1

Secondary Mathematics III: An Integrated Approach

Module 8
Statistics

By

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In partnership with the
Utah State Office of Education

Secondary Mathematics III
Module 8 – Statistics

Classroom Task: 8.1 What is Normal? – A Develop Understanding Task
Understand normal distributions and identify their features **(S.ID.4)**
Ready, Set, Go Homework: Statistics 8.1

Classroom Task: 8.2 Just ACT Normal – A Solidify Understanding Task
Use the features of a normal distribution to make decisions **(S.ID.4)**
Ready, Set, Go Homework: Statistics 8.2

Classroom Task: 8.3 Y B Normal? – A Solidify Understanding Task
Compare normal distributions using z scores **(S.ID.4)**
Ready, Set, Go Homework: Statistics 8.3

Classroom Task: 8.4 Whoa! That's Weird! – A Practice Understanding Task
Compare normal distributions using z scores and understanding of mean and standard deviation **(S.ID.4)**
Ready, Set, Go Homework: Statistics 8.4

Classroom Task: 8.5 Would You Like to Tray a Sample? – A Develop Understanding Task
Understand and identify different methods of sampling **(S.IC.1)**
Ready, Set, Go Homework: Statistics 8.5

Classroom Task: 8.6 Would You Like to Try a Sample? – A Develop Understanding Task
Uses tables, graphs, equations, and written descriptions of functions to match functions and their inverses together and to verify the inverse relationship between two functions. **(S.IC.2)**
Ready, Set, Go Homework: Statistics 8.6

Classroom Task: 8.7 Let's Investigate – A Solidify Understanding Task
Identify the difference between survey, observational studies, and experiments **(S.IC.1, S.IC.2)**
Ready, Set, Go Homework: Statistics 8.7

Classroom Task: 8.8 Slacker's Simulation – A Solidify Understanding Task
Use simulation to estimate the likelihood of an event **(S.IC.2, S.IC.3)**
Ready, Set, Go Homework: Statistics 8.8

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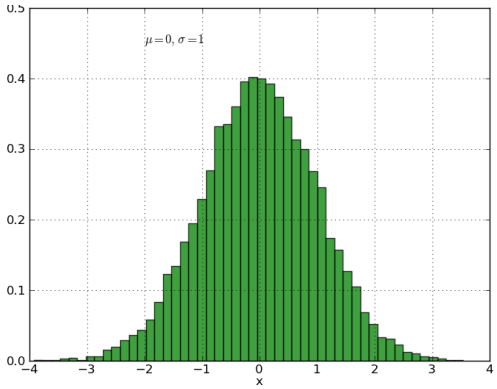
8.1 What is Normal?

A Develop Understanding Task

One very important type of data distribution is called a "normal distribution." In this case the word "normal". In this task, you will be given pair of data distributions represented with histograms and distribution curves. In each pair, one distribution is normal and one is not. Your job is to compare each of the distributions given and come up with a list of features for normal distributions.

1. This is normal:



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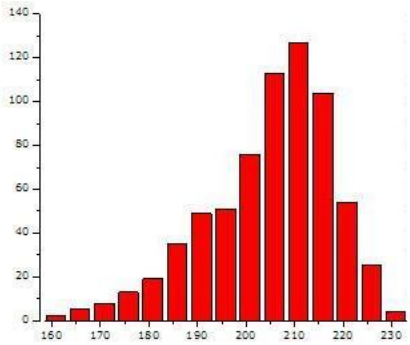

What differences do you see between these distributions?

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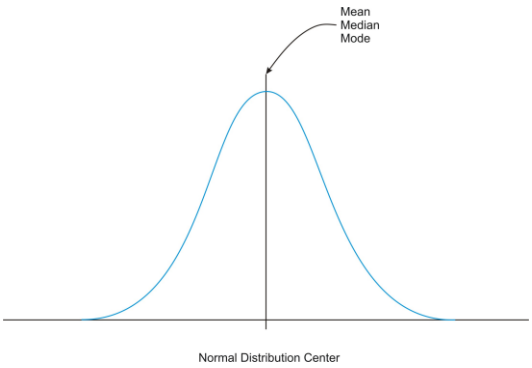
2. This is normal:



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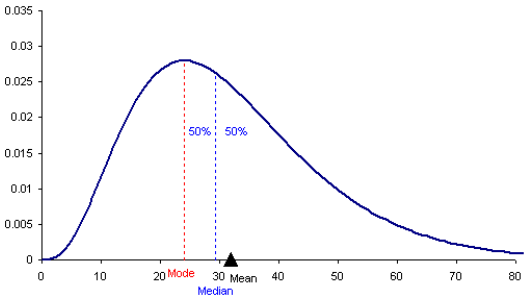

What differences do you see between these distributions?

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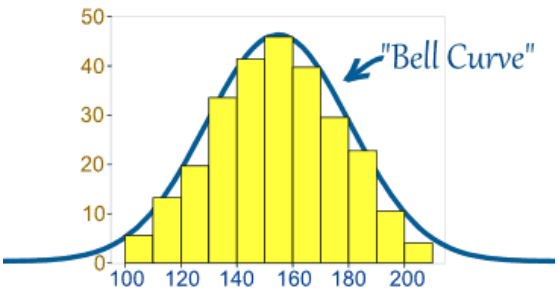
3. This is normal:



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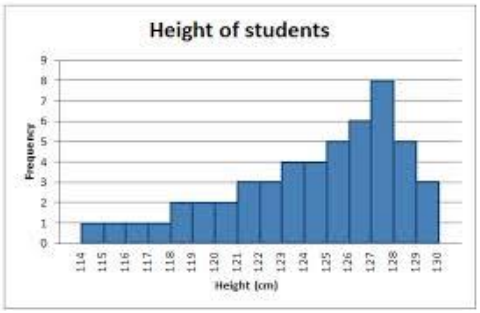

What differences do you see between these distributions?

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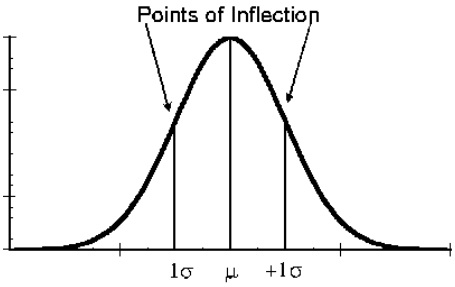
4. This is normal:



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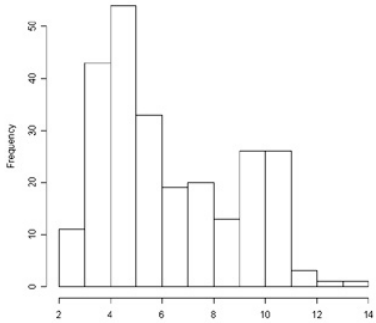

What differences do you see between these distributions?

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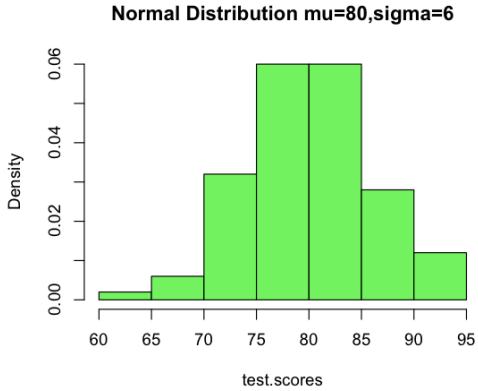
5. This is normal:



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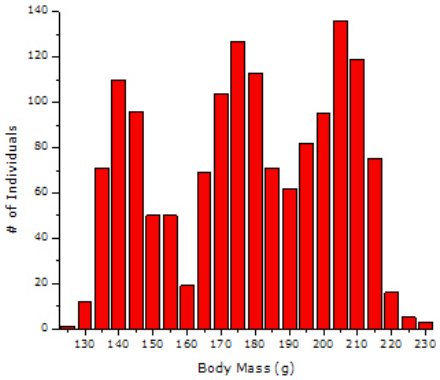

What differences do you see between these distributions?

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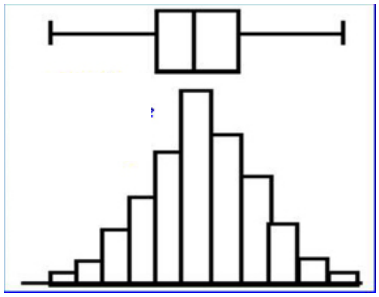
6. This is normal:



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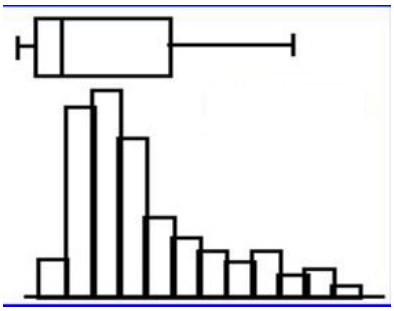

What differences do you see between these distributions?

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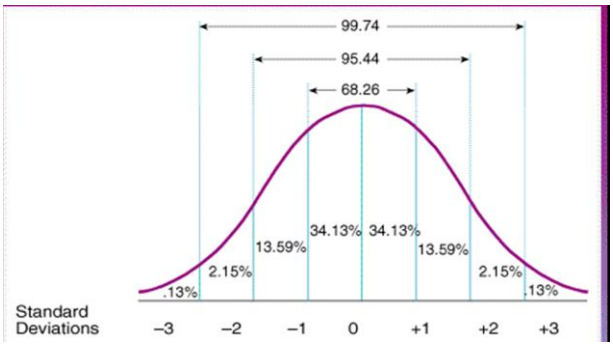
7. This is normal:



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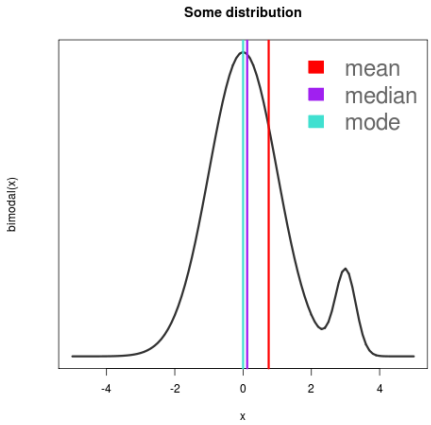


What differences do you see between these distributions?

