Transforming Mathematics Education

Flexible & Engaging Seamless Common Core Companion
What is the Mathematics Vision Project?

• The Mathematics Vision Project is an educator-driven initiative to provide first-rate curriculum and professional development resources to secondary math teachers.

• Built from the ground-up by a team of award-winning, math-loving education experts, the material is flexible, engaging, and a seamless companion to the Common Core.

• Committed to life-long learning, the MVP team also offers educator training and resources to help teachers perform at their peak.
Who are we? Meet the MVP Team

• We believe math is engaging. Our desire is to enable educators to teach their students through accessible, engaging, supportive curriculum.

• We enjoy our work. Our product has grown out of our passion for teaching mathematics and we love sharing it with our clients.

• We know teachers need support. We live the phrase “we’re in this together.” As educators face new challenges and requirements, we’re there to support their next steps and, in turn, the success of their students.
Today’s Agenda

• MVP materials are based on frameworks and have a depth and richness.

• The Teaching Cycle and Learning Cycle.

• Do some tasks, Focus on the Teaching Cycle.

• The power of an entire module.

• Some Geometry?
Standards for Mathematical Practice

Mathematically proficient students:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.
What needs to be in place to make this happen in the classroom?

<table>
<thead>
<tr>
<th>Mathematics Teaching Practices</th>
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<tbody>
<tr>
<td><strong>Establish mathematics goals to focus learning.</strong> Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.</td>
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<tr>
<td><strong>Implement tasks that promote reasoning and problem solving.</strong> Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.</td>
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<tr>
<td><strong>Use and connect mathematical representations.</strong> Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.</td>
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<tr>
<td><strong>Facilitate meaningful mathematical discourse.</strong> Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.</td>
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<tr>
<td><strong>Pose purposeful questions.</strong> Effective teaching of mathematics uses purposeful questions to assess and advance students’ reasoning and sense making about important mathematical ideas and relationships.</td>
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<tr>
<td><strong>Build procedural fluency from conceptual understanding.</strong> Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.</td>
</tr>
<tr>
<td><strong>Support productive struggle in learning mathematics.</strong> Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.</td>
</tr>
<tr>
<td><strong>Elicit and use evidence of student thinking.</strong> Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.</td>
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</tbody>
</table>
A FRAMEWORK for a Lesson or TASK:
Comprehensive Mathematics Instruction Framework

Discuss  Launch  Teaching Cycle  Explore
A FRAMEWORK for a Lesson or TASK:
Comprehensive Mathematics Instruction Framework

1. Anticipate student thinking
2. Monitor student thinking
3. Select student thinking
4. Sequence student thinking
5. Connect student thinking

Launch
Teaching Cycle
Explore
Discuss
A FRAMEWORK for a Lesson or TASK:
Comprehensive Mathematics Instruction Framework

Launch
Establish Mathematical Goals to Focus Learning

Explore
Use and Connect Mathematical Representations
Support Productive Struggle In Learning Mathematics

Discuss
Facilitate Meaningful Mathematics Discourse
Elicit and Use Evidence of Student Thinking
Pose Purposeful Questions

Teaching Cycle
Transforming Mathematics Education
A FRAMEWORK for Task Sequencing:
Moving from a conceptual foundation to procedural fluency

Comprehensive Mathematics Instruction Framework

- Develop Understanding tasks surface student thinking
- Solidify Understanding tasks examine and extend
- Practice Understanding tasks build fluency

Transforming Mathematics Education
A FRAMEWORK for Task Sequencing: Moving from a conceptual foundation to procedural fluency
Comprehensive Mathematics Instruction Framework

- Develop Understanding tasks
  - Low threshold, high ceiling (easy entry, but extendable for all learners)
  - Contextualized (problematic story context, diagrams, symbols)
  - Multiple pathways to solutions or multiple solutions
  - Surface student thinking (misconceptions and correct thinking)
  - Purposeful selection of the vocabulary, numbers, etc. to reveal rather than obscure the mathematics
  - Introduce a number of representations
  - constructing viable arguments and critiquing the reasoning of others
A FRAMEWORK for Task Sequencing: Moving from a conceptual foundation to procedural fluency
Comprehensive Mathematics Instruction Framework

- **Solidify Understanding** tasks

  - Task context, scaffolding questions and constraints focus students’ attention on:
    - looking for patterns and making use of structure
    - looking for repeated reasoning and expressing regularities as generalized methods
    - attending to precision in language and use of symbols
    - constructing viable arguments and critiquing the reasoning of others
    - using representations and tools strategically for the purpose of developing deeper levels of understanding of mathematical ideas, strategies, and/or representations
A FRAMEWORK for Task Sequencing: Moving from a conceptual foundation to procedural fluency

Comprehensive Mathematics Instruction Framework

- **Practice Understanding** tasks

  - Practice tasks focused on **refining understanding**
    - Task allows student to use reasoning habits to contextualize (symbolic to real-world) and decontextualize (real-world to symbolic) problems and situations.
    - Tasks involve sufficient complexity to refine mathematical thinking beyond rote memorization
    - The task requires a high level of cognitive demand because students are required to draw upon multiple concepts and procedures, make use of structure and recognize complex relationships among facts, definitions, rules, formulas and/or models
A FRAMEWORK for Task Sequencing: Moving from a conceptual foundation to procedural fluency

Comprehensive Mathematics Instruction Framework

- **Practice Understanding** tasks

  - Practice tasks focused on acquiring fluency
    - Task involves either reproducing previously learned facts, definitions, rules, formulas or models; OR drawing upon previously learned facts, definitions, rules, formulas or models; OR committing facts, definitions, rules, formulas or models to memory
    - An appropriate vehicle of practice is selected (e.g., routines, games, worksheets, etc.) which allows for reproducing, drawing upon, or committing to memory previously examined mathematics
    - Task focuses on a broad definition of fluency: accuracy, efficiency, flexibility, automaticity
A FRAMEWORK for Coherence and Progression: The Comprehensive Mathematics Instruction Framework

The framework on which MVP curriculum is built

BYU-Cites partnership

Transforming Mathematics Education
A FRAMEWORK for Coherence and Progression: The Comprehensive Mathematics Instruction Framework
The framework on which MVP curriculum is built

Look for and express regularity in repeated reasoning

Construct viable arguments and critique the reasoning of others

Look for and make use of structure

Reason abstractly and quantitatively

Use appropriate tools strategically

Make sense of problems and persevere in solving them

Attending to precision

Discuss Launch

Discuss Launch

Discuss Launch

Discuss Launch

Discuss Launch

Discuss Launch

Discuss Launch

Discuss Launch

Discuss Launch
Check out the Ready – Set – Go!

R, S, G’s  Homework Assignments

**Ready:** Get ready for upcoming lessons

*Topic:*

**Set:** Reinforce what was learned in today’s lesson

*Topic:*

**Go:** Practice previously-learned skills

*Topic:*
READY, SET, GO’s!

- We want our students “immunized against forgetting.” pg.32
- Periodic practice arrests forgetting, strengthens, retrieval routes, and is essential for hanging onto the knowledge you want to gain. When you space out practice at a task and get a little rusty between sessions, or you interweave the practice of two or more subjects, retrieval is harder and feels less productive, but the effort produces longer lasting learning and enables more versatile application of it in later settings. pg.4
3.1 Growing Dots
A Develop Understanding Task

At the beginning
At one minute
At two minutes

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What mathematical ideas that emerged in this task?
A FRAMEWORK for a Lesson or TASK: Moving from a conceptual foundation to procedural fluency

Comprehensive Mathematics Instruction Framework

Discuss → Launch → Teaching Cycle → Explore

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A FRAMEWORK for a Lesson or TASK:
Moving from a conceptual foundation to procedural fluency
Comprehensive Mathematics Instruction Framework

1. Anticipate student thinking
2. Monitor student thinking
3. Select student thinking
4. Sequence student thinking
5. Connect student thinking

Discuss  Launch
Teaching Cycle
Explore

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Five Practices for Facilitating Mathematical Discourse

1. **Anticipate** student thinking – work the task yourself

2. **Monitor** students as they work – circulate around the room and ask students how they are thinking about what they have written.

3. **Select** students for the classroom discussion – have a method for keeping track of who you have selected

4. **Sequence** student work

5. **Connect** the big ideas from the day’s lesson.
Video of Growing Dots
A FRAMEWORK for Coherence and Progression: The Comprehensive Mathematics Instruction Framework

The framework on which MVP curriculum is built

Learning Cycle

Discuss Launch

Teaching Cycle

PRACTICE Understanding

Discuss Launch

Teaching Cycle

EXPLAIN Understanding

Discuss Launch

Teaching Cycle

SOLIDIFY Understanding

Discuss Launch

Teaching Cycle

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3.2 Growing, Growing Dots

A Develop Understanding Task

At the beginning

At one minute

At two minutes

At three minutes

At four minutes
Using Representations

- What do we know about linear and exponential functions based on these tasks?
- How do these ideas emerge from the representations?
3.3 Scott’s Workout
A Solidify Understanding Task
What mathematical ideas were extended and made more concrete in this task?
Video of Scott’s Workout
The Power of Looking at the Entire Module

- Learning to trust the materials
• **Work the task.** Read the Teacher Notes, paying attention to the purpose and the goal. Identify what mathematics is developed, solidified or practiced in this task.

• **Make a poster** labeled with the number and title of the task.  
• Illustrate the mathematical focus of the task (representations).  
• Look at the task before and the task that follows in order to highlight the mathematics that this task contributes to the learning progression.  
• Include topics of the RSG’s how the RSG consolidates the learning.

• When you share your poster, do the Launch for the task, **explain the mathematics** of the task and **what this task contributes** to the learning progression. Anything else you think is important.
How do the big ideas about functions build and progress through secondary Math 1, Math 2, and Math 3?
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Seamless Common Core Companion
Write your response to each question on a post-it note (you will need three post-it notes- one for each question).

1. What can you do to create a classroom environment that promotes discourse?

2. How does understanding the learning cycle help support instructional decisions?

3. Name something you might be doing while the students are engaged in the explore stage of the Teaching Cycle.
Parking lot and other questions

• What questions do you still have?

• What else can we do for you?
The Power of Looking at the Entire Module

• Describe the Learning Cycles in Module 3

• What do we want students to be able to do by the end of the module?

• Keep the progressions in mind and trust the materials. There is more to come!

• Module 4 and Module 5
Writing Warm-ups and Exit Slips

How can I facilitate the progression and flow of learning?

How can I help students see the connections to what they already know?  pg. 88
Warm-ups

**GROWING DOTS 3.1**

1. Write the following expression as a multiplication problem.
   \[ 9 + 9 + 9 + 9 + 9 \]

2. Write the following expression using an exponent.
   \[ 9 \times 9 \times 9 \times 9 \times 9 \]

3. Complete the table of values for \( y = 2x + 5 \).
   Then use the table to graph the equation.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

4. Given: \( f(n) = 2n - 6 \), what do I want to know if I ask you to find the value of \( f(5) \)?
Exit Slips

EXIT SLIP

GROWING DOTS 3.1

1. Name 2 ways to recognize the type of pattern we explored in class today.

2. Identify 3 representations you can use to help you think about a changing pattern.
1.1 Something to Talk About
A Develop Understanding Task

Cell phones often indicate the strength of the phone’s signal with a series of bars. The logo below shows how this might look for various levels of service.
1.1 Something to Talk About  (Math 2)

• Student Work
• Make sense of each student’s work.
• Select 4 of the student work samples.
• Sequence them in the order that you would have them present.
• Be prepared to justify your selections and sequencing choices.
1.3 Scott’s Macho March

A Solidify Understanding Task

After looking in the mirror and feeling flabby, Scott decided that he really needs to get in shape. He joined a gym and added push-ups to his daily exercise routine. He started keeping track of the number of push-ups he completed each day in the bar graph below, with day one showing he completed three push-ups. After four days, Scott was certain he could continue this pattern of increasing the number of push-ups for at least a few months.

![Bar Graph]

1. Model the number of push-ups Scott will complete on any given day. Include both explicit and recursive equations.

2. Estimate the total number of push-ups that Scott will do in a month if he continues to increase the number of push-ups he does each day in the pattern shown above.
1.2 Scotts Macho March (Math 2)

• How does *Scott’s Macho March* build on the work done in *Something to Talk About*?
• How does the work from these tasks build on the work from Math 1?
• What are the big ideas about quadratics that are emerging in this lesson?
### 2.1 Shifty Y’s (Math 2)

<table>
<thead>
<tr>
<th>Matching Equation (A, B, C, or D)</th>
<th>Statement</th>
<th>Function Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The length of each side is increased by 5 units.</td>
<td>A) $A = 5x^2$</td>
</tr>
<tr>
<td></td>
<td>The length of each side is multiplied by 5 units.</td>
<td>B) $A = (x + 5)^2$</td>
</tr>
<tr>
<td></td>
<td>The area of a square is increased by 5 square units.</td>
<td>C) $A = (5x)^2$</td>
</tr>
<tr>
<td></td>
<td>The area of a square is multiplied by 5.</td>
<td>D) $A = x^2 + 5$</td>
</tr>
</tbody>
</table>
3. Predict how the graphs of each of the following equations will be the same or different from the graph of $y = x^2$.

