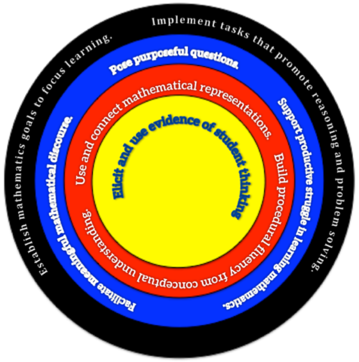
Eliciting and Using Student Thinking to Target the other 7 Effective Teaching Practices



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NCTM San Diego 2019
http://bit.ly/MVPpresentations

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The 8 Effective Teaching Practices (NCTM 2014)

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

The 8th Effective Teaching Practice (NCTM 2014)

Elicit and use evidence of student thinking.

Effective teaching of mathematics uses evidence of student thinking

1) to assess progress towards mathematical understanding and
2) to adjust instruction continually in ways that support and extend learning.

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Effective teaching of mathematics uses evidence of student thinking

3) To affirm students' mathematical identities

4) To build conceptual understanding

5) To promote mathematical discourse

6) To create opportunities for direct instruction

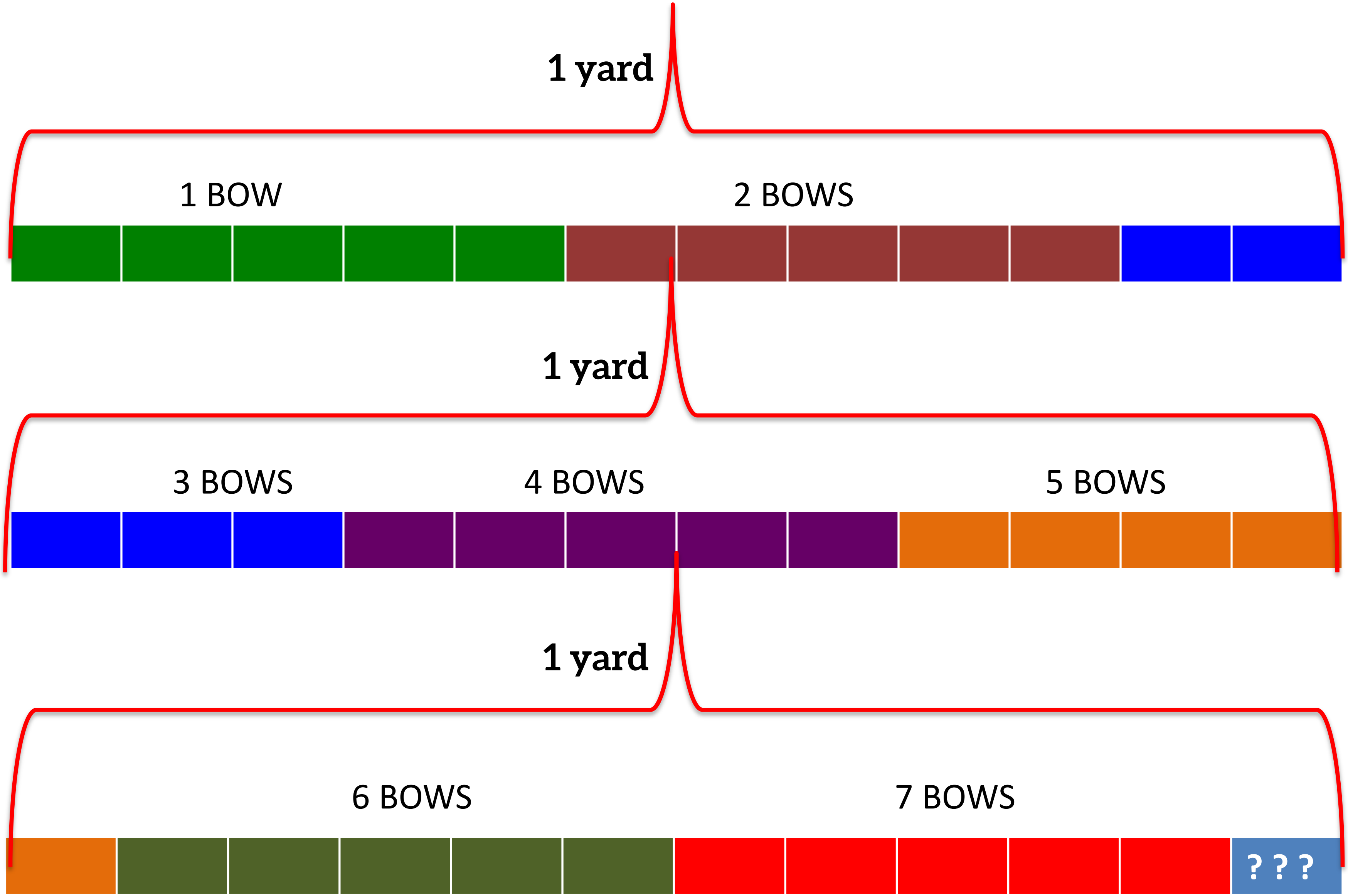
Problem 1

How many bows that are 5/12 yards long can be made from 3 yards of ribbon?

$$7\frac{1}{5}$$

$$7\frac{1}{12}$$

1. Did anyone draw a diagram to help them think about the problem?



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Does anyone still have a question?

Student, "I thought about the problem differently ..."

I know each yard contains 36 inches.
I have 3 yards so that's 108 inches total.
1 bow is 5/12 of a yard. 5/12 times 36 = 15 inches

Number of bows	Inches remaining
0	108 (subtract 15)
1	93 (subtract 15)
2	78 (subtract 15)
3	63 (subtract 15)
4	48 (subtract 15)
5	33 (subtract 15)
6	18 (subtract 15)
7	3 inches leftover

What does the 3 inches represent?

Number of bows	Inches remaining
0	108 (subtract 15)
1	93 (subtract 15)
2	78 (subtract 15)
3	63 (subtract 15)
4	48 (subtract 15)
5	33 (subtract 15)
6	18 (subtract 15)
7	3 inches leftover

1/4 of a foot

1/12 of a yard

1/5 of a bow

What if I had focused only on the "correct" answer in the answer key or how to do the procedure?

How many students would have left that day believing they couldn't do math?

What did I learn from this story?

My actions affirmed my students' math identity.

What kind of questions made this happen?

1. Agree or disagree?
2. Talk to me about this. Tell me more.
3. Did anyone draw a diagram?
4. Did anyone think about the problem differently or approach it in a different way?
5. What do the numbers mean?
6. How can the units help us think about the problem?

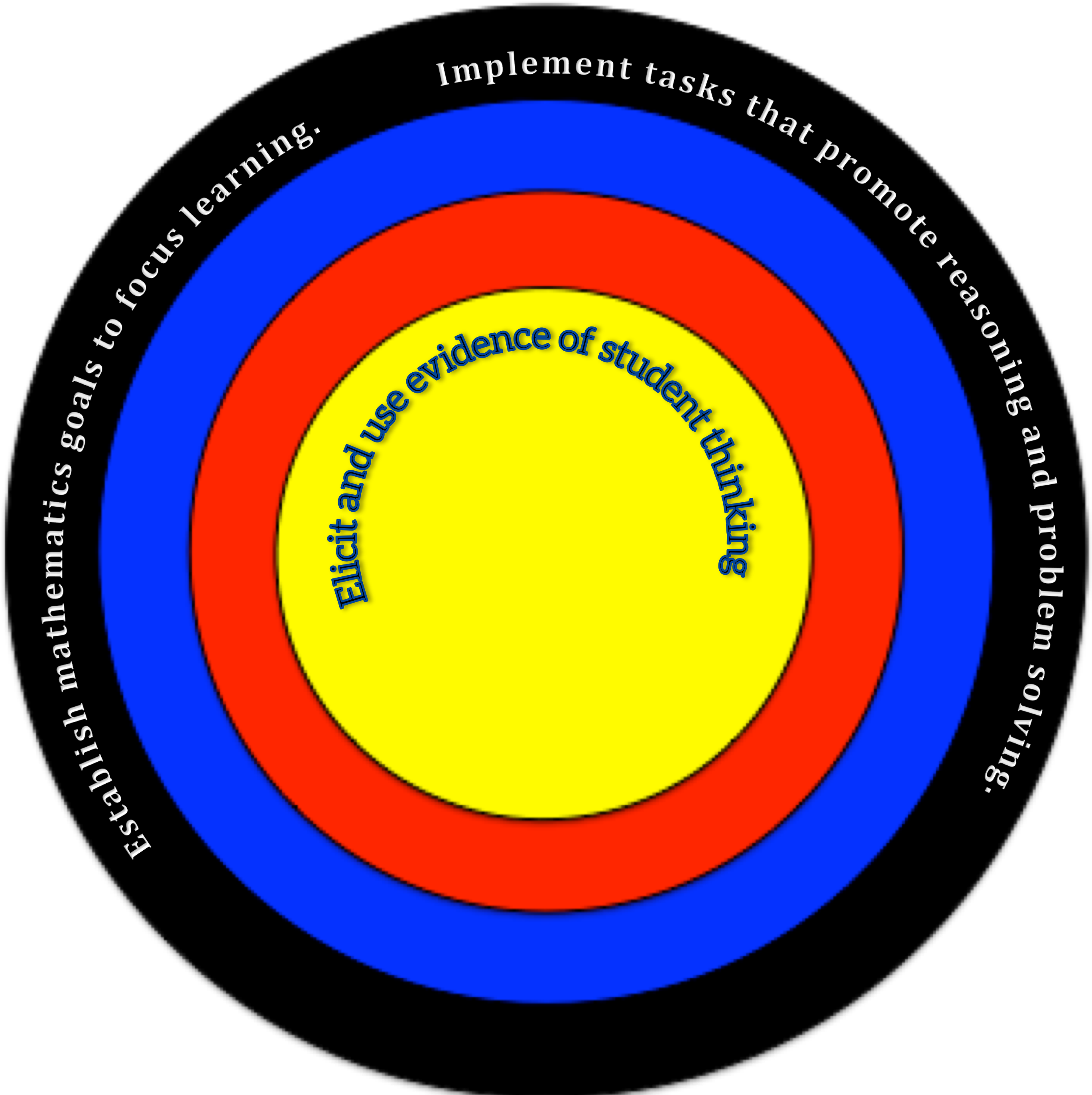
What was right about my teaching that day?

