mathematics vision project

Transforming Mathematics Education

SECONDARY MATH ONE

An Integrated Approach

MODULE 8

Connecting Algebra & Geometry

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

Scott Hendrickson, Joleigh Honey, Barbara Kuehl, Travis Lemon, Janet Sutorius

MODULE 8 - TABLE OF CONTENTS

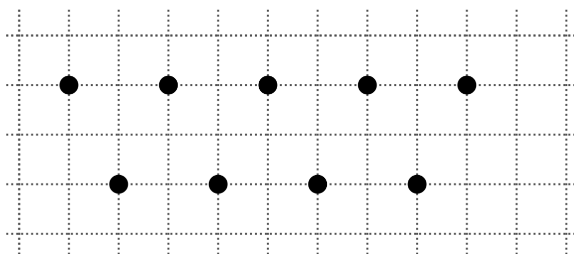
CONNECTING ALGEBRA AND GEOMETRY

8.1 Go the Distance

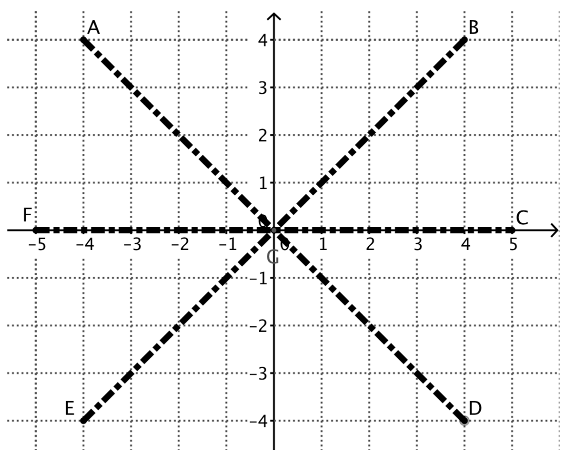
A Develop Understanding Task

CC BY https://flic.kr/p/rFBHk

The performances of the Podunk High School drill team are very popular during half-time at the school's football and basketball games. When the Podunk High School drill team choreographs the dance moves that they will do on the football field, they lay out their positions on a grid like the one below:



In one of their dances, they plan to make patterns holding long, wide ribbons that will span from one dancer in the middle to six other dancers. On the grid, their pattern looks like this:

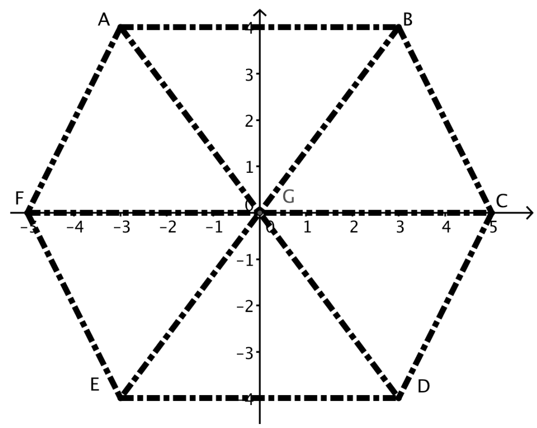


The question the dancers have is how long to make the ribbons. Gabriela (G) is standing in the center and some dancers think that the ribbon from Gabriela (G) to Courtney (C) will be shorter than the one from Gabriela (G) to Brittney (B).

1. How long does each ribbon need to be?

2. Explain how you found the length of each ribbon.

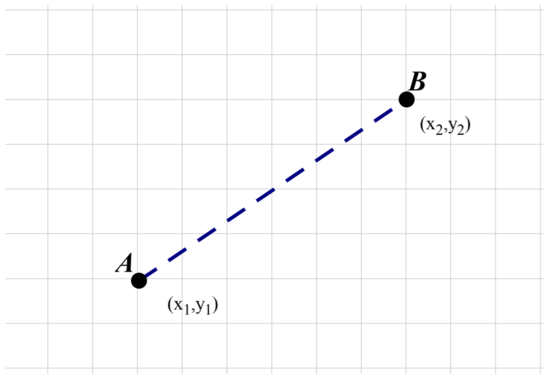
When they have finished with the ribbons in this position, they are considering using them to form a new pattern like this:



3. Will the ribbons they used in the previous pattern be long enough to go between Britney (B) and Courtney (C) in the new pattern? Explain your answer.

Gabriela notices that the calculations she is making for the length of the ribbons reminds her of math class. She says to the group, "Hey, I wonder if there is a process that we could use like what we have been doing to find the distance between any two points on the grid." She decides to think about it like this:

"I'm going to start with two points and draw the line between them that represents the distance that I'm looking for. Since these two points could be anywhere, I named them A (x_1, y_1) and B (x_2, y_2). Hmmmmm. . . . when I figured the length of the ribbons, what did I do next?"



4. Think back on the process you used to find the length of the ribbon and write down your steps here, in terms of (x_1, y_1) and (x_2, y_2).

5. Use the process you came up with in #4 to find the distance between two points located far enough away from each other that using your formula from #4 is more efficient than graphing and counting. For example find the distance between (-11, 25) and (23, -16)

6. Use your process to find the perimeter of the hexagon pattern shown in #3.

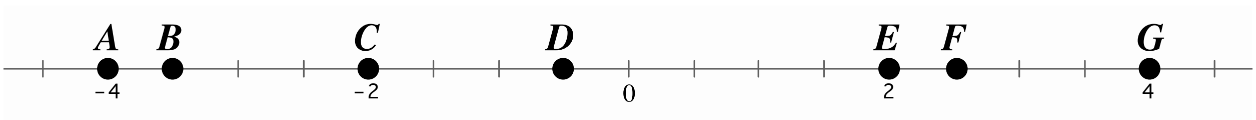
READY, SET, GO! **Name** **Period** **Date**

READY

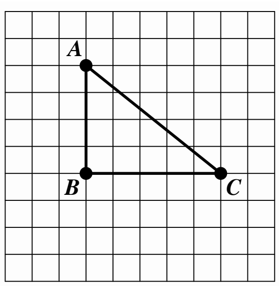
Topic: Finding the distance between two points

Use the number line to find the distance between the given points. (The notation AB means the distance between the points A and B.)

1. AE 2. CF 3. GB 4. CA 5. BF 6. EG



7. Describe a way to find the distance between two points on a number line without counting the spaces.

8. 

a. Find AB.

b. Find BC.

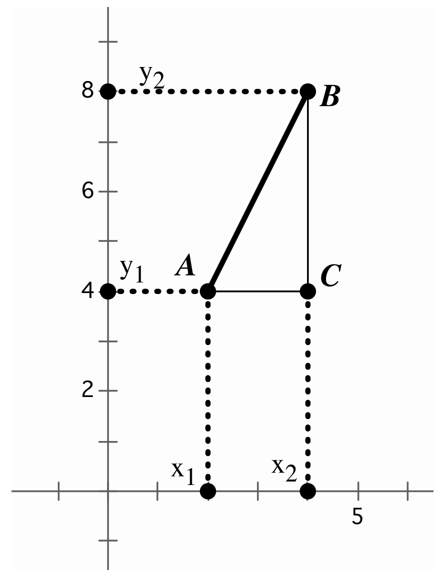
c. Find AC.

9. Why is it easier to find the distance between point A and point B and point B and point C than it is to find the distance between point A and point C?

10. Explain how to find the distance between point A and point C.

SET

Topic: Slope triangles and the distance formula



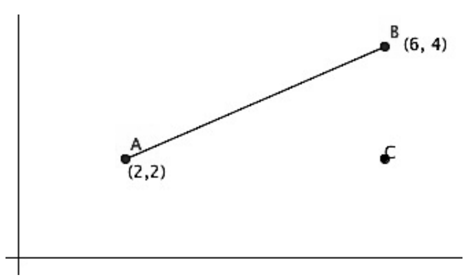
Triangle ABC is a slope triangle for the line segment AB where BC is the rise and AC is the run. Notice that the length of segment BC has a corresponding length on the y-axis and the length of AC has a corresponding length on the x-axis. The slope formula is written as $m = \frac{y_2 - y_1}{x_2 - x_1}$ *where m is the slope.*

11. a. What does the value $(y_2 - y_1)$ tell you?

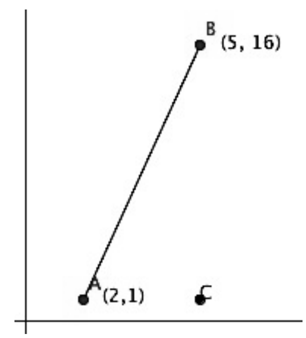
b. What does the value $(x_2 - x_1)$ tell you?

