mathematics vision project

Transforming Mathematics Education

SECONDARY MATH THREE

An Integrated Approach

MODULE 8

Modeling With Functions

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

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MODELING WITH FUNCTIONS

8.1 Function Family Reunion

A Solidify Understanding Task

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During the past few years of math classes you have studied a variety of functions: linear, exponential, quadratic, polynomial, rational, radical, absolute value, logarithmic and trigonometric.

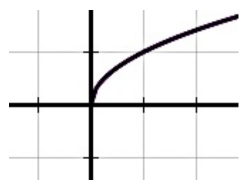
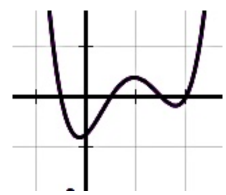
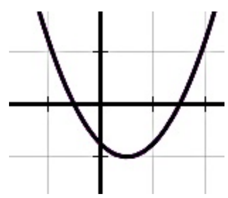
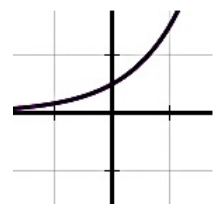
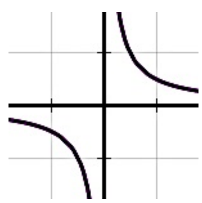
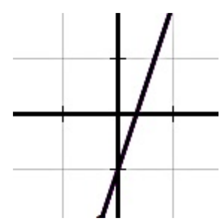
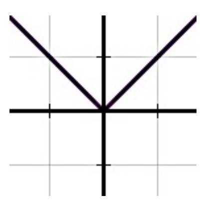
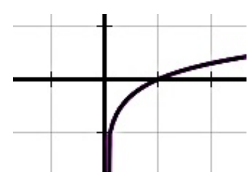
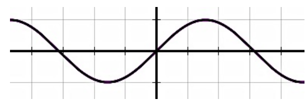
Like a family, each of these types of functions have similar characteristics that differ from other types of functions, making them uniquely qualified to model specific types of real world situations. Because of this, sometimes we refer to each type of function as a "family of functions."

1. Match each function family with the algebraic notation that best defines it.

Function Family Name	***Algebraic Description of the Parent Function***
1. *linear*	**A.** $y = \lvert x \rvert$
2. *exponential*	**B.** $y = a\sin(bx)$ ***or*** $y = a\cos(bx)$
3. *quadratic*	**C.** $y = mx + b$
4. *polynomial*	**D.** $y = \log_b(x)$
5. *rational*	**E.** $y = ax^2 + bx + c$
6. *absolute value*	**F.** $y = \frac{1}{x}$
7. *logarithmic*	**G.** $y = ab^x$
8. *trigonometric*	**H.** $y = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_0$
9. *radical*	**I.** $y = \sqrt[n]{x}$

Just like your family, each member of a function family resembles other members of the family, but each has unique differences, such as being "wider" or "skinnier", "taller" or "shorter", or other features that allow us to tell them apart. We might say that each family of functions has a particular "genetic code" that gives its graph its characteristic shape. We might refer to the simplest form of a particular family as "the parent function" and consider all transformations of this parent function to be members of the same family.

2. Match each function family with the characteristic shape of the graph that fits it.

Function Family Name	***Characteristic Shape of the Graph***	
1. *linear* **2. *exponential***	**A.** 	**B.** 
3. *quadratic* **4. *polynomial***	**C.** 	**D.** 
5. *rational* **6. *absolute value***	**E.** 	**F.** 
7. *logarithmic* **8. *trigonometric***	**G.** 	**H.** 
9. *radical*	**I.** 	

Function family characteristics are passed on to their "children" through a variety of transformations. While the members of each family shares common characteristics, transformations make each member of a family uniquely qualified to accomplish the mathematical work they are required to do.

For each of the following tables, a set of coordinate points that captures the characteristics of a parent graph is given. The additional columns give coordinate points for additional members of the family after a particular transformation has occurred. Write the rule for each of the different transformations of the parent graph. (Note: We can think of each new set of coordinate points (that is, the *image* points) as a geometric transformation of the original set of coordinate points (that is, the *pre-image* points) and use the notation associated with geometric transformations to describe transformation. Or, we can write the rule using algebraic function notation. Use both types of notation to represent each transformation.)

3.

	pre-image (parent graph)	*image 1*	*image 2*	*image 3*
geometric notation	(x, y)	$(x, y) \rightarrow (x,\ y + 2)$		
function notation	$f(x) = x^2$	$f_1(x) = x^2 + 2$		
selected points that fit this image	(-2, 4)	(-2, 6)	(-2, 8)	(-3, 4)
	(-1, 1)	(-1, 3)	(-1, 2)	(-2, 1)
	(0, 0)	(0, 2)	(0, 0)	(-1, 0)
	(1, 1)	(1, 3)	(1, 2)	(0, 1)
	(2, 4)	(2, 6)	(2, 8)	(1, 4)

4.

	pre-image (parent graph)	*image 1*	*image 2*	*image 3*
geometric notation	(x, y)			
function notation	$f(x) = 2^x$			
selected points that fit this image	(-2, $\frac{1}{4}$)	(-2, 1)	(-2, $\frac{-1}{4}$)	(-3, $\frac{1}{4}$)
	(-1, $\frac{1}{2}$)	(-1, 2)	(-1, $\frac{-1}{2}$)	(-2, $\frac{1}{2}$)
	(0, 1)	(0, 4)	(0, -1)	(-1, 1)
	(1, 2)	(1, 8)	(1, -2)	(0, 2)
	(2, 4)	(2, 16)	(2, -4)	(1, 4)

5.

	pre-image (parent graph)	*image 1*	*image 2*	*image 3*
geometric notation	(x, y)			
function notation	$f(x) = \|x\|$			
selected points that fit this image	(-2, 2)	(-2, -4)	(2, 2)	(-5, 2)
	(-1, 1)	(-1, -2)	(3, 1)	(-4, 1)
	(0, 0)	(0, 0)	(4, 0)	(-3, 0)
	(1, 1)	(1, -2)	(5, 1)	(-2, 1)
	(2, 2)	(2, -4)	(6, 2)	(-1, 2)

6.

	pre-image (parent graph)	*image 1*	*image 2*	*image 3*
geometric notation	(x, y)			
function notation	$f(x) = \sin(x)$			
selected points that fit this image	(0, 0)	(0, 2)	(0, 0)	(0, 0)
	($\pi/2$, 1)	($\pi/2$, 3)	($\pi/4$, 1)	($\pi/2$, -2)
	(π, 0)	(π, 2)	($\pi/2$, 0)	(π, 0)
	($3\pi/2$, -1)	($3\pi/2$, 1)	($3\pi/4$, -1)	($3\pi/2$, 2)
	(2π, 0)	(2π, 2)	(π, 0)	(2π, 0)

7.

	pre-image (parent graph)	*image 1*	*image 2*	*image 3*
geometric notation	(x, y)			
function notation	$f(x) = \sqrt{x}$			
selected points that fit this image	(0, 0)	(0, 0)	(0, 0)	(3, 0)
	(1, 1)	(1, ½)	(½, 1)	(4, 1)
	(4, 2)	(4, 1)	(2, 2)	(7, 2)
	(9, 3)	(9, $3/2$)	($9/2$, 3)	(12, 3)
	(16, 4)	(16, 2)	(8, 4)	(19, 4)

READY, SET, GO! Name Period Date

READY

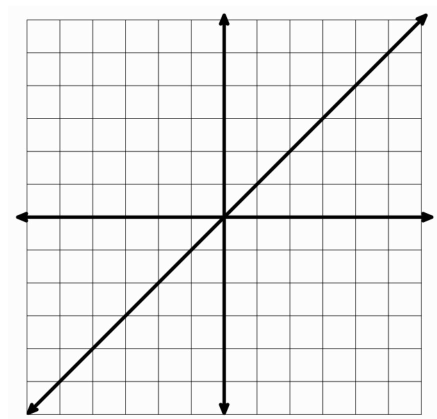
Topic: Reviewing transformations

1. Graph the following linear equations on the grid. The equation y = x has been graphed for you. For each new equation explain what the number 2 does to the graph of y = x. Pay attention to the y-intercept, the x-intercept, and the slope. Identify what changes in the graph and what stays the same.

   a. $y_1 = x + 2$

   b. $y_2 = x - 2$

   c. $y_3 = 2x$



2. Graph the following quadratic equations on the grid. The equation $y = x^2$ has been graphed for you. For each new equation explain what the number 3 does to the graph of $y = x^2$. Pay attention to the y-intercept, the x-intercept(s), and the rate of change. Identify what changes in the graph and what stays the same.