1. I was eliciting and using student thinking.
2. We were engaging in some productive struggle.
3. We were sense-making by connecting mathematical representations
4. We were having meaningful mathematical discourse
5. I was posing some purposeful questions
6. We were building some conceptual knowledge about a procedure that was not clearly understood.

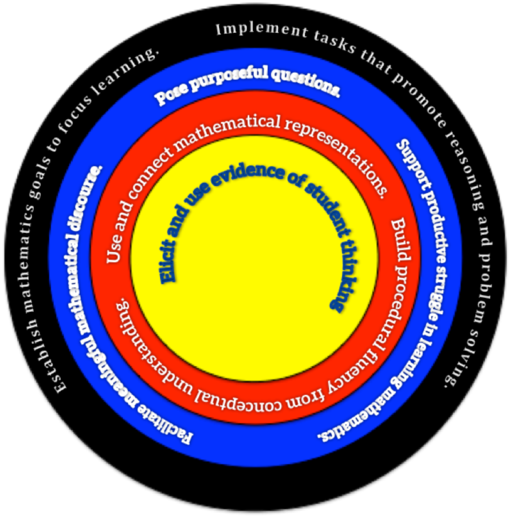
How to begin.

Honor student thinking by *BELIEVING* that they have logical ideas and that they are *TRYING* to make sense of the mathematics.

Then *ASK* about their thinking and truly *LISTEN* to what they say.


Implement tasks that promote reasoning and problem solving.
Establish mathematics goals to focus learning.
Elicit and use evidence of student thinking

Elicit student thinking
To build conceptual understanding



Consider how you might think about the following problems, if you had never learned the rules for inequalities.

Problem 2

If $x > y$, then which is greater? $x + a$ or $y + a$

Make a convincing argument using a representation such as a diagram, story, or number line to support your answer.

From MVP task - 4.4 Greater Than?

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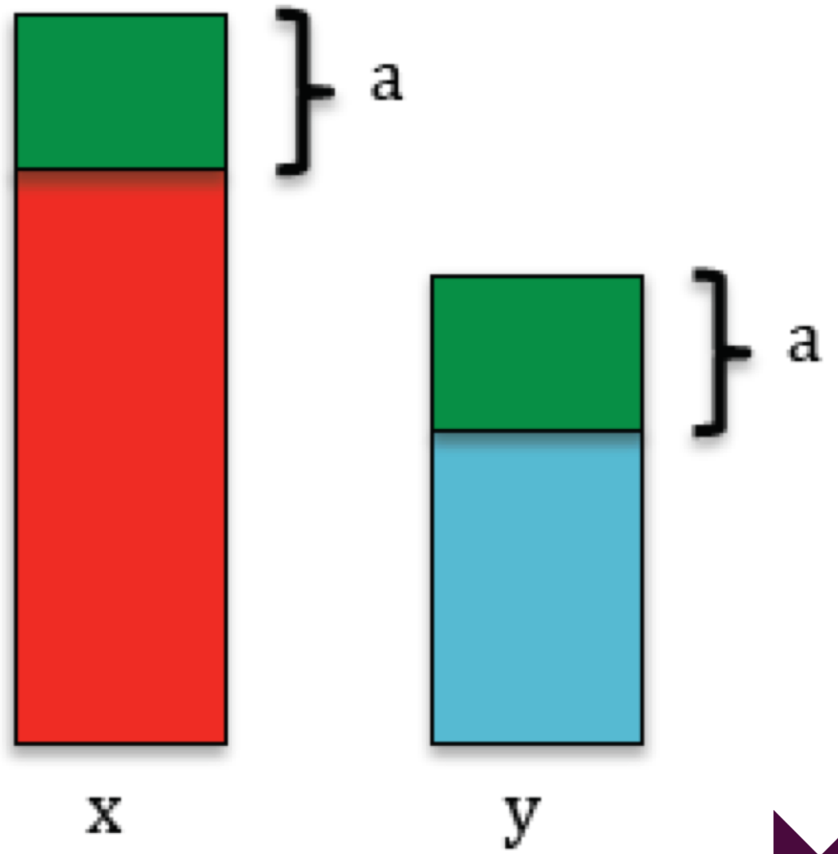
If $x > y$, then which is greater? $x + a$ or $y + a$



If x > y, then which is greater? x + a or y + a

- John is older than Marie.
- 10 years from now John will still be older than Marie.
- 2 years ago, John was older than Marie.

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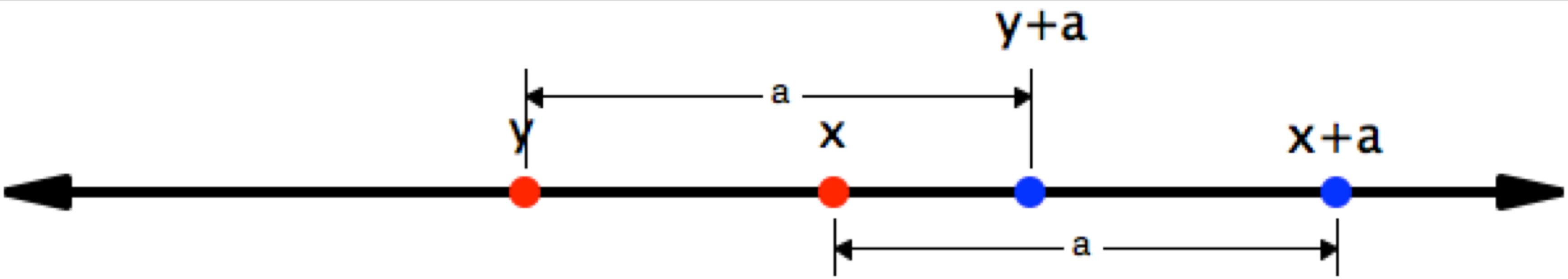
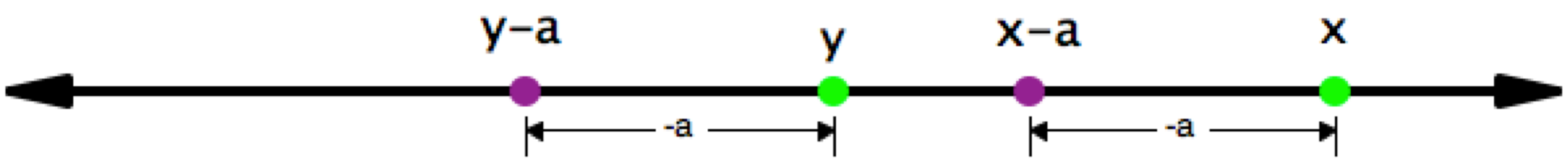
If $x > y$, then which is greater? $x + a$ or $y + a$





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Problem 3

I know that 5 > 4. Which is greater? 5a or 4a

Make a convincing argument using a representation such as a diagram, story, or number line to support your answer.

From MVP task - 4.4 Greater Than?

