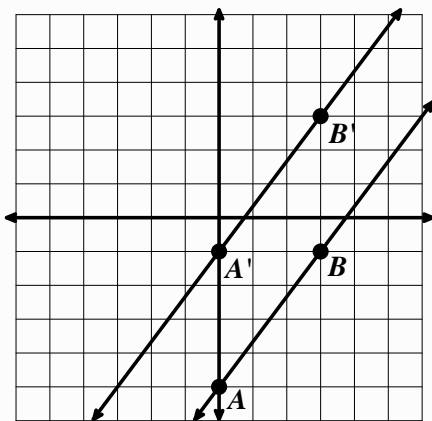
Date \_\_\_\_\_

**READY**

Topic: Transformation of lines, connecting geometry and algebra.

For each set of lines use the points on the line to determine which line is the image and which is the pre-image, write image by the image line and pre image by the original line. Then define the transformation that was used to create the image. Finally find the equation for each line.

1.

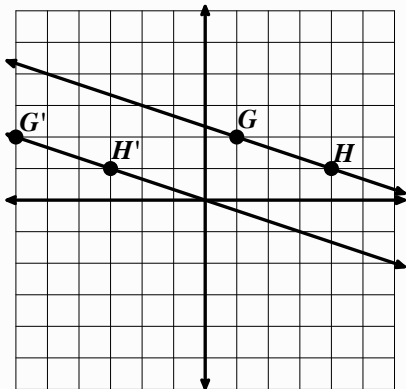


a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

3.

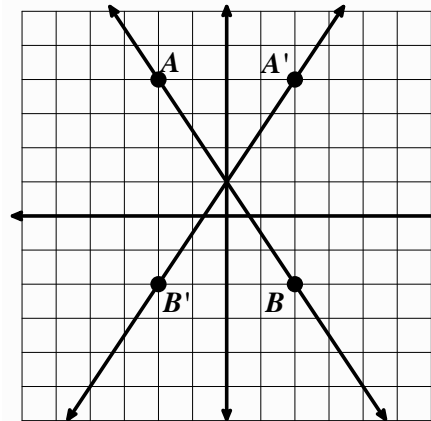


a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

2.

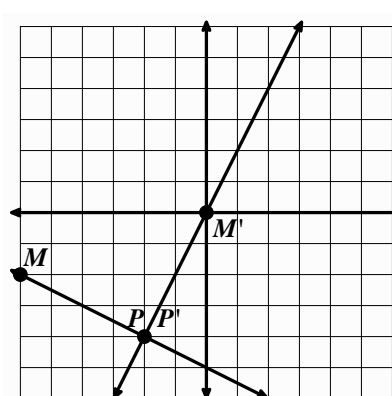


a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

4.



a. Description of Transformation:

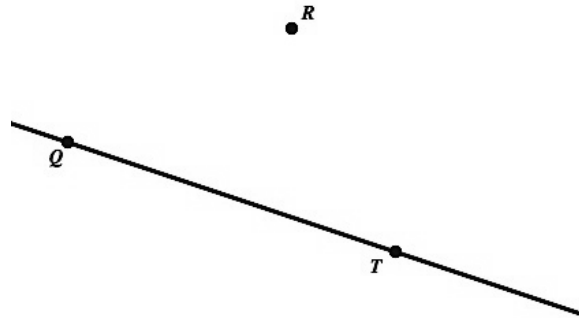
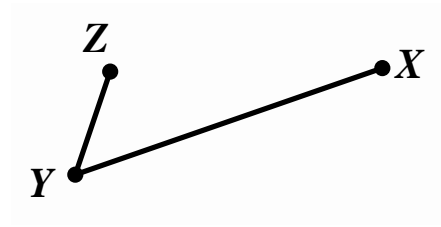
b. Equation for pre-image:

c. Equation for image:

**SET**

Topic: Geometric constructions with compass and straight edge.

5. Construct a parallelogram given sides  $\overline{XY}$  and  $\overline{YZ}$  and  $\angle XYZ$ .      6. Construct a line parallel to  $\overline{QT}$  and through point  $R$ .



7. Given segment  $\overline{AB}$  show all points  $C$  such that  $\triangle ABC$  is an isosceles triangle.



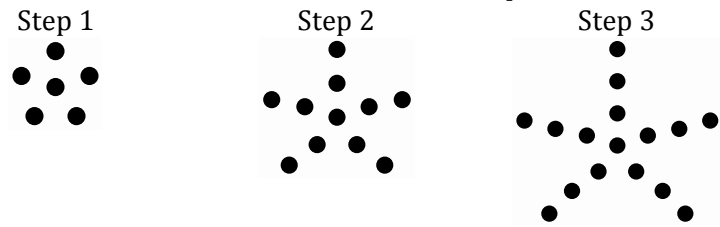
8. Given segment  $\overline{AB}$  show all points  $C$  such that  $\triangle ABC$  is a right triangle.



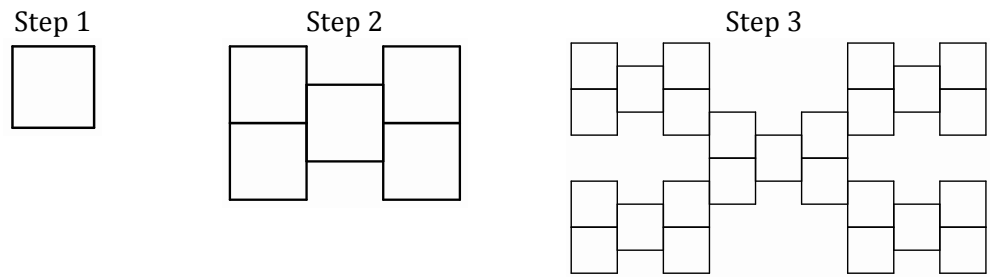
GO

Topic: Creating explicit and recursive rules for visual patterns

9. Find an explicit function rule and a recursive rule for dots in step  $n$ .



10. Find an explicit function rule and a recursive rule for squares in step  $n$ .



Find an explicit function rule and a recursive rule for the values in each table.

11.

Step	Value
1	1
2	11
3	21
4	31

12.

$n$	$f(n)$
2	16
3	8
4	4
5	2

13.

