June 23, 2014

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Transforming Mathematics Education

Flexible & Engaging
Seamless Common Core Companion

The Mathematics Vision Project

- What MVP is
- How MVP fits the Core Standards
- Review the MVP materials

What is the Mathematics Vision Project?

- The Mathematics Vision Project is an educator-driven initiative to provide first-rate curriculum and professional development resources to secondary math teachers.

- Built from the ground-up by a team of award-winning, math-loving education experts, the material is flexible, engaging, and a seamless companion to the Common Core.

- Committed to life-long learning, the MVP team also offers educator training and resources to help teachers perform at their peak.

Who are we?
Meet the MVP Team

- **We believe math is engaging.** Our desire is to enable educators to teach their students through accessible, engaging, supportive curriculum.

- **We enjoy our work.** Our product has grown out of our passion for teaching mathematics and we love sharing it with our clients.

- **We know teachers need support.** We live the phrase "we're in this together." As educators face new challenges and requirements, we're there to support their next steps and, in turn, the success of their students.

What sets us apart from traditional publishers?

- Our materials are created from the ground-up to embody the focus, coherence, and rigor of the Common Core and immerse students in the Standards for Mathematical Practice.
- Our materials respect what is known about how students learn mathematics as well as the key ideas for how knowledge is organized and generated within the discipline of mathematics.
- Our curriculum is free and published under a Creative Commons license. Other support materials and professional development are available at mathematicsvisionproject.org

How are our materials designed?

- To meet the Core Standards
- Engage students in the Standards for Mathematical Practice

Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

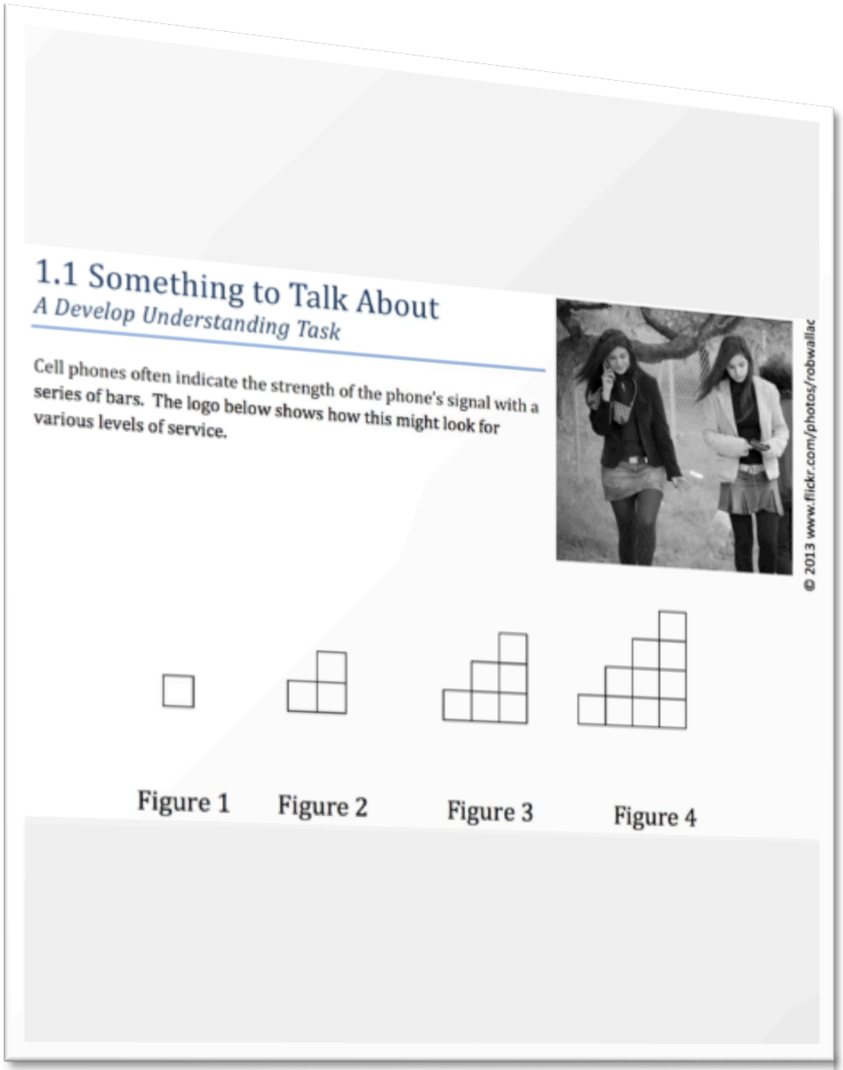
What needs to be in place to make this happen in the classroom?

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Basic Structure of the Materials

- The underlying design of the materials distinguishes between *problems* and *exercises*
- Each problem or exercise has a purpose
  - *teach new knowledge*
  - *bring misconceptions to the surface*
  - *build skill or fluency*
  - *engage students in mathematical practice*
- Assignments are purposefully designed
  - *"Lessons have a few well-designed problems that progressively build and extend understanding."*

(K-8 Publishers' Criteria for the Common Core Standards in Mathematics)



Curriculum

In-Class Tasks

and

Ready, Set, Go! Assignments

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In-Class Tasks and Ready-Set-Go Assignments

- In-class tasks are designed to be facilitated by a teacher, with students working together to build a balance of conceptual understanding and procedural skill.

- Ready-Set-Go assignments are independent practice that reinforce the work done in class and prepare students for upcoming work in class.

For Example:

Module 4 – Linear and Exponential Functions

4.1 Classroom Task: Connecting the Dots: Piggies and Pools – A Develop Understanding Task
Introducing continuous linear and exponential functions **(F.IF.3)**
Ready, Set, Go Homework: Linear and Exponential Functions 4.1

Problem

4.2 Classroom Task: Sorting Out the Change – A Solidify Understanding Task
Defining linear and exponential functions based upon the pattern of change (**F.LE.1, F.LE.2**)
Ready, Set, Go Homework: Linear and Exponential Functions 4.2

Exercise

4.3 Classroom Task: Where's My Change – A Practice Understanding Task
Identifying rates of change in linear and exponential functions (**F.LE.1, F.LE.2**)
Ready, Set, Go Homework: Linear and Exponential Functions 4.3

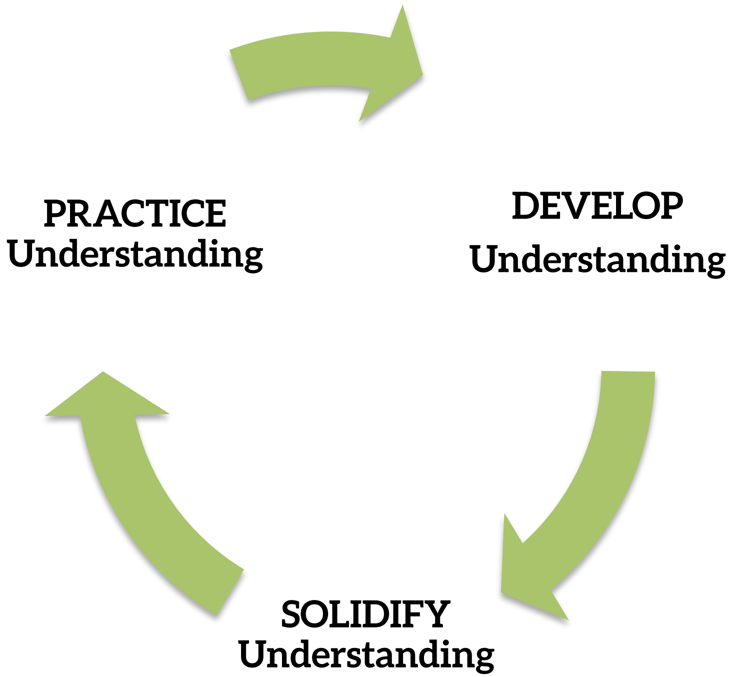
4.4 Classroom Task: Linear, Exponential or Neither – A Practice Understanding Task
Distinguishing between linear and exponential functions using various representations (**F.LE.3, F.LE.5**)
Ready, Set, Go Homework: Linear and Exponential Functions 4.4

4.5 Classroom Task: Getting Down to Business – A Solidify Understanding Task
Comparing the growth of linear and exponential functions (**F.LE.2, F.LE.3, F.LE.5, F.IF.7**)
Ready, Set, Go Homework: Linear and Exponential Functions 4.5

4.6 Classroom Task: Growing, Growing, Gone – A Solidify Understanding Task
Comparing linear and exponential models of population **(F.BF.1, F.BF.2, F.LE.1, F.LE.2, F.LE.3)**
Ready, Set, Go Homework: Linear and Exponential Functions 4.6

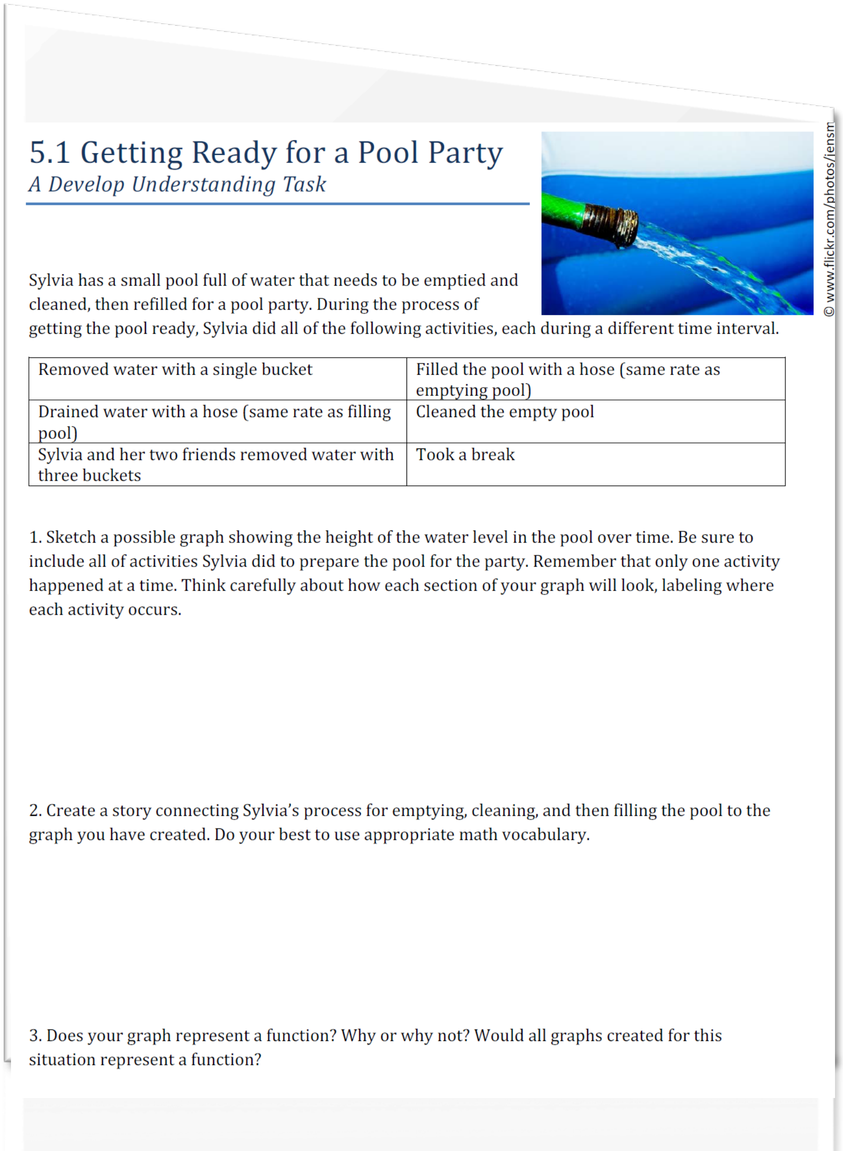
Task Sequencing

Comprehensive Mathematics Instruction Framework



- *Develop Understanding* tasks surface student thinking
- *Solidify Understanding* tasks examine and extend
- *Practice Understanding* tasks build fluency

Getting Ready for a Pool Party



5.1 Getting Ready for a Pool Party

A Develop Understanding Task

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Sylvia has a small pool full of water that needs to be emptied and cleaned, then refilled for a pool party. During the process of getting the pool ready, Sylvia did all of the following activities, each during a different time interval.

Removed water with a single bucket	Filled the pool with a hose (same rate as emptying pool)
Drained water with a hose (same rate as filling pool)	Cleaned the empty pool
Sylvia and her two friends removed water with three buckets	Took a break

1. Sketch a possible graph showing the height of the water level in the pool over time. Be sure to include all of activities Sylvia did to prepare the pool for the party. Remember that only one activity happened at a time. Think carefully about how each section of your graph will look, labeling where each activity occurs.

2. Create a story connecting Sylvia's process for emptying, cleaning, and then filling the pool to the graph you have created. Do your best to use appropriate math vocabulary.

3. Does your graph represent a function? Why or why not? Would all graphs created for this situation represent a function?

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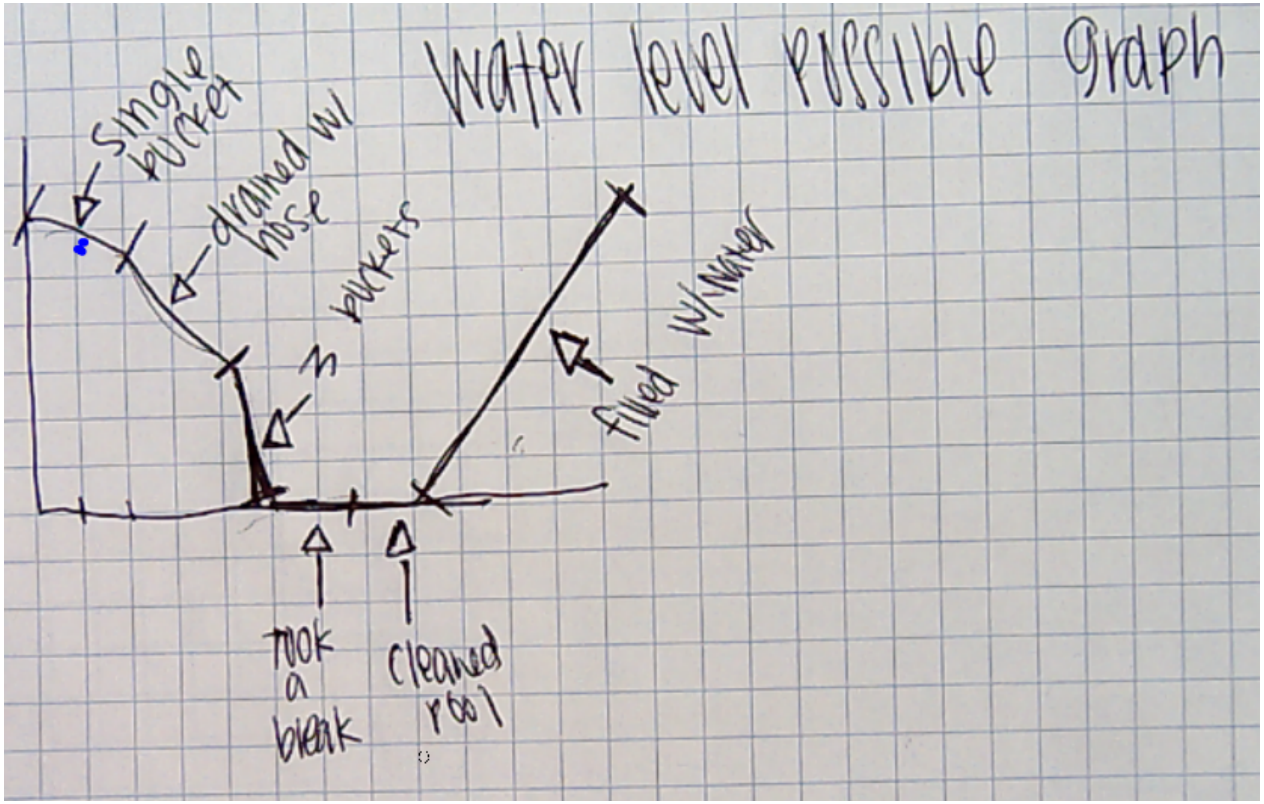
Getting Ready for a Pool Party
Samples of Student Work



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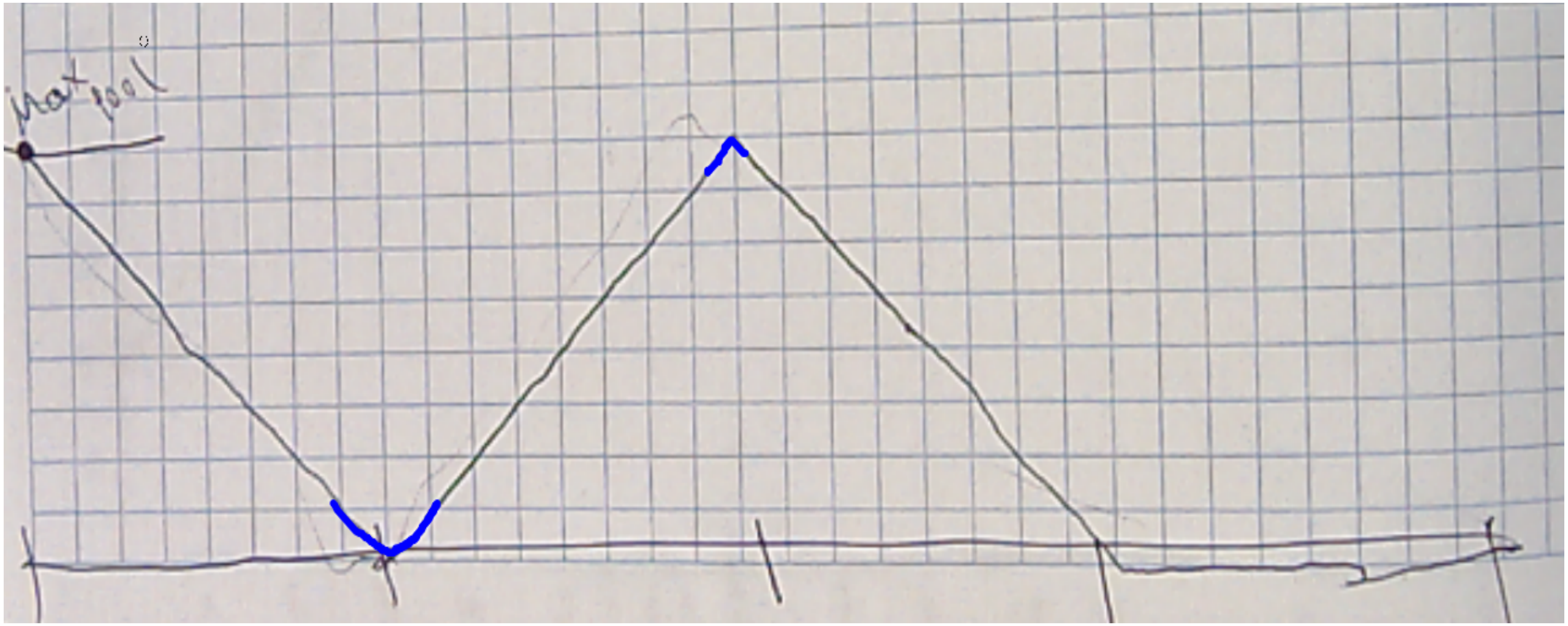

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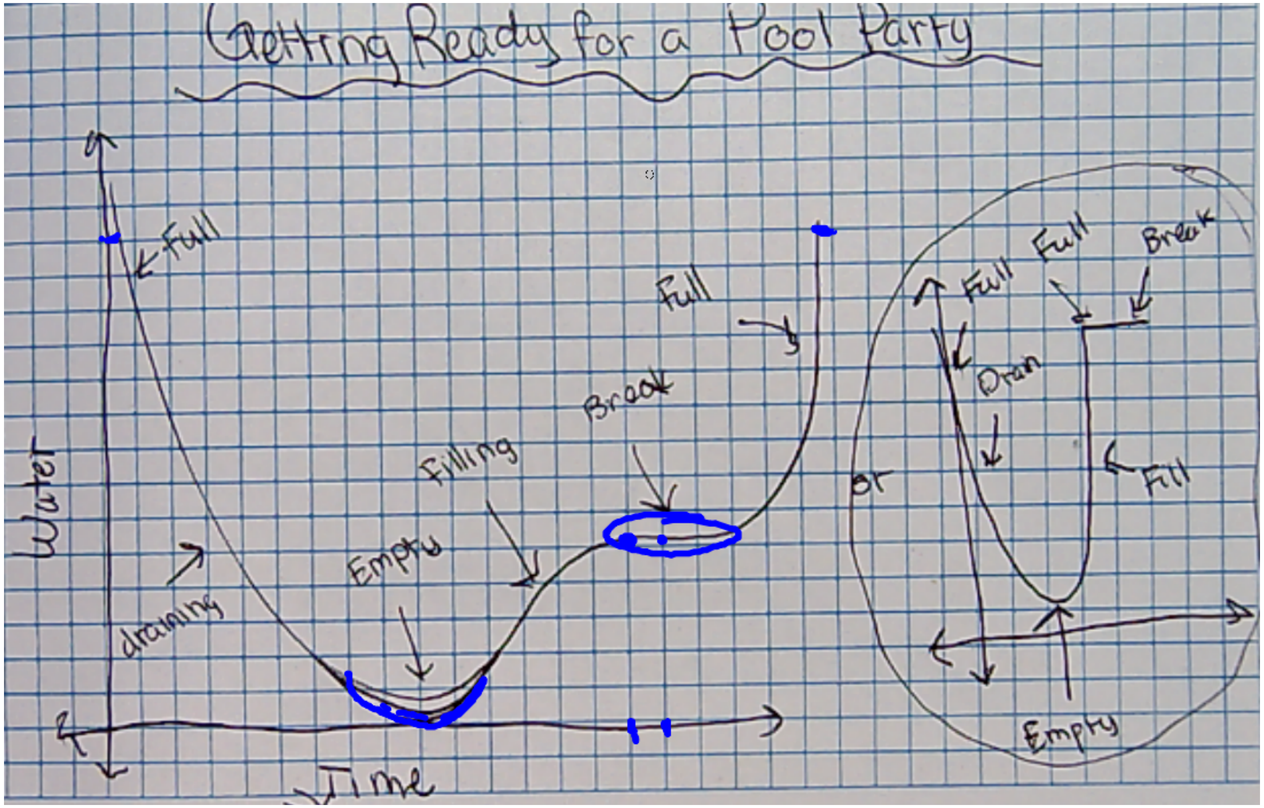
Samples of Student Work



