mathematics vision project

Transforming Mathematics Education

SECONDARY MATH ONE

An Integrated Approach

Standard Teacher Notes

MODULE 8

Connecting Algebra & Geometry

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

Scott Hendrickson, Joleigh Honey, Barbara Kuehl, Travis Lemon, Janet Sutorius

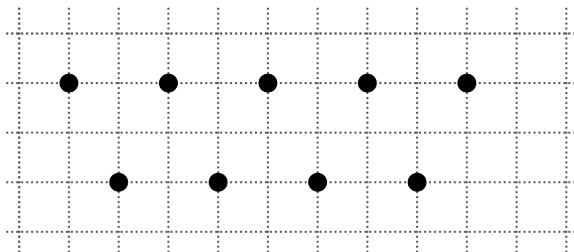
MODULE 8 - TABLE OF CONTENTS

CONNECTING ALGEBRA AND GEOMETRY

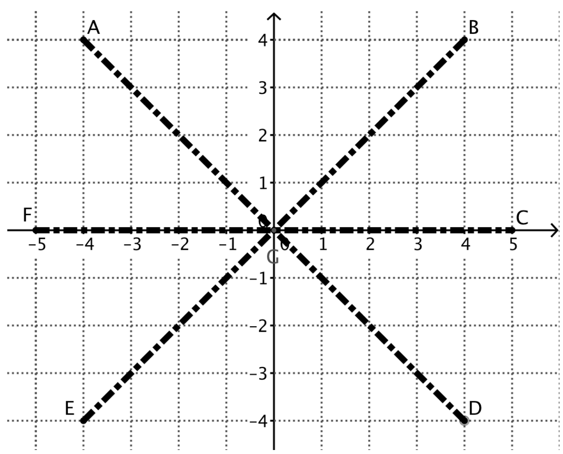
8.1 Go the Distance

A Develop Understanding Task

The performances of the Podunk High School drill team are very popular during half-time at the school's football and basketball games. When the Podunk High School drill team choreographs the dance moves that they will do on the football field, they lay out their positions on a grid like the one below:



In one of their dances, they plan to make patterns holding long, wide ribbons that will span from one dancer in the middle to six other dancers. On the grid, their pattern looks like this:

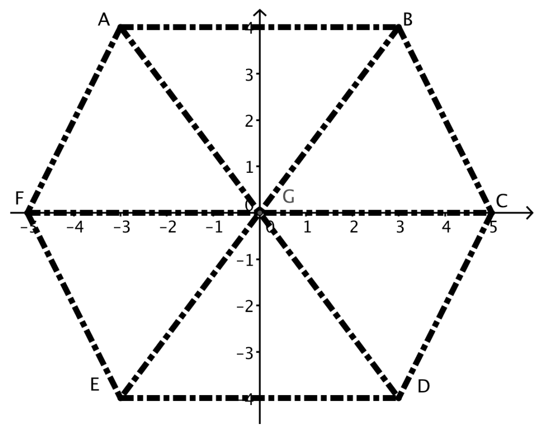


The question the dancers have is how long to make the ribbons. Gabriela (G) is standing in the center and some dancers think that the ribbon from Gabriela (G) to Courtney (C) will be shorter than the one from Gabriela (G) to Brittney (B).

1. How long does each ribbon need to be?

2. Explain how you found the length of each ribbon.

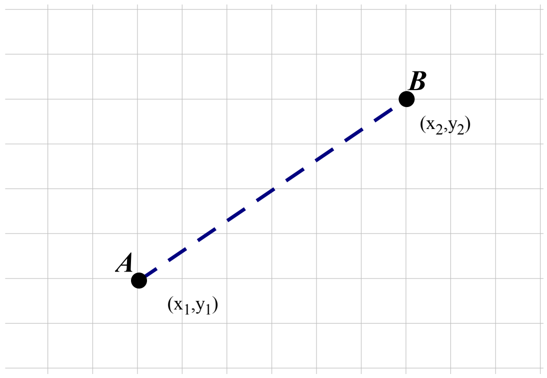
When they have finished with the ribbons in this position, they are considering using them to form a new pattern like this:



3. Will the ribbons they used in the previous pattern be long enough to go between Britney (B) and Courtney (C) in the new pattern? Explain your answer.

Gabriela notices that the calculations she is making for the length of the ribbons reminds her of math class. She says to the group, "Hey, I wonder if there is a process that we could use like what we have been doing to find the distance between any two points on the grid." She decides to think about it like this:

"I'm going to start with two points and draw the line between them that represents the distance that I'm looking for. Since these two points could be anywhere, I named them A (x_1,y_1) and B (x_2,y_2). Hmmmmm. . . . when I figured the length of the ribbons, what did I do next?"



4. Think back on the process you used to find the length of the ribbon and write down your steps here, in terms of (x_1, y_1) and (x_2, y_2).

5. Use the process you came up with in #4 to find the distance between two points located far enough away from each other that using your formula from #4 is more efficient than graphing and counting. For example find the distance between (-11, 25) and (23, -16)

6. Use your process to find the perimeter of the hexagon pattern shown in #3.

8.1 Go the Distance – Teacher Notes

A Develop Understanding Task

Note to Teachers: Calculators facilitate the work for this task.
Purpose: The purpose of this task is to develop the distance formula, based upon students' understanding of the Pythagorean theorem. In the task, students are asked to calculate distances between points using triangles, and then to formalize the process to the distance formula. At the end of the task, students will use the distance formula to find the perimeter of a hexagon.

Core Standards Focus:

G. GPE.4 Use coordinates to prove simple geometric theorems algebraically.

G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Standards for Mathematical Practice of Focus in the Module:

SMP 1 – Make sense of problems and persevere in solving them.

SMP 7 – Look for and make use of structure.

The Teaching Cycle:

Launch (Whole Class):

Begin the task by ensuring that student understand the problem situation. Project the drawing in #1 and ask students which ribbon looks longer, $\overline{GB}$ or $\overline{GC}$. Ask how they can test their claims. Some students may suggest using the Pythagorean Theorem to find the length of GB. Ask what they would need to use the Pythagorean Theorem. At this point, set students to work on the task.

Explore (Small Group):

During the exploration period, watch for students that are stuck on the first part of the problem. You may ask them to draw the triangle that will help them to use the Pythagorean Theorem and how they might find the length of the legs of the triangle so they can find the hypotenuse. As you monitor student thinking on #3, watch for students who are noticing how to find the length of the legs of the triangle when it has been moved away from the origin. Look for students that have written a good step-by-step procedure for #4. It will probably be difficult for them to use the symbols appropriately, so watch for words that appropriate describe the procedure.

Discuss (Whole Class):

Start the discussion by having a group show how they found the length of $\overline{BC}$ in problem #3. Move next to #4 and have a group that has written a step by step procedure. Try walking through the group's procedure with the numbers from problem #3 and see if it gives the appropriate answer. If necessary, work with the class to modify the procedure so that the list of steps is correct. Once the steps are outlined in words, go through the steps using points A (x_1, y_1) and B (x_2, y_2) and formalize the procedures with the symbols. An example:

Steps in words	**Steps in symbols**
Find the length of the horizontal leg of the triangle	$x_2 - x_1$
Find the length of the vertical leg of the triangle	$y_2 - y_1$
Use the Pythagorean Theorem to write an equation	$(x_2 - x_1)^2 + (y_2 - y_1)^2 = c^2$
Solve for ***c***	$(x_2 - x_1)^2 + (y_2 - y_1)^2 = c^2$
Take the square root of both sides of the equation	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{c^2}$
Simplify	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = c$ (c being the desired distance)

Using algebraic notation to model a correct process that is given verbally will result in deriving the distance formula. After going through this process, apply the formula using the points in #5.

Aligned Ready, Set, Go: ***Connecting Algebra and Geometry 8.1***

8.1

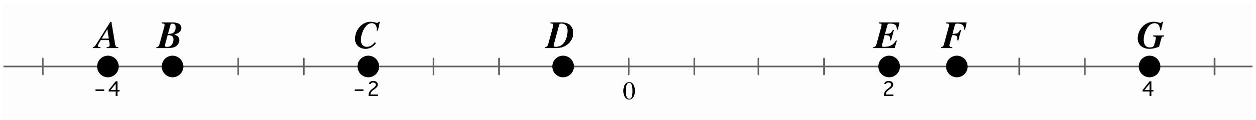
READY, SET, GO! **Name** **Period** **Date**

READY

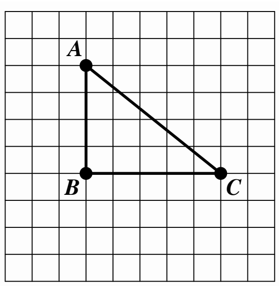
Topic: Finding the distance between two points

Use the number line to find the distance between the given points. (The notation AB means the distance between the points A and B.)

1. AE 2. CF 3. GB 4. CA 5. BF 6. EG



7. Describe a way to find the distance between two points on a number line without counting the spaces.

8. 

a. Find AB.

b. Find BC.

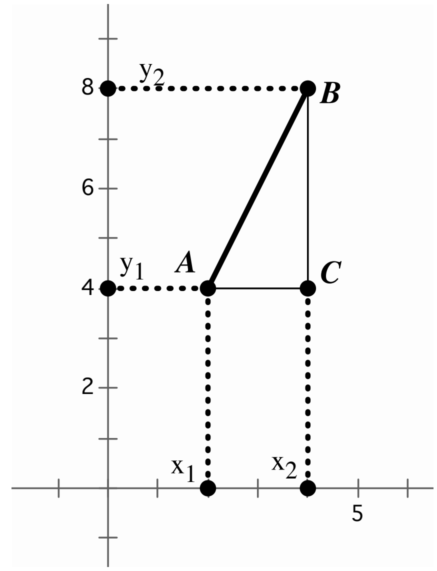
c. Find AC.

9. Why is it easier to find the distance between point A and point B and point B and point C than it is to find the distance between point A and point C?

10. Explain how to find the distance between point A and point C.

SET

Topic: Slope triangles and the distance formula



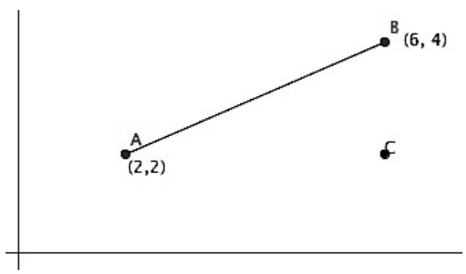
Triangle ABC is a slope triangle for the line segment AB where BC is the rise and AC is the run. Notice that the length of segment BC has a corresponding length on the y-axis and the length of AC has a corresponding length on the x-axis. The slope formula is written as $m = \frac{y_2 - y_1}{x_2 - x_1}$ where m is the slope.