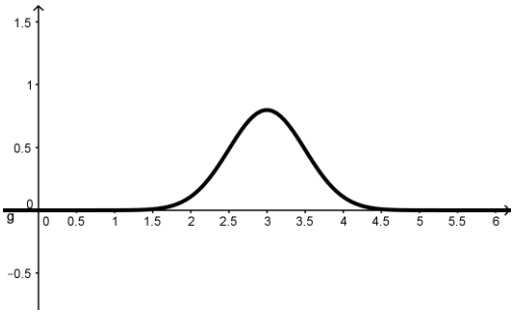
_____

9. Based upon the examples you have seen in #1-7, what are the features of a normal distribution?

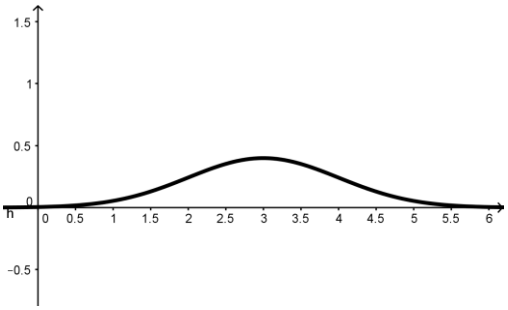
10. a. What does the standard deviation tell us about a distribution?

b. Each of the distributions shown below are normal distributions with the same mean but a different standard deviation.

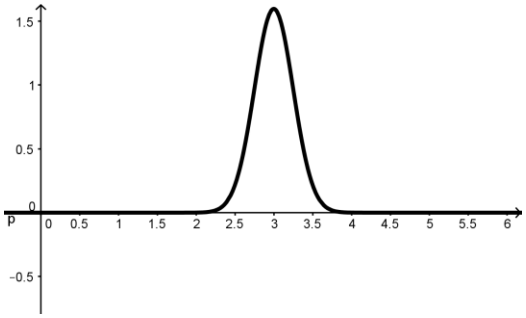
Mean = 3, Standard Deviation = 0.5



Mean = 3, Standard Deviation = 1



Mean = 3, Standard Deviation = 0.25



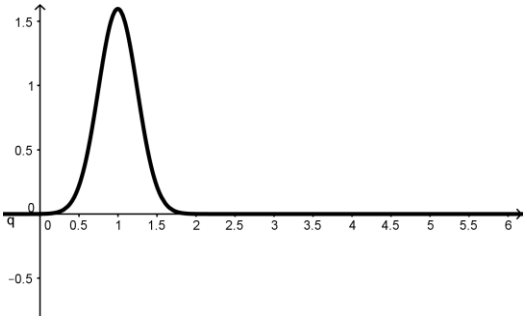
How does changing the standard deviation affect a normal curve? Why does it have this effect?

_____

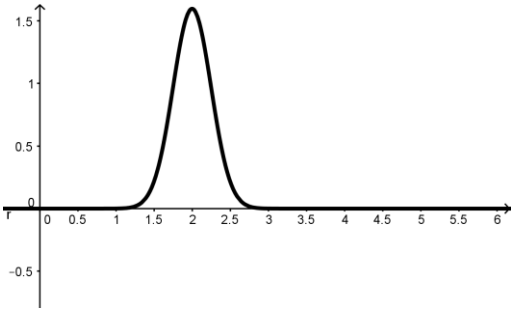
11. a. What does the mean tell us about a distribution?

b. Each of the distributions shown below are normal distributions with the same standard deviation but a different mean.

Mean = 1, Standard Deviation = 0.25

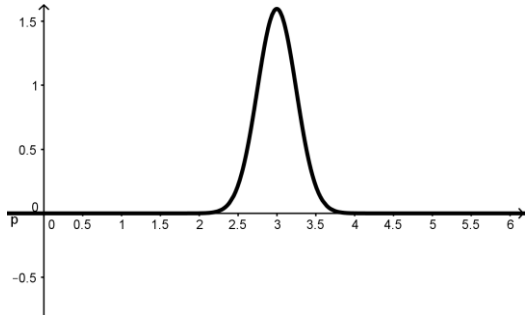


Mean = 2, Standard Deviation = 0.25



7

Mean = 3, Standard Deviation = 0.25



How does changing the mean affect a normal curve? Why does it have this effect?

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12. Now that you have figured out some of the features of a normal distribution, determine if the following statements are true or false. In each case, explain your answer.

a. A normal distribution depends on the mean and the standard deviation.

True/False Why?

b. The mean, median, and mode are equal in a normal distribution.

True/False Why?

c. A normal distribution is bimodal.

True/False Why?

d. In a normal distribution, 50% of the population is within one standard deviation of the mean.

True/False Why?

What Is Normal?– Teacher Notes

A Develop Understanding Task

Purpose: The purpose of this task is to recognize a normal distribution. In the task, students are given multiple examples of frequency distributions, some normal and some that are not. Students are asked to compare the examples and identify features of a normal distribution. The purpose is to understand that the shape of a normal distribution is symmetric, single-peaked, and bell shaped. Students will recognize the effect on the distribution curve of changing the mean and the standard deviation. They will also learn that the mean, median, and mode are equal in a normal distribution. Students will also see the 68 – 95 – 99.7 rule illustrated, and then will use the rule in a later task.

Core Standards Focus:

S.ID.4: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Related Standards: S.ID.1

Vocabulary: point of inflection, distribution curve, normal distribution

Launch (Whole Class): Begin the lesson describing for students the difference between a frequency distribution and a distribution curve. Students should have experience with frequency distributions, but may not have discussed a distribution curve. Explain that a distribution curve "smoothes out the bumps" in a frequency distribution with a theoretical curve that shows how often an experiment with a particular result. Many of the upcoming examples have distribution curves along with other representations. Tell students that they will see several pairs of images that have been grouped to highlight particular features. All of the distributions on the left side are normal distributions, all of the distributions on the right side are not. Their job is to compare the two images in the pair so that they can arrive at a description for a normal distribution.

Explore (Small Group): Monitor students as they are working to see that they are comparing features and identifying differences between the two distributions. As they begin to work on the second part of the task, be sure they are connecting to their previous understanding of the mean and the standard deviation to make sense of the effect of these features on the distribution.

Discuss (Whole Class): Go through each of the examples given in 1-7, highlighting the features of a normal distribution in each pair. The following list suggests features that could be noticed with each pair. By the end of the discussion, all of these features should be identified. Students probably won't use the correct vocabulary as they describe the features, but should be supported in developing and using the vocabulary as the module proceeds.

1. A normal distribution is symmetric.
2. The mean, median, and mode are equal in a normal distribution.
3. The frequency curve of a normal distribution is symmetric.
4. A normal curve has points of inflection at ± 1 standard deviation from the mean.
5. A normal distribution has a single mode.
6. 68% of the distribution will be ± 1 standard deviations from the mean. 95% of the distribution will be within 2 standard deviations from the mean. 99.7% of the distribution will be within 3 standard deviations from the mean.

Ask students for some examples of measurement data that might be distributed normally. Some possibilities:

- The height of adult males or adult females, but not the height of all adults because that distribution would probably have two modes—one for women and one for men.
- The average number of days that it takes for a particular type of seed to sprout.
- The average amount of sleep per night for adults.

Be careful to discuss each example to consider whether or not the distribution would be symmetric, single-peaked, and bell shaped.

Move the discussion to questions 10 and 11. Begin by asking students to explain what the standard deviation describes. Then ask what effect they saw by changing the standard deviation and to explain why that would happen. Repeat with the process when discussing the mean.

Complete the discussion by answering each of the true/false questions and asking students to explain their reasoning. Use this as an opportunity to reinforce vocabulary. Answers with sample student explanations:

12a. True – The mean tells where the peak of the distribution is and the standard deviation determines how spread out the distribution is.

12b. True – That's why normal distributions turn out to be symmetric with a single peak.

12c. False – A bimodal distribution will have two peaks. That's not normal.

12d. False – 68% of the distribution is within 1 standard deviation of the mean in a normal distribution.

Aligned Ready, Set, Go: ***Statistics 8.1***

Name Statistics | 8.1

Ready, Set, Go

Ready

Topic: Standard Deviations, Percentiles

1. Jordan scores a 53 on his math test. The class average is 57 with a standard deviation of 2 points. How many standard deviations below the mean did Jordan score?

2. In Jordan's science class, he scored a 114. The class average was a 126 with a standard deviation of 6 points. How many standard deviations below the mean did Jordan score? In comparison to his peers, which test did Jordan perform better on?

3. Rank the data sets below in order of greatest standard deviation to smallest:

$$A = \{1,2,3,4\} \quad B = \{2,2,2,2,\} \quad C = \{2,4,6,8\} \quad D = \{4,5,6,7\} \quad E = \{1,1.5,2,2.5\}$$

4. Robin made it to the swimming finals for her state championship meet. The times in the finals were as follows:

$$\{2{:}\,10.3, \quad 2{:}\,12.5, \quad 2{:}\,12.7, \quad 2{:}\,12.38, \quad 2{:}\,20.45, \quad 2{:}\,21.43\}$$

If Robin's time was a 2:12.7, what percent of her competitors did she beat?

5. Remember that in statistics, μ is the symbol for mean and σ is the symbol for standard deviation. Using technology, identify the mean and standard deviation for the data set below:

$$\{1.23, 1.3, 1.1, 1.48, 1, 1.14, 5.21, 5.1, 4.63\}$$

$$\mu = \qquad \sigma =$$

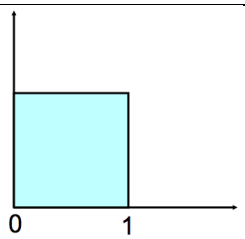
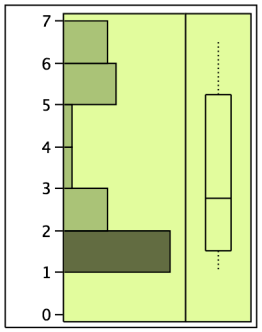
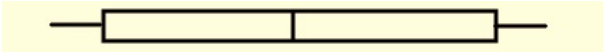
6. For the data in number 5, what time would fall one standard deviation above the mean?