$n$	$f(n)$
1	-5
2	25
3	-125
4	625

## 7.3 Can You Get There From Here?

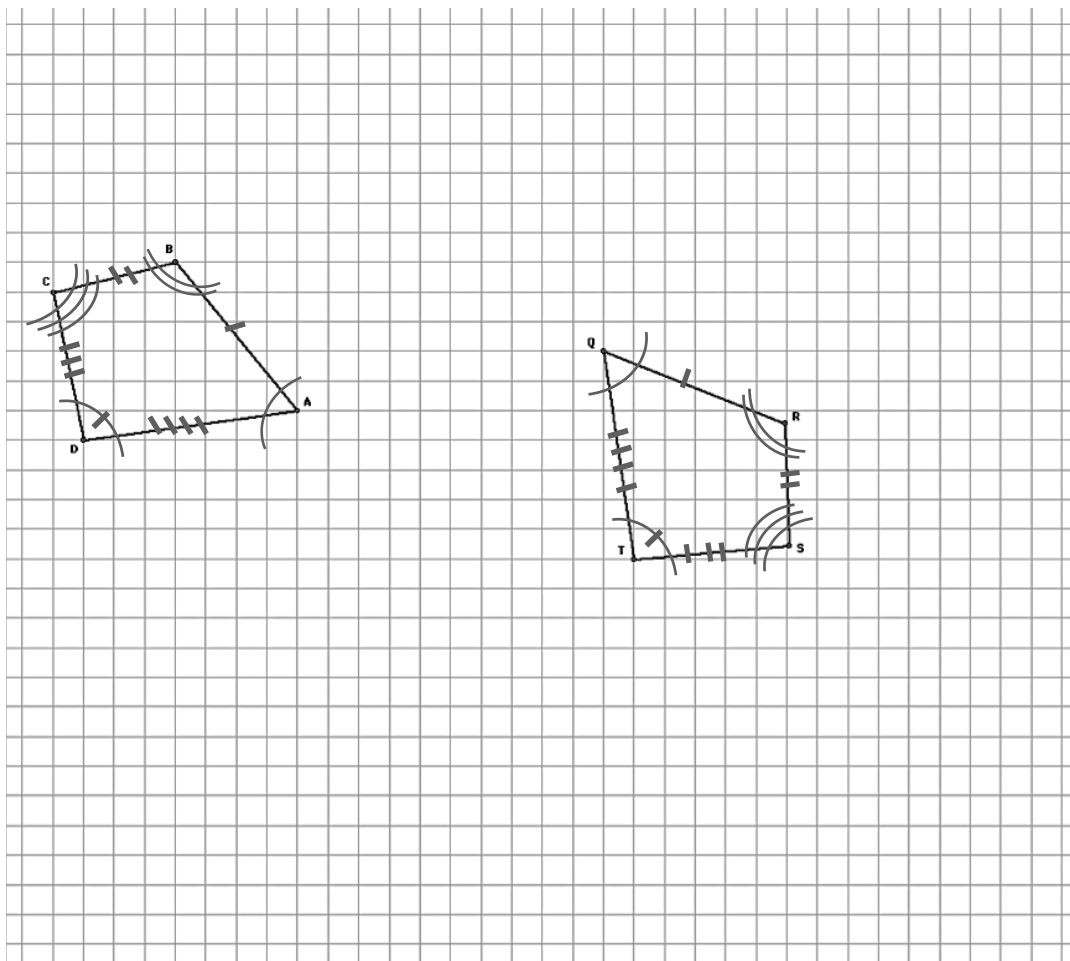


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### *A Develop Understanding Task*

The two quadrilaterals shown below, quadrilateral  $ABCD$  and quadrilateral  $QRST$  are congruent, with corresponding congruent parts marked in the diagrams.

Describe a sequence of rigid-motion transformations that will carry quadrilateral  $ABCD$  onto quadrilateral  $QRST$ . Be very specific in describing the sequence and types of transformations you will use so that someone else could perform the same series of transformations.



READY, SET, GO!

Name \_\_\_\_\_

Period \_\_\_\_\_

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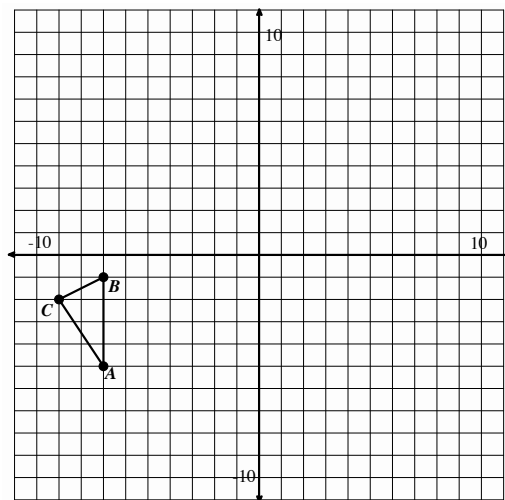
**READY**

Topic: Multiple transformations

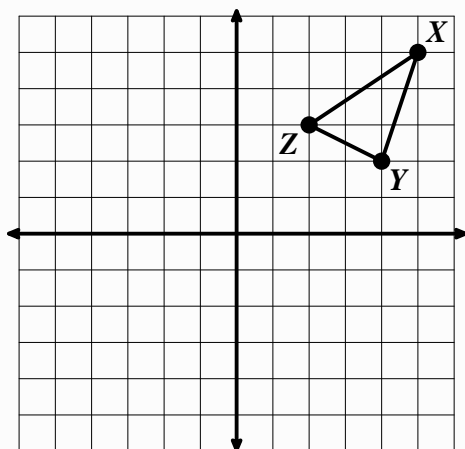
The given figures are to be used as pre-images. Perform the stated transformations to obtain an image. Label the corresponding parts of the image in accordance with the pre-image.

1. Reflect triangle  $ABC$  over the line  $y = x$  and label the image  $A'B'C'$ .

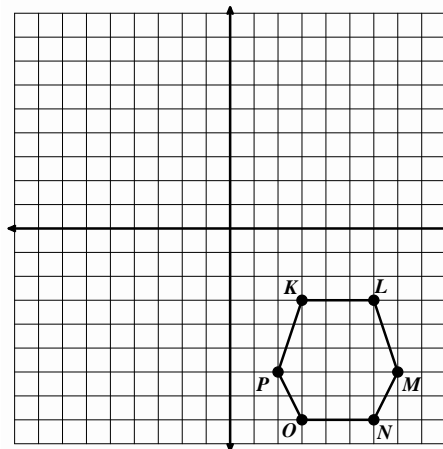
Rotate triangle  $A'B'C'$   $180^\circ$  counter clockwise around the origin and label the image  $A''B''C''$ .