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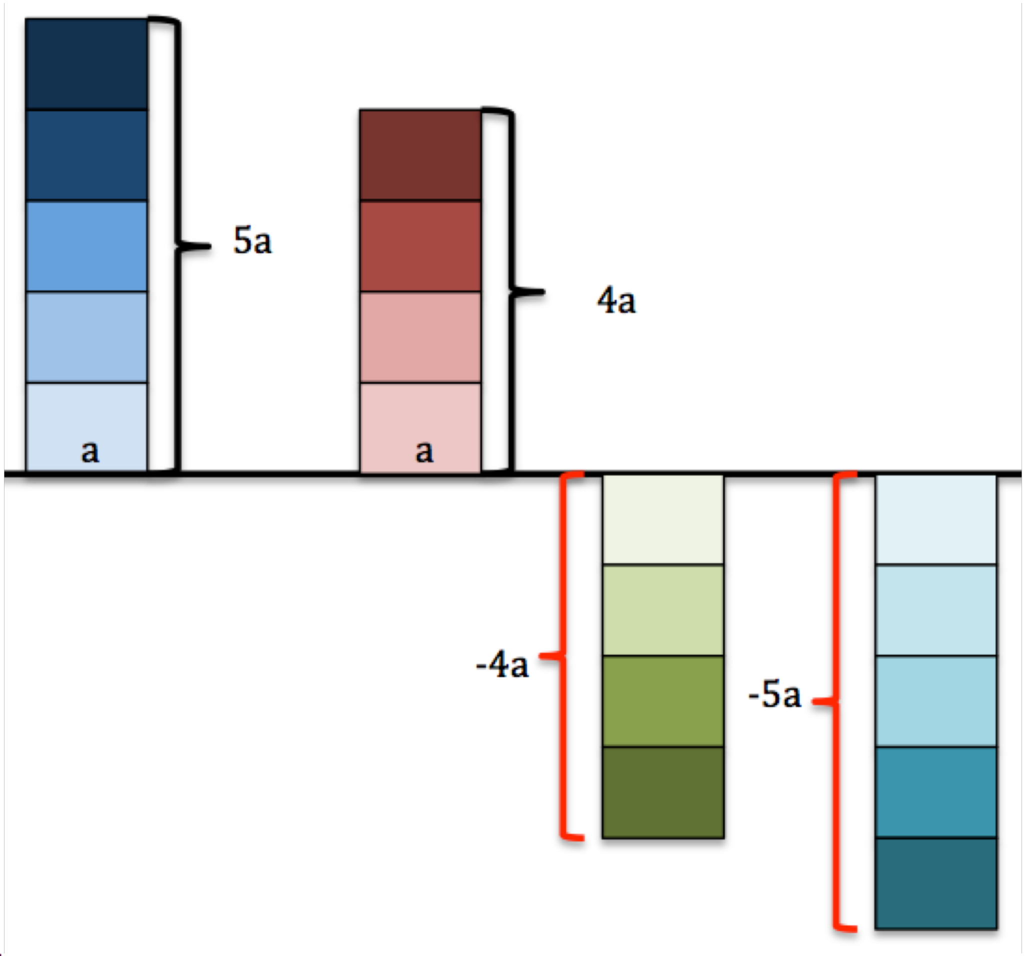
I know that 5 > 4. Which is greater? 5a or 4a



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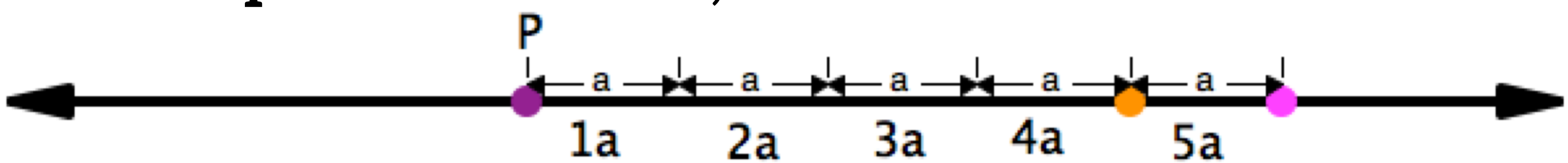
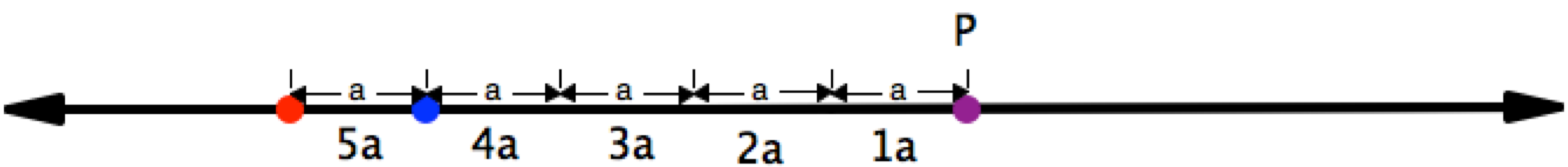
I know that 5 > 4. Which is greater? 5a or 4a

If "a" is a positive number, 5a > 4a because



If a is a negative number, 5a < 4a because



If a = 0, then 5a = 4a because 0 = 0.

From MVP task - 4.4 Greater Than?

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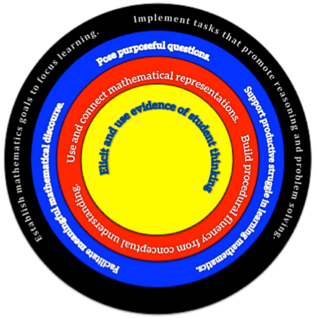
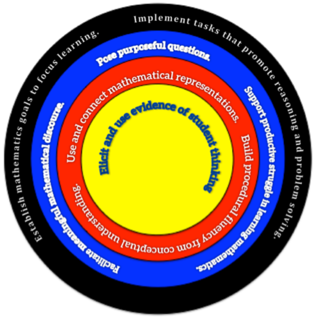
By presenting students with these 2 foundational questions and eliciting and using student thinking



1. Students engaged in some productive struggle.
2. They were making sense of the math by connecting mathematical representations
3. Through sharing their thinking, students engaged in meaningful mathematical discourse
4. Students and I posed some purposeful questions
5. We were working on building a foundation of conceptual understanding about a mathematical procedure.

Let's add in a purpose and a task while still thinking about eliciting and using student thinking.



Purpose: In this task students will develop insights into how to extend the process of solving equations—which they have previously examined for one- or two-step equations—so that the process works with multistep equations. (MVP)

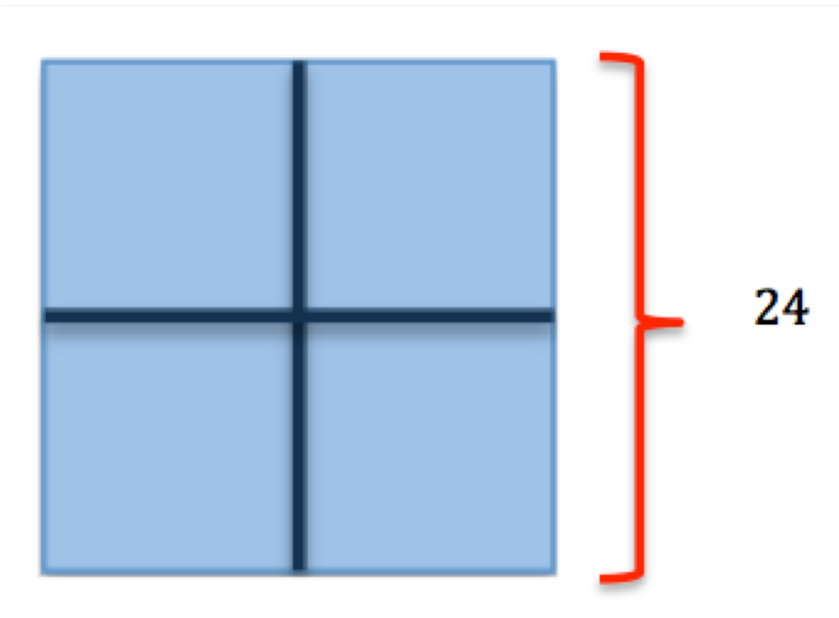
A.REI.1 Explain each step in solving a simple equation … Construct a viable argument to justify a solution method. (CCSS)

4.1 Cafeteria Actions and Reactions

Elvira, the cafeteria manager, has just received a shipment of new trays with the school logo prominently displayed in the middle of the tray. After unloading 4 cartons of trays in the pizza line, she realizes that students are arriving for lunch and she will have to wait until lunch is over before unloading the remaining cartons. The new trays are very popular and in just a couple of minutes 24 students have passed through the pizza line and are showing off the school logo on the trays. At this time, Elvira decides to divide the remaining trays in the pizza line into 3 equal groups so she can also place some in the salad line and the sandwich line, hoping to attract students to the other lines. After doing so, she realizes that each of the three serving lines has only 12 of the new trays.