In the previous unit you found the length of a slanted line segment by drawing the slope triangle and then using the Pythagorean theorem on the two sides of the triangle. In this exercise, try to develop a more efficient method of calculating the length of a line segment by using the meaning of $(y_2 - y_1)$ *and* $(x_2 - x_1)$ *combined with the Pythagorean theorem.*

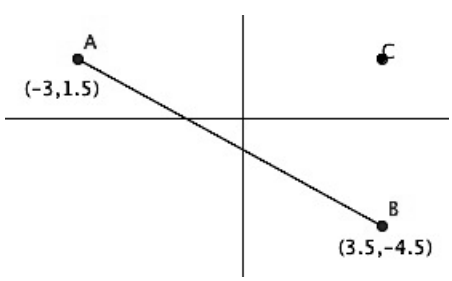
12. Find AB.



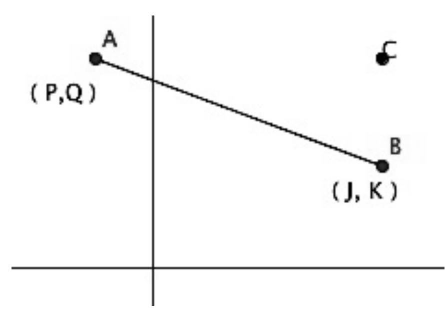
13. Find AB.



14. Find AB.



15. Find AB.



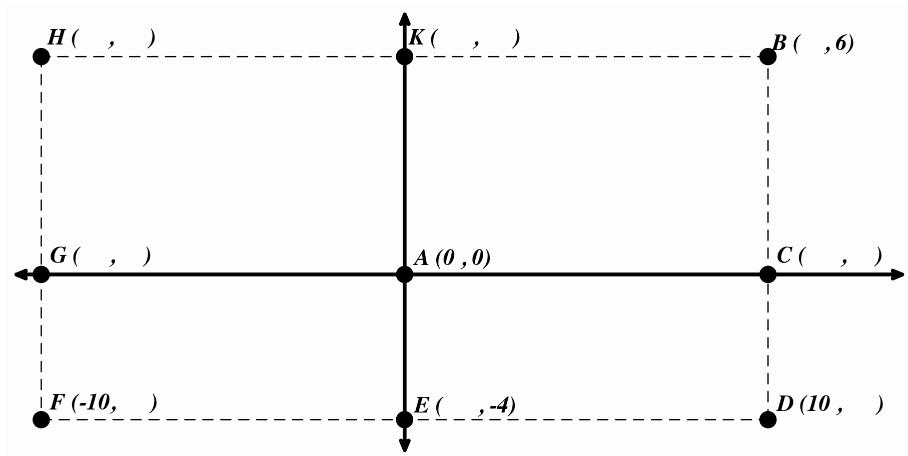
GO

Topic: Rectangular coordinates

Use the given information to fill in the missing coordinates. Then find the length of the indicated line segment.

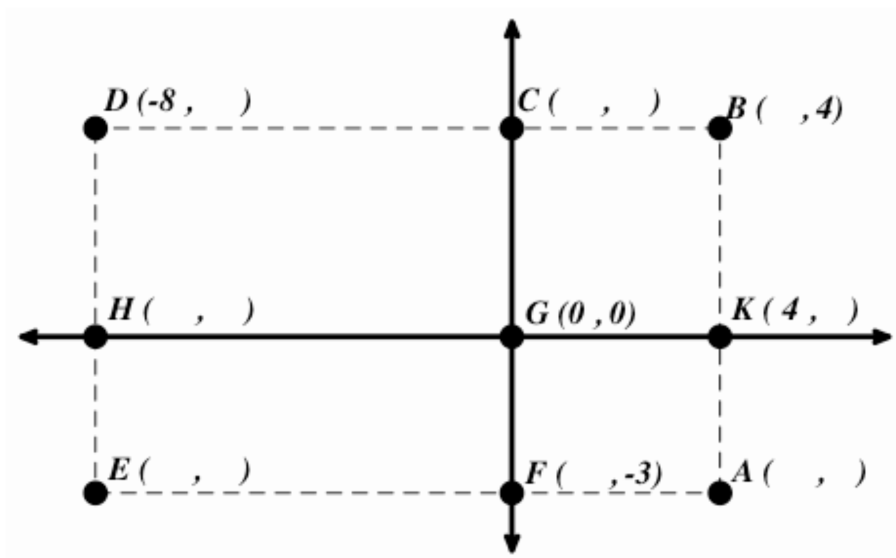
16. a) Find HB.

b) Find BD.



17. a) Find DB

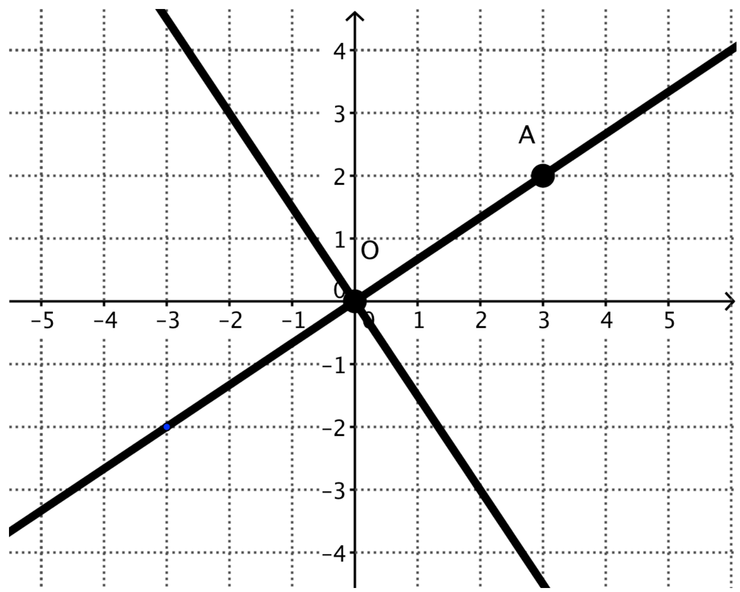
b) Find CF



8.2 Slippery Slopes

A Solidify Understanding Task

While working on "Is It Right?" in the previous module you looked at several examples that lead to the conclusion that the slopes of perpendicular lines are negative reciprocals. Your work here is to formalize this work into a proof. Let's start by thinking about two perpendicular lines that intersect at the origin, like these:



1. Start by drawing a right triangle with the segment $\overline{OA}$ as the hypotenuse. These are often called slope triangles. Based on the slope triangle that you have drawn, what is the slope of $\overleftrightarrow{OA}$?

2. Now, rotate the slope triangle 90° about the origin. What are the coordinates of the image of point A?

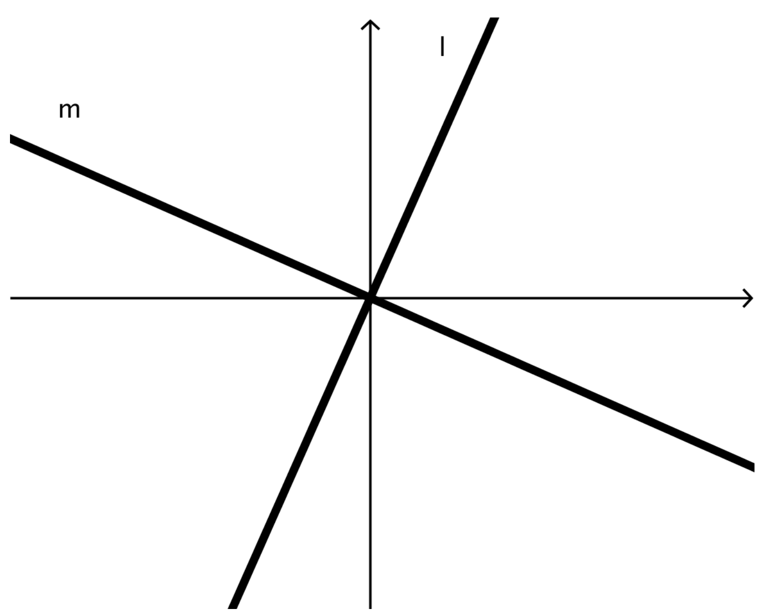
3. Using this new point, A', draw a slope triangle with hypotenuse $\overline{OA'}$. Based on the slope triangle, what is the slope of the line $\overleftrightarrow{OA'}$?

4. What is the relationship between these two slopes? How do you know?

5. Is the relationship changed if the two lines are translated so that the intersection is at (-5, 7)?

   How do you know?

