

**Secondary One Mathematics:  
An Integrated Approach**  
**Module 3**  
**Linear and Exponential  
Functions**

**By**

**The Mathematics Vision Project:**

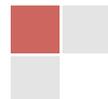
Scott Hendrickson, Joleigh Honey,  
Barbara Kuehl, Travis Lemon, Janet Sutorius  
[www.mathematicsvisionproject.org](http://www.mathematicsvisionproject.org)

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## Module 1 – Systems of Equations and Inequalities

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**Classroom Task:** Connecting the Dots: Piggies and Pools – A Develop Understanding Task  
*Introducing continuous linear and exponential functions (F.IF.3)*

**Ready, Set, Go Homework:** Linear and Exponential Functions 1

**Classroom Task:** Sorting Out the Change – A Solidify Understanding Task  
*Defining linear and exponential functions based upon the pattern of change (F.LE.1, F.LE.2)*

**Ready, Set, Go Homework:** Linear and Exponential Functions 2

**Classroom Task:** Where’s My Change – A Solidify Understanding Task  
*Identifying rates of change in linear and exponential functions (F.LE.1, F.LE.2)*

**Ready, Set, Go Homework:** Linear and Exponential Functions 3

**Classroom Task:** Linear, Exponential or Neither – A Practice Understanding Task  
*Distinguishing between linear and exponential functions using various representations (F.LE.3, F.LE.5)*

**Ready, Set, Go Homework:** Linear and Exponential Functions 4

**Classroom Task:** Getting Down to Business – A Solidify Understanding Task  
*Comparing the growth of linear and exponential functions (F.LE.2, F.LE.3, F.LE.5, F.IF.7)*

**Ready, Set, Go Homework:** Linear and Exponential Functions 5

**Classroom Task:** Growing, Growing, Gone – A Solidify Understanding Task  
*Comparing linear and exponential models of population (F.BF.1, F.BF.2, F.LE.1, F.LE.2, F.LE.3)*

**Ready, Set, Go Homework:** Linear and Exponential Functions 6

**Classroom Task:** Making My Point – A Solidify Understanding Task  
*Interpreting equations that model linear and exponential functions (A.SSE.1, A.CED.2, F.LE.5)*

**Ready, Set, Go Homework:** Linear and Exponential Functions 7

**Classroom Task:** Efficiency Experts – A Solidify Understanding Task  
*Evaluating the use of various forms of linear and exponential equations (A.SSE.1, A.SSE.3, A.CED.2, F.LE.5)*

**Ready, Set, Go Homework:** Linear and Exponential Functions 8

**Classroom Task:** Up a Little, Down a Little – A Solidify Understanding Task  
*Understanding and interpreting formulas for exponential growth and decay (A.SSE.1, A.CED.2, F.LE.5, F.IF.7)*

**Ready, Set, Go Homework:** Linear and Exponential Functions 9

**Classroom Task:** X Marks the Spot – A Practice Understanding Task  
*Solving exponential and linear equations (A.REI.3)*

**Ready, Set, Go Homework:** Linear and Exponential Functions 10



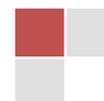
# Connecting the Dots: Piggies and Pools

## *A Develop Understanding Task*



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1. My little sister, Savannah, is three years old and has a piggy bank that she wants to fill. She started with five pennies and each day when I come home from school, she is excited when I give her three pennies that are left over from my lunch money. How much money will Savannah have after 10 days? How many days will it take for her to have at least \$1.50? Justify your answer with a mathematical model of the problem situation.
2. Our family has a small pool for relaxing in the summer that holds 1500 gallons of water. I decided to fill the pool for the summer. When I had 5 gallons of water in the pool, I decided that I didn't want to stand outside and watch the pool fill, so I wanted to figure out how long it would take so that I could leave, but come back to turn off the water at the right time. I checked the flow on the hose and found that it was filling the pool at a rate of 2 gallons every 5 minutes. How many gallons of water will be in the pool after 50 minutes? How many minutes will it take to fill the pool? Justify your answer with a mathematical model of the problem situation.
3. I'm more sophisticated than my little sister. I save my money in a bank account that pays me 3% interest on the money in the account at the end of each month. (If I take my money out before the end of the month, I don't earn any interest for the month.) I started the account with \$50 that I got for my birthday. How much money will I have in the account at the end of 10 months? How many months will it take to have at least \$100? Justify your answer with a mathematical model of the problem situation.
4. At the end of the summer, I decide to drain the swimming pool. I noticed that it drains faster when there is more water in the pool. That was interesting to me, so I decided to measure the rate at which it drains. I found that it was draining at a rate of 3% every 5 minutes. How many gallons are left in the pool after 50 minutes? About how many minutes will it take to have less than 1000 gallons in the pool? Justify your answer with a mathematical model of the problem situation.
5. Compare problems 1 and 3. What similarities do you see? What differences do you notice?
6. Compare problems 1 and 2. What similarities do you see? What differences do you notice?
7. Compare problems 3 and 4. What similarities do you see? What differences do you notice?



# Linear and Exponential Functions | 1

## Ready, Set, Go!



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## Ready

Topic: Recognizing arithmetic and geometric sequences

Predict the next 2 terms in the sequence. State whether the sequence is arithmetic, geometric, or neither. Justify your answer.

1. 4, -20, 100, -500, ...
2. 3, 5, 8, 12, ...
3. 64, 48, 36, 27, ...
4. 1.5, 0.75, 0, -0.75, ...
5.  $40, 10, \frac{5}{2}, \frac{5}{8}, \dots$
6. 1, 11, 1111, 11111, ...
7. -3.6, -5.4, -8.1, -12.15, ...
8. -64, -47, -30, -13, ...
9. Create a predictable sequence of at least 4 numbers that is NOT arithmetic or geometric.

## Set

Topic: Discrete and continuous relationships

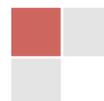
Identify whether the following statements represent a discrete or a continuous relationship.

10. The hair on your head grows  $\frac{1}{2}$  inch per month.
11. For every ton of paper that is recycled, 17 trees are saved.
12. Approximately 3.24 billion gallons of water flow over Niagara Falls daily.

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# Linear and Exponential Functions | 1

13. The average person laughs 15 times per day.
14. The city of Buenos Aires adds 6,000 tons of trash to its landfills everyday.
15. During the Great Depression, stock market prices fell 75%.

## Go

Topic: Slopes of lines

Determine the slope of the line that passes through the following points.

16.  $(-15, 9), (-10, 4)$       17.  $(0.5, 4), (3, 3.5)$       18.  $(50, 85), (60, 80)$

19.

x	y
-5	-20
-4	-17
-3	-14

20.

x	y
-1	-1
0	$\frac{1}{2}$
1	2

21.

x	y
-1	-1
0	$\frac{1}{2}$
1	2

Need Help? Check out these related videos and internet sites:

Discrete vs. continuous: <http://www.mathsisfun.com/data/data-discrete-continuous.html>

Arithmetic and geometric sequences: <http://home.windstream.net/okrebs/page131.html>

Slope: <http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/slope-of-a-line>

Linear relationships:

[http://www.mathsteacher.com.au/year7/ch15\\_linear/04\\_modelling/linear.htm](http://www.mathsteacher.com.au/year7/ch15_linear/04_modelling/linear.htm)



# Sorting Out the Change

## *A Solidify Understanding Task*

- A. Identify the pattern of change in each of the relations and sort each relation into the following categories:
- Equal differences over equal intervals
  - Equal factors over equal intervals
  - Neither
- B. Be prepared to describe the pattern of change and to tell how you found it.



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1.

$x$	$f(x)$
-30	-57
-25	-47
-20	-37
-15	-27
-10	-17
-5	-7
0	3

Type of pattern of change \_\_\_\_\_

How I found the pattern of change:

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2.

$$f(0) = -3, f(n+1) = \frac{5}{3}f(n)$$

Type of pattern of change \_\_\_\_\_

How I found the pattern of change:

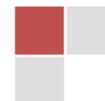
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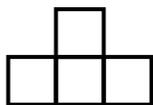
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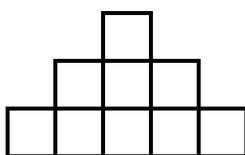
Step 1



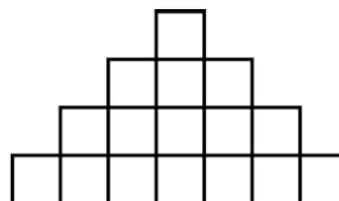
Step 2



Step 3



Step 4



3. The pattern of change in the perimeter of the figures from one step to the next.

Type of pattern of change \_\_\_\_\_

How I found the pattern of change:

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4. The pattern of change in the area of the figures from one step to the next.

Type of pattern of change \_\_\_\_\_

How I found the pattern of change:

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5.  $y = ax - 3$

Type of pattern of change \_\_\_\_\_

How I found the pattern of change:

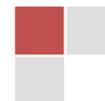
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6.

$x$	$f(x)$
-10	7
-5	7
-0	7
5	7
10	7
15	7
20	7

Type of pattern of change \_\_\_\_\_

How I found the pattern of change:

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7. The height from the ground of a person on a ferris wheel that is rotating at a constant speed.

Type of pattern of change \_\_\_\_\_

How I found the pattern of change:

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8.  $y = x$

Type of pattern of change \_\_\_\_\_

How I found the pattern of change:

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9.

$x$	$f(x)$
0	-5
4	-2
-4	-8
8	1
-8	-11

Type of pattern of change \_\_\_\_\_

How I found the pattern of change:

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10. The algae population in a pond increases by 3% each year until it depletes its food supply and then maintains a constant population.

Type of pattern of change \_\_\_\_\_

How I found the pattern of change:

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11.

$x$	$f(x)$
-5	238
-4	76
-3	22
-2	4
-1	-2
0	-4

Type of pattern of change \_\_\_\_\_

How I found the pattern of change:

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12. The change in the height of the ball from one bounce to the next if the ball is dropped from a height of 8 feet and the ball bounces to 80% of its previous height with each bounce.

Type of pattern of change \_\_\_\_\_

How I found the pattern of change:

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13.

$x$	$y$
3	-5
3	-10
3	-20
3	15
3	35
3	50

Type of pattern of change \_\_\_\_\_

How I found the pattern of change:

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# Linear and Exponential Functions | 2

## Ready, Set, Go!



## Ready

Topic: Rates of change in linear models

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### Which situation has the greatest rate of change?

1. The amount of stretch in a short bungee cord or the amount of stretch in a slinky when each is pulled by a 3 pound weight.
2. A sunflower that grows 2 inches every day or an amaryllis that grows 18 inches in one week.
3. Pumping 25 gallons of gas into a truck in 3 minutes or filling a bathtub with 40 gallons of water in 5 minutes.
4. Riding a bike 10 miles in 1 hour or jogging 3 miles in 24 minutes.

## Set

Topic: linear rates of change

Determine the rate of change in each table below.

5.

x	y
-3	-13
-1	-5
0	-1
3	11

6.

x	y
-4	-4
0	4
2	8
6	16

7.

x	y
-10	-14
5	-8
25	0
50	10



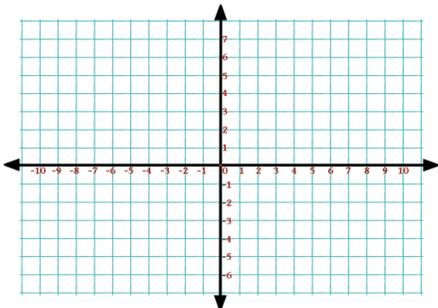
# Linear and Exponential Functions | 2

## Go

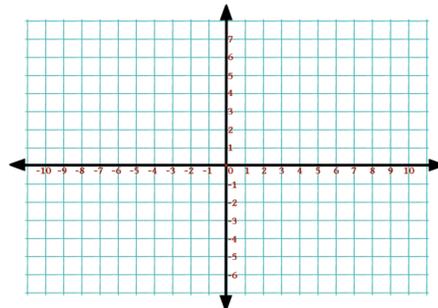
Topic: Graphing linear equations in slope-intercept form.

Graph the following equations:

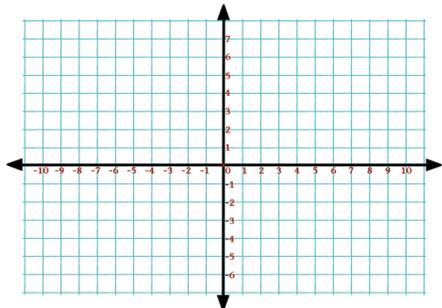
8.  $y = 3x - 1$



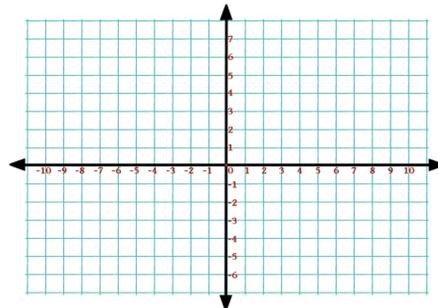
9.  $y = -5x + 4$



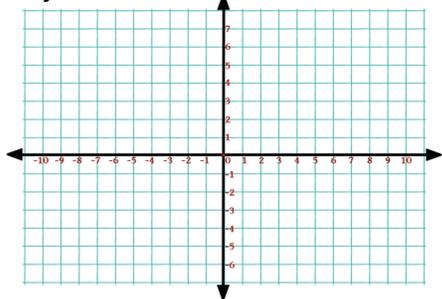
10.  $y = x$



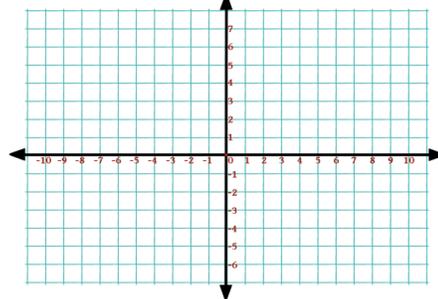
11.  $y = -4$



12.  $y = \frac{1}{2}x - 6$



13.  $x = 3$



Need Help? Check out these related videos:

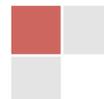
<http://www.algebra-class.com/rate-of-change.html>

<http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphs-using-slope-intercept-form>

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# Where's My Change?

## *A Solidify Understanding Task*

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Look through the problems that you worked with in the “Sorting Out the Change” task.

Choose two problems from your linear category (equal differences over equal intervals) and two problems from your exponential category (equal factors over equal intervals).

Add as many representations as you can to the problem you selected. For instance, if you choose problem #1 which is a table, you should try to represent the function with a graph, an explicit equation, a recursive equation, and a story context.

Identify the rate of change in the function. If the function is linear, identify the constant rate of change. If the function is exponential, identify the factor of change.

How does the rate of change appear in each of your representations?



# Linear and Exponential Functions | 3

## Ready, Set, Go!

### Ready

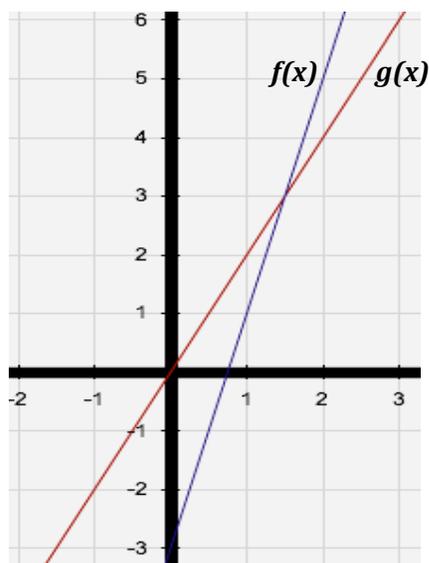
Topic: Recognizing the greater rate of change when comparing 2 linear functions or 2 exponential functions.



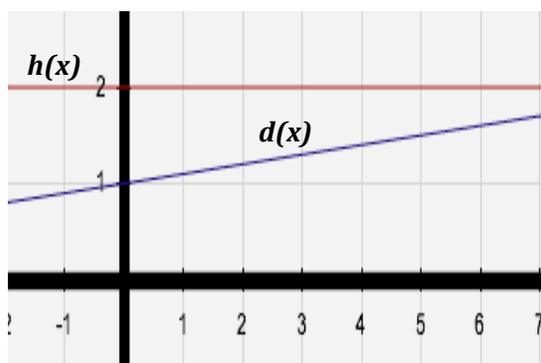
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### Which graph is growing faster?

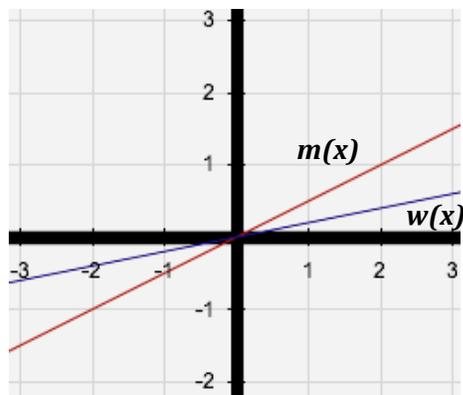
1.



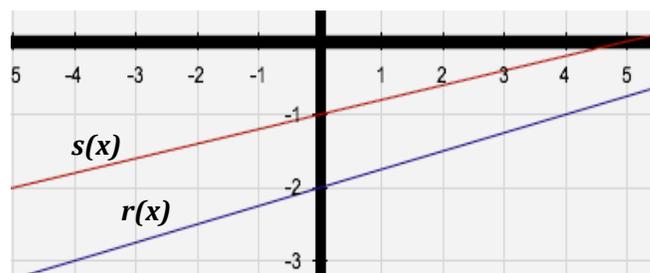
2.



3.



4.



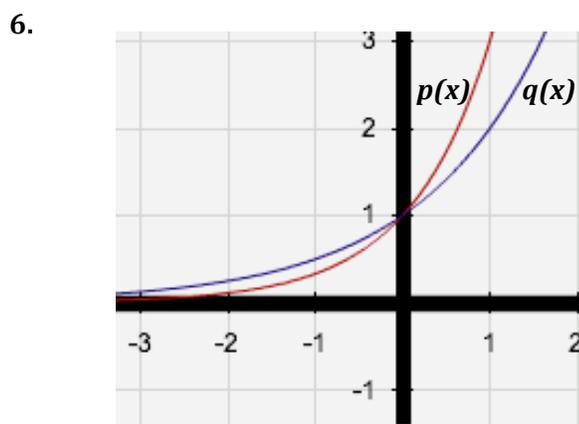
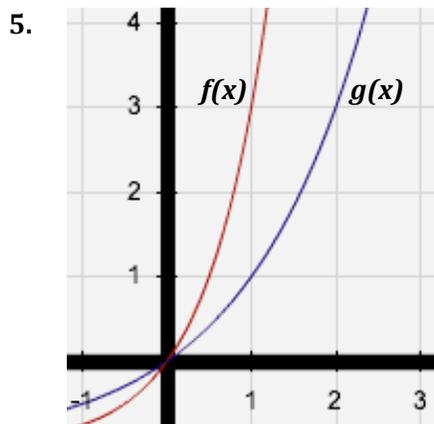
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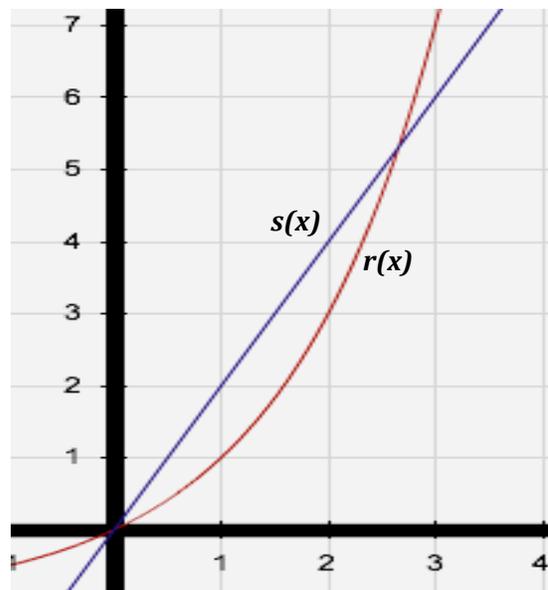


# Linear and Exponential Functions | 3



7. Examine the graph at the right from 0 to 1.  
Which graph do you think is growing faster?

Now look at the graph from 2 to 3.  
Which graph is growing faster in this interval?



## Set

Topic: Representations of linear and exponential functions.

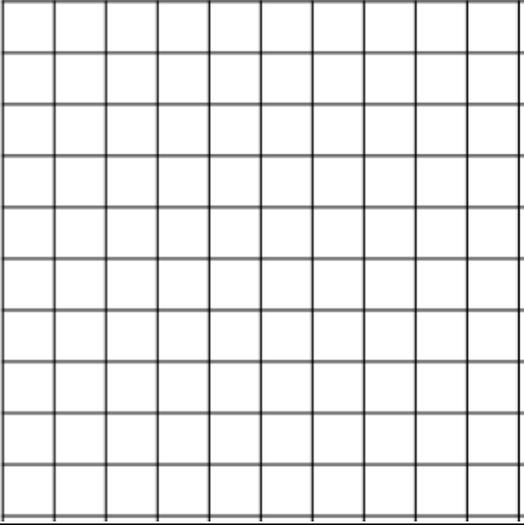


# Linear and Exponential Functions

## 3

**Directions:** In each of the following problems, you are given one of the representations of a function. Complete the remaining 3 representations. Identify the rate of change for the relation.

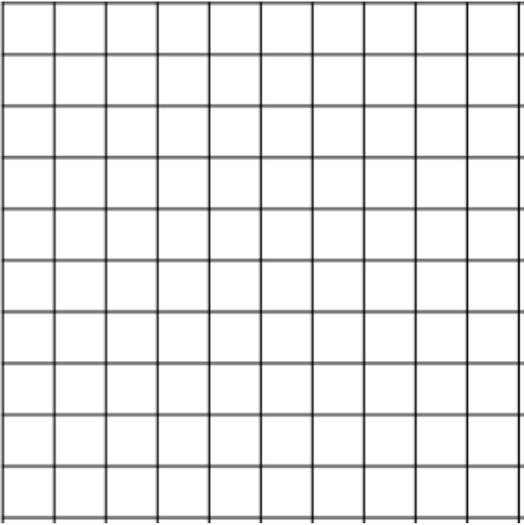
8.

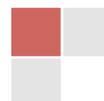
<p><b><u>Context</u></b></p> <p>You and your friends go to the state fair. It costs \$5 to get into the fair and \$3 each time you go on a ride.</p>	<p><b><u>Table</u></b></p> <table border="1" style="width: 100%; height: 100%; border-collapse: collapse;"> <tr><td style="width: 50%; height: 30px;"></td><td style="width: 50%; height: 30px;"></td></tr> <tr><td style="height: 30px;"></td><td style="height: 30px;"></td></tr> </table>																
<p><b><u>Graph</u></b></p> 	<p><b><u>Equation</u></b></p>																



# Linear and Exponential Functions | 3

9.

<p><b><u>Context</u></b></p>	<p><b><u>Table</u></b></p> <p>Be sure to include your units</p> <table border="1"> <thead> <tr> <th>Time</th> <th>Amount</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>18</td> </tr> <tr> <td>2</td> <td>54</td> </tr> <tr> <td>3</td> <td>162</td> </tr> <tr> <td>4</td> <td>486</td> </tr> <tr> <td>5</td> <td>1458</td> </tr> <tr> <td>6</td> <td>4374</td> </tr> </tbody> </table>	Time	Amount	1	18	2	54	3	162	4	486	5	1458	6	4374
Time	Amount														
1	18														
2	54														
3	162														
4	486														
5	1458														
6	4374														
<p><b><u>Graph</u></b></p> 	<p><b><u>Equation</u></b></p>														



# Linear and Exponential Functions

## 3

### Go

Topic: Recursive and explicit equations of geometric sequences.

**Write the recursive and explicit equations for each geometric sequence.**

10. Marissa has saved \$1000 in a jar. She plans to withdraw half of what's remaining in the jar at the end of each month.

11.

Time (days)	Number of bacteria
1	10
2	100
3	1000
4	10000

12.

Folds in paper	Number of rectangles
0	1
1	2
2	4
3	8

13. 1024, 256, 64, 16, ...

14. 3, 9, 27, 81, ...

Need Help? Check out these related videos:

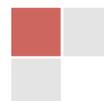
<http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/identifying-exponential-models>

<http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/linear--quadratic--and-exponential-models>

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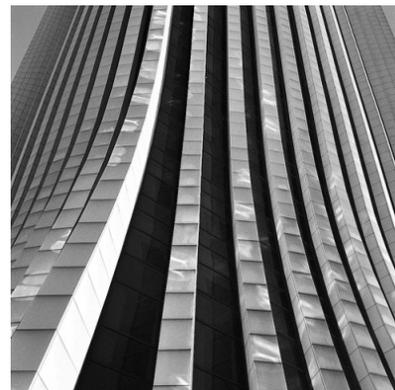
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# Linear, Exponential or Neither?

## A Solidify Understanding Task

For each representation of a function, decide if the function is linear, exponential, or neither. **Give at least 2 reasons for your answer.**



<p>1.</p>	<p>2.</p> <p style="text-align: center;">Tennis Tournament</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Rounds</th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <td>Number of Players left</td> <td>64</td> <td>32</td> <td>16</td> <td>8</td> <td>4</td> </tr> </tbody> </table> <p style="text-align: center;">There are 4 players remaining after 5 rounds.</p>	Rounds	1	2	3	4	5	Number of Players left	64	32	16	8	4
Rounds	1	2	3	4	5								
Number of Players left	64	32	16	8	4								
<p>3.</p> <p style="text-align: center;"><math>y = 4x</math></p>	<p>4.</p> <p>This function is decreasing at a constant rate</p>												
<p>5.</p>	<p>6.</p> <p>A person's height as a function of a person's age (from age 0 to 100)</p>												



7.

$$-3x = 4y + 7$$

8.

$x$	$y$
-2	23
0	5
2	-13
4	-31
6	-49

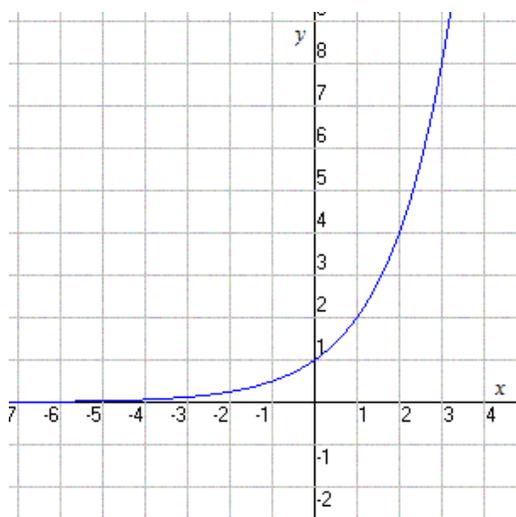
9.

Height in Inches	Shoe Size
62	6
74	13
70	9
67	11
53	4
58	7

10.

The number of cell phone users in Centerville as a function of years, if the number of users is increasing by 75% each year.

11.

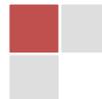


12.

The time it takes you to get to work as a function the speed at which you drive

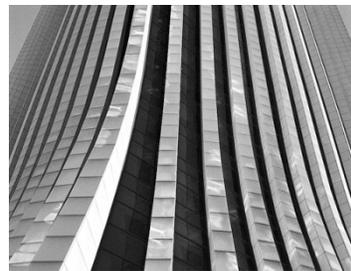


13. $y = 7x^2$	14. Each point on the graph is exactly $\frac{1}{3}$ of the previous point.
15. $f(1) = 7, f(2) = 7, f(n) = f(n-1) + f(n-2)$	16. $f(0) = 1, f(n+1) = \frac{2}{3}f(n)$



# Linear and Exponential Functions | 4

Ready, Set, Go!

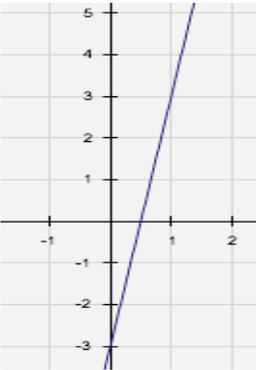
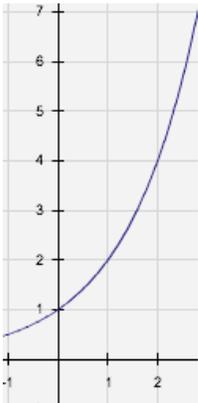
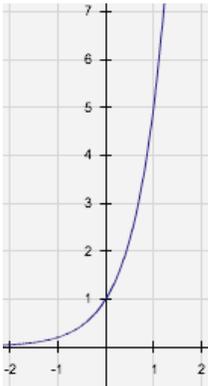


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Ready

Topic: Comparing rates of change in both linear and exponential situations.

Identify whether situation “a” or situation “b” has the greater rate of change.

1.	<p>a.</p> <table border="1" data-bbox="354 730 610 1066"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>-10</td> <td>-48</td> </tr> <tr> <td>-9</td> <td>-43</td> </tr> <tr> <td>-8</td> <td>-38</td> </tr> <tr> <td>-7</td> <td>-33</td> </tr> </tbody> </table>	x	y	-10	-48	-9	-43	-8	-38	-7	-33	<p>b.</p> 
x	y											
-10	-48											
-9	-43											
-8	-38											
-7	-33											
2.	<p>a.</p> 	<p>b.</p> 										

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# Linear and Exponential Functions

## 4

3.	<p>a. Lee has \$25 withheld each week from his salary to pay for his subway pass.</p>	<p>b. Jose owes his brother \$50. He has promised to pay half of what he owes each week until the debt is paid.</p>										
4.	<p>a.</p> <table border="1" style="margin: 10px auto; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">6</td> <td style="padding: 5px;">10</td> <td style="padding: 5px;">14</td> <td style="padding: 5px;">18</td> </tr> <tr> <td style="padding: 5px;">y</td> <td style="padding: 5px;">13</td> <td style="padding: 5px;">15</td> <td style="padding: 5px;">17</td> <td style="padding: 5px;">19</td> </tr> </table>	x	6	10	14	18	y	13	15	17	19	<p>b. The number of rhombi in each shape.</p> <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 20px;"> <div style="text-align: center;">  <p>Figure 1</p> </div> <div style="text-align: center;">  <p>Figure 2</p> </div> <div style="text-align: center;">  <p>Figure 3</p> </div> </div>
x	6	10	14	18								
y	13	15	17	19								
5.	<p>a.</p> $y = 2(5)^x$	<p>b. In the children's book, <i>The Magic Pot</i>, every time you put one object into the pot, two of the same object come out. Imagine that you have 5 magic pots.</p>										

### Set

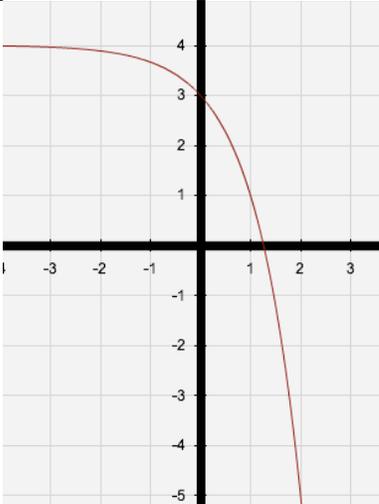
Topic: Recognizing linear and exponential functions.

For each representation of a function, decide if the function is linear, exponential, or neither.

<p>6. The population of a town is decreasing at a rate of 1.5% per year.</p>	<p>7.</p> <table border="1" style="margin: 10px auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="padding: 5px;">Side of a square</th> <th style="padding: 5px;">Area of a square</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">1 inch</td> <td style="padding: 5px;">1 in<sup>2</sup></td> </tr> <tr> <td style="padding: 5px;">2 inches</td> <td style="padding: 5px;">4 in<sup>2</sup></td> </tr> <tr> <td style="padding: 5px;">3 inches</td> <td style="padding: 5px;">9 in<sup>2</sup></td> </tr> <tr> <td style="padding: 5px;">4 inches</td> <td style="padding: 5px;">16 in<sup>2</sup></td> </tr> </tbody> </table>	Side of a square	Area of a square	1 inch	1 in <sup>2</sup>	2 inches	4 in <sup>2</sup>	3 inches	9 in <sup>2</sup>	4 inches	16 in <sup>2</sup>
Side of a square	Area of a square										
1 inch	1 in <sup>2</sup>										
2 inches	4 in <sup>2</sup>										
3 inches	9 in <sup>2</sup>										
4 inches	16 in <sup>2</sup>										
<p>8. <math>3x + 4y = -3</math></p>	<p>9. Joan earns a salary of \$30,000 per year plus a 4.25% commission on sales.</p>										



# Linear and Exponential Functions | 4

<p>10.</p> 	<p>11.</p> <p>“On the 4<sup>th</sup> day of Christmas my true love gave to me, 4 calling birds, 3 French hens, 2 turtledoves, and a partridge in a pear tree.”</p> <p>The number of gifts received each day of “The 12 Days of Christmas” as a function of the day.</p>
----------------------------------------------------------------------------------------------	-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

## Go

Each of the tables below represents a geometric sequence. Find the missing terms in the sequence.

12.

x	1	2	3	4	5
y	2				162

13.

x	1	2	3	4	5
y	1/9			-3	

14.

x	1	2	3	4	5
y	10				0.625



## Linear and Exponential Functions | 4

15.

x	1	2	3	4	5
y	g				$gz^4$

16.

x	1	2	3	4	5
y	-3				-243

Need Help? Check out these related videos and internet sites:

sequences: <http://www.youtube.com/watch?v=THV2Wsf8hro>



# Getting Down to Business

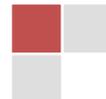
## *A Solidify Understanding Task*

Calcu-rama had a net income of 5 million dollars in 2010, while a small competing company, Computafest, had a net income of 2 million dollars. The management of Calcu-rama develops a business plan for future growth that projects an increase in net income of 0.5 million per year, while the management of Computafest develops a plan aimed at increasing its net income by 15% each year.



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- a. Express the projected net incomes in these two business plans as recursion formulas.
  
- b. Write an explicit equation for the net income as a function of time for each company's business plan.
  
- c. Compare your answers in a and b. How are the two representations similar? How do they differ? What relationships are highlighted in each representation?
  
- d. Explain why if both companies are able to meet their net income growth goals, the net income of Computafest will eventually be larger than that of Calcu-rama. In what year will the net income of Computafest be larger than that of Calcu-rama?



# Linear and Exponential Functions | 5

## Ready, Set, Go!



Topic: Comparing arithmetic and geometric sequences.

## Ready

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The first and 5<sup>th</sup> terms of a sequence are given. Fill in the missing numbers for an arithmetic sequence. Then fill in the numbers for a geometric sequence.

1.

<b>arithmetic</b>	4				324
<b>geometric</b>	4				324

2.

<b>arithmetic</b>	3				48
<b>geometric</b>	3				48

3.

<b>arithmetic</b>	-6250				-10
<b>geometric</b>	-6250				-10

4.

<b>arithmetic</b>	-12				-0.75
<b>geometric</b>	-12				-0.75

5.

<b>arithmetic</b>	-1377				-17
<b>geometric</b>	-1377				-17



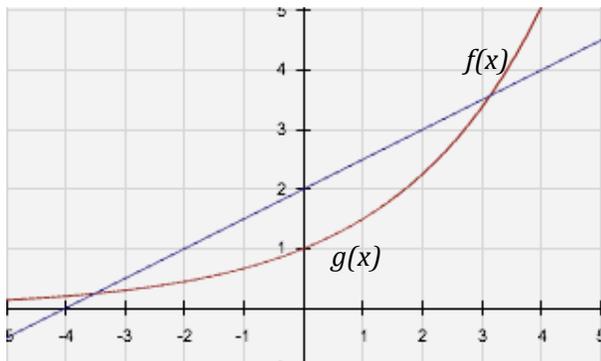
# Linear and Exponential Functions | 5

## Set

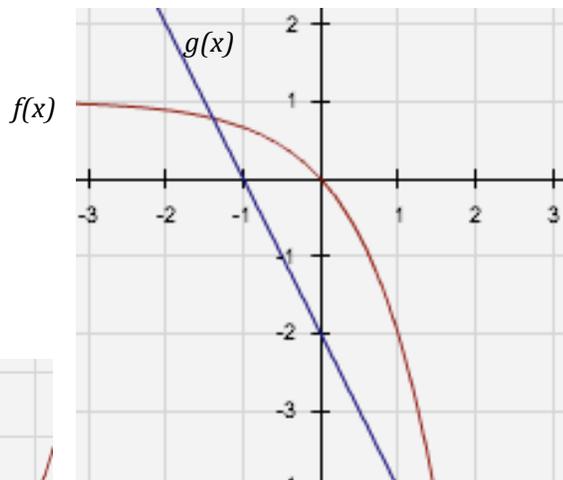
Topic: comparing the rates of change of linear and exponential functions.

Compare the rates of change of each pair of functions by identifying the interval where it appears that  $f(x)$  is changing faster and the interval where it appears that  $g(x)$  is changing faster. Verify your conclusions by making a table of values for each equation and exploring the rates of change in your tables.

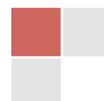
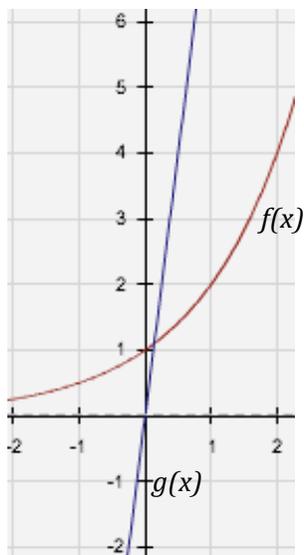
6.  $f(x) = (1.5)^x$   
 $g(x) = \frac{1}{2}x + 2$



7.  $f(x) = -3^x + 1$   
 $g(x) = -2x - 2$



8.  $f(x) = 2^x$   
 $g(x) = 8x$



# Linear and Exponential Functions | 5

Topic: Writing explicit equations for linear and exponential models.

## Go

Write the explicit equation for the tables and graphs below.

9.

$x$	$f(x)$
2	-4
3	-11
4	-18
5	-25

10.

$x$	$f(x)$
-1	$\frac{2}{5}$
0	2
1	10
2	50

11.

$x$	$f(x)$
2	-24
3	-48
4	-96
5	-192

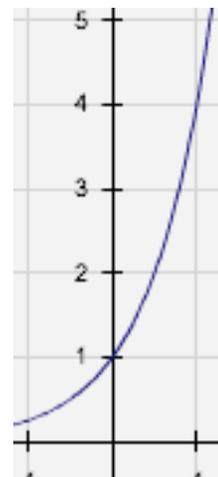
12.

$x$	$f(x)$
-4	81
-3	27
-2	9
-1	3

13.

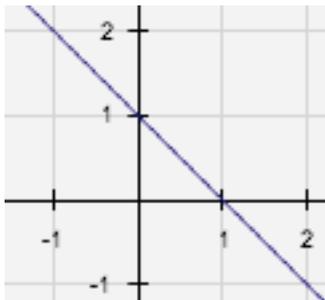


14.

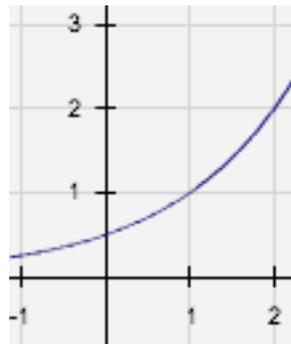


# Linear and Exponential Functions | 5

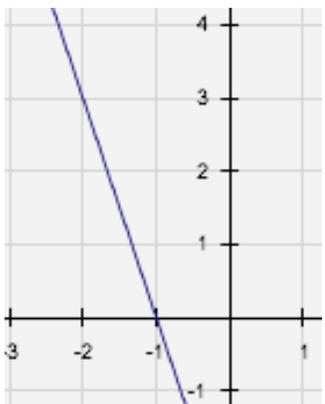
15.



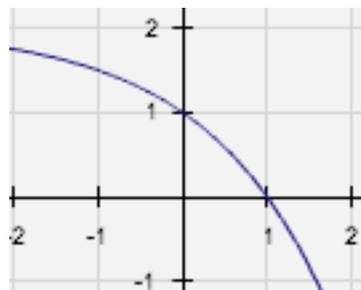
16.



17.



18.



Need Help? Check out these related videos:

<http://www.khanacademy.org/math/algebra/solving-linear-equations/v/equations-of-sequence-patterns>

<http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/geometric-sequences--introduction>

<http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/exponential-growth-functions>

<http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/exponential-decay-functions?v=AXAMVxaxjDg>

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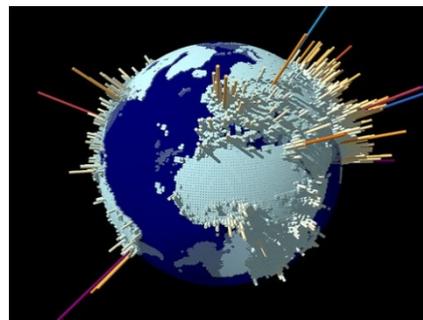
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# Growing, Growing, Gone

## *A Solidify Understanding Task*

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1. The U.S. population in 1910 was 92 million people. In 1990 the population was 250 million. Create both a linear and an exponential model of the population from 1910 to 2030, with projected data points at least every 20 years, starting in 1910.

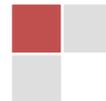
2. The actual U.S. population data (in millions) was:

1930: 122.8

1950: 152.3

1970: 204.9

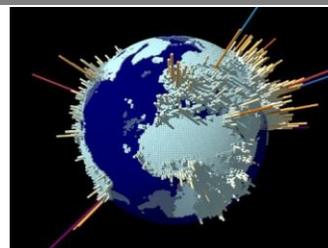
Which model provides a better forecast of the U.S. population for the year 2030? Explain your answer.



# Linear and Exponential Functions | 6

Ready, Set, Go!

Ready



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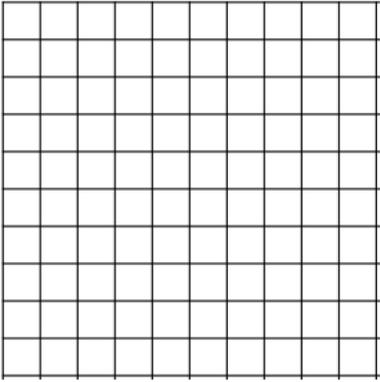
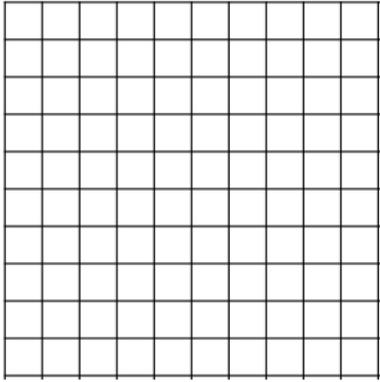
Topic: Comparing Linear and Exponential Models

Describe the defining characteristics of each type of function by filling in the cells of each table as completely as possible.

	<b>linear model</b>		<b>exponential model</b>	
	$y = 4 + 3x$		$y = 4(3^x)$	
1. Describe in words the rule for each type of growth.	linear growth		exponential growth	
2. Identify which kind of sequence corresponds to each model. Explain any differences.				
3. Make a table of values and discuss how you determine the rate of change.	x	y	x	y



# Linear and Exponential Functions | 6

<p>4.</p> <p>Graph each equation. Compare the graphs.</p> <p>What is the same?</p> <p>What is different?</p>		
<p>5. Find the y-intercept for each function.</p>		
<p>6. Find the y-intercepts for the functions on the right.</p>	$y = 3x$	$y = 3^x$
<p>7. Discuss everything you notice about the y-intercept for the models in #1 and the models in #6.</p>		

## Set

Make a table and graph that represent the model. Predict stage 10. Then write the explicit equation.

7.

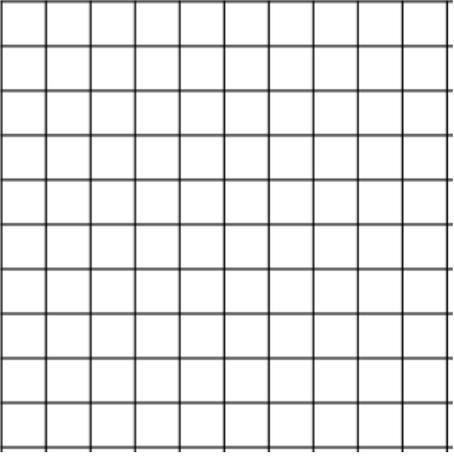
Count the triangles.



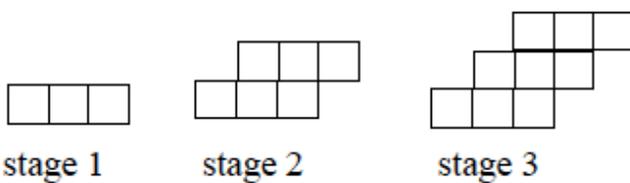
Predict the number at stage 10.	Write the explicit equation.
Graph.	Table

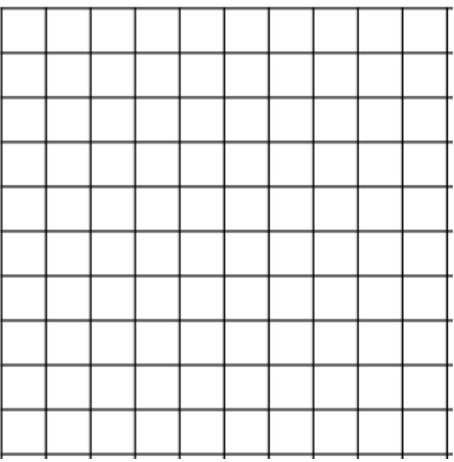


# Linear and Exponential Functions | 6

	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 50%;">x</th> <th style="width: 50%;">y</th> </tr> </thead> <tbody> <tr><td> </td><td> </td></tr> </tbody> </table>	x	y																
x	y																		

8. Count the squares in the stair steps.



<p>Graph.</p> <div style="text-align: center; margin-top: 20px;">  </div>	<p>Table</p> <table border="1" style="width: 100%; border-collapse: collapse; margin-top: 20px;"> <thead> <tr> <th style="width: 50%;">x</th> <th style="width: 50%;">y</th> </tr> </thead> <tbody> <tr><td> </td><td> </td></tr> </tbody> </table>	x	y										
x	y												
<p>Predict the number of squares in stage 10</p>	<p>Write the explicit equation.</p>												



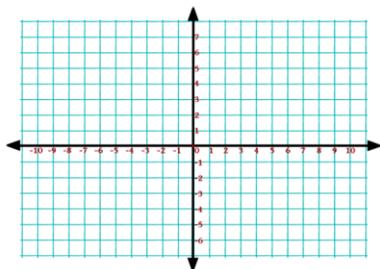
# Linear and Exponential Functions | 6

## Go

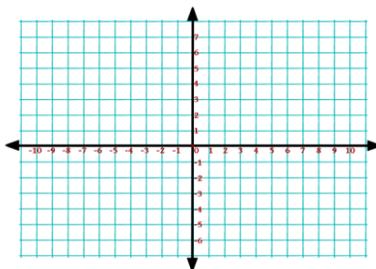
Topic: Solving systems through graphing.

Find the solution of the systems of equations by graphing.

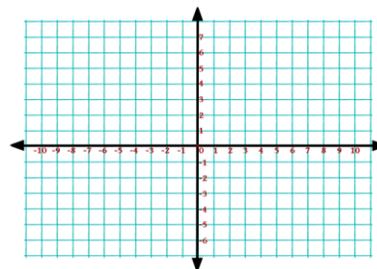
9. 
$$\begin{cases} y = -x \\ y = 3x - 4 \end{cases}$$



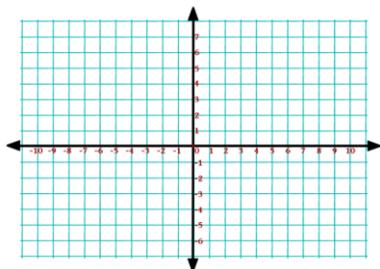
10. 
$$\begin{cases} 2x + y = -6 \\ y = x \end{cases}$$



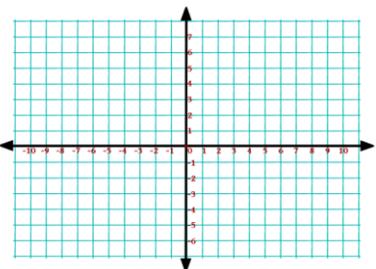
11. 
$$\begin{cases} y = 2x - 2 \\ x + 3y = 15 \end{cases}$$



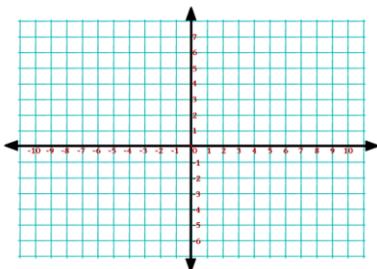
12. 
$$\begin{cases} y + 3 = 6x - 2 \\ y - 2x + 1 = 4(x - 1) \end{cases}$$



13. 
$$\begin{cases} y = -(x - 4) \\ y - 2x - 1 = 0 \end{cases}$$



14. 
$$\begin{cases} y = 3(x - 2) \\ y + x - 2 = 4(x - 1) \end{cases}$$



Need Help? Check out these related videos:

Comparing Linear and exponential functions:

<http://www.khanacademy.org/math/algebra/algebra-functions/v/recognizing-linear-functions>

<http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/identifying-exponential-models>

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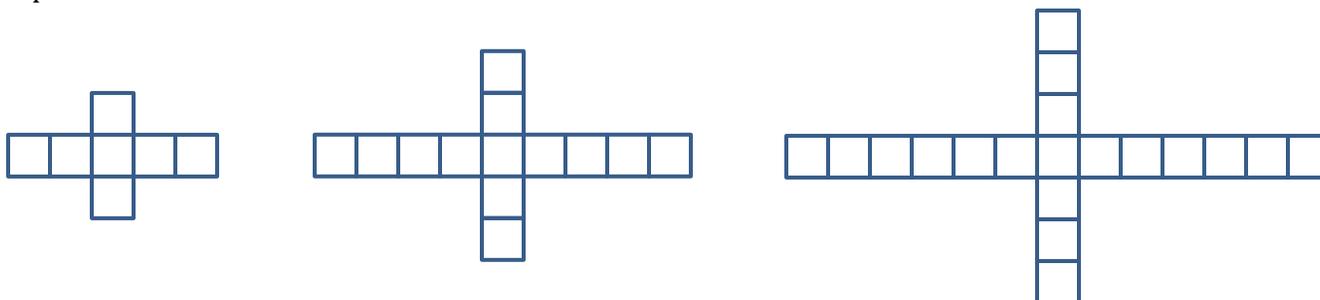




## Making My Point

### A Solidify Understanding Task

Zac and Sione were working on predicting the number of quilt blocks in this pattern:



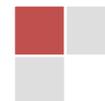
When they compared their results, they had an interesting discussion:

**Zac:** I got  $y = 6n + 1$  because I noticed that 6 blocks were added each time so the pattern must have started with 1 block at  $n = 0$ .

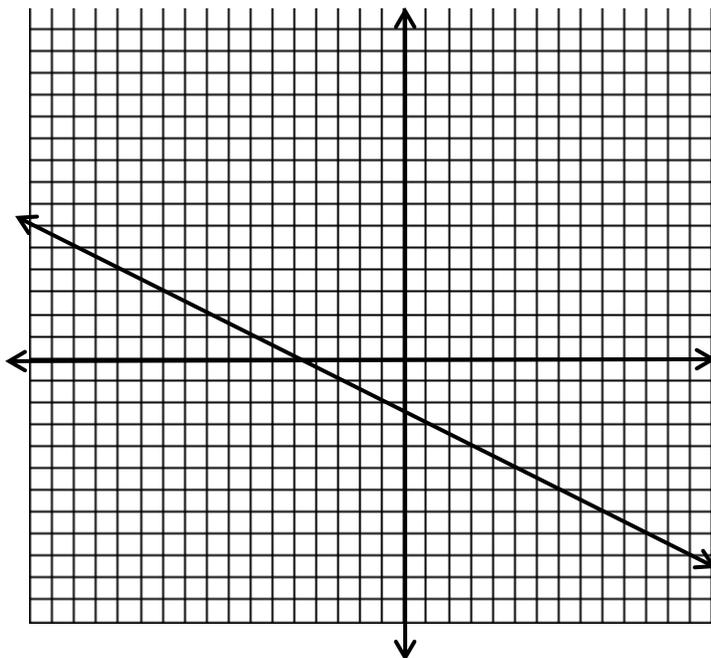
**Sione:** I got  $y = 6(n - 1) + 7$  because I noticed that at  $n = 0$  there were 7 blocks and at  $n = 1$  there were 13, so I used my table to see that I could get the number of blocks by taking one less than the  $n$ , multiplying by 6 (because there are 6 new blocks in each figure) and then adding 7 because that's how many blocks in the first figure. Here's my table:

0	1	2	$n$
7	13	19	$6(n-1) + 7$

1. What do you think about the strategies that Zac and Sione used? Are either of them correct? Why or why not? Use as many representations as you can to support your answer.



The next problem Zac and Sione worked on was to write the equation of the line shown on the graph below.



When they were finished, here is the conversation they had about how they got their equations:

**Sione:** It was hard for me to tell where the graph crossed the y axis, so I found two points that I could read easily,  $(-9, 2)$  and  $(-15, 5)$ . I figured out that the slope was  $-\frac{1}{2}$  and made a table and checked it against the graph. Here's my table:

$x$	-15	-13	-11	-9	$n$
$f(x)$	5	4	3	2	$-\frac{1}{2}(n + 9) + 2$

I was surprised to notice that the pattern was to start with the  $n$ , add 9, multiply by the slope and then add 2.

I got the equation:  $f(x) = -\frac{1}{2}(x + 9) + 2$ .



**Zac:** Hey—I think I did something similar, but I used the points, (7,-6) and (9,-7).

I ended up with the equation:  $f(x) = -\frac{1}{2}(x - 9) - 7$ . One of us must be wrong because yours says that you add 9 to the  $n$  and mine says that you subtract 9. How can we both be right?

2. What do you say? Can they both be right?

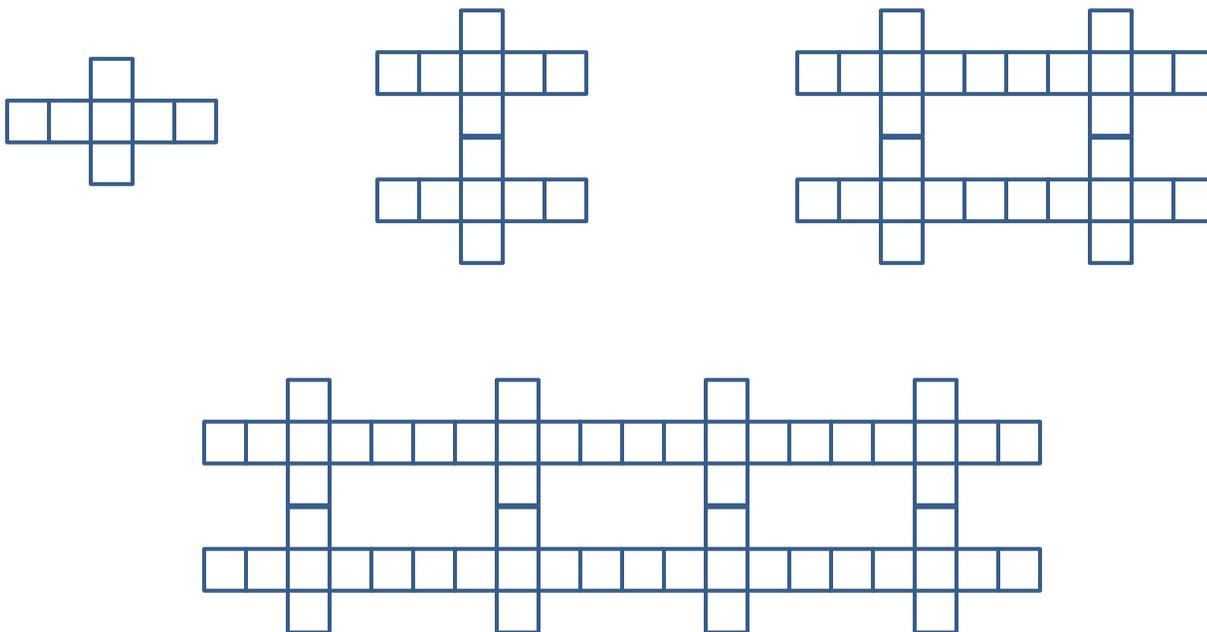
**Zac:** My equation made me wonder if there was something special about the point (9,- 7) since it seemed to appear in my equation  $f(x) = -\frac{1}{2}(x - 9) - 7$  when I looked at the number pattern. Now I'm noticing something interesting—the same thing seems to happen with your equation,  $f(x) = -\frac{1}{2}(x + 9) + 2$  and the point (-9, 2)

3. Describe the pattern that Zac is noticing.
4. Find another point on the line given above and write the equation that would come from Zac's pattern.
5. What would the pattern look like with the point  $(a, b)$  if you knew that the slope of the line was  $m$ ?



6. Could you use this pattern to write the equation of any linear function? Why or why not?

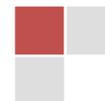
Zac and Sione went back to work on an extension of the quilt problem they were working on before. Now they have this pattern:



**Zac:** This one works a lot like the last quilt pattern to me. The only difference is that the pattern is doubling, so I knew it was exponential. I thought that it starts with 7 blocks and doubles, so the equation must be  $f(x) = 7(2)^x$ .

**Sione:** I don't know about that. I agree that it is an exponential function—just look at that growth pattern. But, I made this table:

$x$	1	2	3	$n$
$f(x)$	7	14	28	$7(2)^{n-1}$



I used the numbers in the table and got this equation:  $f(x) = 7(2)^{x-1}$ .

This seems just like all that stuff we were doing with the lines, but I think that the graphs of these two equations would be different. There is something definitely wrong here.

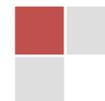
7. What is different about the thinking that Zac and Sione used to come to different equations?
8. How are their results similar to their results on the linear quilt pattern above? How are they different?

**Zac:** I know! Let's try doing the same thing with your exponential function as the linear function. What if we took the point (1, 7) and wrote the equation this way:

$$f(x) = 2^{(x-1)} + 7$$

See what I did? I did the subtract 1 thing with the  $x$  and then added on the 7 from the  $y$  value of the point. I'll bet this is a really good shortcut trick.

9. What do you think of Zac's equation and his strategy? Does it always work? Why or why not?



# Linear and Exponential Functions | 7

## Ready, Set, Go!



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## Ready

Topic: Writing equations of lines.

Write the equation of a line in slope-intercept form:  $y = mx + b$ , using the given information.

1.  $m = -7, b = 4$                       2.  $m = 3/8, b = -3$                       3.  $m = 16, b = -1/5$

Write the equation of the line in point-slope form:  $y - y_1 = m(x - x_1)$ , using the given information.

4.  $m = 9, (0, -7)$                       5.  $m = 2/3, (-6, 1)$                       6.  $m = -5, (4, 11)$

7.  $(2, -5), (-3, 10)$                       8.  $(0, -9), (3, 0)$                       9.  $(-4, 8), (3, 1)$

Topic: Graphing linear and exponential functions

## Set

Make a graph of the function based on the following information. Add your axes. Choose an appropriate scale and label your graph. Then write the equation of the function.

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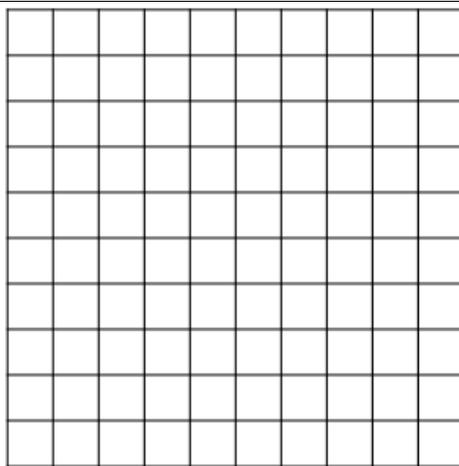
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# Linear and Exponential Functions **7**

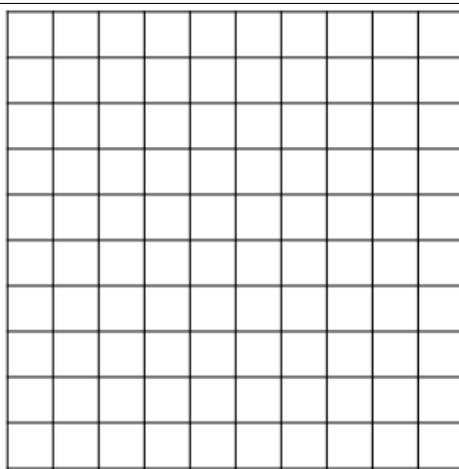
10. The beginning value of the function is 5 and its value is 3 units smaller at each stage.

Equation:



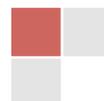
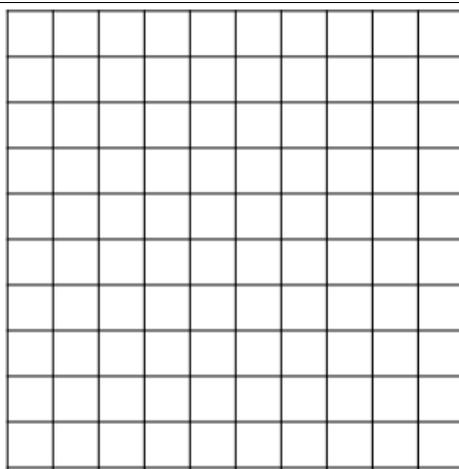
11. The beginning value is 16 and its value is  $\frac{1}{4}$  smaller at each stage.

Equation:



12. The beginning value is 1 and its value is 10 times as big at each stage.

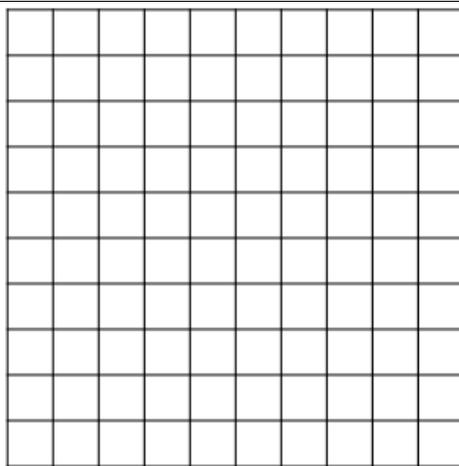
Equation:



# Linear and Exponential Functions | 7

13. The beginning value is -8 and its value is 2 units larger at each stage.

Equation:



## Go

Rewrite the equations in slope-intercept form.

14.  $2y + 10 = 6x + 12$

15.  $5x + y = 7x + 4$

16.  $(y - 13) = \frac{1}{2}(8x - 14)$

17.  $(y + 11) = -7(x - 2)$

18.  $(y - 5) = 3(x + 2)$

19.  $3(2x - y) = 9x + 12$

20.  $y - 2 = \frac{1}{5}(10x - 25)$

21.  $y + 13 = -1(x + 3)$

22.  $y + 1 = \frac{3}{4}(x + 3)$

Need Help? Check out these related videos:

Equations in slope-intercept form: <http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/linear-equations-in-slope-intercept-form>

Equations in point-slope form: <http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/linear-equations-in-point-slope-form>



# Efficiency Experts

## A Solidify Understanding Task



In our work so far, we have worked with linear and exponential equations in many forms. Some of the forms of equations and their names are:

Equation	Name
$y = \frac{1}{2}x + 1$	Slope Intercept Form $y = mx + b$ , where $m$ is the slope and $b$ is the y-intercept
$y - 3 = \frac{1}{2}(x - 4)$	Point Slope Form $y - y_1 = m(x - x_1)$ , where $m$ is the slope and $(x_1, y_1)$ the coordinates of a point on the line
$x - 2y = -2$	Standard Form $ax + by = c$
$f(0) = 1, f(n) = f(n - 1) + \frac{1}{2}$	Recursion Formula $f(n) = f(n - 1) + D$ , Given an initial value $f(a)$ $D =$ constant difference in consecutive terms

1. Verify that the four equations above are equivalent.
2. Explain how you know that the four equations are linear.

You have been appointed as a mathematics efficiency expert. Your job is to compare these four forms of equations for different uses and decide which form is most efficient and effective for each use. The investigation will be conducted in four parts with a report to be written at the end.



### Linear Investigation Part A: Which form best tells the story?

1. In his job selling vacuums, Joe makes \$500 each month plus \$20 for each vacuum he sells. Which equation best describes Joe's monthly income?

$$20x - y = 500$$

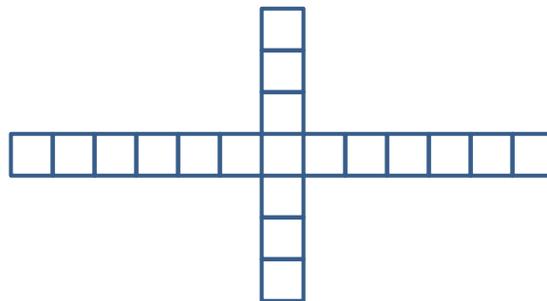
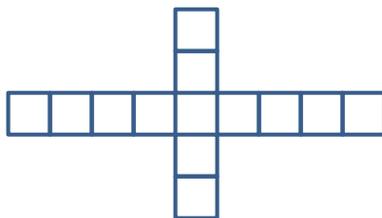
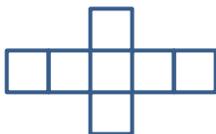
$$y = 20x + 500$$

2. The Tree Hugger Granola Company makes trail mix with candies and nuts. The cost of candies for a trail mix is \$2 per pound and the cost of the nuts is \$1.50 per pound. The total cost of a batch of trail mix is \$540. Which equation best models the quantities of candies and nuts in the mix?

$$2x + 1.5y = 540$$

$$y = \frac{4}{3}x + 360$$

3. Grandma Billings is working on a quilt with blocks in the following pattern. Which equation best models the number of blocks in each pattern?



$$f(n) = 6n + 1$$

$$f(1) = 7, f(n) = f(n - 1) + 6$$

4. What is the important difference between the type of situations that can be modeled with a recursion formula and the other equation forms?



**Linear Investigation Part B: Which is the best form for writing equations?**

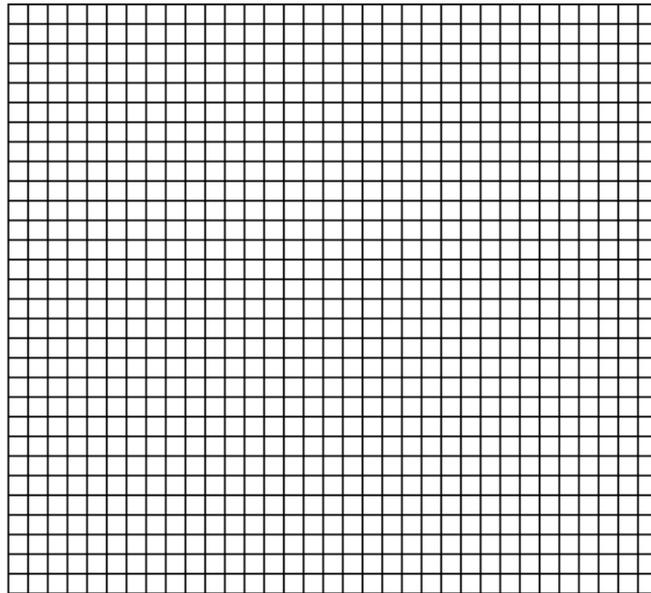
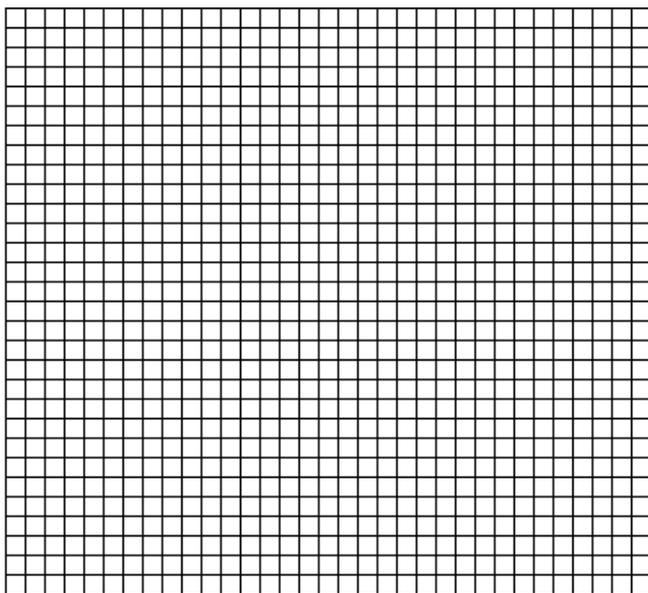
1. Write the equation of the line with a slope of -2 through the point (-2, 5)
2. Write the equation of the line through the points (1, -2) and (4, 1)
3. Write the equation of the arithmetic sequence that starts with -7 and each term decreases by 3.