11. a. What does the value $(y_2 - y_1)$ tell you?

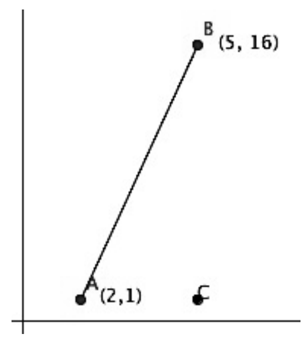
b. What does the value $(x_2 - x_1)$ tell you?

In the previous unit you found the length of a slanted line segment by drawing the slope triangle and then using the Pythagorean theorem on the two sides of the triangle. In this exercise, try to develop a more efficient method of calculating the length of a line segment by using the meaning of $(y_2 - y_1)$ and $(x_2 - x_1)$ combined with the Pythagorean theorem.

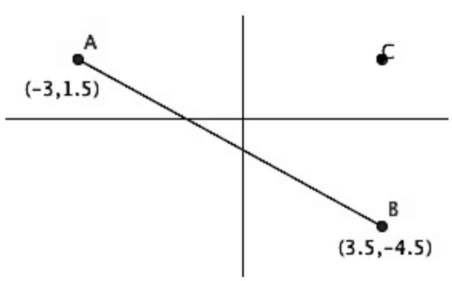
12. Find AB.



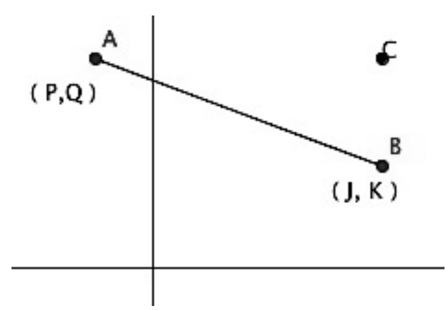
13. Find AB.



14. Find AB.



15. Find AB.

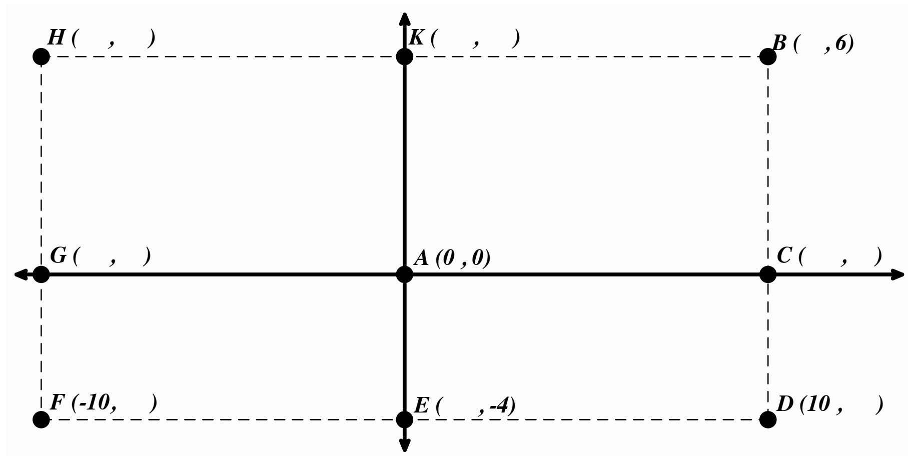


GO

Topic: Rectangular coordinates

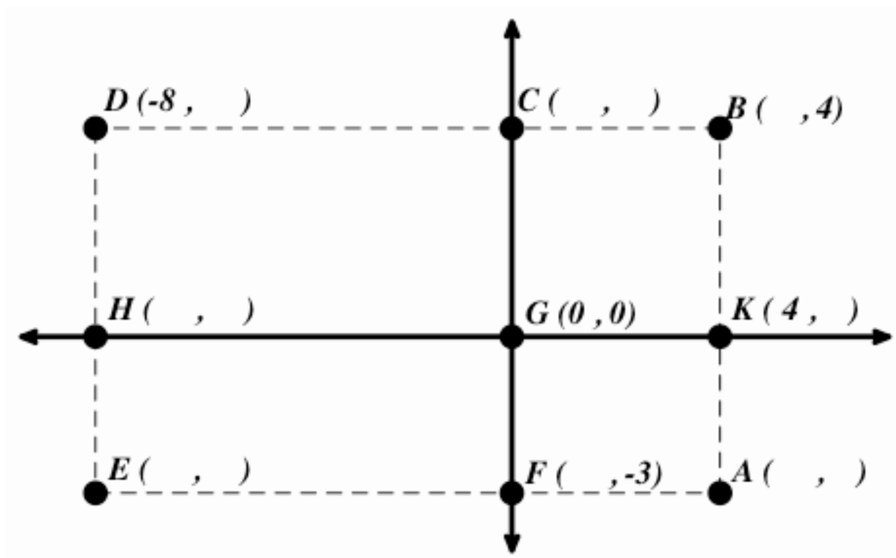
Use the given information to fill in the missing coordinates. Then find the length of the indicated line segment.

16. a) Find HB.



b) Find BD.

17. a) Find DB

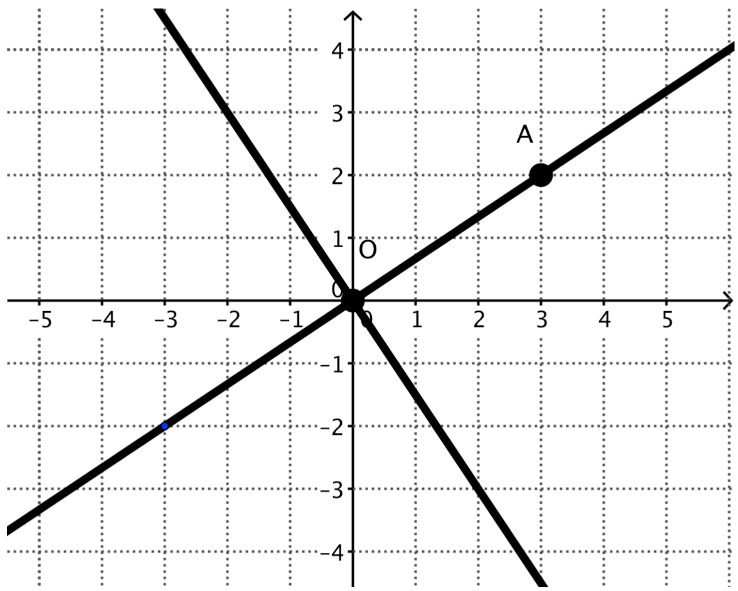


b) Find CF

8.2 Slippery Slopes

A Solidify Understanding Task

While working on "Is It Right?" in the previous module you looked at several examples that lead to the conclusion that the slopes of perpendicular lines are negative reciprocals. Your work here is to formalize this work into a proof. Let's start by thinking about two perpendicular lines that intersect at the origin, like these:



1. Start by drawing a right triangle with the segment $\overline{OA}$ as the hypotenuse. These are often called slope triangles. Based on the slope triangle that you have drawn, what is the slope of $\overleftrightarrow{OA}$?

2. Now, rotate the slope triangle 90° about the origin. What are the coordinates of the image of point A?

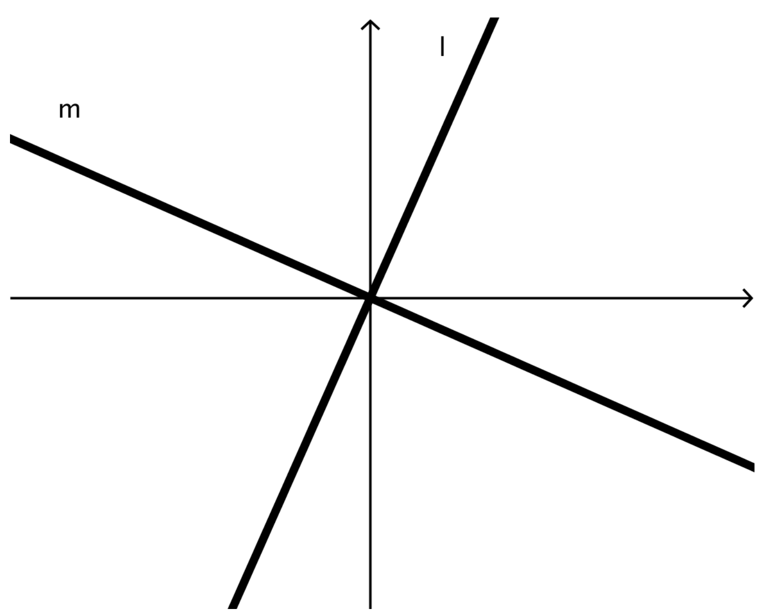
3. Using this new point, A', draw a slope triangle with hypotenuse $\overline{OA'}$. Based on the slope triangle, what is the slope of the line $\overleftrightarrow{OA'}$?

4. What is the relationship between these two slopes? How do you know?

5. Is the relationship changed if the two lines are translated so that the intersection is at (-5, 7)?

   How do you know?

To prove a theorem, we need to demonstrate that the property holds for any pair of perpendicular lines, not just a few specific examples. It is often done by drawing a very similar picture to the examples we have tried, but using variables instead of numbers. Using variables represents the idea that it doesn't matter which numbers we use, the relationship stays the same. Let's try that strategy with the theorem about perpendicular lines having slopes that are negative recipricals.



- Lines *l* and *m* are constructed to be perpendicular.
- Start by labeling a point P on the line *l.*
- Label the coordinates of P.
- Draw the slope triangle from point P.
- Label the lengths of the sides of the slope triangle using variables like *a* and *b* for the run and the rise.

6. What is the slope of line *l*?

Rotate point P 90° about the origin, label it P' and mark it on line m. What are the coordinates of P'?