To prove a theorem, we need to demonstrate that the property holds for any pair of perpendicular lines, not just a few specific examples. It is often done by drawing a very similar picture to the examples we have tried, but using variables instead of numbers. Using variables represents the idea that it doesn't matter which numbers we use, the relationship stays the same. Let's try that strategy with the theorem about perpendicular lines having slopes that are negative reciprocals.



- Lines *l* and *m* are constructed to be perpendicular.
- Start by labeling a point P on the line *l.*
- Label the coordinates of P.
- Draw the slope triangle from point P.
- Label the lengths of the sides of the slope triangle using variables like *a* and *b* for the run and the rise.

6. What is the slope of line *l*?

   Rotate point P 90° about the origin, label it P' and mark it on line m. What are the coordinates of P'?

7. Draw the slope triangle from point P'. What are the lengths of the sides of the slope triangle? How do you know?

8. What is the slope of line *m*?

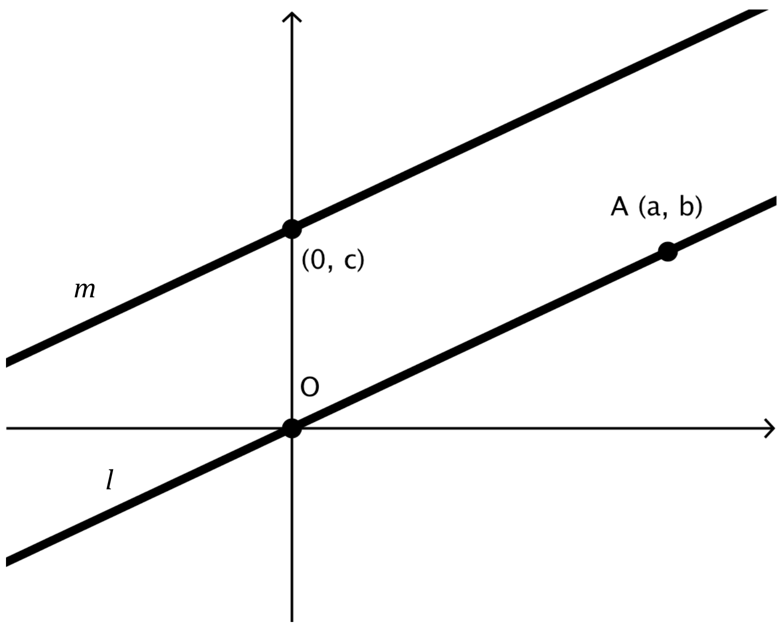
9. What is the relationship between the slopes of line *l* and line *m*? How do you know?

10. Is the relationship between the slopes changed if the intersection between line *l* and line *m* is translated to another location? How do you know?

11. Is the relationship between the slopes changed if lines *l* and *m* are rotated?

12. How do these steps demonstrate that the slopes of perpendicular lines are negative reciprocals for any pair of perpendicular lines?

Think now about parallel lines like the ones below.

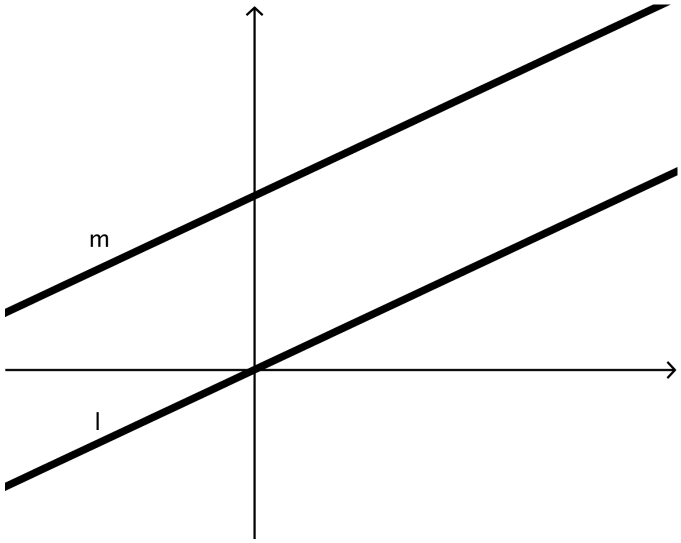


13. Draw the slope triangle from point A to the origin. What is the slope of $\overleftrightarrow{OA}$?

14. What transformation(s) maps the slope triangle with hypotenuse $\overline{OA}$ onto the other line m?

15. What must be true about the slope of line l? Why?

Now you're going to try to use this example to develop a proof, like you did with the perpendicular lines. Here are two lines that have been constructed to be parallel.

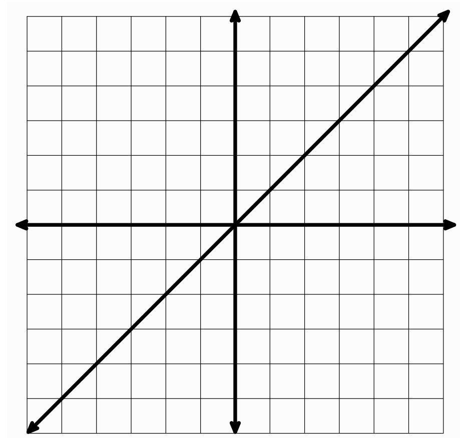


16. Show how you know that these two parallel lines have the same slope and explain why this proves that all parallel lines have the same slope.

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Using translations to graph lines



The equation of the line in the graph is $y = x$.

1. a) On the same grid graph a parallel line that is 3 units above it .

   b) Write the equation for the new line in slope-intercept form.

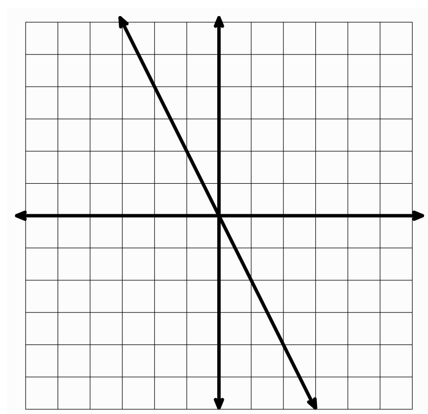
   c) Write the y-intercept of the new line as an ordered pair.

   d) Write the x-intercept of the new line as an ordered pair.

   e) Write the equation of the new line in point-slope form using the *y-intercept.*

   f) Write the equation of the new line in point-slope form using the *x-intercept.*

   g) Explain in what way the equations are the same and in what way they are different.



The graph at the right shows the line $y = -2x$.

2. a) On the same grid, graph a parallel line that is 4 units below it.

   b) Write the equation of the new line in slope-intercept form.

   c) Write the y-intercept of the new line as an ordered pair.

   d) Write the x-intercept of the new line as an ordered pair.

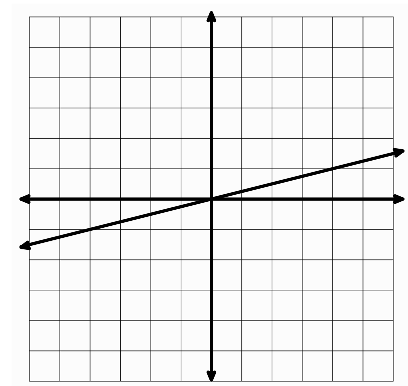
   e) Write the equation of the new line in point-slope form using the *y-intercept.*

   f) Write the equation of the new line in point-slope form using the *x-intercept.*

   g) Explain in what way the equations are the same and in what way they are different.

8.2

The graph at the right shows the line $y = \frac{1}{4}x$.



3. a) On the same grid, graph a parallel line that is 2 units below it.

   b) Write the equation of the new line in slope-intercept form.

   c) Write the y-intercept of the new line as an ordered pair.

   d) Write the x-intercept of the new line as an ordered pair.

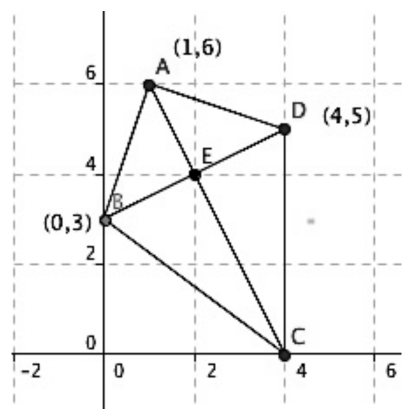
   e) Write the equation of the new line in point-slope form using the *y-intercept.*

   f) Write the equation of the new line in point-slope form using the *x-intercept.*

   g) Explain in what way the equations are the same and in what way they are different.