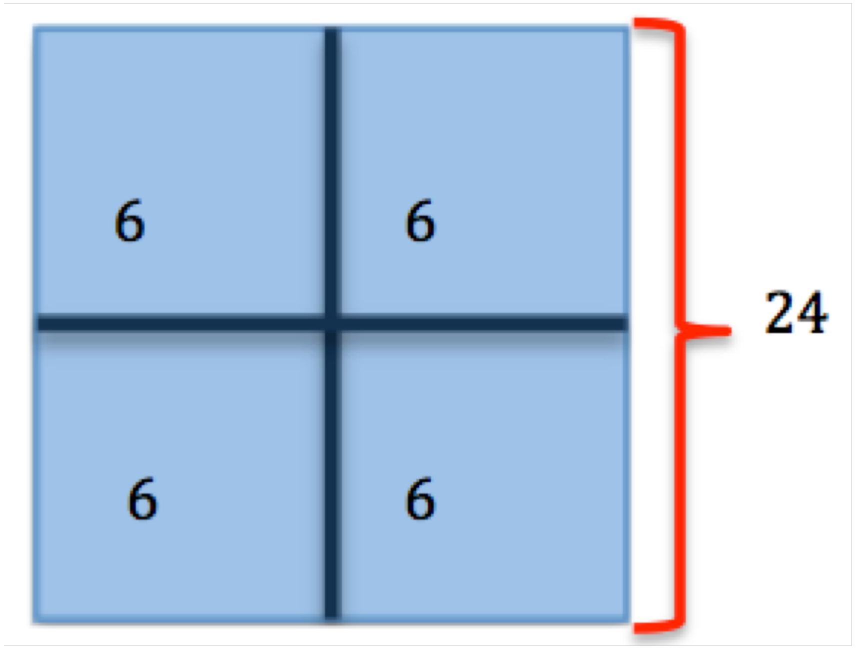
1. **"That's not many trays for each line. I wonder how many trays there were in each of the cartons I unloaded?"**

4.1 Cafeteria Actions and Reactions



I know that Elvira removed 24 trays from the 4 crates.

4.1 Cafeteria Actions and Reactions



24 trays were removed from the 4 crates.

We can think of that as 6 trays having been removed from each crate.

4.1 Cafeteria Actions and Reactions

12

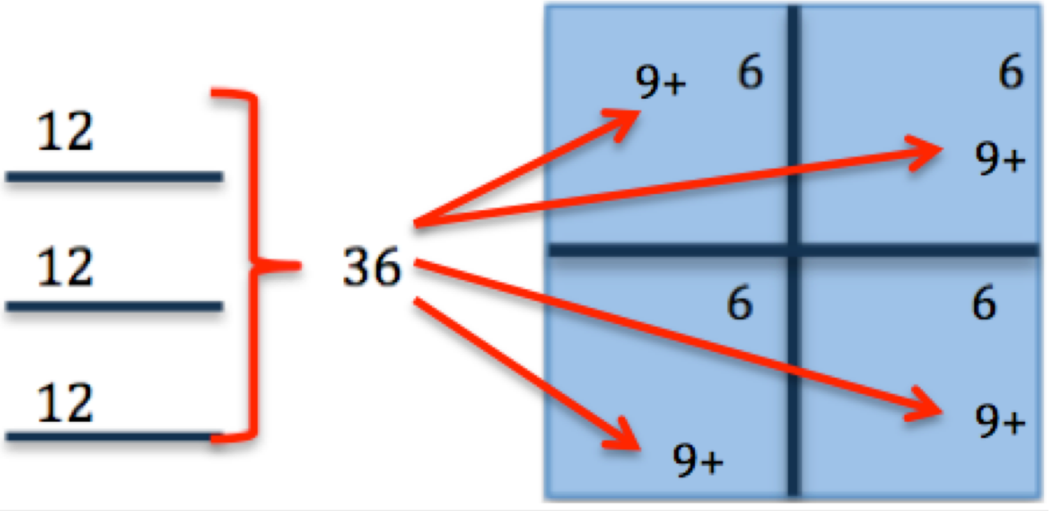
12

12

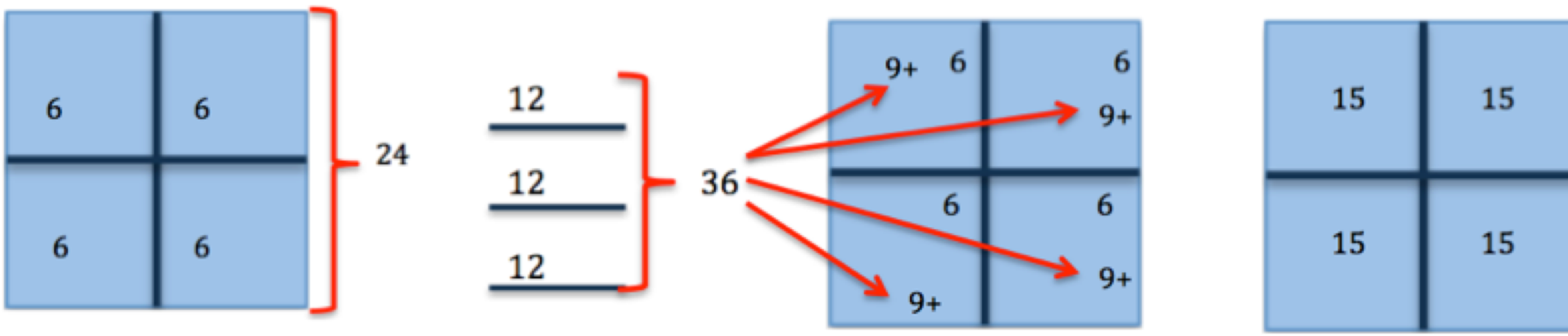
The remaining trays were divided into 3 lines. That means that there were 36 more trays remaining in the crates.

4.1 Cafeteria Actions and Reactions

I can divide those 36 trays by 4 and put them back into the 4 crates to figure out how many trays were in the crates at the beginning.

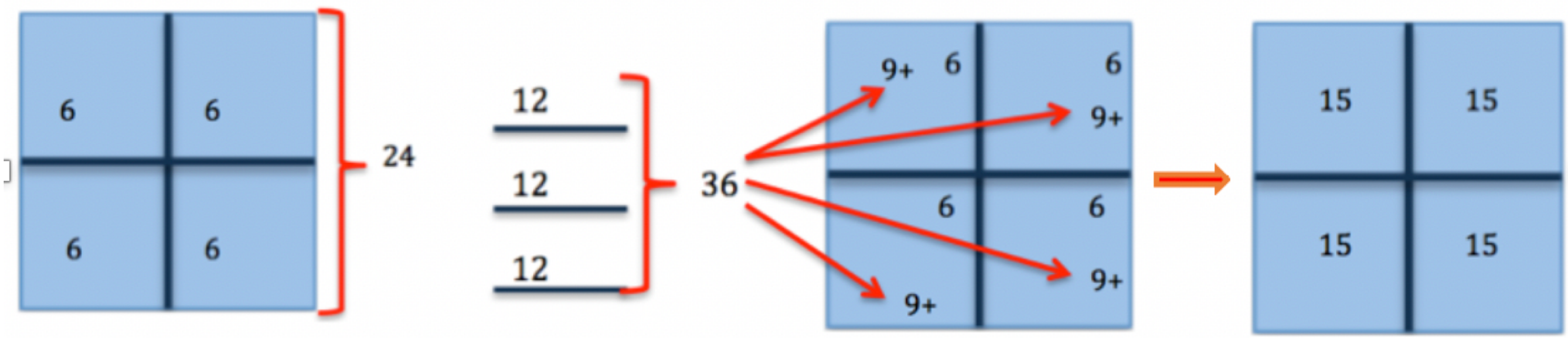


4.1 Cafeteria Actions and Reactions



There were originally 15 trays in each crate.