2. Reflect over the line  $y = -x$ .



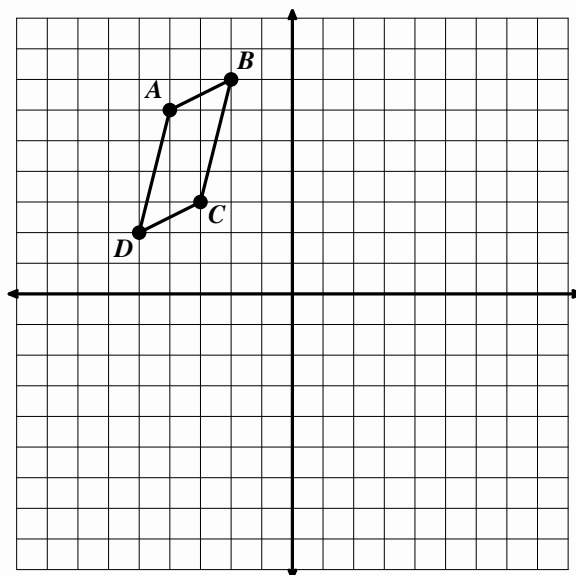
3. Reflect over y-axis and then  
Rotate clockwise  $90^\circ$  around  $P'$ .



## 7.3

4. Reflect quadrilateral ABCD over the line  $y = 2 + x$  and label the image A'B'C'D'.

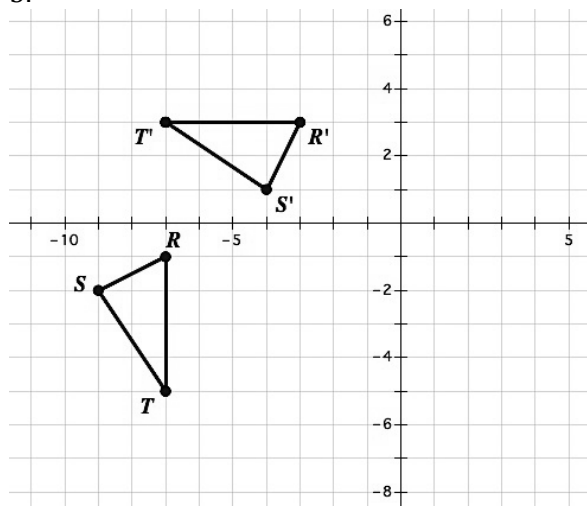
Rotate quadrilateral A'B'C'D' counter-clockwise  $90^\circ$  around  $(-2, -3)$  as the center of rotation label the image A''B''C''D''.

**SET**

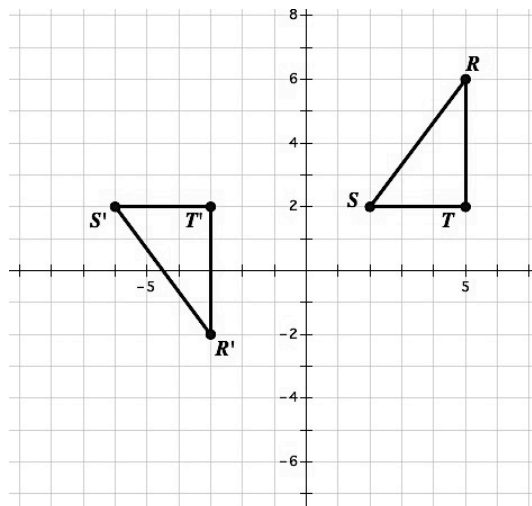
Topic: Find the sequence of transformations.

Find a sequence of transformations that will carry triangle **RST** onto triangle **R'S'T'**. Clearly describe the sequence of transformations below each grid.

5.



6.

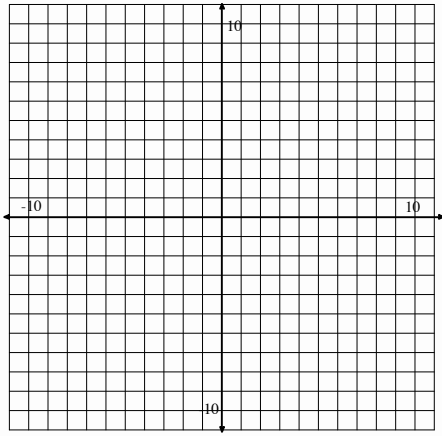


**GO**

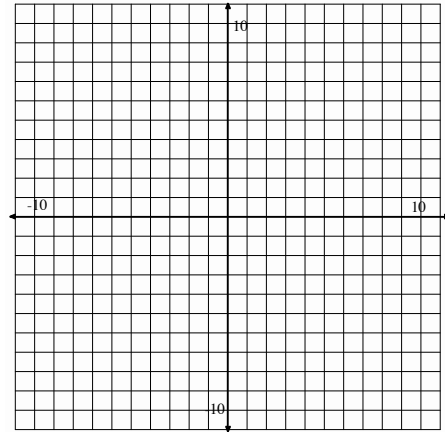
Topic: Graphing systems of functions and making comparisons.

Graph each pair of functions and make an observation about how the functions compare to one another.

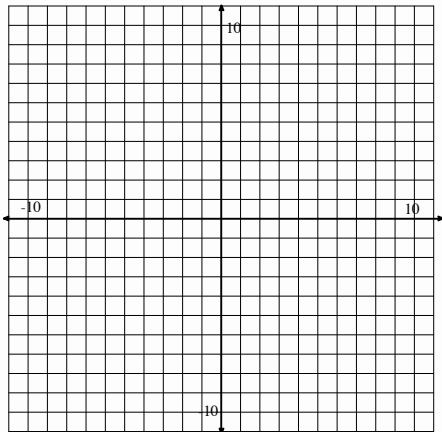
7.  $y = \frac{1}{3}x - 1$   
 $y = -3x - 1$



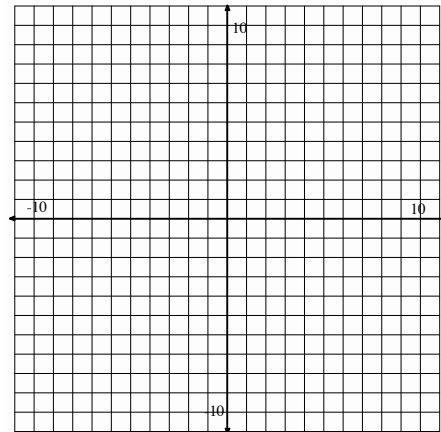
8.  $y = -\frac{2}{3}x + 5$   
 $y = \frac{3}{2}x + 5$