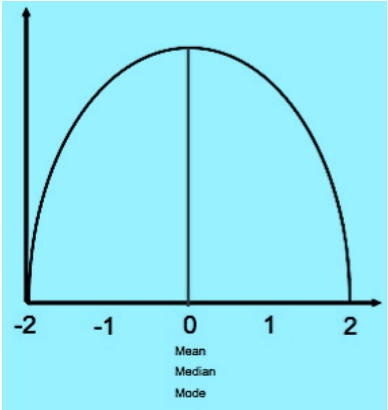
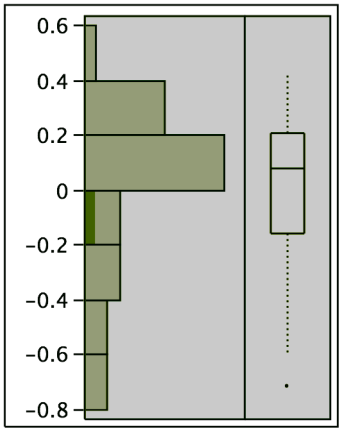
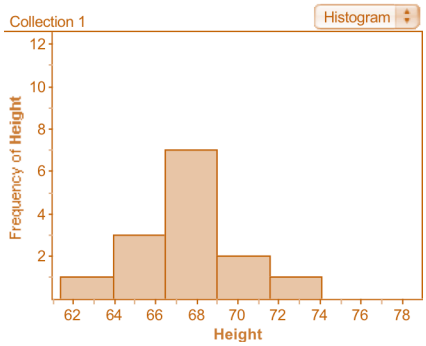
Three standard deviations below the mean?

Set

Topic: Properties of Normal Curves

7. **For each distribution, identify the properties that match with a Normal Distribution, and then decide if the distribution is Normal or not.**

A. 	Normal Properties: Normal? Yes or No
B. 	Normal Properties: Normal? Yes or No
C. 	Normal Properties: Normal? Yes or No

D.  	Normal Properties: Normal? Yes or No
E.  **Mean = 0 Median = 0.1 Mode = 0.1**	Normal Properties: Normal? Yes or No
F  **Mean: 68 Median: 68 Mode: 68**	Normal Properties: Normal? Yes or No

8. If two Normal distributions have the same standard deviation of 4.9 but different means of 3 and 6, how will the two Normal curves look in relation to each other? Draw a sketch of each Normal curve below.

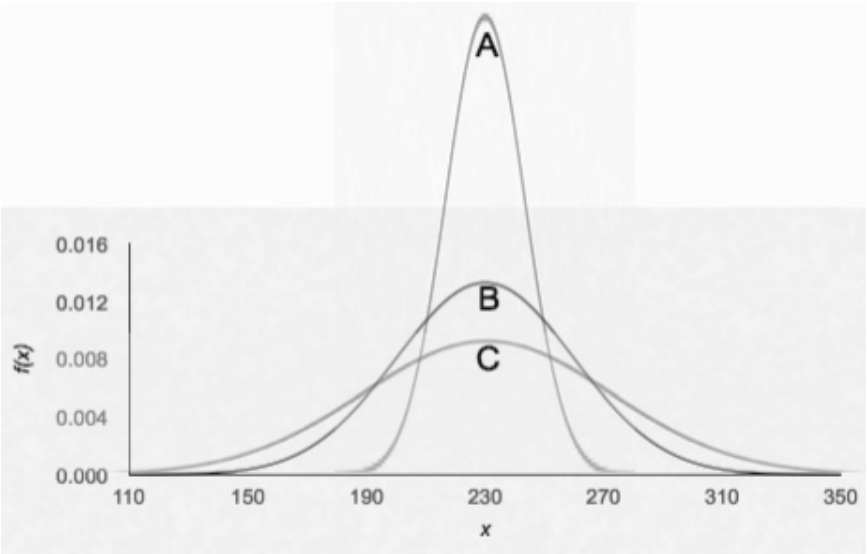
9. If two Normal distributions have the same mean of 3 but standard deviations of 1 and 4, how will they look in relation to each other? Draw a sketch of each Normal curve below.

10. Several Normal curves are given below. Estimate the standard deviation of each one.

A_____

B_____

C_____



Go

Topic: Inverses

Write the inverse of the given function in the same format as the given function:

11. $f(x) = 3x^2 + 2$

12. $g(x) = \frac{2x-7}{4}$

13. $h(x) = 3 + \sqrt{2x - 1}$

14.

X	Y
12	24
14	38
-7	4
13	6
7	0

Determine if the following functions are inverses by finding $f(g(x))$ *and* $g(f(x))$.

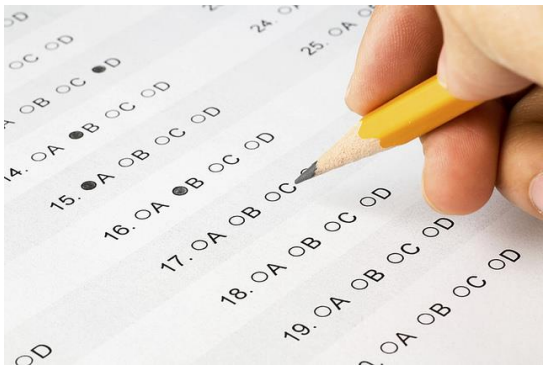
15. $f(x) = 2x + 3$ and $g(x) = \frac{1}{2}x - \frac{3}{2}$

16. $f(x) = 2x^2 - 3$ and $g(x) = \sqrt{\frac{x^2}{2} + 3}$

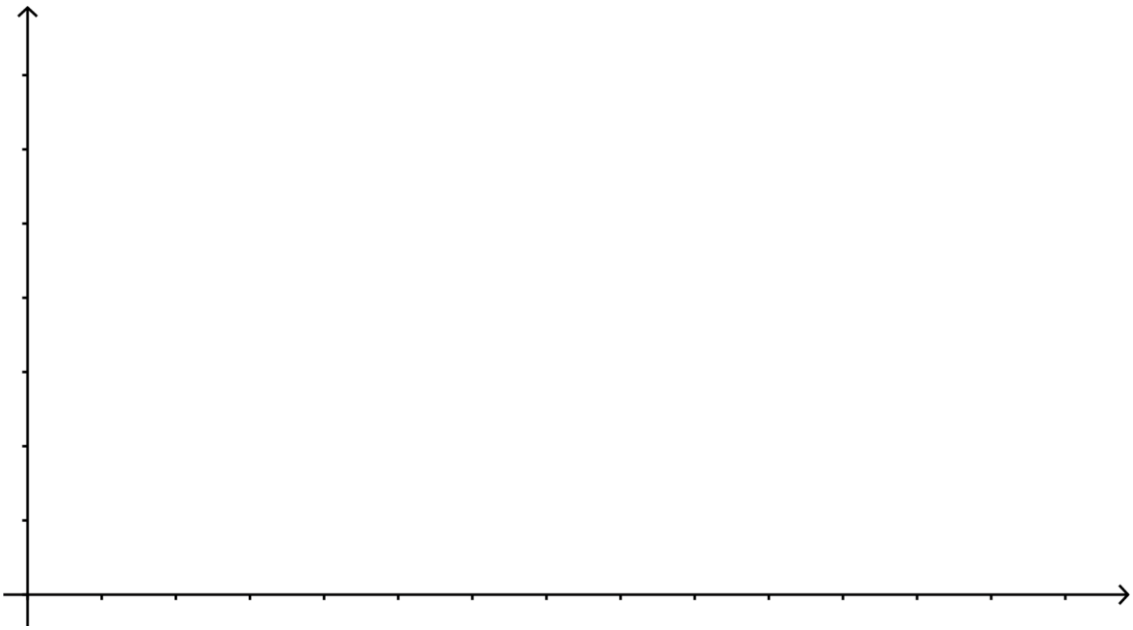
13

8.2 Just ACT Normal

A Solidify Understanding Task



1. One of the most common examples of a normal distribution is the distribution of scores on standardized tests like the ACT. In 2010, the mean score was 21 and the standard deviation was 5.2 (Source: National Center for Education Statistics). Use this information to sketch a normal distribution curve for this test.



2. Use technology to check your graph. Did you get the points of inflection in the right places? (Make adjustments, if necessary.)

3. In "What Is Normal", you learned that the 68 – 95 – 99.7 rule. Use the rule to answer the following questions:

a. What percentage of students scored below 21?

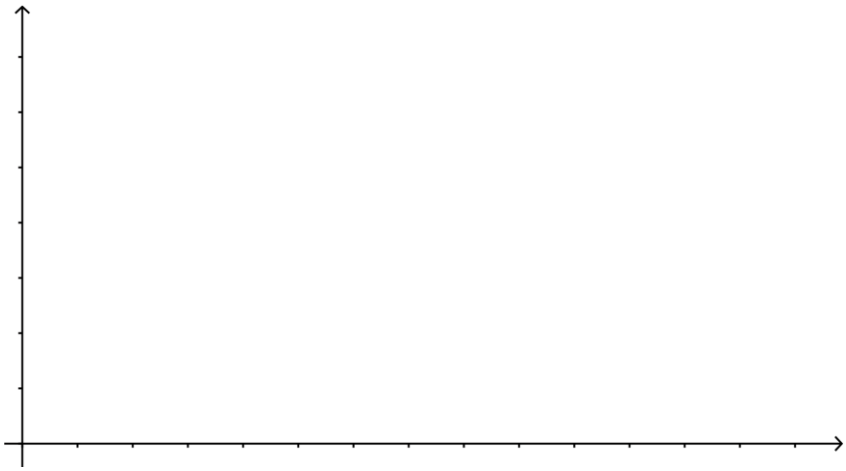
b. About what percentage of students scored below 16?

c. About what percentage of students scored between 11 and 26?

 14

3. Your friend, Calvin, would like to go to a very selective college that only admits the top 1% of all student applicants. Calvin has good grades and scored 33 on the test. Do you think that Calvin's ACT score gives him a good chance of being admitted? Explain your answer.

4. Many students like to eat microwave popcorn as they study for the ACT. Microwave popcorn producers assume that the time it takes for a kernel to pop is distributed normally with a mean of 120 seconds and a standard deviation of 13 for a standard microwave oven. If you're a devoted popcorn studier, you don't want a lot of un-popped kernels, but you know that if you leave the bag in long enough to be sure that all the kernels are popped, some of the popcorn will burn. How much time would you recommend for microwaving the popcorn? Use a normal distribution curve and the features of a normal distribution to explain your answer.



Just Act Normal– Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is for students to use their understanding of the normal distribution to make estimates of frequencies using area. In the task, they will use their knowledge that 1, 2, and 3 standard deviations refer to 68%, 95%, or 99.7% of the distribution respectively to draw conclusions. This task does not require the use of technology or tables.

