





Progression and Practices

Our Session Today

- The need for progressions in learning and curriculum
- A framework for learning that promotes coherence and learning progressions: (CMI)
- Experience and engage with tasks to see and feel the progression and better understand the framework used in the MVP materials



NCTM Curriculum Principle:

A curriculum is <u>more than a collection of activities</u>: it must be <u>coherent</u>, <u>focused</u> on important mathematics, and well <u>articulated across</u> the grades.

(PSSM, 2000)



- *conceptual understanding*—comprehension of mathematical concepts, operations, and relations
- *procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence*—ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification
- *productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

The most important observation we make about these five strands is that they are interwoven and interdependent. This observation has implications for how students acquire mathematical proficiency, how teachers develop that proficiency in their students, and how teachers are educated to achieve that goal.

Conceptual Understanding

Strategic Competence Disposition

Adaptive Reasoning

Procedural Fluency

Intertwined Strands of Proficiency

(Adding it Up, 2001)



Transforming Mathematics Education

"The Common Core State Standards in mathematics were built on progressions...informed both by research on children's cognitive development and by the logical structure of mathematics."

Progression Document Introduction http://ime.math.arizona.edu/progressions/



"With the advent of the Common Core, a decade's worth of recommendations for greater focus and coherence finally have a chance to **bear fruit**. Focus and coherence are two major evidence-based design principles of the Common Core State Standards for Mathematics. These principles are meant to fuel greater achievement in a deep and rigorous curriculum, one in which students acquire conceptual understanding, procedural skill and fluency, and the ability to apply mathematics to solve problems and formulate mathematical models."

(CCSSM Publisher's criteria, pg. 2)



"Coherence is about making math make sense. Mathematics is not a list of disconnected tricks or mnemonics. It is an elegant subject...

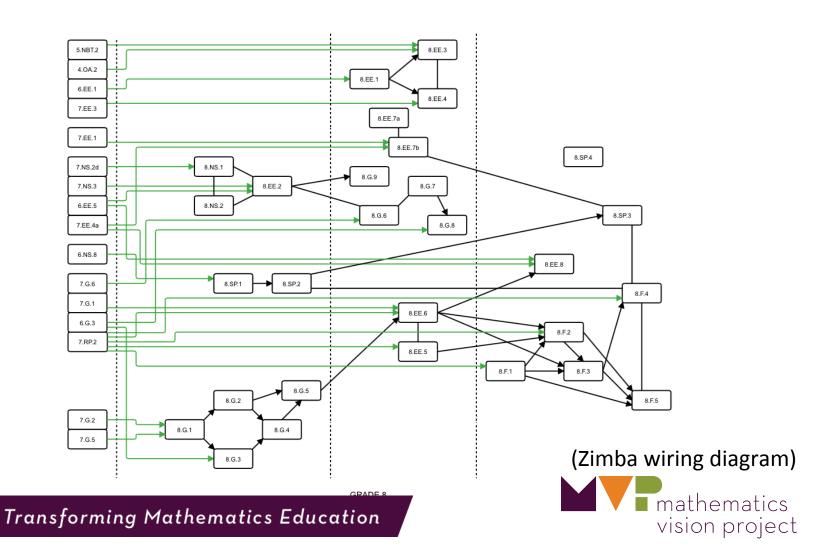
(CCSSM Publisher's criteria, pg. 3)





"Fragmenting the Standards into individual standards, or individual bits of standards, erases all these relationships and produces a sum of parts that is decidedly less than the whole. "

Phil Daro, Jason Zimba, Bill McCallum



Effective Teaching Practices

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Building Procedural Fluency from Conceptual understanding
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking. (NCTM, 2014)



Effective Teaching Practices

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, <u>over</u> <u>time</u>, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.

Effective Teaching Practices

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Building Procedural Fluency from Conceptual understanding
- Support productive struggle in learning mathematics. Effective teaching of mathematics <u>consistently provides</u> students, individually and collectively, with opportunities and supports to engage in productive struggle as they <u>grapple with mathematical ideas and relationships</u>.
- Elicit and use evidence of student thinking.



More to it then what one might first think...

The message about coherence and progression has been building for a decade, or more, but hopefully we have learned from prior experiences in mathematics education reform and will take better advantage of the opportunity we have.

I hear teachers often say things like:

- "So, how often do you do tasks with your students?"
- "I have a couple of hands-on activities that I really like."
- "I love the foldables that I have started doing."
- "I would do more tasks if they didn't take so much time."

We need to do better than this! We need a sustained, persistent press for student thinking, development of ideas over time and productive struggle that occurs on a daily basis.



There is a need for a framework.

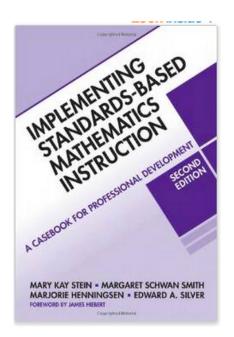
- Chazan and Ball (1999), argue that educators are often left "with no framework for the kinds of specific, constructive pedagogical moves that teachers might make."
- Stein et al. (2008) refer to a *first generation* of instructional reform from which "many teachers got the impression that in order for discussions to be focused on student thinking, they must avoid providing any substantive guidance at all," and they refer to a *second generation* of instructional reform "that re-asserts the critical role of the teacher in guiding mathematical discussions."