9.  $y = \frac{1}{4}x + 2$   
 $y = -\frac{1}{4}x + 2$



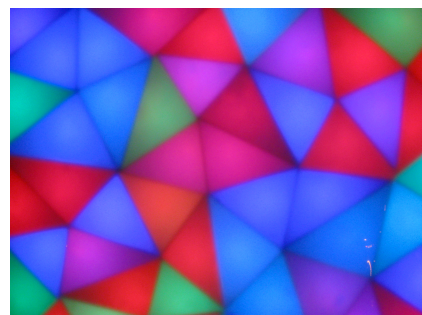
10.  $y = 2^x$   
 $y = -2^x$





## 7.4 Congruent Triangles

### *A Solidify Understanding Task*

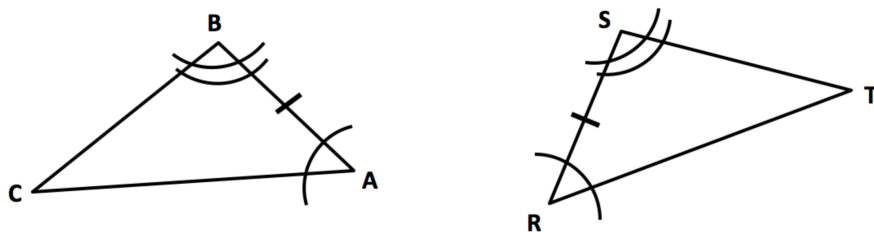


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We know that two triangles are congruent if all pairs of corresponding sides are congruent and all pairs of corresponding angles are congruent. We may wonder if knowing less information about the triangles would still guarantee they are congruent.

For example, we may wonder if knowing that two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of another triangle—a set of criteria we will refer to as ASA—is enough to know that the two triangles are congruent. And, if we think this is enough information, how might we justify that this would be so.

Here is a diagram illustrating ASA criteria for triangles:



1. Based on the diagram, which angles are congruent? Which sides?
2. To convince ourselves that these two triangles are congruent, what else would we need to know?
3. Use tracing paper to find a sequence of transformations that will show whether or not these two triangles are congruent.
4. List your sequence of transformations here:

Your sequence of transformations is enough to show that these two triangles are congruent, but how can we guarantee that *all* pairs of triangles that share ASA criteria are congruent?

Perhaps your sequence of transformations looked like this:

- **translate** point  $A$  until it coincides with point  $R$
- **rotate**  $\overline{AB}$  about point  $R$  until it coincides with  $\overline{RS}$
- **reflect**  $\triangle ABC$  across  $\overline{RS}$

We can use the word “coincides” when we want to say that two points or line segments occupy the same position on the plane. When making arguments using transformations we will use the word a lot.

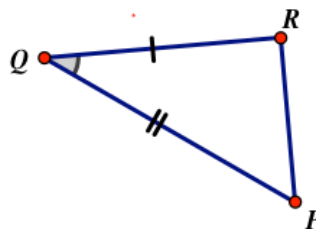
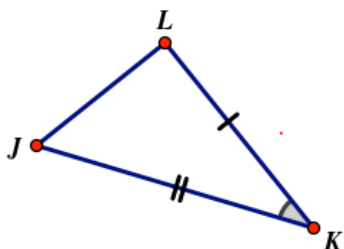
Now the question is, how do we know that point  $C$  has to land on point  $T$  after the reflection, making all of the sides and angles coincide?

5. *Answer this question as best you can to justify why ASA criteria guarantees two triangles are congruent. To answer this question, it may be helpful to think about how you know point  $C$  can't land anywhere else in the plane except on top of  $T$ .*

Using tracing paper, experiment with these additional pairs of triangles. Try to determine if you can find a sequence of transformations that will show if the triangles are congruent. Be careful, there may be some that aren't. If the triangles appear to be congruent based on your experimentation, write an argument to explain how you know that this type of criteria will always work. That is, what guarantees that the unmarked sides or angles must also coincide?

6. Given criteria: \_\_\_\_\_

Are the triangles congruent? \_\_\_\_\_

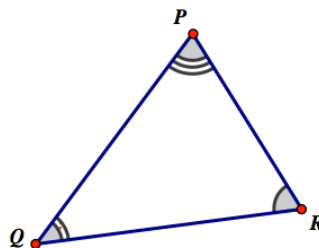
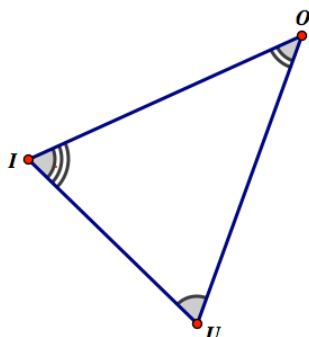


List your transformations in the order performed:

*If the triangles are congruent, justify why this will always be true based on this criteria:*

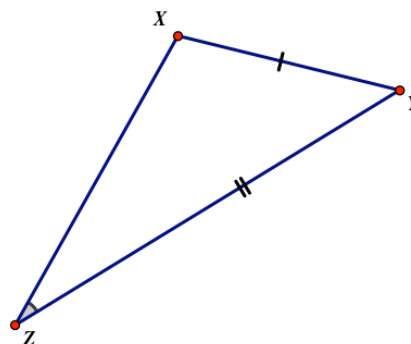
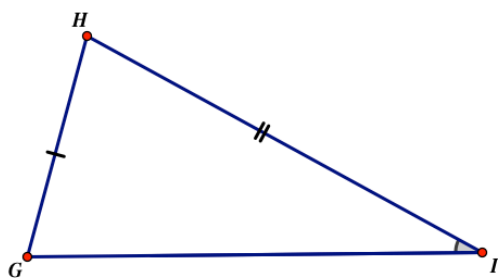
7. Given information: \_\_\_\_\_

Are the triangles congruent? \_\_\_\_\_

List your transformations  
in the order performed:*If the triangles are congruent, justify why this  
will always be true based on this criteria:*

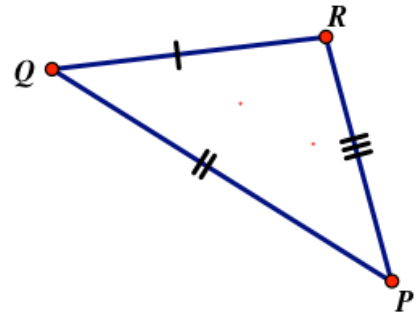
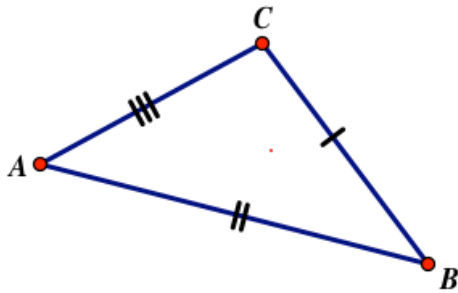
8. Given information: \_\_\_\_\_

Are the triangles congruent? \_\_\_\_\_

List your transformations  
in the order performed:*If the triangles are congruent, justify why this  
will always be true based on this criteria:*

9. Given information: \_\_\_\_\_

Are the triangles congruent? \_\_\_\_\_



List your transformations  
in the order performed:

*If the triangles are congruent, justify why this  
will always be true based on this criteria:*

10. Based on these experiments and your justifications, what criteria or conditions seem to guarantee that two triangles will be congruent? List as many cases as you can. Make sure you include ASA from the triangles we worked with first.
11. Your friend wants to add AAS to your list, even though you haven't experimented with this particular case. What do you think? Should AAS be added or not? What convinces you that you are correct?
12. Your friend also wants to add HL (hypotenuse-leg) to your list, even though you haven't experimented with right triangles at all, and you know that SSA doesn't work in general from problem 8. What do you think? Should HL for right triangles be added or not? What convinces you that you are correct?