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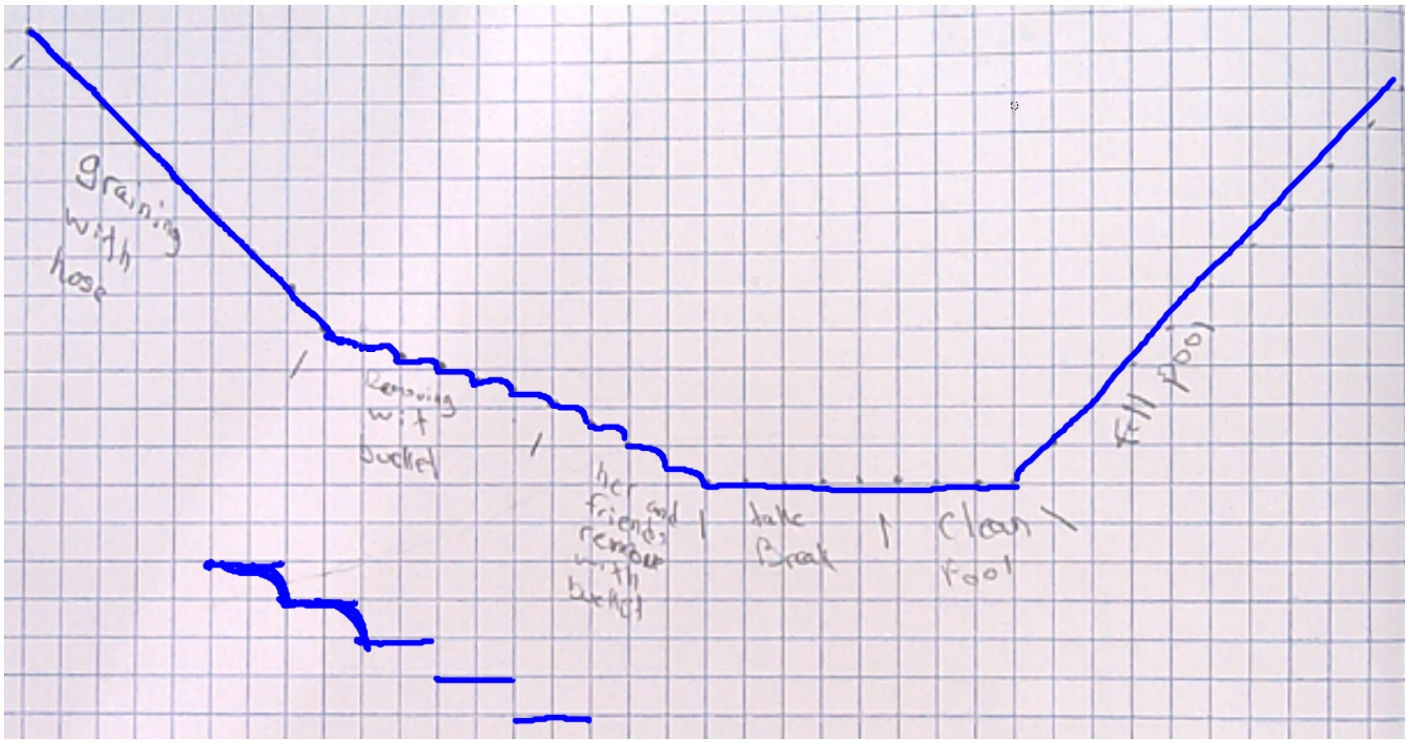
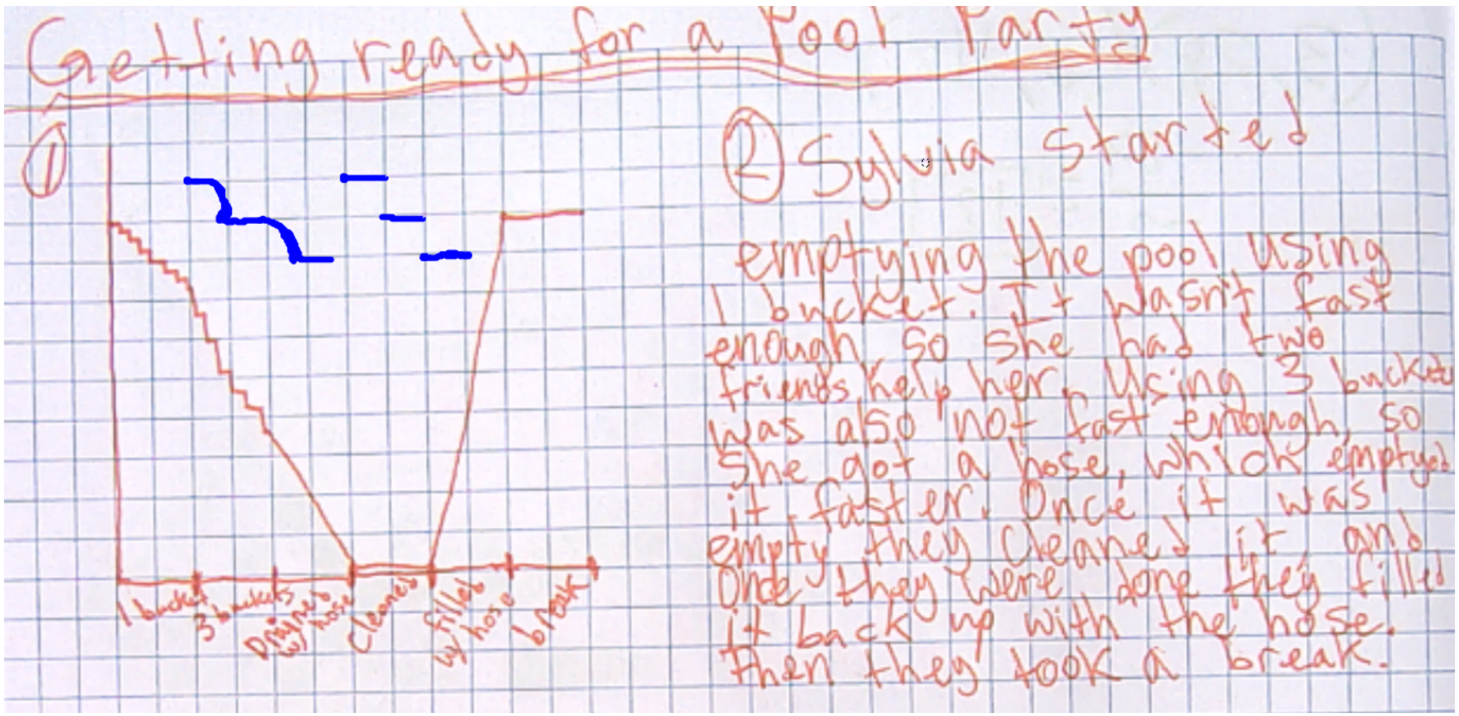


Getting Ready for a Pool Party
Samples of Student Work



Floating Down the Lazy River

5.2 Floating Down the River

A Solidify Understanding Task

Alonzo, Maria, and Sierra were floating in inner tubes down a river, enjoying their day. Alonzo noticed that sometimes the water level was higher in some places than in others. Maria noticed there were times they seemed to be moving faster than at other times. Sierra laughed and said "Math is everywhere!" To learn more about the river, Alonzo and Maria collected data throughout the trip.

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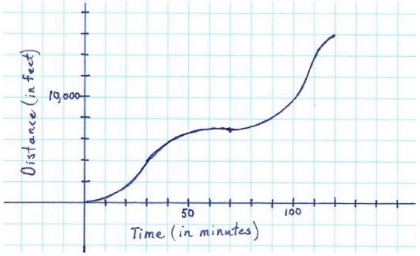
Alonzo created a table of values by measuring the depth of the water every ten minutes.

Time (in minutes)	0	10	20	30	40	50	60	70	80	90	100	110	120
Depth (in feet)	4	6	8	10	6	5	4	5	7	12	9	6.5	5

1. Use the data collected by Alonzo to interpret the key features of this relationship.

Maria created a graph by collecting data on a GPS unit that told her the distance she had traveled over a period of time.



2. Using the graph created by Maria, describe the key features of this relationship.

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Floating Down the Lazy River
Samples of Student Work

Alonzo created a table of values by measuring the depth of the water every ten minutes.

Time (in minutes)	0	10	20	30	40	50	60	70	80	90	100	110	120
Depth (in feet)	4	6	8	10	6	5.	4	5	7	12	9	6.5	5

... to interpret the key features of this relationship.

$0 \le t \le 30$

Increases at a constant rate

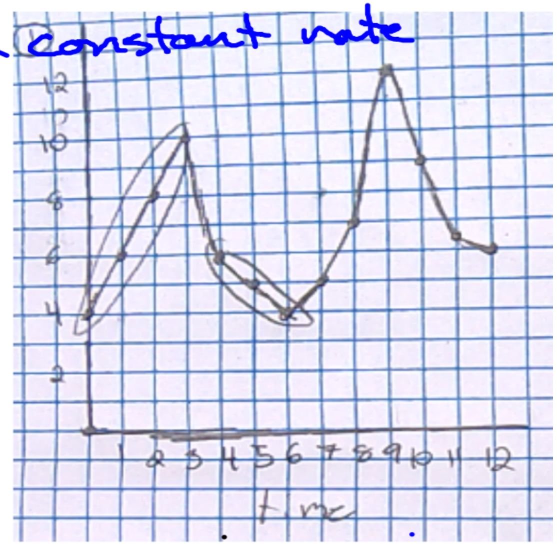
decreases

increases

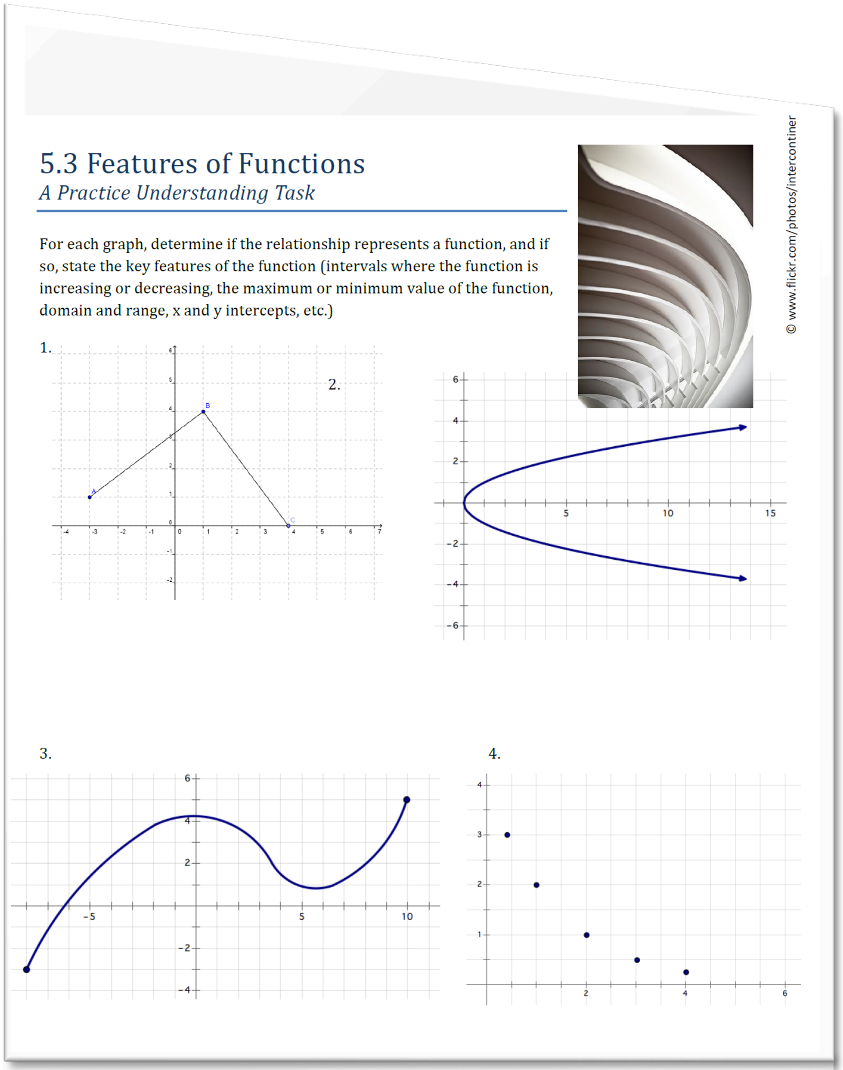
decreases

Continous

- Maximum of 12
- Minimum of 4.



Features of Functions


5.3 Features of Functions

A Practice Understanding Task

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For each graph, determine if the relationship represents a function, and if so, state the key features of the function (intervals where the function is increasing or decreasing, the maximum or minimum value of the function, domain and range, x and y intercepts, etc.)

1.

2.

3.

4.

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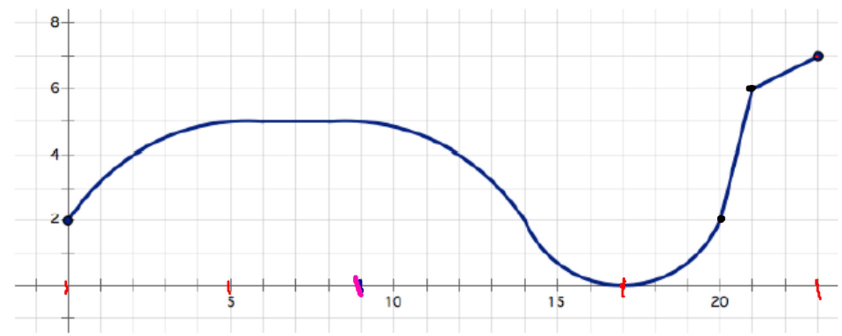
Features of Functions
Samples of Student Work

Given the graph or table for a function list all important features of the function that are requested.

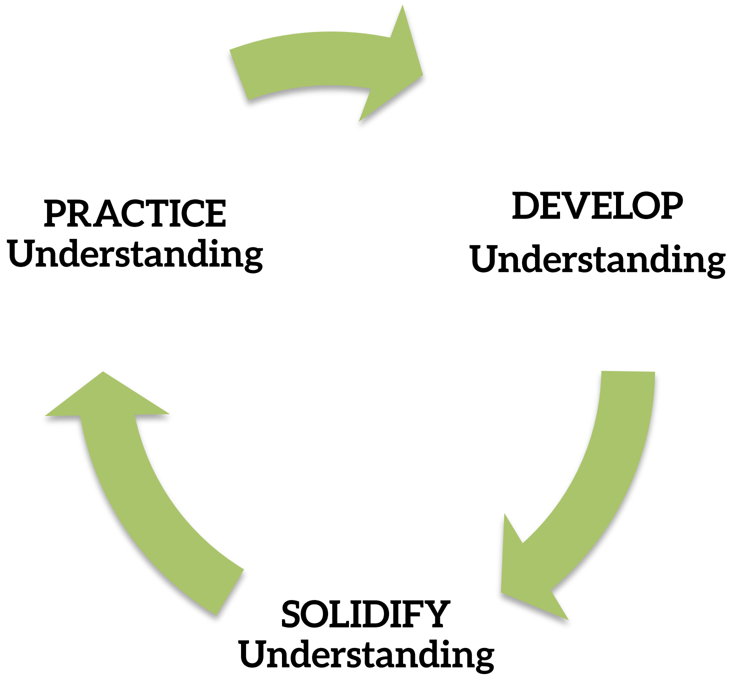


3.

a. On which intervals is the function increasing? (Be specific.) $0 \le x \le 5$ $[0,5]$
b. On which intervals is the function decreasing? (Be specific.) $17 \le x \le 23$ $[17,23]$ — $9 \le x \le 17$ $[9,17]$
c. What is the domain for this function? $\{x \mid x \in \mathbb{R}, 0 \le x \le 23\}$
d. What is the range for this function? $\{y \mid y \in \mathbb{R}, 0 \le y \le 7\}$
e. What are the x- and y-intercepts of this function? x-int = 17 y-int = 2
f. On which intervals does this function have a constant rate of change? (Be specific.) $[5,9]$ $[21,23]$ $[20,21]$
g. On which intervals is this function changing the fastest? $20 \le x \le 21$ $[20,21]$
h. On which intervals is this function changing the slowest? $5 \le x \le 9$ $[5,9]$

Task Sequencing

Comprehensive Mathematics Instruction Framework



- *Develop Understanding* tasks surface student thinking
- *Solidify Understanding* tasks examine and extend
- *Practice Understanding* tasks build fluency

Shopping for Cats and Dogs

2.7 Shopping for Cats and Dogs

A Develop Understanding Task

Clarita is upset with Carlos because he has been buying cat and dog food without recording the price of each type of food in their accounting records. Instead, Carlos has just recorded the total price of each purchase, even though the total cost includes more that one type of food. Carlos is now trying to figure out the price of each type of food by reviewing some recent purchases. See if you can help him figure out the cost of particular items for each purchase, and be prepared to explain your reasoning to Carlos.

1. One week Carlos bought 3 bags of *Tabitha Tidbits* and 4 bags of *Figaro Flakes* for $43.00. The next week he bought 3 bags of *Tabitha Tidbits* and 6 bags of *Figaro Flakes* for $54.00. Based on this information, figure out the price of one bag of each type of cat food. Explain your reasoning.

2. One week Carlos bought 2 bags of *Brutus Bites* and 3 bags of *Lucky Licks* for $42.50. The next week he bought 5 bags of *Brutus Bites* and 6 bags of *Lucky Licks* for $94.25. Based on this information, figure out the price of one bag of each type of dog food. Explain your reasoning.

3. Carlos purchased 6 dog leashes and 6 cat brushes for $45.00 for Clarita to use while pampering the pets. Later in the summer he purchased 3 additional dog leashes and 2 cat brushes for $19.00. Based on this information, figure out the price of each item. Explain your reasoning.

4. One week Carlos bought 2 packages of dog bones and 4 packages of cat treats for $18.50. Because the finicky cats didn't like the cat treats, the next week Carlos returned 3 unopened packages of cat treats and bought 2 more packages of dog bones. After being refunded for the cat treats, Carlos only had to pay $1.00 for his purchase. Based on this information, figure out the price of each item. Explain your reasoning.

5. Carlos has noticed that because each of his purchases have been somewhat similar, it has been easy to figure out the cost of each item. However, his last set of receipts has him puzzled. One week he tried out cheaper brands of cat and dog food. On Monday he purchased 3 small bags of cat food and 5 small bags of dog food for $22.75. Because he went through the small bags quite quickly, he had to return to the store on Thursday to buy 2 more small bags of cat food and 3 more small bags of dog food, which cost him $14.25. Based on this information, figure out the price of each bag of the cheaper cat and dog food. Explain your reasoning.

Summarize the strategies you have used to reason about the price of individual items in the problems given above. What are some key ideas that seem helpful?

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Can You Get to the Point Too?

2.8 Can You Get to the Point, Too?

A Solidify Understanding Task

Part 1

In "Shopping for Cats and Dogs," Carlos found a way to find the cost of individual items when given the purchase price of two different combinations of those items. He would like to make his strategy more efficient by writing it out using symbols and algebra. Help him formalize his strategy by doing the following:

- For each scenario in "Shopping for Cats and Dogs" write a **system of equations** to represent the two purchases.
- Show how your strategies for finding the cost of individual items could be represented by manipulating the equations in the system. Write out intermediate steps symbolically, so that someone else could follow your work.
- Once you find the price of one of the items in the combination, show how you would find the price of the other item.

Part 2

Writing out each system of equations reminded Carlos of his work with solving systems of equations graphically. Show how each scenario in "Shopping for Cats and Dogs" can be represented graphically, and how the cost of each item shows up in the graphs.

Part 3

Carlos also realized that the algebraic strategy he created in part 1 could be used to find the points of intersection for the "Pet Sitters" constraints. Use the **elimination of variables** method developed in part 1 to find the point of intersection for each of the following pairs of "Pet Sitter" constraints.

- *Start-up costs* and *space* constraints
- *Pampering time* and *feeding time* constraints
- Any other pair of "Pet Sitter" constraints of your choice

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Can You Get to the Point, Too?