Core Standards Focus:

S.ID.4: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Related Standards: S.ID.1

Launch (Whole Class): Begin the task by telling students that scores on standardized tests are usually distributed normally. Ask why it makes sense that standardized test scores might be distributed normally. Some responses may be: most students will score around the mean (making that the peak of the distribution), about the same number of students will score above and below the mean (making the distribution symmetric), the tests are designed so that very few students get everything right or everything wrong, which stretches out the distribution and makes the 68%, 95%, 99.7% rule seem reasonable in this context. Use this discussion to draw out some of the features of the normal distribution. Have students sketch the distribution curve and then model how to check it using technology. Be sure to discuss with students that the center of the distribution should be identified on the graph with the mean score, that the points of inflection are ±1 standard deviation away from the mean, and that 2 and 3 standard deviations are appropriately marked.

Explore (Small Group): Monitor students as they work on the remainder of the task. Notice that the standard deviation is 5.2 (not exactly 5), so students are estimating based on the 68%, 95%, 99.7% rule. Encourage students to label their distribution curves and use them to find the percentages. When students are working on #4, listen for students making use of the rule to estimate the percentage of kernels that will be popped. Don't let them get too caught up in how long it might take for the popped kernels to start to burn. The idea is to think that you would want most kernels to be popped, but can't wait for all of them.

Discuss (Whole Class): Begin the discussion with #2. Identify students to explain how they used the distribution curve and the 68%, 95%, 99.7% rule to find the approximate values. Move the

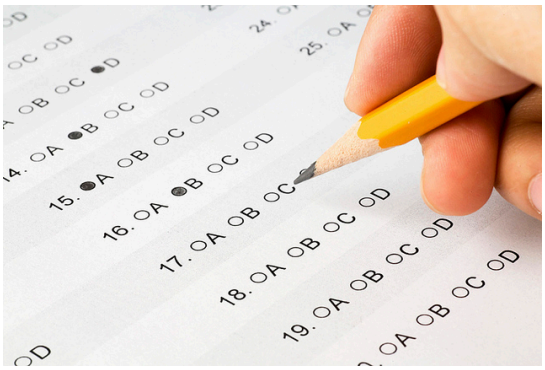
discussion to question #3. Students should be able to argue that Calvin's score is between 2 and 3 standard deviations away from the mean, so it is probably not in the top 1%. Ask students to estimate about what percent of students they think scored below Calvin, based on the distribution curve. Discuss what would be a reasonable answer and then show how it can be calculated using technology.

Discuss question #4, beginning with the distribution curve. Have several students explain and justify their answers.

Aligned Ready, Set, Go: ***Statistics 8.2***

Name Statistics 8.2

Ready, Set, Go!



Ready

Topic: Law of Large Numbers

1. You and your friend are rolling one die over and over again. After 6 rolls, your friend has rolled four fives. Are you surprised by these results? Explain

2. After rolling the die 50 times, you know notice that your rolled a total of 20 fives. Are you surprised now? Explain.

3. You survey 100 people in your school and ask them if they feel your school has adequate parking. Only 30% of the sample feels the school has enough parking. If you have 728 students total in your school, how many would you expect out of all the student body that felt there was enough parking?

Set

Topic: Normal Curves

4. The population of NBA players is Normally distributed with a mean of 6'7" and a standard deviation of 3.9 inches. (wikepdia) Greg is considered unusually tall for his high school at 6' 2".

   a. What percent of NBA players are taller than Greg?

   b. What percent are shorter?

   c. How tall would Greg have to be in order to be in the top 2.5% of NBA player heights?

5. The average height of boy's at Greg's school is 5'6" with a standard deviation of 2 ". If we assume the population is Normal,

a. What percent of students is Greg taller than in his school?

b. What percent of students are between 5' and 5'8"?

6. Jordan is drinking a cup of hot chocolate. From previous research, he knows that it takes a cup of hot chocolate 10 minutes to reach a temperature where his tongue will not burn. The time it takes the chocolate to cool varies Normally with a standard deviation of 2 minutes.

a. How long should he wait to drink his hot chocolate if he wants to be 84% sure that he won't burn himself?

b. If he waits 8 minutes, what percent of the time will he burn his tongue?

Go

Topic: Logarithms

Use the properties of logarithms to expand the expression as a sum or difference, and or constant multiple of logarithms. (Assume all variables are positive.)

7. $\log_2 3x$

8. $\log_x \frac{5}{7}$

9. $\ln\sqrt[3]{x}$

10. $\log \frac{x^2 2y^4}{3z^2}$

11. $\log_3 \frac{16x^2-36}{x^2}$

12. $\log \frac{x^2+12x+20}{5}$

13. $\log_3 27x^7$

14. $\log 10^5\sqrt{y}$

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8.3 Y B Normal?

A Solidify Understanding Task

As a college admissions officer, you get to evaluate hundreds of applications from students that want to attend your school. Many of them have good grades, have participated in school activities, have done service within their communities, and all kinds of other attributes that would make them great candidates for attending the college you represent. One part of the application that is considered carefully is the applicants score on the college entrance examination. At the college you work for, some students have taken the ACT and some students have taken the SAT.

You have to make a final decision on two applicants. They are both wonderful students with the very same G.P.A. and class rankings. It all comes down to their test scores. Student A took the ACT and received a score of 29 in mathematics. Student B took the SAT and received a score of 680 in mathematics. Since you are an expert in college entrance exams, you know that both tests are designed to be normally distributed. A perfect ACT is 36. The ACT mathematics section has a mean of 21 and standard deviation of 5.3. (Source: National Center for Education Statistics 2010) A perfect score on the SAT math section is 800. The SAT mathematics section has a mean of 516 and a standard deviation of 116. (Source: www.collegeboard.com 2010 Profile).

1. Based only on their test scores, which student would you choose and why?

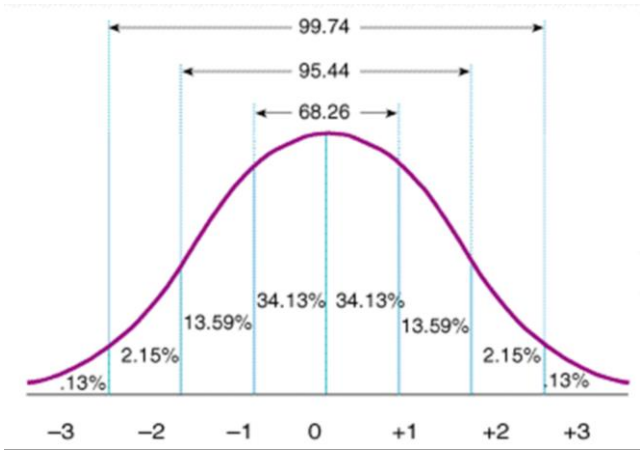
This analysis is starting to make you hungry, so you call your friend in the Statistics Department at the university and ask her to go to lunch with you. During lunch, you tell her of your dilemma. The conversation goes something like this:

You: I'm not sure that I'm making the right decision about which of two students to admit to the university. Their entrance exam scores seem like they're in about the same part of the distribution, but I don't know which one is better. It's like trying to figure out which bag of fruit weighs more

when one is measured in kilograms and one is measured in pounds. They might look like about the same amount, but you can't tell the exact difference unless you put them on the same scale or convert them to the same units.

Statistician: Actually, there is a way to make comparisons on two different normal distributions that is like converting the scores to the same unit. The scale is called the "standard normal distribution". Since it was invented to make it easy to think about a normal distribution, they set it up so that the mean is 0 and the standard deviation is 1.

Here's what your statistician friend drew on her napkin to show you the standard normal distribution:



You: Well, that looks just like the way I always think of normal distributions.

Statistician: Yes, it's pretty simple. When we use this scale, we give things a z-score. A z-score of 1 means that it's 1 standard deviation above the mean. A z-score of -1.3 means that it is between 1 and 2 standard deviations below the mean. Easy-peasy.

What's even better is that when we have a z-score there are tables that will show the area under the curve to the left of that score. For a test score like the ACT or SAT, it shows the percentage of the population (or sample) that is below that score. I've got a z-score table right here in my purse. See, the z-score is -1.3, then 9.68% of the population scored less. You can also say that 90.32% of the population scored better, so -1.3 wouldn't be a very good score on a test.

Try it: Let's say you had two imaginary test takers, Jack and Jill. Jack's z-score was 1.49 and Jill's z-score was 0.89.

2. What percent of the test takers scored below Jack? What percent scored above Jack?

3. What percent of the test takers scored below Jill? What percent scored above Jill?

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4. What percent of the test takers scored between Jack and Jill?

5. Jack and Jill's friend, Jason, scored -1.49. Find the number of test takers that scored above him without using a table or technology. Explain your strategy.

You: That's very cool, but the two scores I'm working with are not given as z-scores. Is there some way that I can transform values from some normal distribution like the scores on the ACT or SAT to z-scores?

Statistician: Sure. The scale wouldn't be so amazing if you couldn't use it for any normal distribution. There's a little formula for transforming a data point from any normal distribution to a standard normal distribution:

$$\text{z-score} = \frac{data\ point - mean}{standard\ deviation}$$

6. So, if you have an ACT score of 23. The mean score on the ACT is 21 and the standard deviation is 5.2. What would you estimate the z-score to be?

7. Let's use the formula to figure it out: z-score = $\frac{23-21}{5.2}$. How was your estimate? Explain why this value is reasonable.

You: That's great. I'm going back to the office to decide which student is admitted.

8. Compare the scores of Student A and Student B. Explain which student has the highest mathematics test score and why.

Y B Normal?– Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to use student understanding of normal distributions to understand the standard normal distribution. In this task, students will be introduced to the z-score formula and using z-score tables to find the percent of the distribution that were above or below a particular point on the distribution. Students will use z-scores to compare scores on the ACT and SAT tests, which are both normally distributed with different means and standard deviations.

Core Standards Focus:

S.ID.4: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Related Standards: S.ID.1

Vocabulary: z-score, standard normal distribution

Note: Students will need some explanation of how to use a z-score table to complete the task. Because of the amount of text and the procedures in the task, this task requires more teacher guidance than usual.

Launch (Whole Class): Begin the discussion by explaining the situation and ensuring that students understand the context of ACT and SAT tests being used for college entrance. Ask students to work individually to decide which score that they believe is better and then discuss arguments from the class. Be sure to bring out the idea that both of the scores are very good (more than 2 standard deviations above the mean), but they are on totally different scales. After this brief discussion, read through the scenario with the class (up to problem #2) and spend some time being sure they understand what a z-score is and how to use a table to look up areas (the percent of the distribution below that point. Introduce the term "standard normal distribution" for the distribution curve. Model drawing a vertical line on the distribution curve through the point under consideration and then looking up the number in the table. Be sure that students see the relationship between the area under the curve and the number in the table.