READY, SET, GO!

Name \_\_\_\_\_

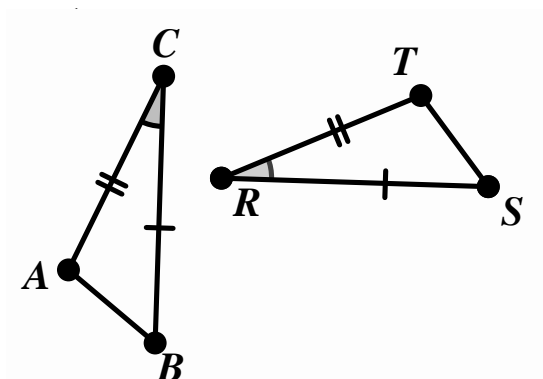
Period \_\_\_\_\_

Date \_\_\_\_\_

**READY**

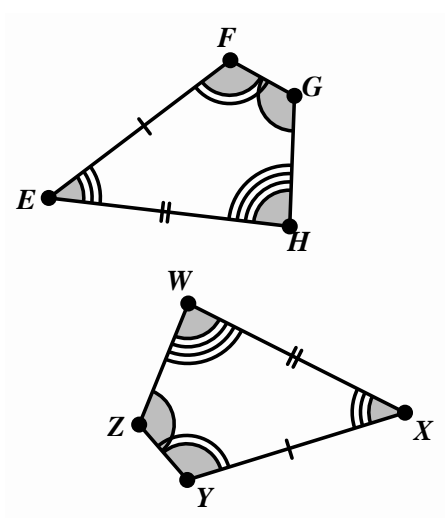
Topic: Corresponding parts of figures and transformations.

Given the figures in each sketch with congruent angles and sides marked, first list the parts of the figures that correspond (For example, in #1,  $\angle C \cong \angle R$ ) Then determine if a reflection occurred as part of the sequence of transformations that was used to create the image.

Congruencies

$$\angle C \cong \angle R$$

Reflected? Yes or No

Congruencies

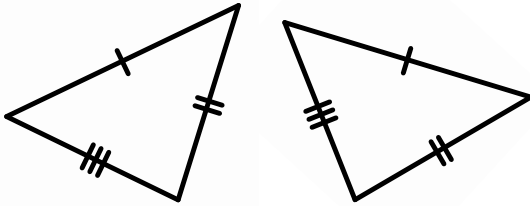
Reflected? Yes or No

**SET**

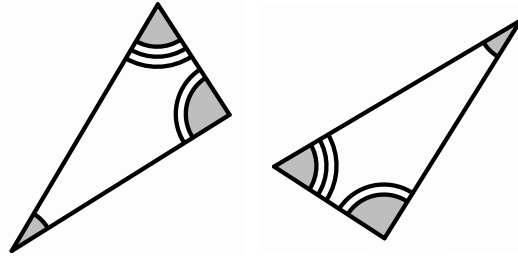
Topic: Triangle Congruence

**Explain whether or not the triangles are congruent, similar, or neither based on the markings that indicate congruence.**

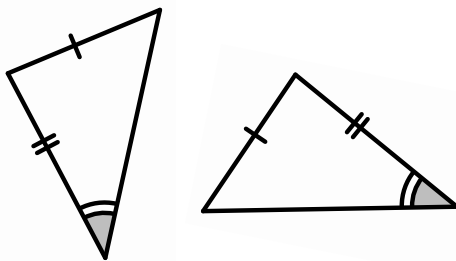
3.



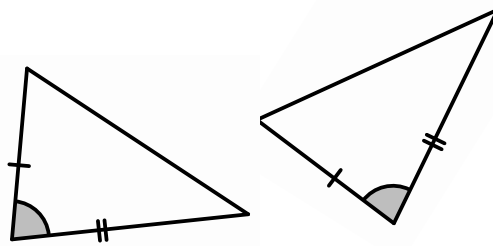
4.



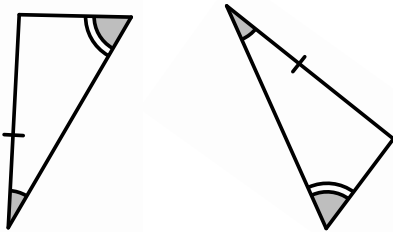
5.



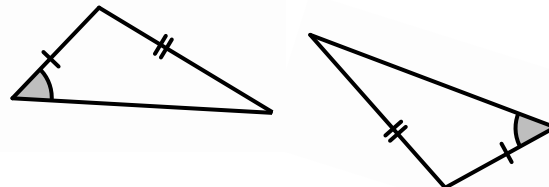
6.



7.



8.



**Use the given congruence statement to draw and label two triangles that have the proper corresponding parts congruent to one another.**

9.  $\triangle ABC \cong \triangle PQR$

10.  $\triangle XYZ \cong \triangle KLM$

**GO**

Topic: Solving equations and finding recursive rules for sequences.

**Solve each equation for  $t$ .**

11.  $\frac{3t-4}{5} = 5$

12.  $10 - t = 4t + 12 - 3t$

13.  $P = 5t - d$

14.  $xy - t = 13t + w$

**Use the given sequence of number to write a recursive rule for the  $n$ th value of the sequence.**

15. 5, 15, 45, ...

16.  $\frac{1}{2}, 0, -\frac{1}{2}, -1, \dots$

17. 3, -6, 12, -24, ...

18.  $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$

## 7.5 Congruent Triangles to the Rescue

### *A Practice Understanding Task*

#### **Part 1**

Zac and Sione are exploring isosceles triangles—triangles in which two sides are congruent:

*Zac:* I think every isosceles triangle has a line of symmetry that passes through the vertex point of the angle made up by the two congruent sides, and the midpoint of the third side.

*Sione:* That's a pretty big claim—to say you know something about *every* isosceles triangle. Maybe you just haven't thought about the ones for which it isn't true.

*Zac:* But I've folded lots of isosceles triangles in half, and it always seems to work.

*Sione:* *Lots* of isosceles triangles are not *all* isosceles triangles, so I'm still not sure.