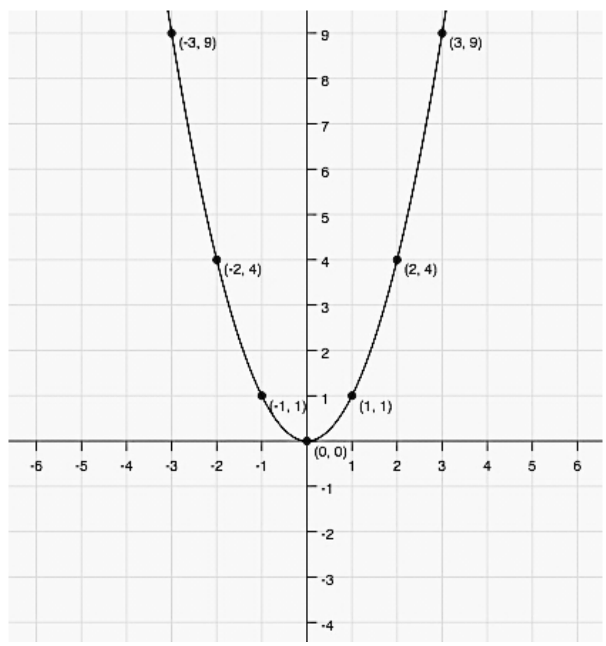
   a. $y_1 = x^2 + 3$

   b. $y_2 = x^2 - 3$

   c. $y_3 = (x - 3)^2$

   d. $y_4 = (x + 3)^2$

   e. $y_5 = 3x^2$

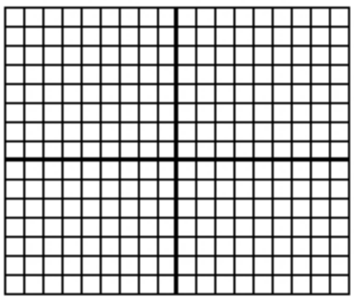


SET

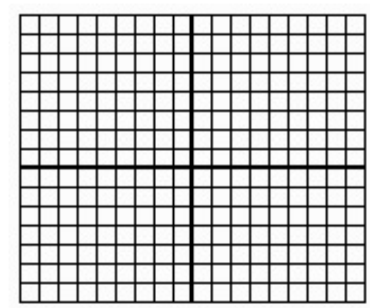
Topic: Transforming parent functions

Sketch the graph of the parent function and the graph of the transformed function on the same set of axes.

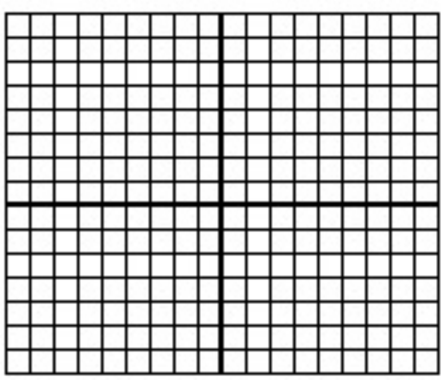
3. $f(x) = |x|, and\ g(x) = |x+3|$



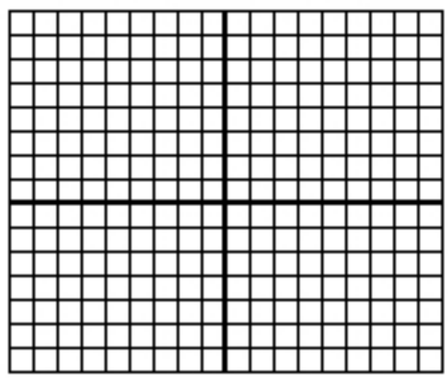
4. $h(x) = 2^x, and\ j(x) = 2^{-x}$



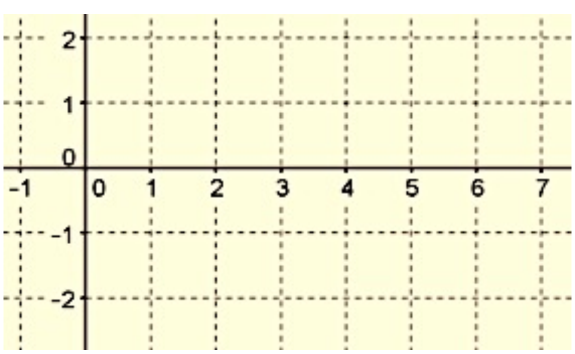
5. $r(x) = x^2, and\ s(x) = -\frac{1}{2}x^2 + 5$



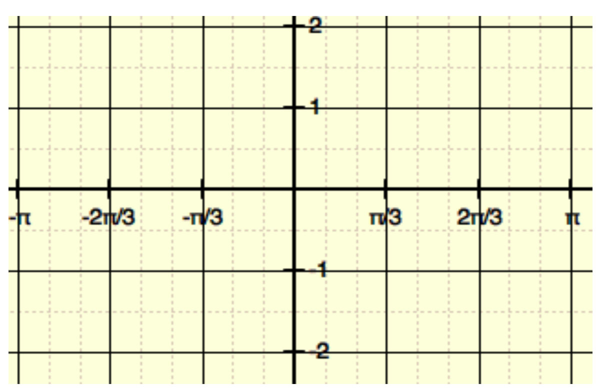
6. $v(x) = \frac{1}{x}, and\ w(x) = -\frac{1}{x}$



7. $k(x) = log(x), and\ m(x) = -1 + log(x)$



8. $p(x) = sin(x), and\ q(x) = 2sin\left(x + \frac{\pi}{3}\right)$



GO

Topic: Evaluating functions

Find the function values: $f(-2), f(0), f(1), and\ f(3)$. Indicate if the function is undefined for a given value of x.

9. $f(x) = |x+5|$

10. $f(x) = |x-2|$

11. $f(x) = x|x|$

12. $f(x) = 3^x$

13. $f(x) = 3^{x+2}$

14. $f(x) = (3^x) + x$

15. $f(x) = \frac{x}{x}$

16. $f(x) = \frac{x}{(x-4)}$

17. $f(x) = \frac{x}{(x+2)} - 5$

18. $f(x) = log_3 x$

19. $f(x) = log_7(7)^x$

20. $f(x) = xlog_{10}1000$

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8.2 Imagineering

A Develop Understanding Task

You are excited to get to vote on the plans for a proposed new thrill ride at a local theme park. The engineers want public input on the design for the new ride. You are one of ten teenagers who have been selected to review the plans based on your good math grades!

As your excitement mounts, the engineers begin their presentation. To your dismay, there are no models or illustrations of the proposed rides—each ride is described only with equations. The equations represent the path a rider would follow through the course of the ride.

Unfortunately, your cell phone—which contains a graphing calculator app—is completely discharged due to too much texting and surfing the internet. So, you are trying hard to keep up with the presentation by trying to imagine what the graphs of each of these equations would look like. While each equation consists of functions you are familiar with, the combination of functions in each equation has you wondering about their combined effects.

For each of the following proposed thrill rides, use your imagination and best reasoning about the individual functions involved to sketch a graph of the path of the rider. Let y represent the height of the rider above the ground and x represent the distance from the start of the ride. Explain your reasoning about the shape of the graph. (Note: Use radians for trigonometric functions.)

Proposal #1: "*The Mountain Climb*"

The Equation: $y = 2x + 5\sin(x)$

My Graph:

My Explanation:

Proposal #2: "*The Periodic Bump*"

The Equation: $y = |10\sin(x)|$

My Graph:

My Explanation:

Proposal #3: "*The Amplifier*"

The Equation: $y = x \cdot \sin(x)$

My Graph:

My Explanation:

Proposal #4: "*The Gentle Wave*"

The Equation: $y = 10(0.9)^x \cdot \sin(x)$

My Graph:

My Explanation:

Proposal #5: "*The Spinning High Dive*"

The Equation: $y = 100 - x^2 + 5\sin(4x)$

My Graph: My Explanation:

When you got home your friends were all anxiously waiting to hear about the proposed new rides. After explaining the situation, your friends all pull out their calculators and they began comparing your imagined images with the actual graphs.