4.1 Cafeteria Actions and Reactions



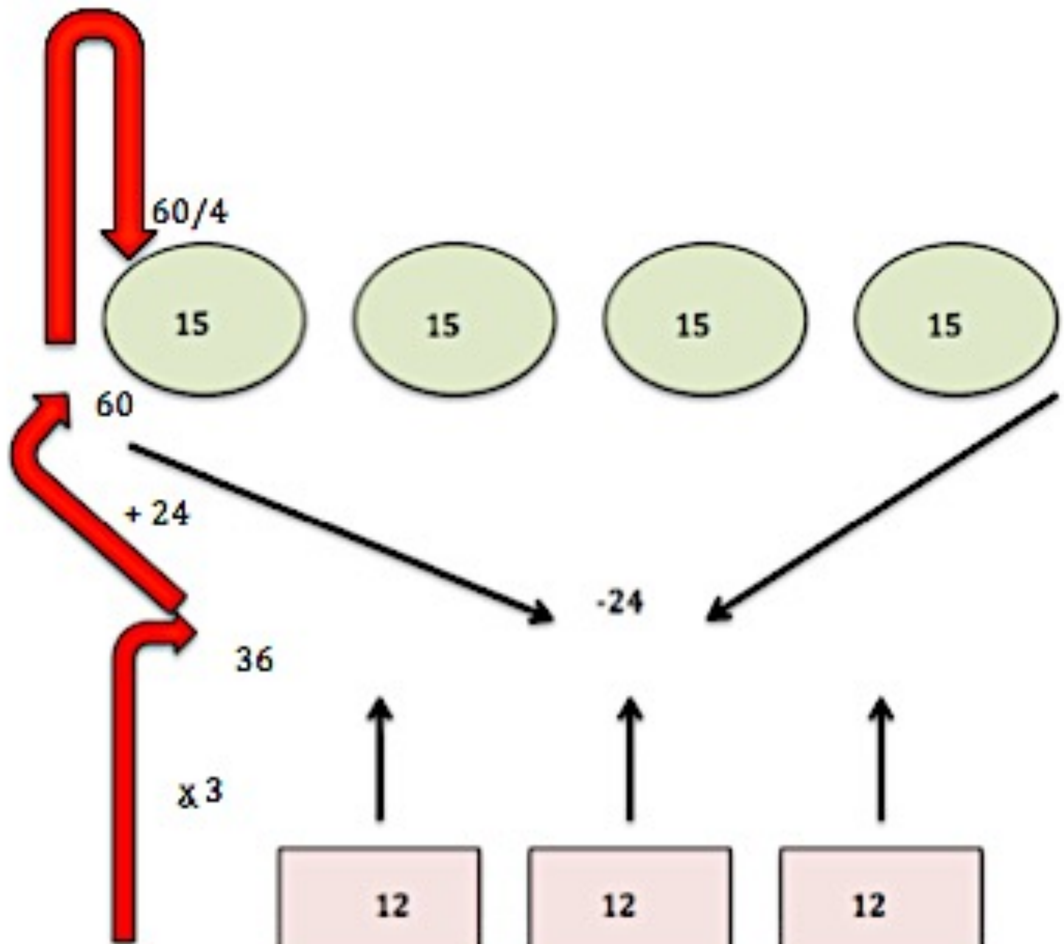
$$\frac{24}{4}$$

$$\frac{3(12)}{4}$$

$$\frac{24}{4}+\frac{3(12)}{4}=x$$

How is this student's thinking different from the previous student?

I started at the end.
I knew I had 12 trays in each of the 3 lines so I multiplied by 3 to get 36.
Since I had already taken 24 trays out of the crates I added 24 to 36. That added up to 60 total trays.
Then I divided 60 by 4 to find out that I had 15 trays in each crate to begin with.



$$\frac{4(x-6)}{3} = 12$$

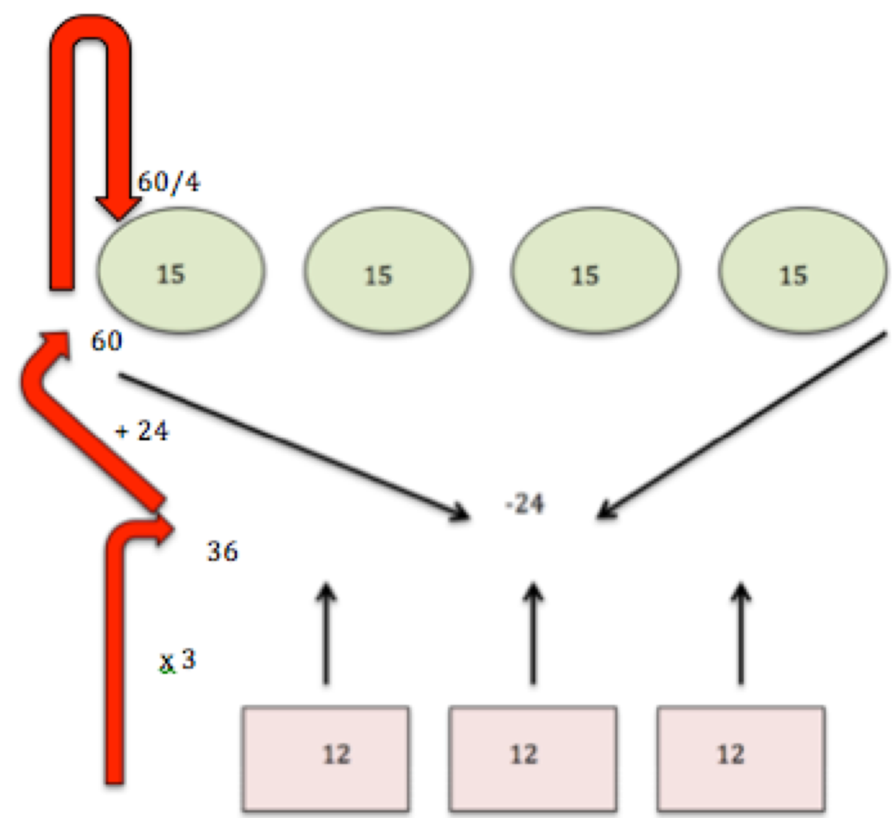
$$4(x-6) = 36$$

$$4x - 24 = 36$$

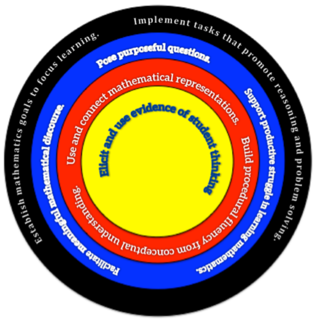
$$4x = 60$$

$$\frac{4x}{4} = \frac{60}{4}$$

$$x = 15$$

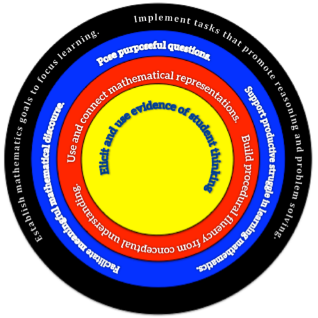


How can I use a task to help me achieve this standard?



- **Core Standards Focus: F.IF.4**
- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

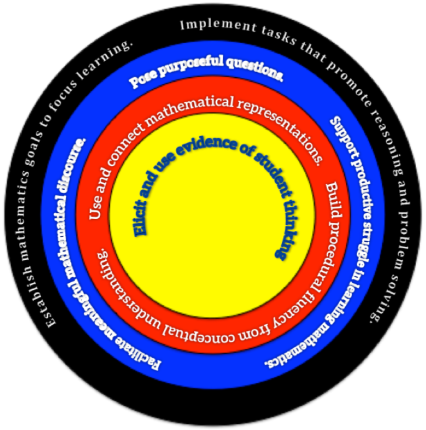
Purpose:



This task is designed to develop the conceptual ideas of the different features of functions using a situation.

Students need to become familiar with the names of these features and use them as tools for analysis during the entire module and whenever they encounter a new function in the future.

How can I use a task to help me achieve this standard?



- *Key features include:*
- *intercepts;*
- *Intervals where increasing/ decreasing,*
- *Positive and negative slope (rates of change)*
- *Relative maximums and minimums*
- *Domain and range*
- *Continuous or discrete*

3.1 Getting Ready for a Pool Party

A Develop Understanding Task

Sylvia has a small pool full of water that needs to be emptied and cleaned, then refilled for a pool party. During the process of getting the pool ready, Sylvia did all of the following activities, each during a different time interval.

3.1 Getting Ready for a Pool Party

A Develop Understanding Task

Sketch a possible graph showing the height of the water level in the pool over time. Be sure to include ALL of the activities Sylvia did to prepare the pool for the party. Remember that only one activity happened at a time. Think carefully about how each section of your graph will look, labeling where each activity occurs.

Removed water with a single bucket	Filled the pool with a hose (same rate as emptying pool)
Drained water with a hose (same rate as filling pool)	Cleaned the empty pool
Sylvia and her two friends removed water with three buckets	Took a break

When you look at student work, ask yourself:

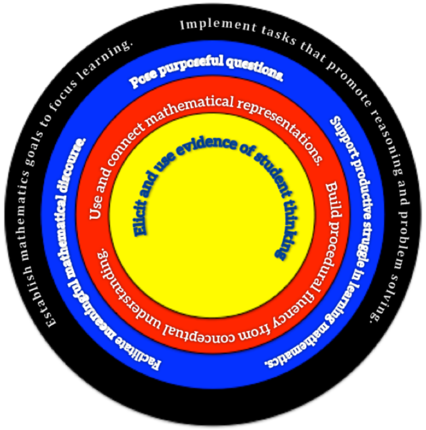
What is right about this work?

What does the student know based on this work?

What would I like the student to tell me about his/her work?

What about this work gives us an opportunity to discuss the mathematics of the purpose of the lesson?

Eliciting student work to create opportunities for direct instruction.



The "student work" in this Presentation so far has been gathered from real student work and thinking!