1. What do you think about Zac's claim? Do you think every isosceles triangle has a line of symmetry? If so, what convinces you this is true? If not, what concerns do you have about his statement?
2. What else would Zac need to know about the crease line through in order to know that it is a line of symmetry? (Hint: Think about the definition of a line of reflection.)
3. Sione thinks Zac's "crease line" (the line formed by folding the isosceles triangle in half) creates two congruent triangles inside the isosceles triangle. Which criteria—ASA, SAS or SSS—could he use to support this claim? Describe the sides and/or angles you think are congruent, and explain how you know they are congruent.
4. If the two triangles created by folding an isosceles triangle in half are congruent, what does that imply about the "base angles" of an isosceles triangle (the two angles that are not formed by the two congruent sides)?





5. If the two triangles created by folding an isosceles triangle in half are congruent, what does that imply about the “crease line”? (You might be able to make a couple of claims about this line—one claim comes from focusing on the line where it meets the third, non-congruent side of the triangle; a second claim comes from focusing on where the line intersects the vertex angle formed by the two congruent sides.)

## **Part 2**

Like Zac, you have done some experimenting with lines of symmetry, as well as rotational symmetry. In the tasks *Symmetries of Quadrilaterals* and *Quadrilaterals—Beyond Definition* you made some observations about sides, angles, and diagonals of various types of quadrilaterals based on your experiments and knowledge about transformations. Many of these observations can be further justified based on looking for congruent triangles and their corresponding parts, just as Zac and Sione did in their work with isosceles triangles.

Pick one of the following quadrilaterals to explore:

- A **rectangle** is a quadrilateral that contains four right angles.
  - A **rhombus** is a quadrilateral in which all sides are congruent.
  - A **square** is both a rectangle and a rhombus, that is, it contains four right angles and all sides are congruent
1. Draw an example of your selected quadrilateral, with its diagonals. Label the vertices of the quadrilateral  $A$ ,  $B$ ,  $C$ , and  $D$ , and label the point of intersection of the two diagonals as point  $N$ .
  2. Based on (1) your drawing, (2) the given definition of your quadrilateral, and (3) information about sides and angles that you can gather based on lines of reflection and rotational symmetry, list as many pairs of congruent triangles as you can find.
  3. For each pair of congruent triangles you list, state the criteria you used—ASA, SAS or SSS—to determine that the two triangles are congruent, and explain how you know that the angles and/or sides required by the criteria are congruent (see the following chart).

Congruent Triangles	Criteria Used (ASA, SAS, SSS)	How I know the sides and/or angles required by the criteria are congruent
If I say $\triangle RST \cong \triangle XYZ$	based on SSS	then I need to explain: <ul style="list-style-type: none"> <li>• how I know that <math>\overline{RS} \cong \overline{XY}</math>, and</li> <li>• how I know that <math>\overline{ST} \cong \overline{YZ}</math>, and</li> <li>• how I know that <math>\overline{TR} \cong \overline{ZX}</math></li> </ul> so I can use SSS criteria to say $\triangle RST \cong \triangle XYZ$

4. Now that you have identified some congruent triangles in your diagram, can you use the congruent triangles to justify something else about the quadrilateral, such as:

- the diagonals bisect each other
- the diagonals are congruent
- the diagonals are perpendicular to each other
- the diagonals bisect the angles of the quadrilateral

Pick one of the bulleted statements you think is true about your quadrilateral and try to write an argument that would convince Zac and Sione that the statement is true.

READY, SET, GO!

Name \_\_\_\_\_

Period \_\_\_\_\_

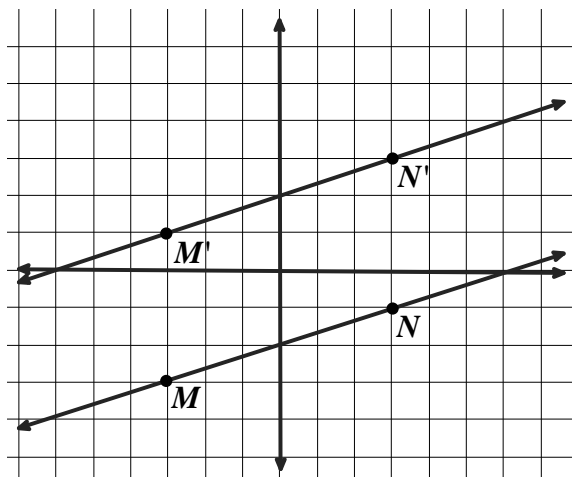
Date \_\_\_\_\_

**READY**

Topic: Transformations of lines, connecting geometry and algebra.

For each set of lines use the points on the line to determine which line is the image and which is the pre-image, write image by the image line and pre image by the original line. Then define the transformation that was used to create the image. Finally find the equation for each line.

1.

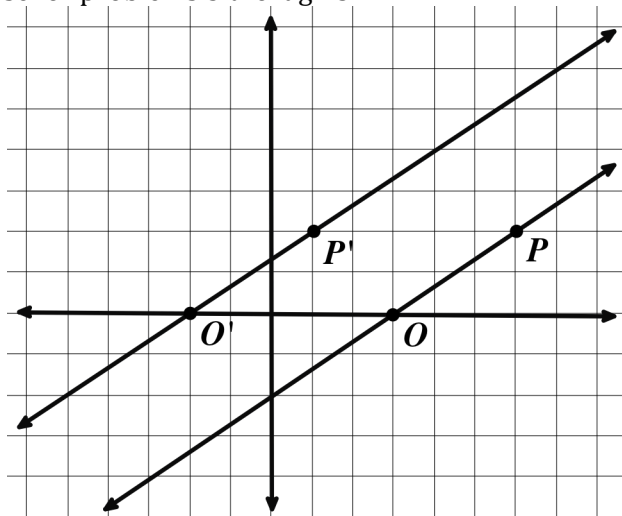


a. Description of Transformation:

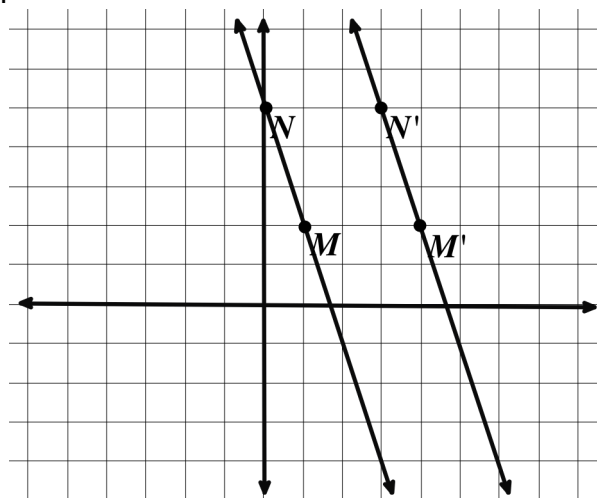
b. Equation for pre-image:

c. Equation for image:

Use for problems 3 through 5.