Some of your friends' graphs differed from the others because of their window settings. Some window settings revealed the features of the graphs you were expecting to see, while other window settings obscured those features.

Examine the actual graphs of each of the thrill ride proposals. Select a window setting that will reveal as many of the features of the graphs as possible. Explain any differences between your imagined graphs and the actual graphs. What features did you get right? What features did you miss?

Proposal #1: "*The Mountain Climb*"

The Equation: $y = 2x + 5\sin(x)$

Actual Graph: What features I got right and what I missed:

Proposal #2: "*The Periodic Bump*"

The Equation: $y = |10\sin(x)|$

Actual Graph:

What features I got right and what I missed:

Proposal #3: "*The Amplifier*"

The Equation: $y = x \cdot \sin(x)$

Actual Graph:

What features I got right and what I missed:

Proposal #4: "*The Gentle Wave*"

The Equation: $y = 10(0.9)^x \cdot \sin(x)$

Actual Graph:

What features I got right and what I missed:

Proposal #5: "*The Spinning High Dive*"

The Equation: $y = 100 - x^2 + 5\sin(4x)$

Actual Graph:

What features I got right and what I missed:

You and your friends decide to propose a different ride to the engineers. Name your proposal and write its equation. Explain why you think the features of this graph would make a fun ride.

My Proposed Ride:

The equation for my ride:

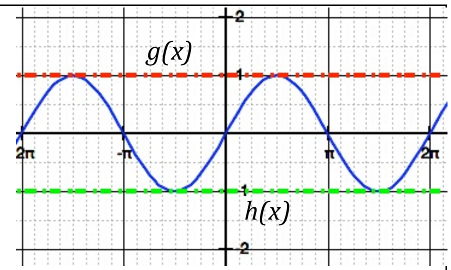
My explanation of my proposal:

READY, SET, GO! Name Period Date

READY

Topic: Exploring the boundaries of functions

1. The curve in the graph at the right shows the graph of $f(x) = sin(x)$. (blue)

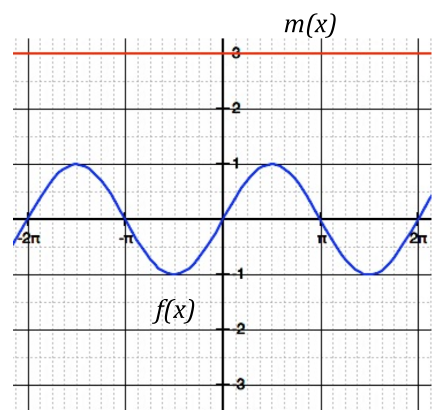


a. Write the equation of the dotted line labeled $g(x)$. (red)

b. Write the equation of the dotted line labeled $h(x)$. (green)

c. List everything you notice about the graphs of $f(x), g(x), and\ h(x)$.

2. The curve in the graph at the right shows the graph of $f(x) = sin(x)$.



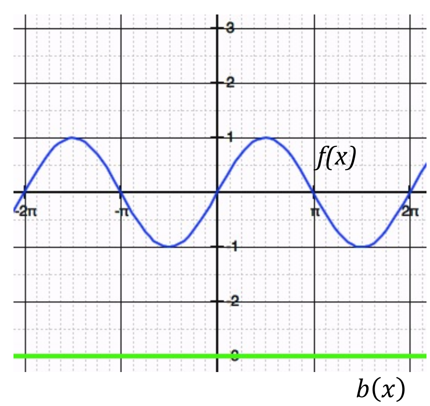
a. Write the equation of the line labeled $m(x)$. (red)

b. Sketch in the graph of $f(x) * m(x)$.

c. What is the equation of $f(x) * m(x)$?

d. Would the line y = -3 also be a boundary line for your sketch? Explain.

3. The curve in the graph at the right shows the graph of $f(x) = sin(x)$. (blue)



a. Write the equation of the line labeled $b(x)$. (green)

b. Sketch in the graph of $f(x) * b(x)$.

c. What is the equation of $f(x) * b(x)$?

d. How is the graph of $f(x) * b(x)$ <u>different</u> from the graph of $f(x) * m(x)$?

e. Would the line y = 3 also be a boundary line for your sketch? Explain.

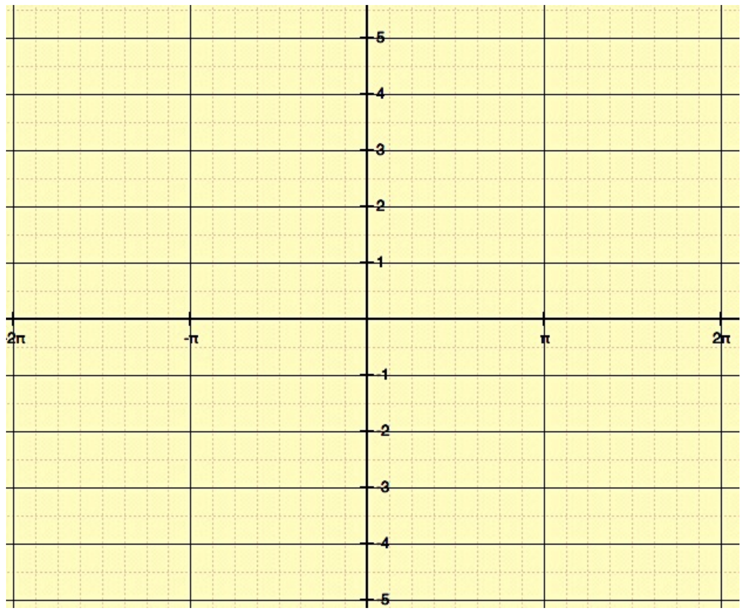
SET

Topic: Combining functions

4. $f(x) = x$ $\quad g(x) = \sin(x)$ $\quad h(x) = f(x) + g(x)$

Fill in the values for *h(x)* in the table. Then graph $h(x) = x + sin(x)$ with a smooth curve.

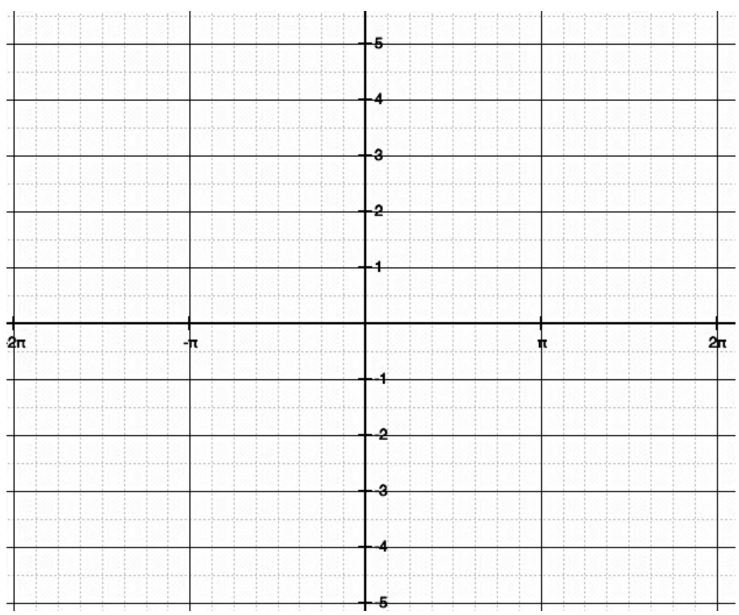
x	*f (x)*	*g(x)*	*h(x)*
-2π	-6.28	0	
$-\frac{3\pi}{2}$	-4.71	1	
-π	-3.14	0	
$-\frac{\pi}{2}$	-1.57	-1	
0	0	0	
$\frac{\pi}{2}$	1.57	1	
π	3.14	0	
$\frac{3\pi}{2}$	4.71	-1	
2π	6.28	0	



5. $f(x) = x$ $\quad g(x) = \sin(x)$

Now graph $k(x) = f(x) * g(x)$ or $k(x) = x \, * sin(x)$

x	*f (x)*	*g(x)*	*k(x)*
-2π	-6.28	0	
$-\frac{3\pi}{2}$	-4.71	1	
-π	-3.14	0	
$-\frac{\pi}{2}$	-1.57	-1	
0	0	0	
$\frac{\pi}{2}$	1.57	1	
π	3.14	0	
$\frac{3\pi}{2}$	4.71	-1	
2π	6.28	0	



Match each equation below with the appropriate graph. Describe the features of the graph that helped you match the equations.

6. $f(x) = |x^2 - 4|$
key features:

7. $g(x) = -x + 5\sin(x)$
key features:

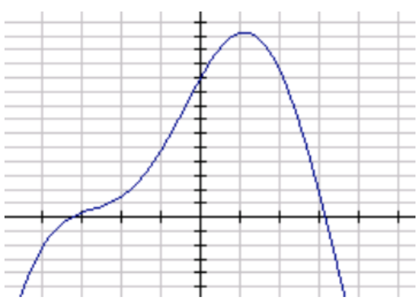
8. $h(x) = 4|sinx|$
key features:

9. $d(x) = 10 - x^2 + 5sin(x)$
key features:

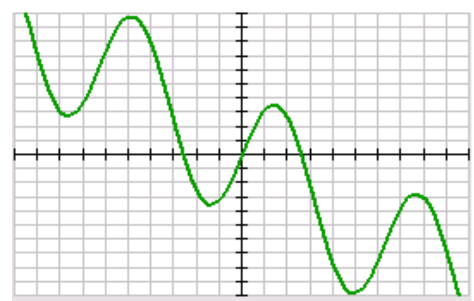
10. $w(x) = -x * 2sin(x)$
key features:

11. $r(x) = (2x - 4) + |x|$
key features:

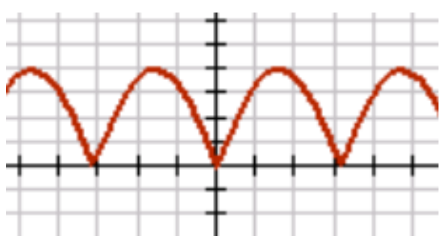
a)



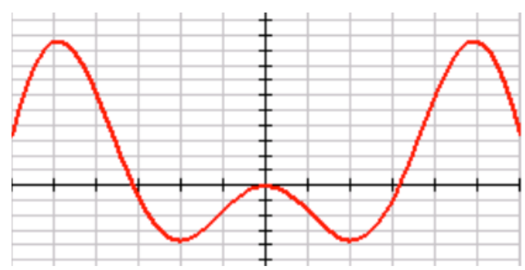
b)



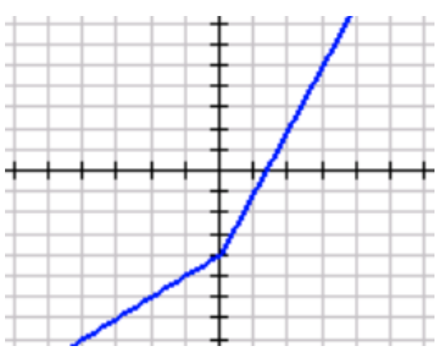
c)



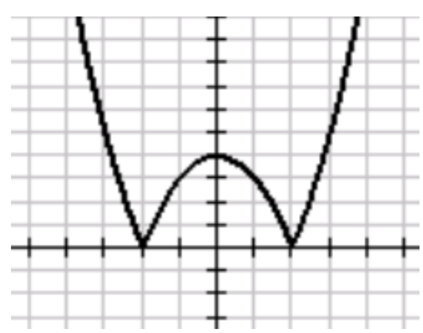
d)



e)



f)



GO

Topic: Identifying key features of function families

The chart below names five families of functions and the parent function. The parent is the equation in its simplest form. In the right hand column is a list of key features of the functions in random order. Match each key feature with the correct function. A key feature may relate to more than one function.

<table>
<tr><th>Family</th><th>Parent(s)</th><th>Key features</th></tr>
<tr><td>12. Linear</td><td>$y = x$</td><td rowspan="5">a) The ends of the graph have the same behavior.
b) The graphs have a horizontal asymptote and a vertical asymptote.
c) The graph only has a horizontal asymptote.
d) These functions either have both a local maximum and minimum or neither a local maximum and minimum.
e) The graph is usually defined in terms of its slope and y-intercept.
f) The graph has either a maximum or a minimum but not both.
g) As x approaches -∞, the function values approach the x-axis.
h) The ends of the graph have opposite behavior.
i) The rate of change of this graph is constant.
j) The rate of change of this graph is constantly changing.
k) This graph has a linear rate of change.
l) These functions are of degree 3.
m) The variable is an exponent.
n) These functions contain fractions with a polynomial in both the numerator and denominator.
p) The constant will always be the y-intercept.</td></tr>
<tr><td>13. Quadratic</td><td>$y = x^2$</td></tr>
<tr><td>14. Cubic</td><td>$y = x^3$</td></tr>
<tr><td>15. Exponential</td><td>$y = 2^x$
$y = 3^x$
etc.</td></tr>
<tr><td>16. Rational</td><td>$y = \frac{1}{x}$</td></tr>
</table>

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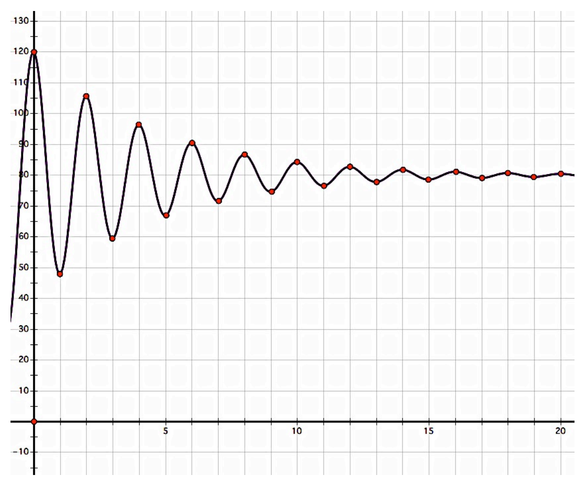
8.3 The Bungee Jump Simulator

A Solidify Understanding Task

As a reward for helping the engineers at the local amusement park select a design for their next ride, you and your friends get to visit the amusement park for free with one of the engineers as a tour guide. This time you remember to bring your calculator along, in case the engineers start to speak in "math equations" again.

Sure enough, just as you are about to get in line for the *Bungee Jump Simulator*, your guide pulls out a graph and begins to explain the mathematics of the ride. To prevent injury, the ride has been designed so that a bungee jumper follows the path given in this graph. Jumpers are launched from the top of the tower at the left, and dismount in the center of the tower at the right after their up and down motion has stopped. The cable to which their bungee cord is attached moves the rider safely away from the left tower and allows for an easy exit at the right.

Your tour guide won't let you and your friends get in line for the ride until you have reproduced this graph on your calculator exactly as it appears in this diagram.



hor	***vert***	***distance from midline***
0	**120**	40
1	48	32
2	**105.6**	25.6
3	59.52	20.48
4	**96.38**	16.38
5	66.89	13.11
6	**90.49**	10.49
7	71.61	8.39
8	**86.71**	6.71
9	74.63	5.37
10	**84.30**	4.30
11	76.56	3.44
12	**82.75**	2.75
13	77.80	2.20
14	**81.76**	1.76
15	78.59	1.41
16	**81.13**	1.13
⋮	⋮	

1. Work with a partner to try and recreate this graph on your calculator screen. Make sure you pay attention to the height of the jumper at each oscillation, as given in the table.

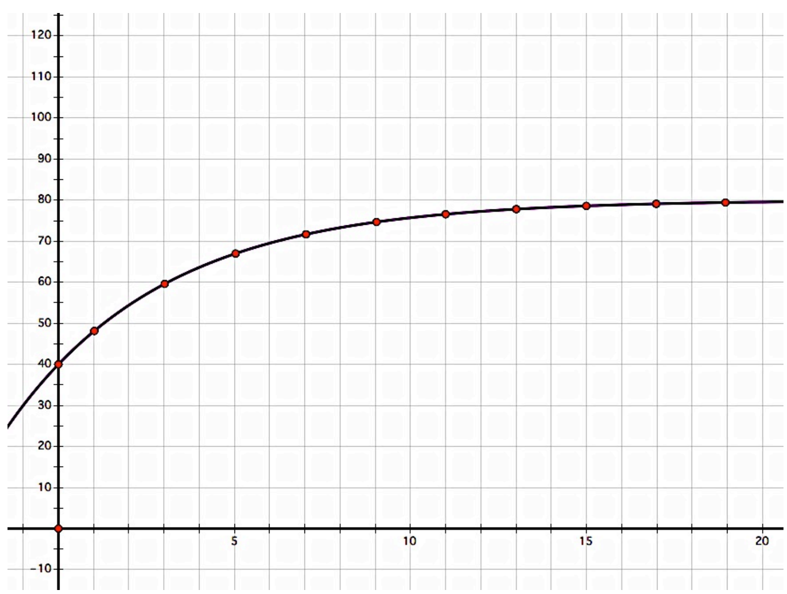
Record your equation of this graph here:

After a thrilling ride on the *Bungee Jump Simulator*, you are met by your host who has a new puzzle for you. "As you are aware," says the engineer, "temperatures around here are very cold at night, but very warm during the day. When designing rides we have to take into account how the metal frames and cables might heat up throughout the day. Our calculations are based on Newton's Law of Heating. Newton found that while the temperature of a cold object increases when the air is warmer than the object, the rate of change of the temperature slows down as the temperature of the object gets closer to the temperature of its surrounding."

Of course the engineer has a graph of this situation, which he says "represents the decay of the difference between the temperature of the cables and the surrounding air."

Your friends think this graph reminds them of the points at the bottom of each of the oscillations of the bungee jump graph.

2. Using the clue given by the engineer, "This graph represents the *decay* of the difference between the temperature of the cables and the surrounding air," try to recreate this graph on your calculator screen. (Hint: What types of graphs do you generally think of when you are trying to model a growth or decay situation? What transformations might make such a graph look like this one?)



Record your equation of this graph here:

READY, SET, GO! Name Period Date

READY

Topic: Evaluating functions

Evaluate each function as indicated. Simplify your answers when possible. State *undefined* when applicable.

1. $f(x) = x^2 - 8x$

   a) $f(0)$ *b)* $f(-10)$ *c)* $f(5)$ *d)* $f(8)$ *e)* $f(x+2)$

2. $g(x) = \frac{3x-5}{x}$

   a) $g(-1)$ *b)* $g(10)$ *c)* $g\left(\frac{1}{3}\right)$ *d)* $g(0)$ *e)* $g(2x+4)$

3. $h(x) = sin(x)$

   a) $h(\pi)$ *b)* $h\left(\frac{3\pi}{2}\right)$ *c)* $h\left(\frac{11\pi}{6}\right)$ *d)* $h\left(\frac{5\pi}{4}\right)$ *e)* $h\left(\cos^{-1}\left(\frac{-1}{2}\right),\ x < \pi\right)$

4. $w(x) = tan(x)$

   a) $w(\pi)$ *b)* $w\left(\frac{3\pi}{2}\right)$ *c)* $h\left(\frac{7\pi}{6}\right)$ *d)* $h\left(\frac{3\pi}{4}\right)$ *e)* $h\left(\cos^{-1}\left(\frac{-1}{2}\right),\ x < \pi\right)$

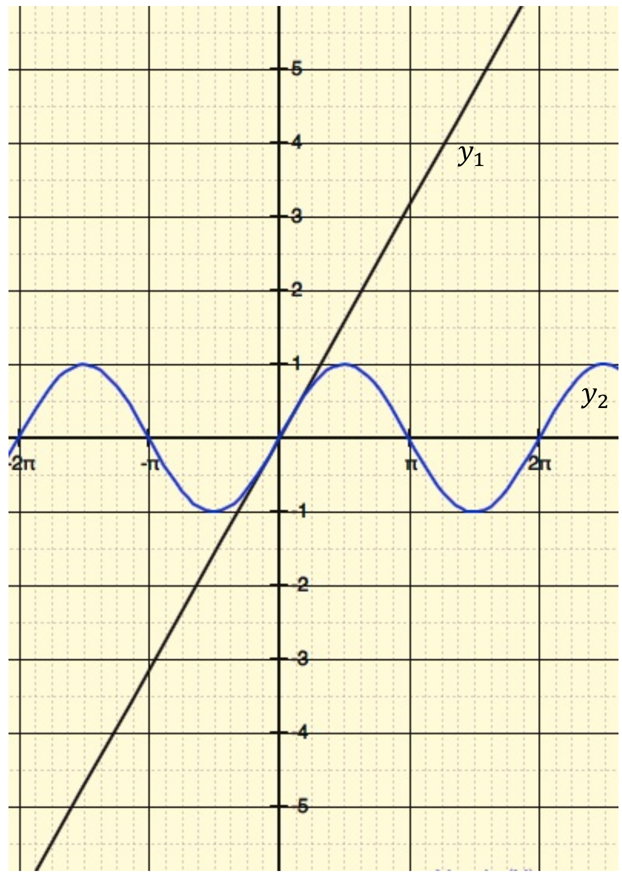
SET

Topic: Damping functions

Two functions are graphed. Graph a third function by multiplying the two functions together. Use the table of values to assist you. It may help you to change the function values to decimals.

5.

x	$y_1 = x$	$y_2 = sinx$	$y_3 = (x)sinx$
-2π			
$-\frac{3\pi}{2}$			
$-\pi$			
$-\frac{\pi}{2}$			
0			
$\frac{\pi}{2}$			
π			
$\frac{3\pi}{2}$			
2π			



6. After you have graphed y_3, graph the line $y_4 = -x$. What do you notice about the graph of y_3 in relation to the graphs of y_1 and y_4?

GO

Topic: Comparing measures of central tendency (mean, median, and mode)

During salary negotiations for teacher pay in a rural community, the local newspaper headlines announced: **Greedy Teachers Demand More Pay!** The article went on to report that teachers were asking for a pay hike even though district employees, including teachers, were paid an average of $70,000.00 per year, while the average annual income for the community was calculated to be $55,000 per household. The 65 schoolteachers in the district responded by declaring that the newspaper was spreading false information.

Use the table below to explore the validity of the newspaper report.

Job Description	Number having job	Annual Salary
Superintendent	1	$258,000
Business Administrator	1	$250,000
Financial Officer	1	$205,000
Transportation Coordinator	1	$185,000
District secretaries	5	$ 55,000
School Principals	5	$200,000
Assistant Principals	5	$175,000
Guidance Counselors	10	$ 85,000
School Nurse	5	$ 83,000
School Secretaries	10	$ 45,000
Teachers	65	$ 48,000
Custodians	10	$ 40,000

7. Which measure of central tendency (mean, median, mode) do you think the newspaper used to report the teachers' salaries? Justify your answer.

8. Which measure of central tendency do you think the teachers would use to support their argument? Justify your answer.

9. Which measure gives the clearest picture of the salary structure in the district? Justify.

10. Make up a headline for the newspaper that would be more accurate.

8.4 Composing and Decomposing

A Develop Understanding Task

As the day at the amusement park get warmer, you and your friends decide to cool off by taking a ride on the *Turbulent Waters Dive*. As you are waiting in line your tour guide explains the mathematics behind designing the waiting area for a ride.

"As you can see," says the engineer, "the waiting area can be enlarged or reduced by moving a few chains around. The area we need for waiting guests depends on the time of day. We collect data for each ride so we can use functions to model the typical wait time and how much waiting area we need to provide for our guests."

And of course, your guide has the functions that represent this particular ride.

- **Average number of people in the *TWD* line as a function of time:** $\boxed{p(t) = 3000\cos\left(\frac{1}{5}(t-3)\right)}$

  $\rightarrow$ ***t*** is the number of hours before or after noon, so $t = 2$ represents 2:00 p.m. and $t = -2$ represents 10:00 a.m.

  $\rightarrow$ ***p*** represents the number of people in line

- **Waiting area required as a function of the number of people in line:** $\boxed{a(p) = 4p + 100}$

  $\rightarrow$ ***a***, the waiting area, is measured in square feet

- **Wait time for a guest as a function of the number of people in line:** $\boxed{W(p) = 60 \cdot \left(\frac{p-1500}{1500}\right)}$

  $\rightarrow$ ***W***, the wait time, is measured in minutes

1. How much waiting area is required for the guests in line for the *Turbulent Waters Dive* at each of the times listed in the following table?

Time of Day	***Waiting Area Required* (sq. ft.)**
10:00 a.m.	
12:00 noon	
2:00 p.m.	
4:00 p.m.	
8:00 p.m.	

   a. For each instant in time you had to complete a series of calculations. Describe how you found the waiting area at different times.

   b. Can you create a single rule that will determine the waiting area as a function of the time of day?

2. What is the wait time for a guest that arrives at the end of the line for the *Turbulent Waters Dive* at each of the times listed in the following table?

Time of Day	***Wait Time* (minutes)**
10:00 a.m.	
12:00 noon	
2:00 p.m.	
4:00 p.m.	
8:00 p.m.	

   a. For each instant in time you had to complete a series of calculations. Describe how you found the wait time at different times of the day.

b. Can you create a single rule that will determine the wait time as a function of the time of day?

To maintain crowd control when the lines get long, cast members dressed as pirates (the *Turbulent Waters Dive* has a pirate theme) mingle with the waiting guests. Their antics distract the guests who listen attentively to their pirate jokes. The number of cast members needed depends on the number of people waiting in the line.

• **Number of cast members needed as a function of the number of people in line:** $\boxed{c(t) = \frac{p}{150}}$

$\rightarrow$ ***p*** represents the number of people in line

$\rightarrow$ ***c*** represents the number of cast members needed

3. How many cast members are needed to entertain and distract the waiting guests at each of the following times of the day?

Time of Day	***Cast Members Needed***
10:00 a.m.	
12:00 noon	
2:00 p.m.	
4:00 p.m.	
8:00 p.m.	
t hours before or after noon ($t < 0$ before noon, $t > 0$ after noon)	

On warm, sunny days misters are used to cool down the waiting guests. The number of misters that need to be turned on depends on the size of the waiting area that has been opened up to contain the number of people in line.

• **Number of misters needed as a function of the waiting area:**

$$\boxed{m(t) = \frac{a}{1000}}$$

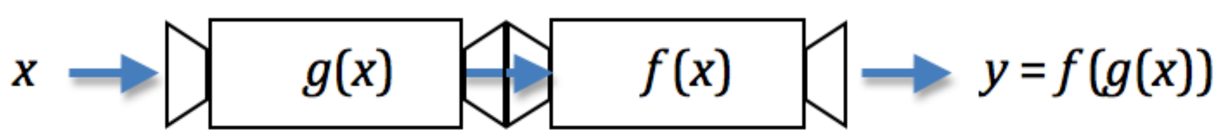
→ ***a***, the waiting area, is measured in square feet

→ ***m*** represents the number of misters to be turned on

4. How many misters need to be turned on to cool the waiting guests at each of the following times of day?

Time of Day	***Misters Needed***
10:00 a.m.	
12:00 noon	
2:00 p.m.	
4:00 p.m.	
8:00 p.m.	
t hours before or after noon ($t < 0$ before noon, $t > 0$ after noon)	

5. Explain how the following diagram might help you think about the work you have been doing on the previous problems. How does the notation used in the diagram support the way you have been combining functions in this task? This way of combining functions is called ***function composition***.



Interpreting the Functions

6. At what time of day is the number of people in line the largest?

7. What is the maximum number of people in line, based on the function for the average number of people in line?

8. When do you think the amusement park opens and closes, based on this function?

9. In terms of the story context, what do you think the 4 and the 100 represent in function rule for waiting area, $a(p) = 4p + 100$?

10. In terms of the story context, what might be the meaning of the 1500 in the function rule for wait time, $w(p) = 60 \cdot \left(\frac{p-1500}{1500}\right)$?

11. In terms of the story context, what might be the meaning of the 150 in the function rule for cast members needed, $c(t) = \frac{p}{150}$?

12. In terms of the story context, what might be the meaning of the 1000 in the function rule for the number of misters needed, $m(t) = \frac{a}{1000}$?

READY, SET, GO! Name Period Date

READY

Topic: Recognizing the order of operations in a composite function

Each expression contains 2 operations. One of the operations will be "inside" the second operation. Identify the "inside" operation as *u* by writing *u* = ________. Then substitute *u* into the expression so that the "outside" operation is being performed on *u*.

Example: $\boldsymbol{Given}$: $\mathbf{5x^3}$.

I can see two operations on x. First the x is being cubed and then x^3 is multiplied by 5. Therefore, if $u = x^3$, then $5x^3 = 5u$.

1. Would the answer in the example have been different if you were given $(5x)^3$? Explain

2. $(x-6)^2$

$u =$

3. $\tan(x+4)$

$u =$

4. $\sqrt[3]{(2x-7)}$

$u =$

5. $-9(x+5)$

$u =$

6. $\dfrac{5}{x^2}$

$u =$

7. $(\sin x)^4$

$u =$

SET

Topic: Creating formulas for composite functions

Recall that $\boldsymbol{f(g(x)) = (f \circ g)(x)}$.

8. Let $f(x) = 2x^2 - 4$ *and* $g(x) = 5x$. Find each and simplify.

   a) $(f \circ g)(1)$ b) $(g \circ f)(1)$ c) $(f \circ f)(-2)$ d) $(g \circ g)(-1)$

9. Let $f(x) = \frac{8}{x-3}$ *and* $g(x) = \frac{15}{x+1}$. Find each and simplify.

   a) $(f \circ g)(x)$ b) $(g \circ f)(x)$ c) $(f \circ f)(x)$ d) $(g \circ g)(x)$

10. Use your answers for a) and b) in problem 9 to calculate the two problems below.

    a) $(f \circ g)(-1)$ b) $(g \circ f)(3)$

11. Now use $f(x) = \frac{8}{x-3}$ *and* $g(x) = \frac{15}{x+1}$ to calculate $(f \circ g)(-1)$ and $(g \circ f)(3)$.

    a) Describe the problem that you encountered when calculating *f(x)* and *g(x)* separately.

    b) Do you think that the answer you derived in #10 is valid based on what happened in #11? Justify your answer.

12. Describe the domains for a) $(f \circ g)(x)$ b) $(g \circ f)(x)$ c) $(f \circ f)(x)$ d) $(g \circ g)(x)$

13. What makes the domain for each composition different?

GO

Topic: Writing equations of polynomials given the degree and the roots

Write the equation of the polynomial with the given features.

	Degree of polynomial	Given roots (you may have to determine others):	Leading coefficient	Equation in standard form:
14.	3	-2, 1, and -1	3	
15.	4	$(2+i), 4, 0$	1	
16.	5	1 multiplicity 2, -1 multiplicity 2, and 3	-1	
17.	4	$(3-i), \sqrt{2}$	-2	

8.5 Translating My Composition

A Solidify Understanding Task

All this work with modeling rides and waiting lines at the local amusement park may have you wondering about the variety of ways of combining functions. In this task we continue building new functions from old, familiar ones.

Suppose you have the following "starter set" of functions.

$f(x) = x + 5$

$g(x) = x^2$

$h(x) = 3x$

$j(x) = 2^x$

$k(x) = x - 1$

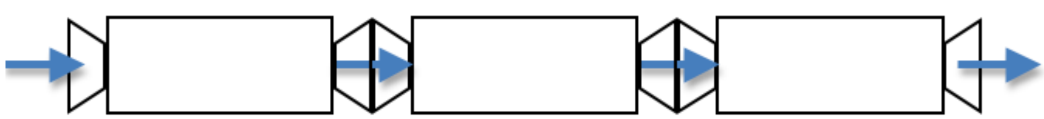
You and a partner will then do the following steps with your given set of functions:

1st. Build a composite function using any three of the above function rules in any order

2nd. Write your final function rule as a single algebraic expression in terms of x

3rd. Give your function rule to your partner, you should also receive a function rule from your partner

4th. Your partner should fill in the following diagram, decomposing your rule into its component parts and combining them in the correct order



1. First, let's try this example:

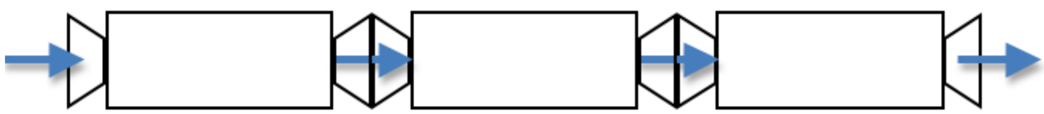
   Your partner gives you $f_1(x) = 3(x+5)^2$. Complete this diagram to decompose this composition into its component parts.

2. To test your decomposition you can try running a number or two through your chain of function machines, and see if you get the same results as when you evaluate the function rule for the same numbers. What do you notice when you do this?

3. Now it's your turn! Create your own function rule using the set of functions given at the beginning of this task and following the four steps given above. Your partner should do the same and give you his or her function rule.

   Record the function rule you received here:

   Complete this diagram to decompose your partner's composition into its component parts.



   Test your decomposition for a few values. Make any adjustments necessary based on your test results.

4. Instead of giving you the function rule, suppose your partner gives you the following input-output table. Can you create the composition function rule based on this information? Describe how you used the numbers in this table to create your rule.

x	*f*(*x*)
0	5 ½
1	6
2	7
3	9
4	13
5	21

5. Is function composition commutative? Give reasons to support your answer.

READY, SET, GO! Name Period Date

READY

Topic: Using a table to find the value of a composite function

Use the table to find the indicated function values.

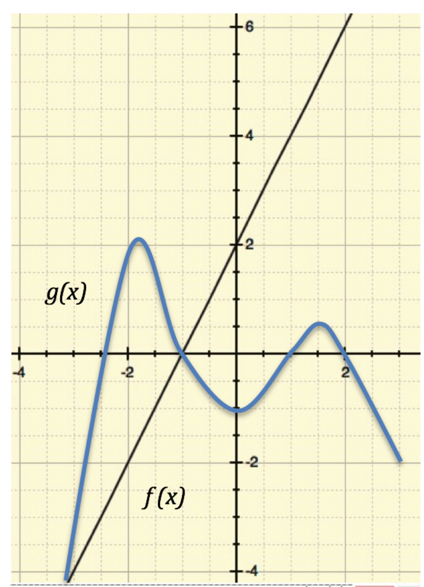
x	$f(x)$	$g(x)$
-2	2	3
-1	1	-2
0	3	-24
1	-1	-1
2	0	-8
3	19	0

1. $f(g(3))$
2. $f(g(1))$
3. $g(f(-2))$
4. $g(f(-1))$
5. $g(f(0))$
6. $g(g(-2))$
7. $f(f(0))$
8. $g(f(1))$

9. Do the graphs of $f(x)$ *and* $g(x)$ described in the table ever intersect each other?

   How do you know?

Use the graph to find the indicated values.



10. $f(g(-2))$
11. $f(g(-1))$
12. $f(g(1.5))$
13. $f(f(0))$

SET

Topic: Creating a composite function given its components

Let $f(x) = x^2, g(x) = 5x, and\ h(x) = \sqrt{x} + 2$.

Express each function as a composite of $f, g, and/or\ h$.

14. $F(x) = x^4$

15. $G(x) = 5x^2$

16. $P(x) = x + 2$

17. $R(x) = 5\sqrt{x} + 10$

18. $Q(x) = 25x$

19. $H(x) = 25x^2$

20. $D(x) = \sqrt{\sqrt{(x)} + 2} + 2$

21. $B(x) = x + 4\sqrt{x} + 4$

22. $K(x) = \sqrt{5x} + 2$

GO

Topic: Finding the zeros of a polynomial

Solve for all of the values of x. Identify any restrictions on x.

23. $x^2 + 6 = 5x$

24. $5x^3 = 45x$

25. $x^4 - 26x^2 + 25 = 0$

26. $1 + \frac{1}{x} = \frac{12}{x^2}$

27. $\frac{x}{6} - \frac{1}{2} - \frac{3}{x} = 0$

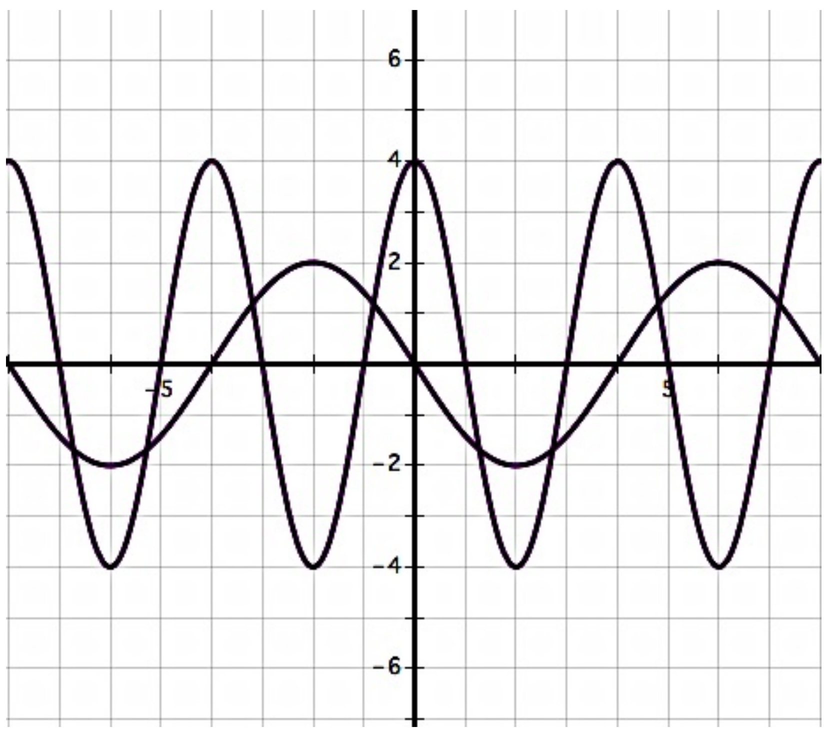
28. $\frac{1}{x^2} = 9$

8.6 Different Combinations

A Practice Understanding Task

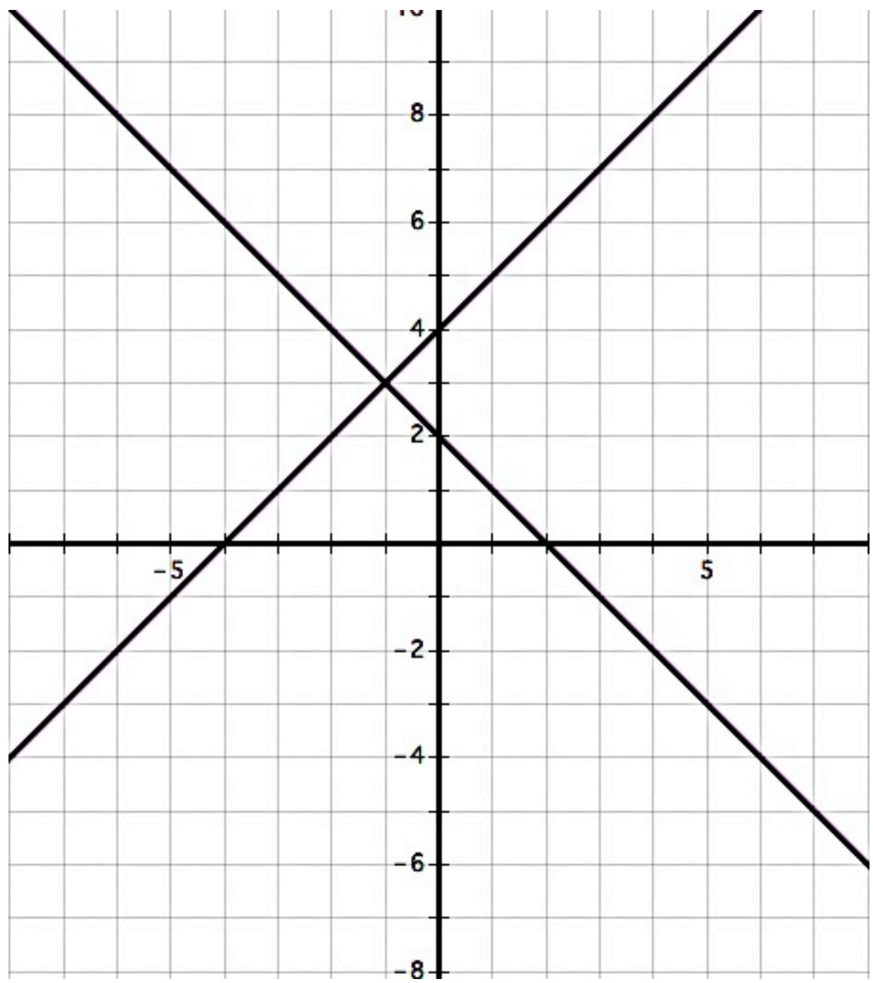
We have found the value of being able to combine different function types in various ways to model a variety of situations. In this task you will practice combining functions when they are described in different ways: graphically, numerically or algebraically.

1. Add the following two functions together graphically. That is, do not write the algebraic rules for each individual function, add them together, and then graph the result. See if you can produce the resulting graph by just working with the points on the two graphs and considering what happens when two functions are combined using the operation of addition.