Levels of Cognitive demand



Lower-level demands	Higher-level demands
Memorization Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas, or definitions to memory	Procedures with connections Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas Procedures with connections Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas
 Cannot be solved by using procedures, because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure 	Suggest, explicitly or implicitly, pathways to follow that are broad general procedures that have close connections to un- derlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts
 Are not ambiguous. Such tasks involve exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated. Have no connection to the concepts or meaning that underlies the facts, rules, formulas, or definitions being learned or reproduced 	Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.
	Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.
Procedures without connections Are algorithmic. Use of the procedure is either specifically called for or is evident from prior instruction, experience, or placement of the task. Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done or how to do it. Have no connection to the concepts or meaning that underlies the procedure being used Are focused on producing correct answers instead of on developing mathematical understanding Require no explanations or explanations that focus solely on describing the procedure that was used	Doing mathematics Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example
	Require students to explore and understand the nature of mathematical concepts, processes, or relationships
	Demand self-monitoring or self-regulation of one's own cog- nitive processes
	Require students to access relevant knowledge and experi- ences and make appropriate use of them in working through the task
	 Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions
	 Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required

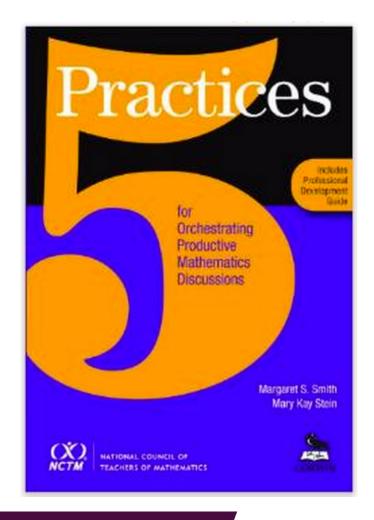


Purple Book (NCTM, 2009)



- Levels of Cognitive demand
- 5 practices for orchestrating discussions





(NCTM, 2011)



- Levels of Cognitive demand
- 5 practices for orchestrating discussions
- Launch, Explore, Summarize model in CMP and other curricula





- Levels of Cognitive demand
- 5 practices for orchestrating discussions
- Launch, Explore, Summarize model in CMP and other curricula
- 3-act tasks





The Three Acts Of A Mathematical Story

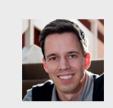
May 11th, 2011 by Dan Meyer

2013 May 14. Here's a brief series on how to teach with three-act math tasks. It includes video.

2013 Apr 12. I've been working this blog post into curriculum ideas for a couple years now. They're all available here.

Storytelling gives us a framework for certain mathematical tasks that is both prescriptive enough to be *useful* and flexible enough to be *usable*. Many stories divide into three acts, each of which maps neatly onto these mathematical tasks.





My name is **Dan Meyer** and I like to teach.

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- Levels of Cognitive demand
- 5 practices for orchestrating discussions
- Launch, Explore, Summarize model in CMP and other curricula
- 3-act tasks
- And more...



- Levels of Cognitive demand
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- And more....
- These are great!



- Levels of Cognitive demand
- 5 practices for orchestrating discussions
- Launch, Explore, Summarize model in CMP and other curricula
- 3-act tasks
- And more....
- These are great!
- They have all assisted with the work of the 2nd generation of reform.

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- The CCSSM publisher's criteria, Principles to Actions, and others, wouldn't urge us to do more if we had arrived.
- We need a sustained, persistent press for student thinking, development of conceptual understanding and procedural fluency, productive struggle that occurs on a daily basis.
- All of the effective mathematics teaching practices which promote the standards for mathematical practice.
- The effort to implement a task needs to lead to the implementation of a progression of tasks and a curriculum that is coherent, rigorous and focused.



NCTM Curriculum Principle:

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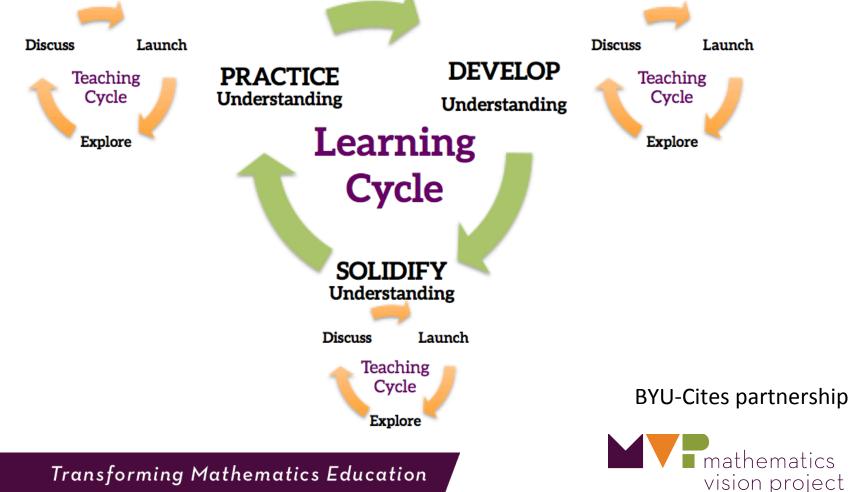
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A FRAMEWORK for Coherence and Progression: The Comprehensive Mathematics Instruction Framework