7. Draw the slope triangle from point P'. What are the lengths of the sides of the slope triangle? How do you know?

8. What is the slope of line *m*?

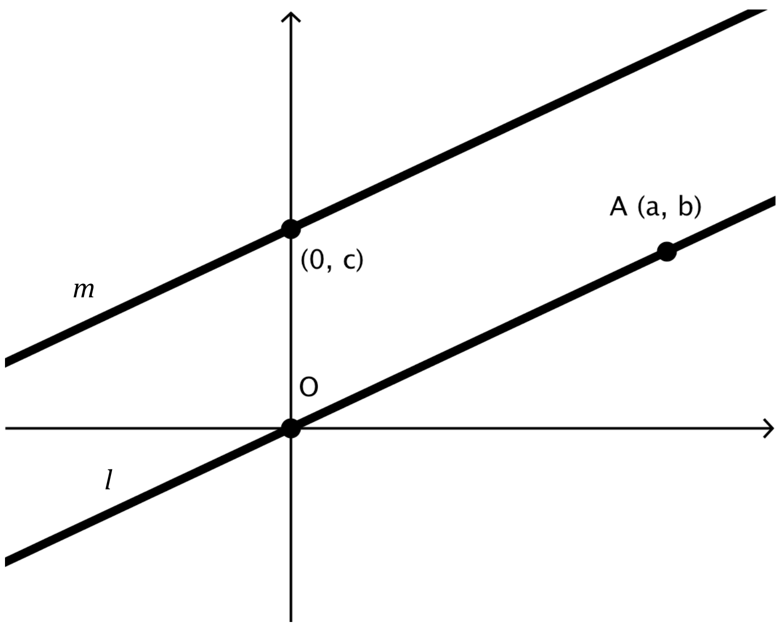
9. What is the relationship between the slopes of line *l* and line *m*? How do you know?

10. Is the relationship between the slopes changed if the intersection between line *l* and line *m* is translated to another location? How do you know?

11. Is the relationship between the slopes changed if lines *l* and *m* are rotated?

12. How do these steps demonstrate that the slopes of perpendicular lines are negative reciprocals for any pair of perpendicular lines?

Think now about parallel lines like the ones below.

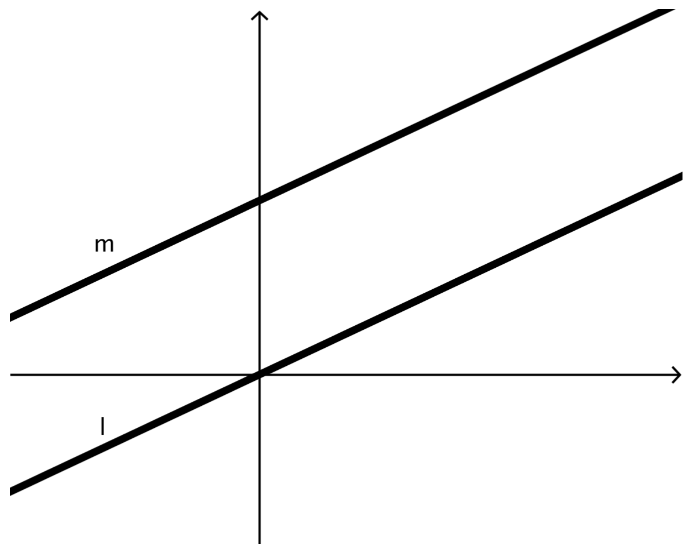


13. Draw the slope triangle from point A to the origin. What is the slope of $\overleftrightarrow{OA}$?

14. What transformation(s) maps the slope triangle with hypotenuse $\overline{OA}$ onto the other line m?

15. What must be true about the slope of line l? Why?

Now you're going to try to use this example to develop a proof, like you did with the perpendicular lines. Here are two lines that have been constructed to be parallel.



16. Show how you know that these two parallel lines have the same slope and explain why this proves that all parallel lines have the same slope.

8.2 Slippery Slopes – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to prove that parallel lines have equal slopes and that the slopes of perpendicular lines are negative reciprocals. Students have used these theorems previously. The proofs use the ideas of slope triangles, rotations, and translations. Both proofs are preceded by a specific case that demonstrates the idea before students are asked to follow the logic using variables and thinking more generally.

Core Standards Focus:

G. GPE Use coordinates to prove simple geometric theorems algebraically.

G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Related Standards: G.CO.4, G.CO.5

Standards for Mathematical Practice of Focus in the Task:

SMP 3 – Construct viable arguments and critique the reasoning of others.
SMP 6 - Attend to precision.

The Teaching Cycle:

Launch (Whole Class):

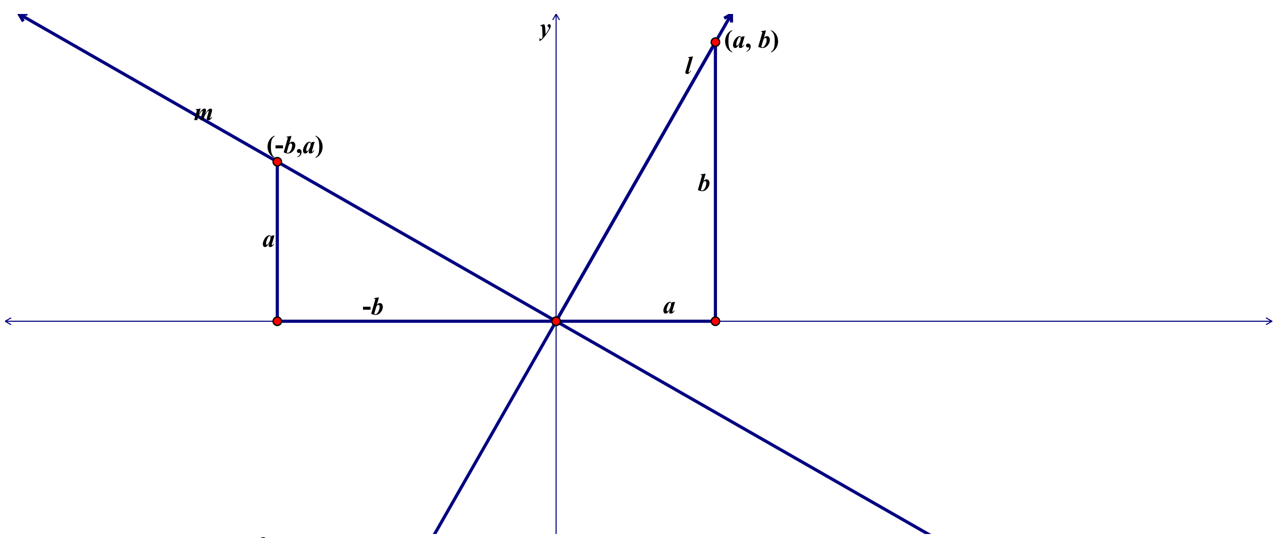
If students haven't been using the term "slope triangle", start the discussion with a brief demonstration of slope triangles and how they show the slope of the line. Students should be familiar with performing a 90 degree rotation from the previous module, so begin the task by having students work individually on questions 1, 2, 3, and 4. When most students have drawn a conclusion for #4, have a discussion of how they know the two lines are perpendicular. Since the purpose is to demonstrate that perpendicular lines have slopes that are negative reciprocals, emphasize that the reason that we know that the lines are perpendicular is that they were constructed based upon a 90 degree rotation.

Explore (Small Group):

The proof that the slopes of perpendicular lines are negative reciprocals follows the same pattern as the example given in the previous problem. Monitor students as they work, allowing them to select a point, label the coordinates and then the sides of the slope triangles. Refer students back to the previous problem, asking them to generalize the steps symbolically if they are stuck. When students are finished with questions 6-12, discuss the proof as a whole group and then have students complete the task.

Discuss (Whole Class):

The setup for the proof is below:



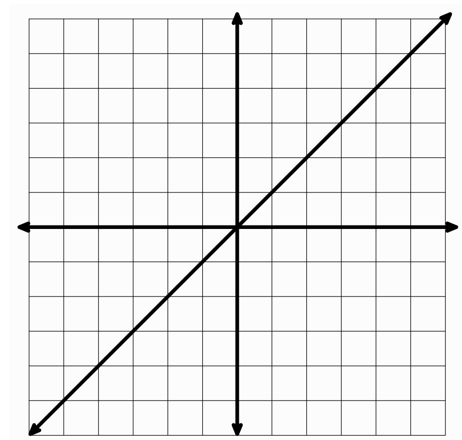
The slope of line l is $\frac{b}{a}$ and the slope of line m is $\frac{a}{-b}$ or -$\frac{a}{b}$. The product of the two slopes is -1, therefore they are negative reciprocals. If the lines are translated so that the intersection is not at the origin, the slope triangles will remain the same. Discuss with the class how questions 6-12 help us to consider all the possible cases, which is necessary in a proof. After students have finished the task, go through the brief proof that the slopes of parallel lines are equal.

Aligned Ready, Set, Go: ***Connecting Algebra and Geometry 8.2***

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Using translations to graph lines



The equation of the line in the graph is $y = x$.

1. a) On the same grid graph a parallel line that is 3 units above it .

   b) Write the equation for the new line in slope-intercept form.

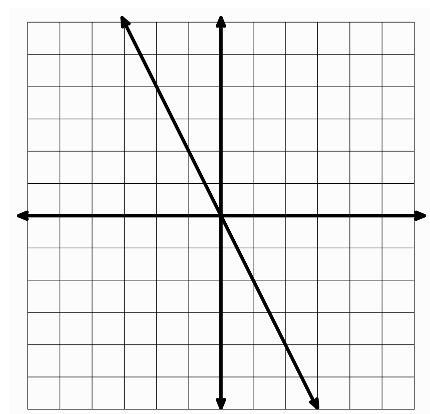
   c) Write the y-intercept of the new line as an ordered pair.

   d) Write the x-intercept of the new line as an ordered pair.

   e) Write the equation of the new line in point-slope form using the *y-intercept.*

   f) Write the equation of the new line in point-slope form using the *x-intercept.*

   g) Explain in what way the equations are the same and in what way they are different.



The graph at the right shows the line $y = -2x$.

2. a) On the same grid, graph a parallel line that is 4 units below it.

   b) Write the equation of the new line in slope-intercept form.

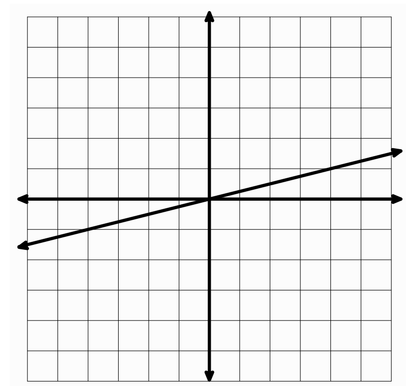
   c) Write the y-intercept of the new line as an ordered pair.

   d) Write the x-intercept of the new line as an ordered pair.

   e) Write the equation of the new line in point-slope form using the *y-intercept.*

   f) Write the equation of the new line in point-slope form using the *x-intercept.*

   g) Explain in what way the equations are the same and in what way they are different.



The graph at the right shows the line $y = \frac{1}{4}x$.

3. a) On the same grid, graph a parallel line that is 2 units below it.

   b) Write the equation of the new line in slope-intercept form.

   c) Write the y-intercept of the new line as an ordered pair.

   d) Write the x-intercept of the new line as an ordered pair.