SET

Topic: Verifying and proving geometric relationships



The quadrilateral at the right is called a **kite.**
Complete the mathematical statements about the kite using the given symbols. Prove each statement algebraically.
(A symbol may be used more than once.)

$\cong \quad \perp \quad \parallel \quad < \quad > \quad =$

Proof

4. $\overline{BC}$_________$\overline{DC}$ ____

5. $\overline{BD}$_________$\overline{AC}$ ____

6. $\overline{AB}$_________$\overline{BC}$ ____

7. ΔABC_____ ΔADC ____

8. $\overline{BE}$________$\overline{ED}$ ____

9. $\overline{AE}$________$\overline{ED}$ ____

10. $\overline{AC}$________$\overline{BD}$ ____

GO

Topic: Writing equations of lines

Use the given information to write the equation of the line in standard form. $(\boldsymbol{Ax + By = C})$

11. $\boldsymbol{Slope: -\frac{1}{4} \quad point\ (12, 5)}$

12. $\boldsymbol{P(11, -3),\ Q(6, 2)}$

13. $\boldsymbol{x - intercept:\ -2;\ \ y - intercept:\ -3}$

14. $\boldsymbol{All\ x\ values\ are\ (-7).\ Y\ is\ any\ number.}$

15. $\boldsymbol{Slope: \frac{1}{2};\ \ x - intercept: 5}$

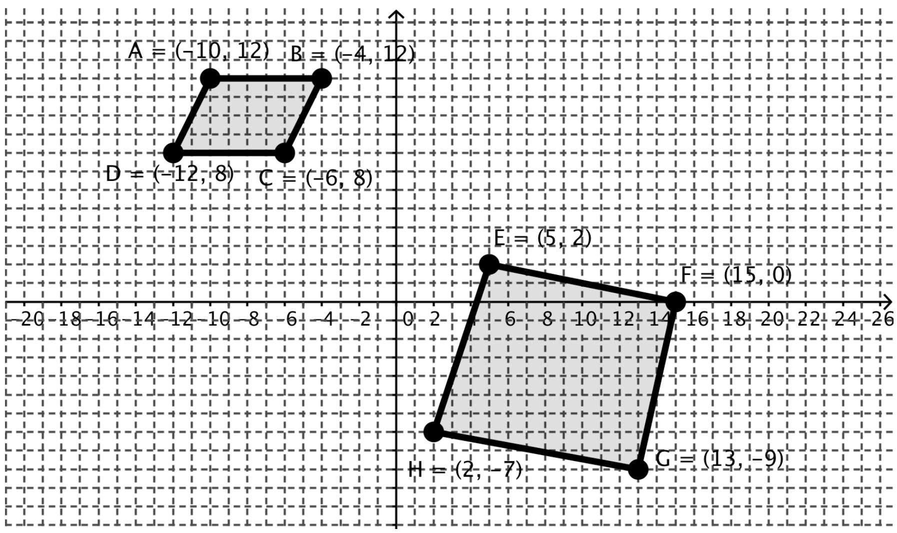
16. $\boldsymbol{E\ (-10, 17),\ G\ (13, 17)}$

8.3 Prove It!

A Practice Understanding Task

In this task you need to use all the things you know about quadrilaterals, distance, and slope to prove that the shapes are parallelograms, rectangles, rhombi, or squares. Be systematic and be sure that you give all the evidence necessary to verify your claim.

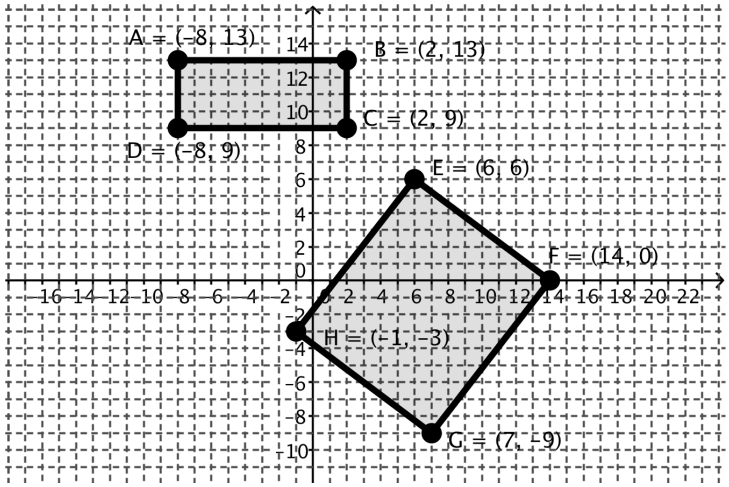
1.



a. Is ABCD a parallelogram? Explain how you know.

b. Is EFGH a parallelogram? Explain how you know.

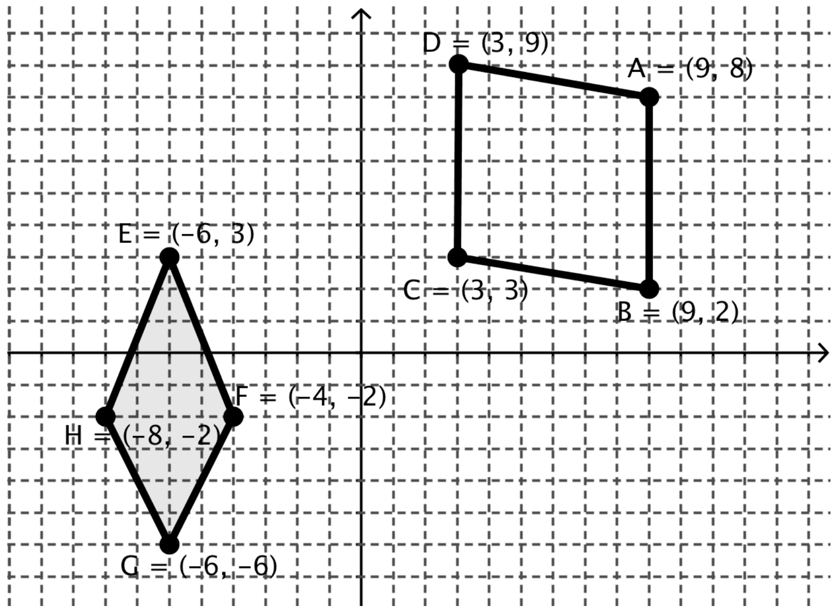
2.



a. Is ABCD a rectangle? Explain how you know.

b. Is EFGH a rectangle? Explain how you know.

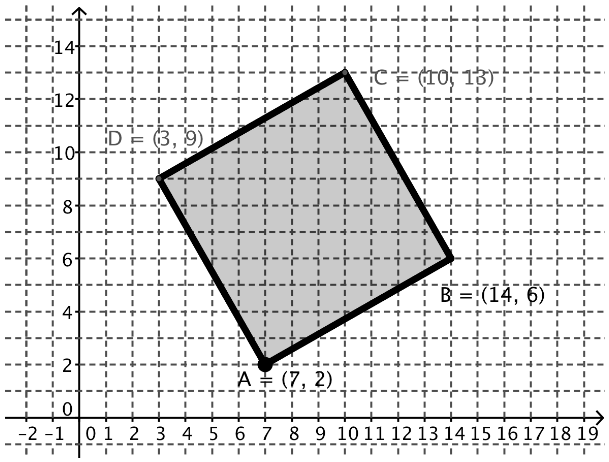
3.



a. Is ABCD a rhombus? Explain how you know.

b. Is EFGH a rhombus? Explain how you know.

4.



a. Is ABCD a square? Explain how you know.

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Interpreting tables of value as ordered pairs.

Find the value of $f(x)$ for the given domain. Write x and $f(x)$ as an ordered pair.

1. $f(x) = 3x - 2$

x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

2. $f(x) = x^2$

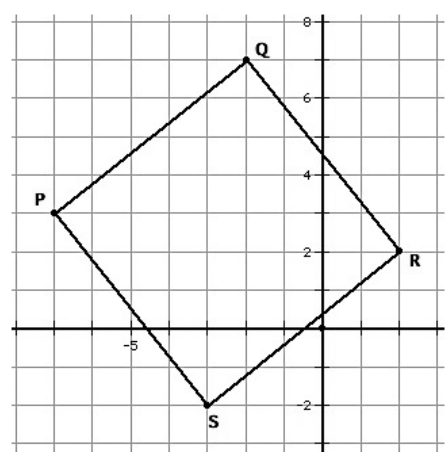
x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

3. $f(x) = 5^x$

x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

SET

Topic: Identifying specific quadrilaterals



4. a) Is the figure at the right a rectangle? Justify your answer.

b) Is the figure at the right a rhombus? Justify your answer.