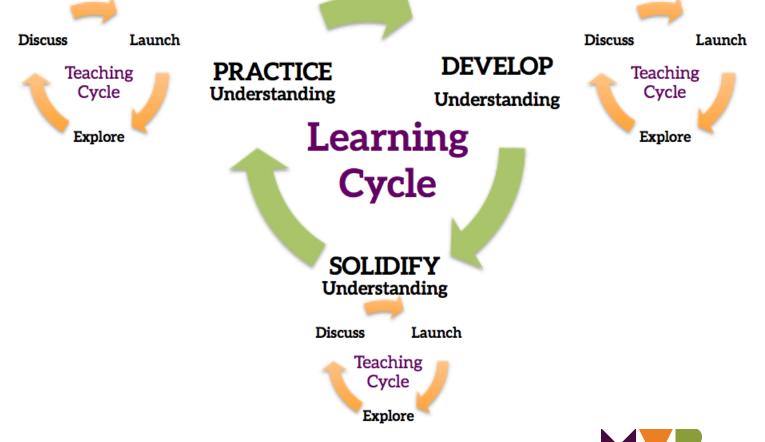
The framework on which MVP curriculum is built



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<u>A FRAMEWORK for Coherence and Progression:</u> The Comprehensive Mathematics Instruction Framework

The framework on which MVP curriculum is built

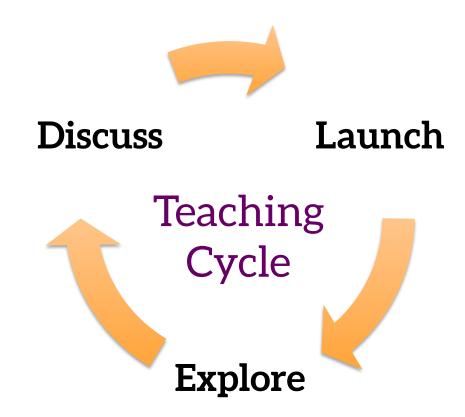




A FRAMEWORK for a Lesson or TASK:

Moving from a conceptual foundation to procedural fluency

Comprehensive Mathematics Instruction Framework





A FRAMEWORK for a Lesson or TASK:

Moving from a conceptual foundation to procedural fluency

Comprehensive Mathematics Instruction Framework

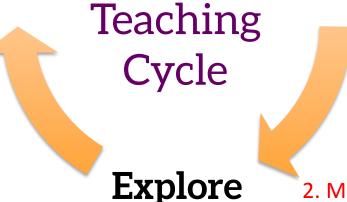
1. Anticipate student thinking

5. Connect student thinking

Discuss

Launch

4. Sequence student thinking



2. Monitor student thinking

3. Select student thinking



A FRAMEWORK for a Lesson or TASK:

Moving from a conceptual foundation to procedural fluency

Comprehensive Mathematics Instruction Framework

Facilitate Meaningful Mathematics Discourse

Discuss

Establish Mathematical Goals to Focus Learning

Launch

Implement Tasks That Promote Reasoning and Problem Solving

Elicit and Use Evidence of Student Thinking

Pose Purposeful Questions

Teaching Cycle

Explore

Support Productive Struggle In Learning Mathematics

Use and Connect Mathematical Representations

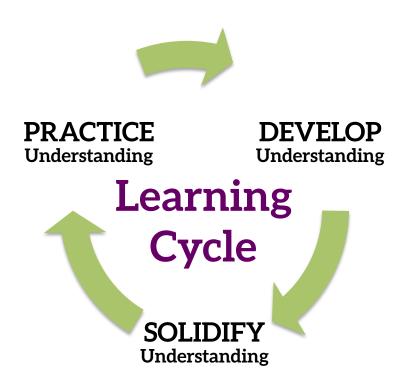


Transforming Mathematics Education

A FRAMEWORK for Task Sequencing:

Moving from a conceptual foundation to procedural fluency

Comprehensive Mathematics Instruction Framework



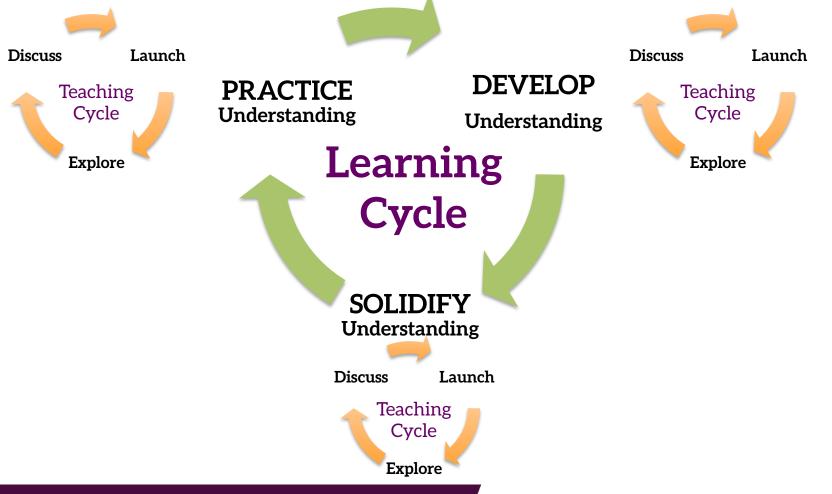
- Develop Understanding tasks surface student thinking
- Solidify Understanding tasks examine and extend
- Practice Understanding tasks build fluency



A FRAMEWORK for Coherence and Progression:

Moving from a conceptual foundation to procedural fluency

Comprehensive Mathematics Instruction Framework

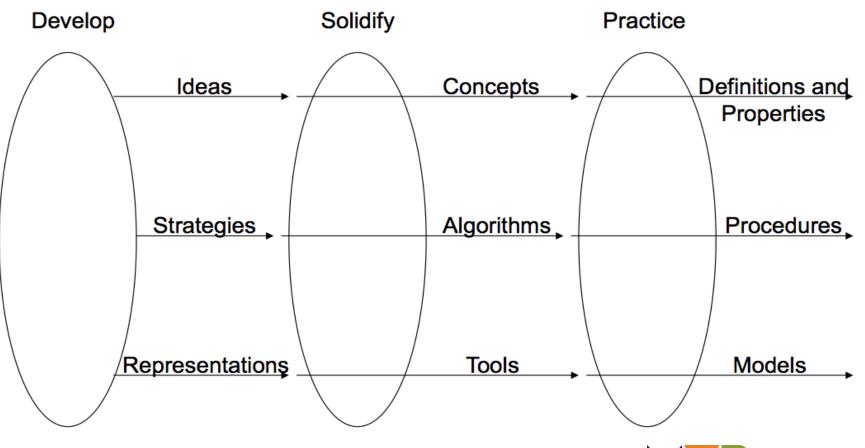


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A FRAMEWORK for Coherence and Progression:

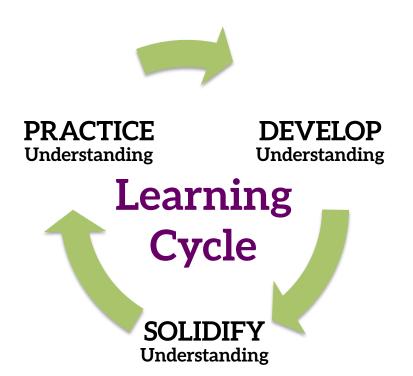
Looking at progression in students' mathematical proficiency



A FRAMEWORK for Task Sequencing:

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- Develop Understanding tasks surface student thinking
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A Develop Understanding Task

Sylvia has a small pool full of water that needs to be emptied and cleaned, then refilled for a pool party. During the process of getting the pool ready, Sylvia did all of the following activities, each during a different time interval.

Removed water with a single bucket	Filled the pool with a hose (same rate as emptying pool)
Drained water with a hose (same rate as filling pool)	Cleaned the empty pool
Sylvia and her two friends removed water with three buckets	Took a break

- 1. Sketch a possible graph showing the height of the water level in the pool over time. Be sure to include all of activities Sylvia did to prepare the pool for the party. Remember that only one activity happened at a time. Think carefully about how each section of your graph will look, labeling where each activity occurs.
- 2. Create a story connecting Sylvia's process for emptying, cleaning, and then filling the pool to the graph you have created. Do your best to use appropriate math vocabulary.
- 3. Does your graph represent a function? Why or why not? Would all graphs created for this situation represent a function?

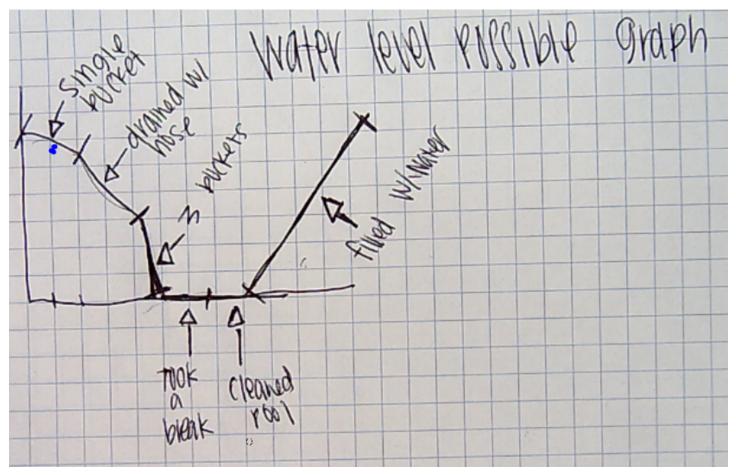
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A Develop Understanding Task

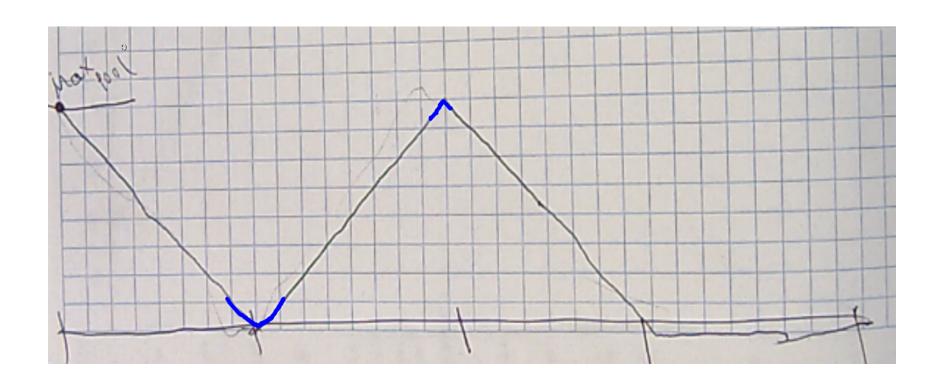
What mathematical ideas are surfaced in this task?

How would you describe the nature of the work?

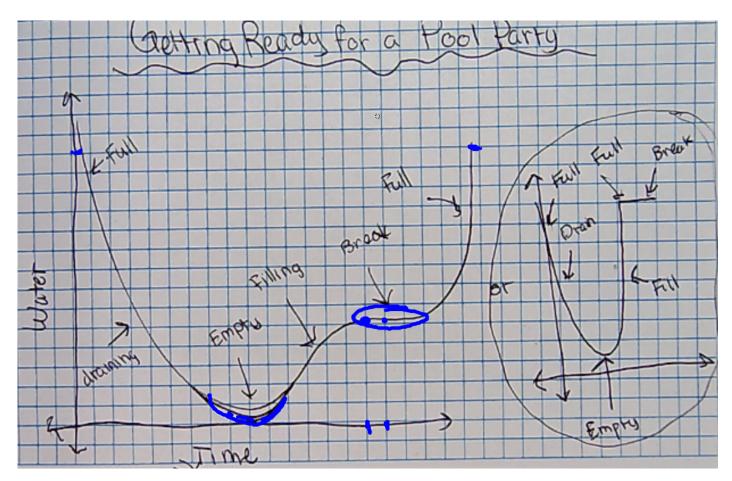




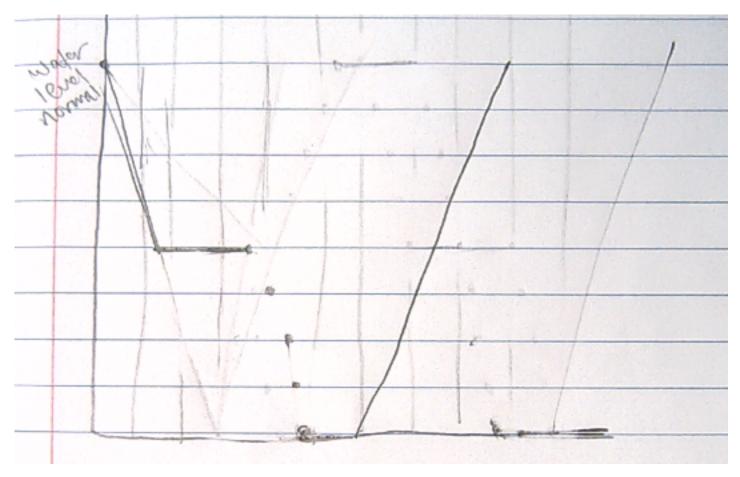




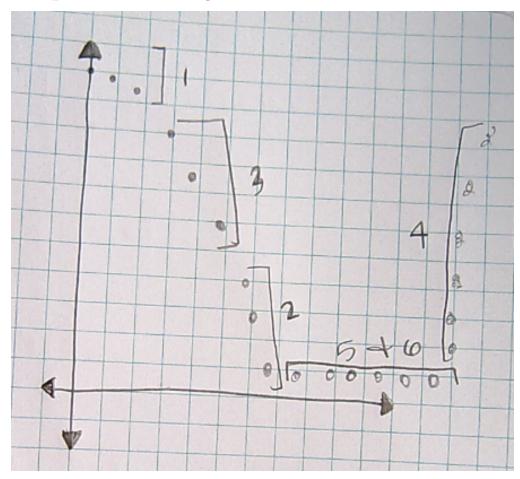




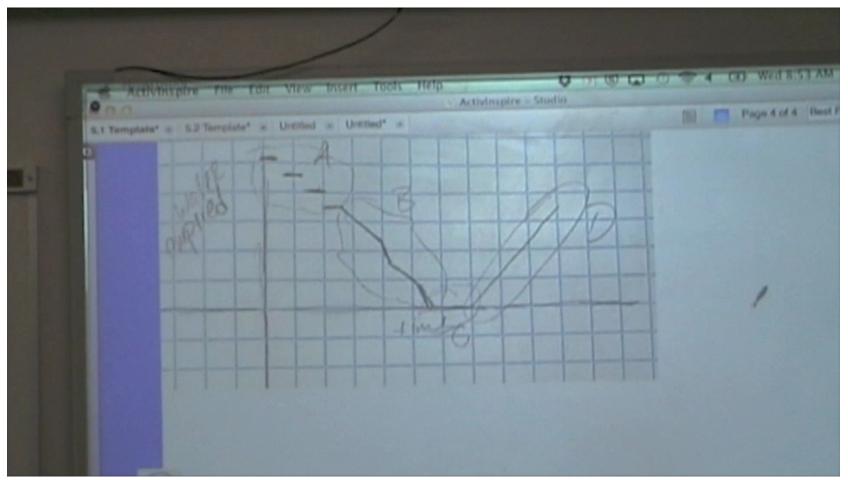














A Develop Understanding Task

What mathematical ideas are surfaced in this task?

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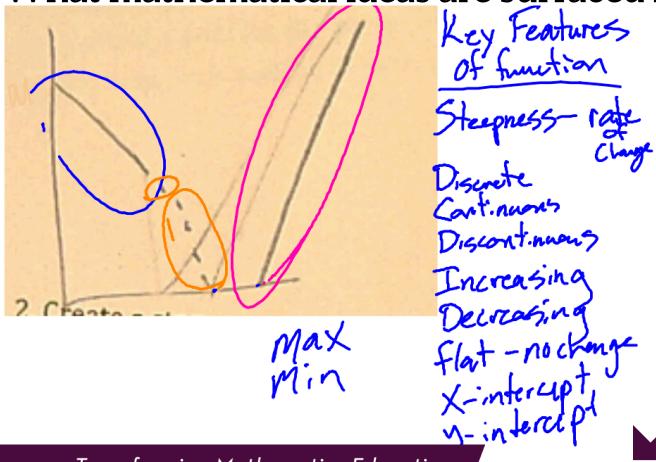


A Develop Understanding Task

What mathematical ideas are surfaced in this task?

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A Develop Understanding Task

How would you describe the nature of the work?

- Develop Understanding tasks
 - Low threshold, high ceiling (easy entry, but extendable for all learners)
 - Contextualized (problematic story context, diagrams, symbols)
 - Multiple pathways to solutions or multiple solutions
 - Surface student thinking (misconceptions and correct thinking)
 - Purposeful selection of the vocabulary, numbers, etc. to reveal rather than obscure the mathematics
 - Introduce a number of representations
 - constructing viable arguments and critiquing the reasoning of others