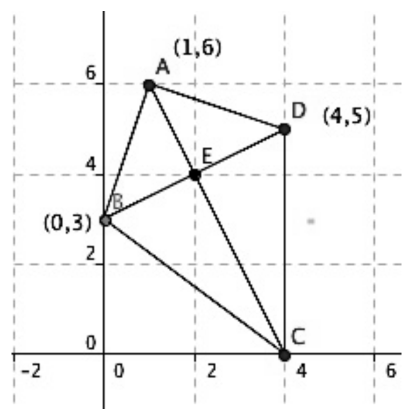
   e) Write the equation of the new line in point-slope form using the *y-intercept.*

   f) Write the equation of the new line in point-slope form using the *x-intercept.*

   g) Explain in what way the equations are the same and in what way they are different.

SET

Topic: Verifying and proving geometric relationships



The quadrilateral at the right is called a **kite.**
Complete the mathematical statements about the kite using the given symbols. Prove each statement algebraically.
(A symbol may be used more than once.)

$\cong \quad \perp \quad \parallel \quad < \quad > \quad =$

Proof

4. $\overline{BC}$________$\overline{DC}$ ______________________________

____

5. $\overline{BD}$________$\overline{AC}$ ______________________________

____

6. $\overline{AB}$________$\overline{BC}$ ______________________________

____

7. ΔABC_____ ΔADC ____

____

8. $\overline{BE}$________$\overline{ED}$ ____

____

9. $\overline{AE}$________$\overline{ED}$ ____

____

10. $\overline{AC}$________$\overline{BD}$ ____

____

GO

Topic: Writing equations of lines

Use the given information to write the equation of the line in standard form. $(\boldsymbol{Ax + By = C})$

11. $\boldsymbol{Slope: -\frac{1}{4} \quad point\ (12, 5)}$

12. $\boldsymbol{P(11, -3),\ Q(6, 2)}$

13. $\boldsymbol{x - intercept: -2;\ \ y - intercept: -3}$

14. $\boldsymbol{All\ x\ values\ are\ (-7).\ Y\ is\ any\ number.}$

15. $\boldsymbol{Slope: \frac{1}{2};\ \ x - intercept: 5}$

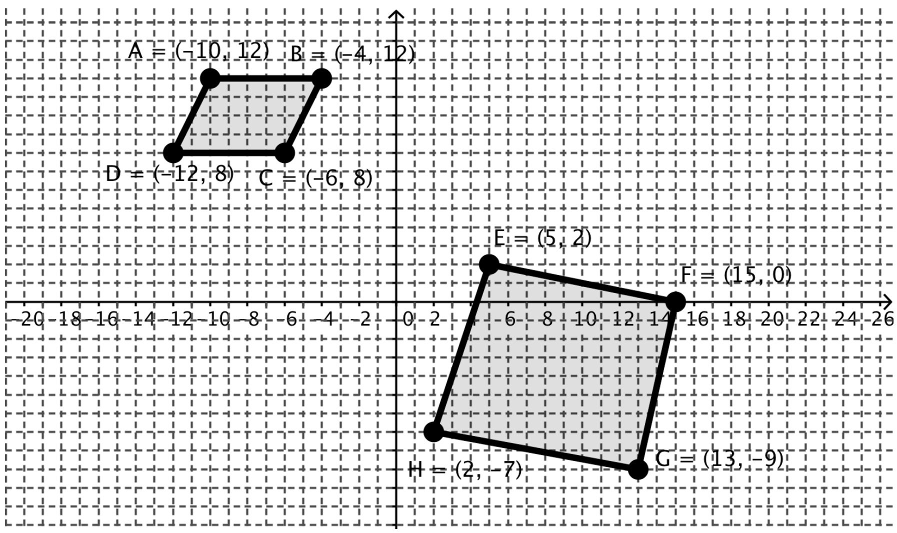
16. $\boldsymbol{E\ (-10, 17),\ G\ (13, 17)}$

8.3 Prove It!

A Practice Understanding Task

In this task you need to use all the things you know about quadrilaterals, distance, and slope to prove that the shapes are parallelograms, rectangles, rhombi, or squares. Be systematic and be sure that you give all the evidence necessary to verify your claim.

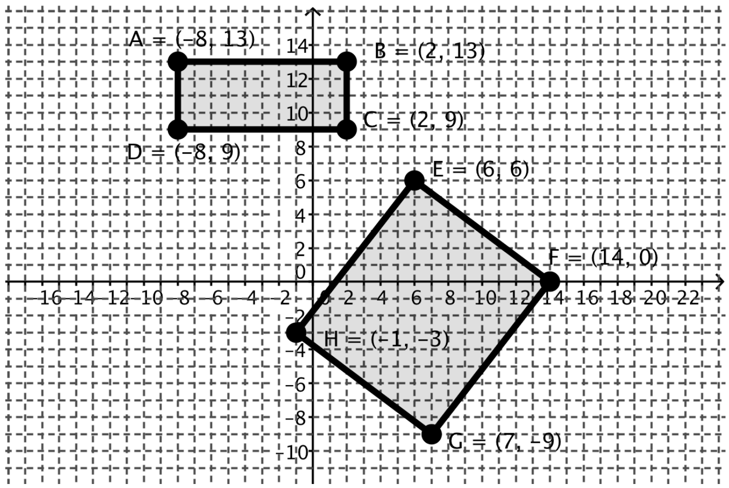
1.



a. Is ABCD a parallelogram? Explain how you know.

b. Is EFGH a parallelogram? Explain how you know.

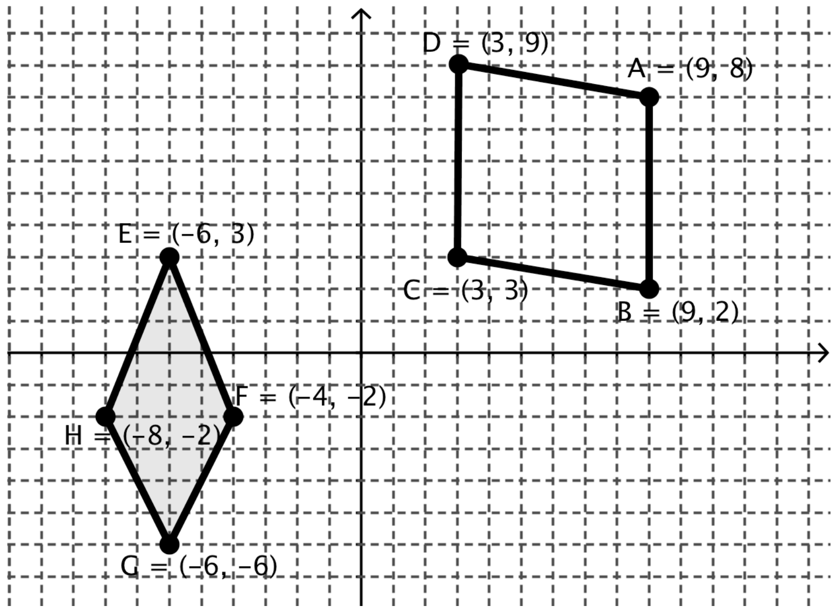
2.



a. Is ABCD a rectangle? Explain how you know.

b. Is EFGH a rectangle? Explain how you know.

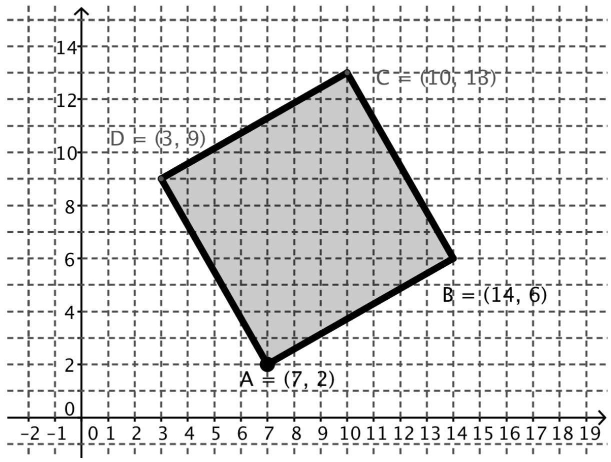
3.



a. Is ABCD a rhombus? Explain how you know.

b. Is EFGH a rhombus? Explain how you know.

4.



a. Is ABCD a square? Explain how you know.

8.3 Prove It! – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is to solidify student understanding of quadrilaterals and to connect their understanding of geometry and algebra. In the task they will use slopes and distance to show that particular quadrilaterals are parallelograms, rectangles, rhombi or squares. This task will also strengthen student understanding of justification and proof, and the need to put forth a complete argument based upon sound mathematical reasoning.

Core Standards Focus:

G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle.

Related Standards: G.GPE.5, G.GPE.7

Standards for Mathematical Practice of Focus in the Task:

SMP 3 – Construct viable arguments and critique the reasoning of others

SMP 6 – Attend to precision

The Teaching Cycle:

Launch (Whole Class):

Launch the task with a discussion of what students know about the properties of quadrilaterals, for instance that a rhombus has two pairs of parallel sides (making it a parallelogram), congruent sides, and perpendicular diagonals. Discuss what you would need to show to prove a claim that a figure is a particular quadrilateral. For instance, it is not enough to show that a shape is a rhombus by showing that the two pairs of sides are parallel, but it would be enough to show that the diagonals are perpendicular. Why?

Explore (Small Group):

Monitor students as they work. It may be helpful to recognize that each set of problems is set up so there is a simple case and a more complicated case. The simple case is designed to help students get ideas for how to prove the more complicated case. Keep track of the various approaches that students use to verify their claims and press them to organize their work so that it communicates to an outside observer. Students should be showing sides or diagonals are parallel or perpendicular using the slope properties, and the distance formula to show that sides or diagonals are congruent. You may also see students try to show one figure to be a particular quadrilateral and then use transformations to show that the second figure is the same type. Select one group of students that have articulated a clear argument for each type of quadrilateral. Be sure to also select a variety of approaches so that students have opportunity to make connections and become more fluent.

Discuss (Whole Class):

The discussion should proceed in the same order as the task, with different groups demonstrating their strategies for one parallelogram, one rectangle, one rhombus, and one square. Select the shape that does not have sides that are on a grid line, so that students are demonstrating the more challenging cases. One recommended sequence for the discussion would be:

1b. Quadrilateral EFGH is not a parallelogram demonstrated by showing that one pair of opposite sides are not parallel using slopes.

2b. Rectangle EFGH demonstrated by using the distance formula to show that the diagonals are congruent.

3a. Showing quadrilateral ABCD is a rhombus because the diagonals are perpendicular and the sides are congruent (or that the sides are congruent and the opposite sides are parallel).