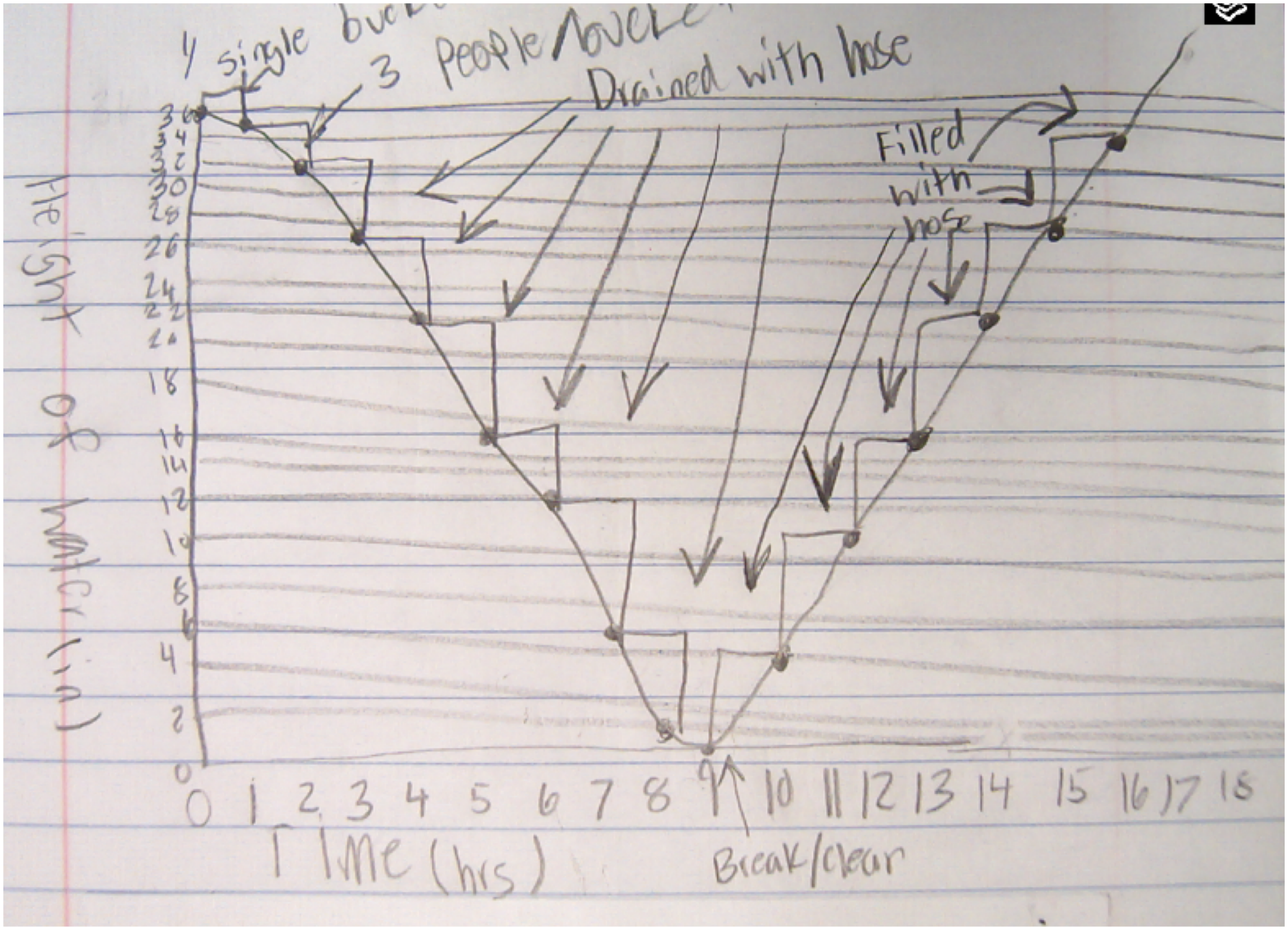
I made it legible and pretty for this presentation. Usually, it's messy and not very pretty.

Opportunities for direct instruction

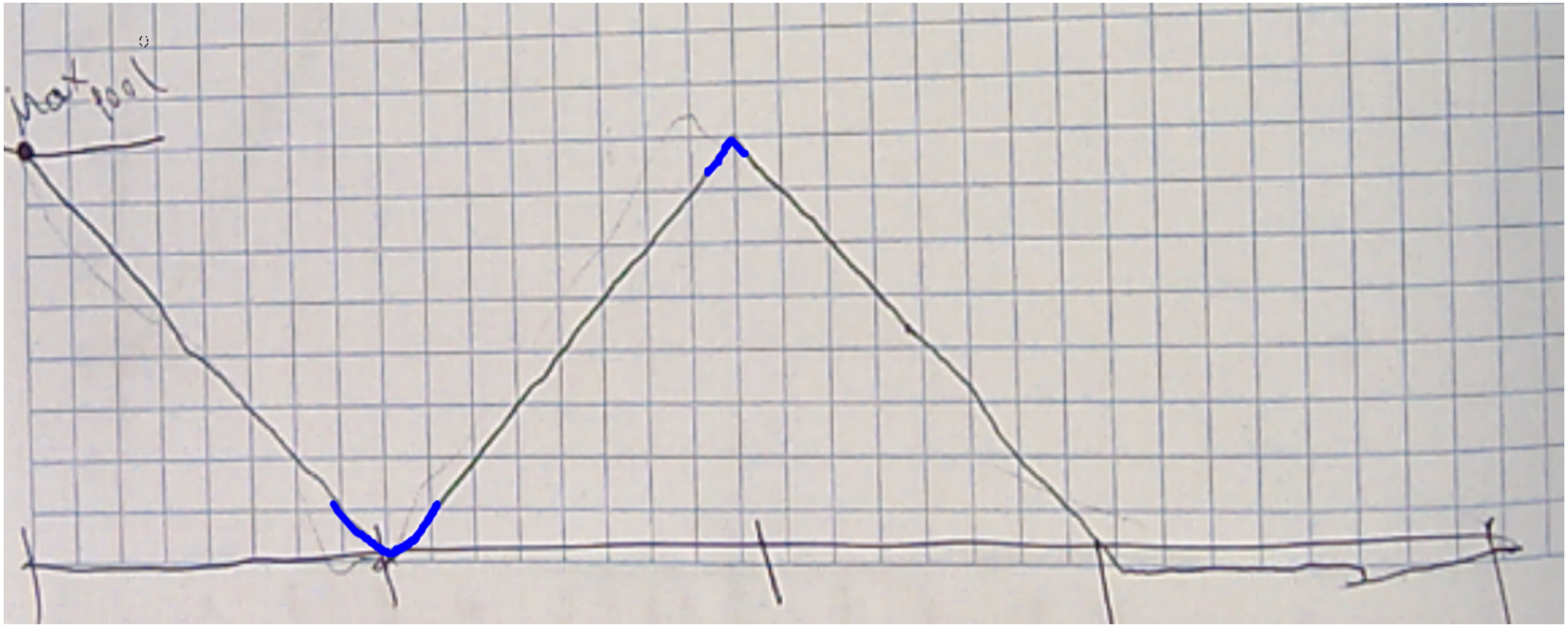
We are not listening for a particular response but listening to students to uncover the "**sparks of rightness**" that exist if we are willing to help them clarify their assumptions.

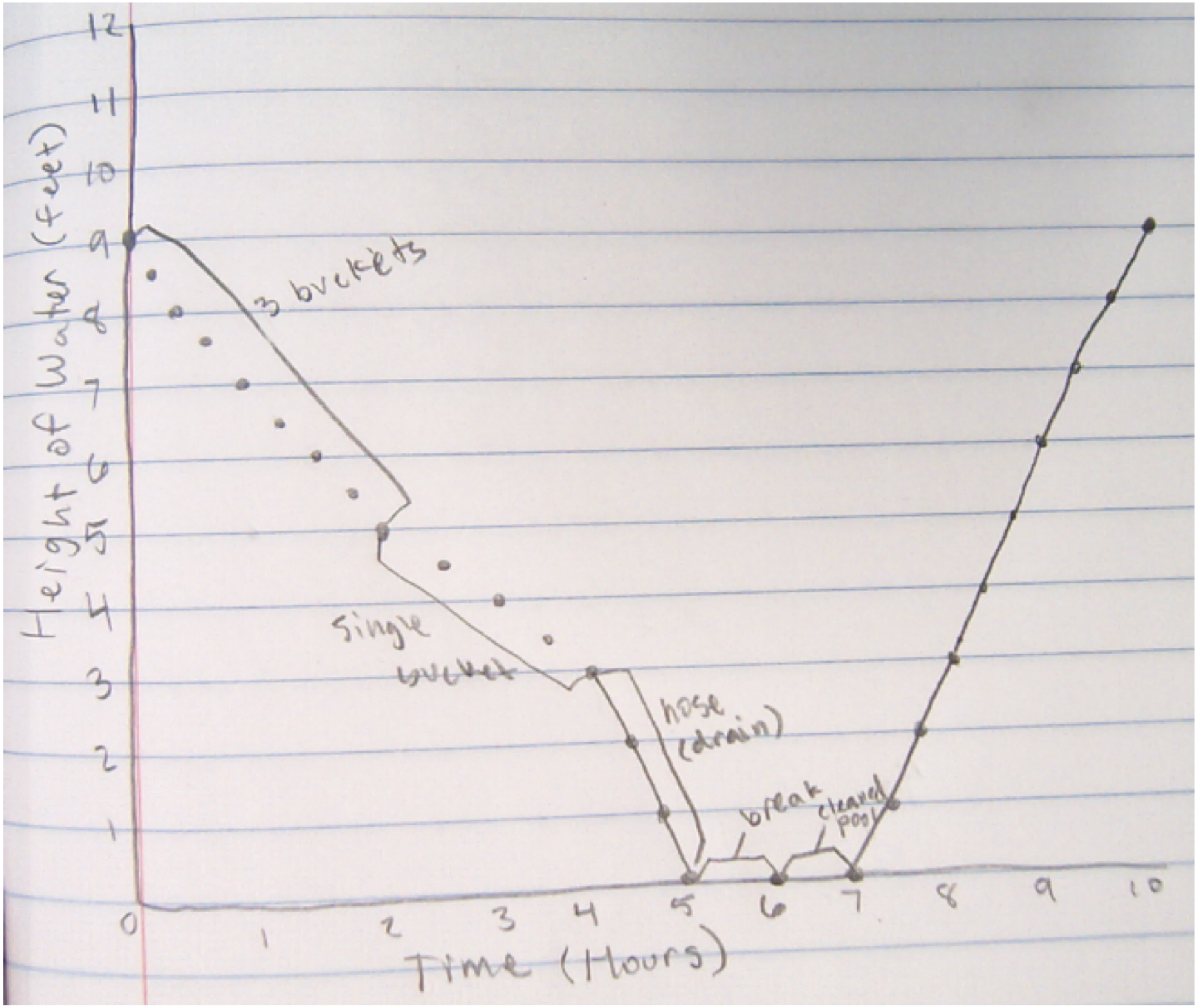
(Harkness 2009, p.244)