2.



a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

3. a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

4. Write an equation for a line with the same slope that goes through the origin.

5. Write the equation of a line perpendicular to these and through the point  $O'$ .

After working with these equations and seeing the transformations on the coordinate graph it is good timing to consider similar work with tables.

6. Match the table of values below with the proper function rule.

I	II	III	IV	V																																																		
<table><tr><th><math>x</math></th><th><math>f(x)</math></th></tr><tr><td>-1</td><td>16</td></tr><tr><td>0</td><td>14</td></tr><tr><td>1</td><td>12</td></tr><tr><td>2</td><td>10</td></tr></table>	$x$	$f(x)$	-1	16	0	14	1	12	2	10	<table><tr><th><math>x</math></th><th><math>f(x)</math></th></tr><tr><td>-1</td><td>14</td></tr><tr><td>0</td><td>12</td></tr><tr><td>1</td><td>10</td></tr><tr><td>2</td><td>8</td></tr></table>	$x$	$f(x)$	-1	14	0	12	1	10	2	8	<table><tr><th><math>x</math></th><th><math>f(x)</math></th></tr><tr><td>-1</td><td>12</td></tr><tr><td>0</td><td>10</td></tr><tr><td>1</td><td>8</td></tr><tr><td>2</td><td>6</td></tr></table>	$x$	$f(x)$	-1	12	0	10	1	8	2	6	<table><tr><th><math>x</math></th><th><math>f(x)</math></th></tr><tr><td>-1</td><td>10</td></tr><tr><td>0</td><td>8</td></tr><tr><td>1</td><td>6</td></tr><tr><td>2</td><td>4</td></tr></table>	$x$	$f(x)$	-1	10	0	8	1	6	2	4	<table><tr><th><math>x</math></th><th><math>f(x)</math></th></tr><tr><td>-1</td><td>8</td></tr><tr><td>0</td><td>6</td></tr><tr><td>1</td><td>4</td></tr><tr><td>2</td><td>2</td></tr></table>	$x$	$f(x)$	-1	8	0	6	1	4	2	2
$x$	$f(x)$																																																					
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A.  $f(x) = -2(x - 1) + 8$

D.  $f(x) = -2(x + 1) + 8$

B.  $f(x) = -2(x - 1) + 12$

E.  $f(x) = -2(x + 1) + 10$

C.  $f(x) = -2(x - 2) + 8$

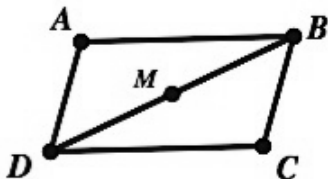
### SET

Topic: Use Triangle Congruence Criteria to justify conjectures.

In each problem below there are some true statements listed. From these statements a conjecture (a guess) about what might be true has been made. Using the given statements and conjecture statement create an argument that justifies the conjecture.

7. True statements:

Point  $M$  is the midpoint of  $\overline{DB}$   
 $\angle ABD \cong \angle BDC$   
 $\overline{AB} \cong \overline{DC}$



Conjecture:  $\angle A \cong \angle C$

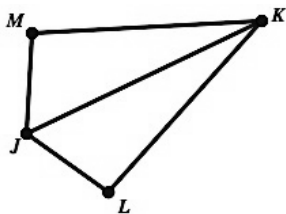
a. Is the conjecture correct?

b. Argument to prove you are right:

8. True statements

$$\angle KJL \cong \angle KJM$$

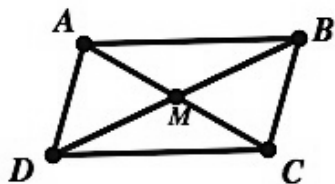
$$\overline{JL} \cong \overline{JM}$$

Conjecture:  $\overline{JK}$  bisects  $\angle MKL$ 

a. Is the conjecture correct?

b. Argument to prove you are right:

9. True statements

 $\triangle ADM$  is a  $180^\circ$   
rotation of  $\triangle CMB$ 
Conjecture:  $\triangle ABM \cong \triangle CDM$ 

a. Is the conjecture correct?

b. Argument to prove you are right:

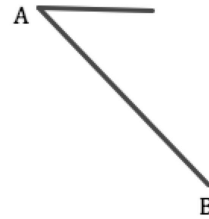
**GO**

Topic: Constructions with compass and straight edge.

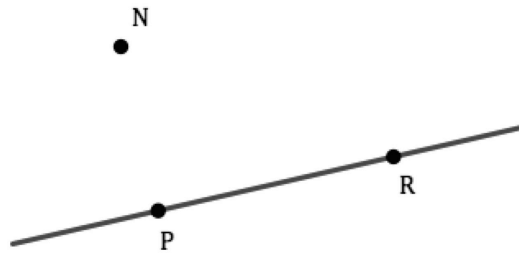
10. Why do we use a geometric compass when doing constructions in geometry?

Perform the indicated constructions using a compass and straight edge.

11. Construct a rhombus, use segment AB as one side and angle A as one of the angles.



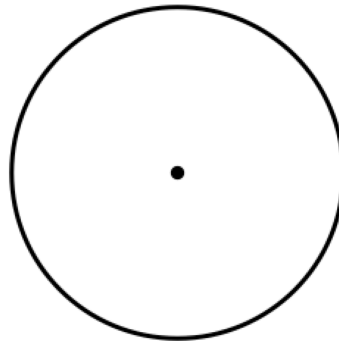
12. Construct a line parallel to line PR and through the point N.



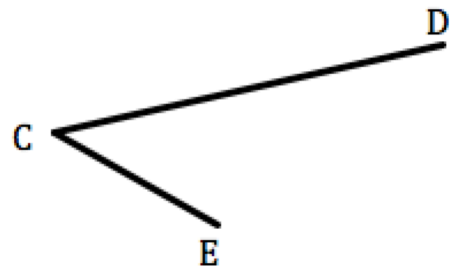
13. Construct an equilateral triangle with segment RS as one side.



14. Construct a regular hexagon inscribed in the circle provided.



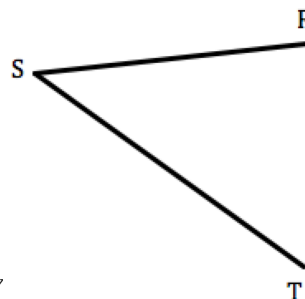
15. Construct a parallelogram using CD as one side and CE as the other side.