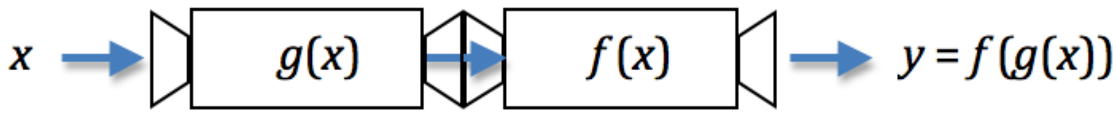
Which points are most helpful in determining the shape of the resulting graph, and why?

2. Multiply the following two functions together graphically. That is, do not write the algebraic rules for each individual function, multiply them together, and then graph the result. See if you can produce the resulting graph by just working with the points on the two graphs and considering what happens when two functions are combined using the operation of multiplication.



Which points are most helpful in determining the shape of the resulting graph, and why?

3. In a previous task we used the following diagram to illustrate function composition. Draw a similar type of diagram to illustrate what happens when two functions are combined by addition or multiplication. Your diagram should clearly show how the output values are obtained for specific input values.



4. Functions f and g are defined numerically in the following table. No other points exist for these functions other than the points given. Find the output values for each of the other combinations of functions indicated. Fill in as many points as are defined based on the give data. Use the same input values for all functions.

x	$f(x)$	$g(x)$	$(f+g)(x)$	$f^{-1}(x)$	$g(f(x))$	$f(g(x))$
0	**0**	**-3**				
1	**2**	**-2**				
2	**4**	**-1**				
3	**6**	**0**				
4	**8**	**1**				
5	**10**	**2**				
6	**12**	**3**				
7	**14**	**4**				
8	**16**	**5**				

5. Remember the race between the tortoise and the hare? Well, their friends and families have come to cheer them on, and have positioned themselves at various places along the course. Because rabbits are quick and eager to know the outcome of the race, more of them have congregated towards the end of the course. Because turtles are slow and more anxious to cheer their champion off to a good start, more of them have congregated at the beginning of the race. In fact, the density (or amount of animals/meter) of turtles and rabbits along the course as a function of the distance from the starting line is given by the following functions.

The tortoise: $a(d) = 243 \cdot \left(\frac{1}{3}\right)^{\frac{1}{20}d}$ (a is in turtles per meter, d in meters)

The hare: $a(d) = 2^{\frac{1}{10}d}$ (a is in rabbits per meter, d in meters)

The distance from the starting line, as a function of the elapsed time since the start of the race, is given for the tortoise and the hare by the following functions.

The tortoise: $d(t) = 2^t$ (d in meters, t in seconds)

The hare: $d(t) = t^2$ (d in meters, t in seconds)

The tortoise and the hare are anxious to know how many of their friends and family they are passing at any instant in time along the race.

- Create functions for the tortoise and for the hare that will calculate the number of turtles or rabbits they will pass at any time, t, after the race begins. Include a reasonable domain for each function.

- If the race is 100 meters long, create a function that will tell how many spectators, rabbits and turtles, are watching at any distance away from the start of the race?

- Who is passing the most friends and families, the tortoise or the hare, 5 seconds after the race began?

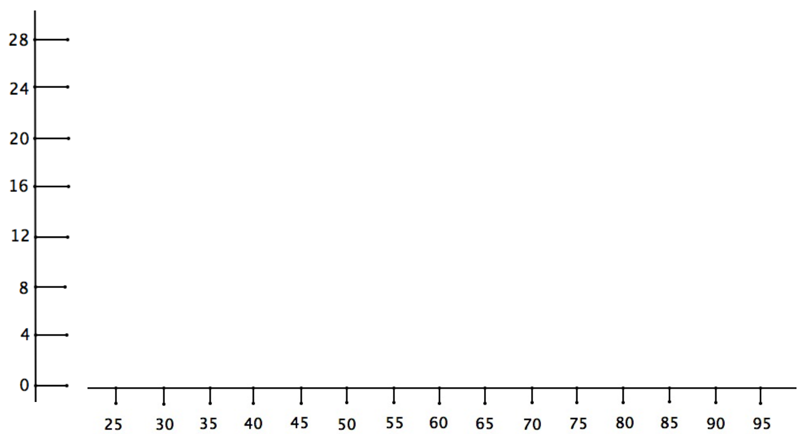
READY, SET, GO! Name Period Date

READY

Topic: Building a histogram

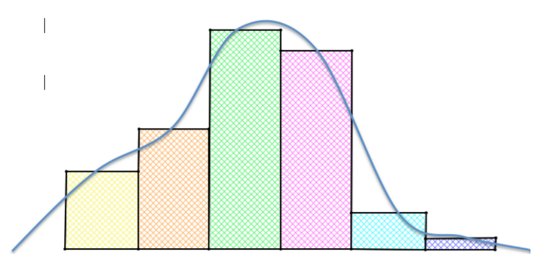
One hundred forty-four college freshmen were given a math placement exam with 100 possible points. The results show that 56 different scores were earned, ranging from 24 to 96. The scores were grouped in intervals as shown in the following table:

1. Make a histogram of the grouped data in the chart. (Note: The midpoint of each cell is given in the horizontal axis. The sides of the cells will match the score interval. Frequency is the vertical height.)



Score interval	Midpoint of interval	Frequency of interval
92.5 – 97.5	95	2
87.5 – 92.5	90	4
82.5 – 87.5	85	10
77.5 – 82.5	80	13
72.5 – 77.5	75	21
67.5 – 72.5	70	26
62.5 – 65.5	65	18
57.5 – 62.5	60	15
52.5 – 57.5	55	12
47.5 – 52.4	50	8
42.5 – 47.5	45	3
37.5 – 42.5	40	3
32.5 – 37.5	35	4
27.5 – 32.5	30	4
22.5 – 27.5	25	1

2. Locate the midpoint at the top of each cell in your histogram and connect each consecutive midpoint with straight line segments. The resulting figure is called a *frequency polygon.* If you smooth the line segments out into a smooth curve, you will create a *frequency curve.* Make a frequency curve on your histogram. It should look something like the figure on the right.



SET

Topic: Identifying the 2 functions that make up a composite function

Find functions f and g so that $f \circ g = H$.

3. $H(x) = \sqrt{x^2 + 5x - 4}$

4. $H(x) = \left(3 - \frac{1}{x}\right)^2$

5. $H(x) = (3x - 7)^4$

6. $H(x) = |5x^2 - 78|$

7. $H(x) = \frac{2}{3 - x^5}$

8. $H(\theta) = (\tan\theta)^2$

9. $H(\theta) = tan(\theta^2)$

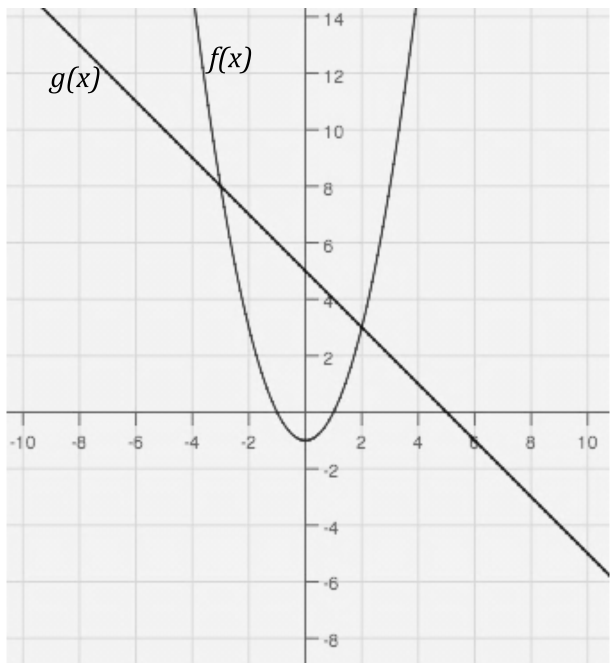
10. $H(x) = \sqrt{\frac{1}{6x}}$

11. $H(x) = 9(4x - 8) + 1$

GO

Topic: Finding function values given the graph

Use the graph to find all of the missing values.



12. $f(\blacksquare) = 8$

13. $g(\blacksquare) = 5$

14. $f(\blacksquare) = -1$

15. $g(\blacksquare) = 0$

16. $f(-1) =$ ____

17. $g(0) =$ ____

18. $f(x) = g(x)$

19. $f(x) - g(x) = 0$

20. $f(x) * g(x) = 0$

21. $f(2) + g(2) = \blacksquare$

22. $f(0) - g(0) = \blacksquare$