<table>
<thead>
<tr>
<th></th>
<th>Similarities to the graph of $y = x^2$</th>
<th>Differences from the graph of $y = x^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = 5x^2$</td>
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</table>
2.3 Building the Perfect Square (Math 2)

2.3 BUILDING THE PERFECT SQUARE

A Solidify Understanding Task

Quadratic Quilts
Optima has a quilt shop where she sells many colorful quilt blocks for people who want to make their own quilts. She has quilt designs that are made so that they can be sized to fit any bed. She bases her designs on quilt squares that can vary in size, so she calls the length of the side for the basic square \( x \), and the area of the basic square is the function \( A(x) = x^2 \). In this way, she can customize the designs by making bigger squares or smaller squares.

1. If Optima adds 3 inches to the side of the square, what is the area of the square?

When Optima draws a pattern for the square in problem #1, it looks like this:
2.4 A SQUARE DEAL

A Solidify Understanding Task

Quadratic Quilts, Revisited

Remember Optima’s quilt shop? She bases her designs on quilt squares that can vary in size, so she calls the length of the side for the basic square $x$, and the area of the basic square is the function $A(x) = x^2$. In this way, she can customize the designs by making bigger squares or smaller squares.

1. Sometimes a customer orders more than one quilt block of a given size. For instance, when a customer orders 4 blocks of the basic size, the customer service representatives write up an order for $A(x) = 4x^2$. Model this area expression with a diagram.
2.5 Be There or Be Square (Math 2)

SECONDARY MATHEMATICS II // MODULE 2

2.5 BE THERE OR BE SQUARE

A Practice Understanding Task

Quilts and Quadratic Graphs

Optima’s niece, Jenny works in the shop, taking orders and drawing quilt diagrams. When the shop isn’t too busy, Jenny pulls out her math homework and works on it. One day, she is working on graphing parabolas and notices that the equations she is working with look a lot like an order for a quilt block. For instance, Jenny is supposed to graph the equation: \( y = (x - 3)^2 + 4 \). She thinks, “That’s funny. This would be an order where the length of the standard square is reduced by 3 and then we add a little piece of fabric that has an area of 4. We don’t usually get orders like that, but it still makes sense. I better get back to thinking about parabolas. Hmm...”

1. Fully describe the parabola that Jenny has been assigned to graph.
2.6 Factor Fixin’ (Math 2)

2.6 Factor Fixin’
A Develop Understanding Task

At first, Optima’s Quilts only made square blocks for quilters and Optima spent her time making perfect squares. Customer service representatives were trained to ask for the length of the side of the block, $x$, that was being ordered, and they would let the customer know the area of the block to be quilted using the formula $A(x) = x^2$.

Optima found that many customers that came into the store were making designs that required a combination of squares and rectangles. So, Optima’s Quilts has decided to produce several new lines of rectangular quilt blocks. Each new line is described in terms of how the rectangular block has been modified from the original square block. For example, one line of quilt blocks consists of starting with a square block and extending one side length by 5 inches and the other side length by 2 inches to form a new rectangular block. The design department knows that the area of this new block can be represented by the expression: $A(x) = (x + 5)(x + 2)$, but they do not feel that this expression gives the customer a real sense of how much bigger this new block is (e.g., how much more area it has) when compared to the original square blocks.

1. Can you find a different expression to represent the area of this new rectangular block? You will need to convince your customers that your formula is correct using a diagram.
Summing up Module 2 (Math 2)

- Different forms of a Quadratic
- Vertex
- Factored
- Standard
- How to see them as equivalent and change from one to the other.
- Connecting them all to a graph.
Math 3

- Solving Quadratic Equations
- Rational Exponents as well