Breaking Tradition:

Transforming Teaching Using the Common Core and Integrated Pathways

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Why Integrated?

Essential Question:

How does the Integrated Pathway lend itself to transforming teaching?

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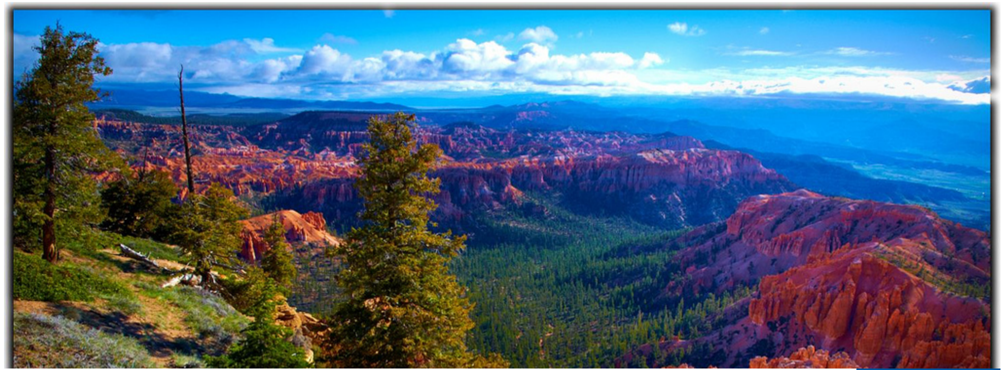
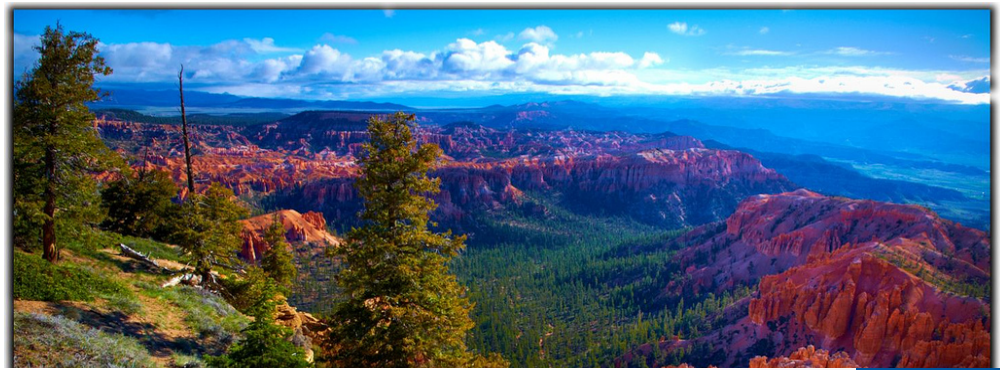
Vision



"The most necessary task of civilization is to teach people how to think. It should be the primary purpose of our public schools . . . The trouble with our way of educating is that it does not give elasticity to the mind. It casts the brain into a mold. It insists that the child must accept. It does not encourage original thought or reasoning, and it lays more stress on memory than observation."

-- Thomas A. Edison

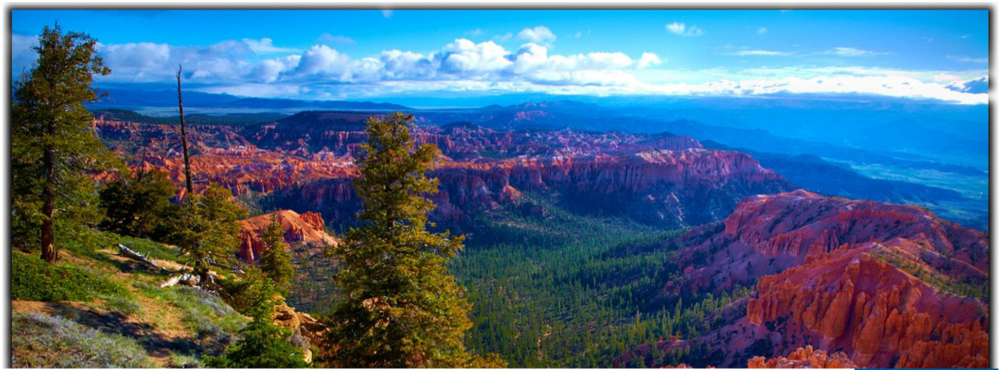
Interrelated



"We have always liked to emphasize, in our work, that the various parts of mathematics are very closely interrelated, and that it is very artificial, and misleading, to keep them rigidly apart." (pg. 81)