The goal of this task is to surface the vocabulary that connects to the graph of a function.



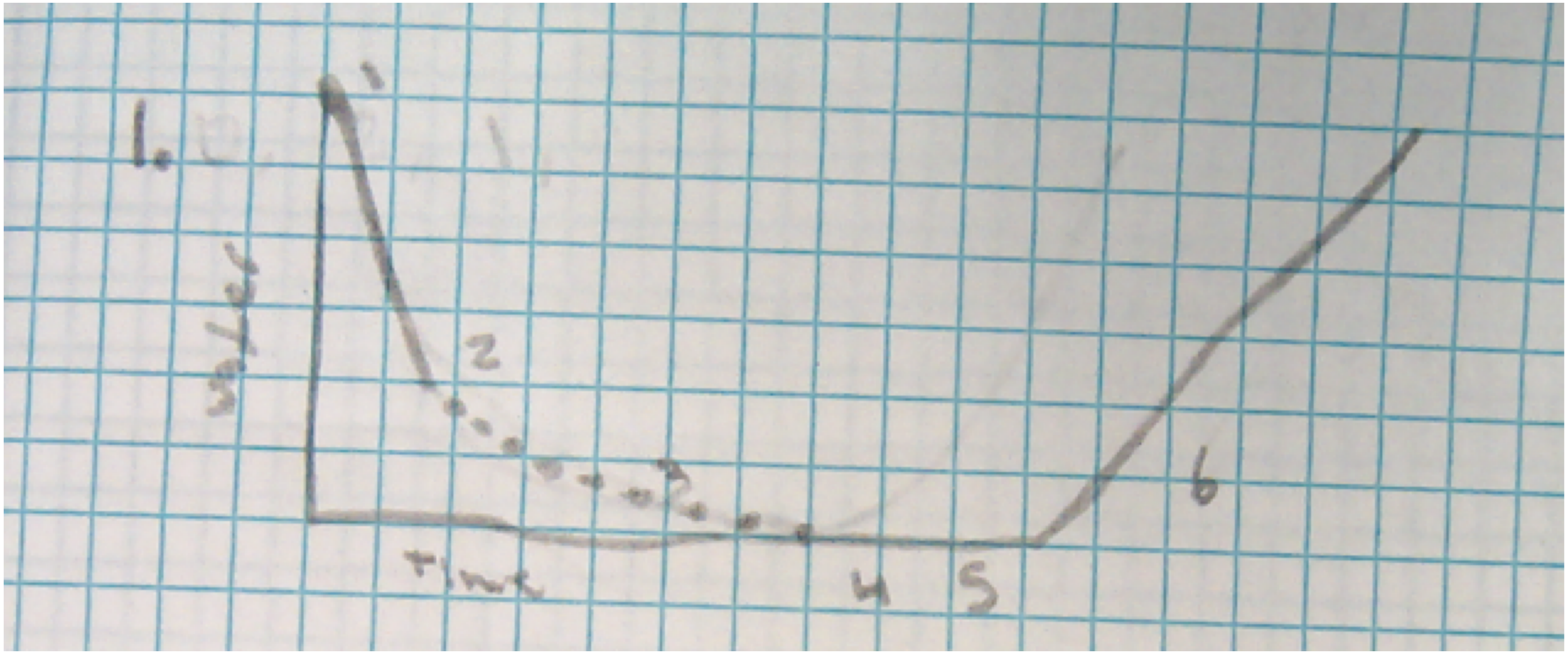
What opportunity does this work provide?



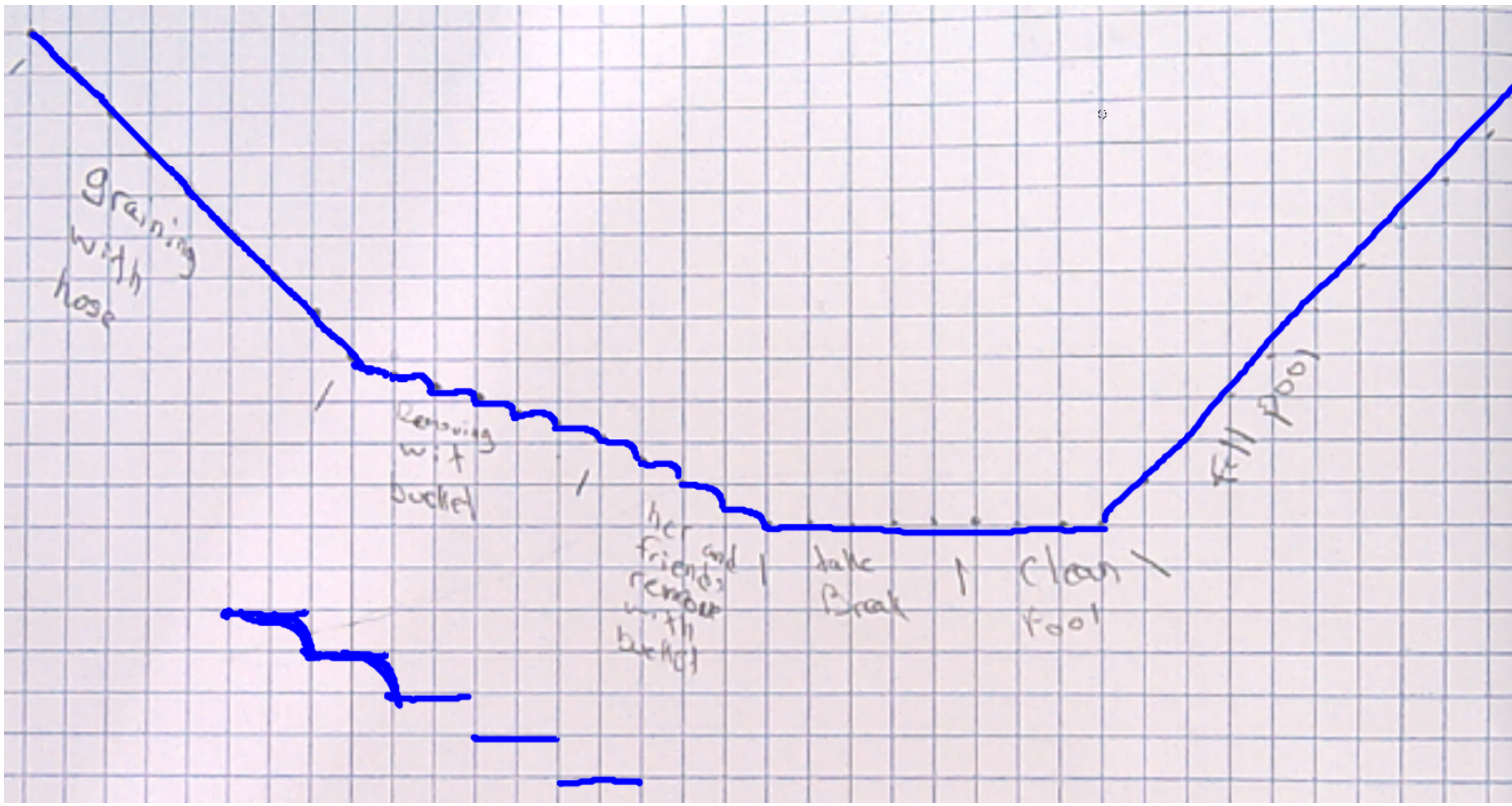


Height of Water (feet)
Time (Hours)
3 buckets
single bucket
hose (drain)
break
cleaned pool
0
1
2
3
4
5
6
7
8
9
10
11
12