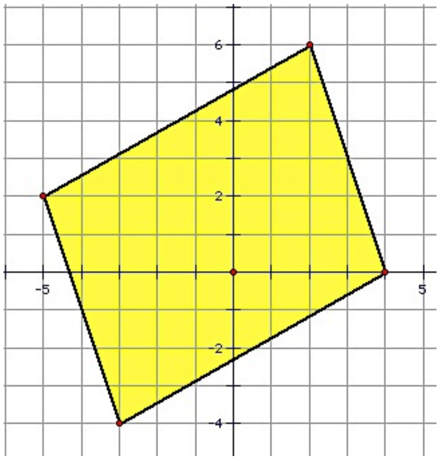
c) Is the figure at the right a square? Justify your answer.

GO

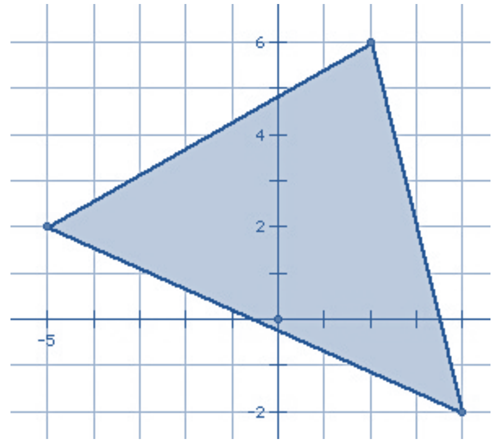
Topic: Calculating perimeters of geometric shapes

Find the perimeter of each figure below. Round answers to the nearest hundredth.

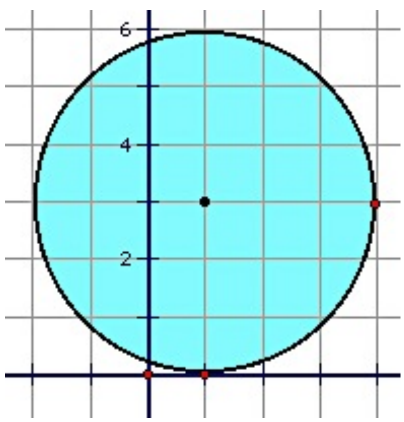
5.



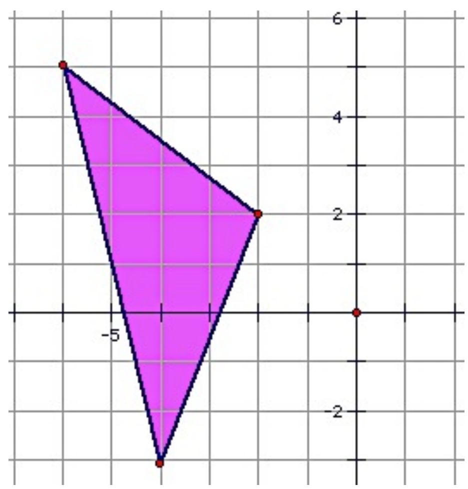
6.



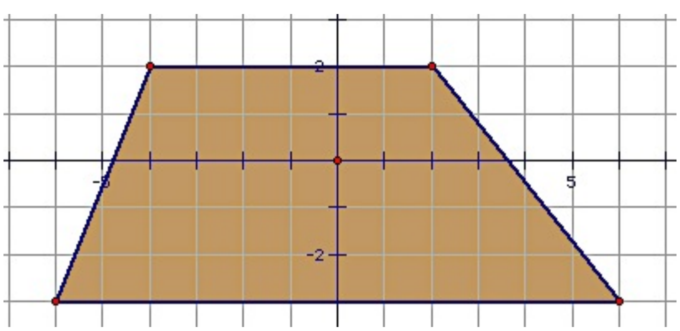
7.



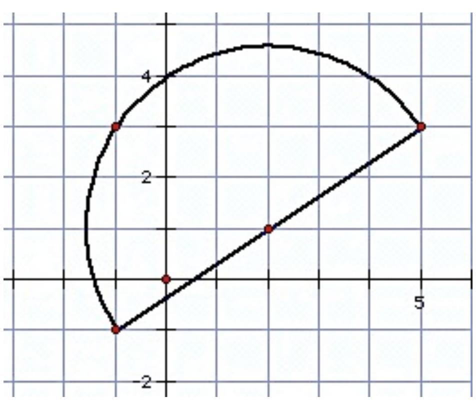
8.



9.



10.



8.4 Training Day

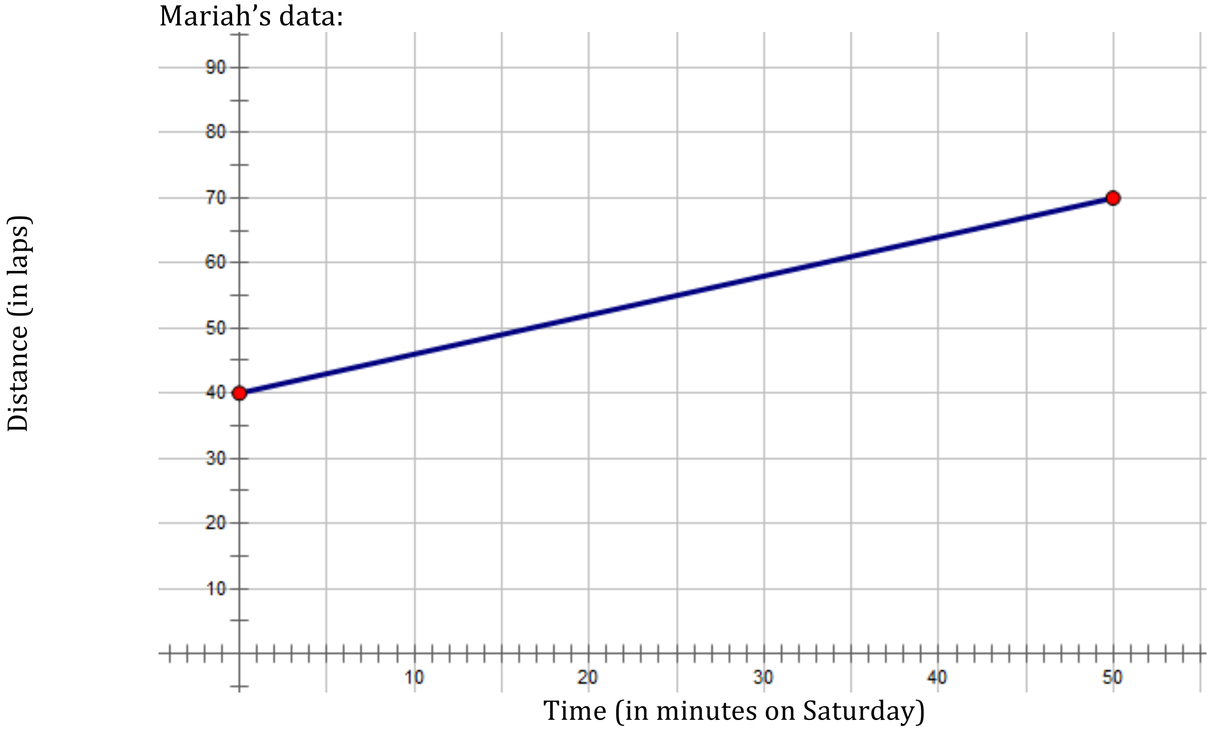
A Develop Understanding Task

Fernando and Mariah are training for six weeks to run in a marathon. To train, they run laps around the track at Eastland High School. Since their schedules do not allow them to run together during the week, they each keep a record of the total number of laps they run throughout the week and then always train together on Saturday morning. The following are representations of how each person kept track of the total number of laps that they ran throughout the week plus the number of laps they ran on Saturday.

Fernando's data:

Time (in minutes on Saturday)	0	10	20	30	40	50
Distance (in laps)	60	66	72	78	84	90

Mariah's data:



1. What observations can be made about the similarities and differences between the two trainers?

2. Write the equation, *m(t),* that models Mariah's distance.

3. Fernando and Mariah both said they ran the same rate during the week when they were training separately. Explain in words how Fernando's equation is similar to Mariah's. Use the sentence frame:

   The rate of both runners is the same throughout the week, however,

   Fernando____.