Part 1

- In "Shopping for Cats and Dogs," Carlos found a way to find the cost of individual items when given the purchase price of two different combinations of those items. He would like to make his strategy more efficient by writing it out using symbols and algebra. Help him formalize his strategy by doing the following:
- For each scenario in "Shopping for Cats and Dogs" write a **system of equations** to represent the two purchases.
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- Once you find the price of one of the items in the combination, show how you would find the price of the other item.
- (Focus on question #3 only for your graphical representation.)

Can You Get to the Point, Too

Part 2

- Writing out each system of equations reminded Carlos of his work with solving systems of equations graphically. Show how each scenario in "Shopping for Cats and Dogs" can be represented graphically, and how the cost of each item shows up in the graphs.

(Focus on question #3 only for your graphical representation.)

Taken Out of Context

Write a shopping scenario similar to those in "Shopping for Cats and Dogs" to fit each of the following systems of equations. Then use the elimination of variables method you invented in "Can You Get to the Point, Too" to solve the system. Some of the systems may have interesting or unusual solutions. See if you can explain them in terms of the shopping scenarios you wrote.

1. $\begin{cases} 3x + 4y = 23 \\ 5x + 3y = 31 \end{cases}$

2. $\begin{cases} 2x + 3y = 14 \\ 4x + 6y = 28 \end{cases}$

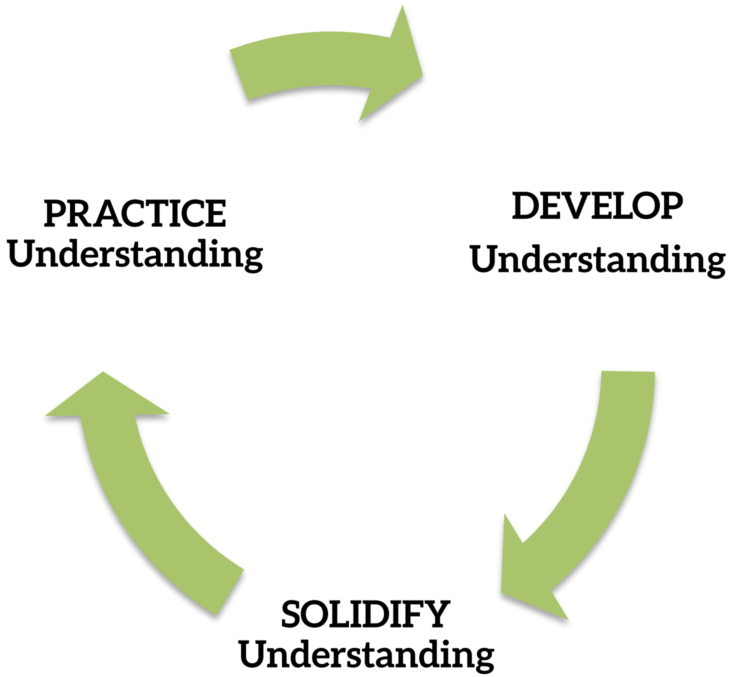
3. $\begin{cases} 3x + 2y = 20 \\ 9x + 6y = 35 \end{cases}$

4. $\begin{cases} 4x + 2y = 8 \\ 5x + 3y = 9 \end{cases}$

Three of Carlos' and Clarita's friends are purchasing school supplies at the bookstore. Stan buys a notebook, three packages of pencils and two markers for $7.50. Jan buys two notebooks, six packages of pencils and five markers for $15.50. Fran buys a notebook, two packages of pencils and two markers for $6.25. How much do each of these three items cost?
Explain in words or with symbols how you can use your intuitive reasoning about these purchases to find the price of each item.

Task Sequencing

Comprehensive Mathematics Instruction Framework



- *Develop Understanding* tasks surface student thinking
- *Solidify Understanding* tasks examine and extend
- *Practice Understanding* tasks build fluency

Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

How does the experience of the learner change throughout the learning cycle?

Information Transmission Model

Show and Tell

- *Teacher's Role:*
- Provide examples, definitions and properties, procedures, and models

Practice

- Students' Role
- Replicate, drill, practice, and memorize

Comprehensive Mathematics Instruction Model

Problem Solving, Reasoning and Proving, Modeling

Develop

- *Teacher's Role*: Provide experiences, orchestrate discussions using the 5 practices
- *Students' Role*: Notice patterns; make conjectures; create arguments

Solidify

- *Teacher's Role*: Provide experiences, orchestrate discussions using the 5 practices
- *Students' Role*: See structure; see regularities; attend to precision; create and critique

Practice

- *Teacher's Role*: Provide a vehicle for practice, provide feedback
- *Students' Role*: Reason quantitatively; work towards efficiency, flexibility, accuracy; Apply (model with mathematics)

Develop Understanding Tasks

- Low threshold, high ceiling (easy entry, but extendable for all learners)
- Contextualized (problematic story context, diagrams, symbols)
- Multiple pathways to solutions or multiple solutions
- Surface student thinking (misconceptions and correct thinking)
- Purposeful selection of the vocabulary, numbers, etc. to reveal rather than obscure the mathematics
- Introduction of a number of representations

Solidify Understanding Tasks

Features of the task (context, scaffolding questions, constraints) focuses students' attention on:

- looking for patterns and making use of structure
- looking for repeated reasoning and expressing regularities as generalized methods
- attending to precision in language and use of symbols
- constructing viable arguments and critiquing the reasoning of others
- using representations and tools strategically for the purpose of developing deeper levels of understanding of mathematical ideas, strategies, and/or representations

Practice Understanding Tasks

Practice tasks focused on acquiring fluency:

- Task involves either reproducing previously learned facts, definitions, rules, formulas or models; OR drawing upon previously learned facts, definitions, rules, formulas or models; OR committing facts, definitions, rules, formulas or models to memory

- An appropriate vehicle of practice is selected (e.g., routines, games, worksheets, etc.) which allows for reproducing, drawing upon, or committing to memory previously examined mathematics

- Task focuses on a broad definition of fluency: accuracy, efficiency, flexibility, automaticity

Practice Understanding Tasks

Practice tasks focused on refining understanding:

- Task allows student to use reasoning habits to contextualize (symbolic to real-world) and decontextualize (real-world to symbolic) problems and situations.
- Tasks involve sufficient complexity to refine mathematical thinking beyond rote memorization
- The task requires a high level of cognitive demand because students are required to draw upon multiple concepts and procedures, make use of structure and recognize complex relationships among facts, definitions, rules, formulas and/ or models

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Ready, Set, Go Homework: Linear and Exponential Functions 4.6

Develop

Solidify

Practice

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A Multi-tasking Approach to Learning

- Each standard is addressed in more than one task.
- Each task addresses more than one standard.
- Because the materials are built from the ground-up, every standard is fully addressed.

Materials cannot match the contours of the standards by approaching each individual content standard as a separate event.

(K-8 Publishers' Criteria for the Common Core Standards in Mathematics)

For Example:

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The same standards appear in multiple tasks

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Ready-Set-Go

Name: Systems | 2.7

Ready, Set, Go!

Ready

Topic: Exponents

Write the following in exponential notation.

1. $4 \times 4 \times 4 \times 4 \times 4$
2. $3x \cdot 3x \cdot 3x \cdot 3x$

Find each value.

3. 2^3
4. 3^3
5. 2^5
6. $(-2)^3$
7. 4^3

Ready-Set-Go

Set

Topic: Solving systems

8. Nadia and Peter visit the candy store. Nadia buys three candy bars and four fruit roll-ups for $2.84. Peter also buys three candy bars, but can only afford one additional fruit roll-up. His purchase costs $1.79. What is the cost of a candy bar and a fruit roll-up individually?

9. A farmer noticed that his chickens were loose and were running around with the cows in the cow pen. He quickly counted 100 heads and 270 legs. How many chickens did he have and how many cows?

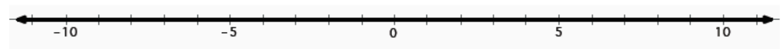
Ready-Set-Go

Go

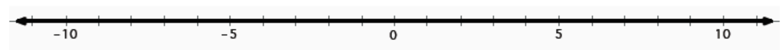
Topic: Solve one variable inequalities.

Solve the following inequalities. Write the solution set in *interval notation* and graph the solution set on a number line.

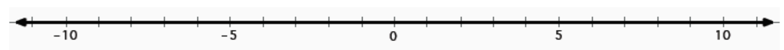
10. $4x + 10 < 2x + 14$



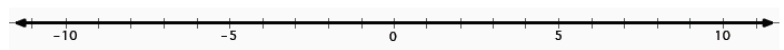
11. $2x + 6 > 55 - 5x$



12. $2(\frac{x}{4} + 3) > 6(x - 1)$



13. $9x + 4 \leq -2(x + \frac{1}{2})$



Solve each inequality. Give the solution in *inequality notation* and *set notation*.

14. $-\frac{x}{3} > -\frac{10}{9}$

15. $5x > 8x + 27$

16. $\frac{x}{4} > \frac{5}{4}$

17. $3x - 7 \geq 3(x - 7)$

18. $2x < 7x - 36$

19. $5 - x < 9 + x$

Check out the Ready – Set - Go

Ready: Get ready for upcoming lessons

Set: Reinforce what was learned in today's lesson

Go: Practice previously-learned skills

What needs to be in place to make it all happen in the classroom?

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
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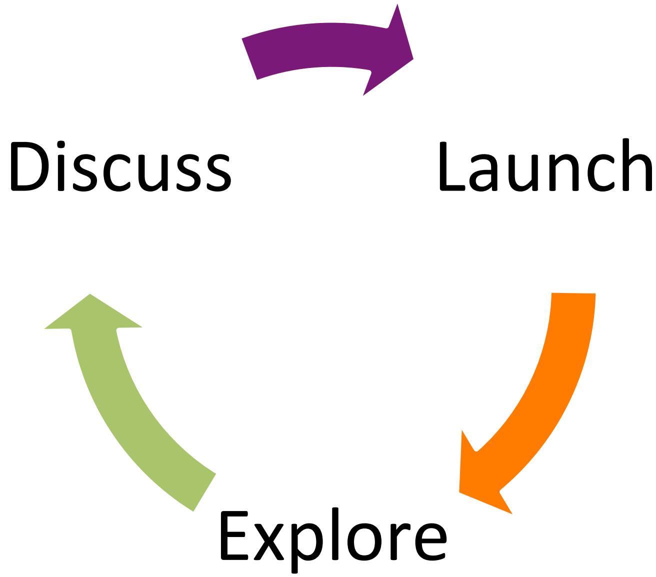
The *Teaching Cycle*



Shopping for Cats and Dogs Video

The *Teaching Cycle*: Launch

How will you . . .

- hook and motivate students;
- provide schema (the problem setting, the mathematical context, and the challenge) for the mathematical task;
- provide tools, information, vocabulary, conventions and notations, as necessary; and
- describe what the expectations are for the finished task without giving away too much of the problem and leaving the potential of the task intact?

The *Teaching Cycle*: Explore

- How will you organize and encourage students to explore, investigate, experiment, look for patterns, make conjectures, collect and record data, participate in group discussions, and revisit and revise their thinking relative to the mathematical ideas intended to be elicited by the task?

- What will you look for and listen for as you observe students?

- What will you accept as evidence of student understanding?

- What questions will you ask to stimulate, redirect, focus, and extend the students' mathematical thinking?

The *Teaching Cycle*: Discuss

- How will you select which students will present and discuss their solutions and strategies?

- How will you determine what ideas to pursue in depth and what to defer for another time?

- How will you decide whether to contribute to the discourse by providing additional information (e.g., vocabulary, conventions, notation), suggesting other models, demonstrating alternative strategies, clarifying difficult issues; or to allow students to continue to struggle to make sense of an idea or concept?

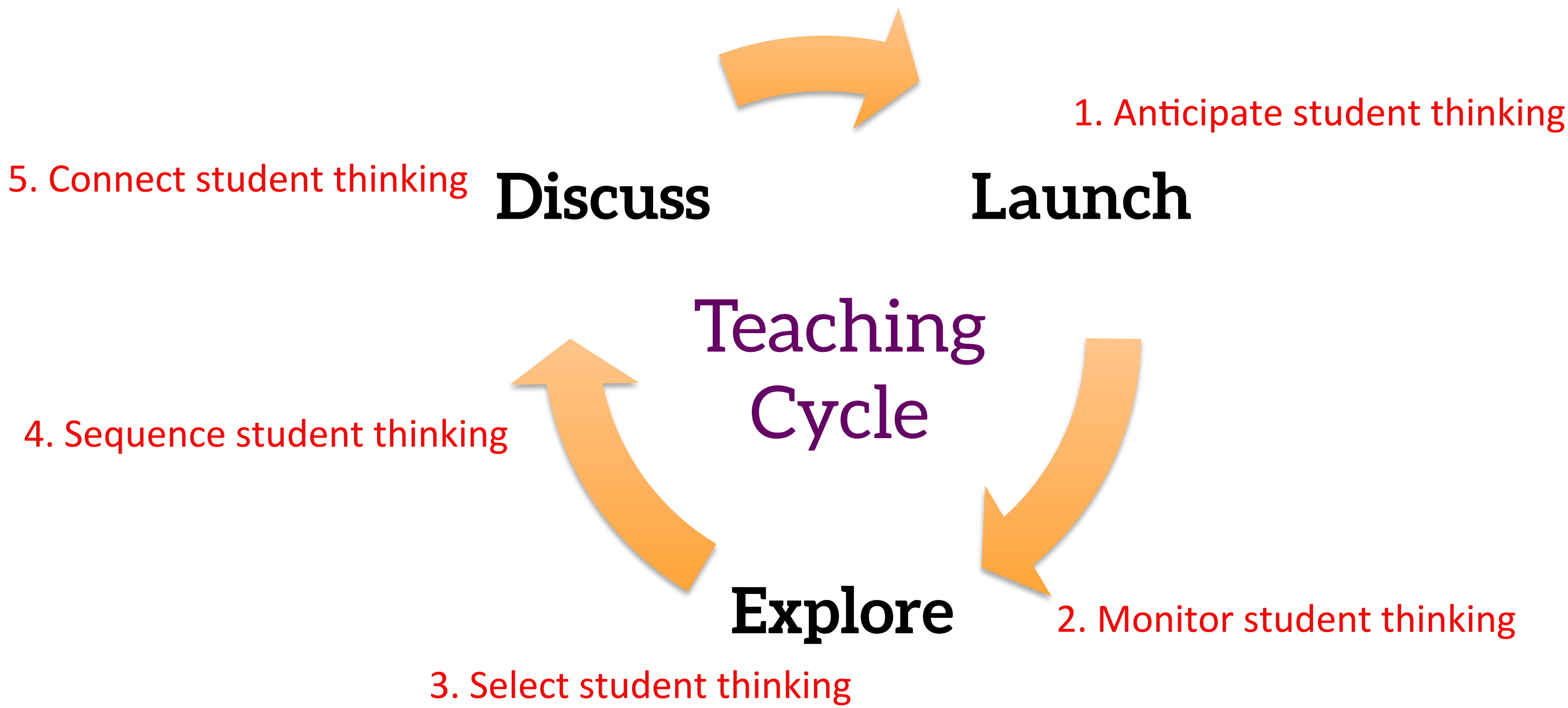
Five Practices for Facilitating Mathematical Discourse

1. Anticipate student thinking – work the task yourself
2. Monitor students as they work – circulate around the room and ask students how they are thinking about what they have written
3. Select students for the classroom discussion – have a method for keeping track of who you have selected
4. Sequence student work
5. Connect – help students to make connections among the ideas presented

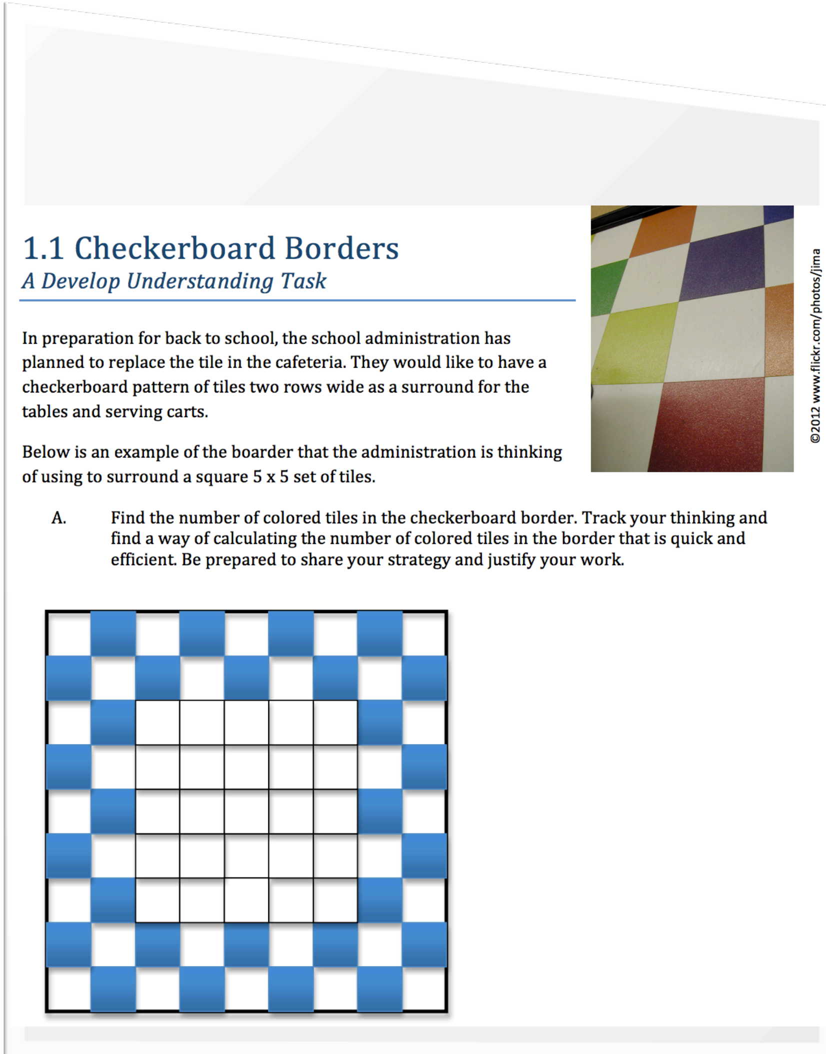
A FRAMEWORK for a Lesson or TASK:

Moving from a conceptual foundation to procedural fluency

Comprehensive Mathematics Instruction Framework



Checkerboard Borders



1.1 Checkerboard Borders
A Develop Understanding Task

In preparation for back to school, the school administration has planned to replace the tile in the cafeteria. They would like to have a checkerboard pattern of tiles two rows wide as a surround for the tables and serving carts.

Below is an example of the boarder that the administration is thinking of using to surround a square 5 x 5 set of tiles.

A. Find the number of colored tiles in the checkerboard border. Track your thinking and find a way of calculating the number of colored tiles in the border that is quick and efficient. Be prepared to share your strategy and justify your work.

Transforming Mathematics Education

mathematics vision project

The 0 Practice

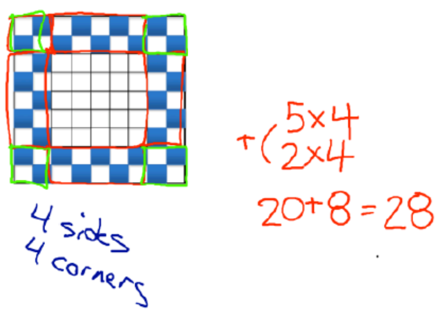
- Have clear mathematical goals and a task that supports the goals.

Checkerboard Borders

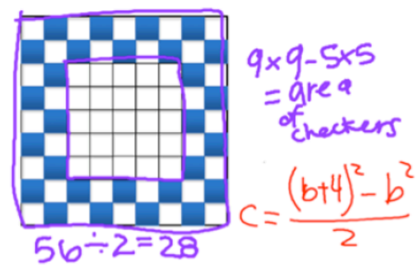
Purpose: The focus of this task is on the generation of multiple expressions that connect with the visuals provided for the checkerboard borders. These expressions will also provide opportunity to discuss equivalent expressions and review the skills students have previously learned about simplifying expressions and using variables.