Explore (Small Group): Let students work on the rest of the problems. As they are working, be sure that they are using the table correctly and interpreting the areas on the distribution curve. As they move on, check on how they are using the formula to convert to z-scores. Identify students for the discussion that are ready to use the z-scores to make an argument about which student should be admitted.

Discuss (Whole Class): Begin the discussion with questions 3, 4, and 5, which are direct applications of using the z-score table. For each question, have the student share how to use the distribution curve to visualize and interpret the areas that they are looking up in the table. Move the discussion to #8. Have a student show how to convert each of the students' scores to z-scores. Mark the scores on the standard normal distribution. Ask a student to show how they used the scores to find the percentage of test-takers that scored above and below Student A and Student B.

Aligned Ready, Set, Go: ***Statistics 8.3***

Name Statistics 8.3

Ready, Set, Go!

Ready

Topic: Probability

At South Beach High School, there are 2500 students attending. Mariana surveys 40 of her friends where they prefer to eat lunch. She created the following two-way table showing her results:

	9th Grade	10th Grade	11th Grade	12th Grade	Totals
School Cafeteria	18	6	2	1	**28**
Off Campus	2	4	3	4	**12**
Totals	**20**	**10**	**5**	**5**	**40**

Mariana plans to use her data to answer the following questions:

I. Do students prefer to eat on campus or off campus overall?
II. Is there a difference between grade levels for where students prefer to eat lunch?

1. In Mariana's sample, what percent of students prefer school lunch?

What percent prefer to eat off campus?

2. For each grade level in her sample, determine the percent of students that prefer school lunch and the percent that prefer off campus lunch. Do you notice anything unusual?

3. Based on her sample, Mariana concludes that students at South Beach High school overall like school lunch. Do you agree or disagree? Why?

Name Statistics 8.3

Set

A company makes a mean monthly income of $20,300 with a standard deviation of $3,200. In one given month the company makes $29,500.

4. Find the z-score.

5. Assuming the companies monthly income is Normal, what percent of the time does the company make more than this amount? Less than?

6. What percent of the time does the company make between $15,000 and $25,000?

7. If the company needs to make $16,400 in order to break even, how likely in a given month is the company to make a profit?

On the Wechsler Adult Intelligence Scale, an average IQ is 100 with a standard deviation of 15 units. (Source: http://en.wikipedia.org/wiki/Intelligence_quotient)

8. IQ scores between 90 and 109 are considered average. Assuming IQ scores follow a Normal distribution, what percent of people are considered average?

9. One measure of Genius is an IQ score of above 135. What percent of people are considered genius?

10. Einstein had an IQ score of 160. What is his z-score?

11. What is the probability of an individual having a higher IQ than Einstein?

Name Statistics | 8.3

Go

Topic: Sketching Polynomials

Without using technology, sketch the graph of a polynomial function with the given characteristics.

12. A quartic function with a leading coefficient of -2 with one double zero and two complex roots.

13. $f(x) = (x+2)^2(x-3)^5$

14. $g(x) = -2(x-3)^2(x+5)(2x-5)^3$

15. A cubic function with a leading coefficient of 4 and three positive roots.

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8.4 Whoa! That's Weird!

A Practice Understanding Task

Each of the stories below are based upon normal distributions. Rank order these stories from most unusual to most average. (1 is the most unusual, 6 is the most average.) In each case, explain your ranking.

A. The number of red loops in a box of Tutti-Frutti-O's is normally distributed with mean of 800 loops and standard deviation 120. Tony bought a new box, opened it, and counted 1243 red loops. (It didn't really matter because all the colors are the same flavor anyway.)

Rank _____ Explanation: _____

B. The weight of house cats is normally distributed with a mean of 10 pounds and standard deviation 2.1 pounds. My cat, Big Boy, weighs 6 pounds.

Rank _____ Explanation: _____

C. The lifetime of a battery is normally distributed with a mean life of 40 hours and a standard deviation of 1.2 hours. I just bought a battery and it died after just 20 hours

Rank _____ Explanation: _____

D. The amount that a human fingernail grows in a year is normally distributed with a mean growth of 3.5 cm and a standard deviation of 0.63 cm. My neighbor's thumbnail grew all year without breaking and it is 4.6 cm long with stars and stripes painted on it.

Rank _____ Explanation: _____

E. My little brother was digging in the garden and found a giant earthworm that was 35 cm long. The length of earthworms is normally distributed with a mean length of 14 cm and a standard deviation of 5.3 cm.

Rank _____ Explanation: _____

F. The mean length of a human pregnancy is 268 days with a standard deviation of 16 days. My aunt just had a premature baby delivered after only 245 days.

Rank _____ Explanation: _____

Whoa! That's Weird! - Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is to practice using the features of a normal distribution to decide how unusual a particular event is. Students will rank order these events from most unusual to least unusual using the 68-95-99.7 rule, z-scores, and logic. Technology and/or z-score tables will be necessary to distinguish between some of the events.

Core Standards Focus:

S.ID.4: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Related Standards: S.ID.1

Launch (Whole Class): Introduce the task by telling students that when something happens that is out of the ordinary, we often tend to think that it is stranger than it actually is. Tell them that this task is about of a number of "strange" occurrences. The need to use their knowledge about normal distributions to rank each of these occurrences from most unusual to least unusual. Tell them that they may want to use technology for some of the problems, but it isn't necessary for all of them.

Explore (Small Group): Monitor students as they are working to hear how they are using the information given to rank the events. It is possible, but not necessary, to come up with the ranking by converting to z-scores and using a table or calculator to find the percent of the distribution to the left of z. Encourage students to do as many as they can without resorting to a procedure. Listen for strategies such as categorizing based on the number of standard deviations away from the mean, so that this can be used in the discussion. There may be some discussion about the idea that some of the "weird" things are on the far right of the distribution and others are on the far left. This may be a great way to start the discussion.

Discuss (Whole Class): Start the discussion by asking students to share their initial strategies for ranking. Sort through the idea that an event that is three standard deviations to the left of the mean is just as unusual as an event that is three standard deviations to the right. Ask a student to share that used the strategy of grouping the events that are within 1 standard deviation, between 1 and 2 standard deviations, between 2 and 3, and greater than 3. Then, ask students how they can rank within those categories. Finally, have students share how they used z-scores to determine the ones that were too close to tell otherwise.

Aligned Ready, Set, Go: ***Statistics 8.4***

Name Statistics 8.4

Ready, Set, Go!

Ready

Topic: Two-Way Tables

The data below is the data from Mrs. Hender's class. Students needed to score a 60% or better to pass the test.

1st hour:	2nd hour:	3rd hour:
72, 83, 56, 63, 89, 92, 92, 67, 88, 84, 67, 97, 96, 100, 84, 82	80, 83, 81, 81, 67, 90, 70, 71, 72, 77, 81, 85, 86, 77, 74, 51	51, 45, 67, 83, 99, 100, 94, 52, 48, 46, 100, 59, 65, 56, 72, 63

1. Make a two-way frequency table showing how many students passed the test and how many failed each class.

	1st	2nd	3rd	Total
Passed				
Failed				

2. What percent of students passed Mrs. Hender's test in each class? What is the total percent that passed?

3. Use the data from all three classes to create a histogram. What properties of the Normal curve does your histogram have?

4. If Mrs. Hender's were going to predict her total pass rate using only 2nd hour, would she have a good prediction? Explain why or why not.

Name Statistics | 8.4

Set Topic: Normal Curves

5. Five track athletes are in the running for the Athletic Performance of the Year award. A panel of coaches is trying to decide which athlete is the most deserving to win the award. Rank each athlete below by the given information. Assume all distributions follow a Normal Curve.

a. Javier threw the Javelin 215 ft. The average Javelin throw is 152.08 ft. with a standard deviation of 15.85 ft.
b. Chance ran a 400m time of 46.99 seconds. The average 400m time was 52.6, with a standard deviation of 1.01 seconds.
c. Derick ran a 36.26 in the 300m Hurdles. The average time was 41.77 with a standard deviation of 1.49 seconds.
d. Chad ran a 100m time of 10.59 seconds. The average time was 11.603 seconds with a standard deviation of .29 seconds.
e. Kayden threw the discus 180 ft. The average throw was 122.4 ft. with a standard deviation of 14.38 ft.

Go Topic: Logarithms

Solve each equation below for x by applying properties for exponents and logarithms.

6. $2^{x-5} = 128$

7. $\left(\frac{x}{243}\right) = 27$

8. $3^{x+2} = 27^{x-3}$

9. $\log(2x+4) - \log(3x) = 0$

10. $\log_2(2x^2+4x-2) - \log_2 10 = 0$

11. $\frac{\ln(x+7)}{\ln(2x-3)} = 1$

12. $\frac{\log(4x+2)}{\log 15} = 1$

13. $\frac{\log_3(3x+6)}{\log_3 81} = 1$

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8.5 Would You Like to Try a Sample?

A Develop Understanding Task

In the task *Whoa! That's Weird!*, you saw a number of statistics for things like the average weight of a house cat. You know it would be impossible to measure all the house cats to find their average weights, but scientists still claim to know it.

You've probably heard it many times before: "Survey results show that 54% of Americans believe that. . ." You're sure that you didn't participate in the survey and neither did anyone you know, and yet, the researchers claim that the survey represents the beliefs of all Americans.

How can this be possible? In the next few tasks, we'll explore how statistics allow us to draw conclusions about an entire group without actually working with the entire group. Sometimes the results make sense and other times you might think that they just can't be right. We will learn how to make judgments about statistical studies, based on the methods that have been used.

First, we need to get our terms straight. When we talk about the entire group that we are interested in, that is called the **population.** When some members of the group are selected to represent the entire group, that is called a **sample**. The thing we are interested in knowing about the population is the **parameter of interest**.

For each of the scenarios below, identify the population, the sample and the population parameter of interest.

1. A grocery store wants to know the average number of items that shoppers purchase in each visit to the store. They decide to count the items in the cart of every twentieth person through the check stand.

   Population _____

   Sample _____

   Parameter of interest _____

2. A team of biologist wants to know the average weight of fish in a lake. They decide to drop a net and measure all the fish caught in three different locations in the lake.

   Population _____

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Sample _____

Parameter of interest _____

3. There are lots of different ways that a sample can be chosen from a population. Group the following examples of ways to select a sample into six categories.