3b. Showing that quadrilateral EFGH is not a rhombus because the sides are not congruent . Ask if the figure is a parallelogram? How do we know?

4a. Showing that quadrilateral ABCD is a square because adjacent sides are perpendicular and sides are congruent. Ask if it sufficient to show that the sides are congruent? How do we know that the opposite sides are parallel?

Aligned Ready, Set, Go: Connecting Algebra and Geometry 8.3

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Interpreting tables of value as ordered pairs.

Find the value of $f(x)$ for the given domain. Write x and $f(x)$ as an ordered pair.

1. $f(x) = 3x - 2$

x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

2. $f(x) = x^2$

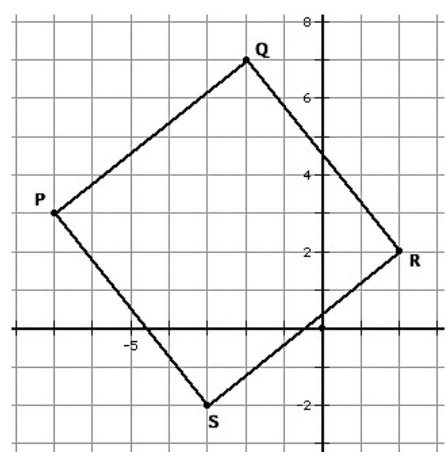
x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

3. $f(x) = 5^x$

x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

SET

Topic: Identifying specific quadrilaterals



4. a) Is the figure at the right a rectangle? Justify your answer.

b) Is the figure at the right a rhombus? Justify your answer.

c) Is the figure at the right a square? Justify your answer.

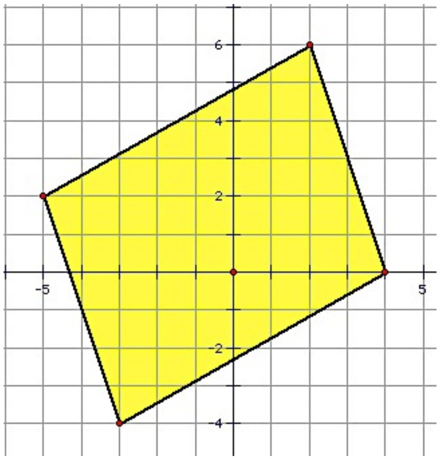
GO

Topic: Calculating perimeters of geometric shapes

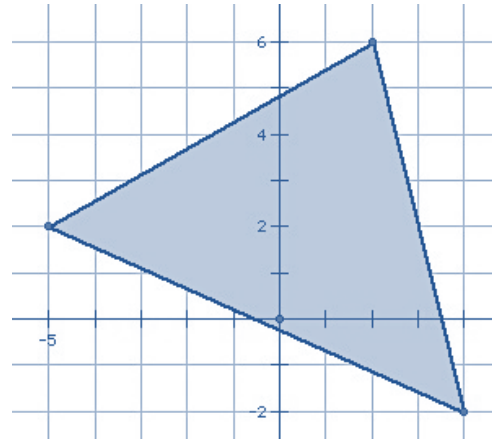
8.3

Find the perimeter of each figure below. Round answers to the nearest hundredth.

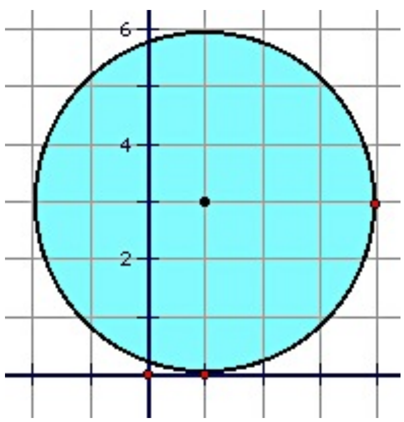
5.



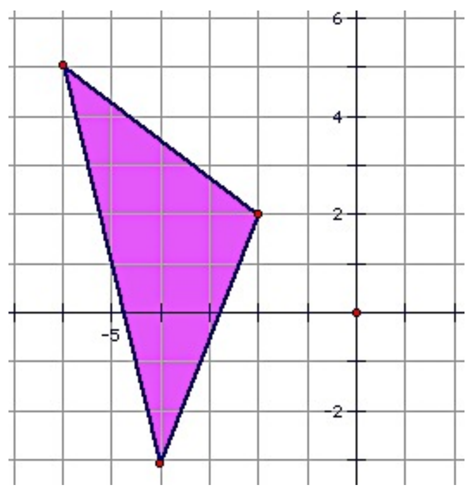
6.



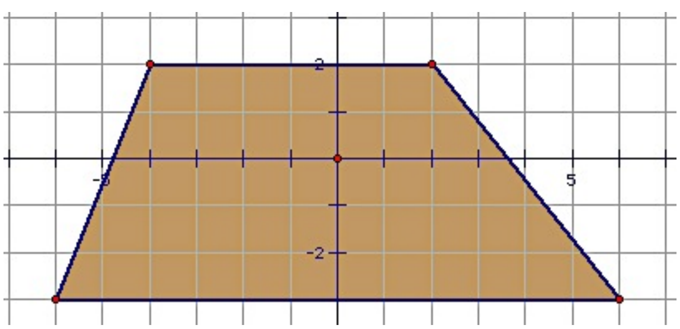
7.



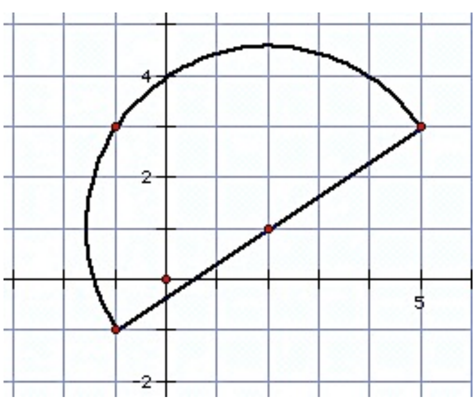
8.



9.



10.



8.4 Training Day

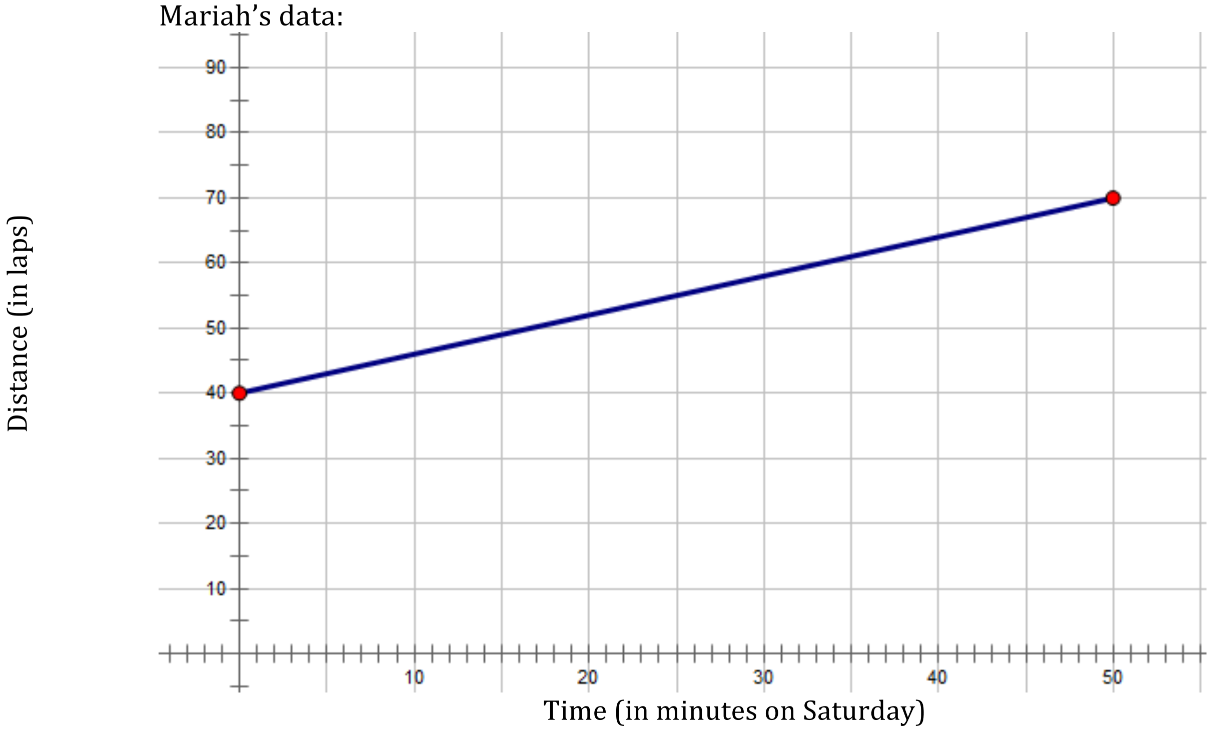
A Develop Understanding Task

Fernando and Mariah are training for six weeks to run in a marathon. To train, they run laps around the track at Eastland High School. Since their schedules do not allow them to run together during the week, they each keep a record of the total number of laps they run throughout the week and then always train together on Saturday morning. The following are representations of how each person kept track of the total number of laps that they ran throughout the week plus the number of laps they ran on Saturday.

Fernando's data:

Time (in minutes on Saturday)	0	10	20	30	40	50
Distance (in laps)	60	66	72	78	84	90

Mariah's data:



1. What observations can be made about the similarities and differences between the two trainers?

2. Write the equation, *m(t),* that models Mariah's distance.

3. Fernando and Mariah both said they ran the same rate during the week when they were training separately. Explain in words how Fernando's equation is similar to Mariah's. Use the sentence frame:

   The rate of both runners is the same throughout the week, however,

   Fernando____.

4. In mathematics, sometimes one function can be used to build another. Write Mariah's equation, *m(t),* by starting with Fernando's equation, *f(t).*

   f(t) =

5. Use the mathematical representations given in this task (table and graph) to model the equation you wrote for number 4. Write in words how you would explain this new function to your class.

8.4 Training Day – Teacher Notes

A Develop Understanding Task

Purpose: Students have had a lot of experience with linear functions and their relationships. They have also become more comfortable with function notation and features of functions. In this task, students first make observations about the rate of change and the distance traveled by the two runners. Using their background knowledge of linear functions, students start to surface ideas about vertical translations of functions and how to build one function from another.

Core Standards Focus:

F.BF.3 Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. (Note: Focus on vertical translations of graphs of linear and exponential functions.)

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

F.BF.1 Write a function that describes a relationship between two quantities.