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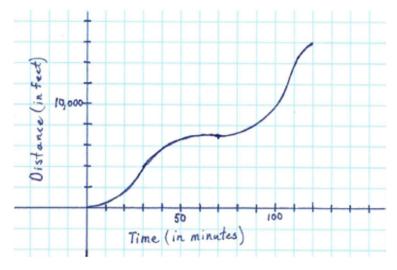
A Solidify Understanding Task

Alonzo, Maria, and Sierra were floating in inner tubes down a river, enjoying their day. Alonzo noticed that sometimes the water level was higher in some places than in others. Maria noticed there were times they seemed to be moving faster than at other times. Sierra laughed and said "Math is everywhere!" To learn more about the river, Alonzo and Maria collected data throughout the trip.

Alonzo created a table of values by measuring the depth of the water every ten minutes. Maria created a graph by collecting data on a GPS unit that told her the distance she had traveled

over a period of time.

Time (in minutes)	0	10	20	30	40	50	60	70	80	90	100	110	120
Depth (in feet)	4	6	8	10	6	5.	4	5	7	12	9	6.5	5



A Develop Understanding Task

What mathematical ideas are examined/extended in this task?

How would you describe the nature of the work?



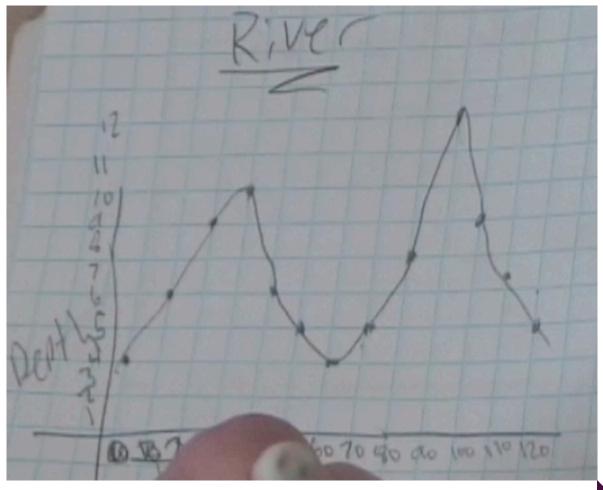
A Solidify Understanding Task

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A Develop Understanding Task

What mathematical ideas are examined/extended in this task?