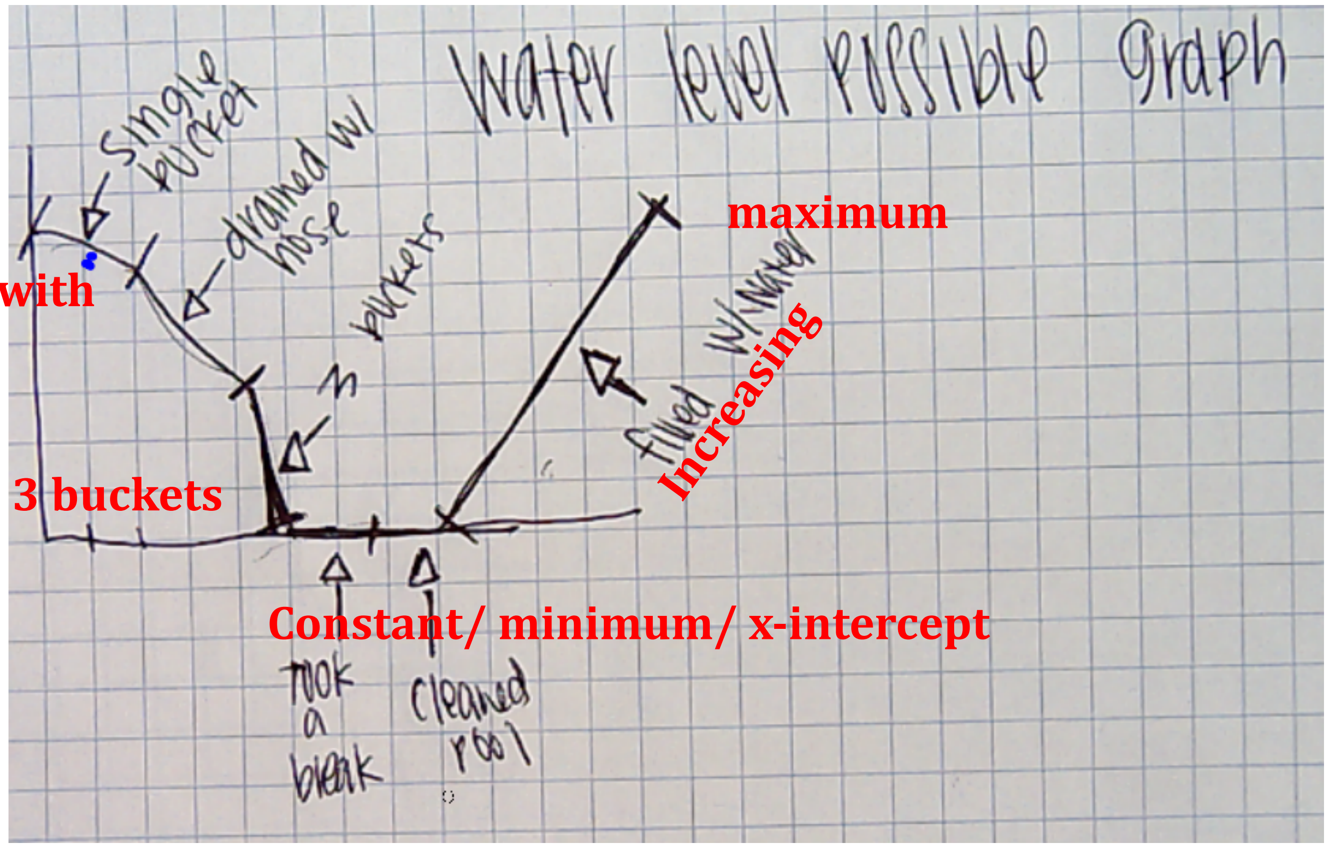
What do you think the student is thinking about in this graph?



What opportunities are provided by this representation?



What would you want to talk about in this graph?



y-intercept

Rate of change with 1 bucket

Rate with 3 buckets

maximum

Increasing

Constant/ minimum/ x-intercept

Learning from each other's work.
Learning from the class discussion.

- What can I learn from my classmate's representation?
- Do I need to rethink my interpretation?
- Do I need to add something to my graph?
- Are there math vocabulary words that I need to add to my notes?
- Do I understand everything that we talked about in class?

Why is sharing student work so important?

Elicit and use evidence of student thinking.

- Student work is student thinking made visible.
- Student work gives students something to talk about.
- It challenges students to make sense of the math.
- It provide opportunities for students to think "out loud," and to change their minds.
- Teaching should be conversations for learning.

Mathematic Vision Project (MVP)

Effective teaching of mathematics uses evidence of student thinking

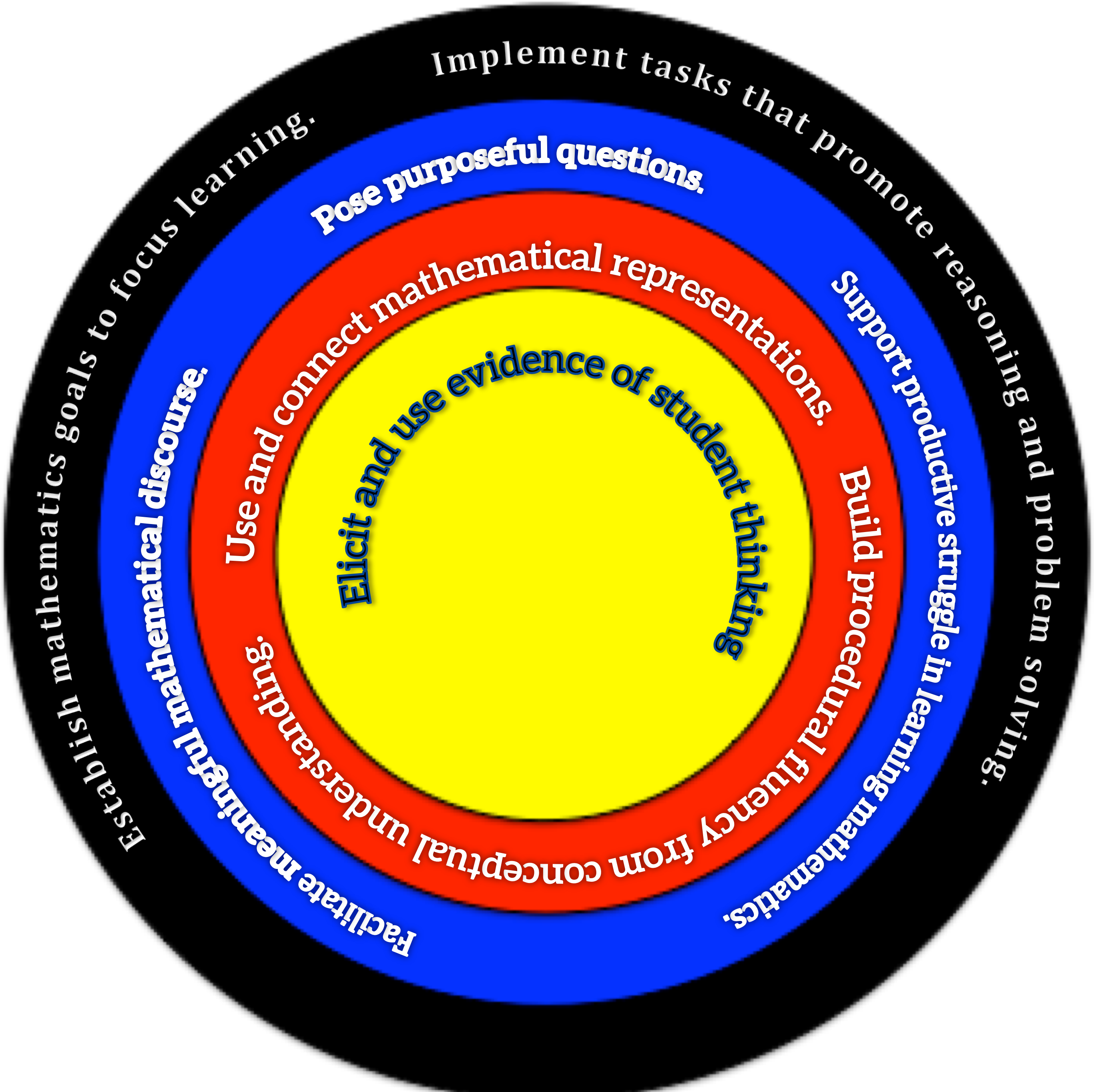
3) To affirm students' mathematical identities

4) To build conceptual understanding

5) To promote mathematical discourse

6) To create opportunities for direct instruction

We've been asked to do this.



Establish mathematics goals to focus learning.
Implement tasks that promote reasoning and problem solving.
Pose purposeful questions.
Support productive struggle in learning mathematics.
Facilitate meaningful mathematical discourse.
Use and connect mathematical representations.
Build procedural fluency from conceptual understanding.
Elicit and use evidence of student thinking

If you place eliciting and using student thinking as the focus of your instruction, you will naturally pull in the other 7 Effective Teaching Practices.

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