Quotes from *Mathematical Reflections, In a Room with Many Mirrors* by Peter Hilton, Derek Holton, Jean Pedersen© 1997 Springer-Verlag New York, Inc

Learning Progressions

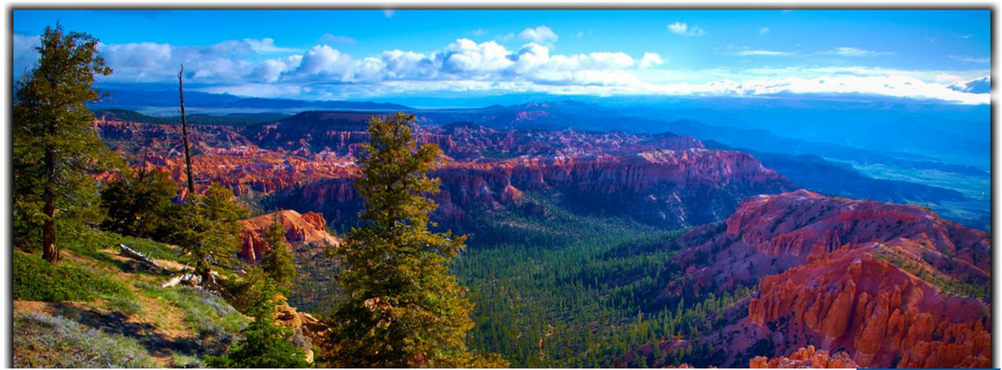


"Trajectories involve hypotheses about the order and nature of the steps in the growth of students' mathematical understanding, and about the nature of the instructional experiences that might support them in moving step by step towards the goals of school mathematics."

Learning progressions cannot be derived solely from "the disciplinary logic of mathematics itself."

Phil Daro, Learning Trajectories in Mathematics: A Foundation for Standards, Curriculum, Assessment and Instruction

Why Integrated?



- Learning Progressions of standards are more developed.

- Connections between domains are easily linked within a grade level (or course).

Geometric Progressions:

- **6.G.3** Draw polygons in the coordinate plane given coordinates for the vertices; **use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate.** Apply these techniques in the context of solving real-world and mathematical problems.
- **8.G.6** Explain a proof of the Pythagorean Theorem and its converse.
- **8.G.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
- **HS(10).G.SRT.4:** Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

Geometric Progressions

- **7.G.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

- **8.G.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

Geometric Progressions

- **8.G.1 Verify experimentally the properties of rotations, reflections, and translations:**
  a. Lines are taken to lines, and line segments to line segments of the same length.
  b. Angles are taken to angles of the same measure.
  c. Parallel lines are taken to parallel lines.
- **8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations**; given two congruent figures, describe a sequence that exhibits the congruence between them.
- **8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures** using coordinates.
- **8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations**; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Geometric Progressions

Geometry: Ninth Grade

- **G.CO.2:** Represent transformations in the plane using, e.g., transparencies and geometry software; **describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not** (e.g. translation versus horizontal stretch).
- **G.CO.3:** Given a rectangle, parallelogram, trapezoid, or regular polygon, **describe the rotations and reflections** that carry it onto itself.
- **G.CO.4: Develop definitions** of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
- **G.CO.5: Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure** using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Geometric Progressions

Geometry: Ninth Grade

- **G.CO.6: Use geometric descriptions of rigid motions to transform figures** and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- **G.CO.7: Use the definition of congruence in terms of rigid motions** to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- **G.CO.8:** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
- **G.CO.12: Make formal geometric constructions** with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

Geometric Progressions

Geometry: Tenth Grade

- **G.CO.9: Prove theorems about lines and angles**. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
- **G.CO.10: Prove theorems about triangles**. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* [Implementation may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for G.C.3 later.]
- **G.CO.11: Prove theorems about parallelograms**. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Geometric Progressions

Geometry: Tenth Grade

- What do you notice about the geometric progressions from grades 8 – 9 – 10 regarding transformations?