4. In mathematics, sometimes one function can be used to build another. Write Mariah's equation, *m(t),* by starting with Fernando's equation, *f(t).*

   f(t) =

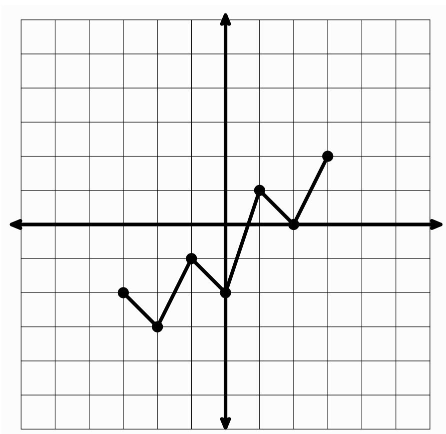
5. Use the mathematical representations given in this task (table and graph) to model the equation you wrote for number 4. Write in words how you would explain this new function to your class.

READY, SET, GO! **Name** **Period** **Date**

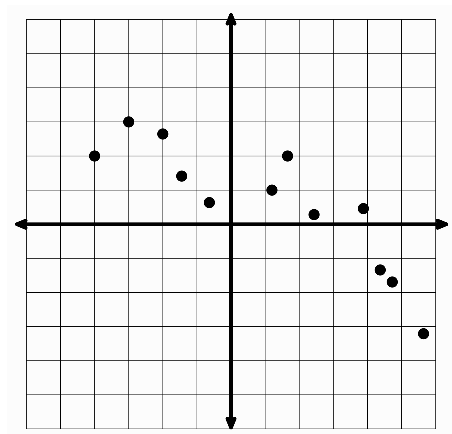
READY

Topic: Vertical transformations on graphs

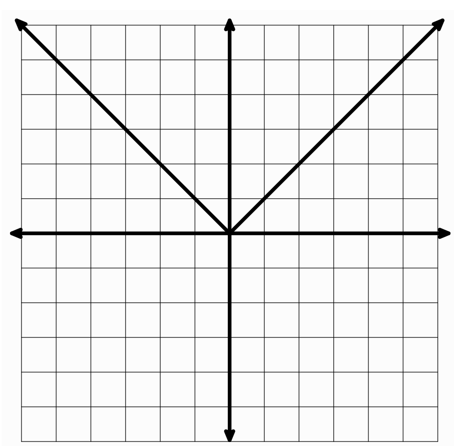
1. Use the graph below to draw a new graph that is translated up 3 units.



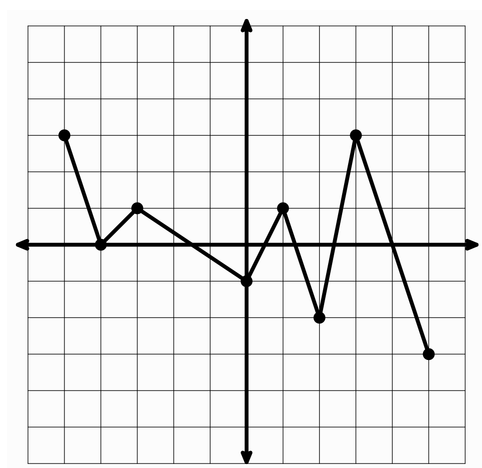
2. Use the graph below to draw a new graph that is translated down 1 unit.



3. Use the graph below to draw a new graph that is translated down 4 units.



4. Use the graph below to draw a new graph that is translated down 3 units.

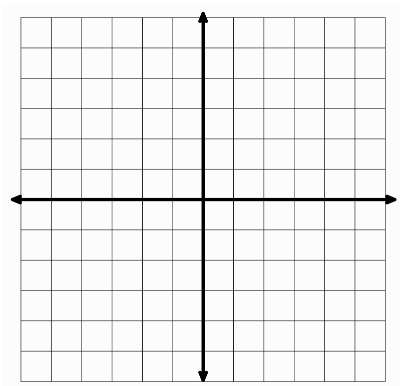


SET

Topic: Graphing transformations and writing the equation of the new graph

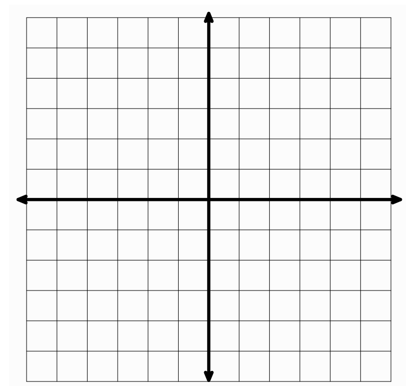
You have been given the equations of $f(x)$ and the transformation $g(x) = f(x) + k$. Graph both $f(x)$ and $g(x)$. Then write the linear equation for $g(x)$ in the space provided.

5. $f(x) = 2x - 4$; $g(x) = f(x) + 3$



$g(x)$ = ______________________________

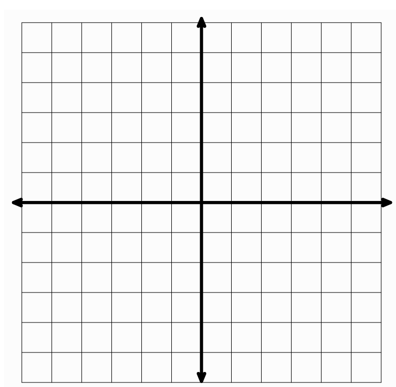
6. $f(x) = 0.5x$; $g(x) = f(x) - 3$



$g(x)$ = ______________________________

Based on the given graph, write the equation of $g(x)$ in the form of $g(x) = f(x) + k$. Then simplify the equation of $g(x)$ into slope-intercept form. The equations of $f(x)$ is given.

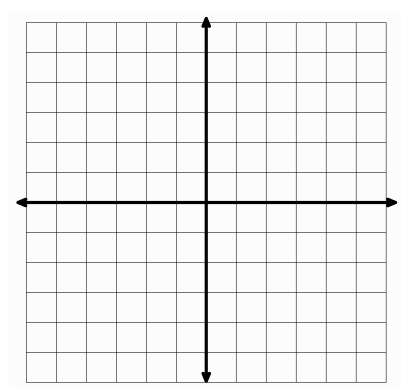
7. $f(x) = \frac{1}{4}x - 3$



a. $g(x) =$ ______________________________
Translation form

b. $g(x) =$ ______________________________
Slope-intercept form

8. $f(x) = -2x + 5$



a. $g(x) =$ ______________________________
Translation form

b. $g(x) =$ ______________________________
Slope-intercept form

GO

Topic: Converting units and making decisions based on data

9. Fernando and Mariah are training for a half marathon. The chart below describes their workouts for the week just before the half marathon. A half marathon is equal to 13.1 miles. If four laps make up one mile, do you think Mariah and Fernando are prepared for the event?

   Describe how you think each person will perform in the race. Include who you think will finish first and predict what you think each person's finish time will be. Use the data to inform your conclusions and to justify your answers.

Day of the week	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Fernando: Distance (in laps)	34	45	52	28	49	36
Time per day (in minutes)	60	72	112	63	88	58
Mariah: Distance (in laps)	30	48	55	44	38	22
Time per day (in minutes)	59	75	119	82	70	45

8.5 Training Day Part II

A Solidify Understanding Task

Fernando and Mariah continued training in preparation for the half marathon. For the remaining weeks of training, they each separately kept track of the distance they ran during the week. Since they ran together at the same rate on Saturdays, they took turns keeping track of the distance they ran and the time it took. So they would both keep track of their own information, the other person would use the data to determine their own total distance for the week.

1. **Week 2**: Mariah had completed 15 more laps than Fernando before they trained on Saturday.
   a. Complete the table for Mariah.

Time (in minutes on Saturday)	0	10	20	30	40	50	60
Fernando: Distance (in laps)	50	56	62	68	74	80	86
Mariah: Distance (in laps)							

   b. Write the equation for Mariah as a transformation of Fernando. Equation for Mariah: $m(t) = f(t)$ ________

2. **Week 3:** On Saturday morning before they started running, Fernando saw Mariah's table and stated, "My equation this week will be $f(t) = m(t) + 30$."
   a. What does Fernando's statement mean?

   b. Based on Fernando's translated function, complete the table.