Related Standards: F.BF.1b, F.IF.1, F.IF.2, A.CED.3

Standards for Mathematical Practice of Focus in the Task:

SMP 1 – Make sense of problems and persevere in solving them

SMP 2 – Reason abstractly and quantitatively

SMP 4 – Model with mathematics

SMP 7 – Look and make use of structure

The Teaching Cycle:

Launch (Whole Class):
To begin this task, read the scenario as a whole group, then ask students to write their answer to the question: "What observations can be made about the similarities and differences between the two trainers?" After a couple of minutes, have students share their observations with a partner. Listen for students to discuss the meaning of the slope and the y-intercept in both situations. If needed, ask the group questions to clarify that the y-intercept is the number of laps each person runs during the week before they meet on Saturday morning and that the slope is the same in both situations. Since the purpose of the lesson is to see how one function can be built from another similar function, these are the two most important ideas to come out of the launch conversation.

Explore (Small Group):
As you monitor, listen for student reasoning about the relationship between the amount of laps run by Mariah and Fernando. Encourage students to explain their reasoning to each other using prior academic vocabulary while working through solutions to problems. If students are incorrect in their thinking, redirect their thinking by asking them to explain how their function relates to the situation.

Discuss (Whole Class):
During the monitoring phase, select students to share their results to strengthen the whole group understanding of the relationship between the 'original function' ($m(t)$) and the 'transformed function' ($f(t)$). You may wish to start the whole group discussion by choosing someone who has graphically shown Fernando and Mariah's graph on the same axes. Have this student share the relationship between the two graphs and press to bring out that at any given time, $f(t)$ is always '20

laps more' than *m(t).* Likewise, have a student share who can explain the relationship using a table. After both representations (table and graph) are shown, ask the whole group to see what connections they can make between the equation, table, and graph.

Aligned Ready, Set, Go: Connecting 8.4

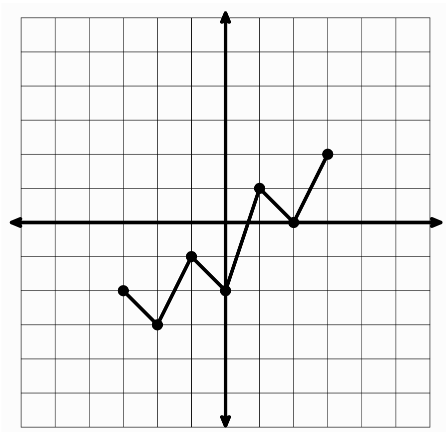
8.4

READY, SET, GO! **Name** **Period** **Date**

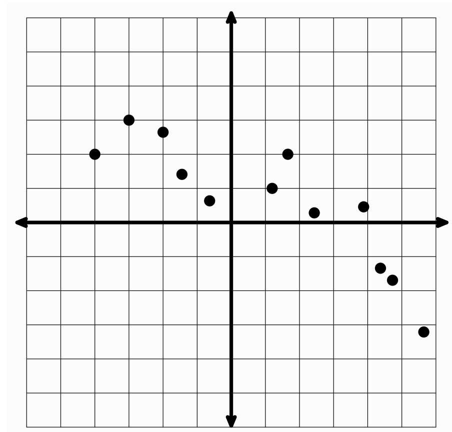
READY

Topic: Vertical transformations on graphs

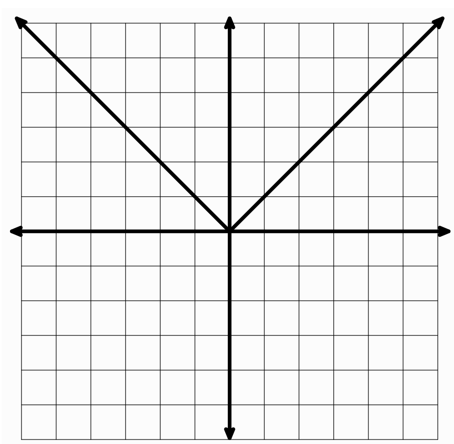
1. Use the graph below to draw a new graph that is translated up 3 units.



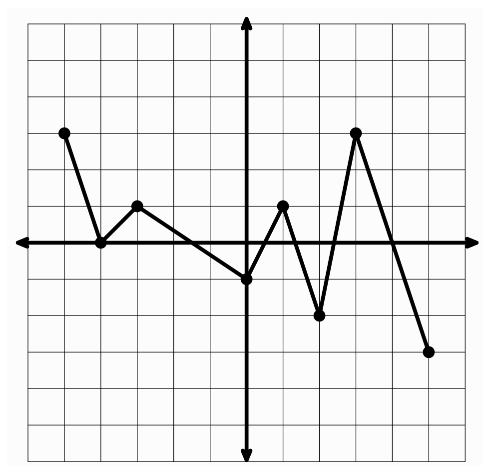
2. Use the graph below to draw a new graph that is translated down 1 unit.



3. Use the graph below to draw a new graph that is translated down 4 units.



4. Use the graph below to draw a new graph that is translated down 3 units.

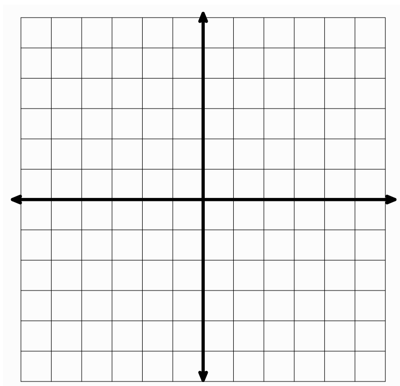


SET

Topic: Graphing transformations and writing the equation of the new graph

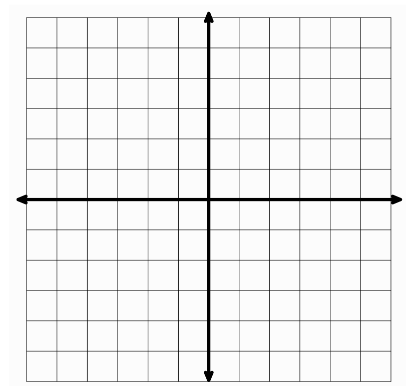
You have been given the equations of $f(x)$ and the transformation $g(x) = f(x) + k$. Graph both $f(x)$ and $g(x)$. Then write the linear equation for $g(x)$ in the space provided.

5. $f(x) = 2x - 4$; $g(x) = f(x) + 3$



$g(x) =$ ______________________________

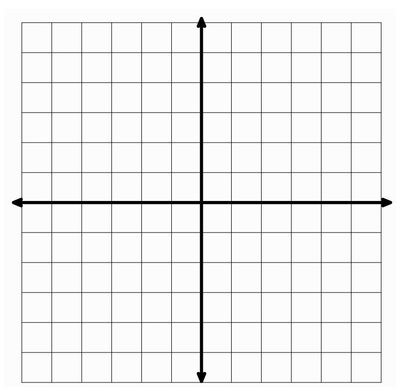
6. $f(x) = 0.5x$; $g(x) = f(x) - 3$



$g(x) =$ ______________________________

Based on the given graph, write the equation of $g(x)$ in the form of $g(x) = f(x) + k$. Then simplify the equation of $g(x)$ into slope-intercept form. The equations of $f(x)$ is given.

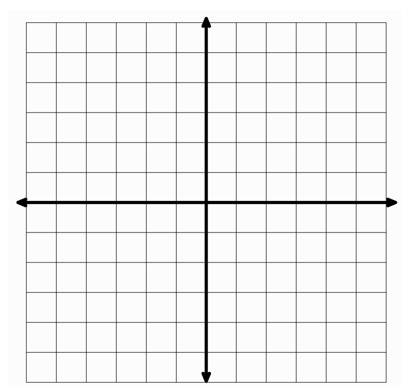
7. $f(x) = \frac{1}{4}x - 3$



a. $g(x) =$ ______________________________
Translation form

b. $g(x) =$ ______________________________
Slope-intercept form

8. $f(x) = -2x + 5$



a. $g(x) =$ ______________________________
Translation form

b. $g(x) =$ ______________________________
Slope-intercept form

GO

Topic: Converting units and making decisions based on data

9. Fernando and Mariah are training for a half marathon. The chart below describes their workouts for the week just before the half marathon. A half marathon is equal to 13.1 miles. If four laps make up one mile, do you think Mariah and Fernando are prepared for the event?

   Describe how you think each person will perform in the race. Include who you think will finish first and predict what you think each person's finish time will be. Use the data to inform your conclusions and to justify your answers.

Day of the week	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Fernando: Distance (in laps)	34	45	52	28	49	36
Time per day (in minutes)	60	72	112	63	88	58
Mariah: Distance (in laps)	30	48	55	44	38	22
Time per day (in minutes)	59	75	119	82	70	45

8.5 Training Day Part II

A Solidify Understanding Task

Fernando and Mariah continued training in preparation for the half marathon. For the remaining weeks of training, they each separately kept track of the distance they ran during the week. Since they ran together at the same rate on Saturdays, they took turns keeping track of the distance they ran and the time it took. So they would both keep track of their own information, the other person would use the data to determine their own total distance for the week.

1. **Week 2**: Mariah had completed 15 more laps than Fernando before they trained on Saturday.
   a. Complete the table for Mariah.

Time (in minutes on Saturday)	0	10	20	30	40	50	60
Fernando: Distance (in laps)	50	56	62	68	74	80	86
Mariah: Distance (in laps)							

   b. Write the equation for Mariah as a transformation of Fernando. Equation for Mariah: $m(t) = f(t)$ ________

2. **Week 3:** On Saturday morning before they started running, Fernando saw Mariah's table and stated, "My equation this week will be $f(t) = m(t) + 30$."
   a. What does Fernando's statement mean?

   b. Based on Fernando's translated function, complete the table.