16. Bisect the line segment LM.



17. Bisect the angle RST.



## 7.6 Justifying Constructions

### *A Solidify Understanding Task*



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Compass and straightedge constructions can be justified using such tools as:


- the definitions and properties of the rigid-motion transformations
- identifying corresponding parts of congruent triangles
- using observations about sides, angles and diagonals of special types of quadrilaterals

Study the steps of the following procedure for *constructing an angle bisector*, and complete the illustration based on the descriptions of the steps.

Steps	Illustration
Using a compass, draw an arc (portion of a circle) that intersects each ray of the angle to be bisected, with the center of the arc located at the vertex of the angle.	
Without changing the span of the compass, draw two arcs in the interior of the angle, with the center of the arcs located at the two points where the first arc intersected the rays of the angle.	
With the straightedge, draw a ray from the vertex of the angle through the point where the last two arcs intersect.	

Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

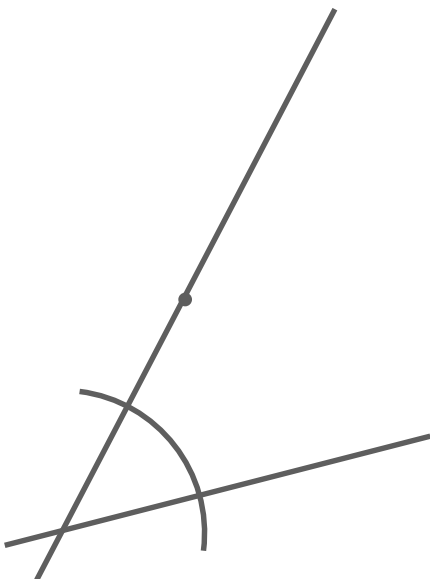
Study the steps of the following procedure for *constructing a line perpendicular to a given line through a given point*, and complete the illustration based on the descriptions of the steps.

Steps	Illustration
Using a compass, draw an arc (portion of a circle) that intersects the given line at two points, with the center of the arc located at the given point.	
Without changing the span of the compass, locate a second point on the other side of the given line, by drawing two arcs on the same side of the line, with the center of the arcs located at the two points where the first arc intersected the line.	
With the straightedge, draw a line through the given point and the point where the last two arcs intersect.	

Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.



Study the steps of the following procedure for constructing *a line parallel to a given line through a given point*, and complete the illustration based on the descriptions of the steps.

Steps	Illustration
Using a straightedge, draw a line through the given point to form an arbitrary angle with the given line.	
Using a compass, draw an arc (portion of a circle) that intersects both rays of the angle formed, with the center of the arc located at the point where the drawn line intersects the given line.	
Without changing the span of the compass, draw a second arc on the same side of the drawn line, centered at the given point. The second arc should be as long or longer than the first arc, and should intersect the drawn line.	
Set the span of the compass to match the distance between the two points where the first arc crosses the two lines. Without changing the span of the compass, draw a third arc that intersects the second arc, centered at the point where the second arc intersects the drawn line.	
With the straightedge, draw a line through the given point and the point where the last two arcs intersect.	

Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

READY, SET, GO!

Name \_\_\_\_\_

Period \_\_\_\_\_

Date \_\_\_\_\_

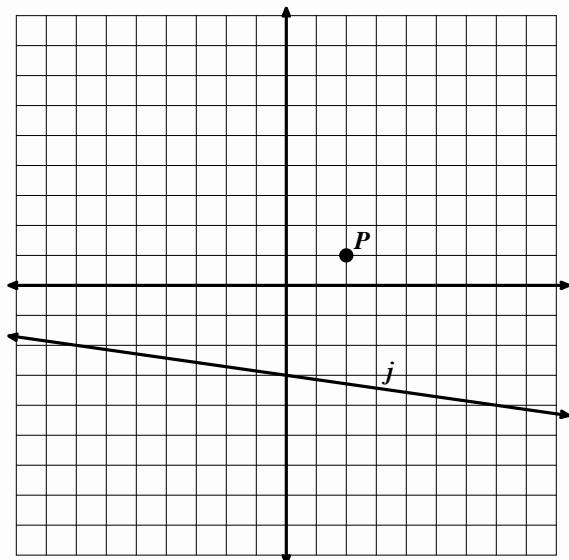
### READY

Topic: Rotational symmetry in regular polygons and with transformations.

1. What angles of rotational symmetry are there for a regular pentagon?
2. What angles of rotational symmetry are there for a regular hexagon?
3. If a regular polygon has an angle of rotational symmetry that is  $40^\circ$ , how many sides does the polygon have?

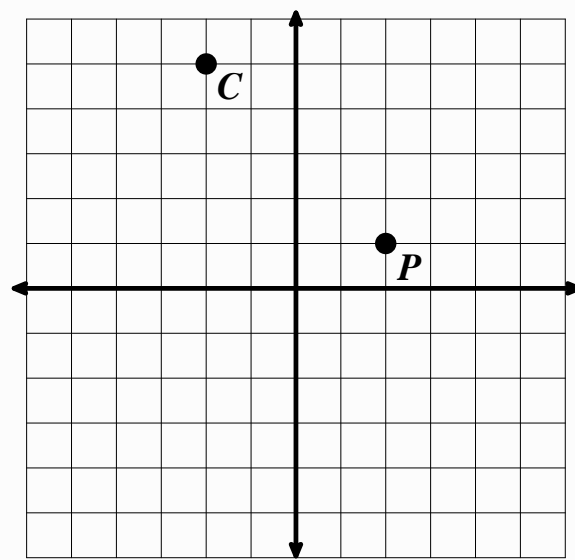
On each given coordinate grid below perform the indicated transformation.

4.



Reflect point  $P$  over line  $j$ .

5.



Rotate point  $P$   $90^\circ$  clockwise around point  $C$ .

**SET**

Topic: Use Triangle Congruence Criteria to justify conjectures.

6. Construct an isosceles triangle that incorporates  $\overline{CD}$  as one of the sides. Construct the circumscribed circle around the triangle.



7. Construct a regular hexagon that incorporates  $\overline{CD}$  as one of the sides. Construct the circumscribed circle around the hexagon.



8. Construct a square that incorporates  $\overline{CD}$  as one of the sides. Construct the circumscribed circle around the square.



**GO**

Topic: Finding Distance and Slope.

**For each pair of given coordinate points find distance between them and find the slope of the line that passes through them. Show all your work.**

9.  $(-2, 8), (3, -4)$

a. Slope:

b. Distance:

10.  $(-7, -3), (1, 5)$

a. Slope:

b. Distance:

11.  $(3, 7), (-5, 9)$

a. Slope:

b. Distance:

12.  $(1, -5), (-7, 1)$

a. Slope:

b. Distance:

13.  $(-10, 31), (20, 11)$

a. Slope:

b. Distance:

14.  $(16, -45), (-34, 75)$

a. Slope:

b. Distance: