

Transforming Mathematics Education

SECONDARY MATH TWO

An Integrated Approach

MODULE 1

Quadratic Functions

MATHEMATICS VISION PROJECT, ORG

The Mathematics Vision Project

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MODULE 1 - TABLE OF CONTENTS

QUADRATIC FUNCTIONS

1.1 Something to Talk About - A Develop Understanding Task

An introduction to quadratic functions, designed to elicit representations and surface a new type of pattern and change (F.BF.I, A.SSE.I, A.CED.2)

READY, SET, GO Homework: Quadratic Functions 1.1

1.2 I Rule - A Solidify Understanding Task

Solidification of quadratic functions begins as quadratic patterns are examined in multiple representations and contrasted with linear relationships (F.BF.I, A.SSE.I, A.CED.2)

READY, SET, GO Homework: Quadratic Functions 1.2

1.3 Scott's Macho March - A Solidify Understanding Task

Focus specifically on the nature of change between values in a quadratic being linear (F-BF, F-LE)

READY, SET, GO Homework: Quadratic Functions 1.3

1.4 Rabbit Run- A Solidify Understanding Task

Focus on maximum/minimum point as well as domain and range for quadratics (F.BF.I, A.SSE.I, A.CED.2)

READY, SET, GO Homework: Quadratic Functions 1.4

1.5 The Tortoise and the Hare- A Solidify Understanding Task

Comparing quadratic and exponential functions to clarify and distinguish between each type of growth as well as how that growth appears in each of their representations (F.BF.I, A.SSE.I, A.CED.2, F.LE.3)

READY, SET, GO Homework: Quadratic Functions 1.5



SECONDARY MATH 2 // MODULE 1 QUADRATIC FUNCTIONS

1.6 How Does it Grow - A Practice Understanding Task

Incorporating quadratics with the understandings of linear and exponential functions (F.LE.1, F.LE.2, F.LE.3)

READY, SET, GO Homework: Quadratic Functions 1.6



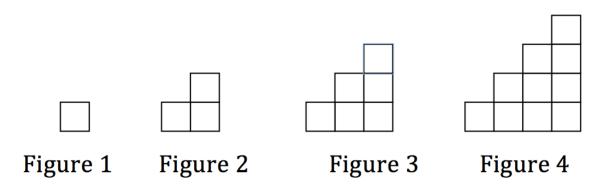
1.1 Something to Talk About

A Develop Understanding Task

Cell phones often indicate the strength of the phone's signal with a series of bars. The logo below shows how this might look for various levels of service.



CC BY Skinny Casual Lover



- 1. Assuming the pattern continues, draw the next figure in the sequence.
- 2. How many blocks will be in the figure 10?
- 3. Examine the sequence of figures and find a rule or formula for the number of tiles in any figure number.

READY, SET, GO!

Name

Period

Date

READY

Topic: Distributive Property

Simplify the following expressions

1. 3(2x + 7)

2. -12(5x-4)

3. 5a(-3a + 13)

4. 9x(6x-2)

- 5. $\frac{2x}{3}(12x + 18)$
- 6. $\frac{4a}{5}(10a 25b)$

7. $\frac{-4x}{11}(121x + 22)$

SET

Topic: Recognizing Linear Exponential and Quadratic Functions

In each set of 3 functions, one will be linear and one will be exponential. One of the three will be a new category of function. List the characteristics in each table that helped you to identify the linear and the exponential functions. What are some characteristics of the new function? Find an explicit and recursive equation for each.

8. Linear, exponential, or a new kind of function?

a.

x	f(x)
6	64
7	128
8	256
9	512
10	1024

h

x	f(x)
6	36
7	49
8	64
9	81
10	100

C

6 11	х	f(x)
7 12	6	11
/ 13	7	13
8 15	8	15
9 17	9	17
10 19	10	19

Type and characteristics?

Type and characteristics?

Type and characteristics?

Explicit equation:

Explicit equation:

Explicit equation:

Recursive equation:

Recursive equation:

Recursive equation:

9. Linear, exponential, or a new kind of function?

d.