Time (in minutes on Saturday)	0	20	40	60	70
Fernando: Distance (in laps)					
Mariah: Distance (in laps)	45	57	69	81	87

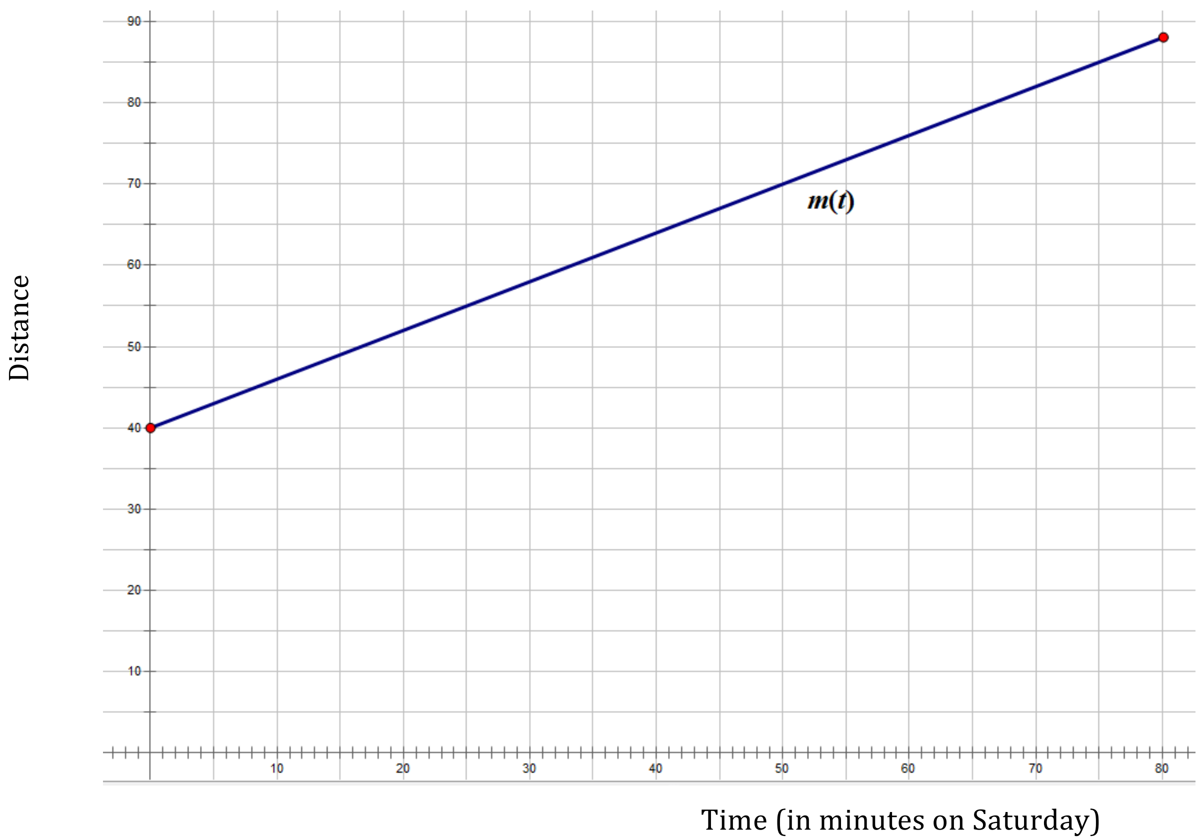
c. Write the equation for both runners in slope-intercept form:

d. Write the equation for Mariah, transformed from Fernando.

e. What relationship do you notice between your answers to parts c and d?

3. **Week 4:** The marathon is only a couple of weeks away!

a. Use Mariah's graph to sketch *f(t)*. $f(t) = m(t) - 10$



b. Write the equations for both runners in slope-intercept form.

c. What do you notice about the two graphs? Would this always be true if one person ran "k" laps more or less each week?

4. **Week 5:** This is the last week of training together. Next Saturday is the big day. When they arrived to train, they noticed they had both run 60 laps during the week.

   a. Write the equation for Mariah on Saturday given that they run at the same rate as the week before.

   b. Write Fernando's equation as a transformation of Mariah's equation.

5. What conjectures can you make about the general statement: "$g(x) = f(x)+k$" when it comes to linear functions?

8.5 Training Day Part II – Teacher Notes

A Solidify Understanding Task

Purpose: Students will solidify their understanding of vertical transformations of linear functions in this task. Goals of this task include:

- Writing function transformations using function notation.
- Recognizing that the general form y = $f(x) + k$ represents a vertical translation, with the output values changing while the input values stay the same.
- Understanding that a vertical shift of a linear function results in a line parallel to the original.

Core Standards Focus:

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. (Note: Focus on vertical translations of graphs of linear and exponential functions.)

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

F.BF.1 Write a function that describes a relationship between two quantities.

Related Standards: F.IF.1, F.IF.2 , A.CED.3

Standards for Mathematical Practice of Focus in the Task:

SMP 2 – Reason abstractly and quantitatively

SMP 3 – Construct viable arguments and critique the reasoning of others

SMP 7 – Look and make use of structure

The Teaching Cycle:

Launch (Whole Class):

Read the initial story and ask students to clarify what this means. If not stated, clarify that the two runners are going the same rate each time they run together on Saturday morning. Students should be able to get started on this task without additional support, since it is similar in nature to the work they did on "Training Day".

Explore (Small Group):

Watch for students who confuse input/output values as well as for students who struggle with making sense of using function notation in the first two problems (weeks 2 and 3). Listen for students who make the connection that the 'shift' is about 'adjusting the output values' and are visually showing the connection to the table and the equations (the distance between the number of laps of the two runners is the same in the table as it is in the "shift" or "+k" value).

Week 4 has students visually see the shift of "k" units with a graphical representation. This is another way students see that each output value is exactly k units away from the other function. It can be noted that the lines are parallel, however, make sure the discussion talks about the distance of the output values (since with future functions, they will not be 'parallel lines'… but that they do maintain a distance of K units.

Discuss (Whole Class):

The goal of this whole group discussion is to highlight the different ways to see vertical translations of linear functions. Have different students go over each week of training, showing how the vertical shift of one function relates to the other. For each week, have students show connections between the context, the mathematical representation, and the transformation function.

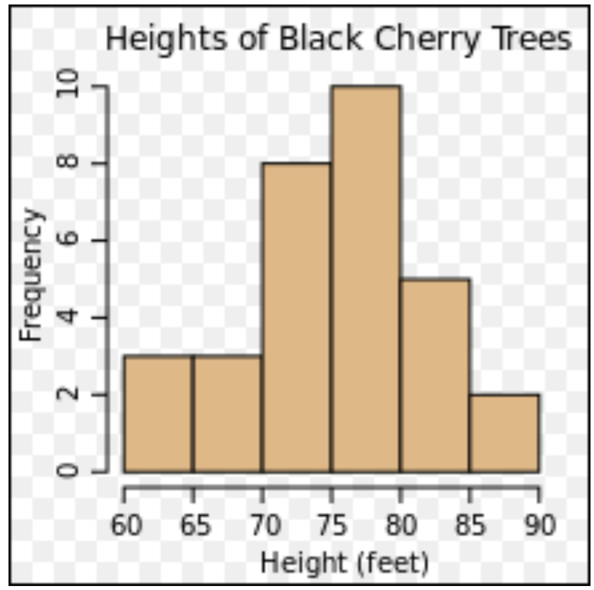
Aligned Ready, Set, Go: Connections 8.5

READY, SET, GO! **Name** **Period** **Date**

READY

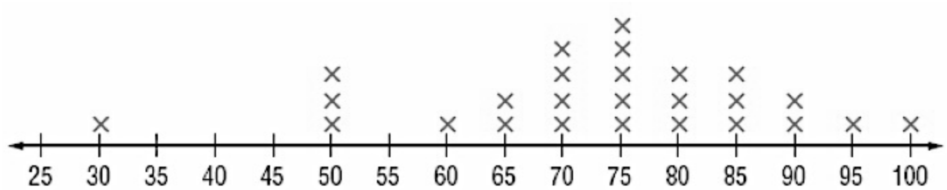
Topic: Describing spread.

1. Describe the spread in the histogram below.

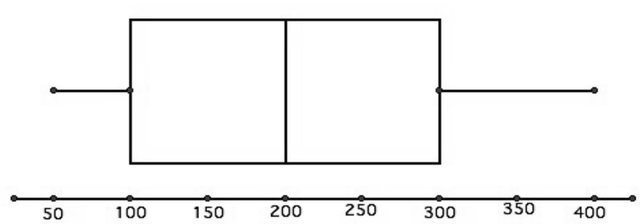


https://commons.wikimedia.org/wiki/File:Black_cherry_tree

2. Describe the spread in the line plot below.



3. Describe the spread in the box and whisker plot.



SET

Topic: Writing functions in translation form.

You are given information about $f(x)$ and $g(x)$.
Rewrite $g(x)$ in translation form: $g(x) = f(x) + k$

4. $f(x) = 7x + 13$
$g(x) = 7x - 5$

$g(x)$ = ______________________
Translation form

5. $f(x) = 22x - 12$
$g(x) = 22x + 213$

$g(x)$ = ______________________
Translation form

6. $f(x) = -15x + 305$
$g(x) = -15x - 11$

$g(x)$ = ______________________
Translation form

7.

x	f(x)	g(x)
3	11	26
10	46	61
25	121	136
40	196	211

$g(x)$ = ______________________
Translation form

8.

x	f(x)	g(x)
-4	5	-42
-1	-1	-48
5	-13	-60
20	-43	-90

$g(x)$ = ______________________
Translation form

9.

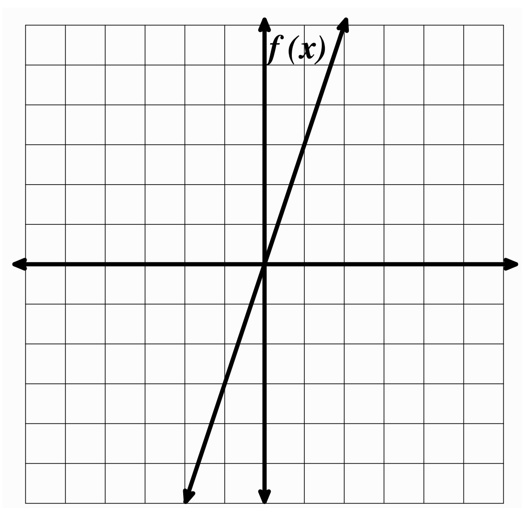
x	f(x)	g(x)
-10	4	-15.5
-3	7.5	-12
22	20	0.5
41	29.5	10

$g(x)$ = ______________________
Translation form

GO

Topic: Vertical and horizontal translations.

10. Use the graph of $\boldsymbol{f(x) = 3x}$ to do the following:



a. Sketch the graph of $\boldsymbol{g(x) = 3x - 2}$ on the same grid.

b. Sketch the graph of $\boldsymbol{h(x) = 3(x - 2)}$

c. Describe how $\boldsymbol{f(x), g(x),}$ ***and*** $\boldsymbol{h(x)}$ are different and how they are the same.

d. Explain in what way the parentheses affect the graph. Why do you think this is so?

8.6 Shifting Functions

A Practice Understanding Task

Part I: Transformation of an exponential function.

The table below represents the property value of Rebekah's house over a period of four years.

Rebekah's Home

Time (years)	Property Value	Common Ratio
0	150,000	
1	159,000	
2	168,540	
3	178,652	
4	189,372	

Rebekah says the function $P(t) = 150{,}000(1.06)^t$ represents the value of her home.