Checkerboard Borders
Student Work

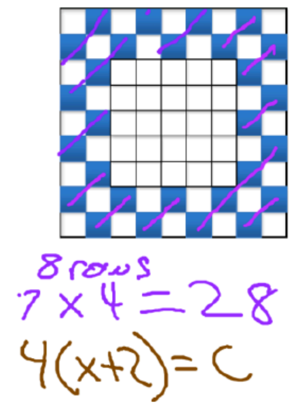
Student A



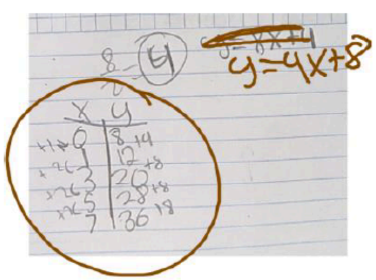
Student B



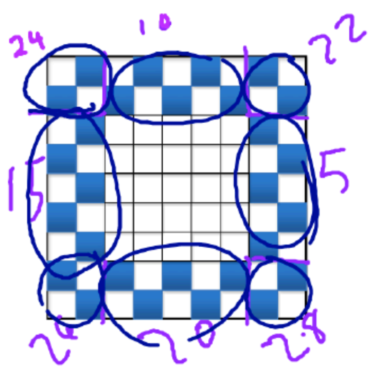
Student C



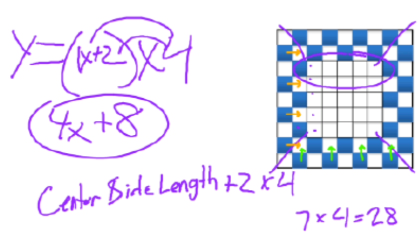
Student D



Student E



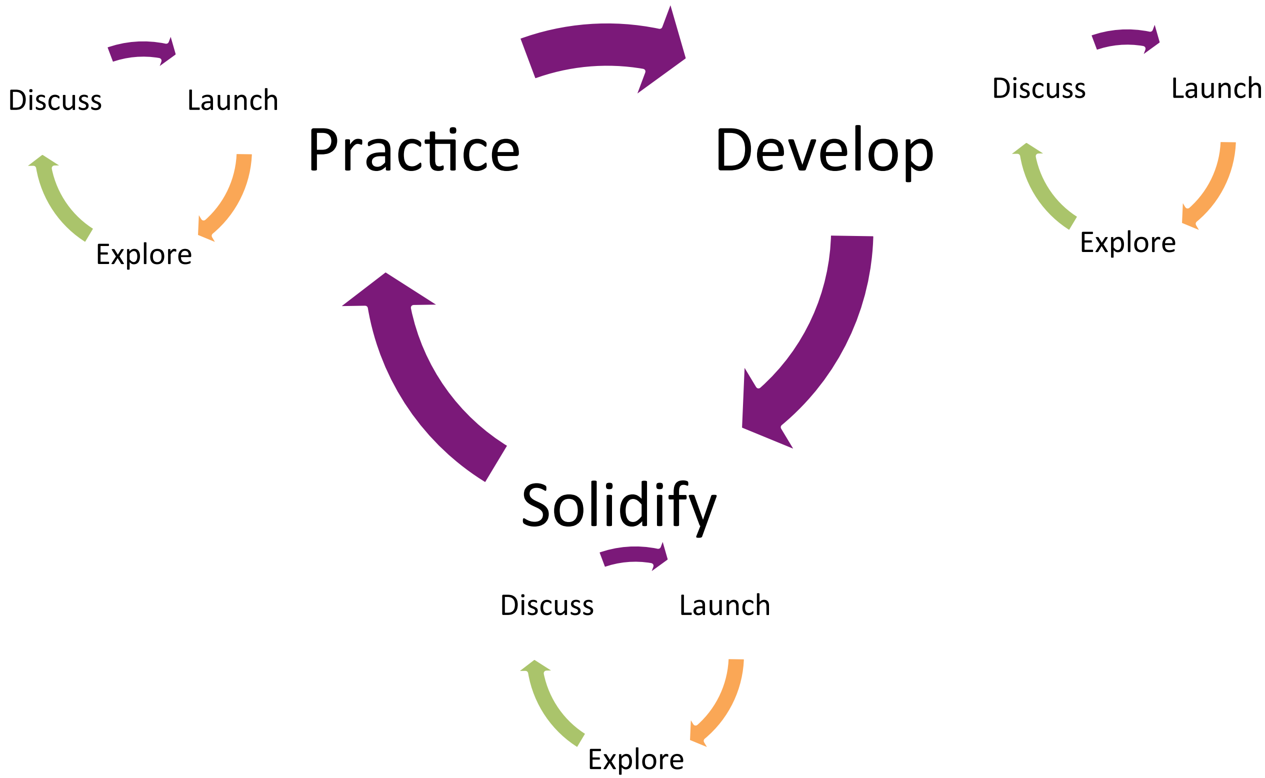
Student F



Looking at Student Work

- Examine the student work using the observation notes guide.
- Select student work that you would want to have shared in the whole class discussion.
- Sequence the work for the whole class discussion.
- Annotate your poster to highlight the connections you would want to make across the samples of student work.

Comprehensive Mathematics Instruction



Write your response to each question on a post-it note (you will need four post-it notes- one for each question).

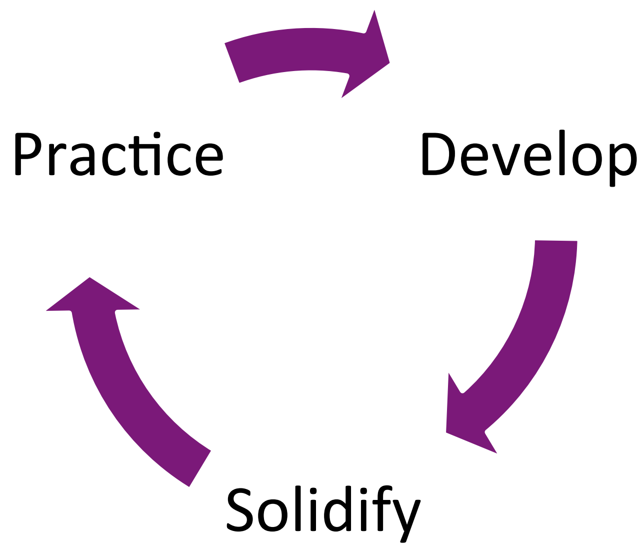
1. What can you do to create a classroom environment that promotes discourse?

2. How does understanding the learning cycle help support instructional decisions?

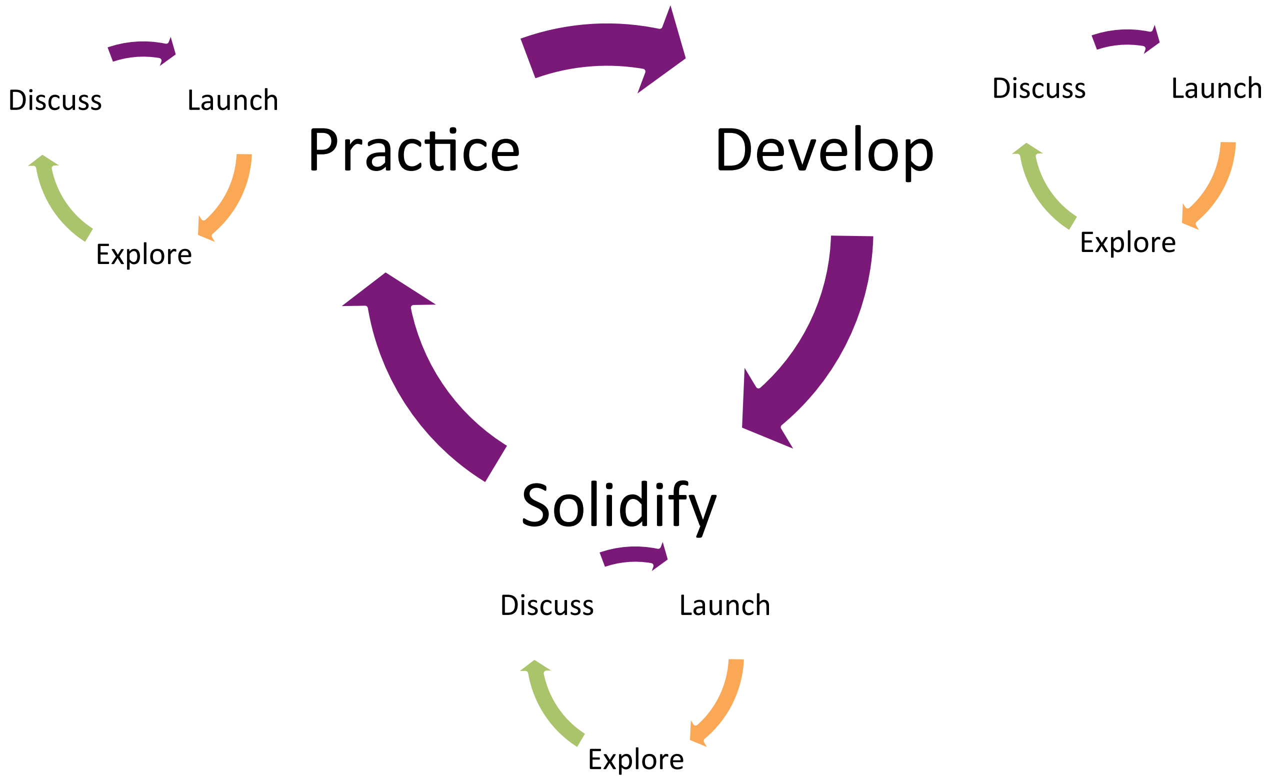
3. How does having a clear focus on a mathematical purpose support instructional decisions?

4. Share a new mathematical connection or insight you have gained since yesterday.

Comprehensive Mathematics Instruction

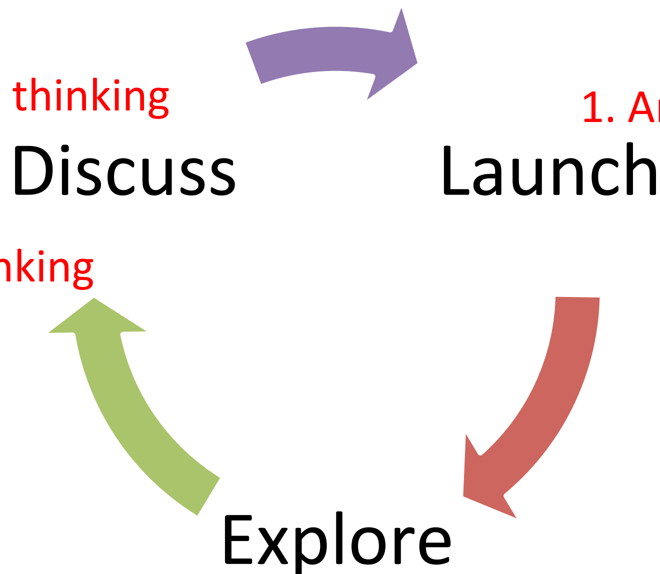


Comprehensive Mathematics Instruction



The *Teaching Cycle*

Connected to the 5 practices of Orchestrating Discussions



5. Connect student thinking

1. Anticipate student thinking

4. Sequence student thinking

2. Monitor student thinking

3. Select student thinking

What's Your Next Step?

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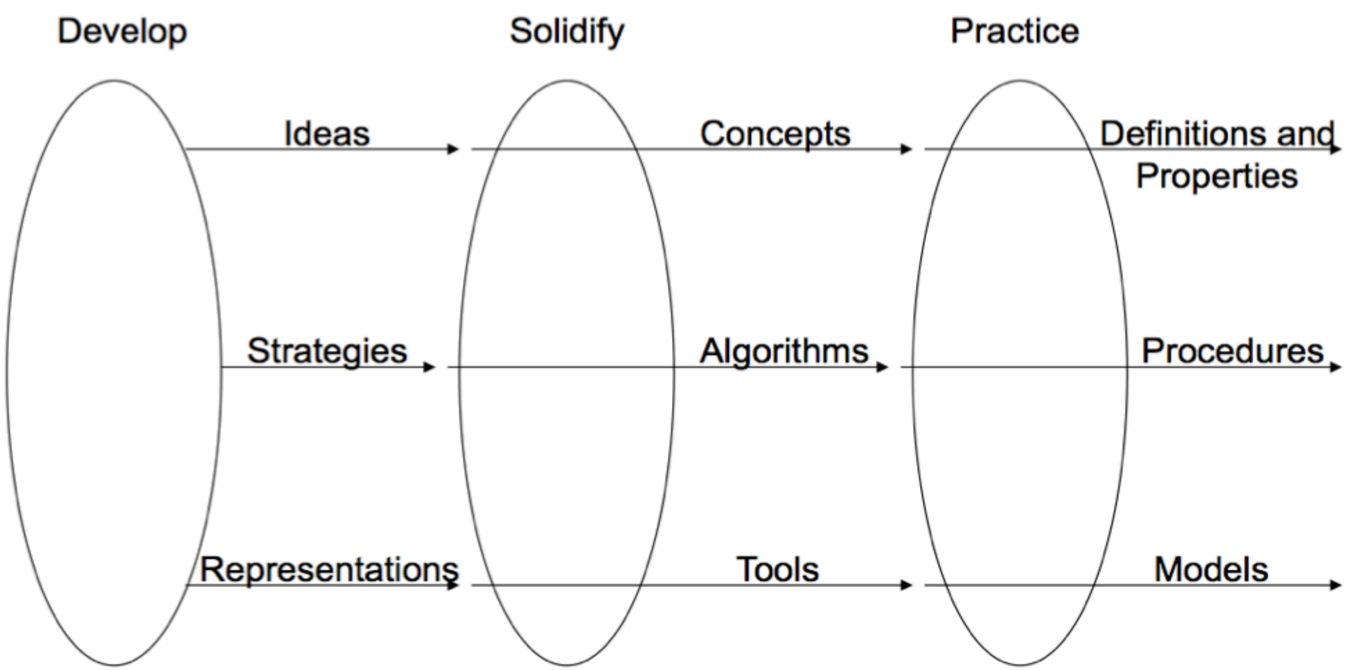
Unpacking the Mathematics of the Learning Cycle

- What is the conceptual, procedural, and representational understanding that is emerging from the work in the learning cycle?
- How does the mathematical understanding change from the beginning to the end of the learning cycle?



Differentiated to meet the needs of all students

- Low-threshold, high ceiling tasks allow all students full participation in the standards
- Story contexts and visual representations support conceptual and procedural understanding
- Tasks are designed to surface students' intuitive understandings and then move them towards extensions, formalization and fluency

SM2: Reason Abstractly and Quantitatively

- Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.
- Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Module 1 Purpose and Progression

Module 1 – Getting Ready

Classroom Task: Checkerboard Borders - A Develop Understanding Task
Defining quantities and interpreting expressions (**N.Q.2, A.SSE.1**)
Ready, Set, Go Homework: Getting Ready 1.1

Classroom Task: Building More Checkerboard Borders – A Develop Understanding Task
Defining quantities and interpreting expressions (**N.Q.2, A.SSE.1**)
Ready, Set, Go Homework: Getting Ready 1.2

Classroom Task: Serving Up Symbols – A Develop Understanding Task
Interpreting expressions and using units to understand problems (**A.SSE.1, N.Q.1**)
Ready, Set, Go Homework: Getting Ready 1.3

Classroom Task: Examining Units – A Solidify Understanding Task
Using units as a way to understand problems (**N.Q.1**)
Ready, Set, Go Homework: Getting Ready 1.4

Classroom Task: Cafeteria Actions and Reactions – A Develop Understanding Task
Explaining each step in the process of solving an equation (**A.REI.1**)
Ready, Set, Go Homework: Getting Ready 1.5

Classroom Task: Elvira's Equations – A Solidify Understanding Task
Rearranging formulas to solve for a variable (**A.REI.3, A.CED.4**)

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Classroom Task: Elvira's Equations – A Solidify Understanding Task
Rearranging formulas to solve for a variable (**A.REI.3**, **A.CED.4**)
Ready, Set, Go Homework: Getting Ready 1.6

Classroom Task: Solving Equations, Literally – A Practice Understanding Task
Solving literal equations (**A.REI.1, A.REI.3, A.CED.4**)
Ready, Set, Go Homework: Getting Ready 1.7

Classroom Task: Cafeteria Conundrums – A Develop Understanding Task
Writing inequalities to fit a context (**A.REI.1, A.REI.3**)
Ready, Set, Go Homework: Getting Ready 1.8

Classroom Task: Greater Than? – A Solidify Understanding Task
Reasoning about inequalities and the properties of inequalities (**A.REI.1, A.REI.3**)
Ready, Set, Go Homework: Getting Ready 1.9

Classroom Task: Taking Sides – A Practice Understanding Task
Solving linear inequalities and representing the solution (**A.REI.1, A.REI.3**)
Ready, Set, Go Homework: Getting Ready 1.10

A.SSE.1 1. Interpret expressions that represent a quantity in terms of its context.

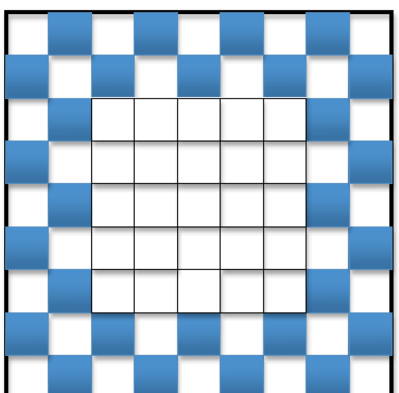
1.1 Checkerboard Borders

A Develop Understanding Task

In preparation for back to school, the school administration has planned to replace the tile in the cafeteria. They would like to have a checkerboard pattern of tiles two rows wide as a surround for the tables and serving carts.

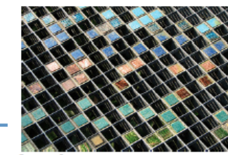
Below is an example of the boarder that the administration is thinking of using to surround a square 5 x 5 set of tiles.

A. Find the number of colored tiles in the checkerboard border. Track your thinking and find a way of calculating the number of colored tiles in the border that is quick and efficient. Be prepared to share your strategy and justify your work.



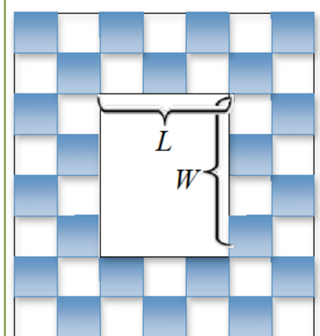
1.2 Building More Checkerboard Borders

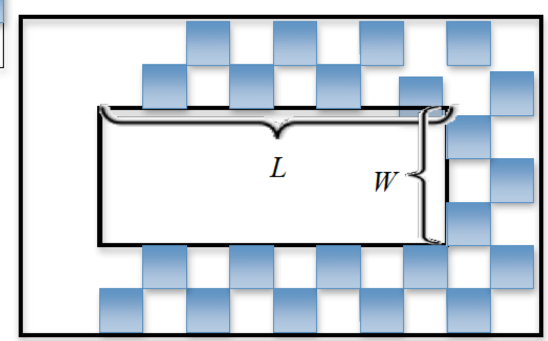
A Develop Understanding Task



As the tile workers started to look more deeply into their work they found it necessary to develop a way to quickly calculate the number of colored border tiles for not just square arrangements but also for checkerboard borders to surround any $L \times W$ rectangular tile center.

Find an expression to calculate the number of colored tiles in the two row checkerboard border for any rectangle. Be prepared to share your strategy and justify your work. Create models to assist you in your work.





Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

1.3 Serving Up Symbols

A Develop Understanding Task

As you look around your school cafeteria, you may see many things that could be counted or measured. To increase the efficiency of the cafeteria, the cafeteria manager, Elvira, decided to take a close look at the management of the cafeteria and think about all the components that affect the way the cafeteria runs. To make it easy, she assigned symbols for each count or measurement that she wanted to consider, and made the following table:

Symbol	Meaning (description of what the symbol means in context)	Units (what is counted or measured)
S	Number of students that buy lunch in the cafeteria each day	*students* or $\frac{students}{day}$
S_M	Number of students who have passed through a line in *M* minutes	
C	Number of classes per lunch period	
P	Number of lunch periods per day	
B	Number of boys that buy lunch each day	*boys* or *students* or $\frac{boys}{day}$
G	Number of girls that buy lunch each day	
F	Number of food servers in the cafeteria	
T	Total number of food items in one lunch (Each entrée, side dish, or beverage counts as 1 item.)	
M	Number of minutes passed since the beginning of the lunch period	
N_e	Number of entrees in each lunch	
N_s	Number of side dishes in each lunch	
N_b	Number of beverages in each lunch	
C_e	Cost of each entrée	
C_s	Cost of each side dish	
C_b	Cost of each beverage	
L	Number of lines in the cafeteria	
W	The number of food servers per line	
i	Average number of food items that a server can serve each minute (Each entrée, side dish, or beverage counts as 1 item.)	
H	Number of hours each food server works each day	
P_L	Price per lunch	

1.4 Examining Units

A Solidify Understanding Task

(Note: This task refers to the same set of variables as used in *Serving Up Symbols*)

Units in Addition and Subtraction

1. Why can you add $N_e + N_s + N_b$ and you can add $B + G$, but you can't add $M + W$?
2. We measure real-world quantities in units like feet, gallons, students and miles/hour (miles per hour).
   a. What units might you use to measure N_e, N_s and N_b? What about the sum $N_e + N_s + N_b$?
   b. What units might you use to measure B? G? What about the sum $B + G$?
   c. What units might you use to measure M? W? What about the sum $M + W$?
3. State a rule about how you might use units to help you think about what types of quantities can be added. How would you use or modify your rule to fit subtraction?

Units in Multiplication, scenario 1

1. Why can you multiply $N_e \times C_e$ and you can multiply $L \times W$, but you can't multiply $G \times C$?
2. Units in multiplication often involve rates like miles/gallon (miles per gallon), feet/second (feet per second), or students/table (students per table).
   a. What units might you use to measure N_e? C_e? What about the product $N_e \times C_e$?
   b. What units might you use to measure L? W? What about the product $L \times W$?
   c. What units might you use to measure G? C? What about the product $G \times C$?
3. State a rule about how you might use units to help you think about what types of quantities can be multiplied.

Understand solving equations as a process of reasoning and explain the reasoning

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

1.5 Cafeteria Actions and Reactions

A Develop Understanding Task

Elvira, the cafeteria manager, has just received a shipment of new trays with the school logo prominently displayed in the middle of the tray. After unloading 4 cartons of trays in the pizza line, she realizes that students are arriving for lunch and she will have to wait until lunch is over before unloading the remaining cartons. The new trays are very popular and in just a couple of minutes 24 students have passed through the pizza line and are showing off the school logo on the trays. At this time, Elvira decides to divide the remaining trays in the pizza line into 3 equal groups so she can also place some in the salad line and the sandwich line, hoping to attract students to the other lines. After doing so, she realizes that each of the three serving lines has only 12 of the new trays.