A. You are in charge of school activities. You want to know what activities students would prefer to participate in during the school year. You decide to put the name of each student in the school into a big bowl. You draw 100 names and ask those students to respond to a survey about the activities they prefer.

B. You are in charge of school activities. You want to know what activities students would prefer to participate in during the school year. You assign each student in the school a number. You randomly select a starting number among the first 10 numbers and then select every tenth student in the list from that point forward.

C. You are in charge of school activities. You want to know what activities students would prefer to participate in during the school year. You use the rolls from each homeroom class. You put the all the names from one class into the bowl and draw two names from the class. You go through each homeroom class, drawing 2 names from each class. You ask those students to respond to a survey about the activities they prefer.

D. You are in charge of school activities. You want to know what activities students would prefer to participate in during the school year. You get the list of all the homeroom classes and randomly select 5 classes. You go to each of the classes selected and survey all the students in that class.

E. You are in charge of school activities. You want to know what activities students would prefer to participate in during the school year. You stand in the cafeteria during your lunch break and ask students in they would be willing to participate in your survey as they walk by.

F. You are in charge of school activities. You want to know what activities students would prefer to participate in during the school year. You use the rolls from each homeroom class. You put the all the names from one class into the bowl and draw two names from the class. You go through each homeroom class, drawing 2 names from each class. You ask those students to respond to a survey about the activities they prefer.

G. You are in charge of school activities. You want to know what activities students would prefer to participate in during the school year. You make a lot of copies of the survey about

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the activities that students prefer and you put them on a table outside the cafeteria. Students can choose to take the survey and drop their responses into a big box on the table.

H. You are interested in finding out the percent of residents in the city that have experienced a robbery in the past year. Using the city property records, you assign each residence a number. You use a random number generator to give you a list of numbers. You contact the residence that corresponds to that number to ask your questions.

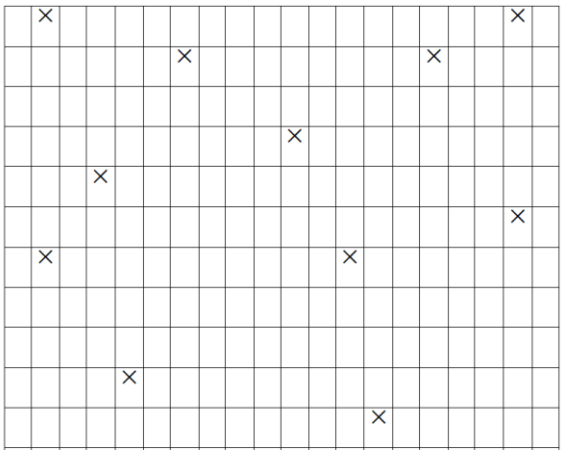
I. You want to know the average number of hours that high school seniors spend playing video games in your state. You randomly select 20 high schools in the state and then ask all the seniors at each of the 20 high schools about their video game habits.

J. An auto analyst is conducting a satisfaction survey, sampling from a list of 10,000 new car buyers. The list includes 2,500 Ford buyers, 2,500 GM buyers, 2,500 Honda buyers, and 2,500 Toyota buyers. The analyst selects a sample of 400 car buyers, by randomly sampling 100 buyers of each brand.

K. A shopping mall management company would like to know the average amount that shoppers in the mall spend during their visit. They post two survey takers near one of the exits who ask shoppers to tell them what they spent as they leave the mall.

L. A restaurant owner wants to find out the average number of dishes ordered at each table served on Friday evenings, their busiest time. She decides to collect and analyze every fifth receipt of the night, starting at 6:00 p.m.

M.



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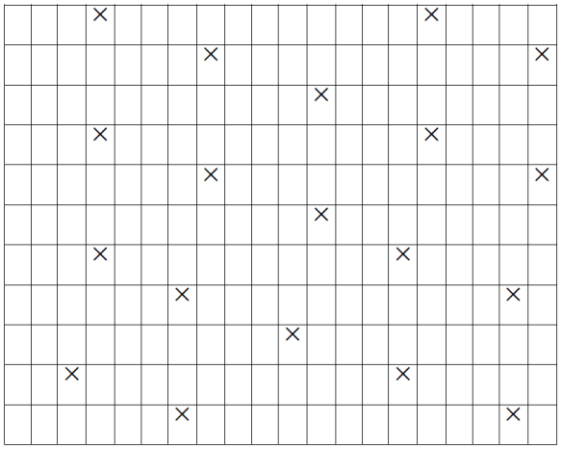

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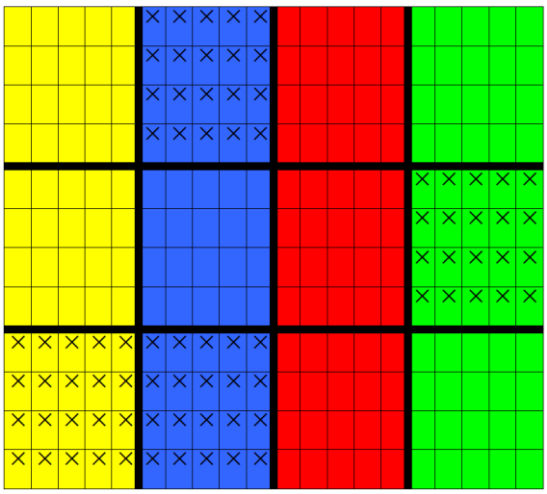

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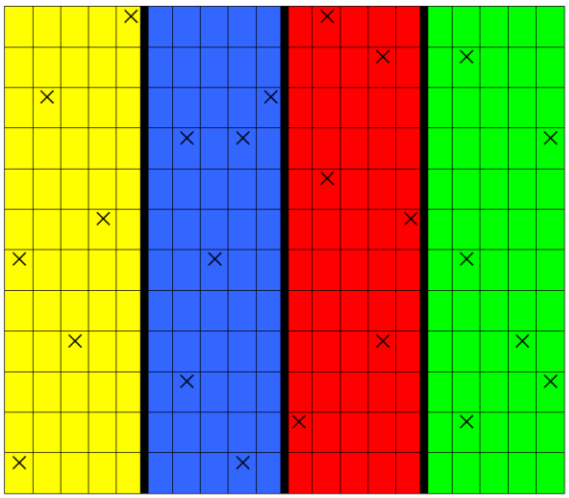


4. What might be some of the advantages and disadvantages of each type?

5. A person you know owns a small theater that shows local dramatic productions. She wants to know the average age of the people that buy tickets to the see the shows so that she can better select which plays to stage. Explain to the owner why selecting the first 20 people that arrive for the show may not be a representative sample.

6. Describe a process for selecting a representative sample of the theater patrons.

Would You Like to Try a Sample?– Teacher Notes

A Develop Understanding Task

Purpose: The purpose of this task is to introduce students to the idea of using a random sample from a population to investigate and make an inference about a parameter of interest. In the task, students sort various sampling methods and consider strengths and weaknesses of each type. The idea that a sample must be truly random to be representative of the population is developed.

Core Standards Focus:

Understand and evaluate random processes underlying statistical experiments.

S.IC.1 Understand that statistics allows inferences to be made about population parameters based on a random sample from that population.

Related Standards:

Vocabulary: population, sample, parameter, inference, simple random sample, systematic sample, cluster sample, stratified random sample, convenience sample, volunteer sample

Launch (Whole Class): Begin the lesson by explaining to students that a lot of statistics are about trying to draw valid conclusions about some topic of interest. Usually, the population is too big to include every member of the population in the investigation, whatever the design. So, investigators usually take a sample that they hope represents the entire population. Work through questions 1-3 with the class to be sure that they understand that there can be many ways to select a sample, with pros and cons to each method. Tell students that for the next part of the task, they will be given examples of several different sample methods. They will need to pay attention to how the participants in the sample are chosen and to sort these methods into categories. It may be useful to cut up the examples so that they can easily be moved around as students sort and re-sort.

Explore (Small Group): Monitor students as they are working, making sure that they are attending to some of the distinguishing details in the examples. There are actually six categories, although students may justify sorting in various ways. Watch for students that have identified two basic categories – random and not random, because this is a fundamental idea. There are four random types here – simple, systematic, cluster, and stratified. There are two non-random types – convenience and volunteer. Encourage students to use the diagrams to help sort the two major groups into the smaller categories.

Discuss (Whole Class): Begin the discussion with a student that has the two basic categories, even if all their sorting is not correct. Ask the class how they might name the two categories based on the features of each. Record the features of the two categories. Then ask students to present that can

successively break down the categories. Continue to note the features of the categories until all six have been identified. Then, give students the names for the categories and discuss some of the merits of each. Ask what might be the problems associated with convenience and volunteer samples, emphasizing that neither of these two types are actually strictly random.

The sampling methods and their examples are:

Simple random sample (A, G, L)

Systematic random sample (B, K, M)

Stratified random sample (C, I, O)

Cluster random sample (D, H, N)

Convenience sample (E, J)

Volunteer sample (F)

Turn the discussion to question #5. Discuss why this is not a random sample and how using this method may skew the results. Ask students which of the random sampling methods might be used in this case and how participants could be selected.

Aligned Ready, Set, Go: ***Statistics 8.5***

Name Statistics 8.5

Ready, Set, Go!

Ready

Topic: Causation

When collecting data, statisticians are often interested in making predictions. Sometimes, statisticians simply want to know if one variable **explains** another variable. Often times, statisticians want to determine if one variable actually **causes** a change in another variable. Given the examples below, decide whether you think the variables simply explain each other, or if you think one variable would cause the other to change.

1. As the amount of food Ollie the elephant eats increases her weight also increases. (Explains/Causes)

2. As Popsicle sales go up in the summer, the number of drownings also increases. (Explains/Causes)

3. As Erika's feet grow longer, she grows taller. (Explains/Causes)

4. As Tabatha gets older, her reading score improves in school. (Explains/Causes)

Set

For the following scenarios, identify the **population**, **sample** and **parameter** of interest.

5. The local school board wants to get parents to evaluate teachers. They select 100 parents and find that 89% approve of their child's teacher.

Population: Sample: Parameter:

6. Jarret wants to know the average height of the students in his school. There are 753 students in his high school; he finds the heights of 52 of them.

Population: Sample: Parameter:

7. A government official is interested in the percent of people at JFK airport that are searched by security. He watches 300 people go through security and observes 42 that are searched.

Population: Sample: Parameter:

Name Statistics 8.5

For each scenario, identify what type of sampling was used to obtain the sample. Explain whether or not you think the sample will be representative of the population it was sampled from:

8. Elvira surveys the first 60 students in the lunch line to determine if students at the school are satisfied with school lunch.