**Linear Investigation Part C: Which is the best form for graphing?**

Graph the following equations:

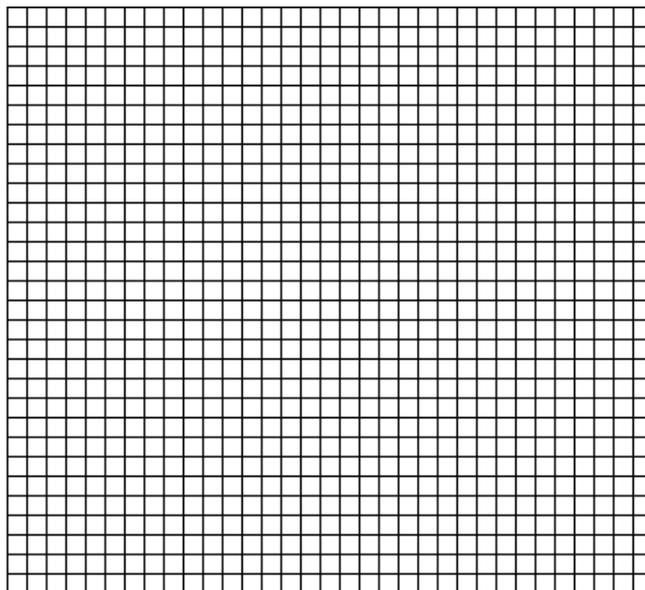
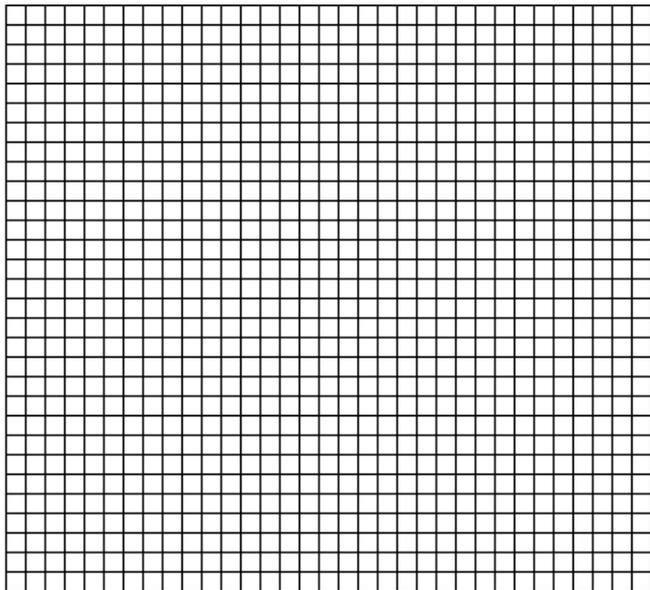
1.  $y = \frac{3}{4}x + 5$

2.  $3x - 5y = 15$



3.  $f(n) = -4(n + 6) + 5$

4.  $f(0) = -2, f(n) = f(n - 1) - 5$

**Linear Investigation Part D: What about tables?**

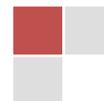
1. Create a table for each equation.

a.  $-x + 2y = 6$

b.  $y = -\frac{1}{2}x - 4$

2. Write an equation for the relation described in this table.

$x$	$y$
12	-3
9	-8
6	-13
3	-18
0	-23



## Your Efficiency Analysis Report, Part 1

Using the results of your investigation, describe the best uses for each form of an equation of a line, with sections for standard form, slope intercept form, point slope form and recursion formulas. Be sure to include a discussion of tables, graphs, and story contexts as part of your report.

### Investigating Exponential Forms

During the course of the year, we have also worked with forms of exponential equations, with a few more coming before the end of the module. The forms of exponential equations that we have seen so far:

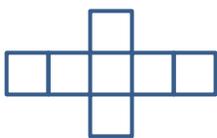
Equation	Name
$y = 10(3)^x$	Explicit Form $y = a(b)^x$
$f(0) = 10, f(n + 1) = 3f(n)$	Recursion Formula $f(n + 1) = Rf(n)$ Given an initial value $f(a)$ $R =$ constant ratio between consecutive terms

Test out the efficiency of these two exponential equation types for these tasks.

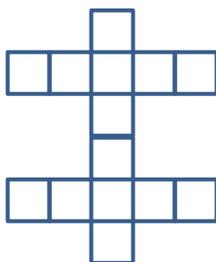
### Exponential Investigation Part A: Which form tells the story best?

- Grandma Billings has started piecing her quilt together and has created the following growth pattern:

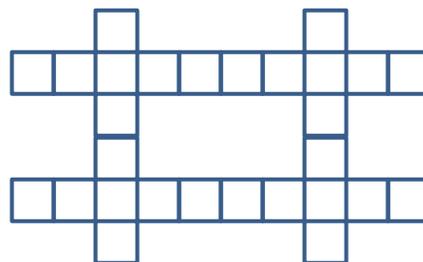
Block 1



Block 2



Block 3



Which equation best models the number of squares in each block?

$$f(n) = 7(2)^{n-1}$$

$$f(1) = 7, f(n) = 2f(n - 1)$$



2. The population of the resort town of Java Hot Springs in 2003 was estimated to be 35,000 people with an annual rate of increase of about 2.4%. Which equation best models the number of people in Java Hot Springs, with  $t$  = the number of years from 2003?

$$f(t) = 35,000(1.024)^t \quad f(0) = 35,000, f(t) = 1.024 \cdot f(t - 1)$$

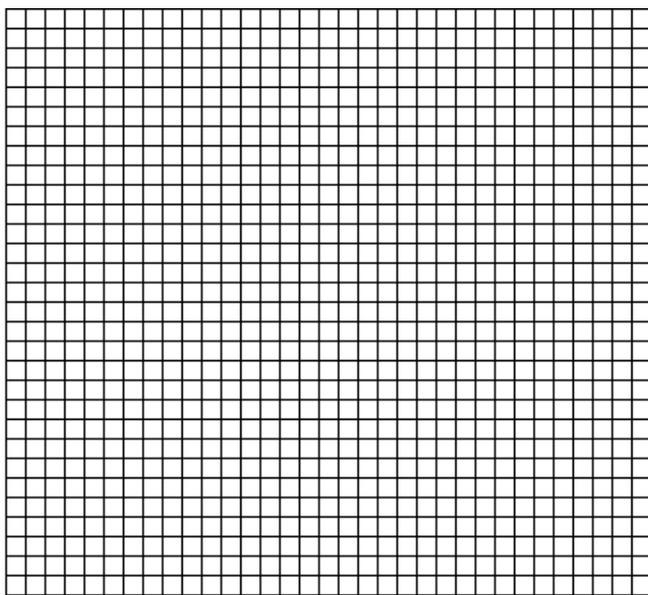
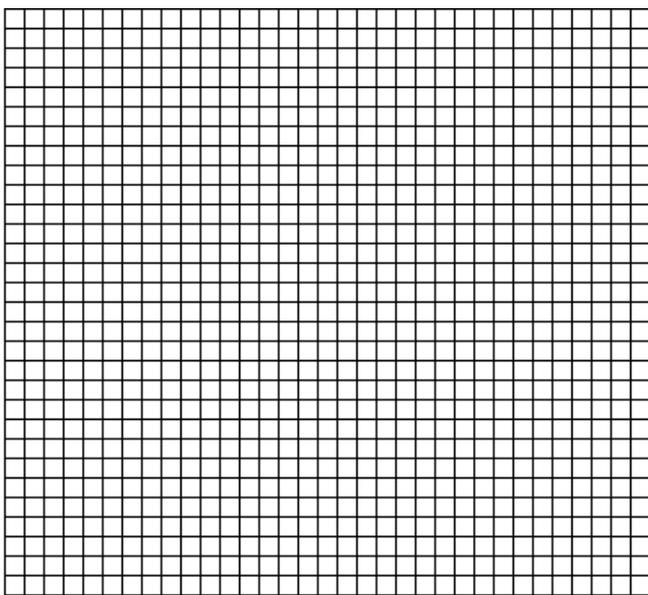
3. How would you have to change the definition of  $t$  in the recursive formula to model the situation?

### Exponential Investigation Part B: Which is the best form for graphing?

Graph each equation:

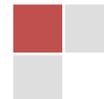
1.  $y = 2(1.8)^x$

2.  $f(0) = 5, f(n) = 0.6 \cdot f(n - 1)$



### Your Efficiency Analysis Report, Part 2

Using the results of your investigation, describe the best uses for each form of an exponential equation, with sections for standard form and recursion formulas. Be sure to include a discussion of tables, graphs, and story contexts as part of your report.



# Linear and Exponential Functions | 8

## Ready, Set, Go!



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### Ready Topic: Simple interest

When a person borrows money, the lender usually charges “rent” on the money. This “rent” is called interest. Simple interest is a percent “ $r$ ” of the original amount borrowed “ $p$ ” multiplied by the time “ $t$ ”, usually in years. The formula for calculating the interest is  $i = prt$ .

Calculate the simple interest owed on the following loans.

- $p = \$1000$      $r = 11\%$      $t = 2$  years     $i =$  \_\_\_\_\_
- $p = \$6500$      $r = 12.5\%$      $t = 5$  years     $i =$  \_\_\_\_\_
- $p = \$20,000$      $r = 8.5\%$      $t = 6$  years     $i =$  \_\_\_\_\_
- $p = \$700$      $r = 20\%$      $t = 6$  months     $i =$  \_\_\_\_\_

5. Juanita borrowed \$1,000 and agreed to pay 15% interest for 5 years. Juanita did not have to make any payments until the end of the 5 years, but then she had to pay back the amount borrowed “ $P$ ” plus all of the interest “ $i$ ” for the 5 years “ $t$ .” Below is a chart that shows how much money Juanita owed the lender at the end of each year of the loan. Compare the simple interest formula to the pattern you see in the chart for the amount owed to the lender. Explain in what ways you see the pattern in the formula.

End of year	Interest owed for the year	Total amount owed to the lender to pay back the loan.
1	$\$1000 \times .15 = \$150$	Principal + interest = \$1150
2	$\$1000 \times .15 = \$150$	$P + i + i = \$1300$
3	$\$1000 \times .15 = \$150$	$P + i + i + i = \$1450$
4	$\$1000 \times .15 = \$150$	$P + i + i + i + i = \$1600$
5	$\$1000 \times .15 = \$150$	$P + i + i + i + i + i = \$1750$

6. At the end of year 5, the interest was calculated at 15% of the original loan of \$1000. But by that time Juanita owed \$1600 (before the interest was added.) What percent of \$1600 is \$150?

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# Linear and Exponential Functions | 8

7. Consider if the lender charged 15% of the amount owed instead of 15% of the amount of the original loan. Make a fourth column on the chart and calculate the interest owed each year if the lender required 15% of the amount owed at the end of each year. Note that the interest owed at the end of the first year would still be \$150. Fill in the 4<sup>th</sup> column.

**Set** Topic: The 4 forms of a linear equation

8. Given are the 4 forms of the same linear equation. In each equation, <sup>A</sup> Circle the rate of change, <sup>B</sup> name the point that describes the y-intercept, and <sup>C</sup> name the point that describes the x-intercept. Be prepared to explain which equation you prefer for finding A, B, and C.

slope-intercept	point-slope	standard	recursive formula	answer B & C
8. $y = 3x - 2$	$y - 13 = 3(x - 5)$	$3x - y = 2$	$f(0) = -2, f(n) = f(n-1) + 3$	B      C
9. $y = \frac{1}{4}x + 7$	$y - 5 = \frac{1}{4}(x + 8)$	$x - 4y = -28$	$f(0) = 7, f(n) = f(n-1) + \frac{1}{4}$	
10. $y = -\frac{2}{3}x + 3$	$y + 1 = -\frac{2}{3}(x - 6)$	$2x + 3y = 9$	$f(0) = 3, f(n) = f(n-1) - \frac{2}{3}$	

**Go** Topic: Solving multi-step equations

11. What does it mean when you have solved an equation?
12. Explain how a linear equation can have more than one solution.

Solve the following equations:

- |                                      |                                 |
|--------------------------------------|---------------------------------|
| 10. $12 + 6x - 4 = 5 + 2(3x - 1)$    | 11. $5(2x + 4) = 3(x + 5) - 19$ |
| 12. $7 - 3(4x + 2) = 6(2x + 3) - 17$ | 13. $2(x + 1) = 6(x - 3)$       |

Need Help? Check out these related videos:

Solving equations: <http://www.purplemath.com/modules/solvein4.htm>

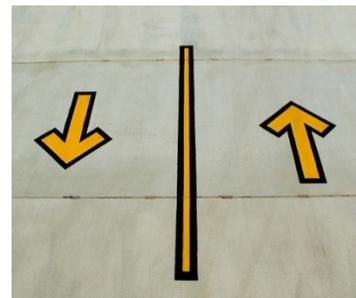
Interest: <http://www.khanacademy.org/finance-economics/core-finance/v/introduction-to-interest>



## Up a Little, Down a Little

### *A Solidify Understanding Task*

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One of the most common applications of exponential growth is compound interest. For example, Mama Bigbucks puts \$20,000 in a bank savings account that pays 3% interest compounded annually. “Compounded annually” means that at the end of the first year, the bank pays Mama 3% of \$20,000, so they add \$600 to the account. Mama leaves her original money (\$20,000) and the interest (\$600) in the account for a year. At the end of the second year the bank will pay interest on the entire amount, \$20,600. Since the bank is paying interest on a previous interest amount, this is called “compound interest”.