Connections within Grades

Grade Six

- Number Systems
  - 6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

- Geometry
  - 6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Connections within Grades

Grade Seven

- Rational Numbers and Proportionality
  - 7.RP. 2 Recognize and represent proportional relationships between quantities.
  - 7.RP. 3 Use proportional relationships to solve multistep ratio and percent problems.

- Statistics
  - 7.S.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

Connections within Grades

Grade Nine (Secondary Math I)

- Geometry (G.GPE.5)
  - Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

- Functions (F.BF.3)
  - Identify the effect on the graph of replacing *f(x)* by *f(x) +k, kf(x), f(kx), and f(x+k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

The Arithmetic of Complex Numbers

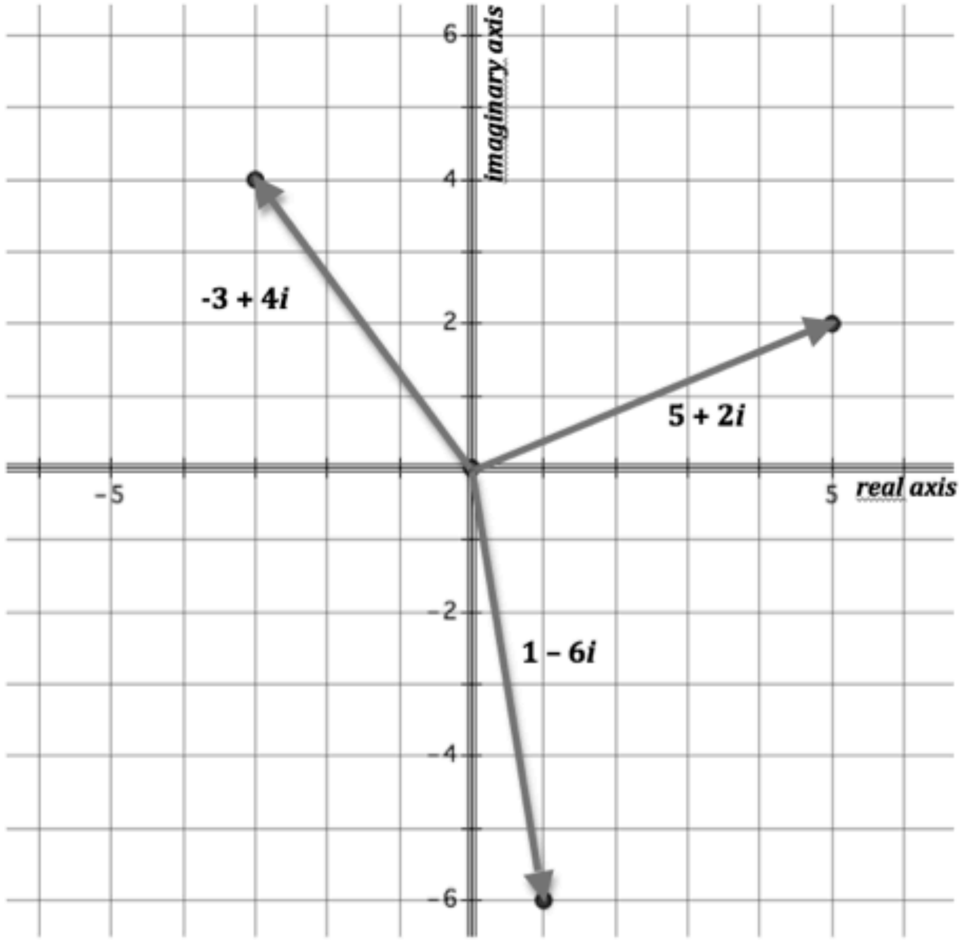
CCSS: Represent complex numbers and their operations on the complex plane.

How does using a geometric representation (the complex plane) add to one's understanding of the arithmetic of complex numbers from an algebraic perspective?