Time (in minutes on Saturday)	0	20	40	60	70
Fernando: Distance (in laps)					
Mariah: Distance (in laps)	45	57	69	81	87

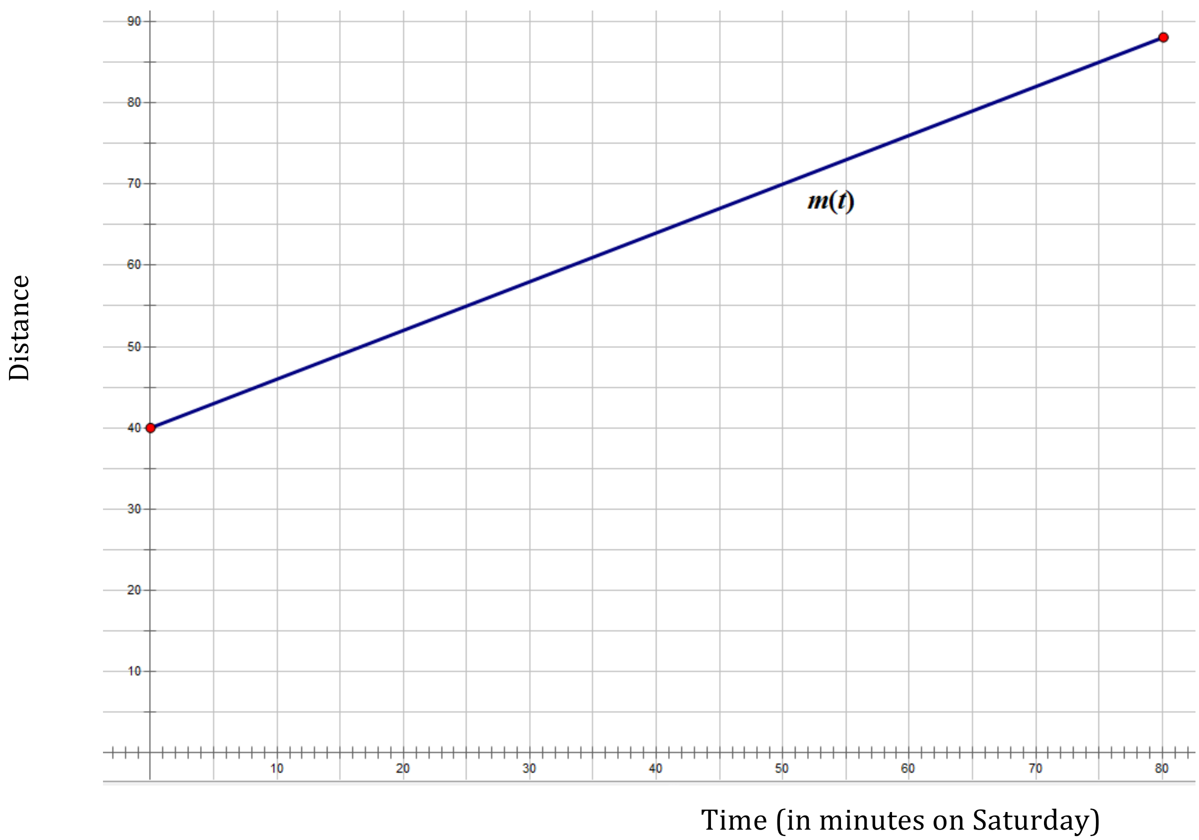
c. Write the equation for both runners in slope-intercept form:

d. Write the equation for Mariah, transformed from Fernando.

e. What relationship do you notice between your answers to parts c and d?

3. **Week 4:** The marathon is only a couple of weeks away!

a. Use Mariah's graph to sketch *f(t)*. *f(t)* = *m(t)* – 10



b. Write the equations for both runners in slope-intercept form.

c. What do you notice about the two graphs? Would this always be true if one person ran "k" laps more or less each week?

4. **Week 5:** This is the last week of training together. Next Saturday is the big day. When they arrived to train, they noticed they had both run 60 laps during the week.

   a. Write the equation for Mariah on Saturday given that they run at the same rate as the week before.

   b. Write Fernando's equation as a transformation of Mariah's equation.

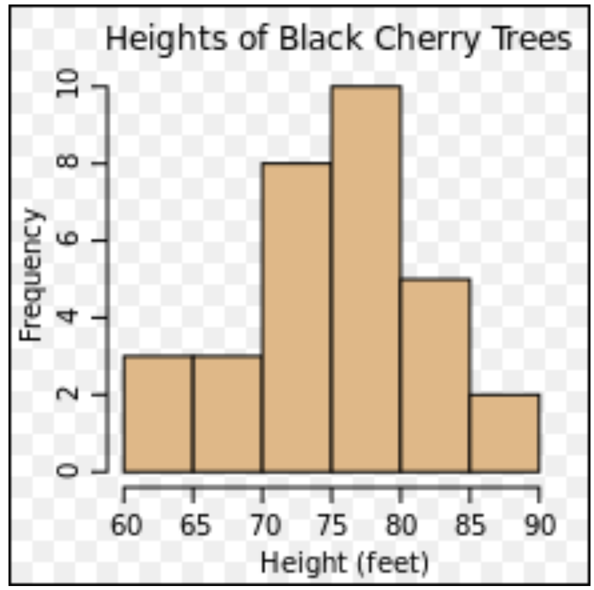
5. What conjectures can you make about the general statement: "$g(x) = f(x)+k$" when it comes to linear functions?

READY, SET, GO! **Name** **Period** **Date**

READY

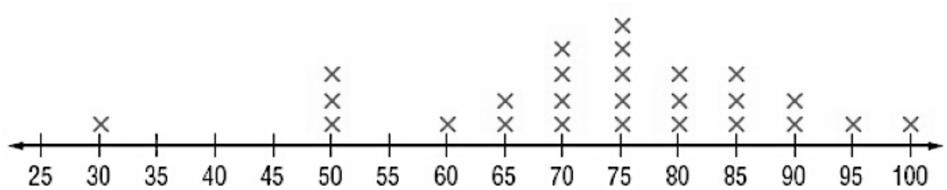
Topic: Describing spread.

1. Describe the spread in the histogram below.

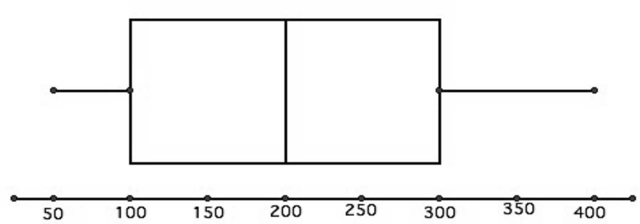


https://commons.wikimedia.org/wiki/File:Black_cherry_tree

2. Describe the spread in the line plot below.



3. Describe the spread in the box and whisker plot.



SET

Topic: Writing functions in translation form.

You are given information about $f(x)$ and $g(x)$.
Rewrite $g(x)$ in translation form: $g(x) = f(x) + k$

4. $f(x) = 7x + 13$
$g(x) = 7x - 5$

$g(x)$ = ______________________
Translation form

5. $f(x) = 22x - 12$
$g(x) = 22x + 213$

$g(x)$ = ______________________
Translation form

6. $f(x) = -15x + 305$
$g(x) = -15x - 11$

$g(x)$ = ______________________
Translation form

7.

x	f(x)	g(x)
3	11	26
10	46	61
25	121	136
40	196	211

$g(x)$ = ______________________
Translation form

8.

x	f(x)	g(x)
-4	5	-42
-1	-1	-48
5	-13	-60
20	-43	-90

$g(x)$ = ______________________
Translation form

9.

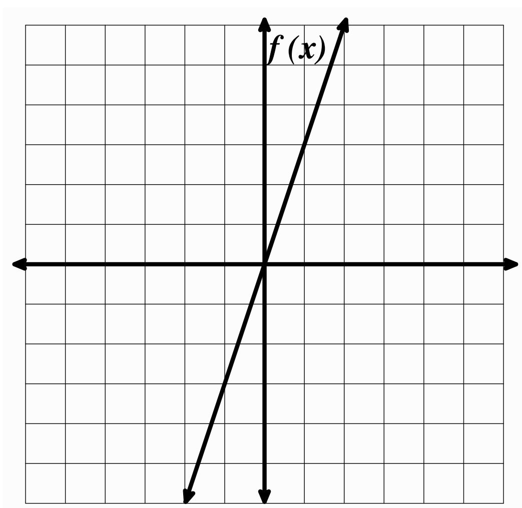
x	f(x)	g(x)
-10	4	-15.5
-3	7.5	-12
22	20	0.5
41	29.5	10

$g(x)$ = ______________________
Translation form

GO

Topic: Vertical and horizontal translations.

10. Use the graph of $\boldsymbol{f(x) = 3x}$ to do the following:



a. Sketch the graph of $\boldsymbol{g(x) = 3x - 2}$ on the same grid.

b. Sketch the graph of $\boldsymbol{h(x) = 3(x - 2)}$

c. Describe how $\boldsymbol{f(x), g(x), and\ h(x)}$ are different and how they are the same.

d. Explain in what way the parentheses affect the graph. Why do you think this is so?