"That's not many trays for each line. I wonder how many trays there were in each of the cartons I unloaded?"

1. Can you help the cafeteria manager answer her question using the data in the story about each of the actions she took? Explain how you arrive at your solution.

Elvira is interested in collecting data about how many students use each of the tables during each lunch period. She has recorded some data on Post-It Notes to analyze later. Here are the notes she has recorded:

- Some students are sitting at the front table. (I got distracted by an incident in the back of the lunchroom, and forgot to record how many students.)
- Each of the students at the front table has been joined by a friend, doubling the number of students at the table.
- Four more students have just taken seats with the students at the front table.
- The students at the front table separated into three equal-sized groups and then two groups left, leaving only one-third of the students at the table.
- As the lunch period ends, there are still 12 students seated at the front table.

Elvira is wondering how many students were sitting at the front table when she wrote her first note. Unfortunately, she is not sure what order the middle three Post-It Notes were recorded in since they got stuck together in random order. She is wondering if it matters.

1.6 Elvira's Equations

A Solidify Understanding Task

(Note: This task refers to the same set of variables as used in *Serving Up Symbols*)

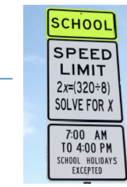
Elvira, the cafeteria manager, has written the following equation to describe a cafeteria relationship that seems meaningful to her. She has introduced a new variable A to describe this relationship.

$$A = \frac{S}{CP}$$

1. What does A represent in terms of the school and the cafeteria?
2. Using what you know about manipulating equations, solve this equation for S. Your solution will be of the form S = *an expression written in terms of the variables A, C and P.*
3. Does your expression for S make sense in terms of the meanings of the other variables? Explain why or why not.
4. Now solve the above equation for C and explain why the solution makes sense in terms of the variables.

1.7 Solving Equations, Literally

A Practice Understanding Task



Solve each of the following equations for x:

1. $\frac{3x+2}{5} = 7$
2. $\frac{3x+2y}{5} = 7$
3. $\frac{4x}{3} - 5 = 11$
4. $\frac{4x}{3} - 5y = 11$
5. $\frac{2}{5}(x+3) = 6$
6. $\frac{2}{5}(x+y) = 6$
7. $2(3x+4) = 4x+12$
8. $2(3x+4y) = 4x+12y$

Write a verbal description for each step of the equation solving process used to solve the following equations for x. Your description should include statements about how you know what to do next. For example, you might write, "*First I* _____________, *because* _______________ . . ."

9. $\frac{ax+b}{c} - d = e$
10. $r \cdot \sqrt{\frac{mx}{n}} + s = t$

Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

1.8 Cafeteria Conundrums

A Solidify Understanding Task

Between serving and preparing delicious school lunches, our cafeteria manager, Elvira, is busy analyzing the business of running the cafeteria. We previously saw the symbols for some of the things that she measured. Now she plans to use those symbols. Help Elvira to consider the pressing questions of the lunch room.

Symbol	Meaning
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L	Number of lines in the cafeteria
W	The number of food workers (servers) per line
i	Average number of food items that a worker can serve each minute (Each entrée, side dish, or beverage counts as 1 item.)
H	Number of hours each food worker works each day
P_L	Price per lunch

1.9 Greater Than?

A Solidify Understanding Task

For each situation you are given a mathematical statement and two expressions beneath it.

1. Decide which of the two expressions is greater, if the expressions are equal, or if the relationship cannot be determined from the statement.
2. Write an equation or inequality that shows your answer.
3. Explain why your answer is correct.

Watch out—this gets tricky!

Example:
Statement: $x = 8$
Which is greater? $x + 5$ or $3x + 2$
Answer: $3x + 2 > x + 5$ because if $x = 8$, $3x + 2 = 26$, $x + 5 = 13$ and $26 > 13$.

Try it yourself:

1. Statement: $y < x$
   Which is greater? $x - y$ or $y - x$

2. Statement: $2x - 3 > 7$
   Which is greater? 5 or x

3. Statement: $10 - 2x < 6$
   Which is greater? x or 2

4. Statement: $4x = 0$
   Which is greater? 1 or x

5. Statement: $a > 0$, $b < 0$
   Which is greater? ab or $\frac{a}{b}$

1.10 Taking Sides

A Practice Task

Joaquin and Serena work together productively in their math class. They both contribute their thinking and when they disagree, they both give their reasons and decide together who is right. In their math class right now, they are working on inequalities. Recently they had a discussion that went something like this:

> Joaquin: The problem says that "6 less than a number is greater than 4." I think that we should just follow the words and write $6 - x > 4$.
>
> Serena: I don't think that works because if x is 20 and you do 6 less than that you get $20 - 6 = 14$. I think we should write $x - 6 > 4$.
>
> Joaquin: Oh, you're right. Then it makes sense that the solution will be $x > 10$, which means we can choose any number greater than 10.

The situations below are a few more of the disagreements and questions that Joaquin and Serena have. Your job is to decide how to answer their questions, decide who is right, and give a mathematical explanation of your reasoning.

1. Joaquin and Serena are assigned to graph the inequality $x \geq -7$.
   Joaquin thinks the graph should have an open dot -7.
   Serena thinks the graph should have a closed dot at -7.
   Explain who is correct and why.

2. Joaquin and Serena are looking at the problem $3x + 1 > 0$.
   Serena says that the inequality is always true because multiplying a number by three and then adding one to it makes the number greater than zero.
   Is she right? Explain why or why not.

3. The word problem that Joaquin and Serena are working on says, "4 greater than x".
   Joaquin says that they should write: $4 > x$.
   Serena says they should write: $x + 4$.
   Explain who is correct and why.

Story Contexts that Support Conceptual Understanding

Pet Sitters

2.1 Pet Sitters
A Develop Understanding Task

The Martinez twins, Carlos and Clarita, are trying to find a way to make money during summer vacation. When they overhear their aunt complaining about how difficult it is to find someone to care for her pets while she will be away on a trip, Carlos and Clarita know they have found the perfect solution. Not only do they have a large, unused storage shed on their property where they can house animals, they also have a spacious fenced backyard where the pets can play.

Carlos and Clarita are making a list of some of the issues they need to consider as part of their business plan to care for cats and dogs while their owners are on vacation.

- *Space*: Cat pens will require 6 ft^2 of space, while dog runs require 24 ft^2. Carlos and Clarita have up to 360 ft^2 available in the storage shed for pens and runs, while still leaving enough room to move around the cages.

- *Start-up Costs*: Carlos and Clarita plan to invest much of the $1280 they earned from their last business venture to purchase cat pens and dog runs. It will cost $32 for each cat pen and $80 for each dog run.

Of course, Carlos and Clarita want to make as much money as possible from their business, so they are trying to determine how many of each type of pet they should plan to accommodate. They plan to charge $8 per day for boarding each cat and $20 per day for each dog.

After surveying the community regarding the pet boarding needs, Carlos and Clarita are confident that they can keep all of their boarding spaces filled for the summer.

So the question is, how many of each type of pet should they prepare for? Their dad has suggested the same number of each, perhaps 12 cats and 12 dogs. Carlos thinks they should plan for more dogs, since they can charge more. Clarita thinks they should plan for more cats since they take less space and time, and therefore they can board more.

What do you think? What recommendations would you give to Carlos and Clarita, and what argument would you use to convince them that your recommendation is reasonable?

Transforming Mathematics Education

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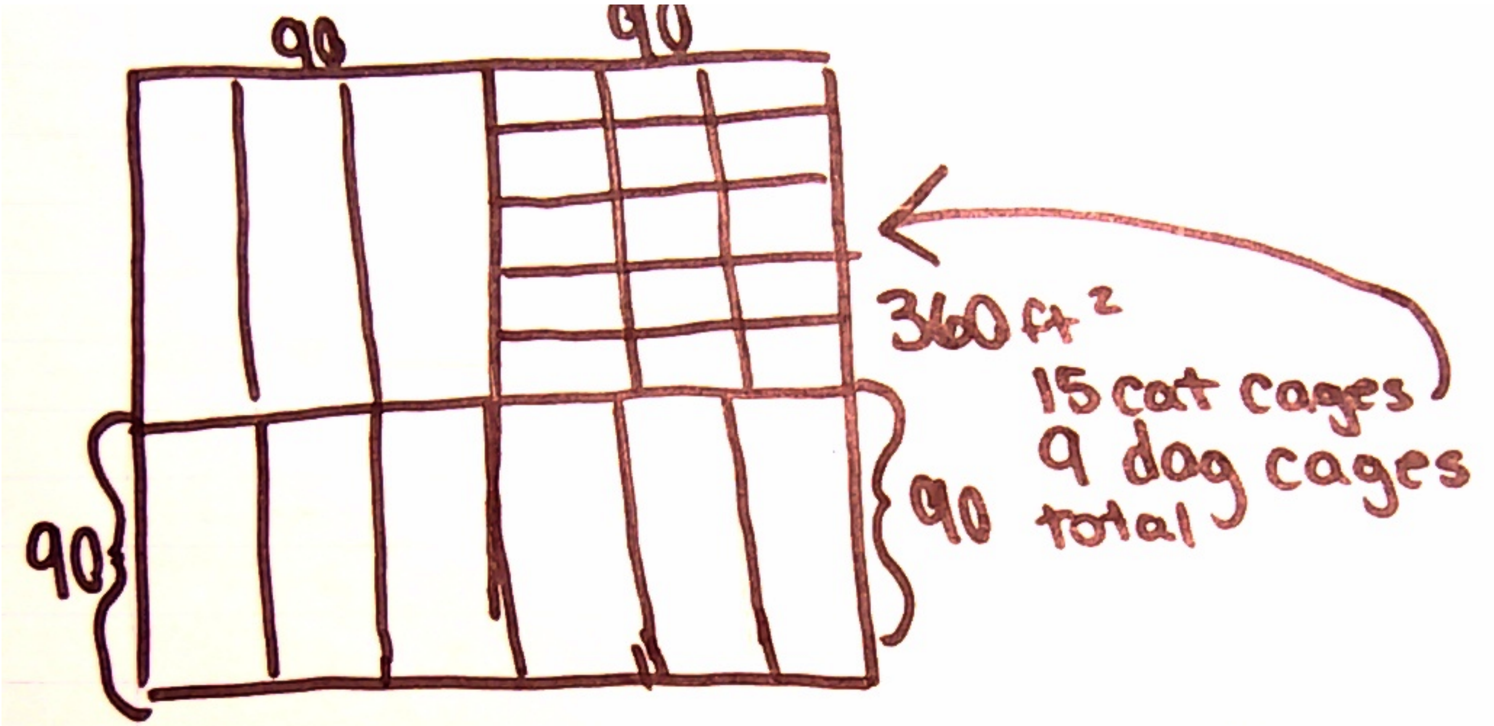
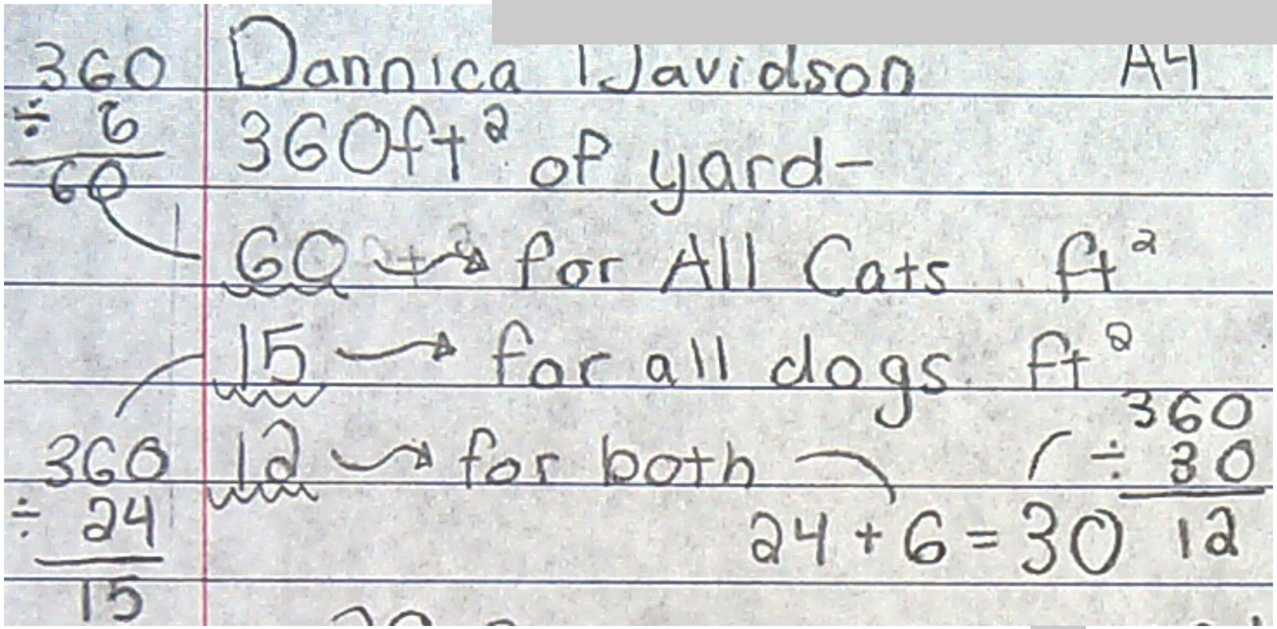
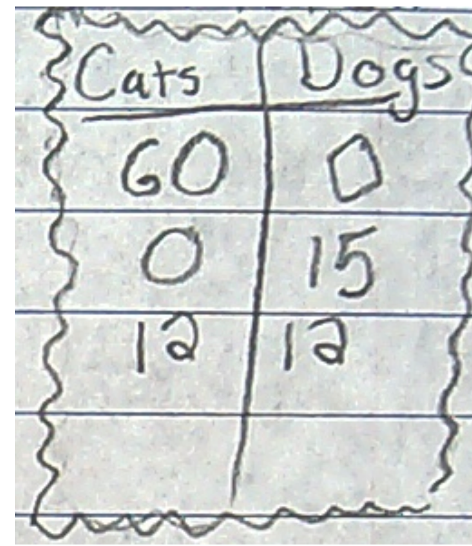
Pet Sitters: Student Work



Pet Sitters: Student Work





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Pet Sitters: Student Work

Space

Cats	Dogs
60	0
0	15
30	7
20	10
12	12

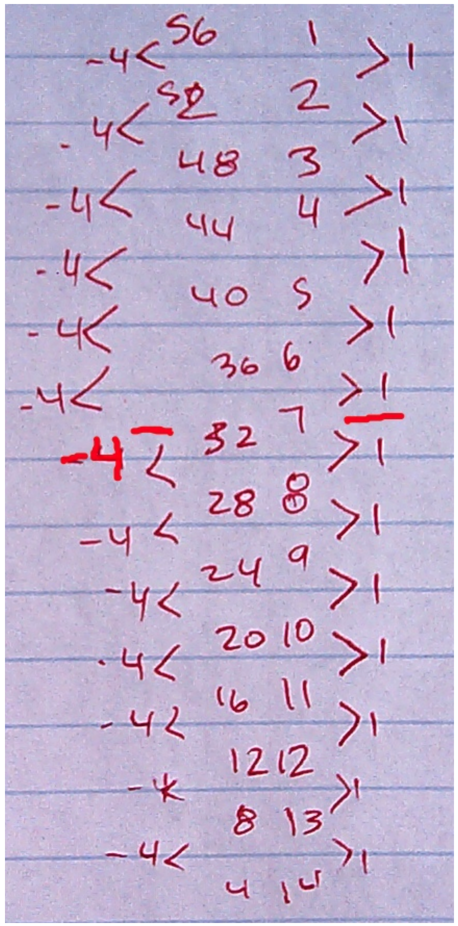
Earn

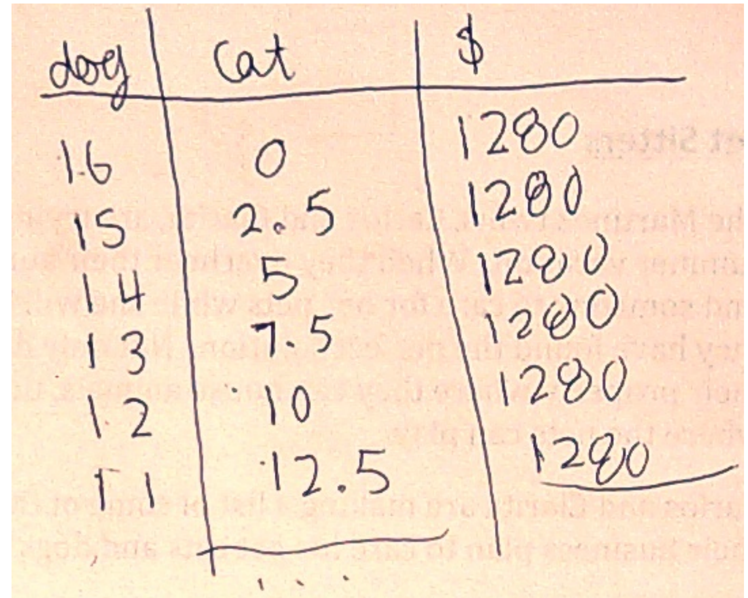
Cats	Dog
3,360	0
0	2,100
1,680	980
1,120	1,400
672	1,680

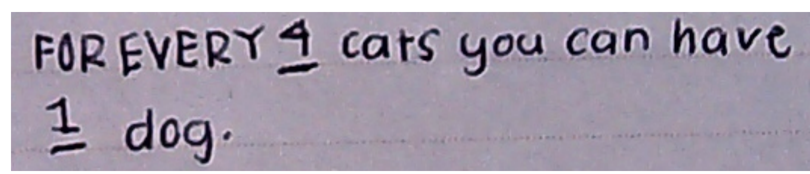
Cost

CAt'S	Dogs
1,920	0 =1920
0	1,200=1,200
960	560=1,520
640	800=1,440
384	960=1,344

Pet Sitters: Student Work







Pet Sitters: Student Work

What do you think? What recommen
what argument would you use to con

dogs cats

space $y = 4x + 60$ cats

dogs

cost $y = 40 - 2.5x$

Student #1

Student #2

amnt. of cats

ace equation: $y = 360 - ((6 \cdot c) + (24 \cdot d))$

cost equation: $y = (c \cdot 32) + (d \cdot 80)$

Dogs equashion: $y = -80x + 1280$

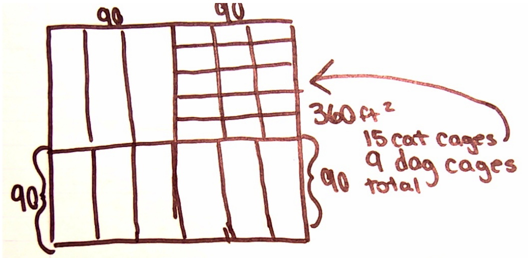
Cats equashion: $y = -32x + 1280$

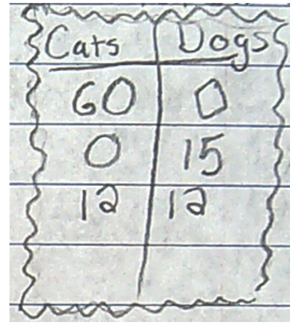
Student #3

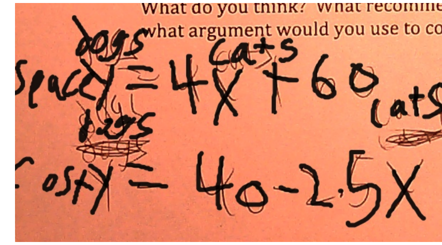
Space : Dog equashion: $y = -24x + 360$

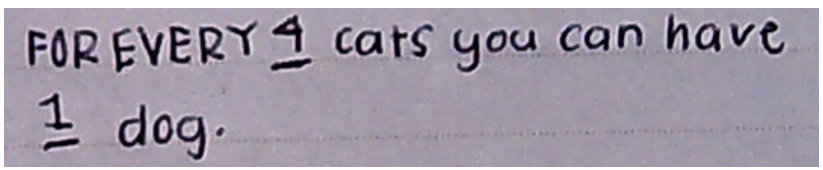
cats equashion: $y = -6x + 360$

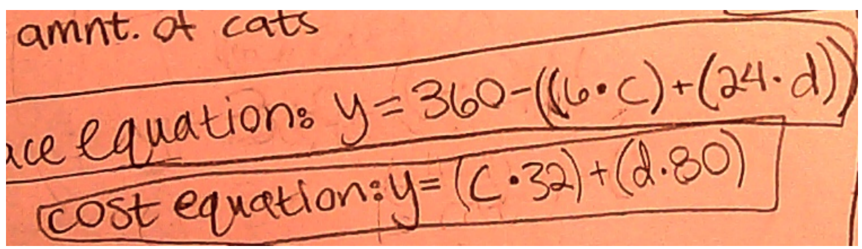
Student Work Demonstrates Multiple Access Points

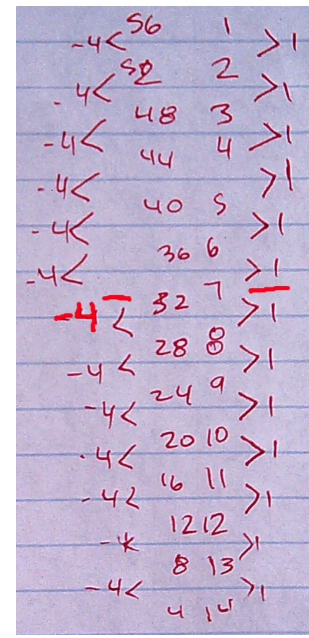




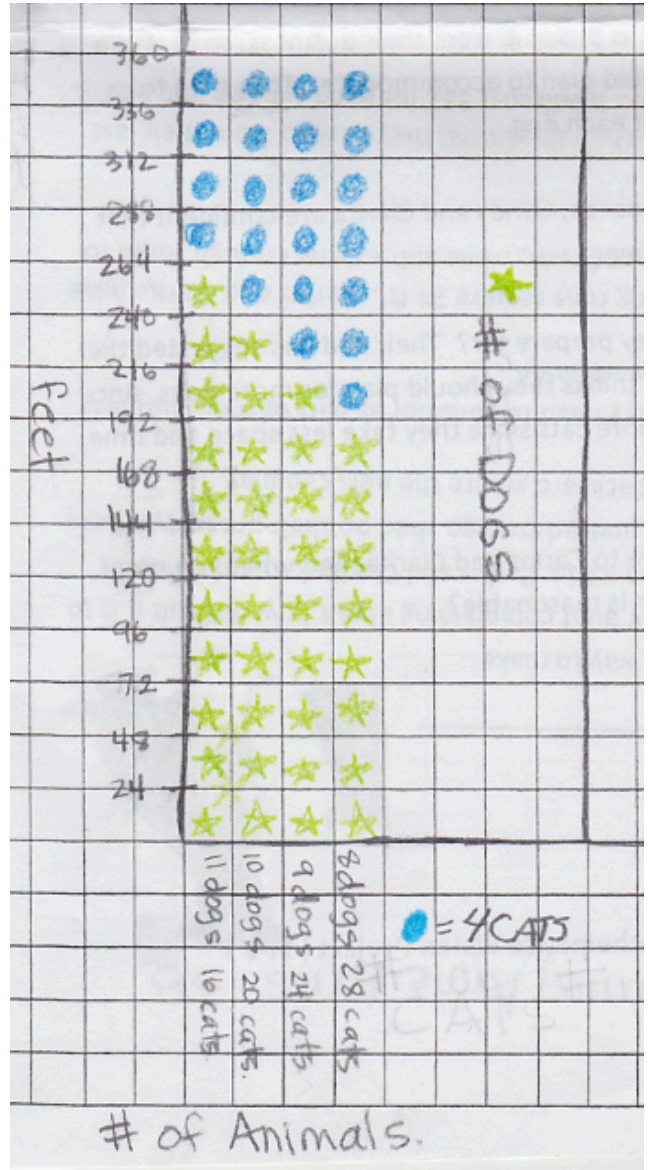








How is this student thinking about the data?