, - I	,
x	f(x)
-2	-17
-1	-12
0	-7
1	-2
2	3
-	-

e.

unction:	
x	f(x)
-2	1/25
-1	1/5
0	1
1	5
2	25

f.

х	f(x)
-2	9
-1	6
0	5
1	6
2	9

Type and characteristics?

Type and characteristics?

Type and characteristics?

Explicit equation:

Explicit equation:

Explicit equation:

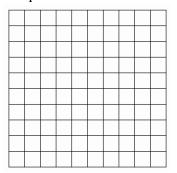
Recursive equation:

Recursive equation:

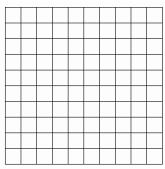
Recursive equation:

10. Graph the functions from the tables in #8 and #9. Add any additional characteristics you notice from the graph. Place your axes so that you can show all 5 points. Identify your scale. Write your explicit equation above the graph.

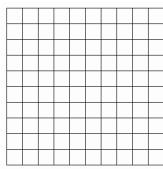
a. Equation:



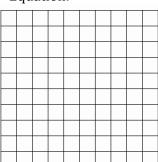
b. Equation:



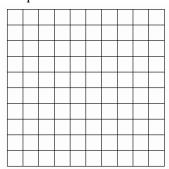
c. Equation:



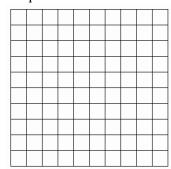
d. Equation:



e. Equation:



f. Equation:



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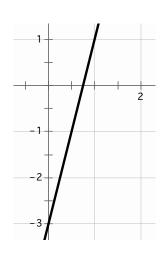
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GO

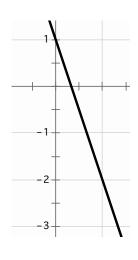
Topic: Rates of Change

Identify the rate of change in each of the representations below.

11.



12.



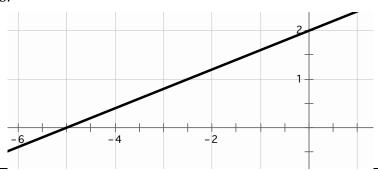
13.

Х	f(x)	
25	65	
26	68	
27	71	
28	74	

14.

$$f(0) = 7; f(n + 1) = f(n) + 5$$

15.



16.

Slope of \overrightarrow{AB}

A(-3, 12) B(-11, -16)

17. George is loading freight into an elevator. He notices that the weight limit for the elevator is 1000 lbs. He knows that he weighs 210 lbs. He has loaded 15 boxes into the elevator. Each box weighs 50 lbs. Identify the rate of change for this situation.

4

18.

Independent variable	4	5	6	7	8
Dependent variable	5	5.5	6	6.5	7

19.

$$f(-4) = 24$$
 and $f(6) = -36$

1.2 I Rule!

A Solidify Understanding Task

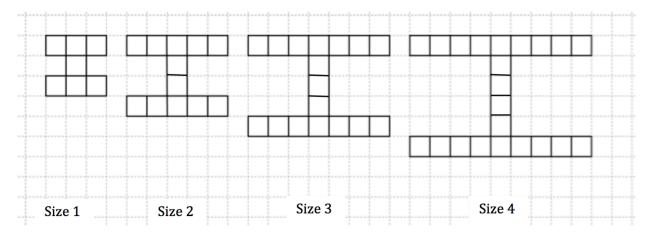
Marco has started a new blog about sports at



Imagination High School (mascot: the fighting unicorns) that he has decided to call "I Site". He created a logo for the web site that looks like this:

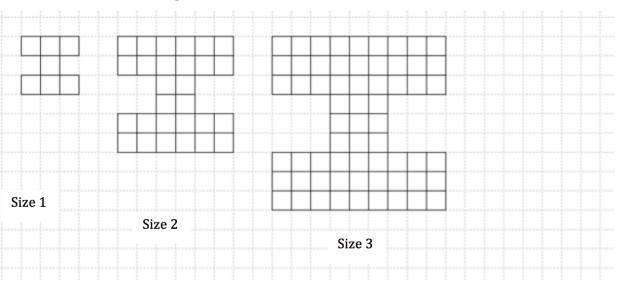


He is working on creating the logo in various sizes to be placed on different pages on the website. Marco developed the following designs:

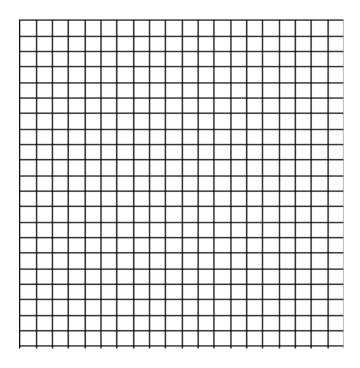


- 1. How many squares will be needed to create the size 100 logo?
- 2. Develop a mathematical model for the number of squares in the logo for size n.

Marco decides to experiment with making his logo "blockier" so that it looks stronger. Here's what he came up with:



3. Assuming that Marco continues with the pattern as it has begun, draw the next figure, size 4, and find the number of blocks in the figure.





SECONDARY MATH II // MODULE 1 QUADRATIC FUNCTIONS - 1.2

4.	Develop a mathematical model for the number of blocks in a logo of size n .
5.	Compare the models that you developed for the first set of logos to the second set of logos. In what ways are they similar? In what ways are they different?

READY, SET, GO!

Name

Period

Date

READY

Topic: Distributive Property

Simplify. First use the distributive property and then combine the like terms.

Example:

$$3x(4x+1) + 2(4x+1) \rightarrow (12x^2 + 3x) + (8x+2) \rightarrow 12x^2 + [3x+8x] + 2 \rightarrow 12x^2 + 11x + 2$$
Simplified form

- 1. 2x(5x + 3) + 7(5x + 3)
- 2. 8x(x + 1) + 2(x + 1)

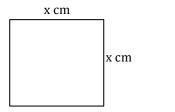
- 3. 6x(x-10) 1(x-10)
- 4. 1x(3x + 4) + 5(3x + 4)
- 5. 3x(8x + 3) 4(8x + 3)
- 6. 5x(2x+6) + 2(2x+6)
- 7. 7x(-5x+2) 13(-5x+2) 8. -4x(12x+3) + 3(12x+3)

SET

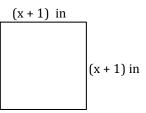
Topic: Comparing Area and perimeter

Calculate the area and perimeter of each figure below. The area may be written as a product. Include the correct unit on your answer. (Your answers will contain a variable.)

9.



10.

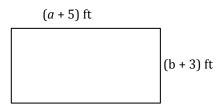


- a. Perimeter: _____
- a. Perimeter: _____

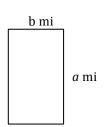
b. Area: _____

b. Area: _____

11.



12.



a. Perimeter: _____

b. Area: _____

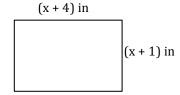
a. Perimeter: _____

b. Area: _____

13.

(x + 3) m	
	(x - 2) m

14.



a. Perimeter: _____

b. Area: _____

a. Perimeter: _____

b. Area: _____

15. Compare the perimeter to the area in each of problems (9-14).

In what way are the numbers and units in the perimeters and areas different?

GO

Topic: Greatest Common Factor

Find the GCF for the given terms.

16. 15abc² and 25a³bc

17. 12x⁵y and 32x⁶y

18. 17pqr and 51pqr³

19. 7x² and 21x

20. 6x², 18x, and -12

21. $4x^2$ and 9x

22. 11x²y², 33x²y, and 3xy² 23. 16a²b, 24ab, and 16b

24. 49s²t² and 36s²t²



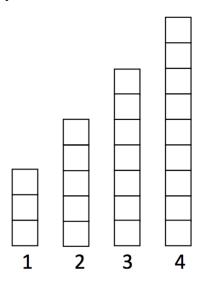
1.3 Scott's Macho March

A Solidify Understanding Task

After looking in the mirror and feeling flabby, Scott decided that he really needed to get in shape. He



joined a gym and added push-ups to his daily exercise routine. He started keeping track of the number of push-ups he completed each day in the bar graph below, with day one showing he completed three push-ups. After four days, Scott was certain he could continue this pattern of increasing the number of push-ups for at least a few months.



1. Model the number of push-ups Scott will complete on any given day. Include both explicit and recursive equations.

SECONDARY MATH II // MODULE 1
QUADRATIC FUNCTIONS - 1.3

Scott's gym is sponsoring a "Macho March" promotion. The goal of "Macho March" is to raise money

for charity by doing push-ups. Scott has decided to participate and has sponsors that will donate

money to the charity if he can do a total of at least 500 push-ups, and they will donate an additional

\$10 for every 100 push-ups he can do beyond that. So now Scott is going to track the total number

of push-ups done up to any given day of the month.

2. Estimate the total number of push-ups that Scott will do in a month if he continues to increase

the number of push-ups he does each day in the pattern shown above.

3. Draw the diagram that shows the total number of pushups that Scot has done in the month at

the end of each day.

4. How many push-ups will Scott have done after a week?



SECONDARY MATH II // MODULE 1 QUADRATIC FUNCTIONS - 1.3

5.	Model the total number of push-ups that Scott has completed on any given day during "Macho March". Include both recursive and explicit equations.

6. Will Scott meet his goal and earn the donation for the charity? Will he get a bonus? If so, how much? Explain.



READY, SET, GO!

Name

Period

Date

READY

Topic: Multiplying two binomials

In the previous RSG, you were asked to use the distributive property on two different terms in the same problem. Example: Multiply and Simplify Simp

You may have noticed that the binomial (4x + 1) occurred twice in the problem.

Here is a simpler way to write the same problem: (3x + 2)(4x + 1).

You will use the distributive property twice. First multiply 3x(4x + 1); then multiply +2(4x + 1). Add the like terms. Write the x^2 term first, the x-term second, and the constant term last.

$$3x(4x+1) + 2(4x+1) \rightarrow (12x^2 + 3x) + (8x+2) \rightarrow 12x^2 + [3x+8x] + 2 \rightarrow 12x^2 + 11x + 2$$
like terms

Simplified form

Multiply the two binomials. (Your answer should have 3 terms and be in this form $ax^2 + bx + c$.)

1.
$$(x+5)(x-7)$$

2.
$$(x + 8)(x + 3)$$

3.
$$(x-9)(x-4)$$

4.
$$(x+1)(x-4)$$

5.
$$(3x-5)(x-1)$$

6.
$$(5x - 7)(3x + 1)$$

7.
$$(4x-2)(8x+10)$$

8.
$$(x+6)(-2x+5)$$

9.
$$(8x-3)(2x-1)$$

SET

Topic: Distinguishing between linear and quadratic patterns

Use first and second differences to identify the pattern in the tables as *linear*, *quadratic*, or *neither*. Write the recursive equation for the patterns that are linear or quadratic.

10.

x	у
-3	-23
-2	-17
-1	-11
0	-5
1	1
2	7
3	13

11.

x	У
-3	4
-2	0
-1	-2
0	-2
1	0
2	4
3	10

12.

y
-15
-10
-5
0
0 5
10
15

- a. Pattern:
- b. Recursive equation:
- a. Pattern:
- b. Recursive equation:
- a. Pattern:
- b. Recursive equation:

13.

x	y
-3 -2	24
-2	22
-1	22 20
0	18
1	16
1 2 3	14
3	12

14.

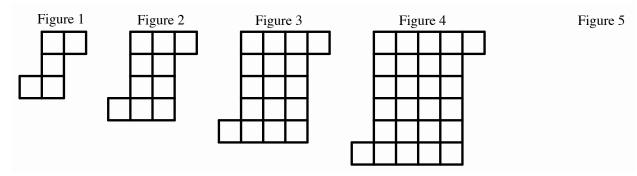
x	y
-3	48
-2 -1	22 6
-1	6
0	0
1	4
3	18
3	42

15.

x	y
-3	4
-2	1
-1	0
0	1
1	4
2	9
3	16

- a. Pattern:
- b. Recursive equation:
- a. Pattern:
- b. Recursive equation:
- a. Pattern:
- b. Recursive equation:

16.



- a. Draw figure 5.
- b. Predict the number of squares in figure 30. Show what you did to get your prediction.

GO

Topic: Interpreting recursive equations to write a sequence

Write the first five terms of the sequence.

17.
$$f(0) = -5$$
; $f(n) = f(n-1) + 8$ 18. $f(0) = 24$; $f(n) = f(n-1) - 5$

18.
$$f(0) = 24$$
: $f(n) = f(n-1) - 5$

14

19.
$$f(0) = 25$$
; $f(n) = 3f(n-1)$

20.
$$f(0) = 6$$
; $f(n) = 2f(n-1)$

1.4 Rabbit Run

A Solidify Understanding Task



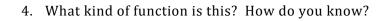
Misha has a new rabbit that she named "Wascal". She wants to build Wascal a pen so that the rabbit has space to move around safely. Misha has purchased a 72 foot roll of fencing to build a rectangular pen.

1. If Misha uses the whole roll of fencing, what are some of the possible dimensions of the pen?

2. If Misha wants a pen with the largest possible area, what dimensions should she use for the sides? Justify your answer.

SECONDARY MATH II // MODULE 1 QUADRATIC FUNCTIONS - 1.4

3.	Write a model for the area of the rectangular pen in terms of the length of one side.
	Include both an equation and a graph.



5. How does this function compare to the second type of block I logos in *I Rule*?



READY, SET, GO!

Name

Period

Date

READY

Topic: Applying slope formula

Calculate the slope of the line between the given points. Use your answer to indicate which line is the steepest.

1. A (-3, 7) B (-5, 17)

2. H (12, -37) K (4, -3)

3. P(-11, -24) Q(21, 40)

4. R (55, -75) W(-15, -40)

SET

Topic: Investigating perimeters and areas

Adam and his brother are responsible for feeding their horses. In the spring and summer the horses graze in an unfenced pasture. The brothers have erected a portable fence to corral the horses in a grazing area. Each day the horses eat all of the grass inside the fence. Then the boys move it to a new area where the grass is long and green. The porta-fence consists of 16 separate pieces of fencing each 10 feet long. The brothers have always arranged the fence in a long rectangle with one length of fence on each end and 7 pieces on each side making the grazing area 700 sq. ft. Adam has learned in his math class that a rectangle can have the same perimeter but different areas. He is beginning to wonder if he can make his daily job easier by rearranging the fence so that the horses have a bigger grazing area. He begins by making a table of values. He lists all of the possible areas of a rectangle with a perimeter of 160 ft., while keeping in mind that he is restricted by the lengths of his fencing units. He realizes that a rectangle that is oriented horizontally in the pasture will cover a different section of grass than one that is oriented vertically. So he is considering the two rectangles as different in his table. Use this information to answer questions 5 – 9 on the next page.

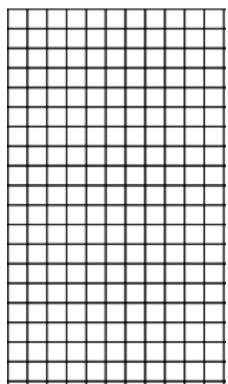
Horizontal

Vertical

5. Fill in Adam's table with all of the arrangements for the fence. (The first one is done for you.)

	Length in fencing" units	idth in "fencing" units	Length in ft.	Width in ft.	Perimeter (ft)	Area (ft) ²
	1 unit	7 units	10 ft	70 ft	160 ft	700 ft ²
a.	2 units				160 ft	
b.	3 units				160 ft	
c.	4 units				160 ft	
d.	5 units				160 ft	
e.	6 units				160 ft	
f.	7 units				160 ft	

- 6. Discuss Adam's findings. Explain how you would rearrange the sections of the porta-fence so that Adam will be able to do less work.
- 7. Make a graph of Adam's investigation. Let length be the independent variable and area be the dependent variable. Label the scale.
- 8. What is the shape of your graph?
- 9. Explain what makes this function be a quadratic.

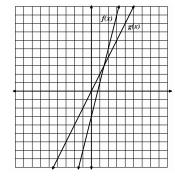


GO

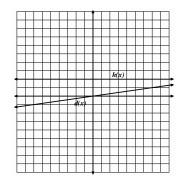
Topic: Comparing linear and exponential rates of change

Indicate which function is changing faster.

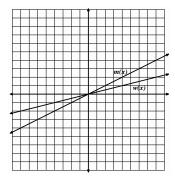
10.



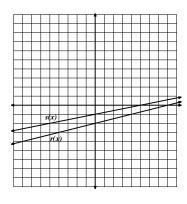
11.



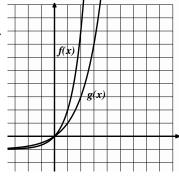
12.



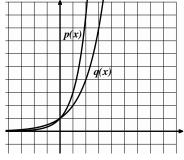
13.

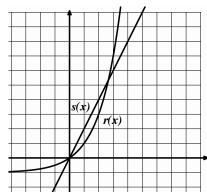


14.



15.





16 a. Examine the graph at the left from 0 to 1. Which graph do you think is growing faster?

b. Now look at the graph from 2 to 3. Which graph is growing faster in this interval?

1.5 The Tortoise and The Hare

A Solidify Understanding Task



In the children's story of the tortoise and the hare, the hare mocks the tortoise for being slow. The tortoise replies, "Slow and steady wins the race." The hare says, "We'll just see about that," and challenges the tortoise to a race. The distance from the starting line of the hare is given by the function:

 $d = t^2$ (d in meters and t in seconds)

Because the hare is so confident that he can beat the tortoise, he gives the tortoise a 1 meter head start. The distance from the starting line of the tortoise including the head start is given by the function:

 $d = 2^t$ (d in meters and t in seconds)

- 1. At what time does the hare catch up to the tortoise?
- 2. If the racecourse is very long, who wins: the tortoise or the hare? Why?
- 3. At what time(s) are they tied?
- 4. If the racecourse were 15 meters long who wins, the tortoise or the hare? Why?

5. Use the properties $d = 2^t$ and $d = t^2$ to explain the **speeds** of the tortoise and the hare in the following time intervals:

Interval	Tortoise $d = 2^t$	Hare $d = t^2$
[0, 2)		
[2, 4)		
[4,∞)		

READY, SET, GO!

Name

Period

Date

READY

Topic: Recognizing Functions

Identify which of the following representations are functions. If the representation is NOT a function state how you would fix it so it was.

- 1. $D = \{(4,-1)(3,-6)(2,-1)(1,2)(0,4)(2,5)\}$
- 2. The number of calories you have burned since midnight at any time during the day.

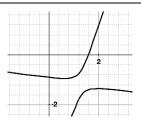
3.



4.

X	-12	-8	-6	-4
f(x)	25	25	25	25

5.



6.



SET

Topic: Comparing rates of change in linear, quadratic, and exponential functions

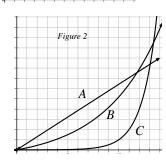
The graph at the right shows a time vs. distance graph of two cars traveling in the same direction along the freeway.

Figure 1

A
B

- 7. Which car has the cruise control on? How do you know?
- 8. Which car is accelerating? How do you know?
- 9. Identify the interval in *figure 1* where car A seems to be going faster than car B.
- 10. For what interval in *figure 1* does car B seem to be going faster than car A?
- 11. What in the graph indicates the speed of the cars?
- 12. A third car *C* is now shown in the graph (*see figure 2*). All 3 cars have the same destination. If the destination is a distance of 12 units from the origin, which car do you predict will arrive first? Justify your answer.

22



GO

Topic: Identifying domain and range from a graph

State the domain and range of each graph. Use interval notation where appropriate.

13a. Domain ___ 14a. Domain _____ 15a. Domain ___ b. Range ____ b. Range _____ b. Range ____ 16a. Domain _ 17a. Domain _____ 18a. Domain _____ b. Range_ b. Range _____ b. Range ____ 19a. Domain __ 20a. Domain __ b. Range_ b. Range

21. Are the domains of #19 and #20 the same? Explain.

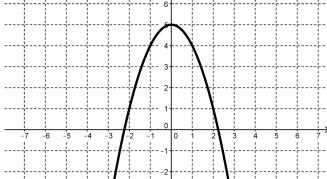
1.6 How Does It Grow?

A Practice Understanding Task

For each relation given:

- Identify whether or not the relation is a function;
- b. Determine if the function is linear, exponential, quadratic or neither;
- c. Describe the type of growth
- d. Create one more representation for the relation.
- 1. A plumber charges a base fee of \$55 for a service call plus \$35 per hour for each hour worked during the service call. The relationship between the total price of the service call and the number of hours worked.

2.

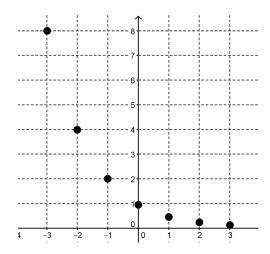




SECONDARY MATH II // MODULE 1

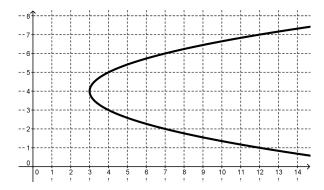
QUADRATIC FUNCTIONS - 1.6

3.



4.
$$y = \frac{1}{3}(x-2)^2 + 4$$

5.

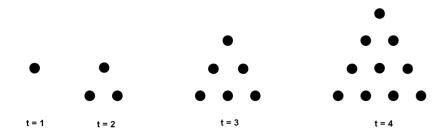


6.
$$y = \frac{1}{3}(x-2) + 4$$

7. The relationship between the speed of a car and the distance it takes to stop when traveling at that speed.

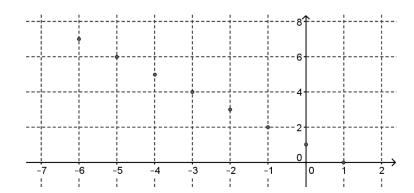
Speed	Stopping
(mph)	Distance
	(ft)
10	12.5
20	50
30	112.5
40	200
50	312.5
60	450
70	612.5

8. The relationship between the number of dots in the figure and the time, *t*.



9. The rate at which caffeine is eliminated from the bloodstream of an adult is about 15% per hour. The relationship between the amount of caffeine in the bloodstream and the number of hours from the time the adult drinks the caffeinated beverage if the initial amount of caffeine in the bloodstream is 500 mg.

10.



11.
$$y = (4x + 3)(x - 6)$$

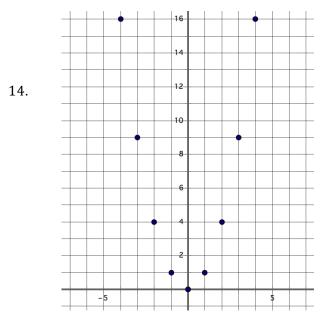
SECONDARY MATH II // MODULE 1

QUADRATIC FUNCTIONS - 1.6

- 12. Mary Contrary wants to build a rectangular flower garden surrounded by a walkway 4 meters wide. The flower garden will be 6 meters longer than it is wide.
- a. The relationship between the width of the garden and the perimeter of the walkway.

b. The relationship between the width of the garden and area of the walkway.