8.6 Shifting Functions

A Practice Understanding Task

CC BY https://flic.kr/p/bFcAWq

Part I: Transformation of an exponential function.

The table below represents the property value of Rebekah's house over a period of four years.

Rebekah's Home

Time (years)	Property Value	Common Ratio
0	150,000	
1	159,000	
2	168,540	
3	178,652	
4	189,372	

Rebekah says the function $P(t) = 150{,}000(1.06)^t$ represents the value of her home.

1. Explain how this function is correct by using the table to show the initial value and the common ratio between terms.

Jeremy lives close to Rebekah and says that his house is always worth $20,000 more than Rebekah's house. Jeremy created the following table of values to represent the property value of his home.

Jeremy's Home

Time (years)	Property Value	Relationship to Rebekah's table
0	170,000	
1	179,000	
2	188,540	
3	198,652	
4	209,372	

When Rebekah and Jeremy tried to write an exponential function to represent Jeremy's property value, they discovered there was not a common ratio between all of the terms.

2. Use your knowledge of transformations to write the function that could be used to determine the property value of Jeremy's house.

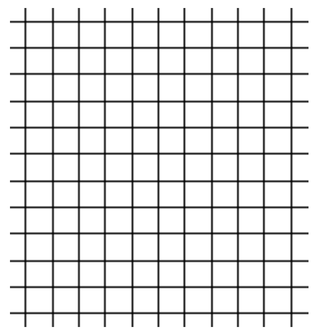
Part 2: Shifty functions.

Given the function *g(x)* and information about *f(x),*

- write the function for *f(x),*
- graph both functions on the set of axes, and
- show a table of values that compares *f(x)* and *g(x)*.

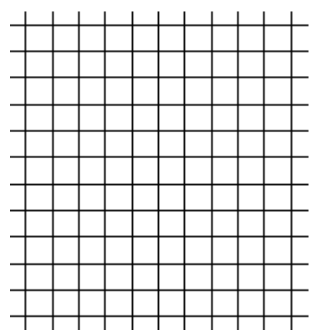
3. $If\ g(x) = 3(2)^x\ and\ \ f(x) =\ g(x) - 5, then\ \ f(x) =$ ______________________

x				
f(x)				
g(x)				



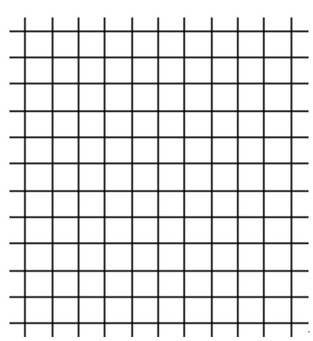
4. $If\ g(x) = 4(.5)^x\ and\ \ f(x) =\ g(x) + 3,\ then\ \ f(x) =$ ______________________

x				
f(x)				
g(x)				



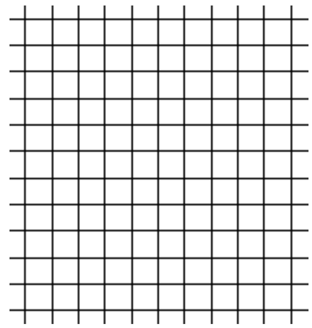
5. $If\ g(x) = 4x + 3\ and\ \ f(x) =\ g(x) + 7, then\ \ f(x) =$ ______________________

x				
f(x)				
g(x)				



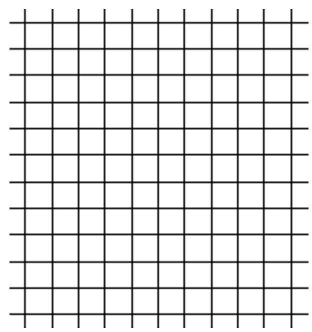
6. $If\ g(x) = 2x + 1\ and\ f(x) = g(x) - 4, then\ f(x) =$ ______________________________

x				
$f(x)$				
$g(x)$				



7. $If\ g(x) = -x\ and\ f(x) = g(x) + 3, then\ f(x) =$ ______________________________

x				
$f(x)$				
$g(x)$				



Part III: Communicate your understanding.

8. If $f(x) = g(x) + k$, describe the relationship between $f(x)$ and $g(x)$. Support your answers with tables and graphs.

READY, SET, GO! Name Period Date

READY

Topic: Finding percentages

Mrs. Gonzalez noticed that her new chorus class had a lot more girls than boys in it. There were 32 girls and 17 boys. (Round answers to the nearest %.)

1. What percent of the class are girls?

2. What percent are boys?

3. 68% of the girls were sopranos.
   a. How many girls sang soprano?

   b. What percent of the entire chorus sang soprano?

4. Only 30% of the boys could sing bass.
   a. How many boys were in the bass section?

   b. What percent of the entire chorus sang bass?

5. Compare the number of girls who sang alto to the number of boys who sang tenor. Which musical section is larger? Justify your answer.

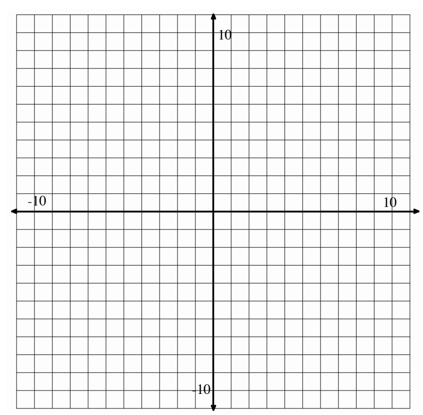
SET

Topic: Graphing exponential equations.

6. Think about the graphs of $y = 2^x$ and $y = 2^x - 4$.

   a. Predict what you think is the same and what is different.

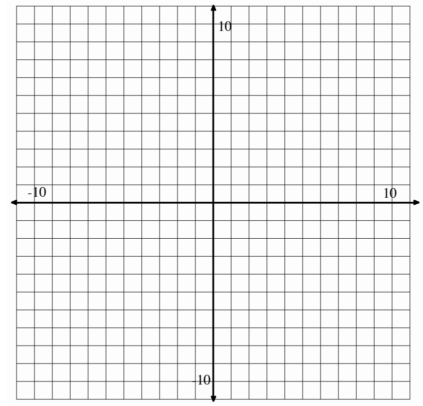
   b. Use your calculator to graph both equations on the same grid. Explain what stayed the same and what changed when you subtracted 4. Identify in what way it changed. (If you don't have a graphing calculator, this can easily be done by hand.)



7. Think about the graphs of $y = 2^x$ and $y = 2^{(x-4)}$.

   a. Predict what you think is the same and what is different.

   b. Use your calculator to graph both equations on the same grid. Explain what stayed the same and what changed. Identify in what way it changed.



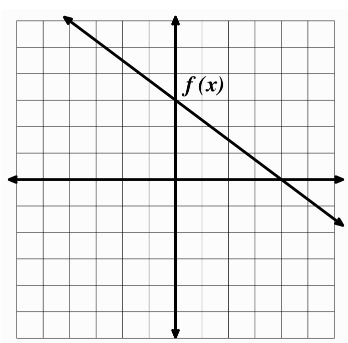
GO

Topic: Vertical translations of linear equations

The graph of $f(x)$ and the translation form equation of $g(x)$ are given. Graph $g(x)$ on the same grid as $f(x)$ and write the slope-intercept equation of $f(x)$ and $g(x)$.

8. $g(x) = f(x) - 5$

   a.

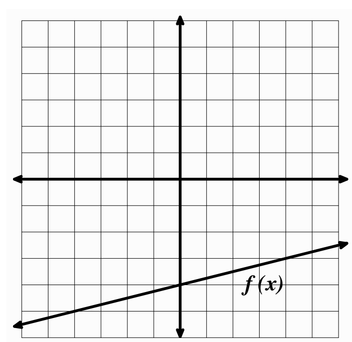


   b. $f(x) =$ ____________________

   c. $g(x) =$ ____________________
   Slope-intercept form

9. $g(x) = f(x) + 4$

   a.

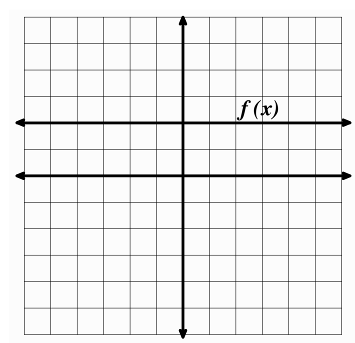


   b. $f(x) =$ ____________________

   c. $g(x) =$ ____________________
   Slope-intercept form

10. $g(x) = f(x) - 6$

   a.



   b. $f(x) =$ ____________________

   c. $g(x) =$ ____________________
   Slope-intercept form