1. Explain how this function is correct by using the table to show the initial value and the common ratio between terms.

Jeremy lives close to Rebekah and says that his house is always worth $20,000 more than Rebekah's house. Jeremy created the following table of values to represent the property value of his home.

Jeremy's Home

Time (years)	Property Value	Relationship to Rebekah's table
0	170,000	
1	179,000	
2	188,540	
3	198,652	
4	209,372	

When Rebekah and Jeremy tried to write an exponential function to represent Jeremy's property value, they discovered there was not a common ratio between all of the terms.

2. Use your knowledge of transformations to write the function that could be used to determine the property value of Jeremy's house.

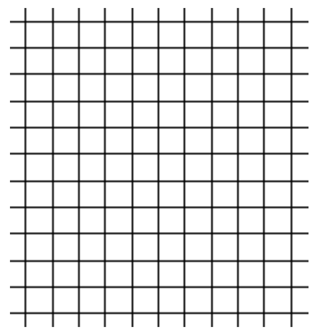
Part 2: Shifty functions.

Given the function *g(x)* and information about *f(x),*

- write the function for *f(x),*
- graph both functions on the set of axes, and
- show a table of values that compares *f(x)* and *g(x)*.

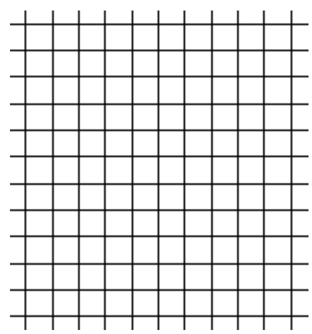
3. $If\ \ g(x) = 3(2)^x\ and\ \ f(x) = \ g(x) - 5, then\ \ f(x) =$ ______________________________

x				
$f(x)$				
$g(x)$				



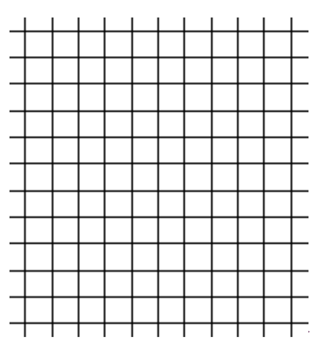
4. $If\ \ g(x) = 4(.5)^x\ and\ \ f(x) = \ g(x) + 3,\ \ then\ \ f(x) =$ ______________________________

x				
$f(x)$				
$g(x)$				



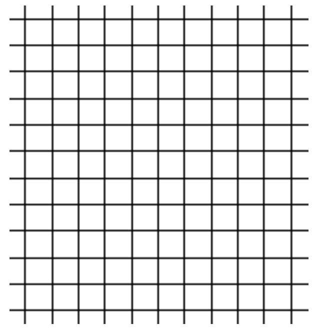
5. $If\ \ g(x) = 4x + 3\ and\ \ f(x) = \ g(x) + 7, then\ \ f(x) =$ ______________________________

x				
$f(x)$				
$g(x)$				



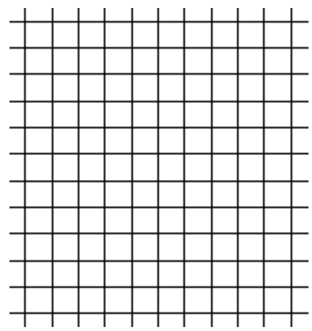
6. $If\ g(x) = 2x + 1\ and\ f(x) = g(x) - 4, then\ f(x) =$ ____________________

x				
f(x)				
g(x)				



7. $If\ g(x) = -x\ and\ f(x) = g(x) + 3, then\ f(x) =$ ____________________

x				
f(x)				
g(x)				



Part III: Communicate your understanding.

8. If *f(x) = g(x) + k,* describe the relationship between *f(x)* and *g(x).* Support your answers with tables and graphs.

8.6 Shifting Functions – Teacher Notes

A Practice Understanding Task

Purpose: Students will solidify their understanding of vertical transformations of exponential functions then practice shifting linear and exponential functions in this task. Goals of this task include:

- Writing function transformations using function notation.
- Recognizing that the general form y = $f(x) + k$ represents a change of k units in output values while the input values stay the same.
- Understanding that a vertical shift of a function creates a function that is exactly k units above or below the original function.
- Connecting equations, graphs, and table values and how the value of k shows up in each representation.

Core Standards Focus:

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. (Note: Focus on vertical translations of graphs of linear and exponential functions.)

F.BF.1 Write a function that describes a relationship between two quantities.★

Related Standards: F.BF.1b, F.IF.1, F.IF.2, F.IF.9, A.CED.3

Standards for Mathematical Practice:

SMP 2 – Reason abstractly and quantitatively
SMP 7 – Look and make use of structure

The Teaching Cycle:

Launch (Whole Class):

Prior to the task, you may wish to access student background knowledge by asking students how they would determine the equation of an exponential function given a table of values or a pair of points. You may wish to have an example to help clarify:

x	0	1	2	3	4
$f(x)$	5	10	20	40	80

Introduce this task by talking about the scenario presented, then let students work independently at first, then with a partner to answer questions 1 and 2 from Part I.

Explore (Small Group):

As you monitor Part I of this task, look for student understanding that exponential functions in the form of $f(x) = a(b)^x$ have a common ratio between terms (If there is confusion, assist the students- the goal of this task is to deepen student understanding of vertical transformations of linear and exponential functions, not to determine that there is a common ratio between terms). Bring students back together, then ask someone to explain why this function, *J(t)* = *R(t)* + 20,000 is a solution to problem 2 (Use your knowledge of transformations to write the function that could be used to determine the property value of Jeremy's house.)
Once you discuss how this is similar to the linear and geometric transformations, have students complete the rest of the task, monitoring for student understanding.

Discuss (Whole Class):

Students work toward becoming fluent with transforming linear and exponential functions. For the whole group discussion, choose problems to discuss that were difficult for students to complete. Goals of this task include:

- Writing function transformations using function notation.
- Recognizing that the general form y = *f(x)*+ *k* represents a change of *k* units in output values while the input values stay the same.
- Understanding that a vertical shift of a function creates a function that is exactly *k* units above or below the original function.
- Connecting equations, graphs, and table values and how the value of *k* shows up in each representation.

Aligned Ready, Set, Go: Connecting 8.6

READY, SET, GO! Name Period Date

READY

Topic: Finding percentages

Mrs. Gonzalez noticed that her new chorus class had a lot more girls than boys in it. There were 32 girls and 17 boys. (Round answers to the nearest %.)

1. What percent of the class are girls?

2. What percent are boys?

3. 68% of the girls were sopranos.
   a. How many girls sang soprano?

   b. What percent of the entire chorus sang soprano?

4. Only 30% of the boys could sing bass.
   a. How many boys were in the bass section?

   b. What percent of the entire chorus sang bass?

5. Compare the number of girls who sang alto to the number of boys who sang tenor. Which musical section is larger? Justify your answer.

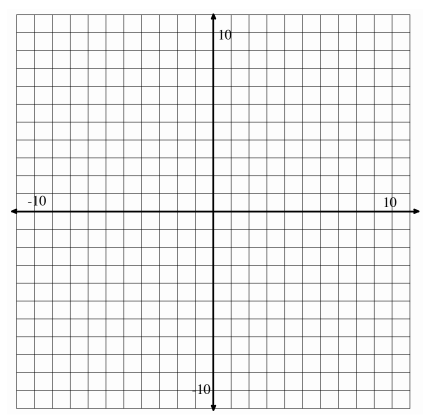
SET

Topic: Graphing exponential equations.

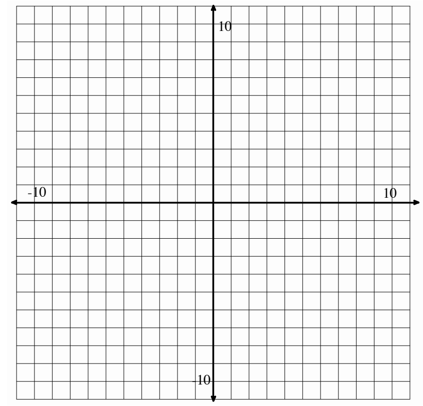
6. Think about the graphs of $y = 2^x$ and $y = 2^x - 4$.

   a. Predict what you think is the same and what is different.

   b. Use your calculator to graph both equations on the same grid. Explain what stayed the same and what changed when you subtracted 4. Identify in what way it changed. (If you don't have a graphing calculator, this can easily be done by hand.)



7. Think about the graphs of $y = 2^x$ and $y = 2^{(x-4)}$.



   a. Predict what you think is the same and what is different.

   b. Use your calculator to graph both equations on the same grid. Explain what stayed the same and what changed. Identify in what way it changed.

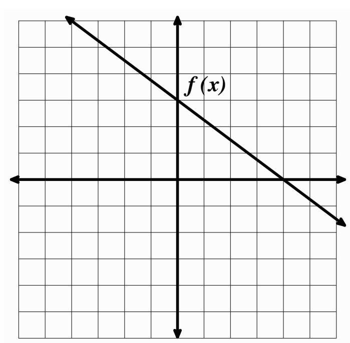
GO

Topic: Vertical translations of linear equations

The graph of $f(x)$ and the translation form equation of $g(x)$ are given. Graph $g(x)$ on the same grid as $f(x)$ and write the slope-intercept equation of $f(x)$ and $g(x)$.

8. $g(x) = f(x) - 5$

a.

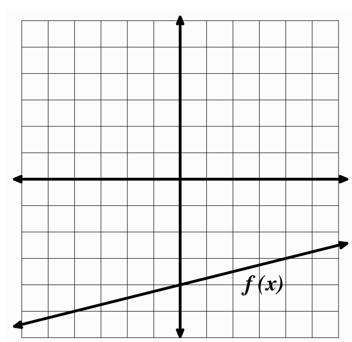


b. $f(x) =$____________________

c. $g(x) =$____________________
Slope-intercept form

9. $g(x) = f(x) + 4$

a.

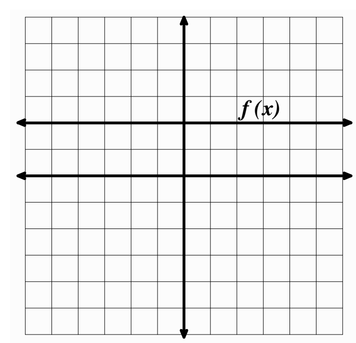


b. $f(x) =$____________________

c. $g(x) =$____________________
Slope-intercept form

10. $g(x) = f(x) - 6$

a.



b. $f(x) =$____________________

c. $g(x) =$____________________
Slope-intercept form