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A Develop Understanding Task

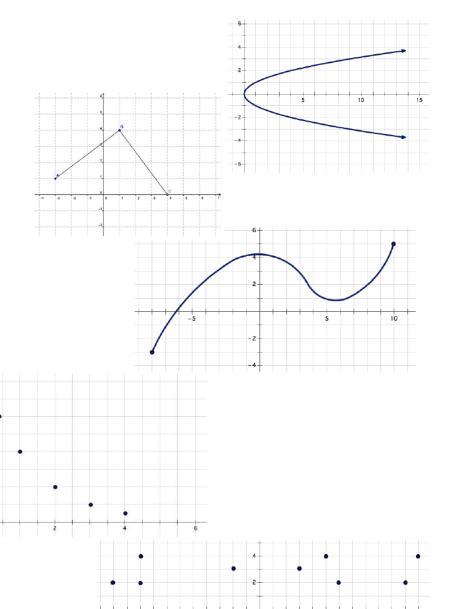
How would you describe the nature of the work?

- Task context, scaffolding questions and constraints focus students' attention on:
 - looking for patterns and making use of structure
 - looking for repeated reasoning and expressing regularities as generalized methods
 - attending to precision in language and use of symbols
 - constructing viable arguments and critiquing the reasoning of others
 - using representations and tools strategically
- for the purpose of developing deeper levels of understanding of mathematical ideas, strategies, and/or representations



A Practice Understanding Task

For each graph, determine if the relationship represents a function, and if so, state the key features of the function (intervals where the function is increasing or decreasing, the maximum or minimum value of the function, domain and range, x and y intercepts, etc.)



ICS

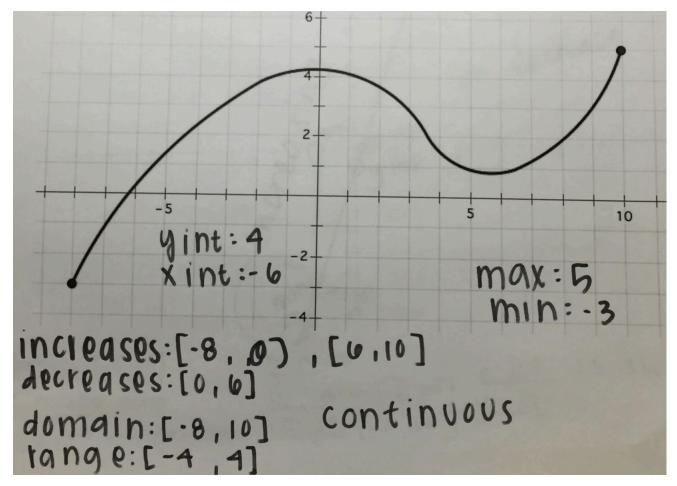
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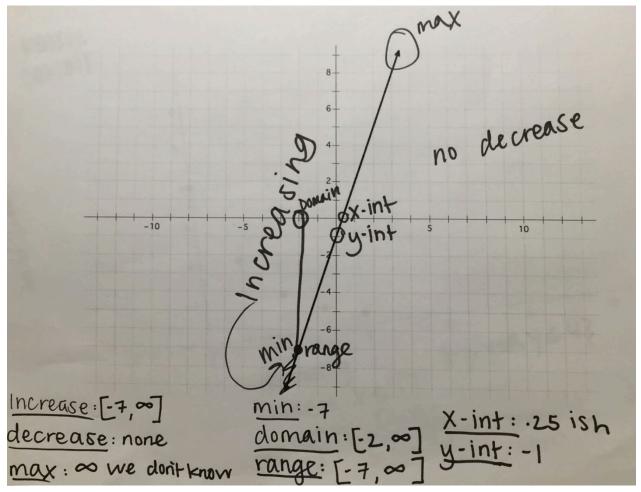
What mathematical ideas are refined in this task?

How would you describe the nature of the work?

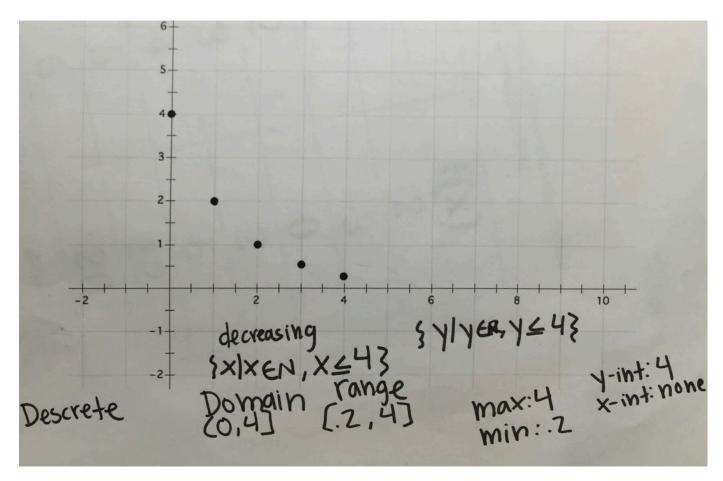














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5	12
6	20
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A Develop Understanding Task

What mathematical ideas are refined in this task?