13.
$$y = \left(\frac{1}{3}\right)^{x-2} + 4$$



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Name

Period

Date

READY

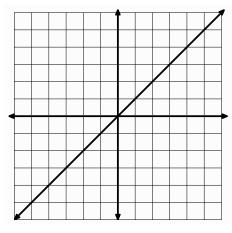
Topic: Transforming lines

1. Graph the following linear equations on the grid. The equation y = x has been graphed for you. For each new equation explain what the number 3 does to the graph of y = x. Pay attention to the y-intercept, the x-intercept, and the slope. Identify what changes in the graph and what stays the same.

a.
$$y = x + 3$$

b.
$$y = x - 3$$





2. The graph of y = x is given. (*See figure 2.*) For each equation predict what you think the number -2 will do to the graph. Then graph the equation.

a.
$$y = x + (-2)$$

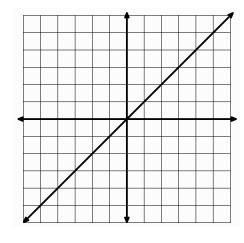
Prediction:

b.
$$y = x - (-2)$$

Prediction:

c.
$$y = -2x$$

Prediction:

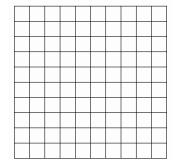


SET

Topic: Distinguish between linear, exponential and quadratic functions

For each relation given:

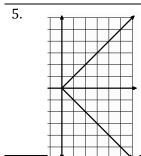
- a. Identify whether or not the relation is a function. (If it's not a function, skip b d.)
- b. Determine if the function is Linear, Exponential, Quadratic or Neither.
- c. Describe the type of growth.
- d. Express the relation in the indicated form.
- 3. I had 81 freckles on my nose before I began using vanishing cream. After the first week I had 27, the next week 9, then 3...



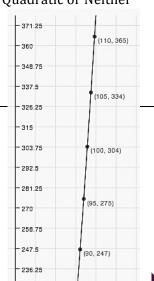
- a. Function?
- b. Linear, Exponential, Quadratic or Neither
- c. How does it grow?
- d. Make a graph. Label your axes and the scale Show all 4 points.

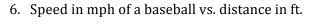
4.	X	у
4.	0	81
	1	$80\frac{2}{3}$
	2	$80\frac{1}{3}$
	3	80
	4	$79\frac{2}{3}$

- a. Function?
- b. Linear, Exponential, Quadratic or Neither
- c. How does it grow?
- d. Write the explicit equation.



- a. Function?
- b. Linear, Exponential, Quadratic or Neither
- c. How does it grow?
- d. Create a table





- a. Function?
- b. Linear, Exponential, Quadratic or Neither
- c. How does it grow?
- d. Predict the distance the baseball flies, if it leaves the bat at a speed of 115 mph.

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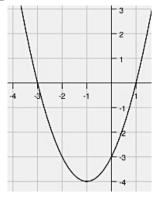
GO

Topic: Matching function representations

Match the function on the left with the equivalent function on the right.

$$_{---}$$
 7. $f(x) = -2x + 5$

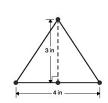
____8.

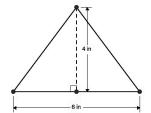


____9. I put \$7000 in a savings account that pays 3% interest compounded annually. I plan to leave it in the bank for 20 years. The amount I will have then.

____10. The area of the triangles below.







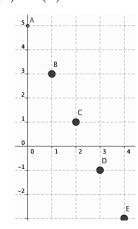
____11. f(0) = 5; f(n) = 2 * f(n-1)

____12. f(0) = 5; f(n) = f(n-1)-2

13.				
X	-7.75	-1/4	1/2	11.6
f(x)	7.75	1/4	-1/2	-11.6

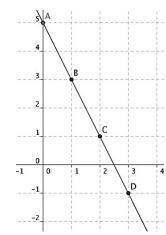
a.
$$f(x) = 5(2)^x$$

b.



c.
$$f(1) = 2$$
; $f(n+1) = f(n) + 2n + 2$

d.



e.
$$y + x = 0$$

f.
$$y = (x - 1)(x + 3)$$

g.
$$A = 7000(1.03)^{20}$$

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