Teacher's Role in Supporting Student Understanding

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Teacher's Role Supported with Teacher Notes

Teacher notes identify the focus for each task and provides full description of the lesson including anticipating student work, and ideas for facilitating discussions.

2.1 Pet Sitters – Teacher Notes

A Develop Understanding Task

Purpose: As students work with the context of making recommendations for how many dogs and cats Carlos and Clarita should plan to accommodate, they will surface many ideas, strategies and representations related to solving systems of equations and inequalities. For example, they will explore the notion of *constraints* since in this task the number of each type of pet that can be accommodated is limited by space and money, but many different combinations of dogs and cats are possible. They may consider the notion of a *system of equations* since each constraint (space, start-up costs) allows for a different set of possibilities—a particular combination of dogs and cats may satisfy one constraint but not another—so both constraints must be considered simultaneously. Finally, they may surface the notion of *a system of inequalities* since Carlos and Clarita don't have to use up all of the available space or money, implying that each constraint may be represented by an inequality.

Core Standards Focus:

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

Related Standards: N.Q.2, A.REI.12

Launch (Whole Class):

After reading and discussing the "Pet Sitters" scenario, challenge students to come up with a combination of dogs and cats that would yield the highest daily income. Give students a few minutes to work independently to find the income for a particular combination of dogs and cats. After a few minutes have students compare their daily income with others and then work with a partner to improve their initial guesses

Explore (Small Group):

For students who don't know where to begin, have them determine the daily income if Carlos and Clarita follow their dad's advice of boarding "the same number of each, perhaps 12 cats and 12

Students who focus on just one constraint and find combinations of cats and dogs that satisfy that constraint have naturally simplified the larger problem into a more manageable task. They will gain valuable insight into the mathematical work of this module, and need not be pressed at this time to consider both constraints, or the bigger issue of maximizing the profit. Press for the work you feel your students can handle, and use this task to assess what ideas, strategies and representations will be available for future work in this module. For example, a student who visualizes the space constraint by drawing possible layouts of how the shed might be occupied with cat pens and dog runs is noticing that there are many such combinations that use up the total available space.

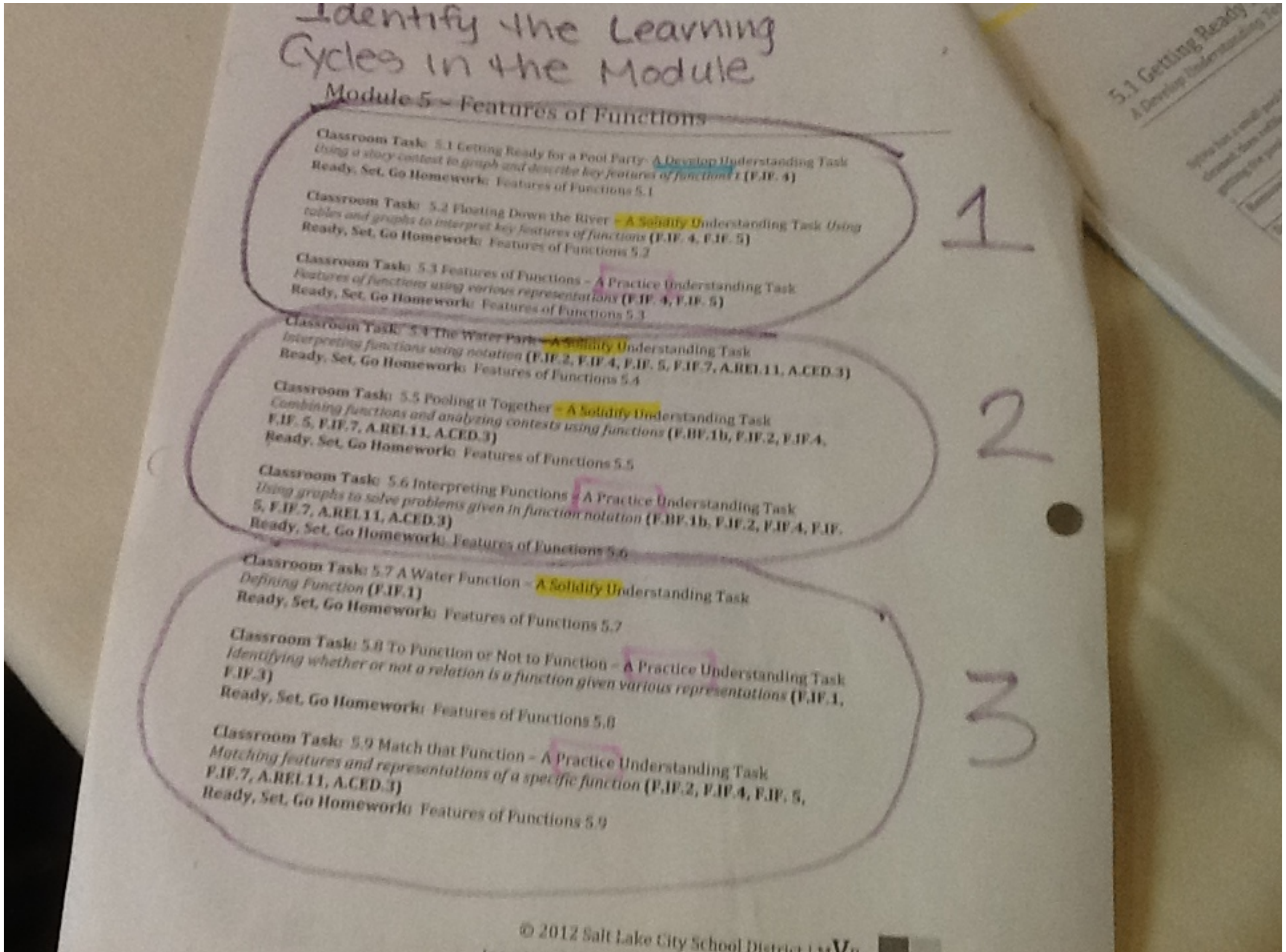
Watch for students who focus on the following ideas, since they underpin essential strategies in future tasks: (This list is sequenced from most likely to less likely to occur; don't be concerned if not all of these ideas are present in your class, since future tasks will elicit each of these ways of thinking about the system of constraints.)

- Students who calculate the intercepts of the constraints; that is, students who consider how they might use up all of the money or all of the space, by boarding just cats or just dogs.
- Students who make note of an "exchange rate" between cats and dogs in terms of either of the constraints. For example, four cats use the same space as one dog.
- Students who create charts to keep track of the combinations of dogs and cats they have tried. Such charts will probably include columns to track the number of dogs, the number of cats, and the money earned. Students will also have to keep track of whether a particular combination of dogs and cats satisfies each of the constraints.
- Students who plot combinations on a coordinate grid to keep track of the combinations they have tested.
- Students who try to write equations or inequalities to represent the constraints.

Discuss (Whole Class):

Begin with a combination of dogs and cats that worked, a second combination that worked and

Teacher's Role Supported with Teacher Notes


Identify the Learning Cycles in the Module

Module 5 – Features of Functions

1

Classroom Task: 5.1 Getting Ready for a Pool Party- A Develop Understanding Task
Using a story context to graph and describe key features of functions (F.IF.4)
Ready, Set, Go Homework: Features of Functions 5.1

Classroom Task: 5.2 Floating Down the River – A Solidify Understanding Task *Using tables and graphs to interpret key features of functions* (F.IF.4, F.IF.5)
Ready, Set, Go Homework: Features of Functions 5.2

Classroom Task: 5.3 Features of Functions – A Practice Understanding Task
Features of functions using various representations (F.IF.4, F.IF.5)
Ready, Set, Go Homework: Features of Functions 5.3

2

Classroom Task: 5.4 The Water Park – A Solidify Understanding Task
Interpreting functions using notation (F.IF.2, F.IF.4, F.IF.5, F.IF.7, A.REI.11, A.CED.3)
Ready, Set, Go Homework: Features of Functions 5.4

Classroom Task: 5.5 Pooling it Together – A Solidify Understanding Task
Combining functions and analyzing contexts using functions (F.BF.1b, F.IF.2, F.IF.4, F.IF.5, F.IF.7, A.REI.11, A.CED.3)
Ready, Set, Go Homework: Features of Functions 5.5

Classroom Task: 5.6 Interpreting Functions – A Practice Understanding Task
Using graphs to solve problems given in function notation (F.BF.1b, F.IF.2, F.IF.4, F.IF.5, F.IF.7, A.REI.11, A.CED.3)
Ready, Set, Go Homework: Features of Functions 5.6

3

Classroom Task: 5.7 A Water Function – A Solidify Understanding Task
Defining Function (F.IF.1)
Ready, Set, Go Homework: Features of Functions 5.7

Classroom Task: 5.8 To Function or Not to Function – A Practice Understanding Task
Identifying whether or not a relation is a function given various representations (F.IF.1, F.IF.3)
Ready, Set, Go Homework: Features of Functions 5.8

Classroom Task: 5.9 Match that Function – A Practice Understanding Task
Matching features and representations of a specific function (F.IF.2, F.IF.4, F.IF.5, F.IF.7, A.REI.11, A.CED.3)
Ready, Set, Go Homework: Features of Functions 5.9

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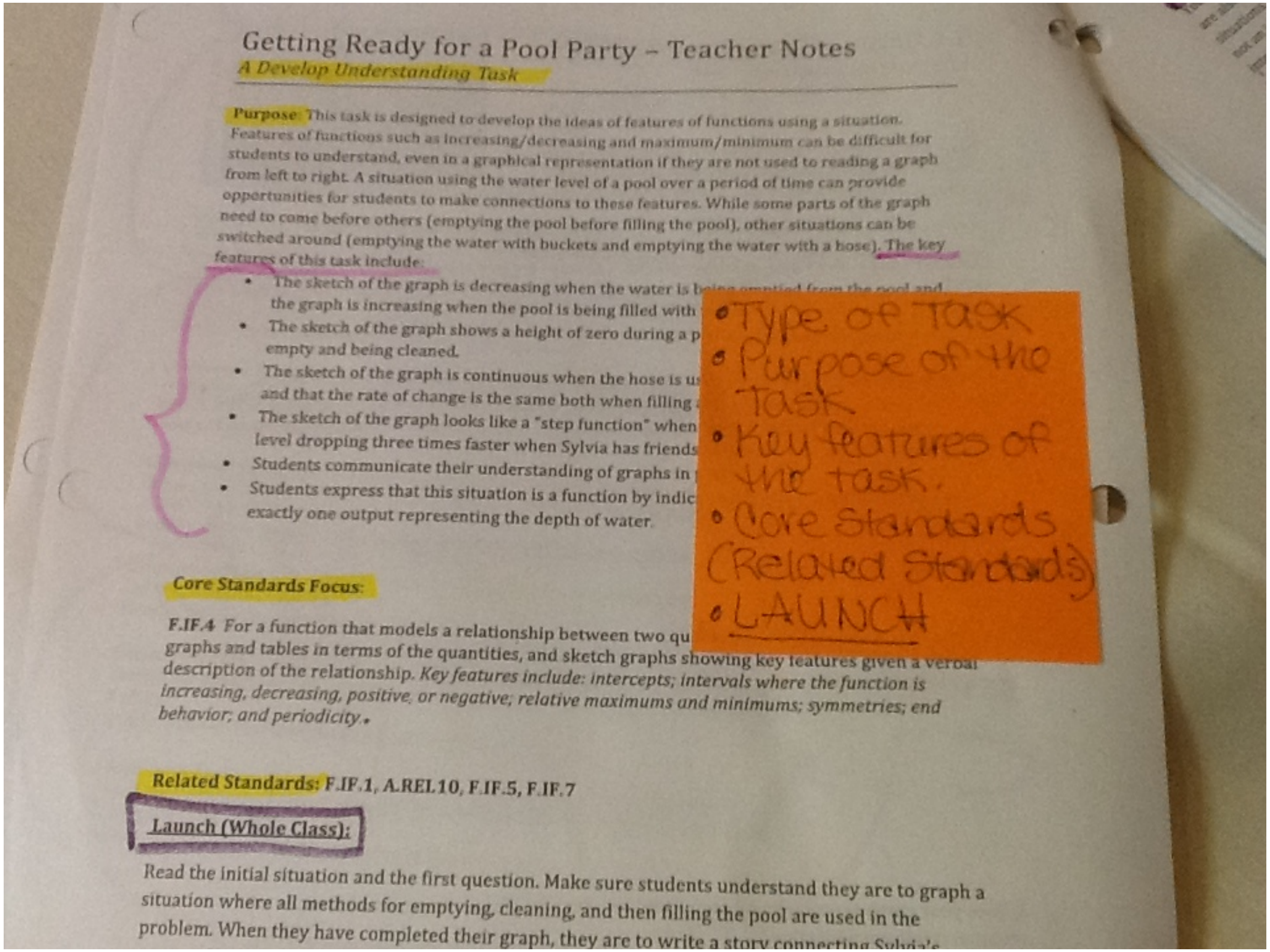
Teacher's Role Supported with Teacher Notes



Getting Ready for a Pool Party – Teacher Notes

A Develop Understanding Task

Purpose: This task is designed to develop the ideas of features of functions using a situation. Features of functions such as increasing/decreasing and maximum/minimum can be difficult for students to understand, even in a graphical representation if they are not used to reading a graph from left to right. A situation using the water level of a pool over a period of time can provide opportunities for students to make connections to these features. While some parts of the graph need to come before others (emptying the pool before filling the pool), other situations can be switched around (emptying the water with buckets and emptying the water with a hose). The key features of this task include:

- The sketch of the graph is decreasing when the water is b... from the pool and the graph is increasing when the pool is being filled with
- The sketch of the graph shows a height of zero during a p... empty and being cleaned.
- The sketch of the graph is continuous when the hose is u... and that the rate of change is the same both when filling
- The sketch of the graph looks like a "step function" when... level dropping three times faster when Sylvia has friends
- Students communicate their understanding of graphs in
- Students express that this situation is a function by indic... exactly one output representing the depth of water.

- Type of Task
- Purpose of the Task
- Key features of the task.
- Core Standards (Related Standards)
- LAUNCH

Core Standards Focus:

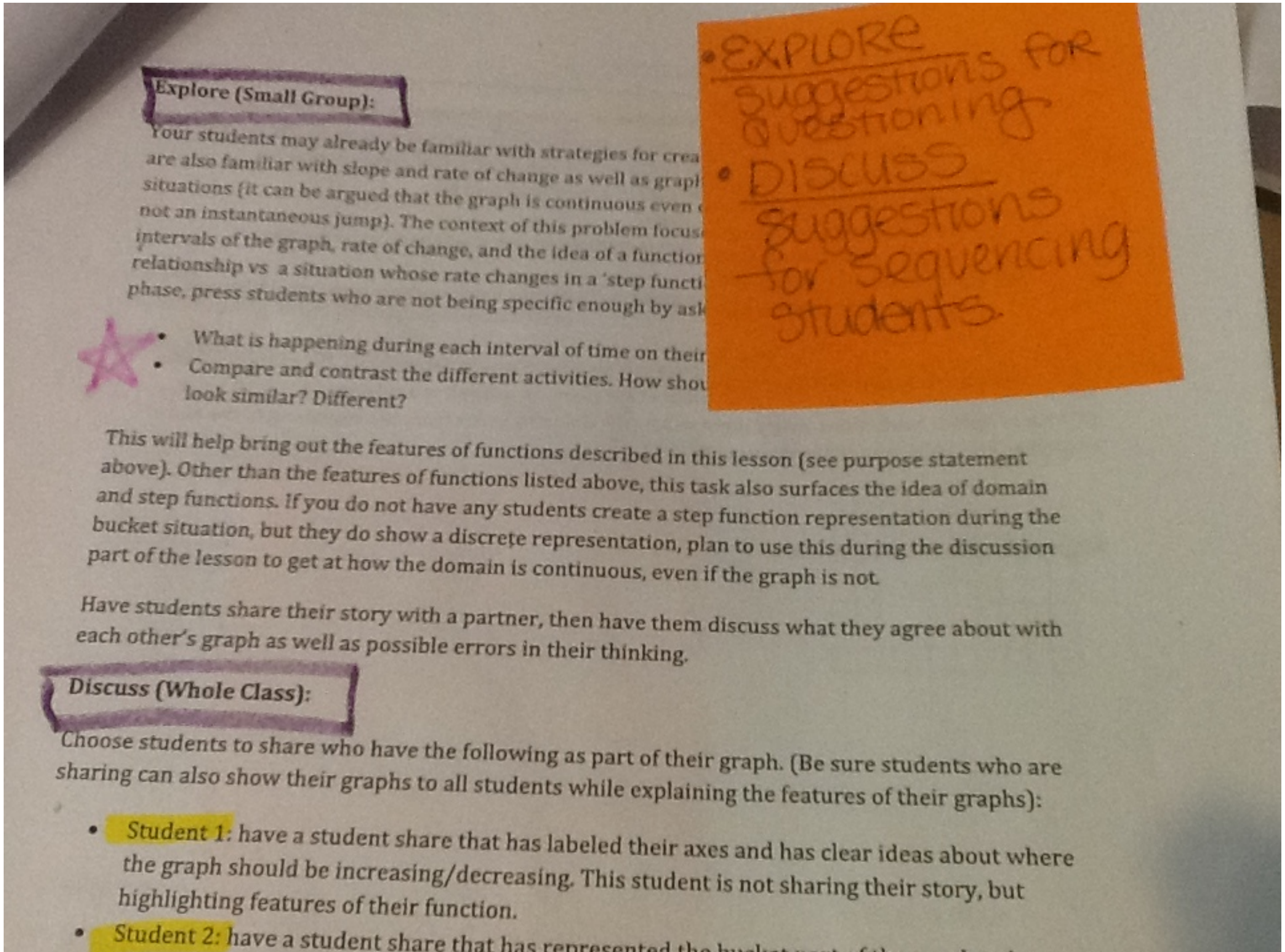
F.IF.4 For a function that models a relationship between two qu... graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

Related Standards: F.IF.1, A.REI.10, F.IF.5, F.IF.7

Launch (Whole Class):

Read the initial situation and the first question. Make sure students understand they are to graph a situation where all methods for emptying, cleaning, and then filling the pool are used in the problem. When they have completed their graph, they are to write a story connecting Sylvia's

Teacher's Role Supported with Teacher Notes



•EXPLORE
suggestions for
Questioning
• DISCUSS
suggestions
for sequencing
students.

Explore (Small Group):

Your students may already be familiar with strategies for crea
are also familiar with slope and rate of change as well as grapl
situations (it can be argued that the graph is continuous even
not an instantaneous jump). The context of this problem focus
intervals of the graph, rate of change, and the idea of a functior
relationship vs a situation whose rate changes in a 'step functi
phase, press students who are not being specific enough by ask

- *What is happening during each interval of time on their*
- *Compare and contrast the different activities. How shou look similar? Different?*

This will help bring out the features of functions described in this lesson (see purpose statement above). Other than the features of functions listed above, this task also surfaces the idea of domain and step functions. If you do not have any students create a step function representation during the bucket situation, but they do show a discrete representation, plan to use this during the discussion part of the lesson to get at how the domain is continuous, even if the graph is not.

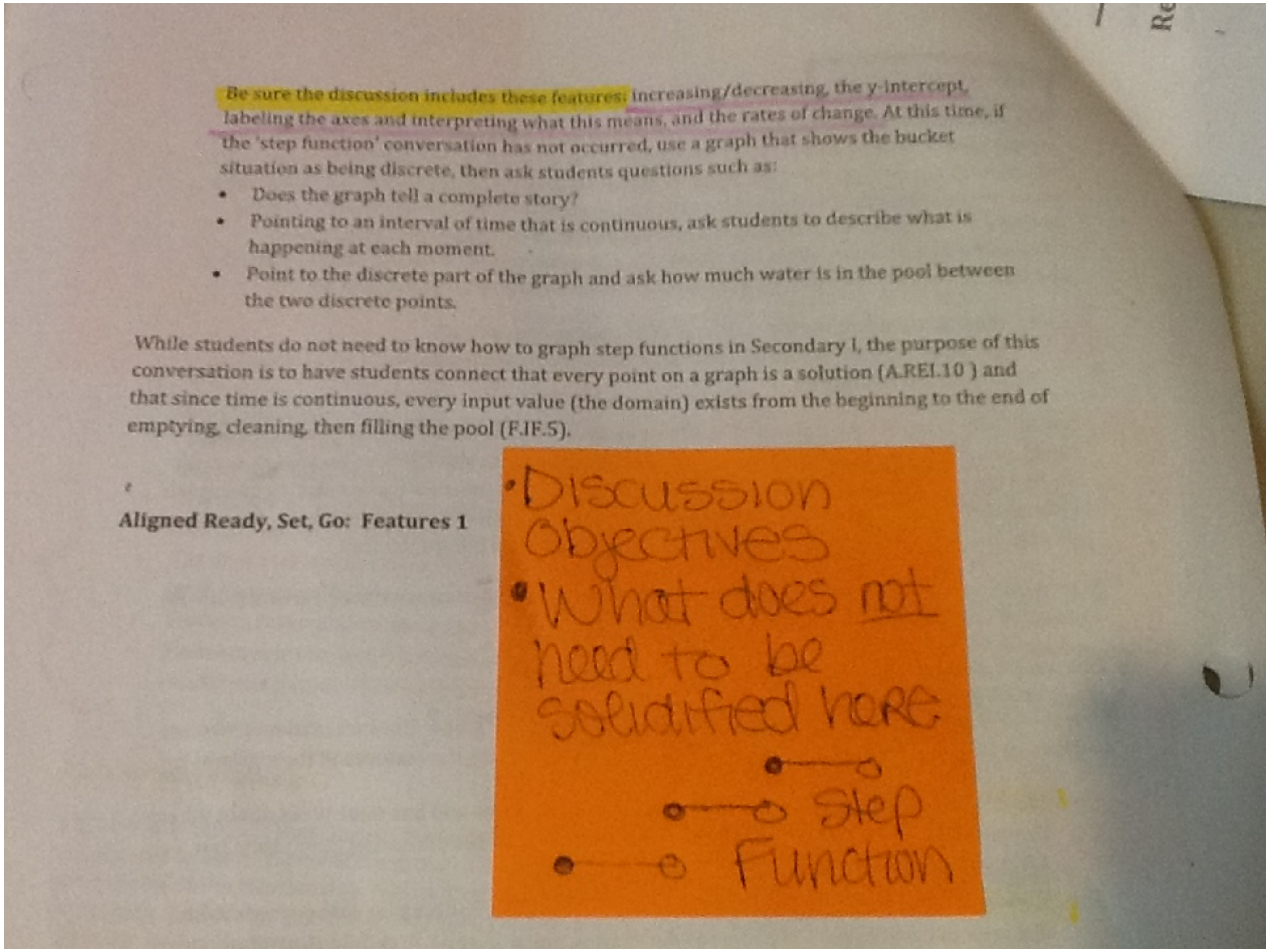
Have students share their story with a partner, then have them discuss what they agree about with each other's graph as well as possible errors in their thinking.

Discuss (Whole Class):

Choose students to share who have the following as part of their graph. (Be sure students who are sharing can also show their graphs to all students while explaining the features of their graphs):

- Student 1: have a student share that has labeled their axes and has clear ideas about where the graph should be increasing/decreasing. This student is not sharing their story, but highlighting features of their function.
- Student 2: have a student share that has represented the bucket part of the graph to be

Teacher's Role Supported with Teacher Notes

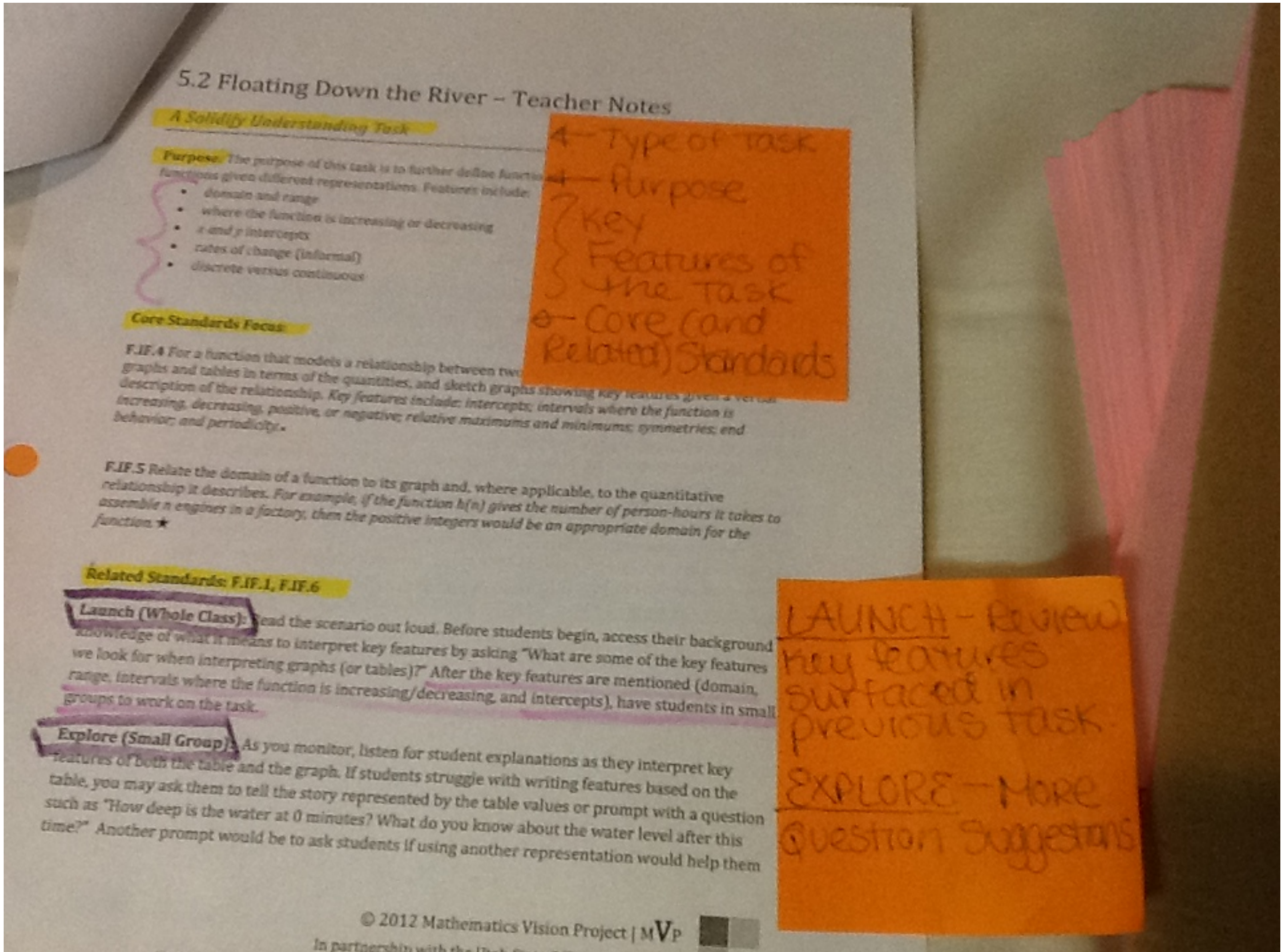

Be sure the discussion includes these features: increasing/decreasing, the y-intercept, labeling the axes and interpreting what this means, and the rates of change. At this time, if the 'step function' conversation has not occurred, use a graph that shows the bucket situation as being discrete, then ask students questions such as:

- Does the graph tell a complete story?
- Pointing to an interval of time that is continuous, ask students to describe what is happening at each moment.
- Point to the discrete part of the graph and ask how much water is in the pool between the two discrete points.

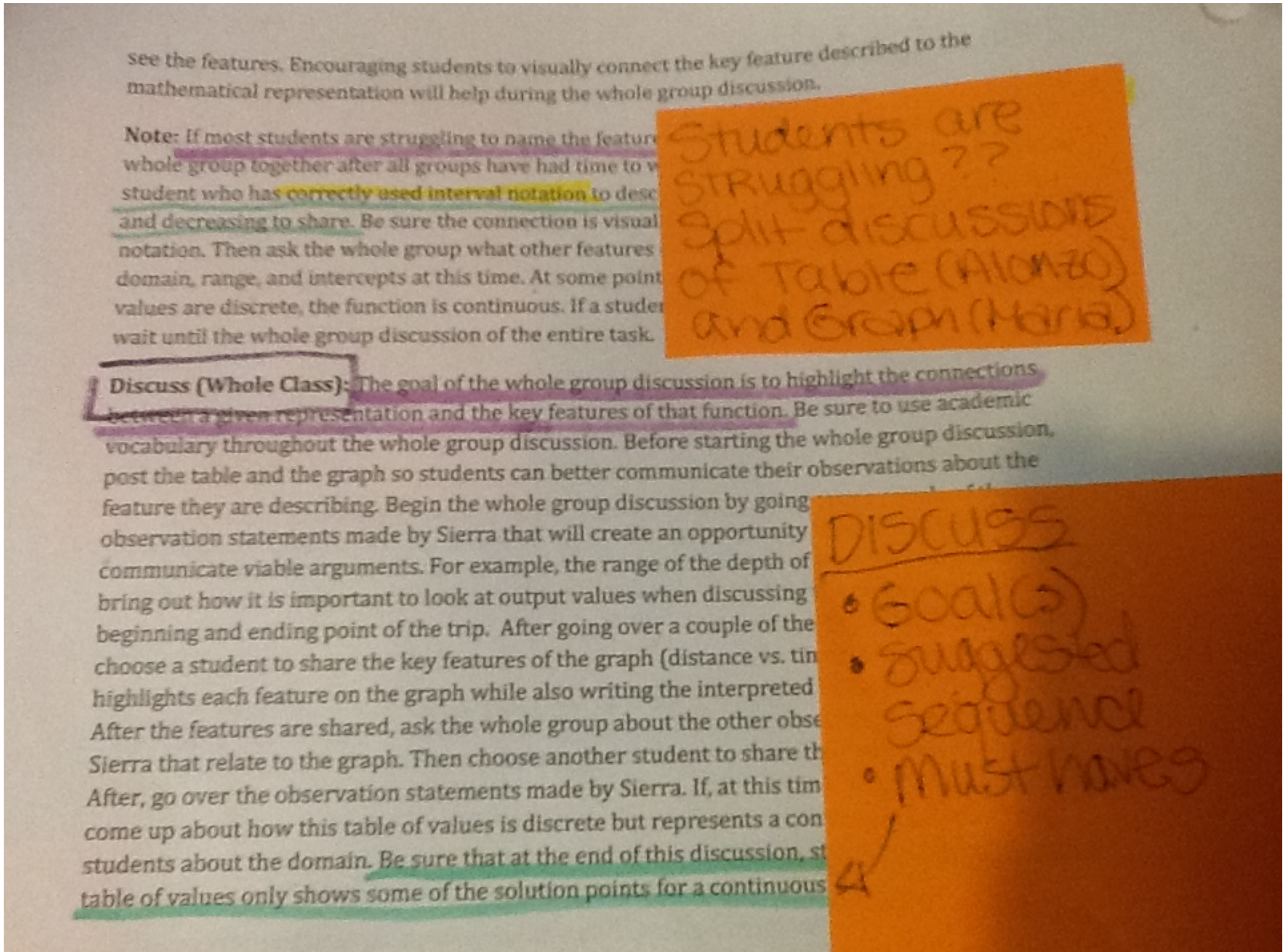
While students do not need to know how to graph step functions in Secondary I, the purpose of this conversation is to have students connect that every point on a graph is a solution (A.REI.10) and that since time is continuous, every input value (the domain) exists from the beginning to the end of emptying, cleaning, then filling the pool (F.IF.5).

Aligned Ready, Set, Go: Features 1

Teacher's Role Supported with Teacher Notes



Teacher's Role Supported with Teacher Notes

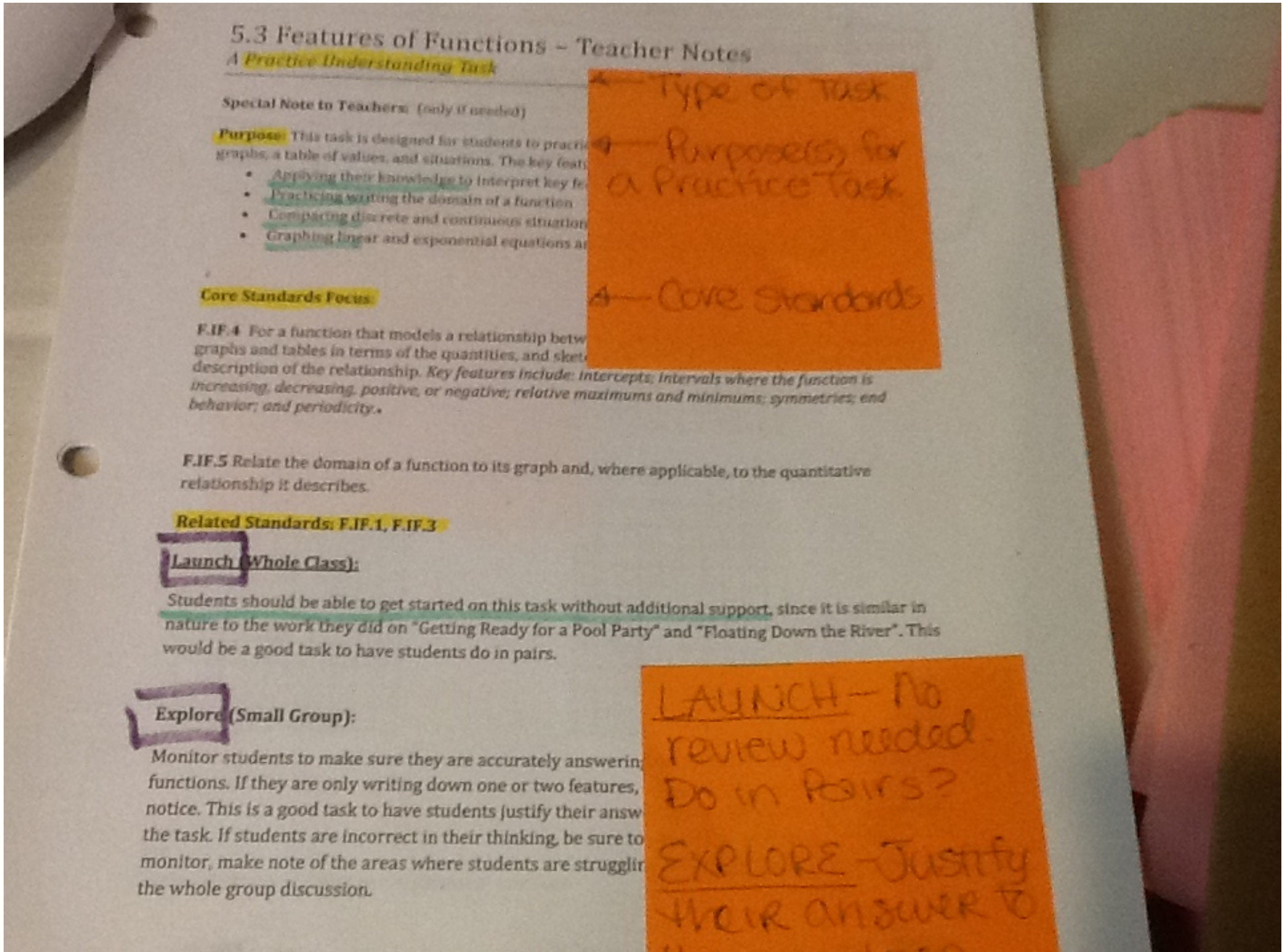

see the features. Encouraging students to visually connect the key feature described to the mathematical representation will help during the whole group discussion.

Note: If most students are struggling to name the featur
whole group together after all groups have had time to w
student who has correctly used interval notation to desc
and decreasing to share. Be sure the connection is visual
notation. Then ask the whole group what other features
domain, range, and intercepts at this time. At some point
values are discrete, the function is continuous. If a studer
wait until the whole group discussion of the entire task.

Discuss (Whole Class): The goal of the whole group discussion is to highlight the connections between a given representation and the key features of that function. Be sure to use academic vocabulary throughout the whole group discussion. Before starting the whole group discussion, post the table and the graph so students can better communicate their observations about the feature they are describing. Begin the whole group discussion by going
observation statements made by Sierra that will create an opportunity
communicate viable arguments. For example, the range of the depth of
bring out how it is important to look at output values when discussing
beginning and ending point of the trip. After going over a couple of the
choose a student to share the key features of the graph (distance vs. tin
highlights each feature on the graph while also writing the interpreted
After the features are shared, ask the whole group about the other obse
Sierra that relate to the graph. Then choose another student to share th
After, go over the observation statements made by Sierra. If, at this tim
come up about how this table of values is discrete but represents a con
students about the domain. Be sure that at the end of this discussion, st
table of values only shows some of the solution points for a continuous

mathematics vision project

Teacher's Role Supported with Teacher Notes



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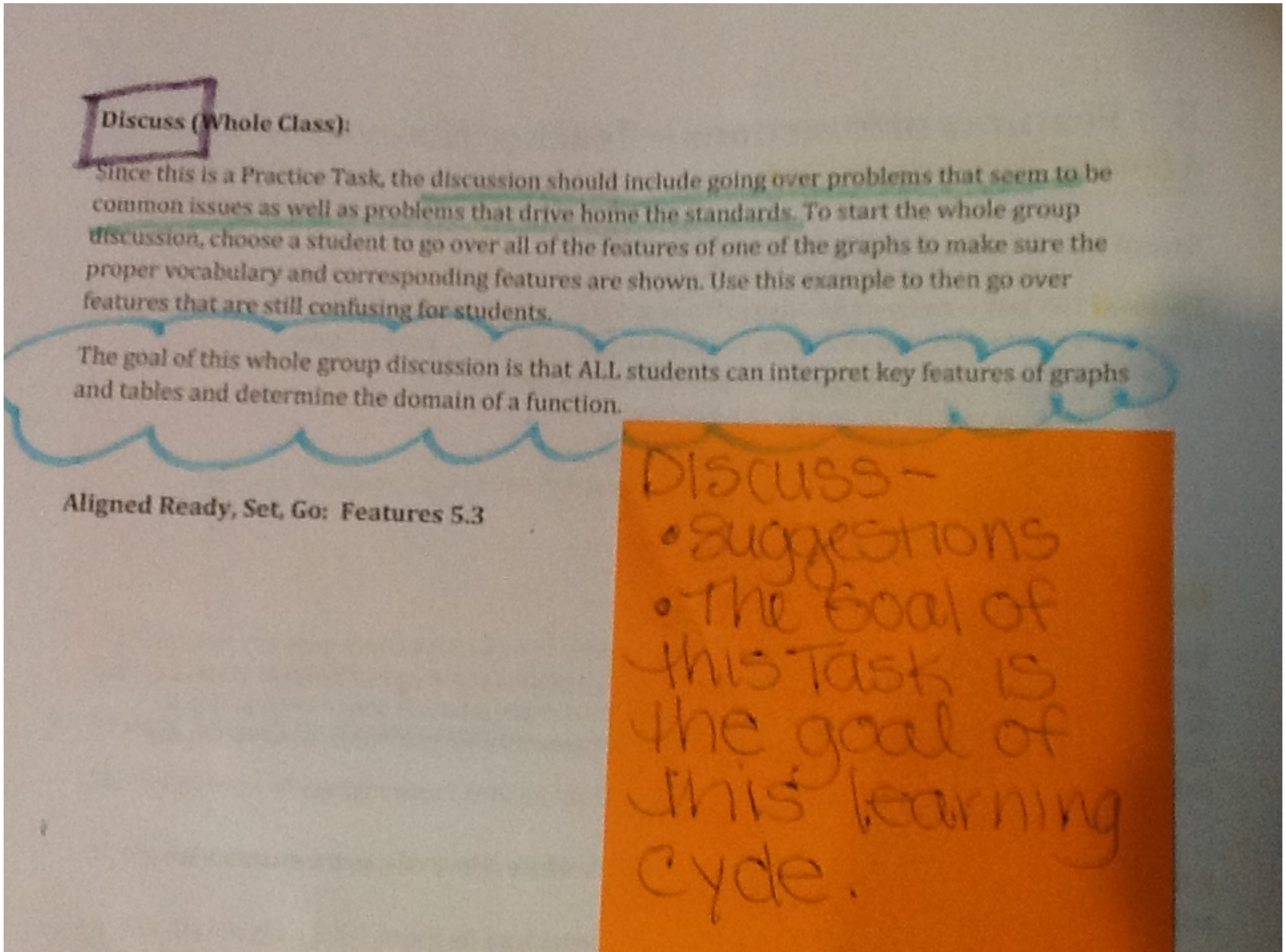