Tools of Euclidean Geometry

- Synthetic (deductive proof)
- Coordinates
- Vectors
- Transformations

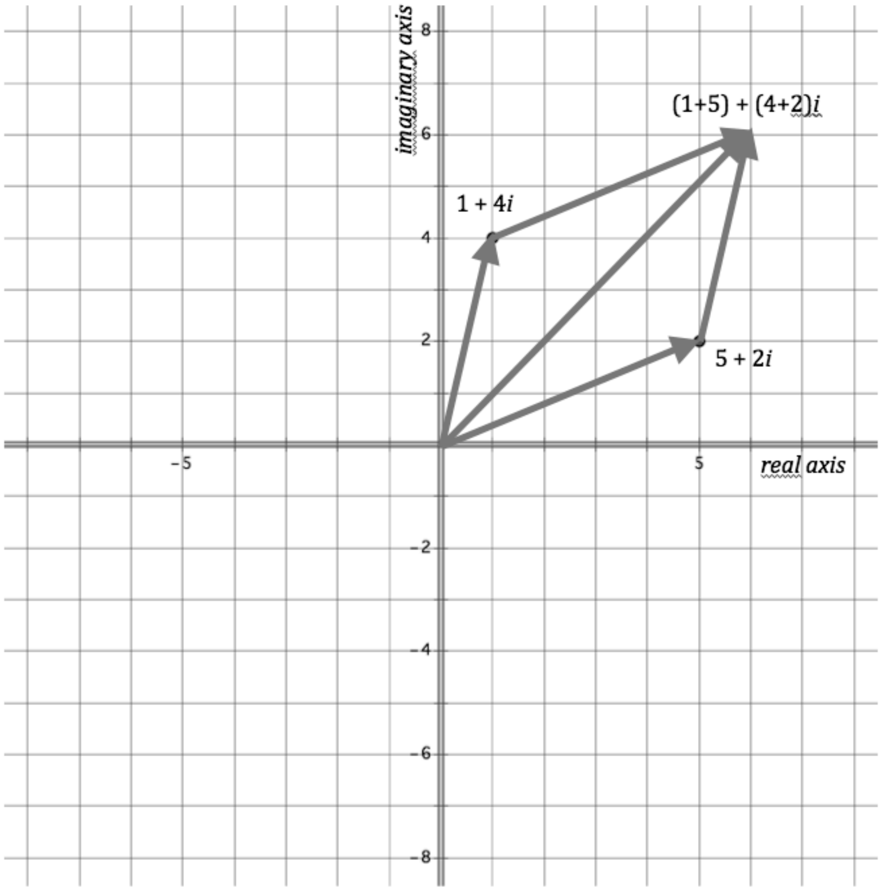
The Complex Plane



The Task, Part 1

Experiment with the vector representation of complex numbers to develop and justify an algebraic rule for adding two complex numbers: (a + b*i*) + (c + d*i*). How do your representations of addition of vectors on the complex plane help to explain your algebraic rule for adding complex numbers?

Unpacking the Mathematics



The Task, Part 2

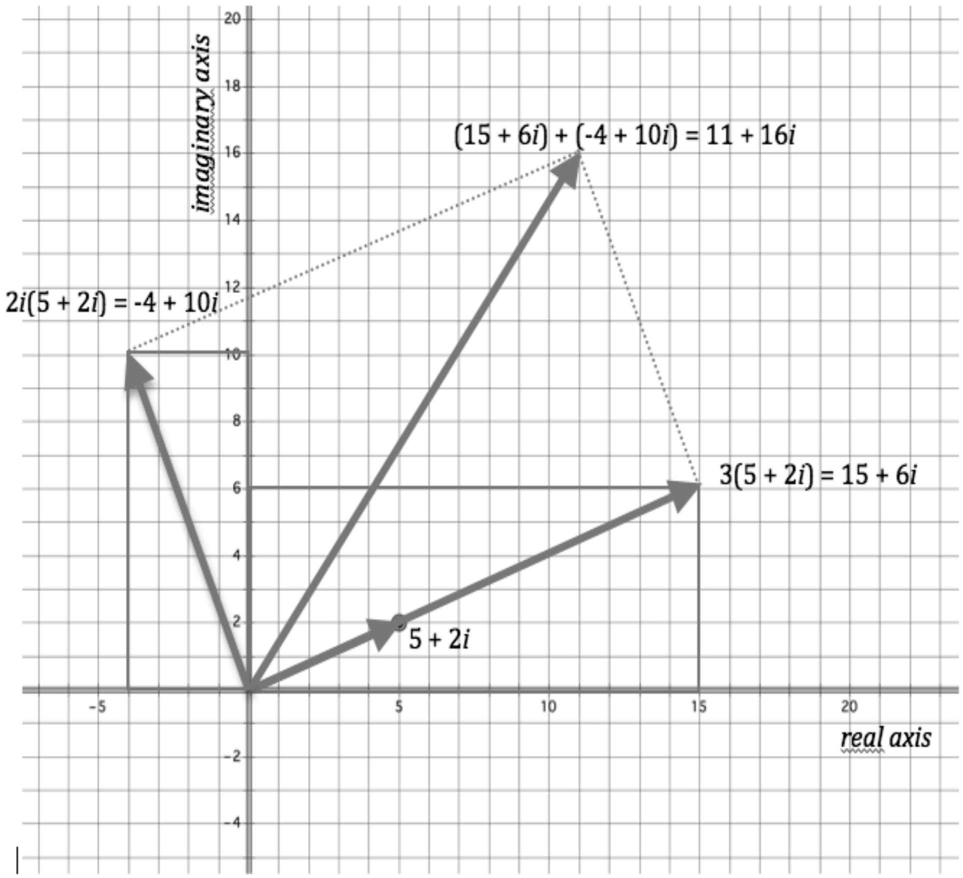
Experiment with the vector representation of complex numbers to develop and justify an algebraic rule for multiplying two complex numbers:

$(a + bi)(c + di)$. How do your representations of multiplication of vectors on the complex plane help to explain your algebraic rule for multiplying complex numbers?

The Task, Part 2 (Scaffolding)

- One way to think about multiplication on the complex plane is to treat the first factor in the multiplication as a scale factor.
- Provide a few examples of multiplying a complex number by a real number scale factor: $a(c + di)$. For example, what happens to the vector representation of a complex number when the scale factor a is 4? ½? -2?
- Provide a few examples of multiplying a complex number by an imaginary scale factor: $bi(c + di)$. For example, what happens to the vector representation when the scale factor bi is i? $2i$? $-3i$?

Unpacking the Mathematics



$(3 + 2i)(5 + 2i) = 3(5 + 2i) + 2i(5 + 2i) = (15 + 6i) + (-4 + 10i) = 11 + 16i$

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Unpacking the Mathematics

- What mathematics concepts did you draw upon in you work with addition and multiplication of complex numbers?
- What connections did you make?

Discussion of Benefits

International

- Eliminates algebra gap (and Geometry gap)
- Better treatment of geometry through integration
- Connections within mathematics topics
- More equitable
- Forces change-no familiar classes
- Separates linear and quadratic mathematics
- Broadens definition of mathematics
- World Class
- Cuts ties with outdated practices, attitudes, and curriculum

Traditional

- Better treatment of geometry through focused curriculum
- More comfortable for teachers and parents
- Parents understand names
- Easier to double enroll (algebra and geometry)

Consistency

- Grades K-8 are Integrated

K	1	2	3	4	5	6	7	8
Geometry								
Measurement and Data						Statistics and Probability		
Number and Operations in Base Ten						The Number System		
Operations and Algebraic Thinking						Expressions and Equations		
Counting and Cardinality			Number and Operations---Fractions			Ratios and Proportional Relationships		Functions

The Geometry Gap

- We really have in mind here geometry at the secondary and undergraduate levels. We claim that it is a serious mistake to regard geometry as just one more topic in mathematics, like algebra, trigonometry, differential calculus, and so on. In fact, geometry and algebra, the two most important aspects of mathematics at these levels, play essentially complementary roles. <u>Geometry is a source of questions; algebra is a source of answers. Geometry provides ideas, inspiration, insight; algebra provides clarification and systematic solution.</u>

- Quotes from *Mathematical Reflections, In a Room with Many Mirrors* by Peter Hilton, Derek Holton, Jean Pedersen© 1997 Springer-Verlag New York, Inc

Students Score Higher

- Research published in the Journal for Research in Mathematics Education (Volume 44, No. 4, July 2013):
- Students in the integrated curriculum scored significantly higher than those in the subject-specific curriculum on the standardized achievement test.
- “What we found was, despite controlling for all these different variables, the curriculum organization was a significant predictor of student outcomes,” Tarr said.

MVP

- Mathematics Vision Project provides tasks that organizes curriculum so that students make connections to important mathematics across conceptual categories.
- MVP provides homework sets (Ready, Set, Go) so that students become masterful with the mathematics they are learning.

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