Type of sample:

Representative? Explain.

9. Elvira selects every 5th student in the lunch line to determine if students at the school are satisfied with school lunch.

Type of sample:

Representative? Explain.

10. Elvira randomly selects 7 different tables in the lunchroom and surveys every student on the table to determine if students at the school are satisfied with school lunch.

Type of sample:

Representative? Explain.

11. Elvira assigns every student in the school a number and randomly selects 60 students to survey to determine if student at the school are satisfied with school lunch.

Type of sample:

Representative? Explain.

12. Elvira wants to determine if students are satisfied with school lunch. She leaves surveys on a table for students to answer as the walk by.

Type of sample:

Representative? Explain.

13. Elvira wants to determine if students are satisfied with school lunch. She wants to include input from each grade level at the high school. She randomly surveys 25 freshman, 25 sophomores, 25 juniors, and 25 seniors.

Type of sample:

Representative? Explain.

Name Statistics | 8.5

Go

Topic: Graphs of Trig Functions

For each function identify the amplitude, period, horizontal shift, vertical shift and the endpoints of the primary interval.

14. $f(t) = 120\cos\left(\frac{\pi}{4}(t-3)\right) + 30$

Amplitude:

Period:

Horizontal Shift:

Vertical Shift:

End Points:

15. $f(t) = 3.5\sin\left(\frac{\pi}{6}t + \frac{1}{3}\right) + 7$

Amplitude:

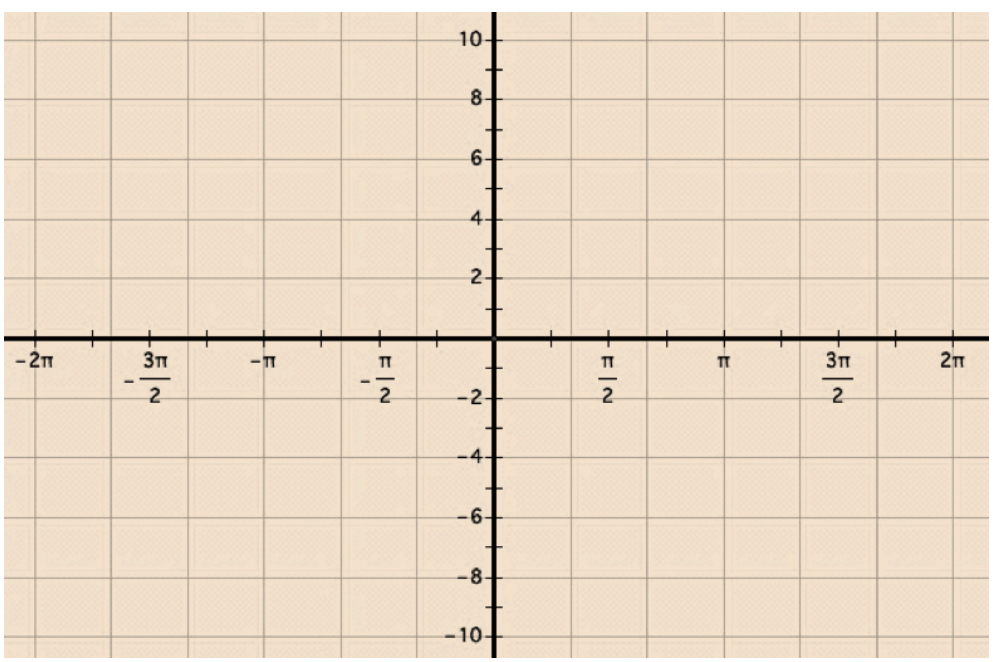
Period:

Horizontal Shift:

Vertical Shift:

End Points:

15. Graph $f(x) = \frac{1}{2}\sin(x-3) + 2$.



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8.6 Let's Investigate

A Solidify Understanding Task

When we want to draw conclusions about some population, there are at least two different statistical ideas to consider. We learned about sampling in *Would You Like to Try a Sample*, since it is usually more practical to sample the population rather than somehow measure everyone or everything in the population.

The second thing to consider is how to measure the parameter of interest, the thing we want to know about the population. Sometimes it's obvious, like if you want to know the average weight of a population, you determine a sample and then put each of the subjects on a scale. Three other techniques are the following:

- **Surveys:** When they want to know how people feel, what their preferences are, what they own, how much they make, etc., researchers often construct a survey to ask the people in the sample about the parameter of interest.
- **Observational Studies:** In this type of study, researchers observe the behavior of the participants/subjects without trying to influence it in any way so they can learn about the parameter of interest.
- **Experiments:** In an experiment, researchers manipulate the variables to try to determine cause and effect.

1. Imagine that you want to know whether a new diet plan is effective in helping people lose weight. You might choose any of the three methods to determine this.

If you used a survey, you could simply ask people that had tried the diet plan in they lost weight.

If you used an observational study, you might monitor volunteers that try the diet plan and measure how much weight they lost.

If you used an experiment, you might randomly assign participants to two groups. One group (the control group) eats as they normally would and the other group (the experimental group) eats according to the diet plan. At the end of two months, the two groups are compared to see the average weight gain or loss in each group.

Based on these three examples,

a. What are some possible advantages and disadvantages of surveys?

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b. What are some possible advantages and disadvantages of observational studies?

c. What are some possible advantages and disadvantage of experiments?

2. Identify which method is illustrated by each example:

a. To determine whether drinking orange juice prevents colds, researchers randomly assigned participants to a group that drank no orange juice or a group that drank two glasses of orange juice a day. They measured the number of colds that each group had over the course of the year and compared the results of the two groups.

b. To determine whether exercise reduces the number of headaches, researchers randomly selected a group of participants and recorded the number of hours each participant exercised and the number of headaches each participant experienced.

c. To determine the effectiveness of a new advertising campaign, a restaurant asked every tenth customer if they had seen the advertisement, and if it had influenced their decision to visit the restaurant.

d. To determine if a new drug is an effective treatment for the flu, researchers randomly selected two groups of people that had the flu. One group was given a placebo (a sugar pill that has no physical effect) and one group was given the new drug. Researchers measured the number of days that participants experienced flu symptoms and compared the two groups to see if they were different.

e. To determine if higher speed limits cause more traffic fatalities, researchers compared the number of traffic deaths on randomly selected stretches of highway with 65 mph speed limits to the number of traffic deaths on an equal number of randomly selected stretches of highway with 75 mph speed limits.

3. Describe how you might select a sample and use a survey to investigate which soft drink people prefer: Soda A or Soda B.

4. Describe how you might select a sample and use an observational study to investigate which soft drink people prefer: Soda A or Soda B.

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5. Describe how you might select a sample and use an experiment to investigate if consuming large quantities of Soda A causes headaches.

6. Describe the method you would use to determine if excessive texting causes bad grades. Explain why you chose that method and what conclusions could be drawn from the study.

That's Simulating! - Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of the task is for student to learn to identify three methods of investigating a parameter of interest: survey, observational study, and experiment. The task begins with an explanation of the three methods including an example of each. Students are then asked to identify which method is illustrated by some other examples and to consider the advantages and disadvantages of each method. Finally, students a given a situation and asked to design an investigation including selecting a sample and a method of investigating the parameter of interest.

Core Standards Focus:

S.IC.3: Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

Related Standards: S.IC.1

Vocabulary: survey, observational study, experiment, control group, experimental group

Launch (Whole Class): Begin the task by reading through the description of the three investigation methods with the class and discussing the examples. Gives students a chance to ask questions and clarify the differences between each of the three types.

Give students a few minutes to think about the advantages and disadvantages of each type and then discuss their answers. During this discussion, include the idea that the validity of the results will partially depend on how the sample was selected as well as the method of the study. For instance, surveys have the advantage of being easy to gauge opinions. A disadvantage may be the difficulty in writing questions that accurately measure the parameter of interest. No matter how good the survey questions are, the survey may not yield useful results if it is not based on a representative sample of the population. After discussing the ideas of each method, then ask students to complete the task.

Explore (Small Group): Listen to students as they are working to see that they are noticing the distinguishing characteristics of each method. Students often have difficulty with the difference between an observational study and an experiment. Be prepared to help them understand that it is not an experimental design if there is not a control group and an experimental (treatment) group.

Try to identify students to present each of the three methods as a study design for #6.

Discuss (Whole Class): Begin the discussion by quickly identifying the method for each of the examples in #2. For each example, clarify the distinguishing features of that method.

Turn the discussion to question #3. Ask a student to present how they might select a sample and design a survey. Ask the class what questions they might ask to obtain an unbiased result.

Have a student present their design for question #4. Ask students how they can ensure that the study participants' preferences are not influenced by the observer or other factors.

Ask students to present their experimental design for question #5. How will students select the two groups so they are comparable? What will be the treatment? What will they maintain the control group?

Finally, discuss question #6. If you have identified students to present each of the three methods, then ask them to share their thinking and have the class critique their design including the sampling method. If there is a method that was not used by any student in the class, facilitate a discussion with the whole class to design a study together using that method.

Aligned Ready, Set, Go: ***Statistics 8.6***

Ready, Set, Go!

Ready

Topic: Finding probabilities from a two-way table.

The following data represents a random sample of boys and girls and how many prefer cats or dogs. Use the information to answer the questions below.

	Cats	**Dogs**	**Total**
Boys	32	68	100
Girls	41	11	52
Total	73	79	152

1. $P(B) =$ 2. $P(G) =$ 3. $P(C) =$ 4. $P(D) =$

5. $P(C|G) =$ 6. $P(C\ or\ B) =$ 7. $P(D|B) =$ 8. $P(B \cap D) =$

9. If this is a random sample from a school, what total percent of boys in this school do you think would prefer dogs?

10. What percent of students at the school would prefer cats?

11. If you sampled a different 152 students, would you get the same percentages? Explain.

12. What would happen to your percentages if you used a larger sample size?

Set

For the following scenarios, identify each situation as a survey, observational study, or an experiment.

13. To determine if a new pain medication is effective, researchers randomly assign two groups of people to use the pain medication in group 1 and a placebo in group 2. Both groups are asked to rate their pain and the results are compared.

14. Officials want to determine if raising the speed limit from 75 mph to 80 mph will have an impact on safety. To determine this, they watch a stretch of the highway when the speed limit is 75 and see how many accidents there are. Then they observe the number of accidents over a period of time on the same stretch of highway for a speed limit of 80 mph. They then compare the difference.