Discuss (Whole Class):

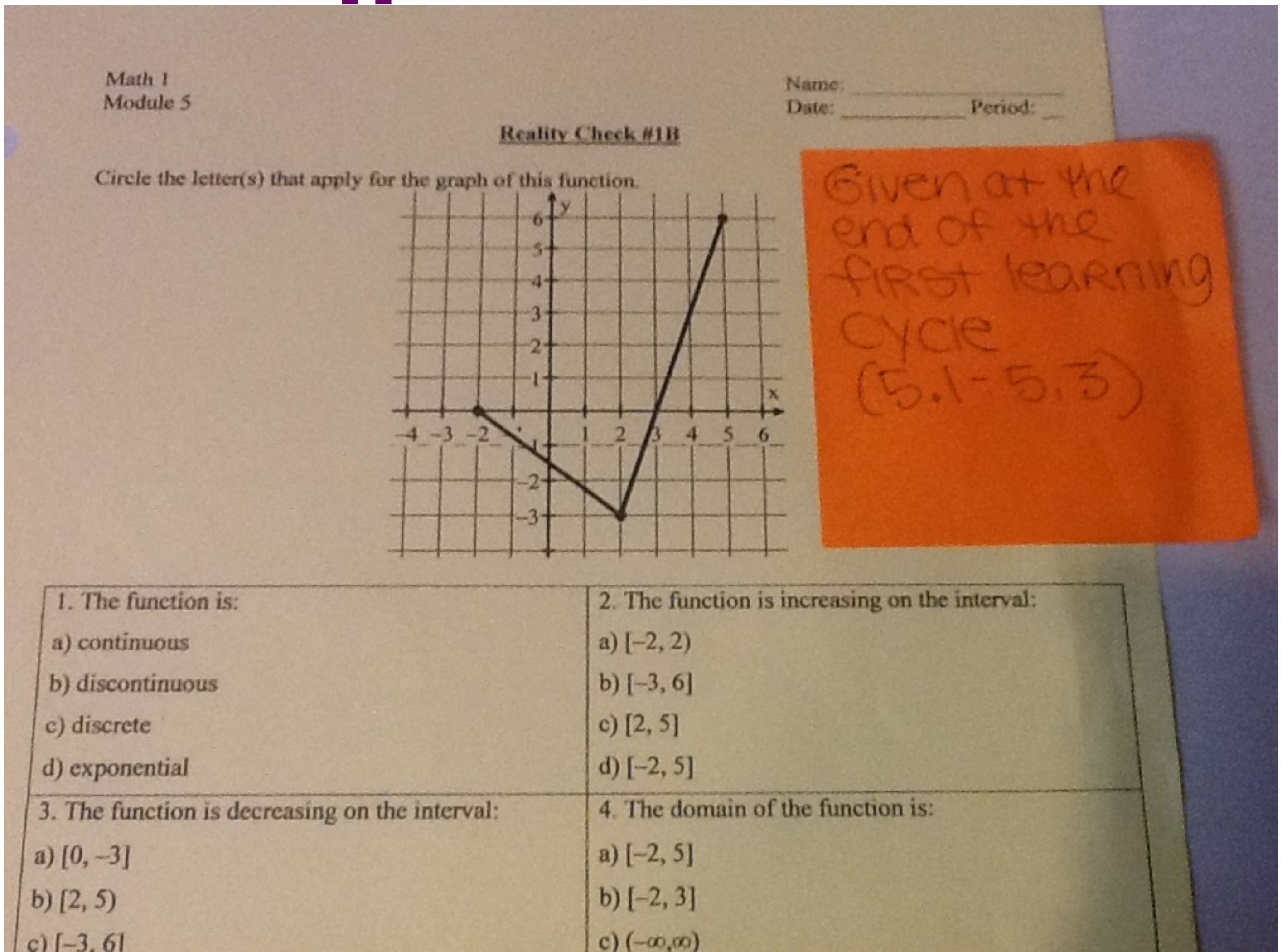
Since this is a Practice Task, the discussion should include going over problems that seem to be common issues as well as problems that drive home the standards. To start the whole group discussion, choose a student to go over all of the features of one of the graphs to make sure the proper vocabulary and corresponding features are shown. Use this example to then go over features that are still confusing for students.

The goal of this whole group discussion is that ALL students can interpret key features of graphs and tables and determine the domain of a function.

Aligned Ready, Set, Go: Features 5.3

Discuss -
- Suggestions
- The Goal of this Task is the goal of this learning cycle.

Teacher's Role Supported with Teacher Notes



Math 1
Module 5

Name: ____________
Date: ________ Period: __

Reality Check #1B

Circle the letter(s) that apply for the graph of this function.

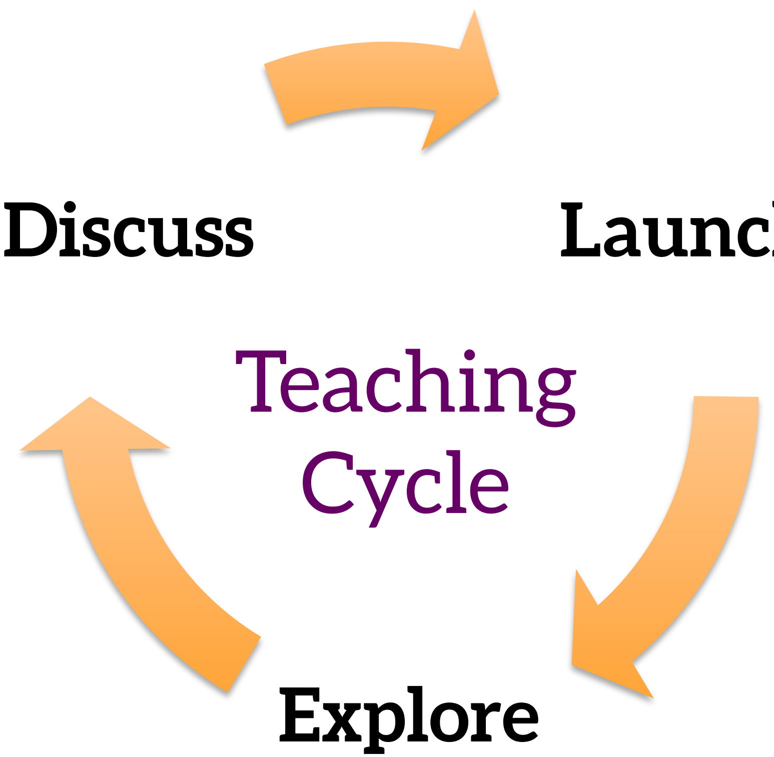
Given at the end of the first learning cycle (5.1-5.3)

1. The function is:	2. The function is increasing on the interval:
a) continuous	a) [–2, 2)
b) discontinuous	b) [–3, 6]
c) discrete	c) [2, 5]
d) exponential	d) [–2, 5]
3. The function is decreasing on the interval:	4. The domain of the function is:
a) [0, –3]	a) [–2, 5]
b) [2, 5)	b) [–2, 3]
c) [–3, 6]	c) (–∞,∞)

A FRAMEWORK for a Lesson or TASK:

Moving from a conceptual foundation to procedural fluency

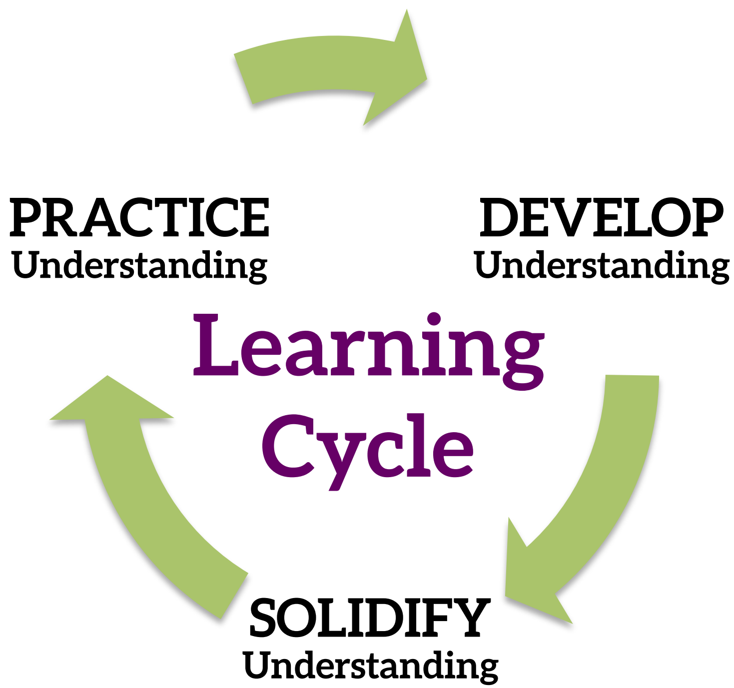
Comprehensive Mathematics Instruction Framework



A FRAMEWORK for Task Sequencing:

Moving from a conceptual foundation to procedural fluency

Comprehensive Mathematics Instruction Framework



- ***Develop Understanding*** tasks surface student thinking
- ***Solidify Understanding*** tasks examine and extend
- ***Practice Understanding*** tasks build fluency

Teacher's Role in Supporting Student Understanding

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

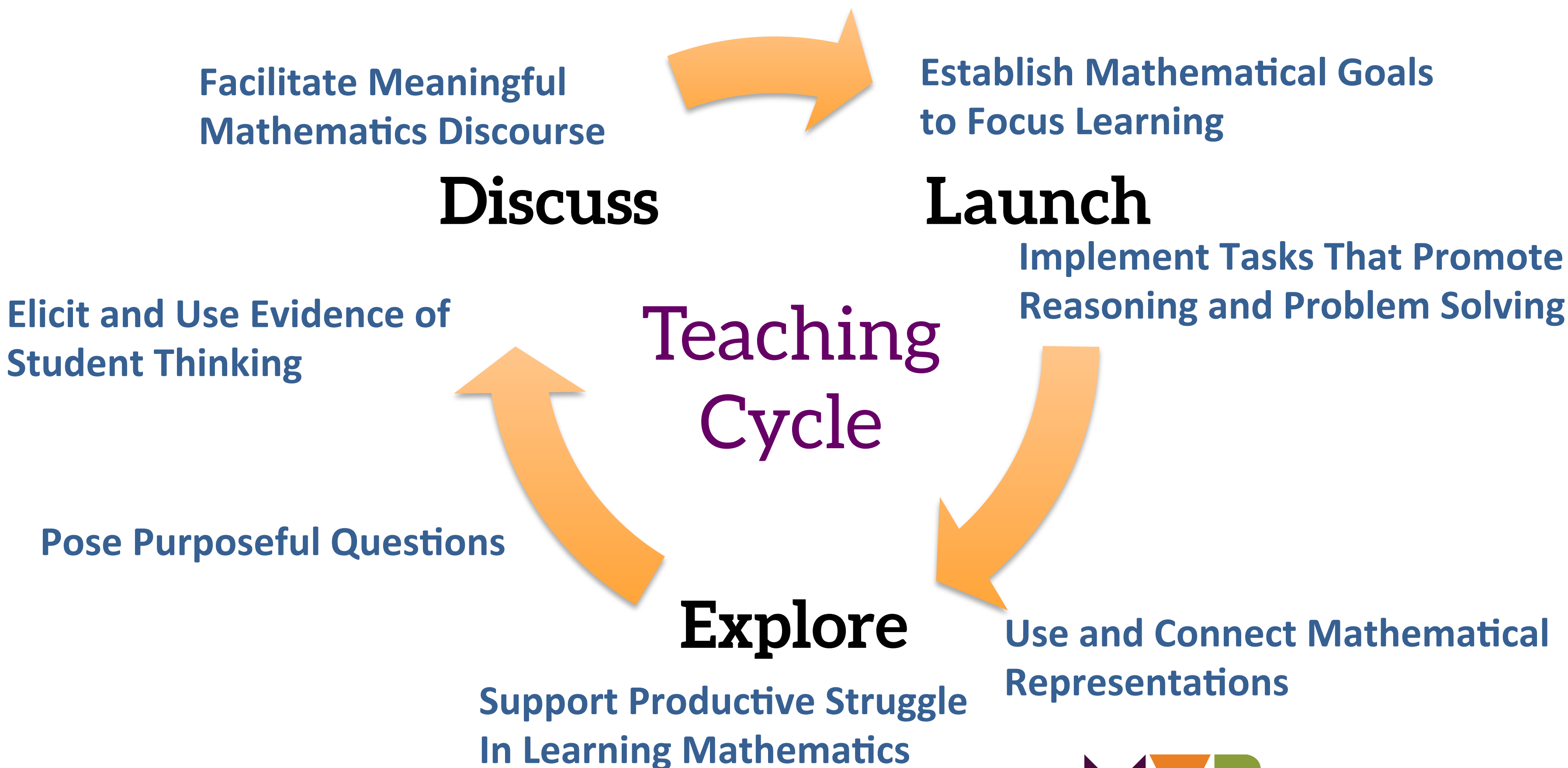
Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

A FRAMEWORK for a Lesson or TASK:

Moving from a conceptual foundation to procedural fluency

Comprehensive Mathematics Instruction Framework



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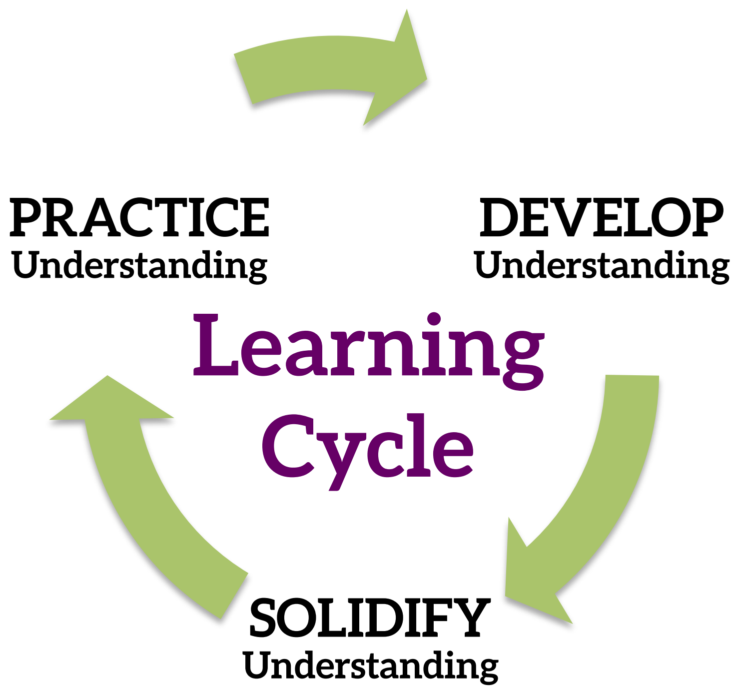
A FRAMEWORK for Task Sequencing:

Moving from a conceptual foundation to procedural fluency

Comprehensive Mathematics Instruction Framework



- ***Develop Understanding*** tasks surface student thinking
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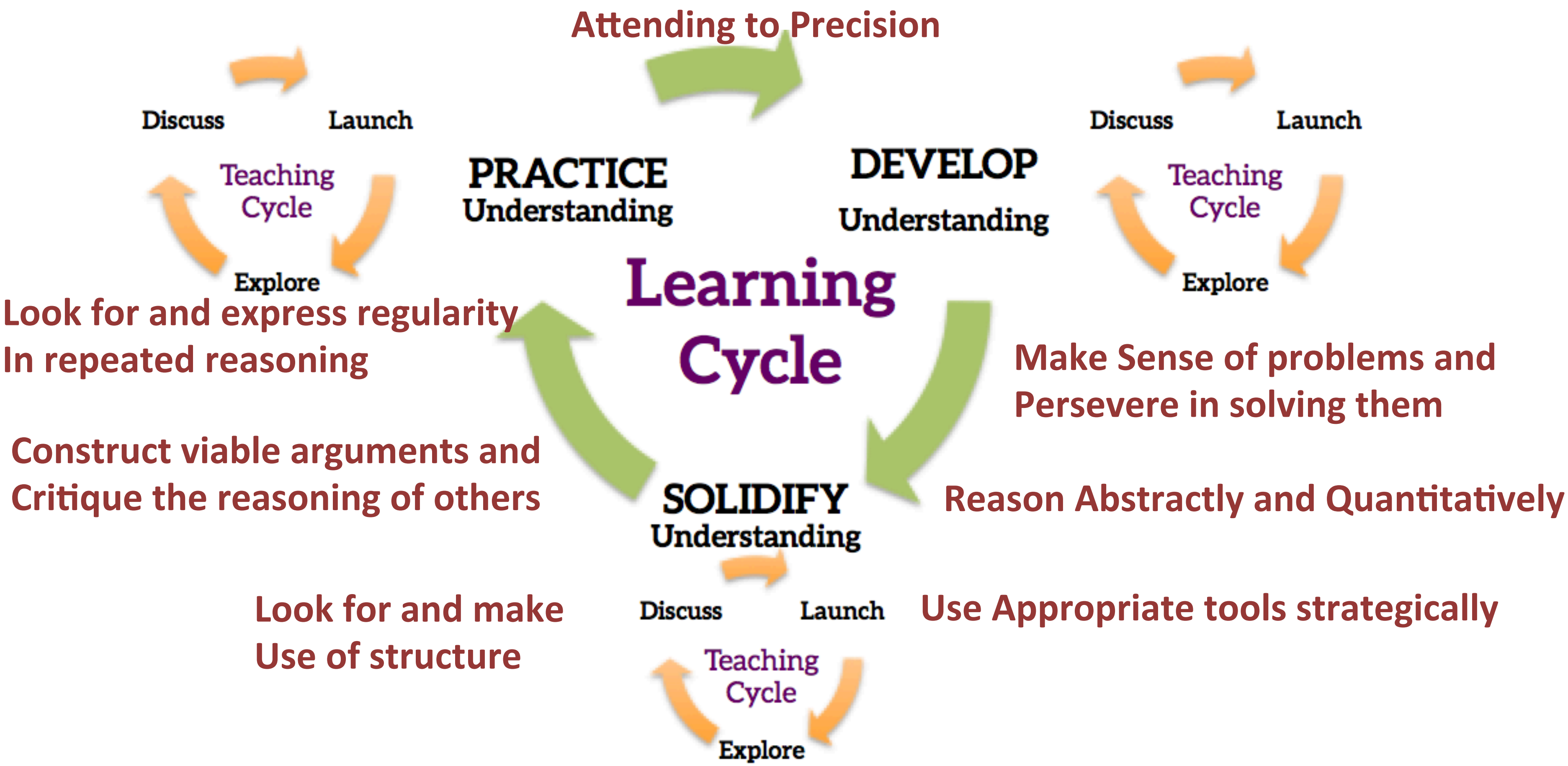
A FRAMEWORK for Coherence and Progression:
The Comprehensive Mathematics Instruction Framework
The framework on which MVP curriculum is built



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Thinking Through a Module of Instruction

- Work the task. Read the Teacher Notes, paying attention to the purpose and the goal. Ask yourself, what mathematics is developed, solidified or practiced in this task?
- Complete the "Module at a Glance" form.
- Make a poster labeled with the number and title of the task. Illustrate the mathematical focus of the task (representations). Look at the task before and the task that follows in order to highlight the mathematics that this task contributes to the learning progression. Include RSG information.

Thinking Through a Module of Instruction

Tracking Student Thinking in MVP Secondary I, Module 2H: *Systems of Equations and Inequalities (Honors)*

Cluster titles ⇒	Create equations that describe numbers or relationships						Solve systems of equations		Represent and solve systems of equations and inequalities graphically		
Description of the nature of student thinking relative to this standard ⇒ in this task ⇓ *(see sample descriptors below*)*	Create equations in two or more variables to represent relationships between quantities (ACED.2)	Graph equations in two variables on coordinate axes with labels and scales (ACED.2)	Represent constraints by equations or inequalities (A.CED.3)	Represent constraints by systems of equations and/or inequalities (A.CED.3)	Interpret solutions to a system of equations and/or inequalities as viable or nonviable options in a modeling context (A.CED.3)	Rearrange formulas to highlight a quantity of interest (A.CED.4)	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. (A.REI.5)	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables (A.REI.6)	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) (A.REI.12)	Graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes (A.CED.4)	Solve systems of equations with matrices (UT Honors)
Pet Sitters											
Too Big or Not Too Big											
Some of One, None of the Other											
Pampering and Feeding Time											
All For One, One For All											
Get to the Point											
Shopping for Cats and Dogs											

June 24, 2014

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Flexible & Engaging
Seamless Common Core Companion