How would you describe the nature of the work?



A Develop Understanding Task

How would you describe the nature of the work?

- Practice tasks focused on <u>acquiring fluency</u>
 - Task involves either reproducing previously learned facts, definitions, rules, formulas or models; OR drawing upon previously learned facts, definitions, rules, formulas or models; OR committing facts, definitions, rules, formulas or models to memory
 - An appropriate vehicle of practice is selected (e.g., routines, games, worksheets, etc.) which allows for reproducing, drawing upon, or committing to memory previously examined mathematics

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Task focuses on a broad definition of fluency: accuracy, efficiency, flexibility, automaticity

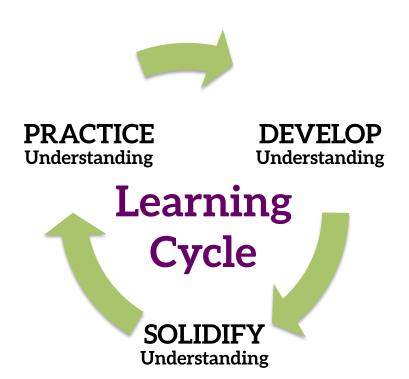
A FRAMEWORK for Task Sequencing: Moving from a conceptual foundation to procedural fluency Comprehensive Mathematics Instruction Framework

- Practice tasks focused on <u>refining</u> understanding
 - Task allows student to use reasoning habits to contextualize (symbolic to real-world) and decontextualize (real-world to symbolic) problems and situations.
 - Tasks involve sufficient complexity to refine mathematical thinking beyond rote memorization
 - The task requires a high level of cognitive demand because students are required to draw upon multiple concepts and procedures, make use of structure and recognize complex relationships among facts, definitions, rules, formulas and/or models



<u>A FRAMEWORK for Task Sequencing</u>: Moving from a conceptual foundation to procedural fluency

Comprehensive Mathematics Instruction Framework



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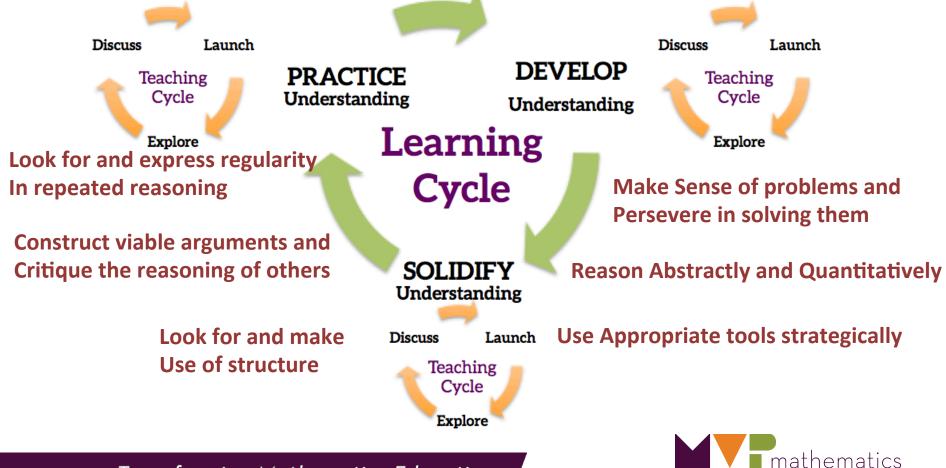
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A FRAMEWORK for Coherence and Progression: The Comprehensive Mathematics Instruction Framework The framework on which MVP curriculum is built

Attending to Precision



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How do learning cycles fit within a module of instruction?

Tracking Student Thinking in MVP Module 4: Features of Functions

		Understand the concept of function and use function notation Interpret functions that arise in applications in terms of context								Analyze functions using different representations	Build a f that moo relation between quantiti	ship 1 two	Represent and solve equations and inequalities graphically	
Description of the nature of student thinking relative to this standard ⇒ in this task ↓ (see sample descriptors below*)	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range (F.HF.1)	If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x; the graph of f is the graph of the equation $y = f(x)$ (F.IF.1)	Use function notation and evaluate functions for inputs in their domains (F.IF.2)	Interpret statements that use function notation in terms of a context (F.IF.2)	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers (F.IF.3)	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities (F.IF.4)	For a function that models a relationship between two quantities, sketch graphs showing key features given a verbal description of the relationship. (F.IF.4)	Relate the domain of a function to its graph (F.IF.5)	Relate the domain of a function (where applicable) to the quantitative relationship it describes (F.IF.5)	Graph functions expressed symbolically and show key features of the graph (F.IF.7)	Write a function that describes a relationship between two quantities (F.BF.1)	Combine standard function types using arithmetic operations (F.BF.1b)	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ (A.REI.11)	
Getting Ready for a Pool Party	D					D	D	D	D					
Floating Down the River	D					S	S	S	s					
Features of Functions	D				P	P	P	P	P					
The Water Park	D	S	S	S		P	P	P	P	S	D		P	
Pooling It Together	D	S	S	S		P	P	P	P	S	S	D		

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