15. To determine if a new sandwich on the menu is liked more than the original, the manager of the restaurant takes a random sample of customers that have tried both sandwiches and asks them which sandwich they prefer.

16. A newspaper wants to know what their customer satisfaction is. They randomly select 500 customers and ask them.

Mrs. Goodmore wants to know if doing homework actually helps students do better on their unit exams.

17. Describe how Mrs. Goodmore could carry out a survey to determine if homework actually helps. Explain the role of randomization in your design.

18. Describe how Mrs. Goodmore could carry out an observational study to determine if homework helps test scores.

19. Describe how Mrs. Goodmore could carry out an experiment to determine if homework helps test scores. Explain how you will use randomization in your design and how you will use a control.

20. If Mrs. Goodmore wants to determine if homework causes test scores to rise, which method would be best? Why?

Go

Topic: Normal Curves

The average resting heart rate of a young adult is approximately 70 beats per minute with a standard deviation of 10 beats per minute. Assuming resting heart rate follows a Normal Distribution, answer the following questions.

21. Draw and label the Normal curve that describes this distribution. Be sure to label the mean, and the measurements 1, 2, and 3 standard deviations out from the mean.

22. What percent of people have a heart rate between 55 and 80 beats per minute? Label these points on your Normal curve above and shade in the area that represents the percent of people with heartbeats between 55 and 80 beats per minute.

23. If a resting heart rate above 80 beats per minute is considered unhealthy, what percent of people have an unhealthy heart rate?

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8.7 Slacker's Simulation

A Solidify Understanding Task

I know a student who forgot about the upcoming history test and did not study at all. To protect his identity, I'll just call him Slacker. When I reminded Slacker that we had a test in the next class, he said that he wasn't worried because the test has 10 true/false questions. Slacker said that he would totally guess on every question, and since he's always lucky, he thinks he will get at least 8 out of 10.

I'm skeptical, but Slacker said, "Hey, sometimes you flip a coin and it seems like you just keep getting heads. You may only have a 50/50 chance of getting heads, but you still might get heads several times in a row. I think this is just about the same thing. I could get lucky."

1. What do you think of Slacker's claim? Is it possible for him to get 8 out of 10 questions right? Explain.

I thought about it for a minute and said, "Slacker, I think you're on to something. I'm not sure that you will get 80% on the test, but I agree that the situation is just like a coin flip. It's either one way or the other and they are both equally likely if you're just guessing." My idea is to use a coin flip to simulate the T/F test situation. We can try it many times and see how often we get 8 out of 10 questions right. I'm going to say that if the coin lands on heads, then you guessed the problem correctly. If it lands on tails, then you got it wrong.

Try it a few times yourself. To save a little time, just flip 10 coins at once and count up the number of heads for each test.

	# Correct (Heads)	# Incorrect (Tails)	% Correct
Test 1			
Test 2			
Test 3			
Test 4			
Test 5			

Did you get 8 out of 10 correct in any of your trials?

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Based on your trials, do you think Slacker has a good chance of getting 80% correct?

Use technology to 50 simulate more tests. Now what do you think of Slacker's chances of getting 80% correct. Explain why.

Slacker's Simulation - Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is for students to decide if a particular result from a model is likely using a simulation. In this task, students simulate the results of a true/false test using a coin flip. The results are collected for the entire class and discussed with the idea that the particular result is possible, but unlikely.

Core Standards Focus:

Understand and evaluate random processes underlying statistical experiments.

S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of five tails in a row cause you to question the model?

Related Standards:

Vocabulary: simulation

Note: There are web applets available that will flip ten coins and collect the data. One such applet is: http://polymer.bu.edu/java/java/1drw/1drwapplet.html. The task is written so that data can be collected from the trials done in class and displayed on a graphing calculator or spreadsheet. Students will need coins or two-colored chips to flip.

Launch (Whole Class): Begin the task by reading the scenario and asking students if they think it is possible for Slacker to get 80%. Some students will argue that he would just get 5/10 right, since the odds of getting a question right is 50%. Ask them, if the odds are 50%, and Slacker gets 5 questions in a row correct, does this make it less likely that he will get the sixth question correct? Reinforce the idea that each question is an independent event.

Explore (Small Group): Give students ten coins or chips to toss for the simulation and ask them to complete questions 2 and 3. Be prepared with a way for the class to collect and display the data from the entire class.

Discuss (Whole Class): Based on the data collected from the class, ask students if they believe that Slacker has a good chance of getting 8/10 correct. The distribution may not look particularly normal, but consider how to use the distribution from the simulation to think about Slacker's idea. Then discuss question 5. Why might we expect that the distribution would be symmetrical? Why might we expect the distribution to center at 5/10? What is the standard deviation of the distribution? Where is 8/10 on the distribution? About what percent of the distribution is below

8/10? Make the point that the normal distribution that occurs for a large number of samples in this case, helps us to understand whether or not a particular sample might represent the entire population.

Aligned Ready, Set, Go: ***Statistics 8.7***

Name Statistics 8.7

Ready, Set, Go!

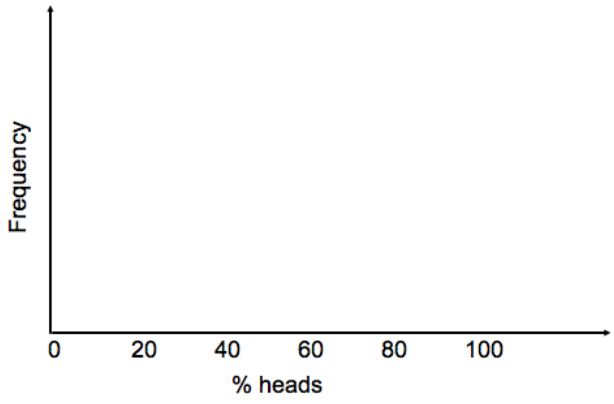
Ready

Topic: Features of Histograms

1. Take a coin and flip it 5 times. Record the number of times the coin landed with heads up. Repeat this process 20 times either by hand or by simulation using technology, http://www.rossmanchance.com/applets/CoinTossing/CoinToss.html each time recording your results in the table below.

# Heads	% Heads	Frequency
0	0%	
1	20%	
2	40%	
3	60%	
4	80%	
5	100%	

2. Create a histogram of your results below. Describe the shape of the histogram (Shape, Center, Spread)

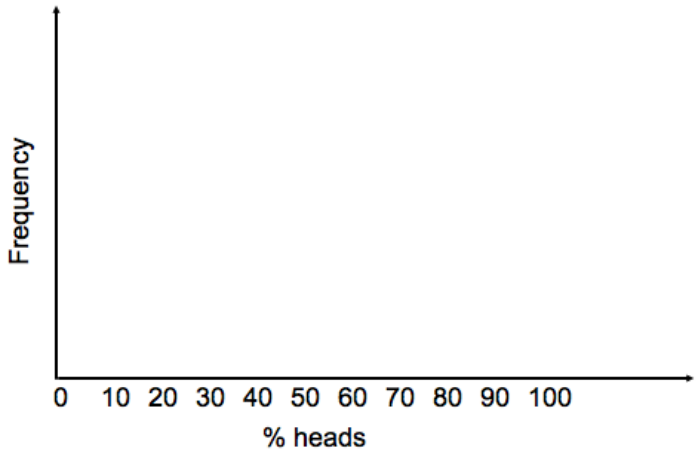


3. Flip a coin 5 times. Record the number of times heads lands side up. Repeat this process 20 times either by hand or by simulation using technology.
http://www.rossmanchance.com/applets/CoinTossing/CoinToss.html

Record your results in the table below.

# Heads	% Heads	Frequency	# Heads	% Heads	Frequency
0	0%		11	55%	
1	5%		12	60%	
2	10%		13	65%	
3	15%		14	70%	
4	20%		15	75%	
5	25%		16	80%	
6	30%		17	85%	
7	35%		18	90%	
8	40%		19	95%	
9	45%		20	100%	
10	50%				

4. Create a histogram of your results below. Describe the shape of the histogram (Shape, Center, Spread)



5. Compare the shape center and spread of each distribution. What do you notice?

6. If you repeated this process with 500 flips instead of 5 or 20, predict what would happen to the shape, spread, and center of the new histogram.

Set

In 1963, NBC started to host a game called *Let's Make a Deal!* Contestants were given three doors to choose from. Behind one door was a prize. After selecting one door, the contestant was shown what was behind one of the doors they did not select. The contestant is then asked if they would like to stick with the door they first selected, or switch to the remaining one.

7. Which strategy do you think would result in the best chance of selecting the winning door? Should the contestant switch doors, or stick with the first one they chose?

Go to the following website: http://nlvm.usu.edu/en/nav/category_g_3_t_2.html

Select the applet stick or switch.

8. Play the game 20 times using the stick method and 20 times using the switch method. Record your wins and losses in the table below:

	Stick	Switch	Total
Win			
Lose			
Total			

9. Based on the simulation, what is $P(winning|stick) =$

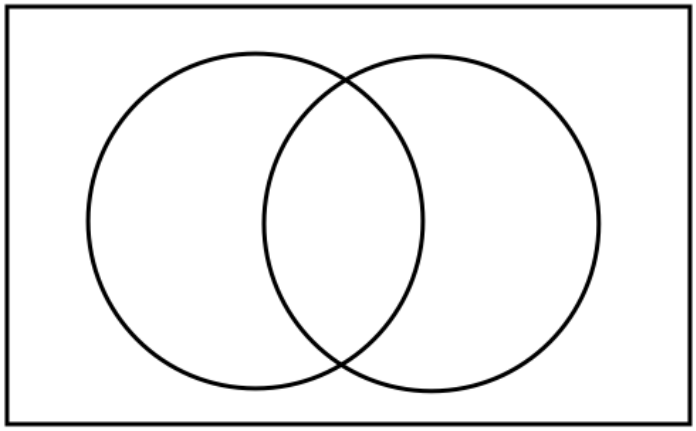
10. Based on the simulation, what is $P(winning|switch) =$

11. Click on the multiple games tab. Simulate 100 games for each strategy. What is the probability of winning using each method?

Go

Topic: Probability

12. For your two-way table in problem 8, create a Venn diagram and a tree diagram below.



13. $P(winning) =$

14. $P(winning \cap sticking) =$

15. $P(winning \cup sticking) =$

16. $P(loosing|sticking) =$

17. $P(winning\ or\ loosing) =$

18. Are the events winning and sticking independent of each other? Justify your answer using probabilities.