Model the amount of money in Mama Bigbucks’ bank account after  $t$  years.

Use your model to find the amount of money that Mama has in her account after 20 years.

A formula that is often used for calculating the amount of money in an account that is compounded annually is:

$$A = P(1 + r)^t$$

Where:

$A$  = amount of money in the account after  $t$  years

$P$  = principal, the original amount of the investment

$r$  = the annual interest rate

$t$  = the time in years

Apply this formula to Mama’s bank account and compare the result to the model that you created.

Based upon the work that you did in creating your model, explain the  $(1 + r)$  part of the formula.



Another common application of exponential functions is depreciation. When the value of something you buy goes down a certain percent each year, it is called depreciation. For example, Mama Bigbucks buys a car for \$20,000 and it depreciates at a rate of 3% per year. At the end of the first year, the car loses 3% of its original value, so it is now worth \$19,400.

Model the value of Mama's car after  $t$  years.

Use your model to find how many years will it take for Mama's car to be worth less than \$500?

How is the situation of Mama's car similar to Mama's bank account?

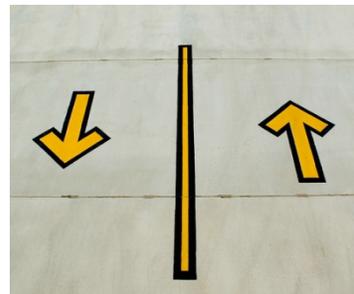
What differences do you see in the two situations?

Consider your model for the value of Mama's car and develop a general formula for depreciation.



# Linear and Exponential Functions | 9

Ready, Set, Go!



Topic: Evaluating equations

Ready

Fill out the table of values for the given equations.

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# Linear and Exponential Functions | 9

## Set

Topic: Evaluate using the formulas for simple interest or compound interest.

Given the formula for simple interest:  $i = Prt$ , calculate the simple interest paid.

$i = \text{interest}$     $P = \text{the principal}$     $r = \text{the interest rate per year as a decimal}$     $t = \text{time in years}$

7. Find the simple interest you will pay on a 5 year loan of \$7,000 at 11% per year.

8. How much interest will you pay in 2 years on a loan of \$1500 at 4.5% per year?

Use  $i = Prt$  to complete the table. All interest rates are annual.

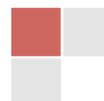
$$i = P \times r \times t$$

9.		\$11,275	12%	3 years
10.	\$1428	\$5100	4%	
11.	\$93.75	\$1250		6 months
12.	\$54		8%	9 months

Given the formula for compound interest:  $A = P(1 + r)^t$ , write a compound interest function to model each situation. Then calculate the balance after the given number of years.

$A = \text{the balance after } t \text{ years}$     $P = \text{the principal}$     $t = \text{the time in years}$   
 $r = \text{the annual interest rate expressed as a decimal}$

13. \$22,000 invested at a rate of 3.5% compounded annually for 6 years.



# Linear and Exponential Functions | 9

14. \$4300 invested at a rate of 2.8% compounded annually for 15 years.
15. Suppose that when you are 15 years old, a magic genie gives you the choice of investing \$10,000 at a rate of 7% or \$5,000 at a rate of 12%. Either choice will be compounded annually. The money will be yours when you are 65 years old. Which investment would be the best? Justify your answer.

Topic: Using order of operations when evaluating equations

## Go

Evaluate the equations for the given values of the variables.

16.  $pq \div 6 + 10$ ; when  $p = 7$  and  $q = -3$
17.  $m + n(m - n)$ ; when  $m = 2$ , and  $n = 6$
18.  $(b - 1)^2 + ba^2$ ; when  $a = 5$ , and  $b = 3$
19.  $y(x - (9 - 4y))$ ; when  $x = 4$ , and  $y = -5$
20.  $x - (x - (x - y^3))$ ; when  $x = 7$ , and  $y = 2$
21.  $an^4 + a(n - 7)^2 + 2n$ ; when  $a = -2$ , and  $n = 4$

Need Help? Check out these related videos:

<http://www.basic-mathematics.com/simple-vs-compound-interest.html>

<http://www.khanacademy.org/finance-economics/core-finance/v/introduction-to-interest>

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# X Marks the Spot

## A Practice Understanding Task

### Table Puzzles

1. Use the tables to find the missing values of  $x$ :

a.

$x$	$y = 0.7x - 3$
-2	-4.2
-10	9
	-6.6
4	-0.6
	1.2

b.

$x$	$y = -\frac{2}{3}x + 4$
10	$-10\frac{2}{3}$
-3	6
5	$7\frac{1}{3}$
	0
	10

c. What equations could be written, in terms of  $x$  only, for each of the rows that are missing the  $x$  in the two tables above?

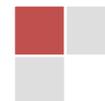
d.

$x$	$y = 3^x$
5	243
	81
-3	$\frac{1}{27}$
	$\frac{1}{3}$
2	9

e.

$x$	$y = \left(\frac{1}{2}\right)^x$
-5	32
	8
	1
2	$\frac{1}{4}$
	$\frac{1}{16}$

f. What equations could be written, in terms of  $x$  only, for each of the rows that are missing the  $x$  in the two tables above?



2. What strategy did you use to find the solutions to equations generated by the tables that contained linear functions?
  
3. What strategy did you use to find the solutions to equations generated by the tables that contained exponential functions?

## Graph Puzzles

4. The graph of  $y = -\frac{1}{2}x + 3$  is given below. Use the graph to solve the equations for  $x$  and label the solutions.

a.  $5 = -\frac{1}{2}x + 3$

$x = \underline{\hspace{2cm}}$

Label the solution with an A on the graph.

b.  $-\frac{1}{2}x + 3 = 1$

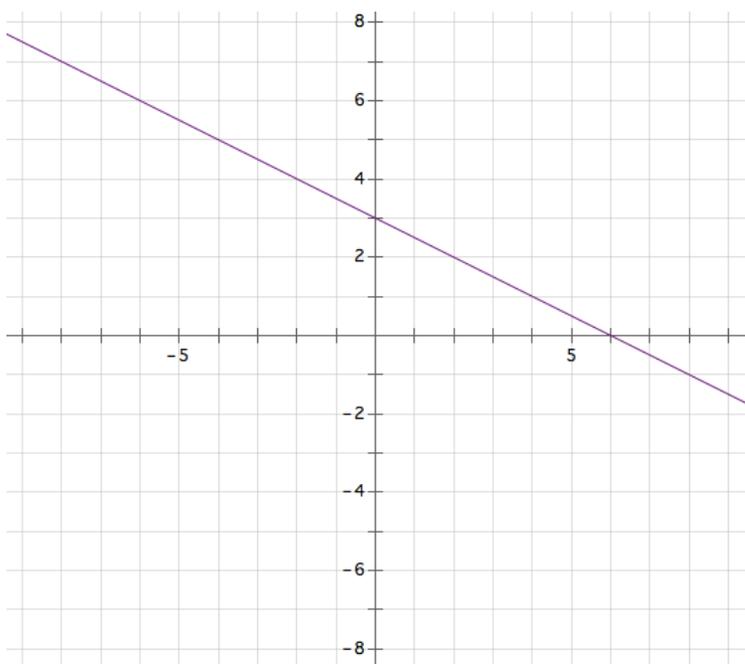
$x = \underline{\hspace{2cm}}$

Label the solution with a B on the graph.

c.  $-0.5x + 3 = -1$

$x = \underline{\hspace{2cm}}$

Label the solution with a C on the graph.



5. The graph of  $y = 3^x$  is given below. Use the graph to solve the equations for  $x$  and label the solutions.

a.  $3^x = \frac{1}{9}$

$x = \underline{\hspace{2cm}}$

Label the solution with an A on the graph.

b.  $3^x = 9$

$x = \underline{\hspace{2cm}}$

Label the solution with a B on the graph.

c.  $3\sqrt{3} = 3^x$

$x = \underline{\hspace{2cm}}$

Label the solution with a C on the graph.

d.  $1 = 3^x$

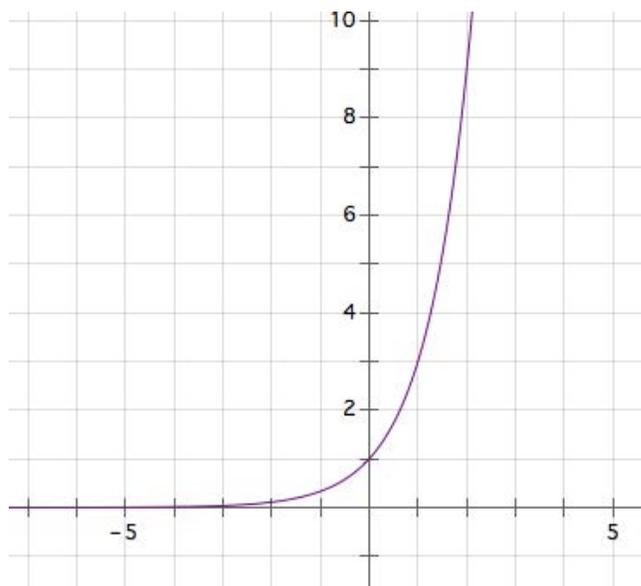
$x = \underline{\hspace{2cm}}$

Label the solution with a D on the graph.

e.  $6 = 3^x$

$x = \underline{\hspace{2cm}}$

Label the solution with an E on the graph.



6. How does the graph help to find solutions for  $x$ ?

## Equation Puzzles:

Solve each equation for  $x$ :

7.  $5^x = 125$

8.  $7 = -6x + 9$

9.  $10^x = 10,000$

10.  $2.5 - 0.9x = 1.3$

11.  $6^x = \frac{1}{36}$

12.  $\left(\frac{1}{4}\right)^x = 16$



# Linear and Exponential Functions | 10

## Ready, Set, Go!

### Ready

1. Give an example of a discrete function.
2. Give an example of a continuous function.

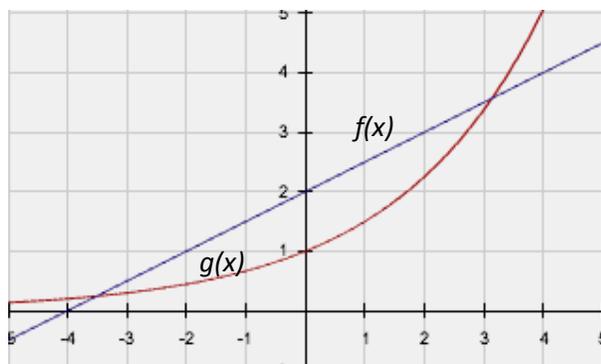


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3. The first and 5<sup>th</sup> terms of a sequence are given. Fill in the missing numbers for an arithmetic sequence. Then fill in the numbers for a geometric sequence.

arithmetic	-6250				-10
geometric	-6250				-10

4. Compare the rate of change in the pair of functions in the graph by identifying the interval where it appears that  $f(x)$  is changing faster and the interval where it appears that  $g(x)$  is changing faster. Verify your conclusions by making a table of values for each function and exploring the rates of change in your tables.



5. Identify the following sequences as linear, exponential, or neither.
  - a. -23, -6, 11, 28, ...
  - b. 49, 36, 25, 16, ...
  - c. 5125, 1025, 205, 41, ...
  - d. 2, 6, 24, 120, ...
  - e. 0.12, 0.36, 1.08, 3.24, ...
  - f. 21, 24.5, 28, 31.5, ...

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